Computer Vision and Image Processing – Fundamentals and Applications

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Lecture No. 04

Shape from Shading

Welcome to NPTEL MOOC course on Computer Vision and Image Processing Fundamentals and Application. In my last class I discussed about the concept of radiometry. Radiometry means the measurement of light, measurement of electromagnetic radiation. Photometry quantifies the sensitivity of camera or sensitivity of human eye. In my last class I mentioned some radiometric quantities like radiant intensity of the source and also I discussed about irradiance of a surface and also radiant intensity and radiance.

Also I discussed about the types of surfaces, the Lambertian surface and diffused surface. So, for this I discussed two parameters, one is the BRDF and another one is the directional hemispheric reflectance. So, today I am going to discuss another topic that is the shape from shading. So, I want to determine the shape information from shading. Shading means the variable levels of darkness. So, from one image I want to determine the shape information that is from shading.

So, before going to this concept, the shape from shading, what I discuss in the last class, I am going to discuss again briefly. So, the first concept I discussed about the concept of the Lambertian surface. And in this case I discussed about the quantity, the quantity is the albedo of a surface.

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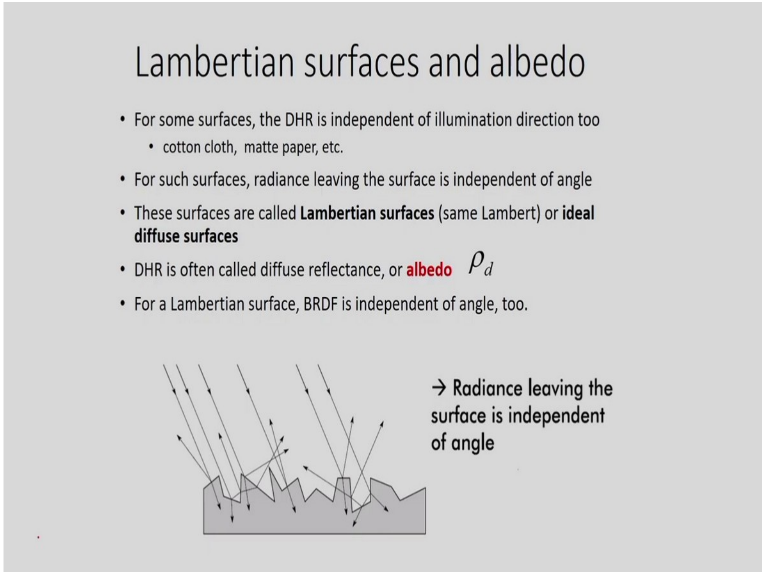

So, what is Lambertian surface? So, example like cotton cloth or may be matte papers, so in Lambertian surface, radiance leaving the surface is independent of angle. And this Lambertian surface is also called the diffused surface. And in case of the Lambertian surface, we have defined a parameter, the perimeter is albedo that is called the diffused reflectance. And for a Lambertian surface, BRDF is independent of angle.

So, in this example I have shown one surface that is the Lambertian surface. You have seen here that these are the incoming light directions and if you see the radiance leaving the surface it is independent of the angle. In all the directions radiance are leaving. So, this is the definition of Lambertian surface, that is, radiance leaving the surface is independent of angle.

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In this case I had given one example of the Lambertian surface that is the diffused surface. So, if you see this surface or maybe the, this surface, the example of Lambertian surfaces.

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In this example I have given the example of the specular surface, that is the non-Lambertian surface. If you see this portion, this is the mirror-like portion, so that is nothing but the specular surface that is the mirror-like surface. This is one example of the non-Lambertian surface.

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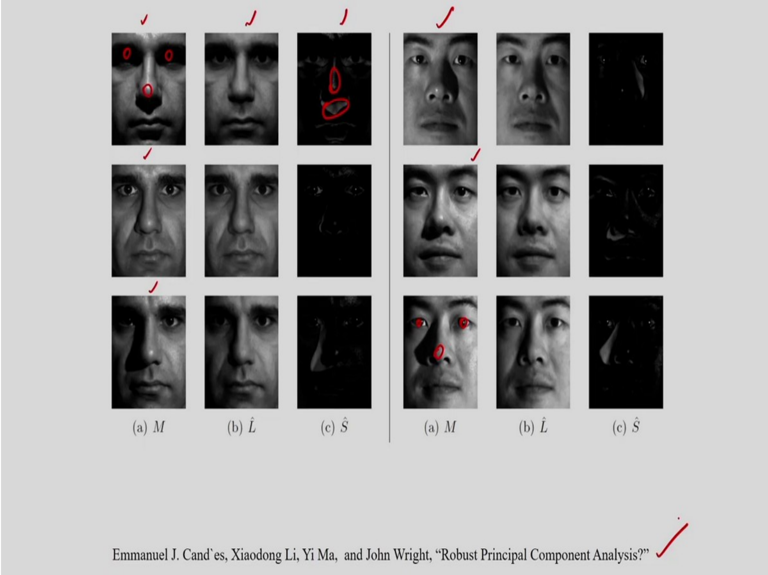

Last class also I discussed about the, this concept, in face recognition how to identify the specular portion of the image. You have seen these are my input images you can see these are my input images and in these input images I have two components, one is the diffused surface, another is the specular surface. So, specular portion is the, mainly the nose portion and also the eyeballs. So, in this paper, this Lambertian surface and the specular surfaces are separated. One is this, this surface is the diffused surface and these are specular surface. So say, specular surface is the specular portion of this face if you see the specular portion of the face that is separated from the original image.

Similarly, in the second example also I have considered the input images like this and in this case I am determining the specular portion of the image. So, this is my specular portion like this. So, you can see this paper to understand this concept. So, how to segment out the specular portion of the image.

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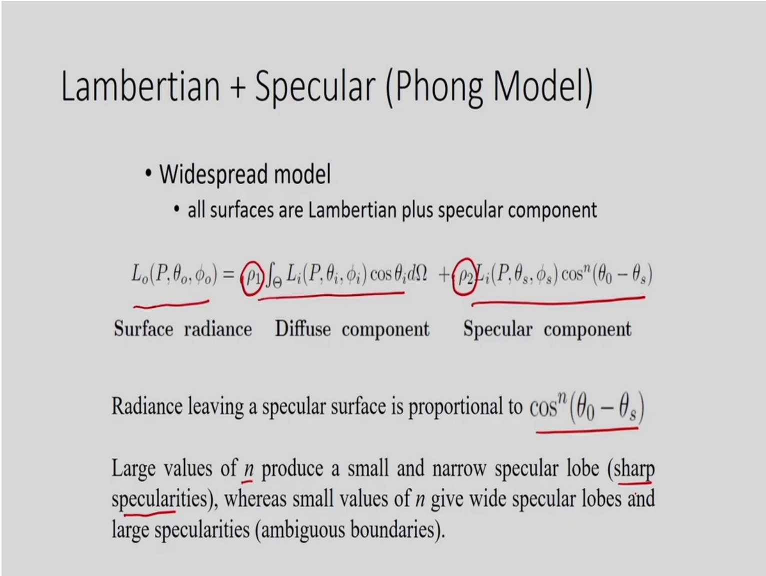

Now one standard model is; the model is the Phong model. So, for real surfaces we can consider both the components, one is the diffused component, another one is the specular component. So, for real surfaces we can consider the Lambertian plus specular that is called the Phong model. So, in this case you can see the surface radiance is represented like this. This is surface radiance. And I have two components, one is the diffused component. The first one is the diffused component and second one is the specular component.

Here the ρ_1, it corresponds to the diffuse albedo and ρ_2 corresponds to specular albedo. So, one is ρ_1, and another one is ρ_2. One is the diffused albedo, another one is the specular albedo. And in this case radiance leaving a specular surface is proportional to $\cos^n(\theta_0-\theta_s)$. Radiance leaving a specular surface is proportional to this term, $\cos^n(\theta_0-\theta_s)$. And in this case one parameter is important, the parameter is n.

So, if I consider large values of n, the large values of n produce a small and narrow specular lobe that corresponds to sharp specularities, that corresponds to the sharp specularities. And if I consider small value of n that give wide specular lobes, that is the large specularity. So, that is the meaning of this, the parameter. The parameter is n. So, based on this n I have the sharp specularity or maybe the ambiguous boundaries. This Phong model is very important because in this case we consider both the parameters, one is the diffused component we have considered, another one is specular component we have considered.

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And one example I can show. The specularity removal in colonoscopic image, maybe in the endoscopic image also we can apply this one. So, we have the images, the colonoscopic images and you can see some specular components like the specular, the mirror-like surfaces. So, by using some algorithms, I am not discussing about the algorithms, we can segment out the specular components like this the specular components are determined in the endoscopic image, in the colonoscopic image and we can show the results. This is one application why the specularity is important, the specularity removal in colonoscopic images.

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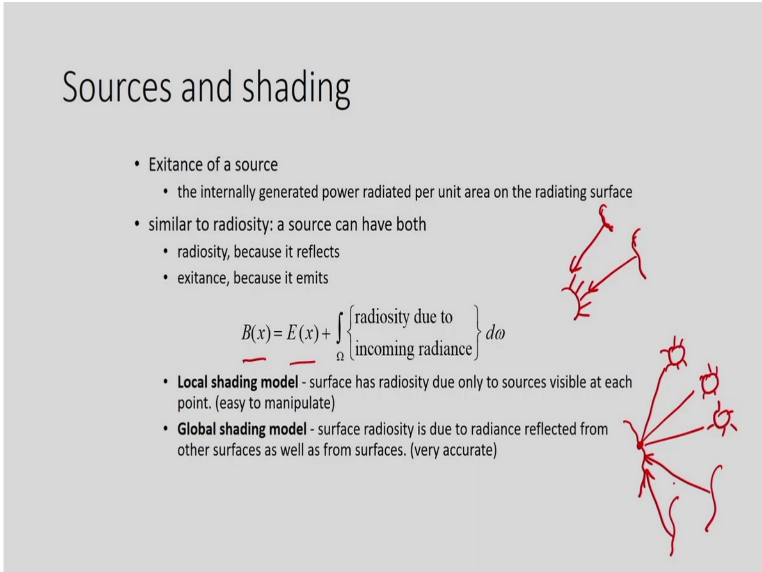

Now next concept is sources and the shading. The one parameter I want to, I want to explain, the exitance of a source. What is the meaning of the exitance of a source? The internally generated power radiated per unit area on the radiating surface that is called the exitance of a source. That is, the power internally generated by the source. Now in this case the source can have both radiosity and the exitance. Radiosity means because it reflects and exitance means because it emits.

So, here you see, in this expression I have considered the radiosity, radosity is B(x). We have considered the exitance. Exitance is E(x). Exitance means the power internally generated by the source and also we have to consider the power reflected by the surfaces, other surfaces. So, if I consider, suppose this is the source, the source is a emitting power. And also I have to consider other surfaces like this. The power coming from other surfaces I have to consider. That is called inter-reflection. So, radiosity due to incoming radiance.

So, I have to consider these two components, one is the exitance, another one is radiosity due to incoming radiance. So, I have considered these surfaces, these surfaces I have considered and I have considered radiosity due to incoming radiance. And one thing is important there are two models, one is the local shading model, another one is there global shading model. So, if I consider a particular surface, suppose if I consider this surface and suppose I am considering this source, sources, the light sources corresponding to these points suppose. These sources are visible. These sources are visible.

So, surface has radiosity only to source visible at each point. So, suppose corresponding to this point these sources are visible. And we have to consider radiosity only to the sources visible at each point so that we have considered that is called the local shading model. But in the global shading model, I have to consider surface radiosity due to the radiance reflected from other surfaces.

Suppose in this case if I consider other surfaces like this, the light is reflected from these surfaces that also I have to consider. That is, the surface radiosity is due to the radiance reflected from other surfaces we have to consider that is inter, inter-reflection and that is also called the ambient illumination. So, in this case I am considering the local shading model, another is the global shading model. In local shading model we are only considered the sources. In case of the global shading model I am considering the radiance reflected from other surfaces.

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Now the next concept is the shape from shading. That is a very important concept. So, how to determine shape information from shading?

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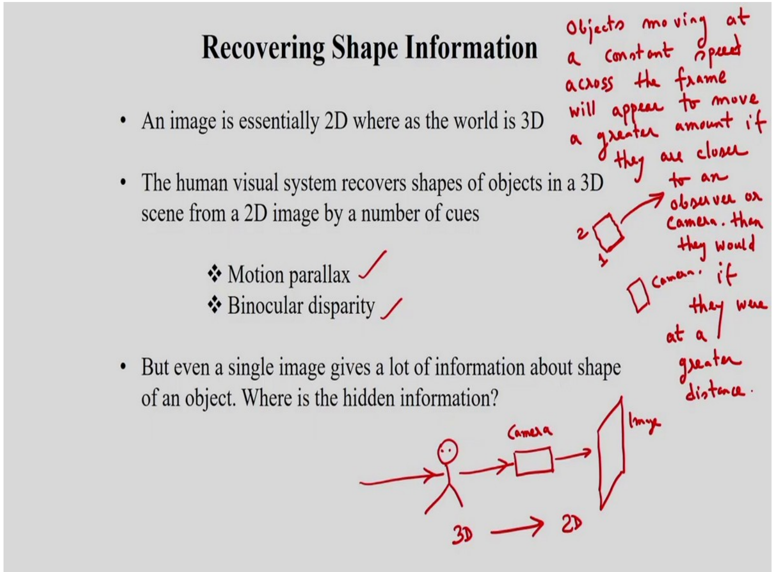

So, an image is essentially 2D, but in this case, the, our world is 3D. That is nothing but the image formation process. So, suppose image formation process already I have discussed. This is my object and this is my imaging system. Imaging system is nothing but the camera and I have the image here in the image plane. Light is coming in this direction and like it I am getting the image here. This process is nothing but the 3D to 2D projection this is nothing but the 3D to 2D projection.

In case of the human visual system, human brain reconstructs the 3D information. There are many cues. The one cue is the motion parallax. Another one is the binocular disparity. So, by using this cue the human visual system recovers the shape information from the 2D images. So, what is motion parallax? I want to explain. Suppose I have this camera and suppose my object is, this is my object. Now object is moving suppose, like this object is moving. So, my one surface plane is 1, another one is surface 2. The surface 1, this surface 1 is close to the camera, the camera is here, as compared to the surface 2.

So, from the camera the surface 1 or the plane 1 moves quickly as compared to the plane 2 that is the motion parallax. So, that means I can write like this. What is the motion parallax? Objects moving at a constant speed, constant speed across the frame will appear to move a greater distance, or greater amount I can write, if they are closer to an observer or camera, then they would, if they were at a greater distance, so that is the motion parallax.

The objects moving at a constant speed across the frame will appear to move a great amount if they are closer to an observer or a camera, then they would if they were at a greater distance. So, this concept already I have explained, that is the motion parallax. So, based on the motion parallax we can get the shape information.

Another cue is the binocular disparity. In binocular disparity we consider two cameras and by using these two cameras, we will get two images. So, from these two images we can determine the disparity values and from the disparity values, we can determine the depth information, disparity in observation. So, by using these two cues human visual systems recovers shape of objects in a 3D scene from the 2D image. So, these two concepts I have explained. One is the motion parallax, another one is the binocular vision.

In binocular vision, already I have explained that we have two cameras and we are getting two images, one is the left image, another one is a right image. And from these two images I can determine the disparity, disparity in observation. And after this we can determine the depth information. That is the binocular vision that we are going to discuss in next classes, the stereo vision.

Now in this problem, the shape from shading problem I have only one image. So, from one image I want to get the shape information. So, how to do this? We will discuss now.

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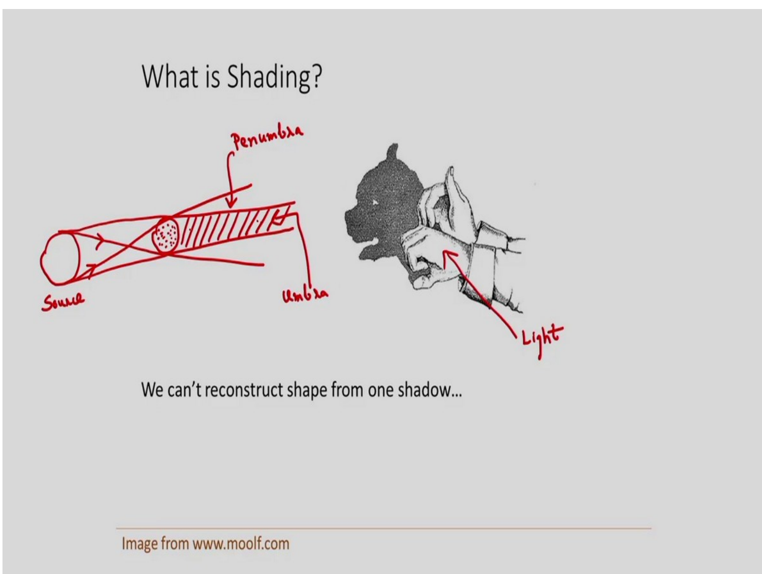

So, in this slide I have shown the shading, one example of the shading. But in this case it is not the shading, it is a shadow. So, we cannot reconstruct shape from one shadow. This is one example of the shadow, because if I consider suppose light is coming in this direction, this is the light, and because of this light I am getting one shadow here.

So, shadow principle is something like this, suppose I have a source, light source. And suppose I am considering one obstacle, this is my source and I am considering one object here. This is my object. And I am getting the shadow here. So, this portion is called penumbra. And this, this portion is called umbra. Umbra means this portion which cannot see the source at all. So, from these points, suppose I cannot see the source completely. What is the penumbra?

A penumbra which can see a part of the source; that is the definition of penumbra. One is the umbra, another one is the penumbra. So, this is the formation of shadow. So, now I have discussed the concept of the shadow. So, in computer vision application, one application is removal of the shadow. Suppose one example I can give, object tracking. In object tracking there may be shadow, the shadow of the object. So, when the, when the object is moving the shadow is also moving that is the cast shadow. So, in this case I have to remove the shadow. Now in this class I am not discussing about the shadow. I am discussing about the shading. Shading means variable levels of darkness. Now let us see what is shading.

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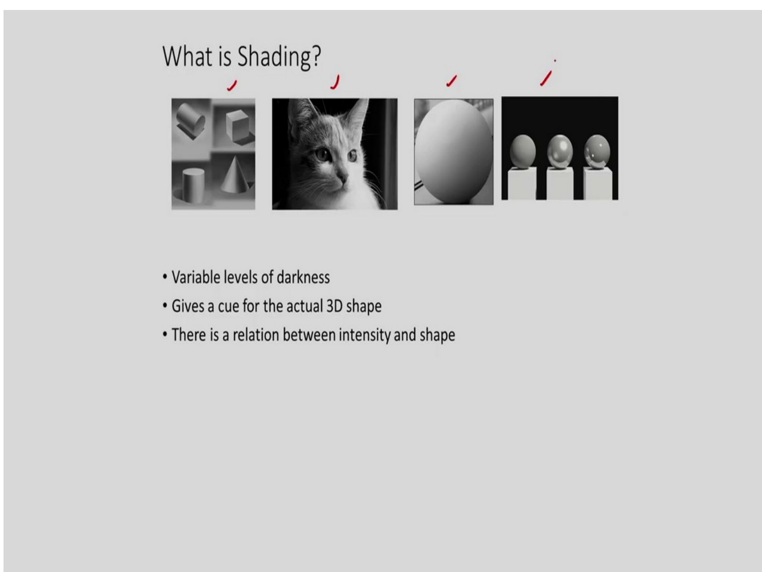

I have given some examples of the shading. If you see these images, if you see all these images, then this variable levels of darkness you can observe. This gives a cue for the actual 3D shape. And in this case there is a relationship between the intensity and the shape. So, if you see these images I, then in this case, I am getting a feeling of the 3D shape. That is the shape from shading.

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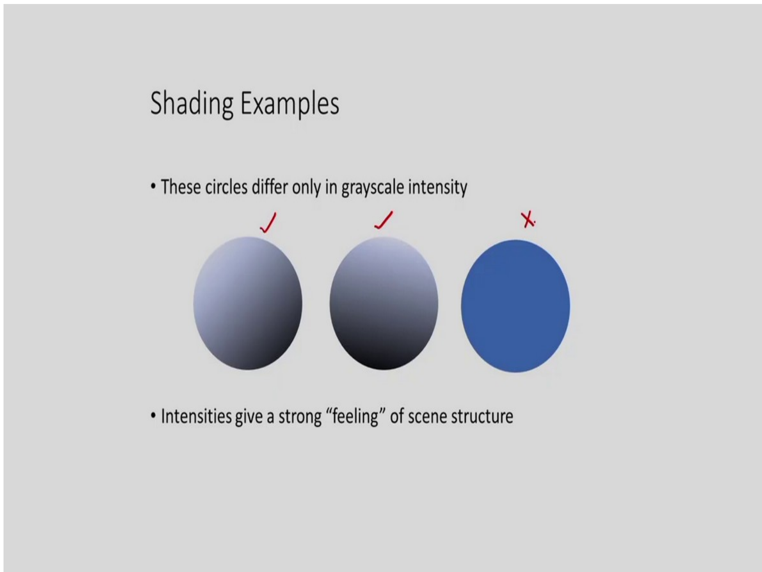

Another example I am giving here. If I see these images you can see the variable levels of darkness. But in this case this image is constant intensity value. So, if I consider the first two images the intensity gives a strong feeling of the scene structure. So, because of the shading I am getting a feeling of the 3D shape. That is the shading example. I am giving this example.

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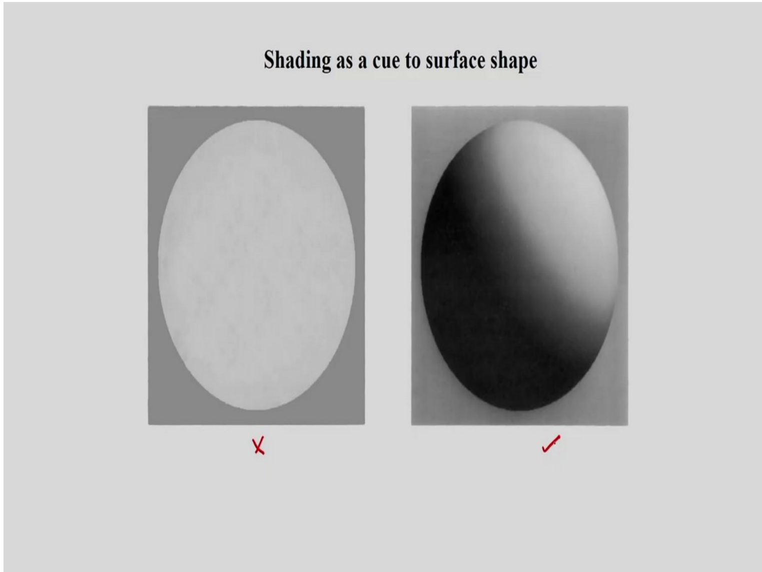

Another example I am giving. This is not the shading because it is a constant intensity value. But if you see this image, because of the shading I am getting a feeling of the 3D shape. So, this is another example of shading. So, from the shading information how to get the shape information.

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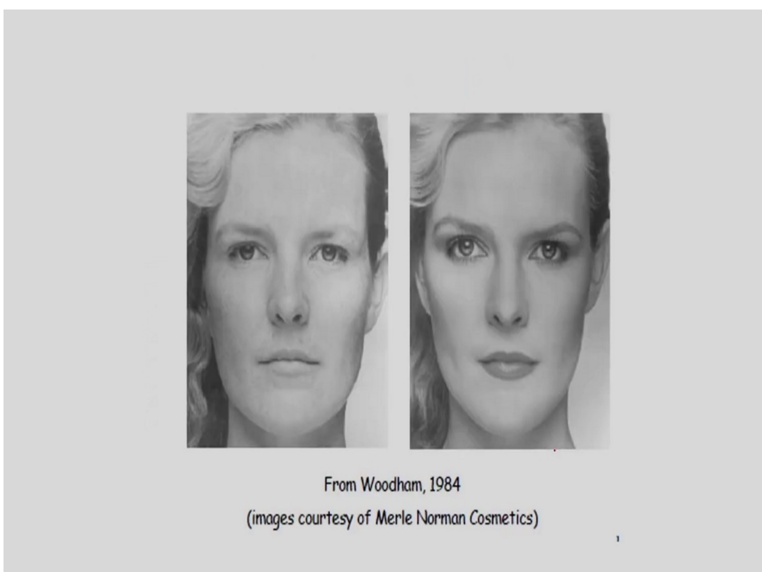

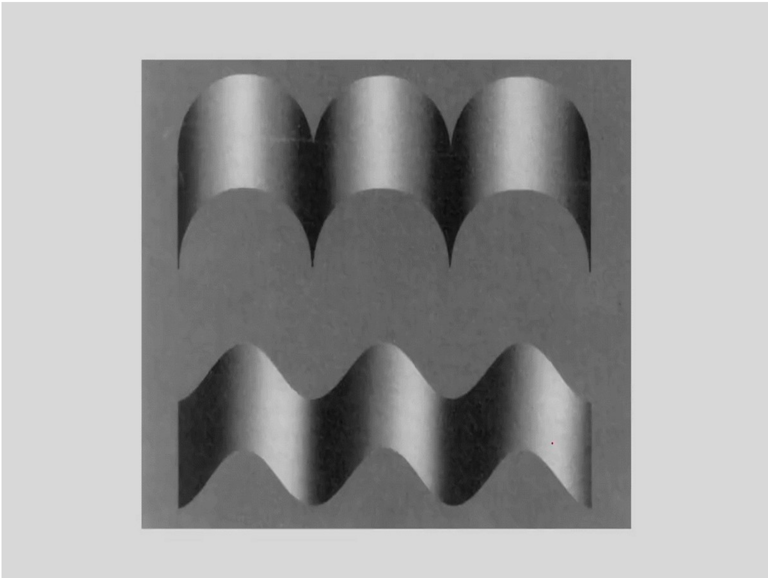

Another example I am giving, the shading example. So, from the shading you can get a feeling of the 3D shape. Here also I am giving another example of the shading. So, here you can see, you have a feeling of the 3D shape that is from the shading. So, shading means, already I have mentioned, shading means variable levels of darkness.

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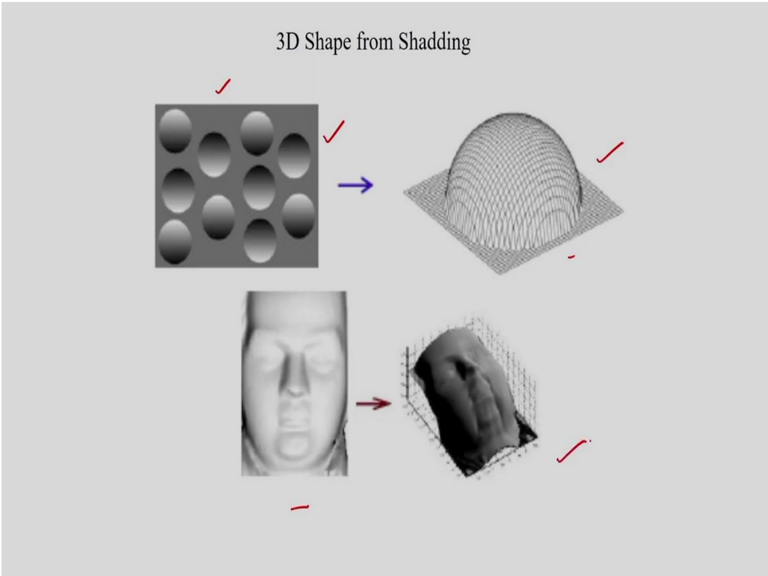

The 3D shape from shading problem is, from one image, I only one image, I want to get the shape information. In this case the shading information is available. If you see this image you can see the shading. And in this case from the one image I want to get 3D shape information. So, 3D shape information I am getting here.

Similarly, in the second example I am considering one face image. In this case also I am, I am having the shading. Now from the single image I want to get the 3D shape information. So, that is the problem of shape from shading.

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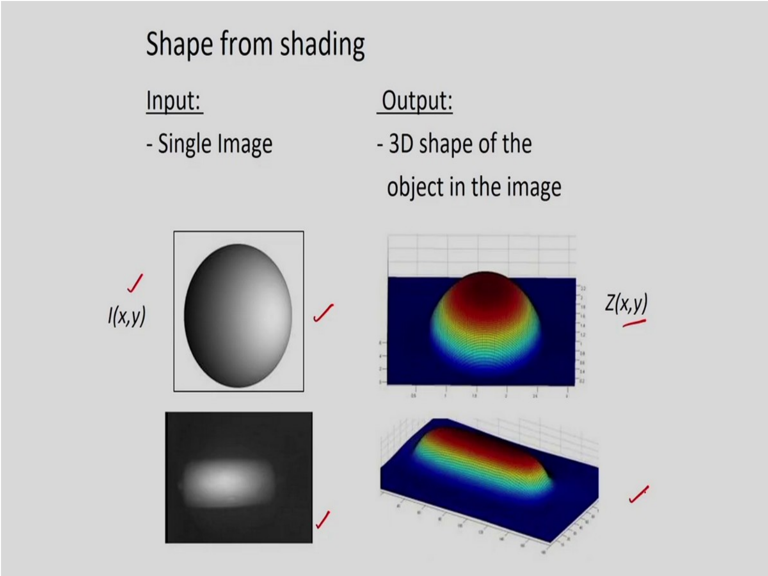

So, what is the concept of shape from shading? So, I have only one image, single image that is I(x, y), one image. So, from one image I want to get the 3D shape information, that is the Z(x,y), that is the depth information I want to determine. 3D shape of the object in the image I want to determine. Similarly, in the second example also I am considering one image. And from this image I want to get the 3D shape information. That is the problem of shape from shading. So, shading gives a feeling of the 3D shape.

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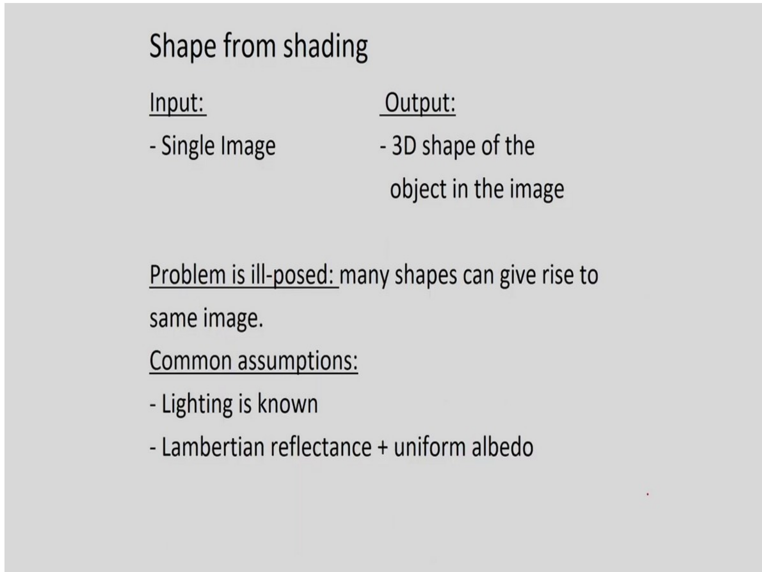

Shape from shading problem is, input is my single image, output is 3D shape of the object in the image. But the problem is ill-posed. Why it is ill-posed? Because many shapes can give rise to same image. That is the problem is ill-posed. I have to consider some common assumptions. The lighting is known that means I know the direction of the light source and also lighting conditions.

Also another assumption is, I have to consider the Lambertian reflectance and also I have to consider uniform albedo. The albedo lies between 0 to 1, already I have explained. So, these two assumptions I have to consider. One is the lighting, another one is the Lambertian reflectance, also uniform albedo.

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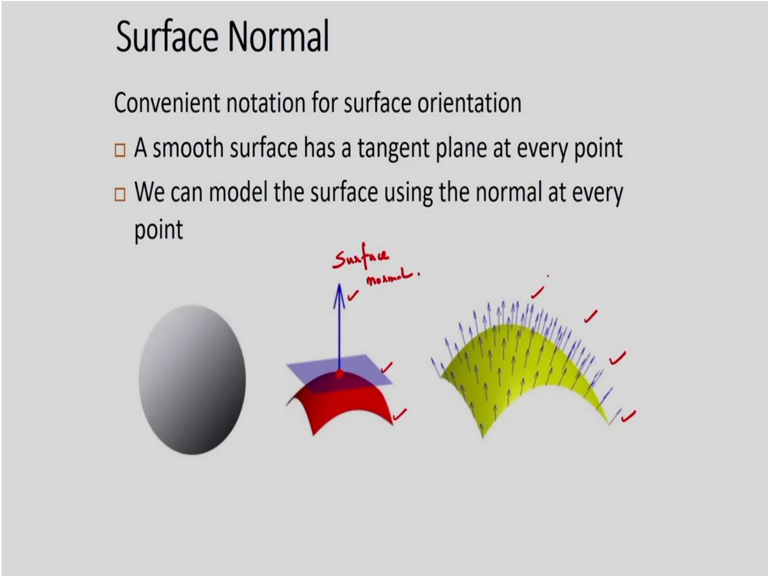

Now, how to represent a particular surface? So, in this example I have considered one surface, this surface I am considering, and corresponding to this surface suppose I am considering one point here. This point I am considering, this is the point. Corresponding to this point, I can draw a tangent plane. So, my tangent plane is this. And corresponding to this tangent plane I have a surface normal. So, my surface normal is this, this is my surface normal.

So, I am explaining again. Corresponding to this red surface and corresponding to this particular point, I can consider one tangent plane and corresponding to this tangent plane, how to represent the tangent plane? The tangent plane is represented by the surface normal. So, this is my surface normal. So, this surface, the particular surface can be represented by surface normals. So, I have the surface normals like this. So, I have the surface normals. So, the surface is represented by surface normals.

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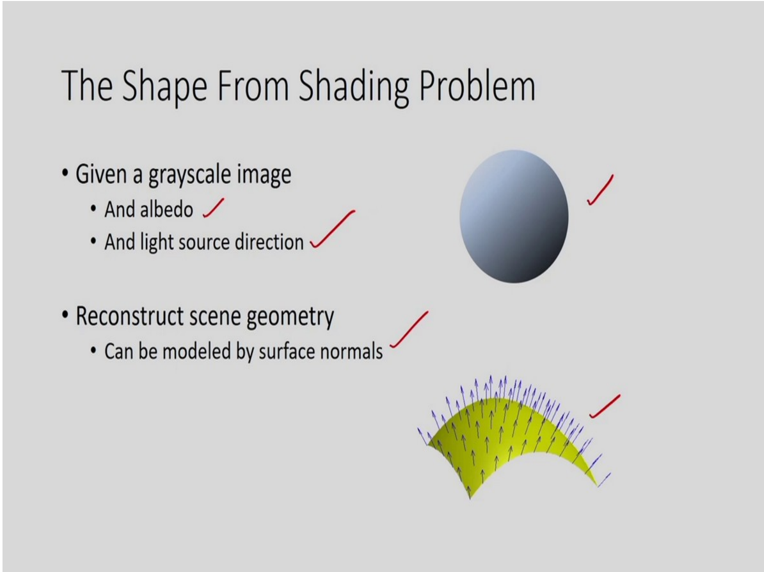

The shape from shading problem is, given a grayscale image, I have a grayscale image and the albedo, and the light source direction is available, then in this case I want to get the scene geometry. So, that means this case, I have to get the information of the surface normals. So, these are the surface normals. So, this, the grayscale image is available. In this case you can see the variable levels of darkness. So, from this I want to determine the surface normals. If I can determine the surface normals that means I can reconstruct scene geometry and this is the shape from shading problem.

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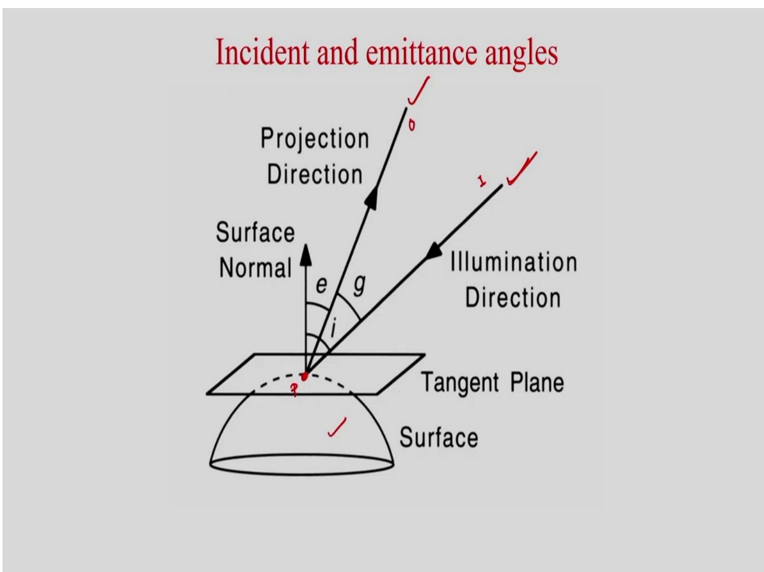

Here, I have shown the incident and the emitance angles. So, light is coming from the source like this. So, this is the direction of illumination and I am considering a surface and corresponding to this point, I am considering a tangent plane and this is the outgoing direction. This is my incoming direction, this is my incoming and this is my outgoing direction. And corresponding to this point, the point is suppose, P, I am considering the surface normal. So, my objective is basically, I have to get all the surface normals so that I can get the orientation information of the surface.

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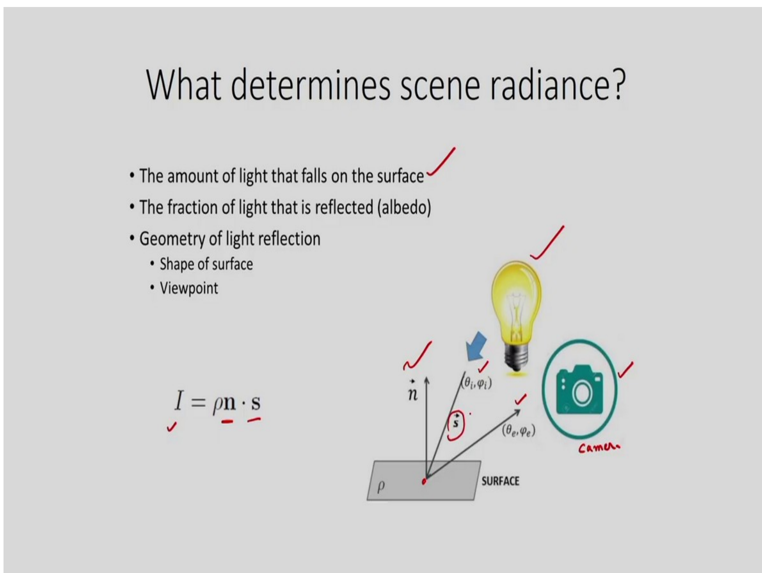

So, what determines the scene radiance? So, here, in this case the scene radiance actually depends on the amount of light that falls on the surface. So, this is the first case. The amount of light that falls on the surface that determines the scene radiance. The fraction of light that is reflected, so that depends on the albedo of the surface. So, here if you see, the light is coming from the source so, this is the direction, incident direction and this is the outgoing direction. And this is my camera. So, this is my camera. And what is this corresponding to this point?

This is my surface normal. So, this vector s, it corresponds to the source vector. n is the surface normal and this ρ is the albedo of the surface. So, that means, in this case what will be my image? I is the image that is the irradiance, the image will be the albedo. And after this I have to take that dot product of the surface normal and the source vector, source vector is this.

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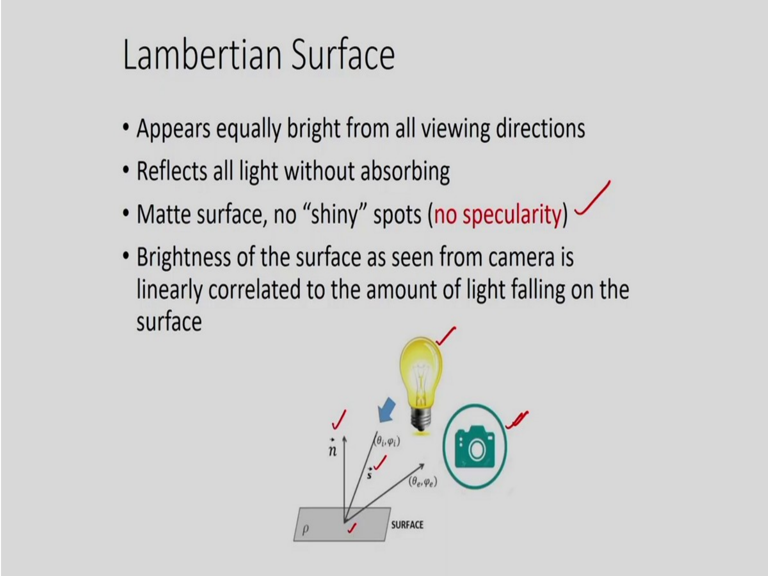

So, in case of the Lambertian surface, already I have explained the Lambertian surface that means the radiance leaving a surface is independent of the angle. That is the definition of Lambertian surface. Now in case of the Lambertian surface, it reflects all light without absorbing and I am not considering the specular surface, I am not considering the specularity. Brightness of the surface as seen from the camera is linearly correlated to the amount of light falling on the surface.

So, brightness of the surface, the brightness of the surface is observed by the camera. So, the brightness actually depends on the amount of light falling on the surface. So, amount of light coming from the source, this is a source and light is reflected back to the camera and this is the surface normal, this is the source vector and I have shown the incoming direction and the outgoing direction. So, brightness of the surface as seen from a camera is linearly correlated to the amount of light falling on the surface. So, this is very clear.

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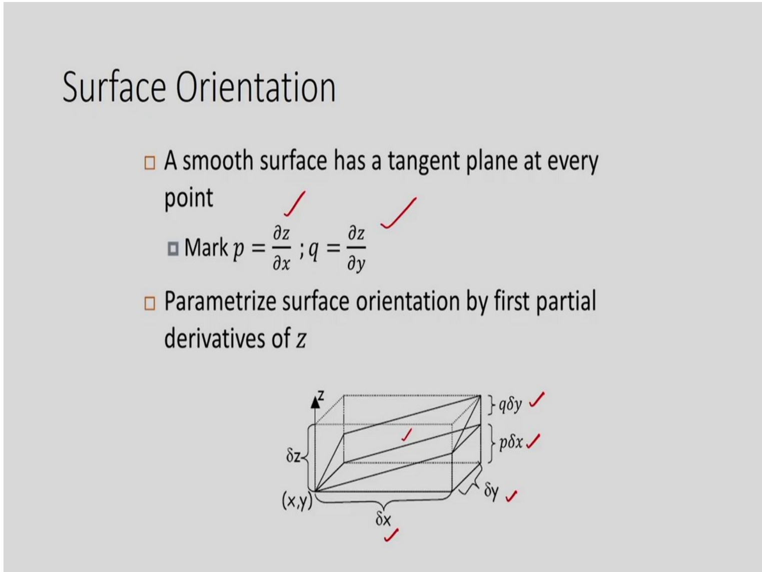

Now how to, how to represent the surface orientation? So, already you know that a smooth surface has a tangent plane at every point. So, this surface, suppose I am considering. So, I have the x co-ordinate, y coordinate and the z coordinate, this surface orientation is represented by this gradient value. Gradient is p and q. What is p? p is the partial derivative of z with respect to x; that is the p, the change of z with respect to x.

And what is q? The q is the partial derivative of z with respect to y. That means with y direction what is the change of z that I am considering. So, this is δx this is δ y. And this case that will be p δ x, this will be equal to q δ y, like this. So, the surface orientation is represented by this gradient. The gradient is p and q.

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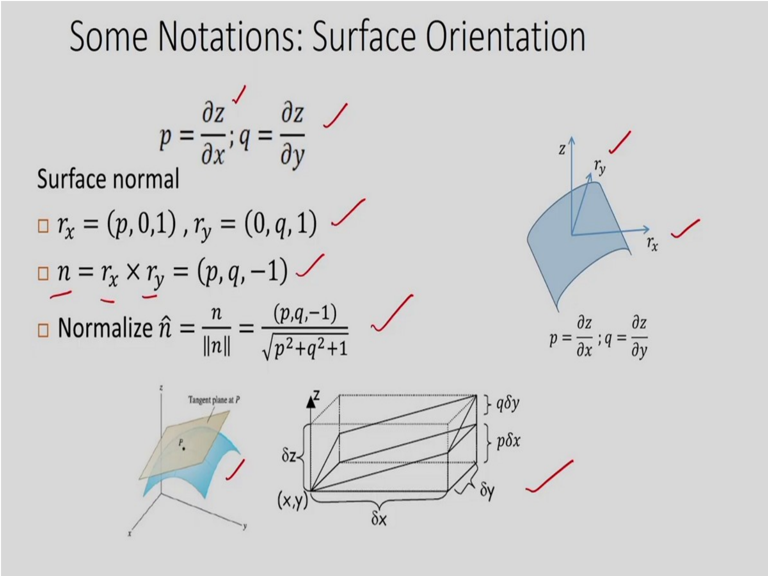

Here I have shown again, the surface orientation is represented by this gradient, one is, is gradient is p, that is the partial derivative of z with respect to x. And q is the partial derivative of z with respect to y. So, I am defining like this that is the surface orientation. And in this case I am considering the surface vectors, one is in this direction r_x, that is (p, 0, 1), that is the surface vector. Another surface vector is r_y that is (0, q, 1) that is r_y. And I want to determine the surface normal.

The surface normal you can determine by the cross product of r_x and r_y. The surface normal is n. So, if I take the cross product of this, I will be getting (p, q, -1). And this can be normalized. So, if I do the normalization of this, then I will be getting this value. (p, q, -1) divided by $\sqrt{p^2+q^2+1}$, normalized value of the surface normal.

So, in this diagram, again I am showing, the tangent plane I am showing corresponding to the point p. And here again I am showing what is gradient value corresponding to a particular surface orientation p is equal to δ that is the partial derivative of z with respect to x. And what is q? The partial derivative of z with respect to y. So I have, I have defined. So, that means the surface orientation is represented by the gradient value p and q. And from this I can determine the surface normal.

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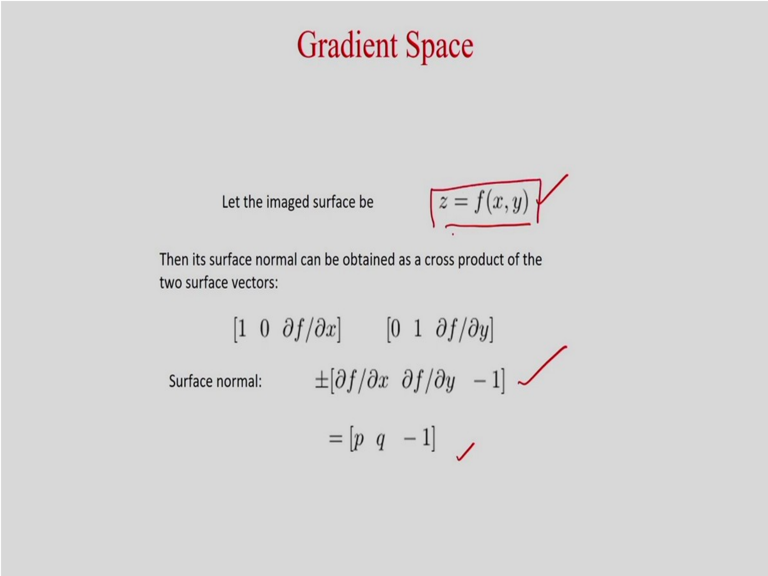

So, same thing I am showing here. So, let the image surface be z is equal to f (x, y). In next slide I am going to explain what is z =f (x, y)? Then the surface normal can be obtained as a cross product of the two surface vector. So, in my last slide I have shown the surface vector is r_x and r_y. And from this you can determine the surface normal. The surface normal is (p, q, - 1). So, this, this concept I am going to discuss in the, my next slide, what is the image surface that z equal to f(x, y)?

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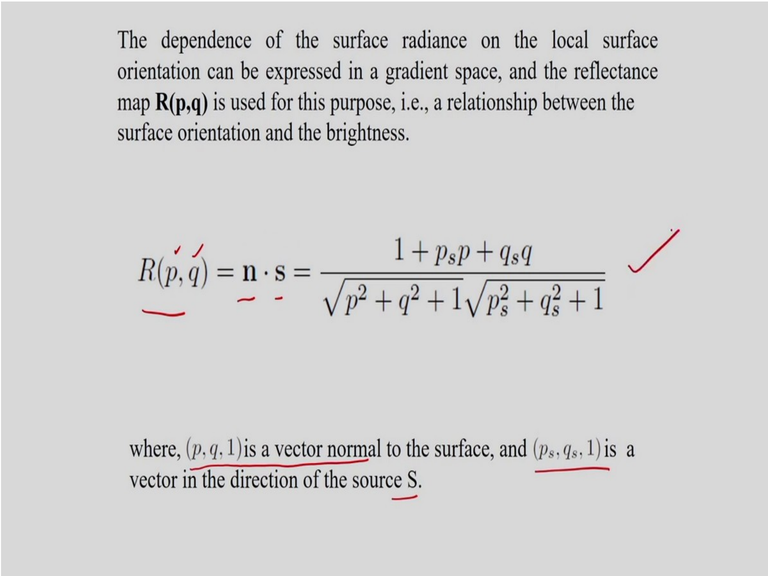

The dependence of the surface radiance on the local surface orientation can be expressed in a gradient space. And the reflectance map R (p, q) used for that purpose. That means what is R (p, q)? r p q means it gives the relationship between surface orientation and the brightness. That is called reflectance map. So, R (p, q) means that it is a relationship between the surface orientation. The surface orientation is given by the parameters p and the q. And the brightness also I am considering. So, for this, what I am doing? I am just doing the dot product between the surface normal and the source vector, s is a source vector.

So, in this case I am considering (p, q, 1) is a vector normal to the surface. There is a surface normal and I am considering the source vector, the (p_s ,q_s ,1), that is the vector in the direction of the source. And if I take the dot product between this, surface normal and the source vector, then in this case I will be getting this one. So, this is the reflectance map.

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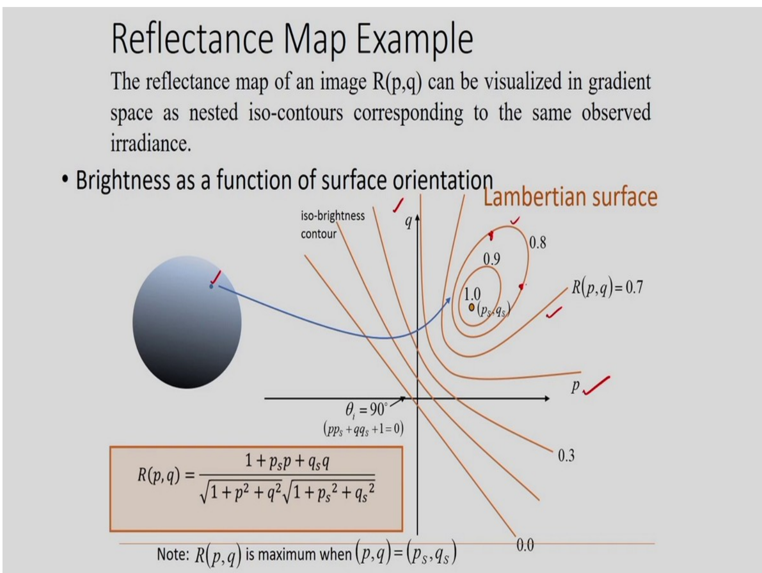

So, in this case I am giving one example of a reflectance map. The reflectance map of an image R (p, q) can be visualized in the gradient space as nested iso-contours corresponding to the, to same observed irradiance. So, in this case if I consider, suppose the concept of brightness as a function of the surface orientation. So, corresponding to this point I have one contour. The contour is, this is the contour. The brightness value, the brightness value is suppose 0.8.

For different values of p and q, so you see corresponding to different values of p and q, the brightness is constant. The brightness is 0.8. Corresponding to this one, the reflectance map is

0.7. So, that means I am showing the brightness as a function of surface orientation. Suppose corresponding to this orientation the brightness is 0.8. So, that is why I am considering iso-contours that corresponds to the same observed irradiance. Irradiance means the brightness. So, this is; I have given one example of the reflectance map.

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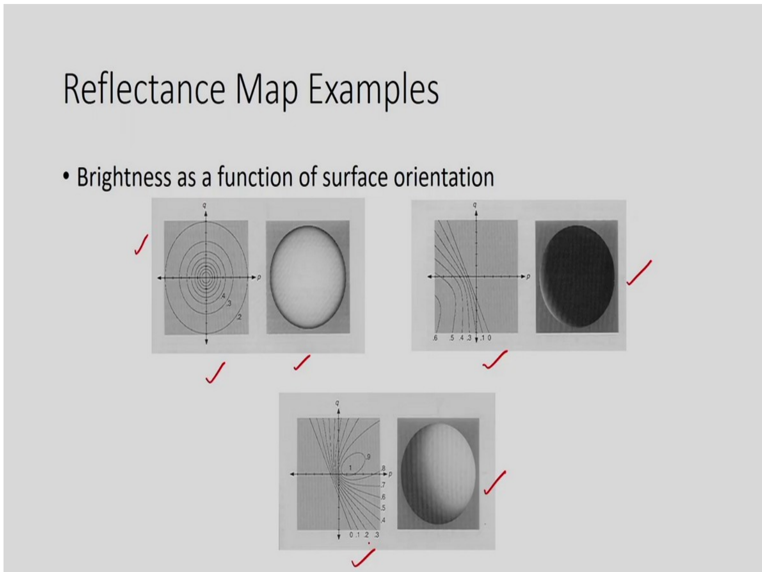

Here, I have given some reflectance map examples, brightness as a function of surface orientation. In this example if you see, this is almost uniform brightness. So, corresponding to this I am getting the reflectance map like this. Corresponding to this dark object, I am getting the reflectance map something like this. And corresponding to this image this will be the reflectance map. The reflectance map, you can you can, plot.

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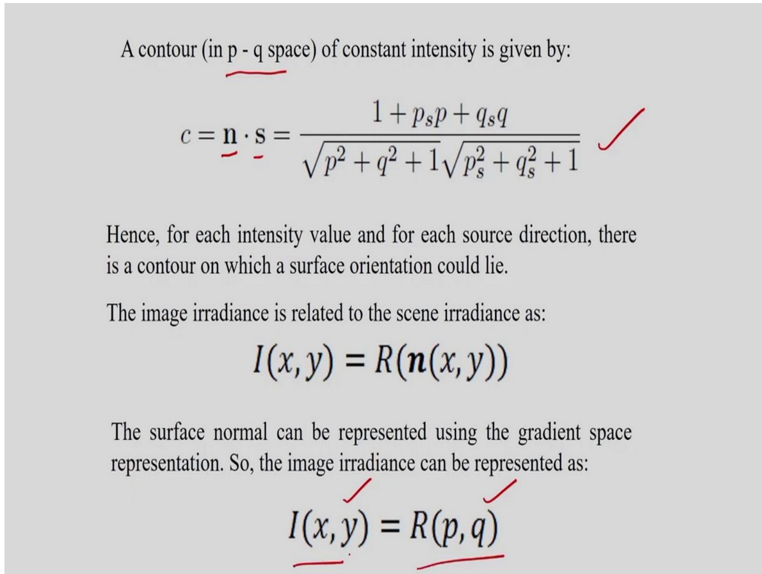

A contour of constant intensity is given by a contour that is, p -q space means it is a gradient space, p -q space called the gradient space of constant intensity is given by c is equal to the dot product between the surface normal and source vector. So, this is a surface normal and this is a source vector. So, I am getting the contour of constant intensity. So, already in my last slide I have shown this expression. Hence for each intensity value and for each source direction there is a contour on which the surface orientation could lie.

Now one important thing is the image irradiance is related to the scene irradiance. So, that is why I can show this image irradiance, I (x, y) is image irradiance; that is related to the reflectance map. So, image irradiance is related to the scene irradiance. So, that means the image irradiance is related to reflectance map. So, this is my reflectance map. So, this is the image irradiance.

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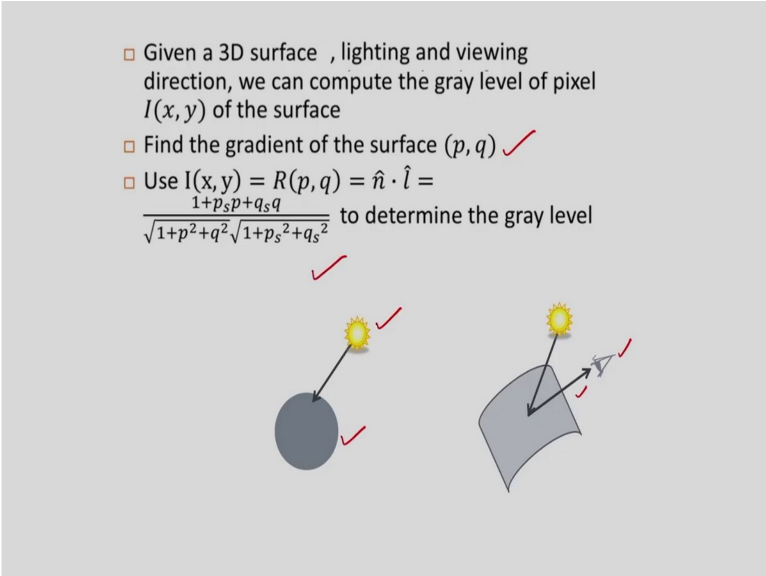

So, here again I am showing the same concept. Light is coming from the source. And this is the surface and the light is reflected by the surface and this is my observation. Observation may be camera. So, given a 3D surface, lighting and viewing direction we can compute the gray level of pixel I(x, y) of the surface. If I know these values lighting and the viewing direction, we can compute the gray level pixel, value that is I (x, y) of the surface I can compute. So, in this case find the gradient of the surface. The gradient of the surface is p and q that I can determine. And in this case by using this expression I can determine gray levels.

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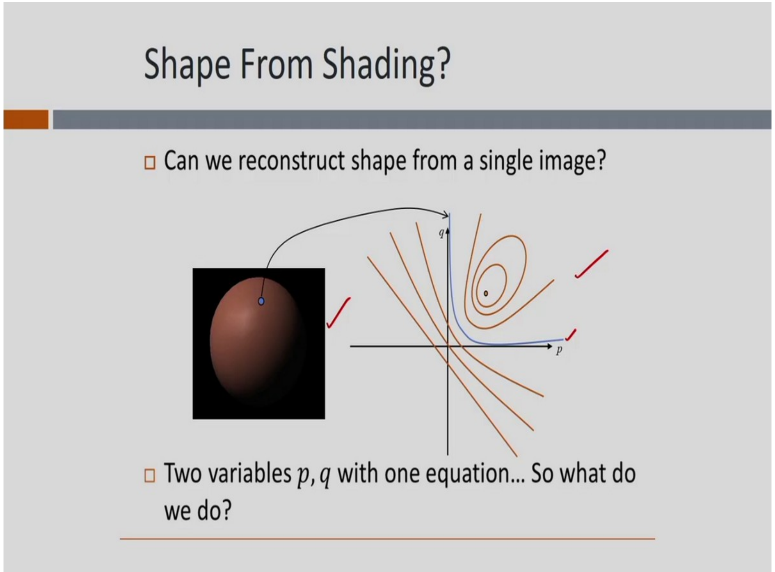

Now shape from shading, can we reconstruct the shape from a single image? So, I have only one image and corresponding to, suppose, this point I have, this is the reflectance map corresponding to this. Suppose this blue one, the blue one is the reflectance map. But the problem, if you see the previous equation, I have shown the previous equation, here p and q, two variables are available, p and q; but equation is only one equation. If you see that only one equation is available but I have two variables. So, how to compute this?

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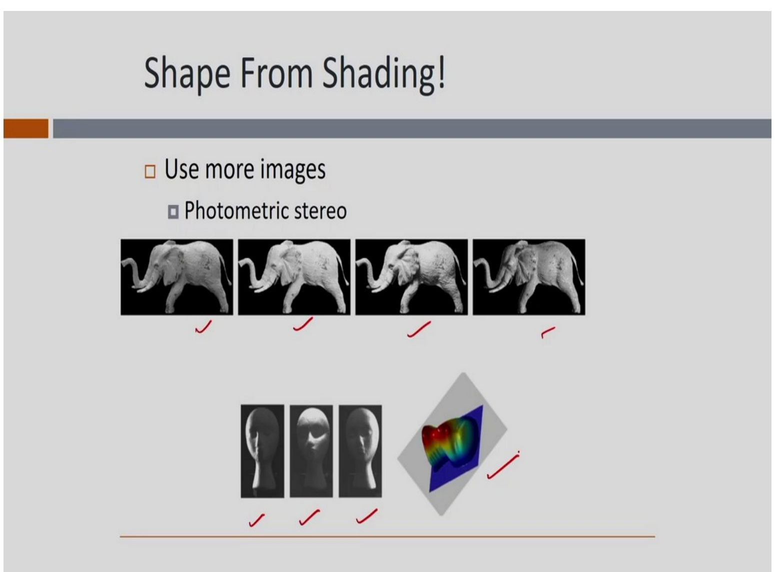

So, one solution is I can consider more images of different brightness. I can consider this image. I can consider this image. I can consider this image. I can consider this image. Like this I can consider. And from this I want to get the 3D information. Similarly, I have shown this example. Different images I am considering 2, 3 images maybe 4 images, first 7 images like this of different brightness, different illumination condition. And from this I am getting the 3D information.

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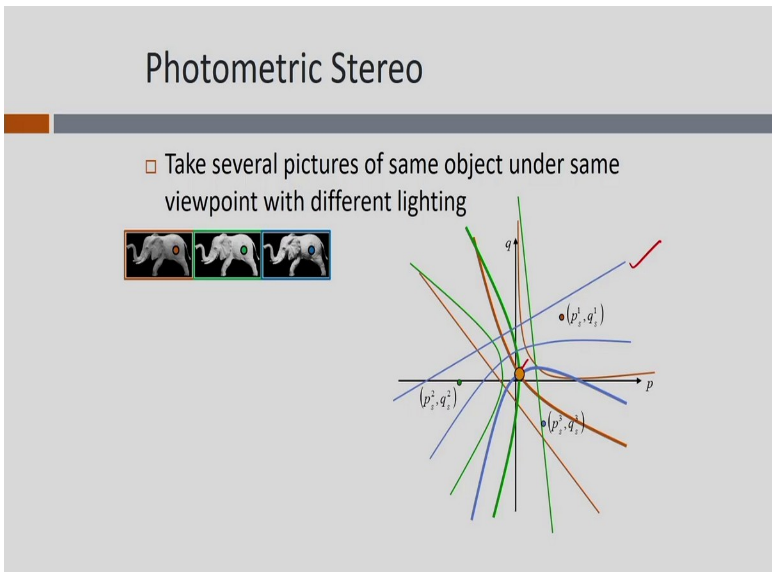

So, like this, take several pictures of the same object under different viewpoint with different lighting conditions. So, I am taking one image corresponding to a particular lighting condition, this image I am considering. Corresponding to the first image I am getting the reflection map like this. This color is same color I am considering the reflectance I am getting. Corresponding to the second image, corresponding to another lighting condition I am getting the reflectance map that is given by these green colors. And again I am considering another image, the blue image. Let us suppose, corresponding this, this one. In different lighting conditions, then in this case I am getting the, this blue reflectance map.

And all these are intersecting at this point. If you see, all these are intersecting at this point, this reflectance map corresponding to that particular point. So, from this I can determine this value, the p and q value. That is the p and q value is mainly the orientation of the surface. So, this is one technique. This technique is called photometric stereo.

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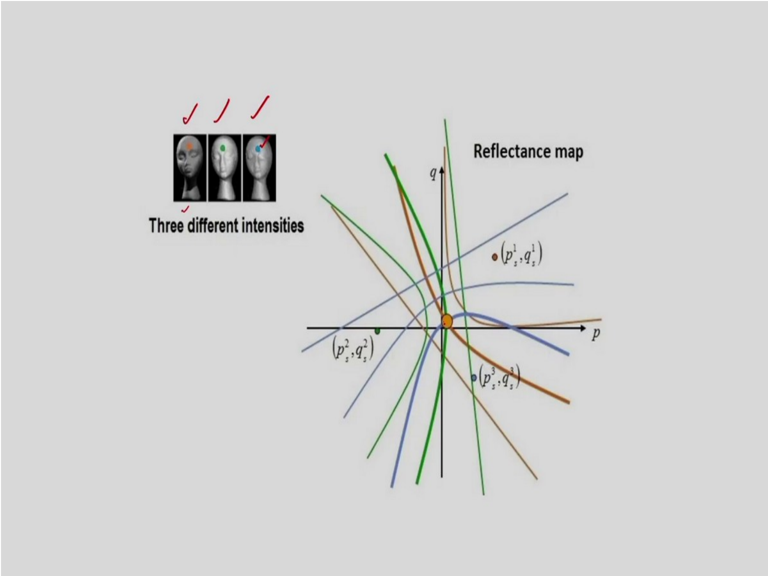

Here again I am showing this another example. Three different lighting conditions, same scene and in this case what I am getting? All these are intersecting at this point corresponding to that particular point. So, these are intersecting. And this gives the value of p and q. So, I am getting three reflectance maps, one is corresponding to the, this image, the first image. Corresponding to the second image I am getting another reflectance map. Corresponding to the third image I am getting another reflectance map, and from this I want to determine the solution. The solution is p and q.

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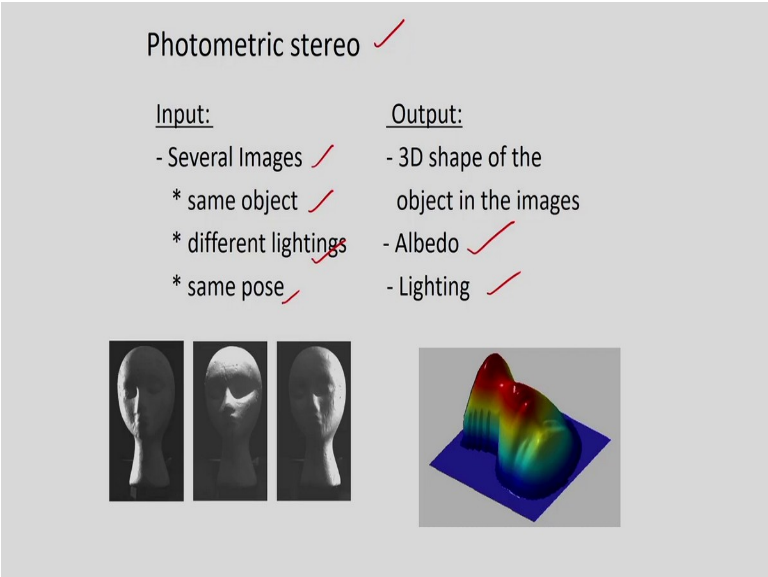

So now, this shape from shading, there are many algorithms to get, to get the solution of this. And one very important algorithm is photometric stereo. The photometric stereo is one very important application, one very important algorithm of computer vision. So, in this case input is several images of the same object but different lighting conditions and pose is the same. This is my case that I have several images of the same object but different lighting conditions, same pose. And what will be my output? The output is the 3D shape of the object in the image. I can also determine the albedo of the surface, and lighting also I can determine. So, this is called the photometric stereo.

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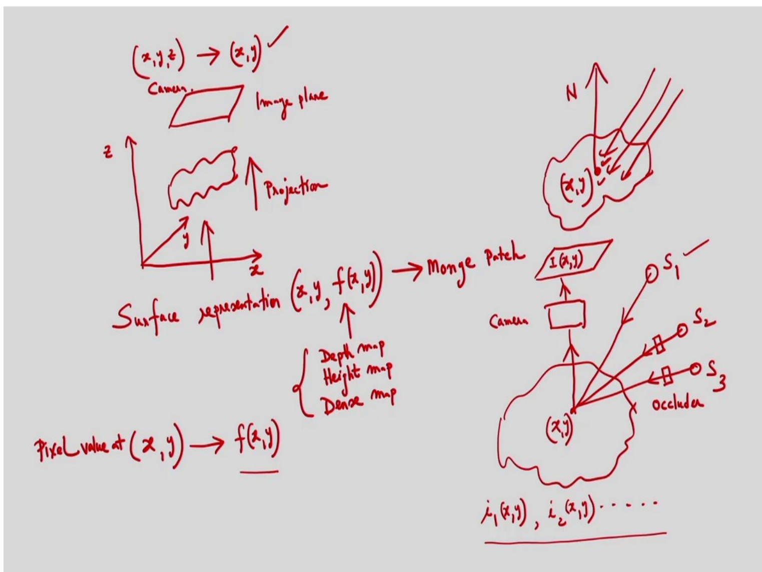

So, already I have explained that in case of the camera I have (x, y, z) that is projected into (x, y) coordinate. That I am doing the projection. So, this is the principle of the camera. That means in this case the depth information is lost. So, that means I can show like this, this is z direction. This is x direction and this is y direction. I am considering one arbitrary surface, this surface I am considering. And for imaging I am doing the projection. This is my image plane. So, I am getting the image. So, this is the direction of the light source, light is going from this direction and I am getting the image.

So, due to this projection I am getting the image, the image is (x , y). Only x and, y coordinate I am getting. The z coordinate is missing. Now the surface representation, how to represent a particular surface? The surface representation, a surface is represented like this. x and y coordinate, there is a spatial coordinate and f (x y), in my previous slide I told you what is f(x y). So, a surface is represented like this. x and y is the spatial coordinate. This f(x, y), this is called the depth map. Or sometimes it is called the height map; sometimes it is called the dense map.

So, this term, depth map, height map and a dense map, this representation is called, one standard term is there, this is called the M, Monge Patch, the representation of a surface is equal to Monge Patch. So, what is (x, y)? The pixel value at the point (x, y) is actually f (x, y). Now in this case what is the concept of the photometric stereo? So, in case of the photometric stereo, suppose this is the surface, light is coming from the source like this and this is the surface normal. And suppose this point (x , y).

So, the photometric stereo concept is, for different views; different views means that is the number of sources, the light sources I am considering, and we will see f (x, y), and find it the depth map. So, this point is illuminated, this point is illuminated by different light sources. From this I want to see f (x, y). So, I am getting the pixel value f (x, y)and from this I want to determine the depth map. So, I will show how to do this photometric stereo.

So, the setup off the photometric stereo is, suppose this is the surface the surface is like this, this is the surface. Light is coming from a source corresponding to the point (x, y), suppose, this point, the particular point. So, I am considering a source, the light source is S_1. Like this I am considering many, many light sources. Another source may be S_2, another source may be S_3. So, I am getting number of images and this light will be reflected by, so this is my camera, this is my camera and after this, I am getting the image. The image is I(x, y) I am getting. So, this is the photometric stereo setup. So, light is coming from the source S_1, S_2, S_3.

But you remember, only at a particular time only one source is considered. So, at a particular time only source, the first source is considered, S_1. And corresponding to this I am getting one image. So, for this the S_2 is not considered. Other sources are not considered. So, that is why I have to obstruct the light, I have to obstruct the light. That is called occluder, occluder I am considering. So, at a particular time one source is considered, so S_1 is considered. Corresponding to S_1, this point is illuminated by the light source and I am getting one image. The image is suppose I_1 (x, y).

Next one is I am considering the second source, from the second source also I am getting another image. The image is I_2 (x, y). So like, I am getting number of images, and in this case from these images, from these images I want to get the information. That information is f(x, y), f (x, y) means it is depth map I want to determine. So, this is the case. So, I have explained the concept of photometric stereo.

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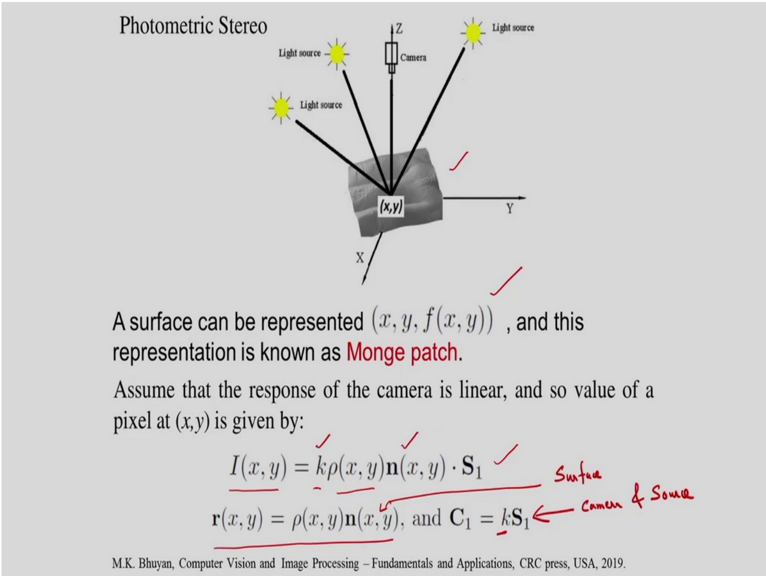

So, let us see the mathematical derivation of this photometric stereo. So, here if you see, I have the surface. I am considering the light sources. And I am showing the camera. Camera is in z direction. The point (x, y) I am considering. Surface is represented by (x, y, f(x, y)). That is already I have explained. This is Monge's Patch. Now, what will be the, my image? If I consider my camera is linear then in this case what will be my pixel value the particular point? The point is (x, y). This is the pixel value I (x, y). I am getting the image.

And if I consider the linear camera so k is considered about linear camera, and this is the surface albedo because light is reflected by the surface. So, I have to consider albedo of the surface. This is the surface normal. And I am, consider the source vector because I know the direction of the source. So, now I am defining r (x, y). r (x, y) is ρ (x, y, n (x, y)). What is actually it means? It corresponds to the surface characteristics because I am considering the albedo and surface normal. This corresponds to characteristics of the camera because I am considering linear camera. Linearity I am considering by this k. And source I am considering, these are the characteristics of the source. The source is S_1. So, I am getting this equation I (x, y) is equal to the pixel value I am getting.

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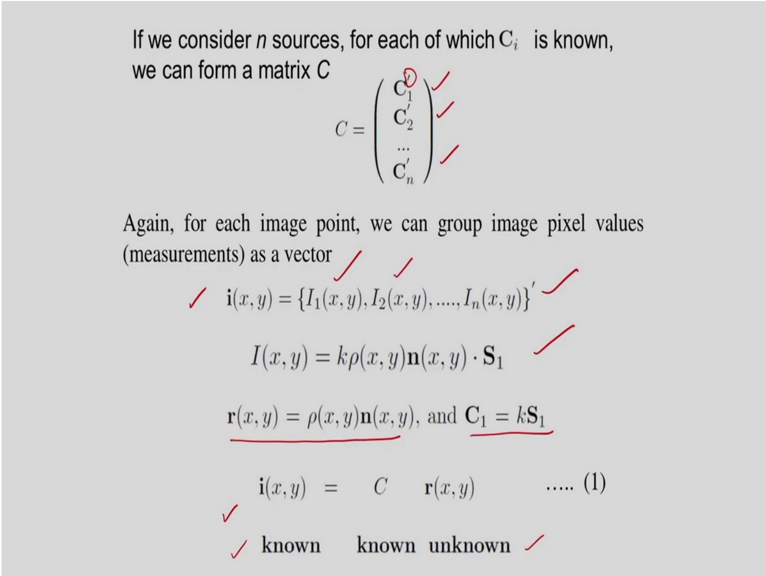

So, if we consider n sources for each of the, for which C_i is known. So, suppose if I consider n number of sources, then in this case I am considering the source because the C, because if you see the previous slide, what is the source? Source and the camera is represented by this C_1. The factor is C_1. So, if I consider n sources, for each of which I am getting C_i. So, for each n of the source I am getting C_1, C_2, C_3 like this.

This is the transpose operation. If you see this bar, the transpose, the transpose of this. And corresponding to these sources I am getting the images. So, I am getting the images. Corresponding to the first source I am getting one image. Corresponding to the second source I am getting the second image. Like this I am getting the number of images. But you remember, only one source at a particular time.

So, already I have explained this equation, already in the previous slide it is there. So, this represents the surface characteristics. This represents the characteristics of the camera and the source. This equation I can write in the vector form. I(x, y) =C r(x, y). What is I (x, y)? I (x, y) is this one. Because I will get number of images corresponding to number of sources. C, this is the basically the characteristics of the source and the camera. I am considering the linear camera. So, this is, C is known. I am getting number of images. I is also known.

What is unknown? The unknown is r (x, y). r (x, y) is basically the surface characteristics. So, I have to, I can determine the albedo of the surface. And my main objective is to determine the surface normal, because surface normal means it gives the orientation of the surface. So, that means the shape of the surface I can determine.

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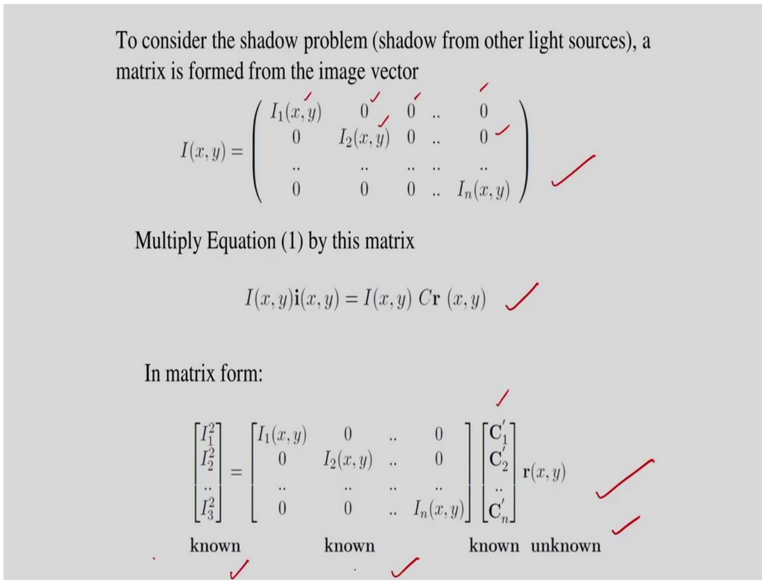

Now I am considering one matrix. This matrix I am considering. So, in this case, since at a particular time only one source is considered and I have shown in my previous diagram that is occluder is considered. So, corresponding to a particular source, suppose S1 is considered I am getting the image I_1. And for the rest of the sources I am getting 0, 0, 0.

If I consider the second source, corresponding to the second source I am getting the image I_2 (x, y). And for the remaining sources I am getting zeroes, zeroes like this. So, that concept already I have explained. So, I am getting the matrix. The matrix is I (x, y). So, multiply equation 1 by this matrix. What is the equation number 1? Equation number 1 is this. So, this equation is multiplied by the matrix. So, matrix I am considering to consider the shadow effect. It has the effect of zeroing the contribution from shadow regions. So, this is, multiplying equation by this I am getting this one. And in the matrix form I can write this one.

So, in this equation which one is known? This is known, this is known and this is unknown. The unknown is r (x, y). So, I know the image. This is, this matrix I know and this is the source matrix I know. I have to determine r (x, y). So, that is the concept of the photometric stereo.

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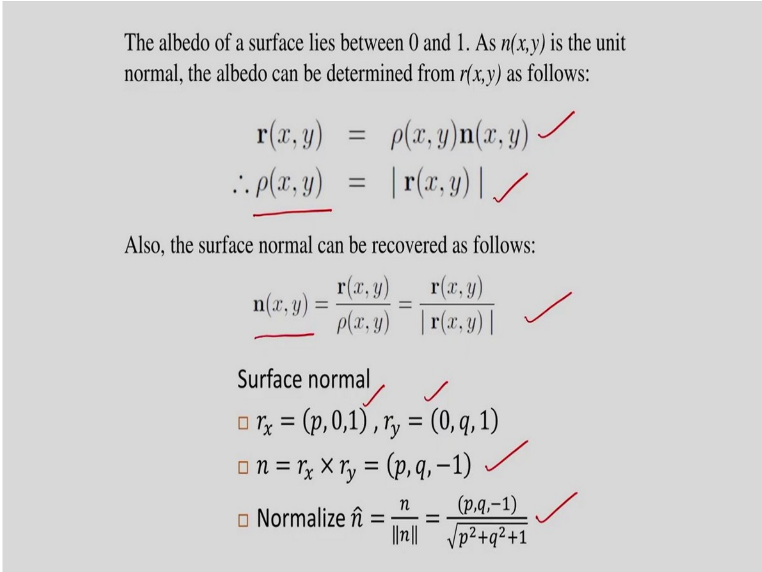

And in this case the albedo of the surface lies between 0 and 1. So, in this case this expression, you know, r (x, y) is equal to ρ(x, y) n (x, y). That is the characteristics of the surface albedo and surface normal. And from this you can determine the, the albedo of the surface. The albedo of the surface is this. And from this also you can determine the normal. Now surface normal also you can determine.

So, what is the surface normal? Already I have explained, I have the surface vectors r_x and the r_y. And what is the surface normal? The surface normal is (p, q, - 1). And I can normalize it. So after normalization I am getting this one. So, from these equations you can determine the albedo of the surface and also you can determine the surface normal.

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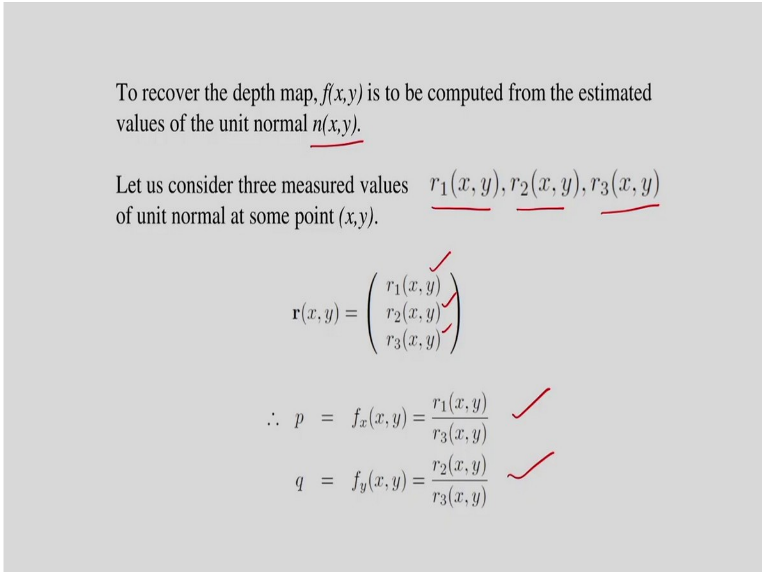

And to recover the depth map, f (x, y) is to be computed from the estimated value of the unit normal. That means from unit normal I have to determine the f (x, y) value. So, for this I am considering 3 measured value of r (x, y). So, first value is r1 x1, second one is r2, third is r3. So I am considering this r (x, y) is, I am considering three values. r1, r2 and r3 I am considering. So from this I want to determine p. The p is determined like this. p is the gradient value. Gradient value is r1 divided by r3. Next value I can determine the q value. So I am considering three measured value of r x y. So, I am considering r1, r2 and r3. So, from r1, r2 and r3 I can determine the gradient value. The gradient value p n q.

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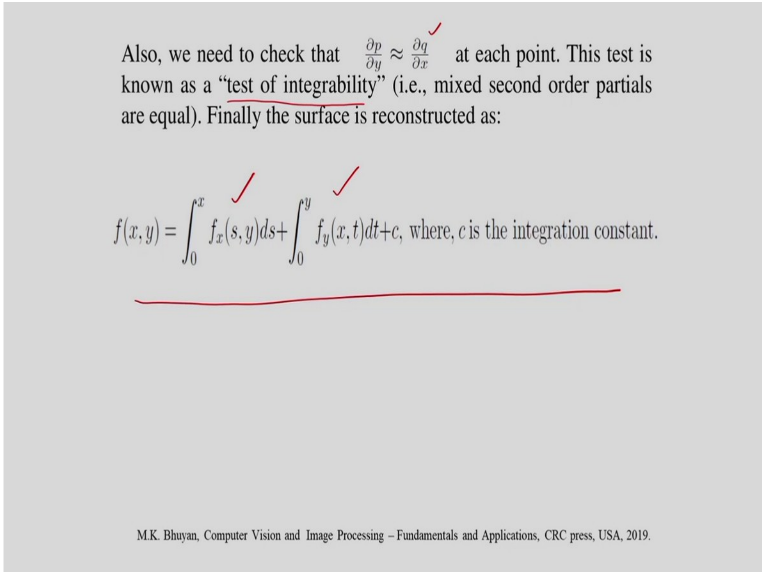

And after this we have to consider this check that is the partial derivative of p with respect to y should be approximately equal to the partial derivative of q with respect to x. This is called the test of integrability. So, this test we have to consider. The mixed second order partials are equal. And finally, I can reconstruct the surface by using this equation. So, I have to do the integration. This f x gradient is available, f y is available. And c is integration constant. So, from this I can determine f (x, y), f (x, y) is the surface I am getting.

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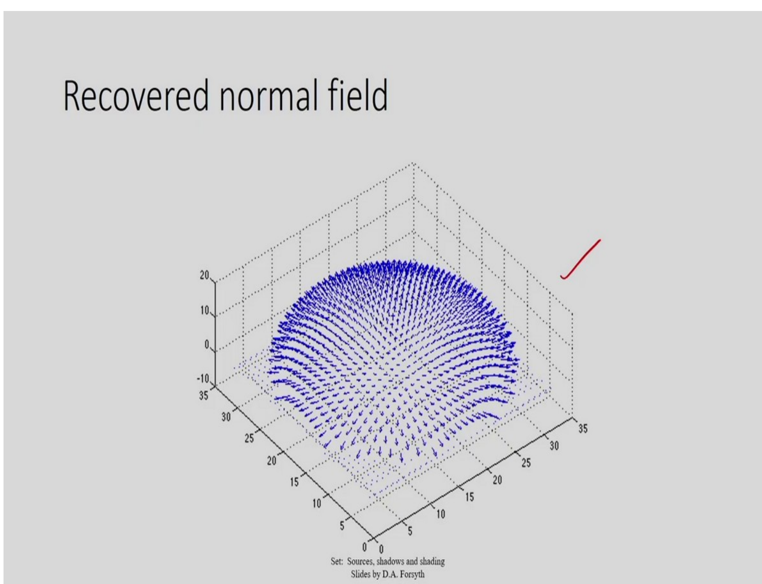

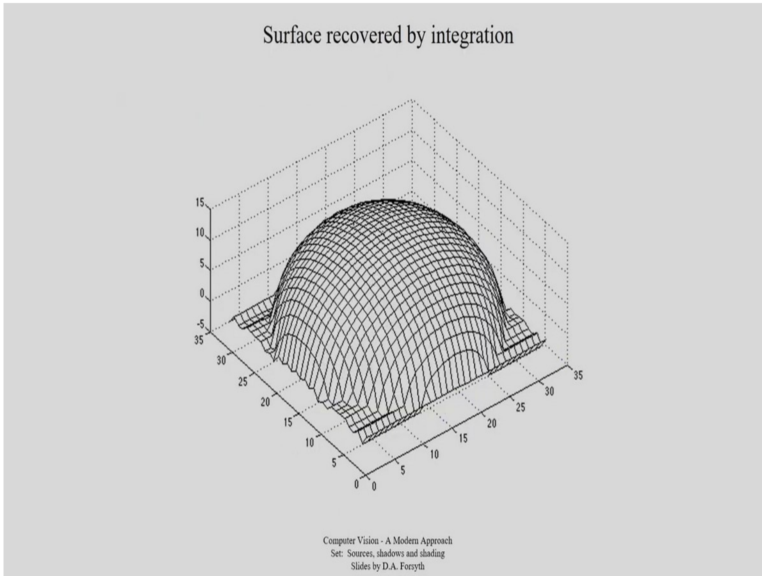

So, the normals are recovered by using these concepts. So, these are the normals, the surface normals. And from the surface normal if I do the integration, then in this case the surface is recovered. The shape of the surface is recovered by integration.

So, in this class I discussed the concept of the photometric stereo. So, before that, I discussed the concept of the shape from shading. There are many algorithms to solve this problem, but one popular algorithm is the photometric stereo. So,, in this case I am considering the number of light sources. And surface is illuminated by the light sources but one at a particular time. And I am getting number of images and from these images I want to get the surface normal. From the surface normal I can get the, the information of the shape of the surface. The surface can be reconstructed. So, this is the main concept, that the concept of the shape from shading.

In my next class I will discuss some geometric transformation like the affine transformation. So, let me stop here today. Thank you.