

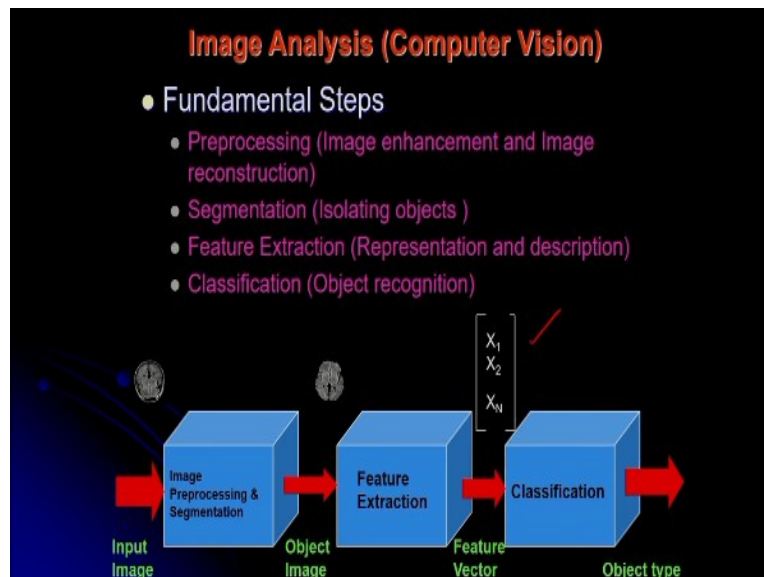
**Computer Vision and Image Processing - Fundamentals and Applications**  
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**Indian Institute of Technology, Guwahati**  
**Lecture-20**  
**Image Segmentation**

Welcome to NPTEL MOOCs course on Computer Vision and Image Processing- Fundamentals and Applications. Today, I am going to discuss the concept of image segmentation. Image segmentation means the partitioning of an image into connected homogeneous region. Homogeneity may be defined in terms of gray value, color, texture, shape, motion. So, based on this I can do image segmentation.

So, today I am going to discuss some very fundamental image segmentation concepts, the algorithms like the thresholding technique and the split and merge technique, region growing technique, active contour, watershed algorithms, K-mean clustering. The advanced the segmentation algorithms, I will be discussing when I will discuss the applications of computer vision.

Image segmentation is a preprocessing step of a computer vision system, so after image segmentation I can extract image features for image classification, image recognition, let us see what is image segmentation.

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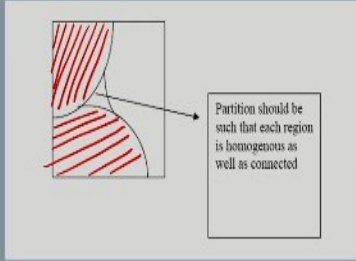


In this case, I have shown one computer vision, the image analysis system and you can see I have the input image and after this I have to do some preprocessing and we have to do the segmentation. Segmentation means the partitioning of an image into connected homogeneous region. After this we can extract features from the images and this is the feature vector, you can see. And after this we can do classification of the images or maybe we can go for object recognition. So, this is a typical block diagram of a computer vision system, the image analysis system.

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**Definition - Segmentation**

- Partitioning of an image into connected homogenous regions.
- Homogeneity may be defined in terms of:
  - Gray value
  - Colour
  - Texture
  - Shape
  - Motion

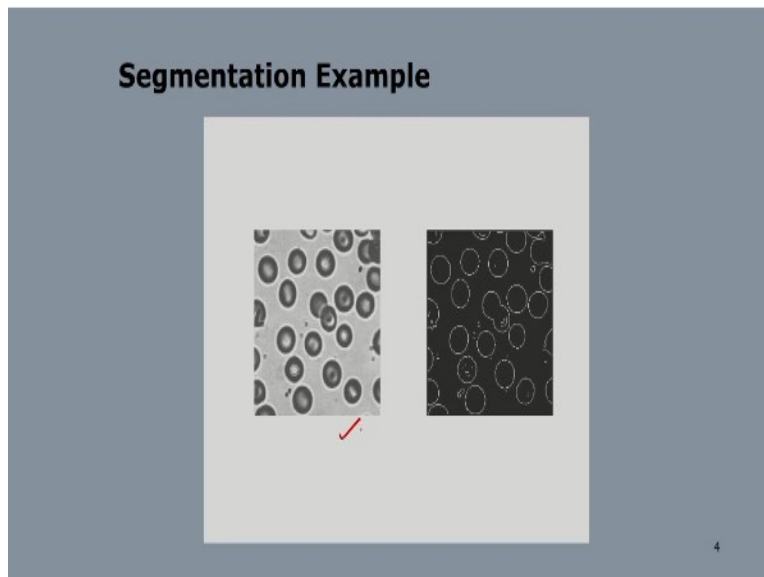


Partition should be such that each region is homogenous as well as connected

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So, what is image segmentation? Partitioning of an image into connected homogeneous region now, homogeneity may be defined in terms of gray value, color, texture, shape and emotion. Suppose, if I consider color, so suppose in this portion the color is almost same, that means it is homogeneous. Similarly, if I consider that this region, corresponding to this region the color is also almost same, that means it is homogeneous. So, that means, based on this that gray value, color, texture, shape and the motion information, I can do image segmentation.

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Here I have shown one example of image segmentation. My input image, you can see first is the input image, the second one is the segmented, image the segmented output.

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The slide is titled "Segmentation". It contains the following text and list:

Mathematically, the segmentation problem is to partition the image  $I$  into regions  $R_1, R_2, \dots, R_N$  Such that

1.  $I = R_1 \cup R_2 \cup \dots \cup R_N$
2.  $R_i \cap R_j = \emptyset, i \neq j$
3. There exists predicate  $P$  such that  $P(R_i) = TRUE$  and for adjacent  $i, j$   
 $P(R_i \cup R_j) = False$

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So, for segmentation how to define mathematically, the mathematical definition of image segmentation. So, mathematically the segmentation problem is to partition the image I. So, I am

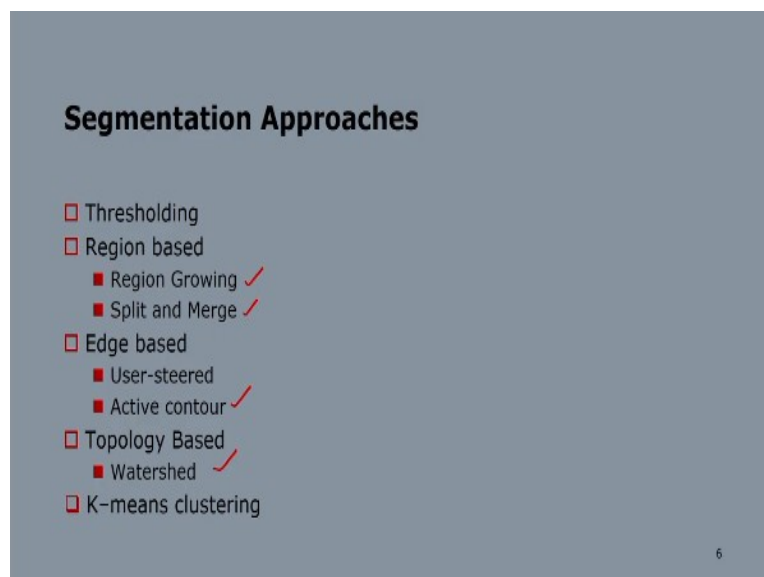
considering the image  $I$  into regions, the regions are  $R_1, R_2, R_n$  like this. So,  $I$  is equal to  $R_1$  union,  $R_2$  union,  $R_3$  like this.

So, that will be my image and in this case, if I consider  $R_i$  intersection of  $R_j$  two different regions, then in this case it will be  $\phi$ , the null set,  $i$  is not equal to  $j$ . So, this region is defined like this, that is if I consider all the regions then I will be getting the image and if I consider two different regions and, that means in this case,  $R_i$  intersection with  $R_j$  will be  $\phi$ .

For determining the homogeneity, I am considering one measure, that measure is called a predicate. So, predicate of a region, the predicate of a region is true that I have considered for the homogeneous region. And in this case, if I consider the adjacent regions, that is two regions,  $R_i$  and  $R_j$ , the predicate of  $R_i$  union  $R_j$  will be false, because it is not homogeneous. So, based on the homogeneity, I am defining this term, the term is the predicate.

So, for the homogeneous region the predicate will be true and for the non-homogeneous region the predicate is false. So, this is the mathematical definition of image segmentation and based on the predicate I can decide whether a particular region is homogeneous or not the homogeneous.

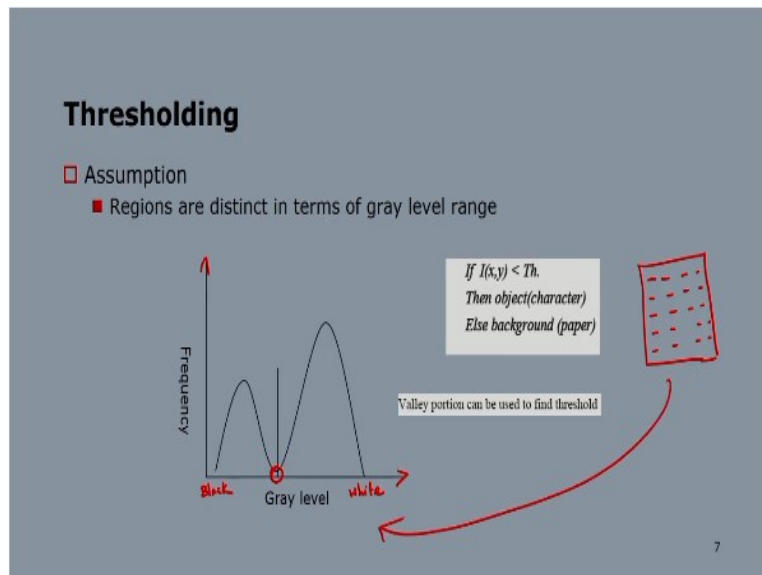
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So, in this class I will discuss these techniques. One is the thresholding technique, one is a region based technique, that is region growing technique, another one is the split and merge technique, and for the edge based technique I will discuss the active contour based technique. And for

topology based approach, I can discuss the watershed algorithm. And finally, I will discuss the K-means clustering algorithm for image segmentation. For advanced segmentation principles that I will discuss when I will discuss the computer vision applications.

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So, first one is thresholding, so here you can see the image histogram I have shown. So, this side is the frequency and this side is the gray levels, all the gray levels and based on this algorithm you can see if  $I(x, y)$  is less than threshold, then object is character and else the background is paper. So, in this case what I am considering, suppose one paper is there, so the background is a white background suppose, and I have the character here, I am writing some character in the paper, that is the black.

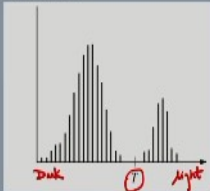
So, in the histogram you can see this portion is the black portion and you can see the white is this side. So, in this case, you can see number of pixels in the black that will be less than the number of pixels of the white region, because white is basically the background, the background is white, and if I consider the characters that is the black.

So, based on this algorithm, if you select the threshold at this point, the valley portion can be used to find a threshold. So, suppose based on this if I select the threshold, then what I can do, I can separate the objects that is the character from the paper, the paper is the background. And corresponding to this case, this is my histogram corresponding to this case.

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### Thresholding

- Suppose that an image,  $f(x,y)$ , is composed of light objects on a dark background, and the following figure is the histogram of the image.



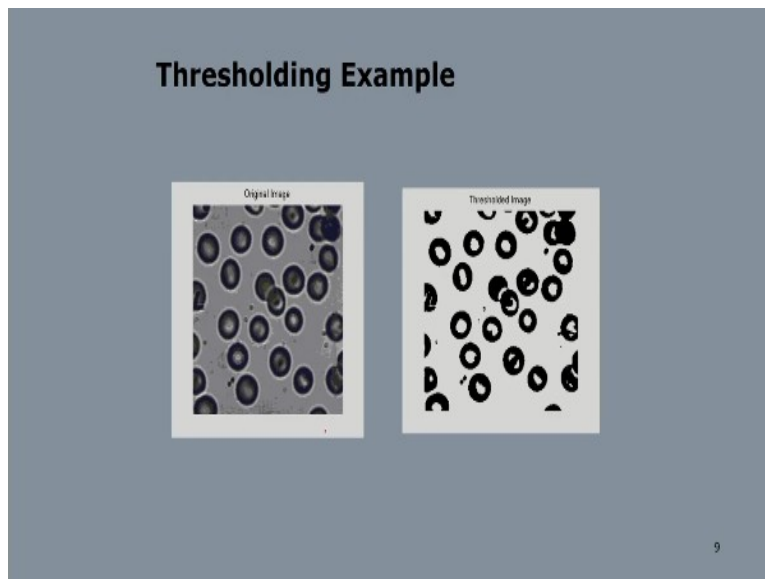
- Then, the objects can be extracted by comparing pixel values with a threshold  $T$ .

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Suppose that an image  $f(x, y)$  is composed of light objects on that dark background. So, here you can see this side is the dark and this side is the light. So, I am showing the histogram. So, the image is composed of light objects on a dark background and corresponding to this, this is the histogram.

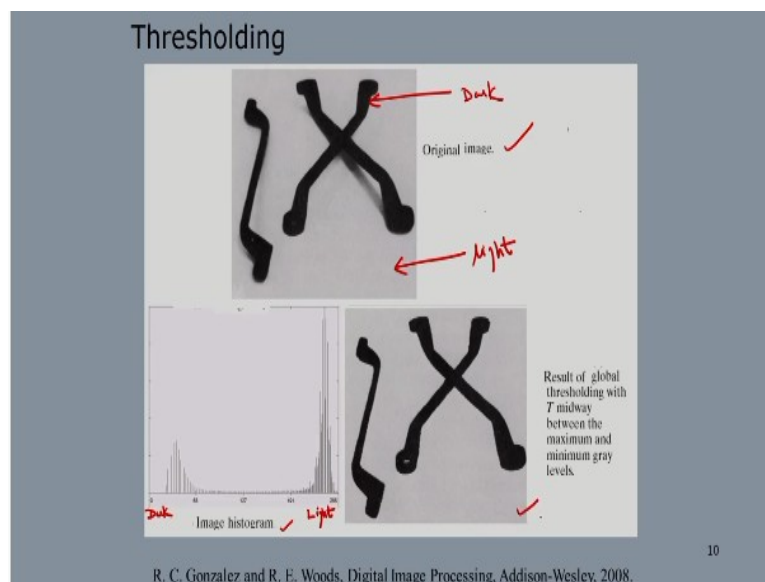
Then, if I consider the threshold at the valley, the  $T$  is the threshold suppose, then objects can be extracted by comparing pixel values with a threshold, the threshold is  $T$ . So, this is a principle of thresholding.

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And corresponding to this principle you can see, this is the input image, original image and after this I am applying the thresholding principle. And you can see thresholded image you can get as an output image.

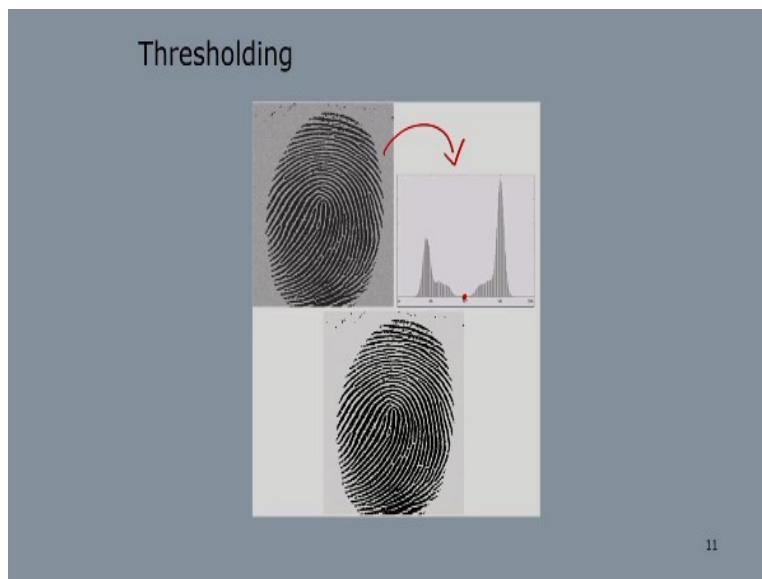
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And in this case, this is another example of the thresholding. You can see the original image and corresponding to this original image, you can see the image histogram and already I have

mentioned this is the dark side and this is the light side, this is the light side that is the background. And object is the foreground, that is the dark. So, you can see the number of pixels in the light portion is greater than the number of pixels in the dark regions. And if I apply the thresholding principle, then corresponding to this you can get the segmented output like this. So, this is a segmented output.

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And this is another example of thresholding, that is the fingerprint segmentation. So, corresponding to this input image, this is my histogram and I can select the threshold, suppose here. And based on the threshold, I can do the segmentation, the segmentation of the fingerprint from the background that I can do.



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Thresholding

- ❑ Non-uniform illumination may change the histogram in a way that it becomes impossible to segment the image using a single global threshold.
- ❑ Choosing local threshold values may help.

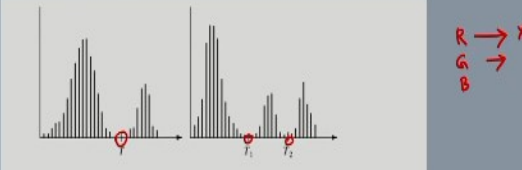
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But the problem of the thresholding is that non-uniform illumination may change the histogram, that is one problem. So, that is why it is very difficult to segment the image using only one global threshold. So, for this we have to select the local threshold, maybe we have to select. So, that is the problem with the thresholding, that then in this case for a non-uniform illumination can change the histogram of the image, and in this case it is impossible to segment the image with the help of a single threshold. So, that is why we have to select a local thresholds that we have to consider.

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### Thresholding

□ It is also possible to extract objects that have a specific intensity range using multiple thresholds.



Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Extension to color images is straightforward: There are three color channels, in each one specify the intensity range of the object... Even if objects are not separated in a single channel, they might be with all the channels... Application example: Detecting/Tracking faces based on skin color...

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So, in this case, you can see the gray level histogram I have shown and you can see in the first figure I have shown only one threshold. In the second figure I have shown two threshold,  $T_1$  and  $T_2$  for segmentation. So, this principle can be extended for color image also, because the color image has three channels, already I have defined, R, G and B three color channels.

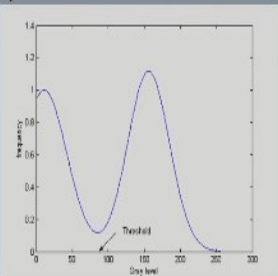
So, suppose even one object which is not separated in a single channel, suppose it is not separated in a single channel, suppose in the R channel it is not separated, that might be separable in other channels, suppose corresponding to the G channel if I consider that G channel, the separation is possible, the segmentation is possible.

In case of the R channel, the objects cannot be separated in a single channel. So, that is why we have to consider all the channels. So, this is one application that is detecting tracking phases based on color, the skin color. So, for this I have to consider RGB channels, the red channel, green channel and the blue channels for the color image.

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### Optimal Thresholding

- Approximate the Histogram by weighted sum of two Gaussians
- The Valley portion represents the threshold




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Another technique is or the optimal thresholding. So, in this case approximate the histogram by weighted sum of two Gaussians, that I can do. The valley portion represents the threshold, so threshold can be selected from the valley portion and this is called the optimal thresholding.

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### Region Growing

- A region satisfies certain homogeneity measure
- The user is required to select a point within a feature
- the region is expanded , adding pixels that are similar in brightness or colour



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Now, I will discuss another technique, region based technique, that is a region growing technique. So, for this what I have to consider a region satisfies, certain homogeneity measure.

So, already I have defined a homogeneity in terms of gray value, maybe in terms of color value, maybe in terms of texture value like this. So, this is based on these parameters, these attributes that is based on the color information based on the gray value, based on the texture information, I can define the homogeneity.

And in this case the user is required to select a point within a particular feature that means, suppose within a particular region or within a particular feature means suppose corresponding to this color, a particular color in a particular region I have to select a particular point. The region is expanded adding pixels that are similar in brightness or color.

So, the concept is like this, suppose let us consider one image, this is the image and suppose corresponding to this portion of the image we have some regions. Corresponding to this portion of the image I am fixing a point, I am selecting a point. After this what I am considering, I have to compare the nearby pixel with this pixel and if the difference in the gray value or maybe the difference in the color value is less than a particular threshold, then in this case the second point I can merge with the first point and I can grow the region.

And like this if I consider another neighborhood I can compare that pixel value with the other pixel value, if this value is less than a particular threshold then I have to merge that one, otherwise I have to neglect that one. So, like this, based on this point I can grow the region, in a particular image I can consider a number of such points. So, these points are called the seed points.


So, based on these seed points I have to compare the seed point with the neighborhood pixels, if it is within a particular threshold, then in this case I have to consider that point, the neighborhood point and that is mainly I am determining the homogeneity, if the difference in the gray value or the color value of the seed point with the neighborhood pixel is less than a particular threshold, then in this case I have to consider that point and I have to grow the region. So, this you can see in the next slide what I have to consider.

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### Region Growing

Given the image and seed points  $S_1, S_2, S_3, \dots$ , predicate will be in terms of the seed points, example, the difference in intensity with respect to a seed point is less than 5.

- Starting-signal point regions consisting of the seed points.
- Add the seed points to the pixels for which the predicate is true.
- Repeat until all the pixels are segmented.


$$I(x, y) \rightarrow R_1, G_1, B_1$$
$$I(x', y') \rightarrow R_1', G_1', B_1'$$
$$\sqrt{(R_1 - R_1')^2 + (G_1 - G_1')^2 + (B_1 - B_1')^2}$$

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So, for this image, I have to select some seed points  $S_1, S_2, S_3$ , and the predicate will be in terms of seed points. So, what will be my predicate? The difference in intensity with respect to the seed point is less than five, this is one example I am considering. So first what we have to consider, starting signal point regions consisting of the seed points. So, first I have to consider the regions with the seed points. So, already I have explained this one.

So, I have to select some seed points like this and the seed points to the pixel for which the predicate is true. So, if I consider corresponding to this seed point the seed point is  $S_1$  I want to compare that seed point with the neighborhood pixel. And if the predicate is true, then in this case I have to add the seed points to the pixels. Like this repeat until all the pixels are segmented. So, I have to do like this. In case of the color images, these comparisons I have to make in terms of RGB value.

So, how to compare? Suppose, one pixel, suppose the seed point is  $I(x, y)$ , and corresponding to  $I(x, y)$  the R value is  $R_1, G_1$  and  $B_1$ , that is corresponding to the seed point. And I have to compare the seed point with the neighborhood pixel, the neighborhood pixel is suppose  $I(x', y')$ , and this value is  $R_1', G_1'$ ,  $B_1'$ . And how to compare? So, for comparison I can apply the Euclidean distance, so  $R_1 - R_1'$  whole square plus  $G_1 - G_1'$  whole square plus  $B_1 - B_1'$  whole square.

So, like this I can find a similarity between two pixels, that is the predicate I am considering. And this predicate is considered to determine the homogeneity condition at the seed points to the pixels for which the predicate is true and this procedure I have to repeat until all the pixels are segmented out. This is the concept of the region growing.

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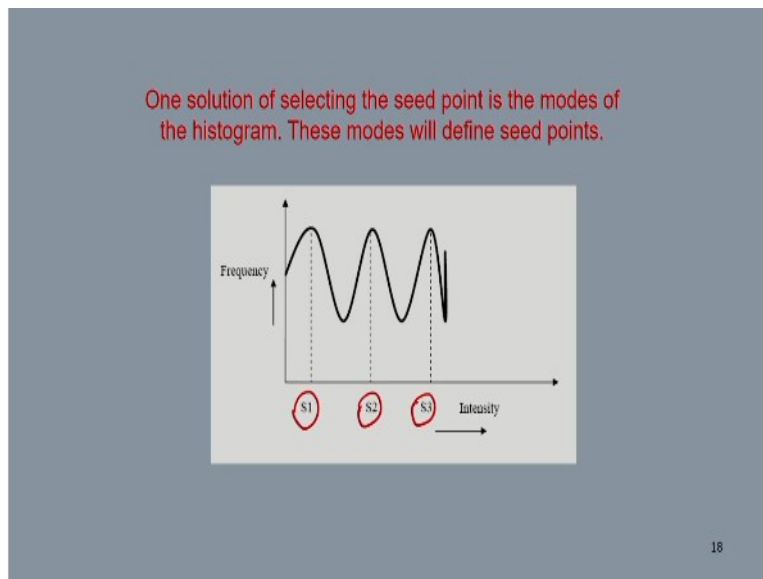
### Region Growing

- Pros
  - Simple
  - Adaptive to gradual changes/Some noise
- Cons
  - Choosing of initial seed points. ✓
  - Initial seed points may be obtained by human interpretation/intervention
  - Seed point selection from the modes of the histogram.

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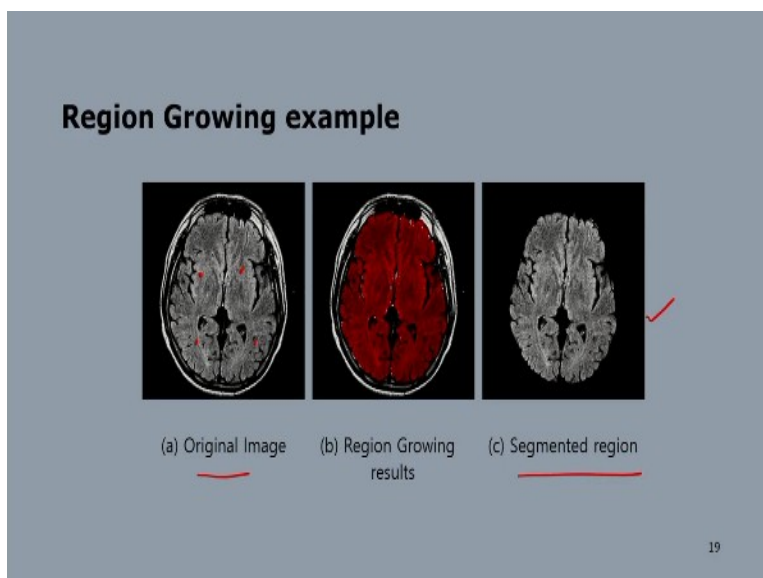
So, what are the advantages and disadvantage of this technique? It is a very simple technique, adaptive to gradual changes and also it is adaptive to noises, some noises. But the main problem is how to select the seed points, the initial seed points, that is the main difficulty. So, initial seed points may be obtained by human interpretation, intervention, seed points election from the modes of the histogram that can be also done, so I have the histogram and from the modes of the histogram, I can select the seed points.

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So, in this case you can see this is the histogram of the image. So, what I am considering, I am selecting the seed point S1, S2, S3. So, selecting the seed point S1, S2, S3 from the modes of the histogram, that is one technique.

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And this is the one example of a region growing segmentation technique. So, I have shown the original image and you can see the region growing results, for this I have to select the seed

points, maybe some points like this we can select some seed points after this I have to grow the regions for segmentation and finally, I am getting the segmented region like this in the final image. So, in this case manually I have to select the seed points in the image or maybe I can consider depth technique, that is from the modes of the histogram I can select the seed point. After this I have to grow that region and to do the segmentation.

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In this case, I have shown one example, I have shown iteration 5, iteration 10, like this 20, 40, iteration 70, iteration 90. And just growing the region and finally, I am getting the segmented output after iteration 90, that is one example of that region growing technique.



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## Region Splitting

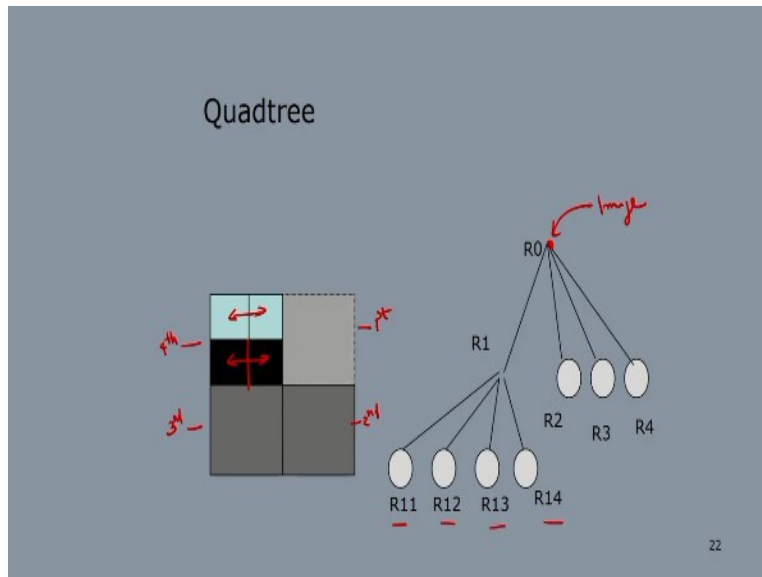
- The opposite approach to region growing is region shrinking ( splitting ).
- It is a top-down approach and it starts with the assumption that the entire image is homogeneous
- If this is not true , the image is split into four sub images
- This splitting procedure is repeated recursively until we split the image into homogeneous regions

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The another technique, that is also another very important technique that is region splitting technique. The opposite approach to region growing is region shrinking, that is the splitting. It is a top down approach and it starts with the assumption that entire image is homogeneous.

So, the assumption is the entire region is homogeneous and that this approach is opposite to the region growing technique. If this is not true, the image is split into four sub images. This splitting procedure is repeated recursively until we split the image into homogeneous regions. So this concept I am going to explain in the next slide, what is the meaning of this.

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So, for this first I am assuming that the entire image is homogeneous. So, in this case, I am showing that entire image is homogeneous. If it is not true, the image is divided into four regions. So, here you can see this is my first region, this is my second region, this is my third region and this is the fourth region.

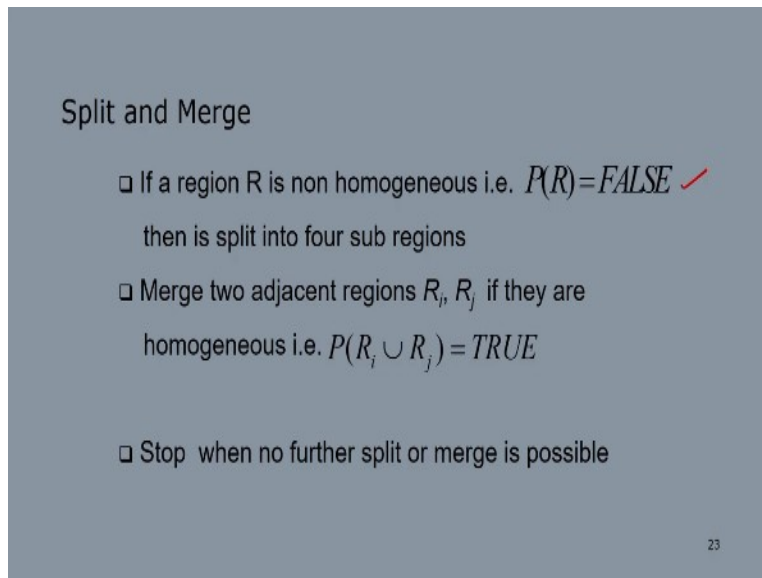
After this you can see in the first region, that is homogeneous, so no need to do splitting. Second region is also homogeneous, so no need to do the splitting. The third region is also homogeneous, so no need to do the splitting. And if I consider the fourth region, that is not homogeneous, so what I have to do, I have to do the splitting. So, I have to do the splitting like this into four regions.

And if you consider these two regions, they are homogeneous, so these two can be merged together. And if you see these two regions, these are homogeneous that can be merged. So, corresponding to this procedure, you can see the Quadtree, so this is the root of this, that is a region that is a corresponding to the image, corresponding to the entire image this is the node R naught.

The R naught is splitted into four regions, the regions are R1, R2, R3, R4. Now, R2 is homogeneous so no need for splitting, R3 is homogeneous so no need to do splitting, R4 is homogeneous so no need to do splitting. But R1 is not homogeneous, so that is why it is divided

into four regions. The regions are R11, R12, R13, R14, so this procedure I have to repeat and also we have to do the merging, the splitting and merging we have to do.

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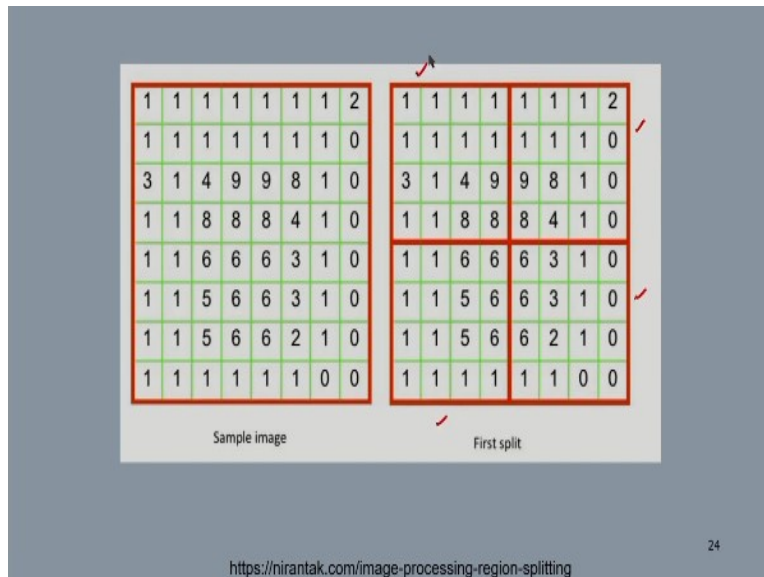
Split and Merge

- If a region R is non homogeneous i.e.  $P(R) = FALSE$  ✓  
then is split into four sub regions
- Merge two adjacent regions  $R_i, R_j$  if they are  
homogeneous i.e.  $P(R_i \cup R_j) = TRUE$
- Stop when no further split or merge is possible

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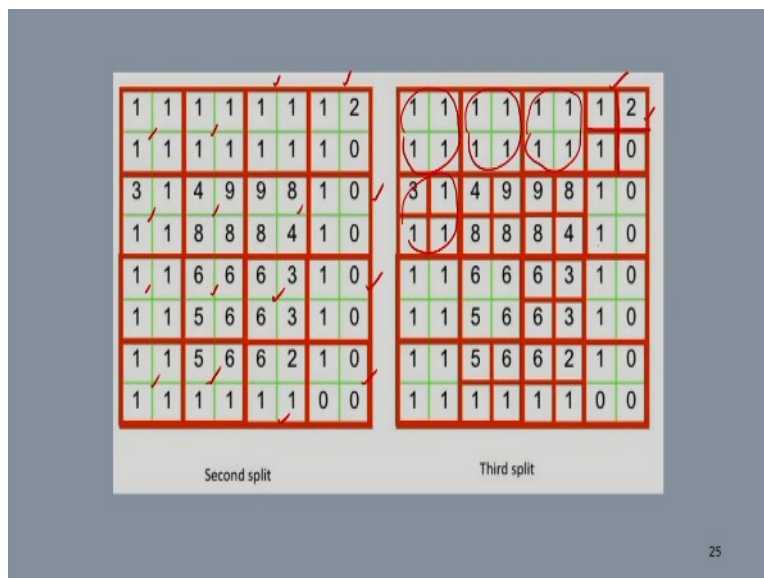
So, for this the algorithm will be like this. If a region R is not homogeneous, then the predicate R will be false, the predicate of the region is false. Then the split into four sub regions, if the region is not homogeneous, that is the predicate of R is false, then the split regions into four sub regions. After this merge two adjacent regions  $R_i$  and  $R_j$  if they are homogeneous, that is the predicate of  $R_i$  union  $R_j$  is equal to true. Then if it is homogeneous, I can do the merging. Stop when no further splitting or merging is possible. So, I have to repeat this and finally I will be getting the segmented image.

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So, here I have given one example. You can see I am considering one image, the sample image. Since it is not homogeneous, so I am dividing it into four regions, you can see four regions here, the first region, second region, third region and the fourth region I am doing the splitting.

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Next slide you can see the first region what I have considered in my previous slide, that is not homogeneous, so that is why I am splitting into four, the four is the first region, second region,

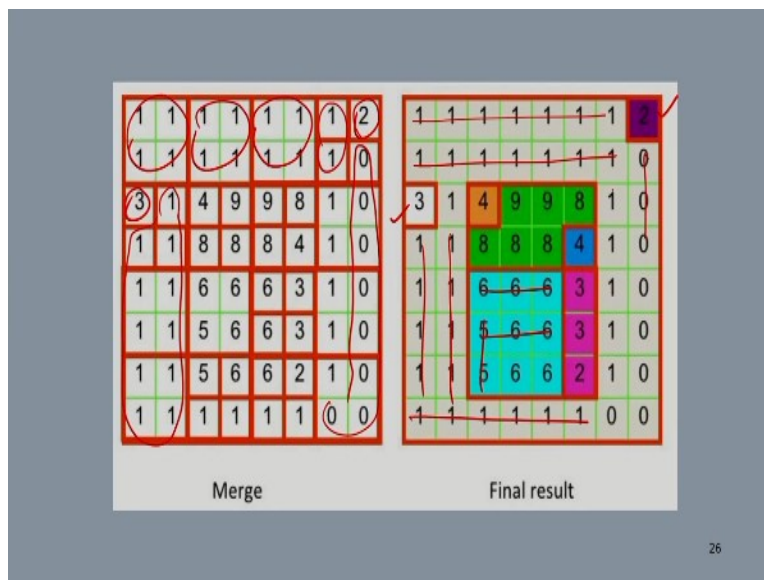
third region, fourth region like this. And similarly, other regions also that is not homogeneous, so that is why I am splitting it again. So, this region, this region, this region and this region.

And similarly, for other region also, since it is not homogeneous, so I am splitting into four regions, this region, this region, this region and this region. And like this the final, if I considered this region, final region, that is also not homogeneous, so that is why it is splitted into four regions.

In the third split what I am considering, so this is homogeneous, this portion is homogeneous. So then in this case no need for splitting. But this region is not homogeneous. So, that is why I am doing the splitting, the splitting is done. So, this region I am doing the splitting.

This region is homogeneous, so that is why no need to split this region is homogeneous, no need to split. But this region is not homogeneous, so that is why I am doing the splitting like this I am doing, I am considering all the regions and I am doing the splitting, so this is the third split.

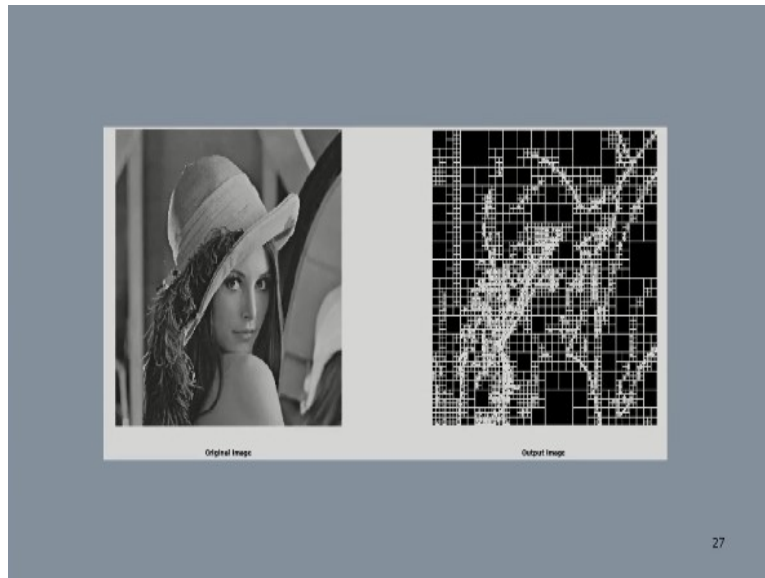
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And after this I am doing the merging. So if you consider, this region can be merged with this, this can be merged with this because this is homogeneous and also this can be merged with this. But if you consider this portion, these two, two I cannot merge. Similarly, if I consider this, all the 0, 0, 0, 0, that can be merged. Similarly, if I consider this 1, 1, 1, 1, 1 this can be merge, 3 will be isolated that I cannot merge, like this I have to do the merging.

So, in the final region what you will be getting, two will be isolated, three is isolated that cannot be merged, and you can see the merged pixel. So, you can see 1, 1, 1, all these are merged, so these are merged, all these are merged, zeros all these are merged, ones this merged, this is merged. So, I am just doing the merging, like the 6 I am doing the merging, the 5 is different. So, I can do the merging of the pixels, so I am getting the final results.

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So, this is the technique of the splitting and merging. And corresponding to this image, original image I am applying this technique and you can see the output.

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### Active Contour (snakes)

- Method
  - A salient curve is generated by minimizing snake energy
    - ✓ Internal energy: Curve bending and continuity
    - ✓ External energy: Image energy (Gradient)
    - ✓ Constraint energy: Measure of external constraints either from higher level shape information or user applied energy.



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Next one important technique I am going to discuss that is active contour based technique. So, what we have to consider, suppose if I want to do the segmentation of an object, one image is there and I have one object, this object. So what I can consider? I can consider one contour, this contour I can consider around this object and this is called a contour around the object and this is my object.

And this contour, or the snake, it is also called a snake. The snake has the internal energy that corresponds to curve bending and the continuity. The snake has the external energy, that is the image energy. And I am also considering some constraint energies, some constraints I am considering, that is measure of external constraints either from higher level shape information or user applied energy that I am considering.

So, these energies I am considering, one is the internal energy, external energy and the constraint energy. And in this case, what I have to do, I have to minimize this energy corresponding to the contour. When the energy is minimum the contour touches the boundary of the objects.

So, my problem is I have to minimize this energy. So, the total energy has three components, internal energy, external energy and the constraint energy. So, what I have to do, I have to minimize this energy and corresponding to this minimum energy, the contour touches the

boundary of the object, that is the active control. So, how to define this energy, one is the internal energy, external energy and the constraint energy?

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**Modeling**

- The contour is defined in the  $(x, y)$  plane of an image as a parametric curve  
$$v(s)=(x(s), y(s))$$
- Contour is said to **possess an energy** ( $E_{snake}$ ) which is defined as the sum of the three energy terms.  
$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$
- The energy terms are defined **cleverly** in a way such that the final position of the contour will have a minimum energy ( $E_{min}$ )
- Therefore our problem of detecting objects reduces to an energy minimization problem.

So, the contour is defined in the  $x, y$  plane of an image as a parametric curve. So, this is the definition of the contour, the  $v$  is equal to  $x$  comma  $y$ . And corresponding to this snake or the contour I am considering these energies, one is the internal energy, one is the external energy and also I am considering the constraints. So, we have three energy terms.

Now, in this case the energy terms are defined cleverly in such a way that the final position of the contour will have the minimum energy. So, whenever the contour touches the boundary of the object, the energy will be minimum. So, the problem is the energy minimization problem, the third problem is the energy minimization problem.



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**Internal Energy ( $E_{int}$ )**

- Depends on the intrinsic properties of the curve.
- Sum of elastic energy and bending energy.

**Elastic Energy ( $E_{elastic}$ ):**

- The curve is treated as an elastic rubber band possessing elastic potential energy.
- It discourages stretching by introducing tension.

$$E_{elastic} = \frac{1}{2} \int_s \alpha(s) |v_s|^2 ds \quad v_s = \frac{dv(s)}{ds}$$

- Weight  $\alpha(s)$  allows us to control elastic energy along different parts of the contour. Considered to be constant  $\alpha$  for many applications.
- Responsible for shrinking of the contour.

So, how to define the internal energy? So, internal energy depends on the intrinsic property of the curve, that is the sum of the elastic energy and the bending energy. So, what is the elastic energy? That means the internal energy has two components, one is the elastic energy, another one is the bending energy.

So, what is the elastic energy? The curve is treated as an elastic rubber band possessing elastic potential energy. So, the curve is considered as an elastic rubber band possessing elastic potential energy. It discourages stressing by introducing tension. So the elastic energy can be defined like this, the weight is alpha s that allows us to control the elastic energy along different parts of the contour.

And for many applications, this alpha s is considered as constant. And this elastic energy is responsible for shrinking of the contour. So, first I am defining the elastic energy and you can see, so how to determine v s? v s is nothing but I am just taking the gradient, dv s divided by ds, that gradient I am considering. So, v s is defined like this. So, first term is the elastic energy I am considering.

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**Bending Energy ( $E_{bending}$ ):**

- The snake is also considered to behave like a thin metal strip giving rise to bending energy.
- It is defined as sum of squared curvature of the contour.  
$$E_{bending} = \frac{1}{2} \int_s \beta(s) |v_{ss}|^2 ds$$
- $\beta(s)$  plays a similar role to  $\alpha(s)$ .
- Bending energy is minimum for a circle.
- Total internal energy of the snake can be defined as  
$$E_{int} = \underbrace{E_{elastic}} + \underbrace{E_{bending}} = \int_s \frac{1}{2} (\alpha |v_s|^2 + \beta |v_{ss}|^2) ds$$

The second term is the bending energy. What is the bending energy? The snake is also considered to behave like a thin metal strip giving rise to bending energy. Now, this bending energy is obtained by this, it is defined as sum of squared curvature of the contour that I can define the bending energy. And  $\beta$  is very similar to  $\alpha$ , in many applications  $\beta$  can be considered as constant.

Bending energy is minimum for a circle, that is obvious. So, bending energy for a circle will be minimum. So, what will be my total energy? For total energy I have to consider these two terms, one is the elastic energy, another one is the bending energy.

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External energy of the contour ( $E_{ext}$ )

- It is derived from the image.
- Define a function  $E_{image}(x,y)$  so that **it takes on its smaller values at the features of interest, such as boundaries.**

$$E_{ext} = \int_s E_{image}(v(s)) ds$$

Key rests on defining  $E_{image}(x,y)$ . Some examples

- $E_{image}(x,y) = -|\nabla I(x,y)|^2$
- $E_{image}(x,y) = -|\nabla(G_\sigma(x,y) * I(x,y))|^2$

Next one is how to define the external energy of the contour. The external energy of the contour is derived from the image. So, in this case you can see the definition of the external energy,  $E_{external}$  is equal to, integration I am taking for the variable  $s$ ,  $E_{image}$  into  $v$   $s$   $ds$ .

So, what is  $E_{image}$ ?  $E_{image}$  is nothing but it is the energy obtained from the image. So, define a function  $E_{image}$  so that it takes on a smaller value as a feature of the interest such as boundaries. So, that means, I have to define the function  $E_{image}$  so that it takes on its smaller value at the features of interests such as boundaries.

So, at the boundaries we have to consider. So, this is the definition of the external energy and that is derived from the image. So, like this if I take the gradient of the image and after this is squared, so I can determine  $E_{image}$ . Also I can consider this expression also, the image is contoured with the Gaussian and after this, I am just taking the gradient of the image. So, the image is contoured with the Gaussian to blur the image and after this I am taking the gradient and from this you can determine the image energy.

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**Energy and force equations**

- The problem at hand is to find a contour  $v(s)$  that minimize the energy functional

$$E_{make} = \int \frac{1}{2} (\alpha(s) |v_s|^2 + \beta(s) |v_{ss}|^2) + E_{image}(v(s)) ds$$

- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation

$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

- Equation can be interpreted as a force balance equation.
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

So, finally, the problem at the hand is to find the contour  $v$  s, that minimizes the energy functional. So, the problem is we have to minimize the energy. So, energy here I am considering the internal energy and the external energy. The external energy is derived from the image itself. And in this case, we have to apply the Euler Lagrange differential equation.

So, this is Euler Lagrange differential equation I have to apply and I will be getting this the energy minimization problem. So, this equation can be interpreted as a force balance equation, each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces, and whenever the contour touches the boundary of the objects, the energy will be minimum.

So, this is a case. So, the problem is the energy minimization problem and for this we have considered the internal energy, external energy and in this expression, I have not considered the constraints. So, only I have shown that two energy, one is the internal energy another only the external energy.

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### Active Contour

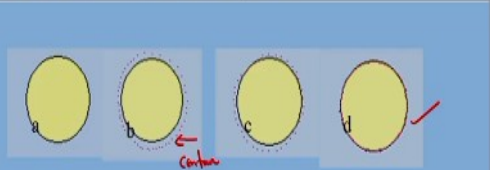
- Pros
  - Works good for ambiguous boundary regions
- Cons
  - Trade off between noise reduction and detailed boundaries
  - Initial curve ✓

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So, what are the advantage of this technique? So, it works good for ambiguous boundary regions, but the problem is how to define the initial curve, that is the also the problem of this method and also the trade off between noise reduction and the detail boundaries.

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### Active Contour Example



a - object  
b - initial contour  
c - iteration steps of snake energy minimization  
d - final contour

Boundary detection using active contour model

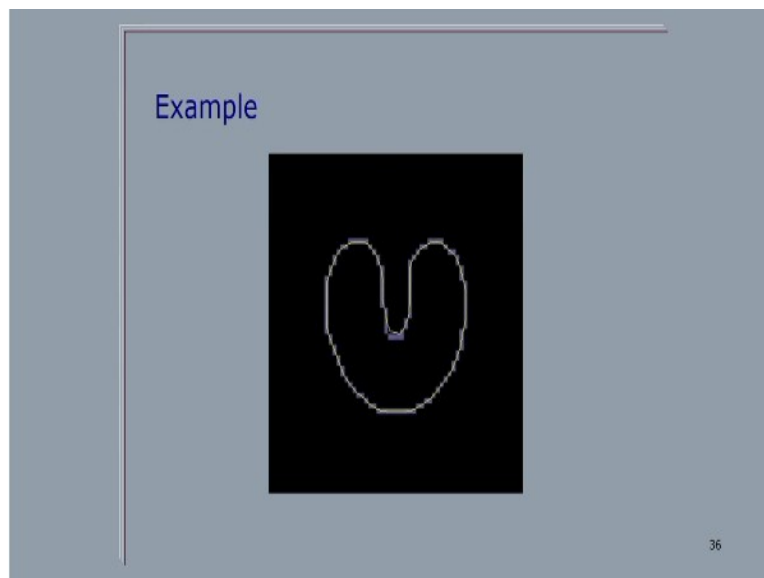
35

So, I can give some examples of the active contours. You can see a is the object and initially I have to define the contour. So, b is the contour here, if you see the dotted line that is the contour.

And after this what I have to consider I have to consider the energy of the contour, that is the internal energy and the external energy. And after this, with each and every iteration, I have to minimize the energy that is the snake energy minimization I have to do.

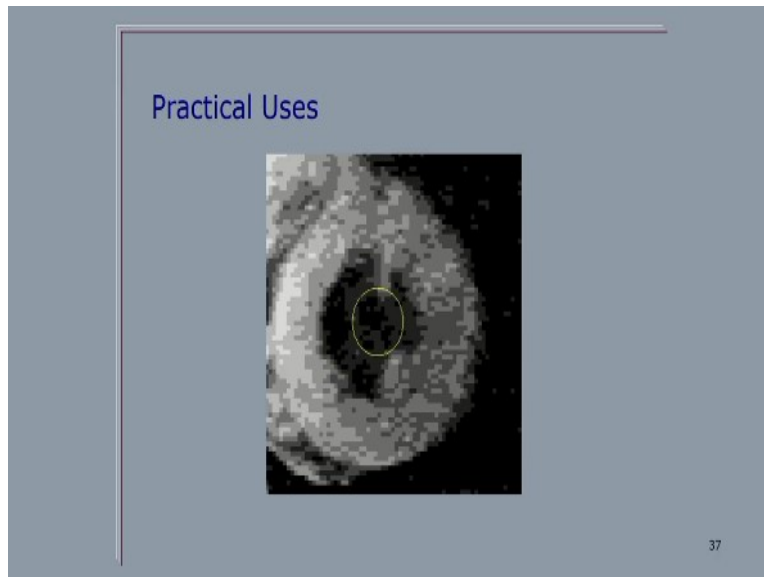
And whenever the contour touches the boundary of the object, the energy will be minimum. And that corresponds to the final contour,  $d$  is the final control, and that is the segmented object I will be getting. So, the segmented object I will be getting when the contour touches the boundary of the object. This is the boundary detection using active contour model.

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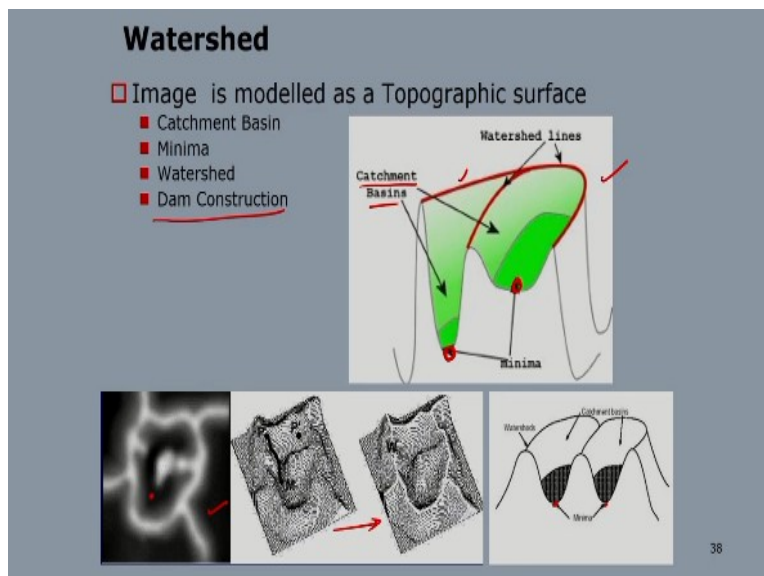
And here I have given another example, you can see the initial contour and after this I am doing the energy minimization, energy is nothing but the internal energy and the external energy and whenever the contour touches the boundary of the object the energy will be minimum.

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The another example I can show, this is the segmentation of the mammogram and I am defining the contour. And you can see that with each and every iteration the contour just touches the boundary of the objects. So, I can do the segmentation by using active contour.

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The another popular technique is the image segmentation, that is the topographic base segmentation technique, that is the watershed algorithm. So, in this case the image is modeled as

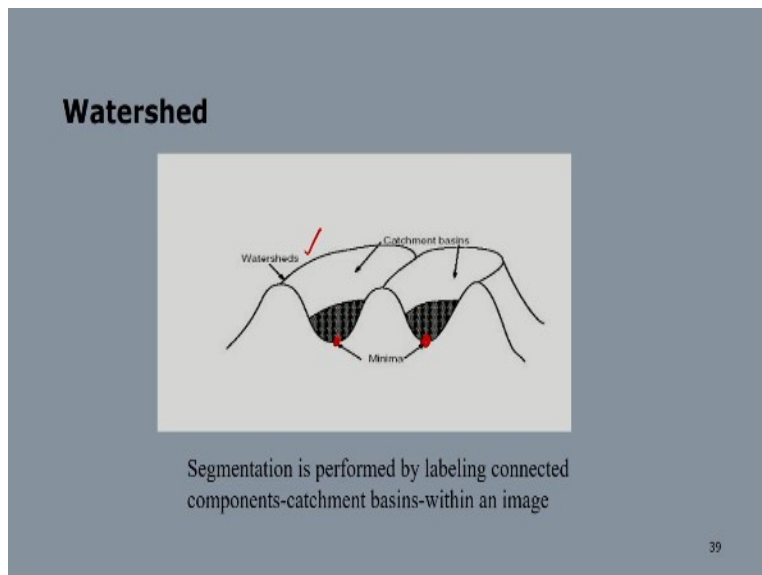
a topographic surface. So, here in this figure, the first figure, you can see I am showing the topographic surface. Corresponding to this topographic surface you can see a two catchment basins, you can see two catchment basins are available.

And corresponding to these two catchment basins, I have the minima, this is one minimum corresponding to the first catchment basin and you have another minima corresponding to the second catchment basin and between these two catchment basins, I have the watershed lines, you can see the watershed lines, this are the red watershed lines.

So, that means, the main concept is what is the catchment basin, what is the local minima and what is the watershed lines, and I will discuss about the dam construction, how to construct the dam. So, that concept I am going to discussed. So, the main concept is the image is model as a topographic surface.

So, corresponding to the figure one you can see two minima and I have considered two catchment basins and corresponding watershed lines. So, corresponding to this image, grayscale image, you can see the catchment basins, maybe the catchment basins here, and in this case, I am considering a topographic surface, this is a topographic surface. And corresponding to this topographic surface, you can see two minima corresponding to two catchment basins. And again you can see that watershed lines.

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So, in case of the watershed segmentation, the segmentation is performed by labeling connected components-catchment basins within an image. So, in this case what I have to consider suppose corresponding to this the first catchment basin, this is the minimum corresponding the second catchment basin this is the minimum. Now, the objective is to find the watershed lines between the catchment basins. So, that concept I will explain in my next slide.

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### Watershed Segmentation Algorithm

- Visualize an image in 3D: spatial coordinates and gray levels.
- In such a topographic interpretation, there are 3 types of points:
  - Points belonging to a regional minimum
  - Points at which a drop of water would fall to a single minimum. (→The *catchment basin* or *watershed* of that minimum.)
  - Points at which a drop of water would be equally likely to fall to more than one minimum. (→The *divide lines* or *watershed lines*.) ✓



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Here you can see, so visualize an image in 3D, spatial coordinates and the gray levels I have to consider. In such a topographic interpretation and there are three types of points, points belonging to a regional minima, so here in this case you can see suppose this point as a regional minima. So, in the second figure also you can see, we have the regional minima.

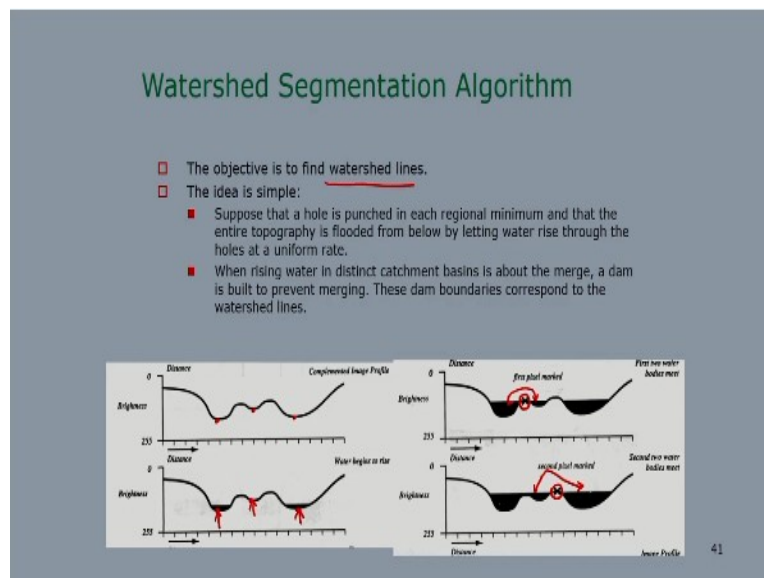
The second point is, point at who is a drop of water would fall to a single minima. Suppose, in this case if I put some water here suppose, corresponding to this, the drop of water would fall to a single minima, single minima suppose drop of water it falls into this minima, point at which a drop of water would fall to a single minima that I am considering, that is the catchment basin or the watershed of that minima. So, corresponding to this minima, if I put some water at this point, it would drop to this single minima.

Third point is, point at who is a drop of water would be equally likely to fall into more than one minima. Suppose, if I put a drop of water here, this drop of water would be equally likely to fall

to more than one minima, so this drop of water may fall to this minima or may fall to this minima.

So, you can see, we have three types of points, the points belonging to a regional minima, that already I have shown the regional minima. Point at which a drop of water would fall to a single minima, so I am considering this point. And points at which the drop of water would equally likely to fall to more than one minima, so that I am considering this one.

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So, idea is how to determine the watershed lines. So, objective is to find the watershed lines. So idea is very simple, suppose in this case I have shown in this figure the brightness profile with respect to the distance. Now what I am considering suppose, in this brightness profile of the image a hole is punched in each of the regional minima, so in this case, this is my original minima, this is my regional minima, this is my regional minima like this.

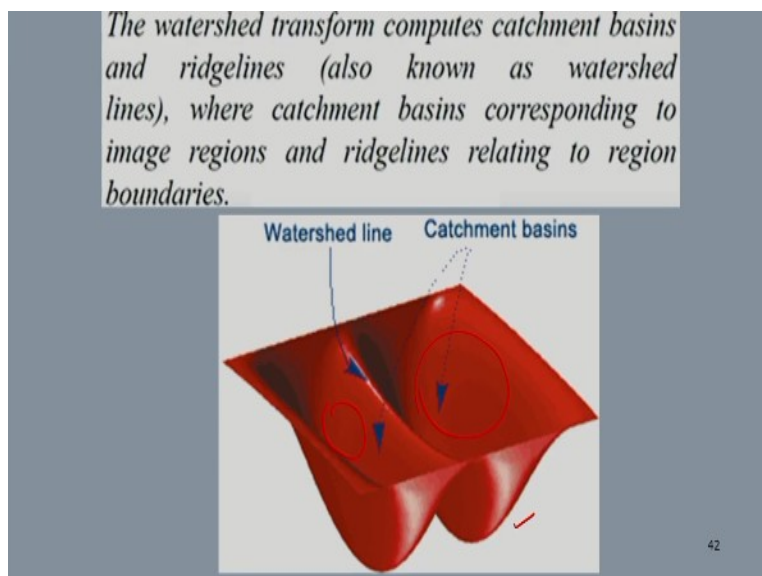
So, a hole is punched here, maybe here, maybe here, the hole is punched, in each regional minima that the entire typography is flooded from the below. So, it is flooded from the below by letting water rise through the holes at a uniform rate. So, just I am doing the flooding. So how to do the flooding? The flooded from below by letting water rise through the holes at a uniform rate. So, for this I am considering the holes like this, this one hole, another hole, like this. So, water is rising now.

so whenever water is rising, what will happen, at a particular time or a particular point, this water is about to merge to this water about to merge, when rising water in distinct catchment basins is about to merge, so how to prevent this flooding? Then a dam is built to prevent merging. So, that is why I am making a dam here so that water cannot flow from one catchment basin to another catchment basin. So, that is why I am constructing a dam, so that the water cannot merge from one region to another region, these dam boundaries corresponds to the watershed lines.

Similarly, corresponding to that these two regions, this is one region and another region is this, the flooding is going on what I am doing flooding from below by letting water rise through the holes at uniform rate. So, what will happen, to prevent merging of water from this both the regions, I have to construct a dam. So, dam construction is going on like this, these dam boundaries corresponds to the watershed lines.

So, that means, the objective is to find the watershed lines. So, that is called a dam construction, dam construction is done so that the merging is not possible, that is water is rising from the bottom and I want to prevent water coming from one region to another region. So, for this a dam is constructed so that water cannot merge from one region to another region. And I have to do the dam construction like this and this dam boundaries corresponds to the watershed lines.

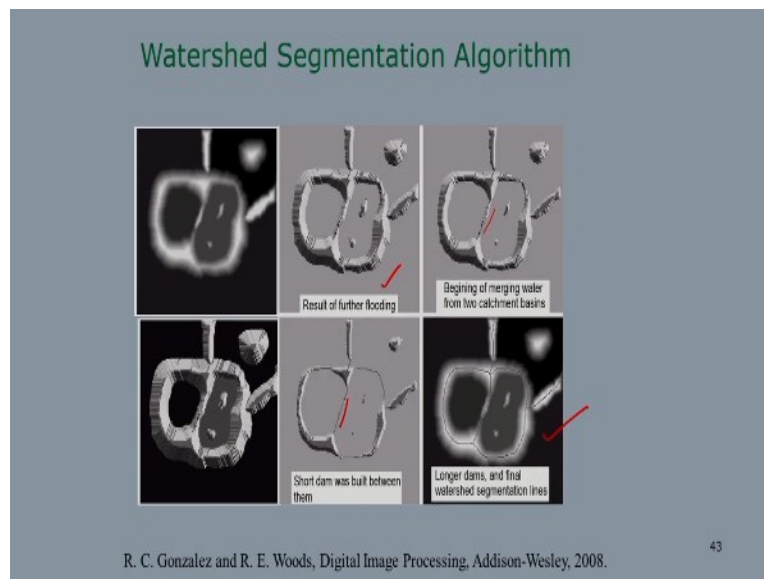
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So, this concept again I am showing here. I have shown in this figure catchment basins I have shown, and also I have shown the watershed lines. So, objective is to determine the watershed lines, the watershed transform compute catchment basins and the ridgelines. These ridgelines are also known as watershed lines, where the catchment basins corresponds to the image regions and the ridgelines corresponds to the region boundaries.

So, that means I have to identify the watershed lines based on this principle. And in this case the catchment basin corresponds to the image region. So, this is the image region, like this, this is the image region. And if I consider the ridgeline that is the watershed line, that corresponds to the region boundaries. So, if you consider this one this is the region boundaries.

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So, in this example, I have shown how to construct the dam. So, you can see the flooding is going on, result of the flooding. And for this the water is about to merge from two catchment basins, so the water is about to merge you can see. So, for this is a short dam constructed, so that the flooding is not possible from one region to another region. So, I am constructing a dam, so that water cannot merge from one region to another region.

And finally you can see I am constructing a longer dam, that is the watershed lines, so, that water cannot merge from one region to another region, water cannot flow from one region to another

region, that is I am determining the watershed lines. So, you can see the final results I have shown, the longer dams are constructed and that is the final watershed segmentation lines.

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### Watershed Segmentation Algorithm

- Start with all pixels with the lowest possible value.
  - These form the basis for initial watersheds ✓
- For each intensity level k:
  - For each group of pixels of intensity k
    - If adjacent to exactly one existing region, add these pixels to that region
    - Else if adjacent to more than one existing regions, mark as boundary
    - Else start a new region


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So, algorithm will be like this whatever I have explained here, start with all pixels with the lowest possible value, these form the basis for initial watersheds. Now, for each intensity level k, for each group of pixel of intensity k is adjacent to exactly one existing region add these pixels to that region. Else, if adjacent to more than one existing regions, mark as boundary that means, I am determining the watershed lines, else start a new region. So, that is the concept that means, I have to find the watershed lines.

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## Watershed

- Pros
  - Closed boundary
  - Correct boundary achievable
- Cons
  - Difficulties in correct/efficient implementation of watershed
  - Oversegmentation Problem ✓
  - Boundary May not be smooth



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So, what are the advantages of the watershed segmentation? Closed boundary is obtained and correct boundary achievable, and one main problem is over segmentation problem. So, what is over segmentation problem? Suppose, if I want to do the segmentation of these objects, this object suppose what I have to determine I have separated the objects from the background, so this is my object and I have to separate it from the background.

But what is the over segmentation? Because of the over segmentation, what happens objects being segmented from the background are themselves segmented into sub components, so that means this will be segmented out, so that means it is again segmented into sub components, that is not desired, so over segmentation is a big problem in watershed algorithm. The over segmentation means, again I am repeating, the objects being segmented from the background are themselves segmented into sub components, that is the over segmentation.

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## Watershed

- Over-segmentation ✓
  - Noise
  - Small Catchment Basins
- Solution Approaches ✓
  - Filtering ✓
  - Weak edge removal ✓

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So, this over segmentation is a problem. And what is the approach? The solution is I have to do the filtering and by filtering I can remove weak edges, that is one method. Over segmentation is a big problem in watershed algorithm, so that I can consider by filtering, so by filtering I can remove the weak edges.

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## K-Means Clustering

*begin*  
*initialize  $m, C, \mu_1, \mu_2, \dots, \mu_c$*   
*do classify  $m$  samples*  
*according to nearest  $\mu_i$*   
*recompute  $\mu_i$*   
*until no change of  $\mu_i$*   
*return  $\mu_1, \mu_2, \dots, \mu_c$*   
*end*

Partition the data points into  $K$  clusters randomly. Find the centroids of each cluster.

For each data point:

- Calculate the distance from the data point to each cluster.
- Assign the data point to the closest cluster.

Recompute the centroid of each cluster.

Repeat steps 2 and 3 until there is no further change in the assignment of data points (or in the centroids).

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And finally, I want to discuss the algorithm that algorithm is the K-means clustering. So what is the K-means clustering? Partition the data points into K number of clusters randomly. Find the centroid for each cluster, I have to find a centroid for each cluster, for each data point calculate the distance from the data points to each other cluster, assign the data point to the closest cluster. Recompute the centroid of each cluster, repeat step two and three until there is no further change the assignment of the data points or in the centroids.

Now, I am going to discuss about the K-means clustering. So, what is K-means clustering? So I will explain, so begin, so first I have to initialize n number of data points, or maybe the samples, c number of clusters. And corresponding to the c number of clusters, I have the centroid, the centroid are  $\mu_1$ ,  $\mu_2$ , up to  $\mu_c$ , c number of centeroids.

After this, what we have to do, do classify n samples according to nearest  $\mu_i$ . So I have to do the classification and samples according to nearest  $\mu_i$ . And after this, again, I have to recompute the centroid, the centroid this  $\mu_i$  until no change of  $\mu_i$ , no change of  $\mu_i$  and what will be my return value, return  $\mu_1$ ,  $\mu_2$  like this, I have c number of centroids end.

So this is the algorithm for the K-means clustering, I have to initialize a c number of centroids, so I am considering c number of classes suppose, so I have to initialize c number of centroids, n means the number of samples. Do classify n samples according to nearest of  $\mu_i$ . After this I have to recompute  $\mu_i$ , until no change of the centroid, the centroid is  $\mu_i$ .

And finally I will be getting the centroids of the cluster,  $\mu_1$ ,  $\mu_2$ ,  $\mu_c$  like this. That means this procedure is the clustering of the data points. So, this algorithm already I have written. So I am considering K number of clusters randomly. And for each data point what I have to consider, calculate the distance from the data point to each cluster I have to consider.

Assign the data point to the closest cluster and recompute the centroid of the each cluster that I am considering, recomputing until no change of the  $\mu_i$  that I have to do this iteration. And finally I will be getting the centroids, the centroids are  $\mu_1$ ,  $\mu_2$ ,  $\mu_c$  like this. So this concept I can explain in my next slide.



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So you can see I am considering suppose the cluster are like this, and randomly I am selecting two clusters centroid, one is the red another one is the green. Two cluster centroids I am selecting. After this, what I am considering, I am considering the distance between the sample points, and the cluster centers. That means the cluster center means the mean.

So in this case, by finding the distance between the sample points you can see at this point is assigned to this cluster. Similarly, this point is assigned to this cluster, this point is assigned to this cluster like this, based on the minimum distance I can determine, whose sample point is closer to a particular mean.

Similarly, corresponding to this point, this point is close to this, close to this mean, this point is close to this mean, and like this I can decide whether these sample points belong to a particular cluster, particular centroid that I can decide. Similarly, this point is closer to this, as compared to the red centroid. So based on this, I can assign the sample points to the centroid, red centroid and the green centroid.

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After this I have to recompute, recompute the centroid. You can see the previous centroid is here and after this I am recomputing the centroid, so next centroid will be this. This procedure I have to repeat again and again you can see, again, I can see whether these sample points belong to this, or this I have to see, that is based on the distance I can do this. And again, I can see whether this is close to this, or this, I have to find like this.

And based on this you can see these two points are now assigned to the red centroid. And again, I am recomputing the centroids, I am recomputing the centroid and after this again like this and finally I am getting the centroid like this and I am getting the two clusters. So this is one cluster, corresponding to the centroid, centroid is the red centroid and corresponding to green centroid I have another cluster, so the is another cluster.

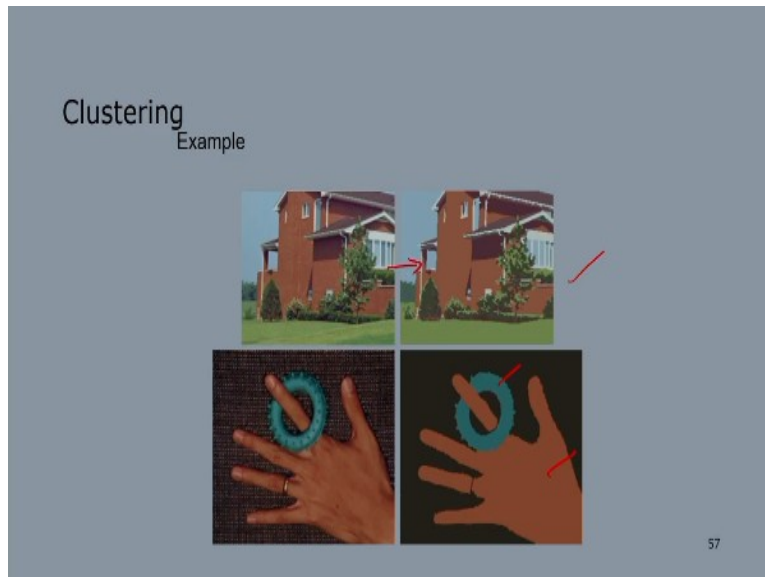
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This K-means algorithm, the procedure I have shown here. So initially, I have to randomly define the cluster centroids, that is the mean. After this I have to assign the sample points to a particular centroid based on the minimum distance, I can consider the Euclidean distance between the sample point and the centroid and based on this, a particular centroid can be assigned to a particular mean, like this I can consider.

And finally, if there is no change of the centroid after all this, then I have to stop the iteration, and I will be getting the clusters corresponding to all the centroids, I have to do like this. So you can see here in this figure, you can see all the iterations and finally I will be having the centroid like this. So that centroid will be like this after all the iterations, because I have to update the centroids after each and every iteration.

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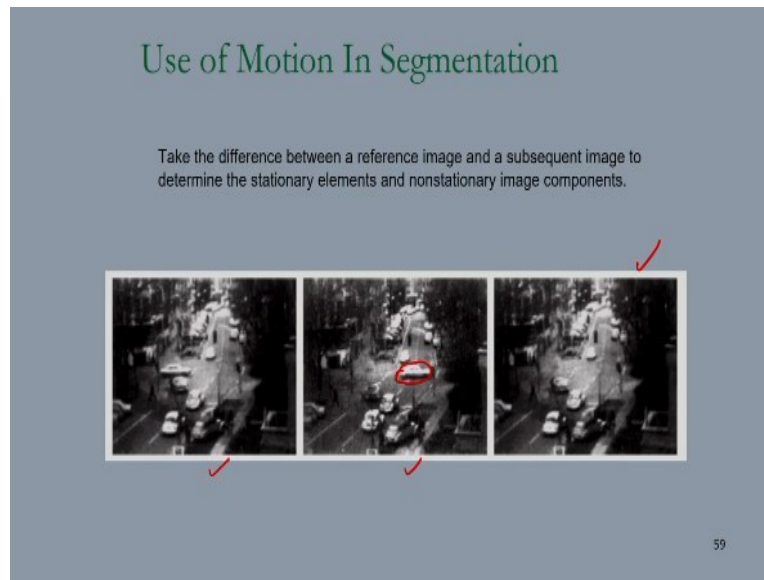
So this is one example of the clustering, that you can see, I have the input images, and you can see I am applying the K-means clustering algorithms for image segmentation. So in this case I have only two clusters. One is this cluster and other one is this cluster and similarly you can see this is the result of the K-means clustering.

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I can give another example, this is the original image and in this case I am considering  $K$  is equal to 5, that means 5 centroids are considered and in the second case I am considering  $K$  is equal to 11. That means 11 centroids are considered, that means, 11 clusters I am considering.  $K$  equal to 5 means the 5 clusters I am considering, and the corresponding segmented images you can see in the results. This is about the  $K$  means clustering.

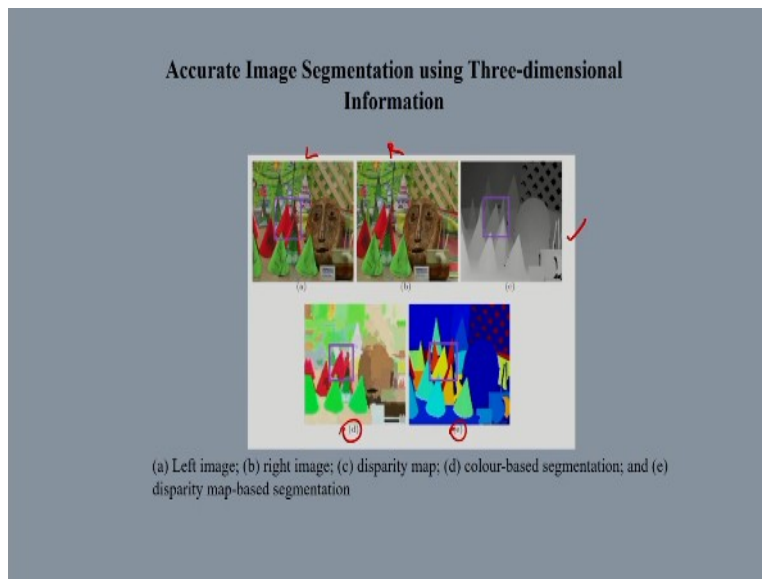
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And also I can use the motion for image segmentation. So just I am giving one simple example. Suppose I am considering one video sequence and suppose this is one frame and this is another frame of the sequence. And you can see one moving car is there, you can see, one moving car is there and if I take the difference between a reference image and the subsequent image, then what I will be getting? I can get the stationary elements and also I can determine the non stationary elements.

So in this case, one moving car is available and in this case, this corresponds to the stationary elements, the stationary background I will be getting. So this is a concept of the motion in segmentation, just taking the difference between the reference image and the subsequent image of the video sequence to determine that stationary elements and the non stationary image components that I can determine.

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And also in stereo imaging, I have the left image and the right image and we can determine the disparity map. And from the disparity map I can do the image segmentation. So, here you can see the first one is the left image, and the right image is available and I am calculating the disparity map here, c is the disparity map and based on the color I can do the segmentation. So d is the result of color based segmentation and e is the result that is obtained from disparity map based image segmentation. So I can also use disparity information for image segmentation.

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**Conclusion**

- ❑ A brief outline of Segmentation techniques is presented
- ❑ Discussion does not include
  - Edge-detection based approaches
  - Statistical based segmentation methods
- ❑ Semi-Automatic approaches seem to be more usable
- ❑ No single effective segmentation method for all scenarios

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So, briefly I have outlined the segmentation techniques, the edge detection based approaches, the statistical based segmentation methods are not considered here in this discussion. And one point is that no single effective segmentation methods for all the applications, it is not possible.

So in this class I have discussed the image segmentation concepts and I have discussed some image segmentation algorithms. One is the thresholding technique, one is the split and merge technique, one is the region growing technique that is also very important, for region growing technique I have to consider initial seed points.

And after this I discussed the concept of active contours, so how to select the active contours and what are the energies, internal energy, external energy, and also the constraints. So the problem is nothing but the energy minimization problem. When the contour touches the boundary of the objects, then the energy will be minimum, that point I am considering in case of the active contour.

After this I discussed about watershed image segmentation algorithms and finally I discussed about the K-means based image segmentation technique. So let me stop here today. Thank you.