

**Computer Vision and Image Processing**  
**Professor Dr. M.K. Bhuyan**  
**Department of Electronics and Electrical Engineering**  
**Indian Institute of Technology, Guwahati**  
**Lecture 3**  
**Image Formation - Radiometry**

Welcome to NPTEL MOOC's course on Computer Vision and Image Processing, Fundamentals and Applications. In my last class, I discussed the concept of image processing. Image is represented by  $f(x, y)$ ;  $f(x, y)$  means the intensity at a particular point. The point is  $(x, y)$ .

What is digital image? Digital image is nothing but the 2D area of quantize intensity values and digital image processing means the manipulation of these numbers. So, in my last class, I discussed two operations; one is the changing the range of an image, and also the changing the domain of an image.

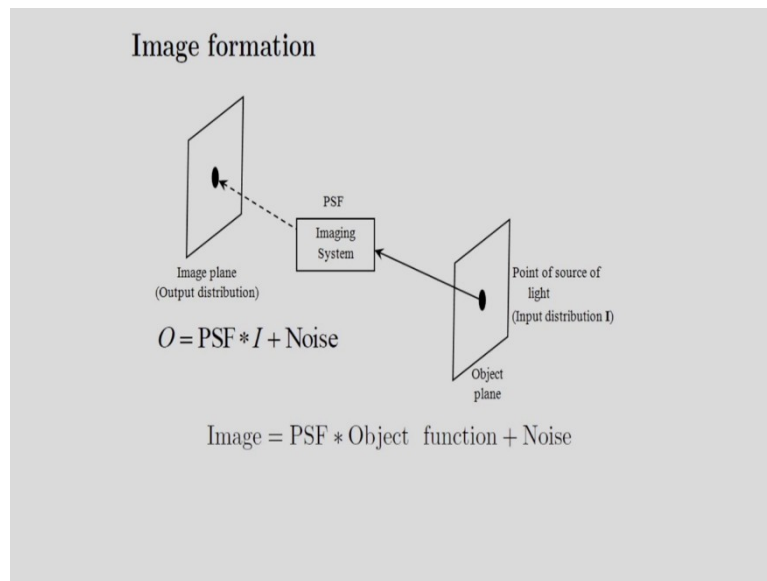
$f(x, y)$  is the image, so I can change the range of an image, and also I can change the domain of an image. Suppose, if I change the pixel values that means, I am changing the range of an image. And if I consider, suppose zooming of an image or if I do the scaling of an image or maybe if I do the rotation of an image, that means, I am changing the domain of an image.

Also, I discussed about the concept of image enhancement, image restoration, image reconstruction. So, image enhancement means to improve the visual quality of an image. So today, I am going to discuss about the image formation concept, and first I will discuss the concept of radiometry.

Radiometry means the measurement of light. So, you know that the pixel intensity value at a, in an image depends on the amount of light reflected by the surface. So that is why the measurement of light is quite important and that measurement is called the radiometry. I can also measure the sensitivity of camera, and also the sensitivity of human eye, that is called photometry. You can see the difference between photometry and the radiometry.

Now, I will discuss the concept of image formation and after this, I will discuss some radiometric parameters.

(Refer Slide Time: 02:35)

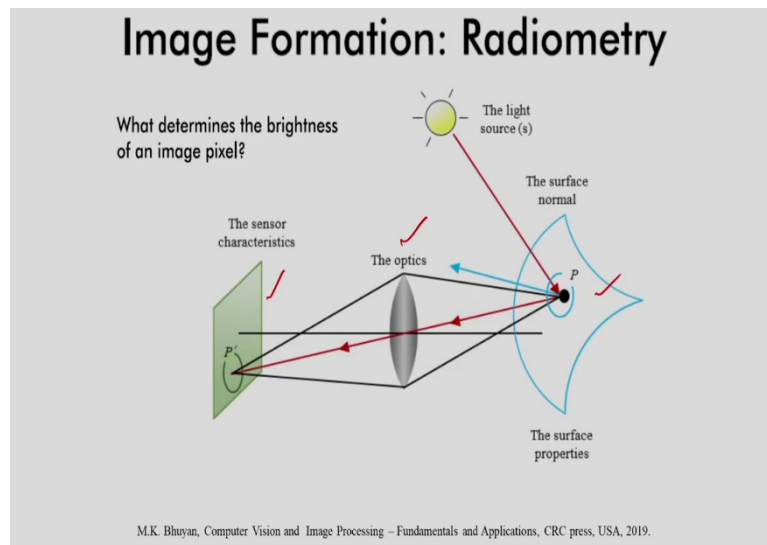


So, in my next slide, you can see I have shown one image formation principle. So, I have considered the point of light source, that is, the input light distribution, I am considering I. I am considering the imaging system; the imaging system is characterized by PSF. The PSF is called the point spread function. PSF is the point spread function and I have, I am getting image in the image plane, that is, the output distribution.

The object function is an object or a scene that is being imaged. Light, the light from the source is incident on the scene or the object surface and it is reflected back to the camera by the imaging system. The point spread function that is nothing but the impulse response of the system, and in this case, if I consider the point light source, PSF actually indicates the spreading of the object function and it is a characteristic of the imaging system; the imaging system is the camera.

A good imaging system generally has a narrow PSF and whereas a poor imaging system has a broad PSF. So, in this diagram, you can see I am getting the output image; output image is nothing but the PSF, it is convolve with the image plus noise. So, this image formation principle I have explained that PSF is quite important. So, it is the characteristics of an imaging system and I am considering the noise; the noise of the imaging system.

(Refer Slide Time: 04:11)



And in this figure, I have shown the image formation concept that is the radiometry. So, light is coming from the source and it is reflected by the surface. So, I have the optics; optics means the, it is the camera lens and I am getting the image in the sensor. The sensor converts the optical photons into electrical signal.

So, this, the pixel intensity depends on the amount of light reflected by the surface. So, that is why I have to determine the amount of light reflected by the surface and the surface property, that is called the reflectance property of the surface, it is also called the albedo of the surface. So, this measurement is quite important. The measurement of light, and also the surface property, the reflectance property of the surface. So, in this diagram, I have shown the image formation concept.

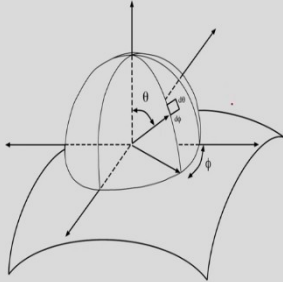
So, light is coming from the source and it is reflected by the surface and I have the optics that is the lens. And I am getting the output, that means, in the sensor, I am getting the response. So, that, the light photon is converted into electrical signal. So here, if you see, the light is coming from the source.

Now, the surface property is important. This is the surface; the surface property is called the reflectance property of the surface. Sometimes, it is called the albedo of the surface. So, light is reflected by the surface and we have the optics, and I am getting the electrical signal in the sensor. So, this is a typical image formation process.

(Refer Slide Time: 05:47)

## Image Formation: Radiometry

- Brightness of a surface:
  - how "bright" will surfaces be?
  - what is "brightness"? ✓
    - measuring light ✓
    - interactions between light and surfaces ✓
- Core idea - think about light arriving at a surface



Computer Vision - A Modern Approach  
Set: Radiometry  
Slides by D.A. Forsyth

For image formation, the concept of the radiometry; radiometry means the measurement of light, so in this case, I have to determine the brightness. That means, I have to do the measurement; the measurement of the light. And another one is the interactions between the light and the surface that I have to consider.

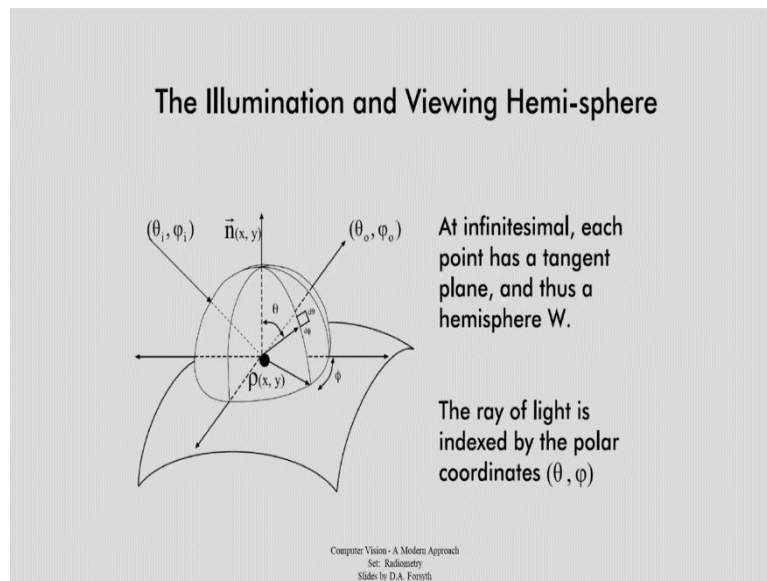
So, in my next diagram, you can see. In this case, I have shown one plane and, in this case, I am considering the spherical coordinate system. So, in the next slide, you can understand this concept.

(Refer Slide Time: 06:14)

## Radiometric Quantities

So, I am going to the next slide. So let us discuss about some radiometric quantities.

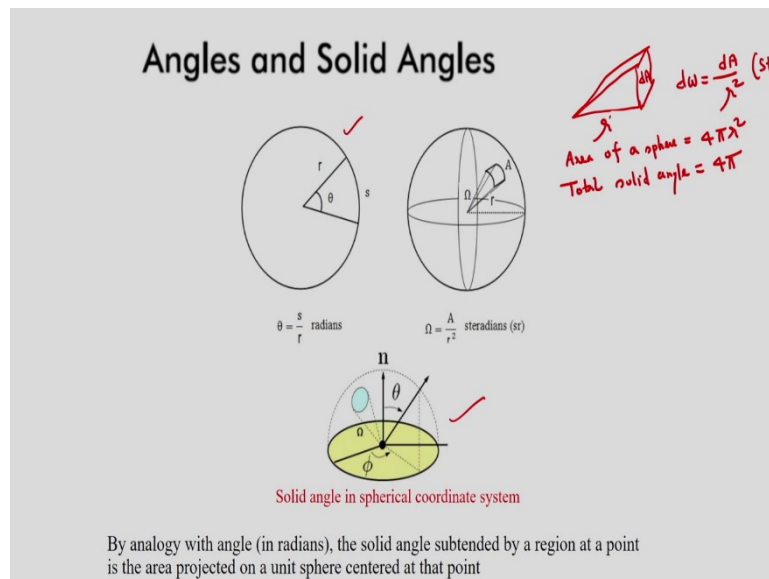
(Refer Slide Time: 06:19)



So, this diagram here, if you see here, so if I consider a point, a particular point, that point has a tangent plane. So, if I consider this point here, this is the point, this point has a tangent plane and thus a hemisphere. So, this is the hemisphere I am considering.

Now, I have shown, this is the incoming light;  $\theta_i$  and  $\varphi_i$ , that is the incident angle. And if I consider this one, this is the outgoing angle, that is, light is reflected by the surface. So, this is the incoming ray and this is the outgoing ray. So, in this case, I am considering the polar coordinates. The polar coordinate is  $\theta$  and  $\varphi$ .

(Refer Slide Time: 06:56)



Now, let us define the angles. So, in this figure, if you see, the first one is, this first one is the plane angle. So, you know that what is the definition of the plane angle; the  $\theta = \frac{s}{r}$  radian. In

the second case, I am considering the concept of the solid angle. Solid angle is  $\frac{A}{r^2}$  steradian, so I will show you.

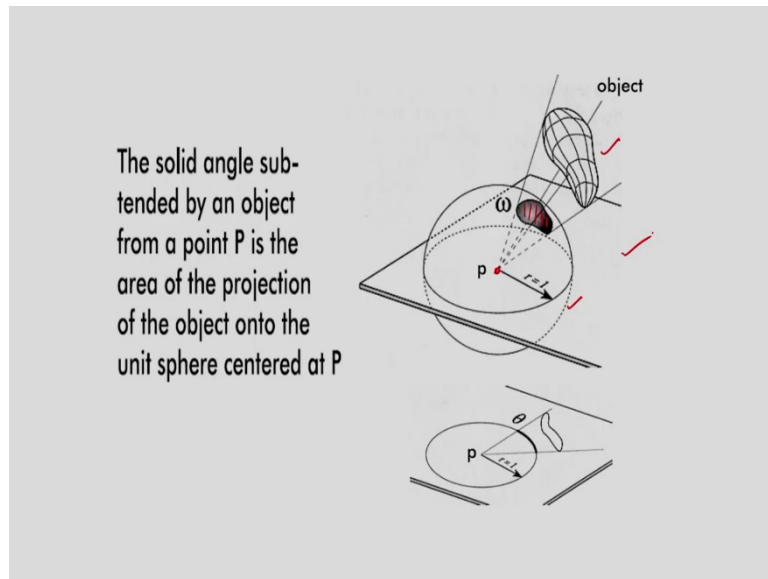
So what is the definition of the solid angle. So suppose, if I consider this, the area is something like this. The idea is suppose  $dA$ ;  $dA$  is the area and this is our, so the solid angle

is defined like this.  $d\omega = \frac{dA}{r^2}$ . And the unit is a steradian, unit is steradians.

So, you know the area of the, area of a sphere is equal to  $4\pi r^2$ , this is the area of a sphere. And that why, what will be a total solid angle? Total solid angle, total solid angle will be equal to  $4\pi$  steradian. Total solid angle will be equal to  $4\pi$  steradian. Area of a sphere is  $4\pi r^2$ . So total solid angle will be  $4\pi$ .

So in this case, I have shown this diagram. If you see here, the solid angle in spherical coordinate system. So the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point. So this definition I can see in the next slide, what is the meaning of this.

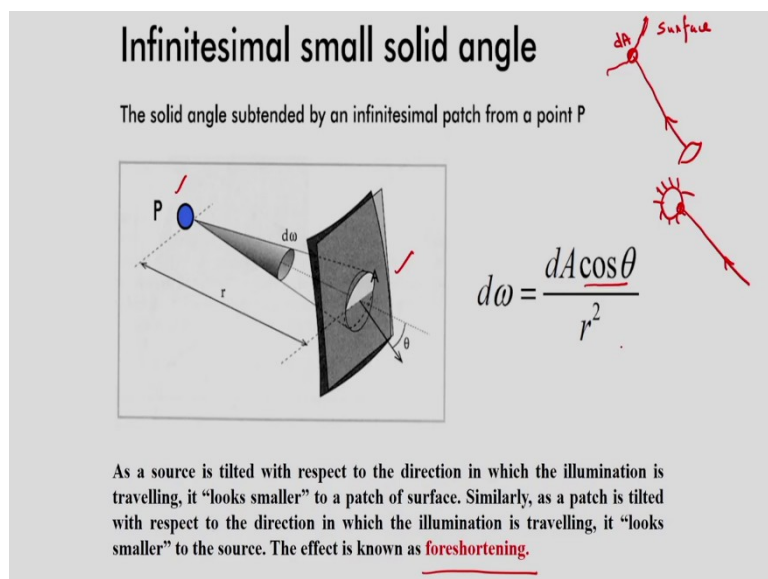
(Refer Slide Time: 08:49)



So, the solid angle subtended by an object from a point p is I am considering and this is the object. And I am considering 1-unit sphere. The radius is equal to 1 and I am considering the projection of this; projection of the objects onto the unit sphere centered at the point P.

So solid angle, you can define like this. Solid angle subtended by an object from a particular point P is the area of the projection of the object onto the unit sphere centered at the point P. So, this is the definition of the solid angle. The definition of the solid angle is very important because based on the solid angle concept, you can define radiometric quantities.

(Refer Slide Time: 09:36)



And one important point is here. A source is tilted with respect to the direction in which the illumination is traveling. It looks smaller to a patch of surface. Similarly, as a patch is tilted

with respect to the direction in which the illumination is traveling, it looks smaller to the source. This effect is called a foreshortening effect.

So, suppose if I consider, suppose a surface is something like this surface. So, if I see from this side, suppose I am observing from the side, since it is tilted with respect to my observation, so it is my camera suppose, then this surface area, suppose this area, small area I am considering the  $dA$ , it looks smaller. And similarly, if I consider one light source, suppose is a light source and light is traveling suppose in this direction, if I see from this side, then in this case, if I consider this small area, then this looks smaller. So, this is called a foreshortening effect.

So, to consider the foreshortening effect, I am considering this factor,  $\cos \theta$  I am considering. The  $\cos \theta$  is basically the foreshortening factor. So, in this diagram, you can see what is the effective area. The effective area is  $dA \cos \theta$ , that is the actual area. If you see from the point,

the point is P and this is the surface is this. So that is why the solid angle will be  $\frac{dA \cos \theta}{r^2}$

steradian. So, this foreshortening effect is very important.



(Refer Slide Time: 11:11)

$$\text{Radiant Intensity, } I = \frac{d\phi}{d\Omega} \quad \checkmark \quad \text{optical flux} = \phi \text{ (watt)}$$

$$\text{Irradiance, } E = \frac{d\phi}{dA}$$

$$\text{Radiance, } L = \frac{d^2\phi}{dA \cos\theta d\Omega}$$

Surface  $dA$       Source  
 Radiant Intensity =  $\frac{d\phi}{d\Omega}$   
 (radiance)  $E = \frac{d\phi}{dA} \left(\frac{W}{m^2}\right)$   

$$\text{Radiance} = L = \frac{dE}{d\Omega}$$

$$= \frac{\frac{d\phi}{dA}}{d\Omega}$$

$$= \frac{d^2\phi}{dA \cos\theta d\Omega}$$

Now, let us define some radiometric quantities, the first quantity is radiant intensity. Radiant intensity is  $I = \frac{d\phi}{d\omega}$ . So, what is the meaning of this? The meaning of this is suppose I am considering optical flux, optical flux is equal to  $\phi$  that is the unit is watt.

Now, let us consider this diagram. Suppose this is a surface, one surface and this is the light source. This is the light source I am considering. Light is going from the source and it is reflected by the surface. So, this is the surface or surface or the object I am considering and the light is reflected from here.

So, based on this, what is the definition of the radiant intensity? Radiant intensity, radiant intensity of the source is equal to the flux, suppose,  $\frac{d\phi}{d\omega}$ . So corresponding to this source, the light source, the radiant intensity is  $d\phi$ , the amount of flux per unit solid angle. So, this, this is the corresponding to the source, I am defining the radiant intensity.

After this, the light is falling on the surface. So in this point I can determine the irradiance, that is the illumination, I can determine. Irradiance means the illumination. So what is the definition of the irradiance? Irradiance, I can determine, irradiance means the illumination. Irradiance is equal to  $E$  equal to  $d\phi$ , the amount of flux on this particular area there.

So to suppose this area is  $dA$ , this area is  $dA$ . So,  $\frac{d\phi}{dA}$  that is the illumination. What will be the unit? Unit will be watt per meter square. So this is the unit; watt per meter square. After this, the light is reflected the light is reflected by the surface. So now, I consider it as a source.

Now, again, I want to determine the radiance, what is the definition of the radiance. So

radiance is equal to,  $L$  is equal to  $\frac{dE}{d\omega}$ . So that is equal to, because if I consider it is a source,

then in this case, it would be  $\frac{dE}{d\omega}$ . So if I put this value,  $\frac{dE}{d\omega}$ , so  $dt$  is  $\frac{d^2\phi}{d\omega}$ .

So, this is our definition of the radiance. From the irradiance, you can determine the radiance.

And if I consider the foreshortening effect, then in this case, I have to consider  $d^2\phi \, dA \cos\theta$  and I have to consider the  $d\omega$  also;  $d\omega$  here;  $d\omega \, dA \cos\theta$ .  $dA \cos\theta$ , I am considering because of the foreshortening effect;  $dA \cos\theta$ .

So these are the basic parameters. First I have to understand the concept of the solid angle and after this, I have defined the radian intensity, after this I have defined the irradiance; irradiance means the illumination at a particular point and after this, I have defined radiance. So, these parameters I have to understand first; these are the parameters for the measurement of light.

(Refer Slide Time: 14:47)

## Radiance

- Used to measure the distribution of light in space
- Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian ( $\text{wm}^{-2}\text{sr}^{-1}$ ) ✓
- In a vacuum, radiance leaving  $p$  in the direction of  $q$  is the same as radiance arriving at  $q$  from  $p$

$$\text{Radiance, } L = \frac{d^2\phi}{dA \cos\theta d\Omega} \quad \checkmark$$

$$L(\underline{x}, \underline{\theta}, \underline{\phi}) \quad \checkmark$$

So, again, you see what is the radiance. Radiance measure the distribution of light in space; radiant power per unit foreshortened area per unit solid angle. That is the definition of the radiance. And already I have discussed what is unit. Unit is watt meter minus 2 steradian minus 1.

So, in a vacuum, radiance leaving a particular point p in the direction of q is the same as the radiance arriving at q from p, because there is no loss of power. So, this is one important concept. So already I have defined the radiance.

Now, in case of the spherical coordinate, I can consider a radiance like this. So, L is the radiance and  $\theta$  and  $\varphi$ , I am considering the angles of the spherical coordinate system.

(Refer Slide Time: 15:32)

Radiance is constant along straight lines

- Power 1->2, leaving 1:
 
$$L(x_1, \theta, \varphi) (dA_1 \cos \theta_1) \left( \frac{dA_2 \cos \theta_2}{r^2} \right)$$
- Power 2->1, arriving at 2:
 
$$L(x_2, \theta, \varphi) (dA_2 \cos \theta_2) \left( \frac{dA_1 \cos \theta_1}{r^2} \right)$$
- But these must be the same, so that the two radiances are equal

Radiance is constant along a straight line. So, in this diagram, if you see, this is suppose point 1 and this is point 2. So, power from 1 to 2, this power leaving 1, so I can determine the power, this is the power. So, first one is the radiance and after this, I am considering the foreshortening area and this is the solid angle.

So, if I multiply this, then I will be getting the power from 1 to 2 and the next one is power from 2 to 1. So, this should be, this would be actually it would be power from 2 to 1. It should be power from 2 to 1 arriving at 2.

So, what is the power arriving at 2? So, the, similarly, I can calculate the power from arriving at two. So, I can calculate this and these two powers will be equal because there is no loss of

radiance, there is no loss of radiance. Radiance is constant along a, along the straight line if I consider vacuum as the medium.

(Refer Slide Time: 16:31)

### Irradiance

- How much light is arriving at a surface?
- A surface experiencing radiance  $L(x, \theta, \phi)$  coming in from  $d\omega$  experiences irradiance  $L(x, \theta, \phi) \cos \theta d\omega$
- Total power arriving at the surface is given by adding irradiance over all incoming angles. Total power is

$$\int_{\Omega} L(x, \theta, \phi) \cos \theta d\omega$$

Now, what is the definition of an irradiance? So already I have defined the irradiance. Irradiance means illumination. So, a surface experience radiance coming in from a solid angle, the solid angle is  $d\omega$ , then the irradiance will be radiance  $\cos \theta d\omega$ . That is the irradiance.

And what will be the total power arriving at the particular surface? Then in this case, I have to calculate by adding irradiance over all incoming angles. So in this case, I am considering all the angles I am considering. This is the solid angle, total solid angle I am considering. I am calculating the total power arriving at the surface, so that I can determine.

(Refer Slide Time: 17:12)

- In spherical coordinate system, radiance at a point  $P$  is represented as  $L(P, \theta, \phi)$ .
- If a small surface patch  $dA$  is illuminated by radiance  $L_i(P, \theta_i, \phi_i)$  coming from a region with a solid angle  $d\Omega$  at angles  $(\theta_i, \phi_i)$ , then irradiance at the surface would be  $L_i(P, \theta_i, \phi_i) \cos \theta_i d\Omega$ .
- So, irradiance is obtained by multiplying the radiance by foreshortening factor  $\cos \theta$  and the solid angle  $d\Omega$ . To get irradiance, the radiance is integrated over the entire hemisphere.

$$E(P) = \int_{\Omega} L(P, \theta_i, \phi_i) \cos \theta_i d\Omega$$
$$= \int_0^{2\pi} \int_0^{\pi/2} L(P, \theta_i, \phi_i) \cos \theta_i \sin \theta_i d\theta d\phi$$
$$d\Omega = \sin \theta_i d\theta d\phi$$

So, in spherical coordinate system, radiance at a point P is represented by  $L(P, \theta, \varphi)$ . So, this is the radiance  $L(P, \theta, \varphi)$ , and if a small surface patch  $dA$  is illuminated by radiance, the incoming radiance is this and the solid angle is  $d\Omega$ ; the incident angle is this. Then in this case, what will be the irradiance? The irradiance is already I explained, this is the irradiance in terms of radiance.

And in this case, the  $\cos\theta$  already I have explained that is the foreshortening factor. So, to get irradiance, the radiance is integrated over the entire hemisphere. So, here, you see I am calculating the irradiance. So the, I am calculating the irradiance.

So I am just integrating the radiance for the total solid angle. The solid angle is  $d\Omega$ , so this is the radiance.  $\cos\theta$ , I am considering because of the foreshortening factors, so I am calculating the irradiance. The radiance is integrated over the entire hemisphere. So that is why for the entire hemisphere, the integration is from 0 to twice pi, 0 to pi by 2, then in this case I can calculate this one. This  $d\Omega = \sin\theta d\theta d\varphi$ .

(Refer Slide Time: 18:29)

### To sum up

**Radiance ( $L$ ): energy carried by a ray**

- Power per unit area perpendicular to the direction of travel, per unit solid angle
- Units: Watts per square meter per steradian ( $\text{W m}^{-2} \text{sr}^{-1}$ ) ✓
- PROPERTY: Radiance is constant along straight lines (in vacuum)

**Irradiance ( $E$ ): energy arriving at a surface**

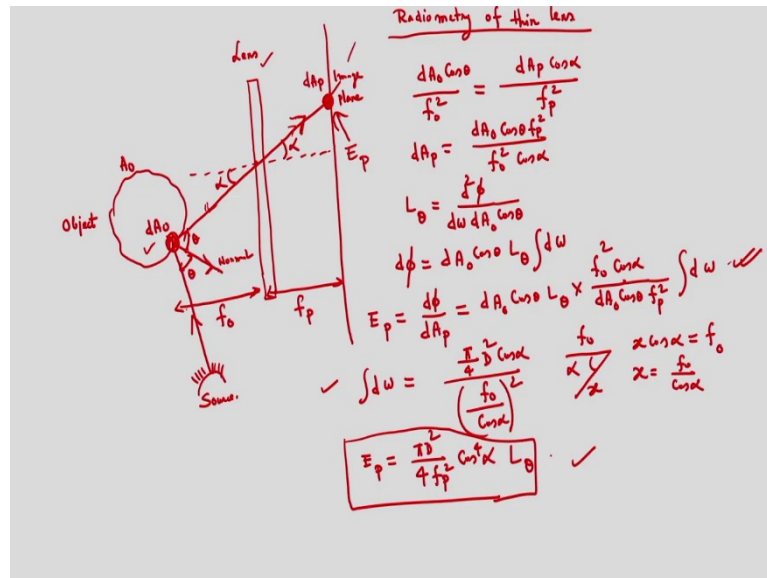
- Incident power in a given direction per unit area
- Units:  $\text{W m}^{-2}$  ✓

So, finally, the summary is the radiance means the energy carried by a ray. So power per unit area perpendicular to the direction of the travel per unit solid angle. So that is the definition of a radiance. So, what is the unit of the radiance? The unit is watt per meter square steradian. And radiance is constant along the straight line, that is in vacuum.

And irradiance, energy arriving at a surface; incident power in a given direction per unit area. So, unit is watt per meter square. So, this is the definition of the irradiance. Now, next, I am

considering one important derivation, that is, the derivation for radiometry of thin lenses; radiometry of thin lenses. So, let us consider one diagram.

(Refer Slide Time: 19:15)



Suppose I am considering one surface like this and I am considering and this is the lens of the camera, and suppose this is the image plane. This is the image plane. This is my lens of the camera. So here, what I am considering? The concept is radiometry of thin lens. So, I am going to explain this concept, radiometry of thin lens.

So, this is a surface. So, this area I am considering suppose, this small area I am considering the surface. This is the object I am considering, object is this. This small area I am considering, this small area is  $dA_0$ ; the total area is  $A_0$  suppose. The total area is  $A_0$  and at this point, so this is the surface, suppose surface normal; the surface normal is this normal to the surface.

So, this is my incident ray and this is my, the ray it will be something like this. So, this angle is  $\theta$  and this angle is also  $\theta$ . This will be  $\theta$  and if I draw like this, so this angle will be  $\alpha$ , this angle will be  $\alpha$ . In this case, you see the  $\alpha$  is not equal to  $\theta$  this is the normal to the surface.

So, this distance, I can consider, this distance I can consider as  $f_p$  suppose. The distance from the lens to the image plane. And suppose this distance, distance from the lens to the, this object point is I can consider as  $f_0$ ; I can consider this distance is  $f_0$ . So I am considering the radiometry of that thin lens.

Now, the respective solid angle seen from the lens, so this is my lens. So I, if I see from the side, I can see one solid angle. If I see, if I see from this side, I will be getting one solid angle and, in this case, also I am getting one solid angle. And suppose this area I am considering, this area I am considering  $dA_p$  small area I am considering in the image plane,  $dA_p$ .

So light is coming from the source, light source and it is incident on the surface and it is reflected back and this is the camera. This is the camera and I am getting the image in the image plane. So light is coming from the source, it is reflected by the surface and it is going to like this.

Now, if you see here, there are respective solid angles seen by the, seen from the lens. So, if I seen from the lens, so I have two solid angle I can see in this direction or in this direction, the both the direction I can see. So, this solid angle will be equal. So, in this case, if I consider this as one,  $dA_0$ ; solid angle, then it  $\cos \theta f_0^2$  is equal to  $dA_p \cos \alpha f_p^2$ .

So respective solid angle seen from the lens; so I am calculating. And from this, you can calculate  $dA_p$ .  $dA_p$  you can calculate;  $dA_p$  will be equal to simple  $dA_0 \cos \theta f_p^2$  is and  $f_0^2 \cos \alpha$ . After this, I want to determine the radiance at this point.

So, what is the radiance?  $L_\theta$  will be equal to  $d^2 \varphi$  as per the definition of radiance. So  $d\omega dA_0 \cos \theta$ . So from this, I can determine I have to do the integration. So,  $d\varphi$  is equal to  $d\varphi$  i have to determine.  $d\varphi$  is equal to  $dA_0 \cos \theta L_\theta$  and  $d\omega$ ; that is the total solid angle.

So, after this, the light is reflected by the surface. So, I am getting the irradiance at this point. So what will be the irradiance? The irradiance at this point  $E_p$  is equal to  $d\varphi$  divided by  $dA_p$ , that I can determine. So in this case, I have to put this value;  $dA_0 \cos \theta L_\theta$  and into multiplied by  $f_0^2$ , this below I am putting  $f_0^2 \cos \alpha dA_0 \cos \theta f_p^2$  and it is  $d\omega$ ,  $d\omega$

So what is the solid angle, total solid angle? The solid angle of the lens, I can consider  $d\omega$ , is equal to, first, I have to consider the area; area of the lens will be  $\pi d^2 \cos \alpha$ . And I have to consider the distance. So distance will be  $f_0 \cos \alpha$  whole square.

So in this case, I am considering suppose this is  $f_0$  and suppose this is  $x$ , this distance  $x$ , so this is suppose  $\alpha$ , this angle. So I am considering  $x \cos \alpha$  is equal to  $f_0$ . So from this, what is  $x$ ?  $x$  is nothing but,  $x$  is nothing but  $f_0$  divided by  $\cos \alpha$ ;  $\cos \alpha$ , I am getting.



So, this is the total solid angle I am determining. So, after putting this value in this equation, if I put this value in this equation, then in this case, I will be getting  $E_p$ , that is, the  $E_p$  means irradiance. I am calculating the irradiance at this point. So,  $E_p$  will be just put the value

$$\frac{\pi d^4}{4f_p^2} \cos^4 \alpha L \theta. \text{ So, this expression I am getting.}$$

So, this is one important expression;  $\frac{\pi d^4}{4f_p^2} \cos^4 \alpha L \theta$ . So, this expression I am getting. That is irradiance I am determining.

(Refer Slide Time: 26:59)

The area of the lens is  $\frac{\pi D^2}{4} \cos \alpha$ , and the solid angle of the lens is given by (foreshortening effect is considered):

$$\int d\Omega = \frac{\frac{\pi D^2 \cos \alpha}{4}}{\left(\frac{f_0}{\cos \alpha}\right)^2} \quad \checkmark$$

So, the irradiance  $E_p$  is given by

$$\begin{aligned} E_p &= \frac{d\phi}{dA_p} \\ &= dA_0 \cos \theta L_\theta \times \frac{f_0^2 \cos \alpha}{dA_0 \cos \theta f_p^2} \times \int d\Omega \\ &= \left(\frac{f_0}{f_p}\right)^2 L_\theta \cos \alpha \int d\Omega \\ &= \left(\frac{f_0}{f_p}\right)^2 L_\theta \cos \alpha \frac{\pi D^2 \cos \alpha \cos^2 \alpha}{4 f_0^2} \\ E_p &= \frac{\pi D^2}{4 f_p^2} \cos^4 \alpha L \theta \quad \checkmark \end{aligned}$$

So in this case, if you see here, in this expression I am showing the same thing. So first, I am considering the equating respective solid angles seen from the lens. So already, I've explained this one and after this, I am calculating  $dA_p$ .

After this, I am calculating the radiance at this point I am calculating. Radiance I am calculating, so  $d\phi$  will be this one. Now, what will be the solid angle? The solid angle already I have explained, the solid angle is this. These are the  $\omega$ , solid angle is this.

And finally, I am getting  $E_p$ .  $E_p$  means irradiance. Irradiance I'm getting this one. So this is

one important expression. So  $E_p$  will be  $\frac{\pi d^4}{4f_p^2} \cos^4 \alpha L \theta$ . So I am getting this one.

(Refer Slide Time: 27:40)

$$E_{total} = \frac{\pi D^2}{4f_p^2} \cos^4 \alpha L$$

$$L = L_{\theta_1} + L_{\theta_2} + \dots, \text{ and } E_{total} = E_{p_1} + E_{p_2} + \dots$$

- So, image irradiance is proportional to the scene radiance, i.e., gray values of an image depend on  $L$ .
- Also, the irradiance is proportional to the area of the lens. It is inversely proportional to the distance between its centre and the image plane.
- The term  $\cos^4 \alpha$  indicates a systematic lens optical defect called **vignetting**.
- The vignetting effect means that the optical rays with larger span-off angle  $\alpha$  are attenuated more, and hence, the pixels closer to the image borders will be darker.
- The vignetting effect can be compensated by a radiometrically calibrated lens.

Now, in this case, what will be the  $E_{total}$ ; that means, total irradiance, what will be the  $E_{total}$ ?  $E_{total}$  is this? So in this case, what I am considering. I am considering different angles; theta 1, theta 2; that means radiance from all these angles I am considering. And after this, I am calculating  $E_{p1}$  for this  $L_{\theta_1}$ ,  $E_{p2}$  for  $L_{\theta_2}$ , like this I am calculating. And I am calculating  $E_{total}$ .  $E_{total}$  means total irradiance at this point.

So like this I can calculate. So this expression already I told you, this is the important expression, that is, irradiance at the image plane. This image irradiance is proportional to the scene irradiance. If you see this expression, the image irradiance is proportional to the scene irradiance. That means the grayscale value of an image, the pixel value of an image depends on  $L$ .

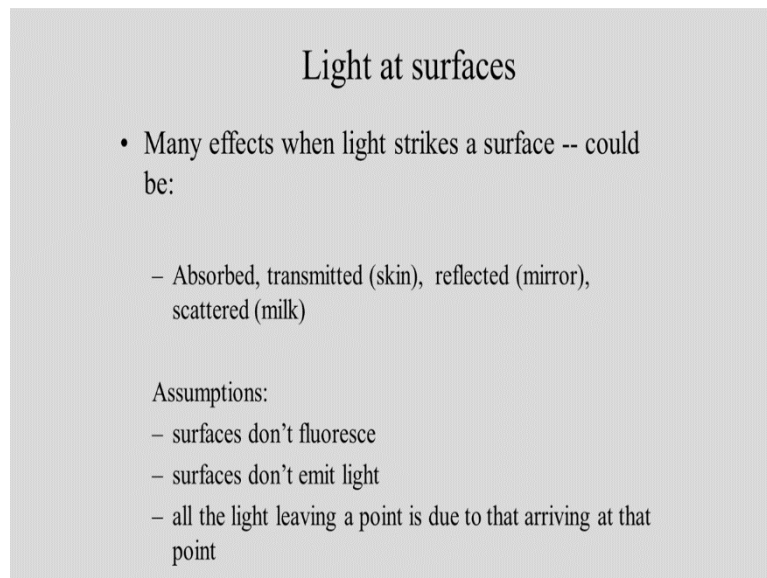
Also, the irradiance is proportional to the area of the lens. So irradiance is proportional to area of the lens. Area of the lens is  $\frac{\pi d^4}{4}$ , and it is inversely proportional to the distance between the center and the image plane. The term  $\cos^4 \alpha$  is very important.

So, it indicates a systematic lens optical defect. This is called a vignetting. So what is the vignetting effect? So optical rays with larger span of angle alpha, if the angle alpha is more, that are attenuated more. Hence, the pixels closer to the image border will be darker. So this effect is called the vignetting effect and the vignetting effect can be compensated by a radiometrically-calibrated lens.

So, this is one important observation, that is, we have to consider the radiometry of a thin lens. And in this case, I have considered that you can calculate  $E_{\text{total}}$  that is the irradiance you can calculate and that is equal to the area of the lens and also it is inversely proportional to the distance between the center and in the image plane. And I have considered another term  $\cos^4 \alpha$ , and after this,  $L$ ;  $L$  is the irradiance.

This  $\cos^4 \alpha$  is very important because it corresponds to the systematic defects of the lens that is called the vignetting. So because of this, if  $\alpha$  is more, then in this case, what will happen? The in the border area of the image, that the pixel will be darker as compared to the central portion of the image.

(Refer Slide Time: 30:09)



**Light at surfaces**

- Many effects when light strikes a surface -- could be:
  - Absorbed, transmitted (skin), reflected (mirror), scattered (milk)

Assumptions:

- surfaces don't fluoresce
- surfaces don't emit light
- all the light leaving a point is due to that arriving at that point

Now, let us consider the light at surfaces. The many effects when light strike at the surface, the light may be absorbed or it may be transmitted, it may be reflected, something like the mirror, or it may be scattered, the example is milk.

So some assumptions like surface does not fluoresce, surface does not emit light, and all the light leaving a point is due to the, due to that arriving at that point. So we have considered these assumptions and light maybe something like the absorbed, transmitted, reflected, and scattered.

(Refer Slide Time: 30:44)

**Bidirectional Reflectance Distribution Function**  
**BRDF**

The ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{dL(\theta_o, \phi_o)}{dE(\theta_i, \phi_i)}$$
$$\rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(P, \theta_o, \phi_o)}{L_i(P, \theta_i, \phi_i) \cos \theta_i d\omega} \text{ (sr}^{-1}\text{)}$$

All the light leaving a point is due to that arriving at that point

Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another

So, to represent the characteristics of a surface, I am defining one radiometric quantity. That definition of the radiometric quantity is BRDF, the bidirectional reflectance distribution function. So, I am defining one, the parameter and that is bidirectional reflectance distribution function. So, this is basically to see the property of a surface.

So, what is the definition of the BRDF? The ratio of radiance in outgoing direction, so this is the radiance in the outgoing direction, to the incident irradiance. So incident irradiance is this because light is reflected by the surface. So, this is your, this is the incident irradiance, the incident irradiance and this is the radiance in the outgoing direction.

So, in this case, I am considering the spherical coordinate systems, this  $\theta_i$  and  $\phi_i$  that corresponds to incident angle, and this  $\theta_o$  and  $\phi_o$  is corresponds to the outgoing direction. So, this is I am writing in the spherical coordinate systems  $L_o$  divided by  $L_i \cos \theta_i$ . So  $\cos \theta$  I am considering because of the foreshortening effect.

(Refer Slide Time: 31:56)

**BRDF**

- Radiance leaving a surface in a particular direction:

$$L_o(P, \theta_o, \phi_o) = \rho_{brd}(\underline{x}, \theta_o, \phi_o, \theta_i, \phi_i) L_i(\underline{x}, \theta_i, \phi_i) \cos \theta_i d\omega_i$$

- Radiance leaving a surface due to its irradiance:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{brd}(\underline{x}, \theta_o, \phi_o, \theta_i, \phi_i) L_i(\underline{x}, \theta_i, \phi_i) \cos \theta_i d\omega_i$$

If we consider discrete light sources (a number of point light sources), the integral is replaced with a summation

$$L_o(P, \theta_o, \phi_o) = \sum_i \rho_{brd}(\theta_o, \phi_o, \theta_i, \phi_i) L_i(P, \theta_i, \phi_i) \cos \theta_i$$

Now, the radiance leaving a surface in a particular direction, that you can determine from the BRDF. So this radiance you can calculate; radiance leaving a surface in a particular direction. So this BRDF, already we have defined, and this is your incoming radiance;  $\cos \theta$  is the foreshortening factor,  $d\omega$  is the solid angle. So from this, you can determine radiance leaving a surface in a particular direction.

And radiance leaving a surface due to its irradiance, then in this case, I have to consider all the contributions from every incoming direction. So in this case, I can calculate the radiance leaving a surface due to irradiance I can determine this one. So in this case, I am considering, all the angles I am considering because that is where I am considering the integration.

The integration, I am considering because I am considering the contribution from every incoming directions and if I consider the discrete light source, a number of point light sources, then the integral is replaced by summation. So this integral is replaced by the summation. So this is the BRDF.

(Refer Slide Time: 32:58)

### Suppressing Angles - Radiosity

- In many practical cases, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface ( $\text{Wm}^{-2}$ ) ✓
  - note that this is independent of the direction
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions, multiplying by a cosine because this is per unit area not per unit foreshortened area

Diffuse ✓  
Specular ✓

$$B(\underline{x}) = \int_{\Omega} L_o(x, \theta, \phi) \cos \theta d\omega$$

And another important definition is radiosity. So suppose, if I consider some surfaces like cotton cloth; in cotton cloth, the reflected light is not dependent on angle. That means, the radiance leaving the surface is independent of angle. That is one important point.

Radiance leaving the surface is independent of the angle, so appropriate radiometric unit is radiosity. So what is the definition of the radiosity? Total power leaving a point on the surface per unit area on the surface. So that is the unit watt per meter square.

And this is independent of the direction because in this case, I am considering the cotton cloth. So, this type of surface is called the diffused surface. I will get two types of surface; diffused or the Lambertian surface, I will discuss about the Lambertian surface. Another one is the specular surface. Specular means the mirror-like surface. So one is the Lambertian or the diffuse surface another one is the specular surface. So I will explain what is the diffuse surface, what is a specular surface.

So, radiosity, you can determine from radiance, that is, if you see this expression, I am determining the radiosity. Radiosity means total power leaving a point on the surface per unit area on the surface. That means, sum the radiance leaving the surface over all the exit angles. So that is why I am taking the integration and I am considering all the exit angles, total solid angle I am considering, so I can determine the radiosity from radiance.

(Refer Slide Time: 34:24)

### Radiosity

- Radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth) ✓

$$\begin{aligned} B(\underline{x}) &= \int_{\Omega} L_o(\underline{x}, \theta, \varphi) \cos \theta d\omega \quad \checkmark \\ &= L_o(\underline{x}) \int_{\Omega} \cos \theta d\omega \\ &= L_o(\underline{x}) \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos \theta \sin \theta d\varphi d\theta \\ &= \pi L_o(\underline{x}) \end{aligned}$$

So if you see this, here, expression, radiosity of a surface whose radiance is independent of the angle, that is, something like the cotton cloth, that is, the diffuse surface. In case of the diffused surface, radiance leaving the surface is independent of angle. This is the definition of a diffused surface, but if I consider a mirror-like surface, it is not the case.

So here, you see, I am calculating the radiosity, the radiosity. And since this is independent of angle, so this will be constant, so I am taking it out from the integration. So this is  $\cos \theta$  and I am considering these angle because it is a solid angle I am considering. And in this spherical coordinate system, integration is from 0 to pi by 2, 0 to twice pi. So I am calculating the radiosity for the diffuse surface.

(Refer Slide Time: 35:12)

### Directional hemispheric reflectance

- For many surfaces, light leaving the surface is largely independent of exit angle
- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - Range is 0-1, note that DHR varies with incoming direction

$$\rho_{dh}(\theta_r, \varphi_r) = \frac{\int_{\Omega} L_o(x, \theta_o, \varphi_o) \cos \theta_o d\omega_o}{L_i(x, \theta_i, \varphi_i) \cos \theta_i d\omega_i}$$

$$= \int_{\Omega} \rho_{bd}(x, \theta_o, \varphi_o, \theta_r, \varphi_r) \cos \theta_o d\omega_o$$

Another important definition is directional hemispheric reflectance. That is also very important definition that many surfaces, for many surfaces, light leaving the surface is largely independent of exit angle. So for this, I am defining one parameter, that parameter is directional hemispheric reflectance.

The range is 0 to 1. So what is the definition of the directional hemispheric reflectance? The fraction of the incident irradiance in a given direction that is reflected by surface. So in this case, this is the incident irradiance in a particular direction that is reflected by the surface for all the angles I am considering.

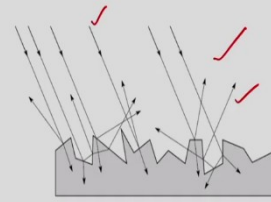
So this is an incident irradiance in a particular direction that I am considering and the light is reflected by the surface in all directions. And so that is why I am considering the integration. And already I have defined a BRDF, so this BRDF, just I am putting the BRDF here,  $\cos \theta d\omega$ . So you see the relationship between the directional hemispheric reflectance and the BRDF.



(Refer Slide Time: 36:10)

### Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of illumination direction too
  - cotton cloth, matte paper, etc.
- For such surfaces, radiance leaving the surface is independent of angle
- These surfaces are called **Lambertian surfaces** (same Lambert) or **ideal diffuse surfaces**
- DHR is often called diffuse reflectance, or **albedo**  $\rho_d$
- For a Lambertian surface, BRDF is independent of angle, too.



→ Radiance leaving the surface is independent of angle

The diagram shows a rough, grey surface with several incident rays (black arrows) coming from the top left. These rays hit the surface and are reflected in various directions (black arrows) across the surface. Red checkmarks are placed above the reflected rays to indicate that the radiance is independent of the angle of reflection. A text box to the right of the diagram states: '→ Radiance leaving the surface is independent of angle'.

So I have shown the diffuse surface that is called the Lambertian surface. Lambertian surface or the diffuse, ideal diffuse surface, that means, the radiance leaving the surface is independent of angle. So if you see this diagram, radiance leaving the surface is independent of the angle. In different angles, the light is reflected back. So these are the incoming lights and these are the reflected lights. So radiance leaving the surface is independent of angle.

And in case of the diffuse surface, that directional hemispheric reflectance is called the albedo. So this definition is very important, albedo. Albedo means the reflectance property of the surface. If I considered a diffuse surface, then it is called a diffuse reflectance that is the albedo.

(Refer Slide Time: 36:55)

$$\begin{aligned} \text{For a Lambertian surface } \rho_{brd}(\theta_o, \varphi_o, \theta_i, \varphi_i) &= \rho \\ \rho_d &= \int_{\Omega} \rho_{brd}(\theta_o, \varphi_o, \theta_i, \varphi_i) \cos \theta_o d\omega_o \\ &= \int_{\Omega} \rho \cos \theta_o d\omega_o \\ &= \rho \int_0^{\pi} \int_0^{2\pi} \cos \theta_o \sin \theta_o d\theta_o d\varphi_o \\ &= \pi \rho = \text{Albedo} \end{aligned}$$

In general,  $\rho_{brd} = \frac{\rho_d}{\pi}$

So, for Lambertian surface, the BRDF is equal to because it is independent of direction, so it is constant, it does not depend on direction. And in this case, I can consider the directional hemispheric reflectance.

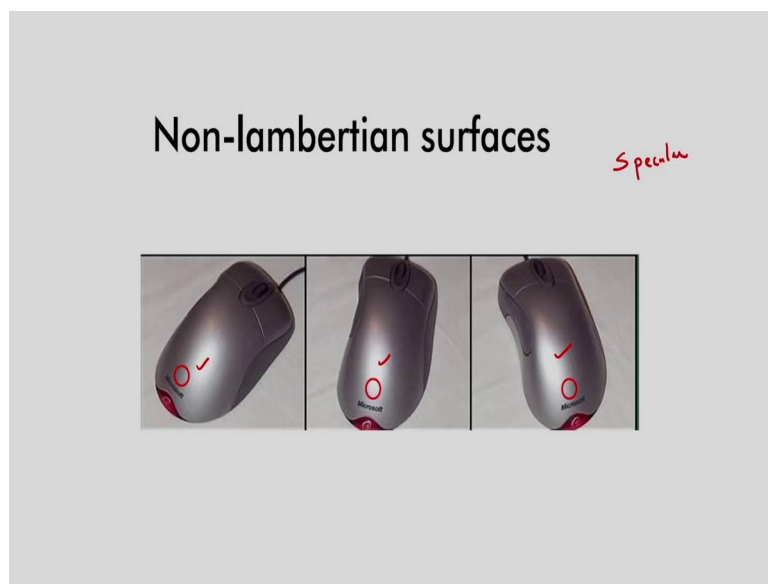
So already, I have explained that directional hemispheric reflectance and the BRDF is constant so I am considering this BRDF is constant because it is independent of direction, constant term I am taking out and after this, it is  $\pi\rho$ . So this  $\cos \theta \sin \theta$ , this I am getting a solid angle. So here, you see, this is the albedo. This is a directional hemispheric reflectance. So the BRDF is equal to the directional hemispheric reflectance divided by  $\pi$ . So this expression is important.

(Refer Slide Time: 37:39)



So I am giving some examples of the Lambertian surface. These are the Lambertian surface, the radiance leaving the surface is independent of angles.

(Refer Slide Time: 37:51)



And in the second example, I have given some example of the non-Lambertian surface that is called non-Lambertian means the specular; specular surface, something like the mirror-like surface. So this portion is the specular, this portion is specular, this portion is specular.

So up till now, what I have considered, the definition of some radiometric parameters. The first parameter I have discussed about the radiance, and after this, I have discussed about the

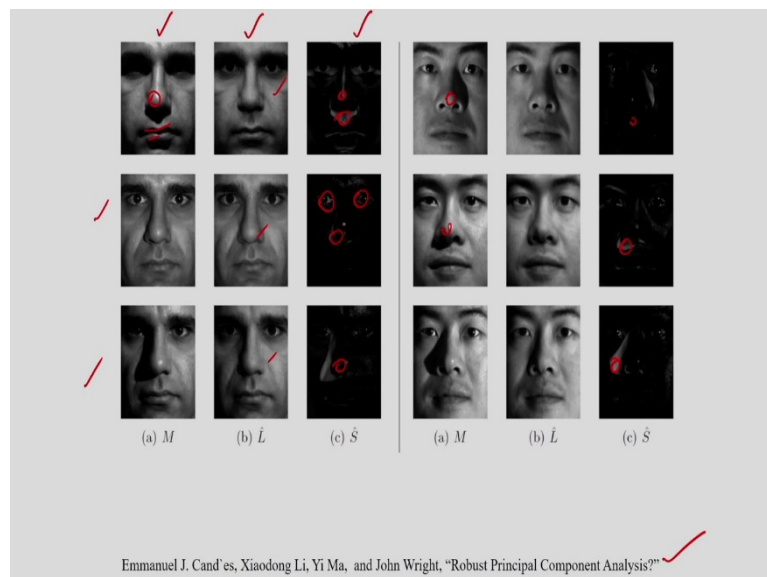
irradiance. And after this, I have discussed the concept of the BRDF. So what is the BRDF, the bi directional reflectance distribution functions.

And after this, I have considered the direction and hemispheric reflectance. And in this case, I have defined the concept of the diffuse surface that is the Lambertian surface and also the specular surface. In these two examples, I have given two examples, I have shown the diffuse surfaces and the specular surfaces. There are many applications in computer vision in which you can, you have to determine the specular surface.

Suppose, if I consider in case of the medical image processing, so if I take some images, some videos of the internal organs of the human, then in this case, you will be getting some specular surfaces. Then I have to remove the specular surfaces for image segmentation. So, identification of the specular surface and the diffuse surface is important.

So, for this, to understand the definition of the albedo. Albedo is the reflectance property of the surface. So, I am considering the diffuse surface. So, in the next slide, I am going to show one application of this in face recognition.

(Refer Slide Time: 39:36)



If you see this here, this is from a paper, this is the paper. So here, I am considering, this is the input image. If you see, these are the input image. From this input image, I am determining, I am actually, the extracting the specular areas.

So, if you see, the nose is the specular portion. This portion, the specular. If you see these lips, lips are also specular. So, specular I am determining. Similarly, in this case, the nose, this portion is the specular. So, specular portion is determined.

So, you can see this paper, in this paper, that the extraction of the specular portion of the face that information you will get from this paper. So, in face recognition also, the application of the specularity is important. So, what is the diffused surface because this surface if you consider, these are the diffused areas if I consider.

And this if you consider, eyes like this, these are the specular parts. So, in this class, I have discussed about the concept of the radiometry, then one important derivation I have derived that is image formation in the thin lens. And after this, I defined some radiometric quantities like radiance, irradiance, and BRDF, and also the directional hemispheric reflectance, and also the concept of the specularity. So, let me finish here today. Thank you.