Computer Vision and Image Processing - Fundamentals and Applications

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Lecture – 17

Image Filtering

Welcome to NPTEL MOOCs course on Computer Vision and Image Processing: Fundamentals and Applications. In my last class I discussed the concept of spatial filtering, I discussed the concept of low pass filter, high pass filter and the high boost filter. Also, I discussed one filter that is the non-linear filter, the median filter. The median filter can be used to remove impulse noises. That is called as the salt and pepper noises.

Also I discussed one important concept the filter is bilateral filter. So, in the bilateral filtering we consider range and the domain of an image. Today I am going to discuss the concept of image filtering in frequency domain. In frequency domain what we have to do, we have to modify the Fourier Transform of the image. So, for this we have filters like the low pass filter, high pass filter, maybe the band pass filter and band stop filter. And this concept I am going to discuss today, the concept of the frequency domain filtering.

So, for this the first step is we have to take the Fourier Transform of the image. So, before determining the Fourier Transform of an images I have to do some preprocessing, that means I have to multiply the image by minus 1 to the power x plus y. So, this step I am going to explain now. So, what is the concept of the frequency domain image filtering?

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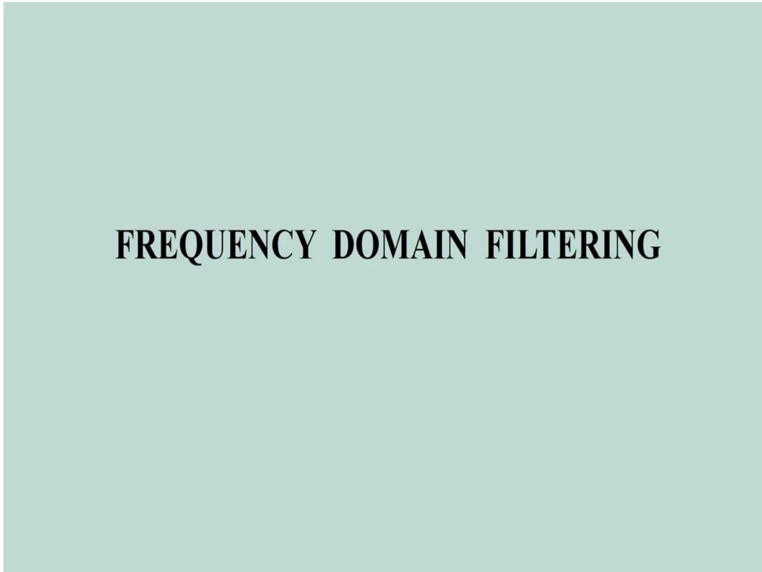

So, I will discuss the concept of the frequency domain filtering.

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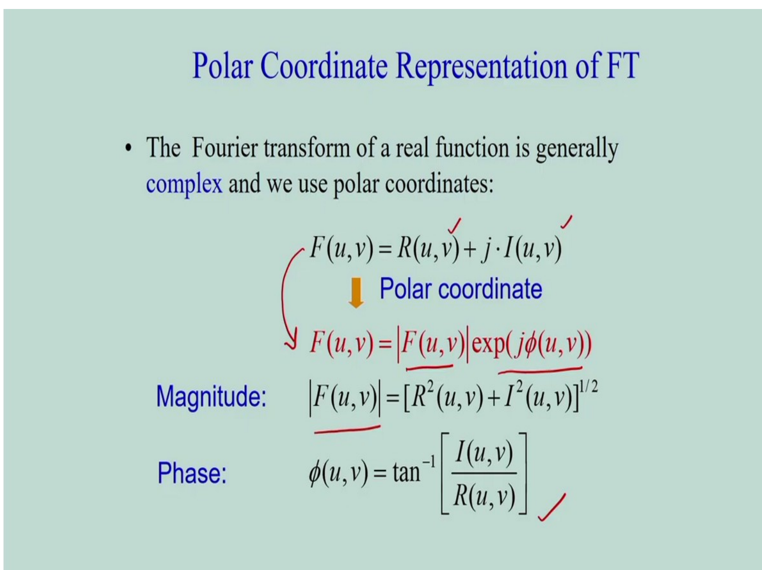

So, already I have explained this concept. If you see F u v, that is the Fourier Transform of an image. If I consider the Fourier Transformation, it has two parts, one is the real part another one is the imaginary part. So, in this case you can see first one is the real part, the next part is the imaginary part.

This Fourier Transform can be represented in the polar coordinate, like this. So, we have the magnitude component and another component is the phase angle component. In case of the

Fourier Transform, I can determine the magnitude of the Fourier Transformation. The magnitude I can determine like this, so I have the real component and I have the imaginary component.

And also I can determine the phase angle. The phase angle is nothing but tan inverse I u v, that is the imaginary part, divided by the real part, the real part is R u v. And in this case u and v is the special frequency along the x direction and the y direction respectively. So, in this case you can see the Fourier Transform I am considering, and I am considering the magnitude of the Fourier Transformation and the phase angle of the Fourier Transform.

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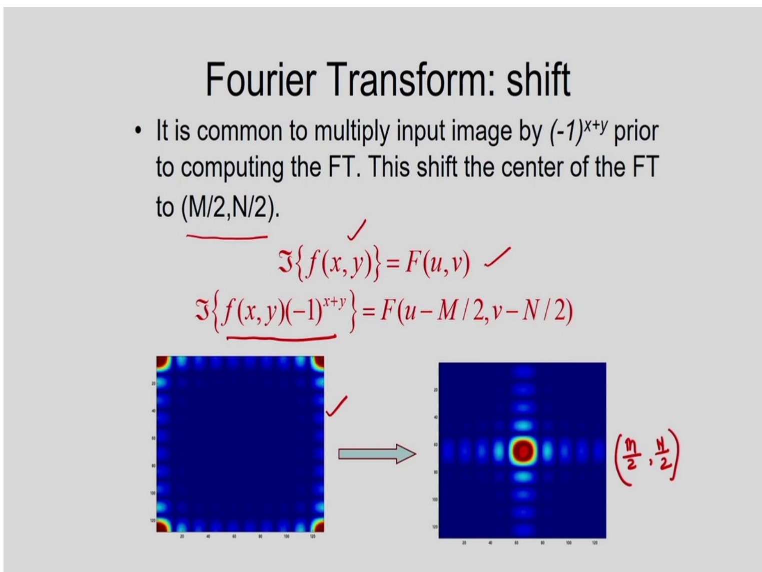

And for the preprocessing what I have to do, if you can see here the Fourier Transform of an image the image is f x y and corresponding to this the Fourier Transform is F u v. So, in this case the input image is multiplied by minus 1 to the power x plus y. Then in this what is the objective of this preprocessing step?

This shifts the center of the Fourier Transform to the point that point is M by 2, N by 2. So, corresponding to the Fourier Transformation, then in this case the center of the Fourier Transform is this and the coordinate is M by 2 and N by 2 that is the coordinate of the center of the Fourier Transform.

So, for shifting I have to multiply the image by minus 1 to the power x plus y, and after this I have to determine the Fourier Transform of the image. This is called the preprocessing. So, first I have to do the preprocessing and after this I have to determine the Fourier Transform of the image. And for displaying the Fourier Transformation already I have explained that I have to consider the log transformation. That is to compress a dynamic range of the image.

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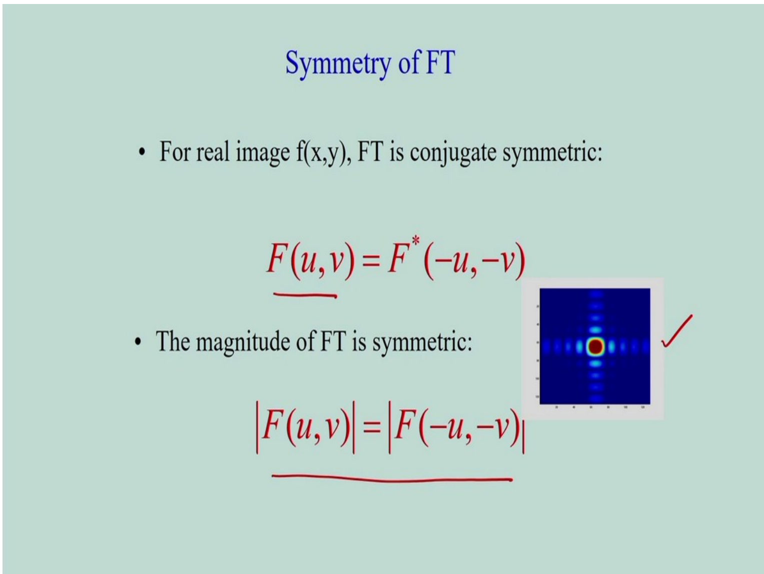

And one important property of Fourier Transform is the symmetric property. If you see here, the F u v is equal to F complex conjugate minus u, minus v that is the conjugate symmetric property. And also if I considered a Fourier Transformation, the magnitude of the Fourier Transform is symmetric. And that is why I am getting the Fourier Transform similar to like this, because of the symmetric. The magnitude of the Fourier Transform is symmetric and also I have to consider the Fourier Transform is conjugate symmetric. So, that is why I am getting this Fourier Transformation.

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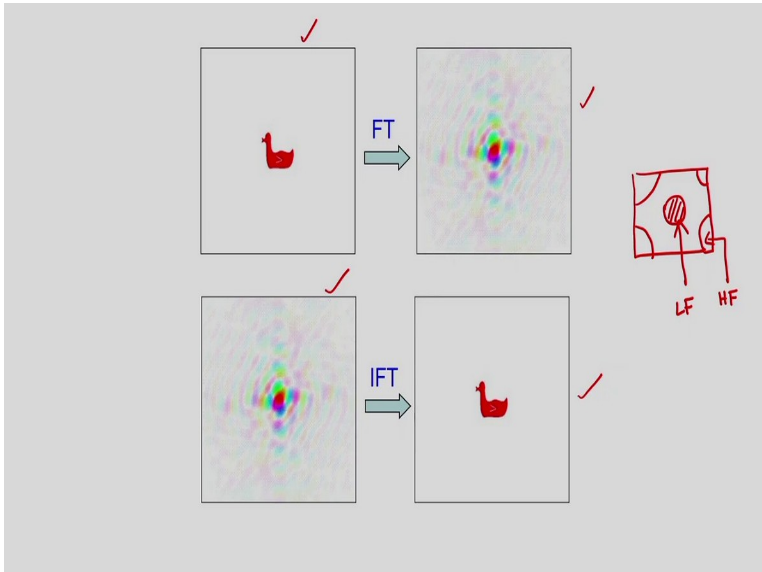

And in this case, I have already explained this one in one of my classes. That is if I take the Fourier Transform of the input image, my input image is this and I am determining the Fourier Transform of the image, so corresponding to this I am getting the Fourier Transform of this. And in this case, if I want to reconstruct the original image, I have to apply the inverse Fourier Transformation.

So, in this case this is the Fourier Transform of the image, and after this I am applying the inverse Fourier Transformation to get back the image, that is the reconstructed image. And in this case the perfect reconstruction is possible, because I am considering all the frequency information presented in the image.

And in case of the Fourier Transformation, the central part, supposes if I considered the Fourier spectrum here, this central portion corresponds to the low frequency part, this is the low frequency part, that is the low frequency information. And if I consider the outside portion, like this that corresponds to the high frequency information. The central position is the low frequency information and the outside if you see, that is the high frequency information.

In this case I am considering all the frequency information for reconstruction so that is why the perfect reconstruction is possible in this case.

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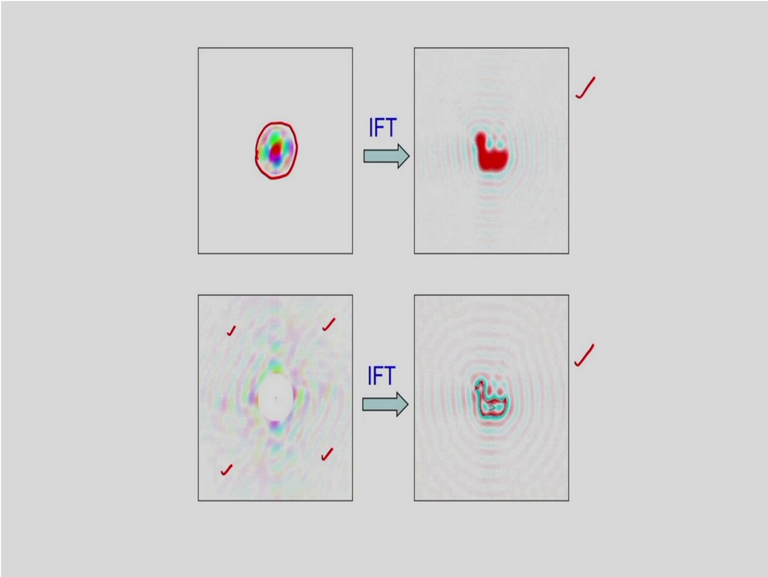

In this example you can see I am considering only the central portion of the Fourier Transformation, that corresponds to the low frequency information. And if I apply the inverse Fourier Transformation, then you can see the reconstructed image. So, this is the reconstructed image.

And in the second example what I am considering, I am neglecting the low frequency information, that means the central portion I am not considering, I am considering only the outer portion of the Fourier Spectrum. That means I am considering the high frequency information. And if I determine the inverse Fourier Transformation, then in this case this is my reconstructed image.

So, in the first case what you will be getting that because of the low frequency component it gives general appearance of the image. Only I have the low frequency information so it gives general appearance of the image. But in the second case I am considering the high frequency information, that corresponds to fine details of the image.

Fine details mean like the edges, boundaries the fine information present in the image.

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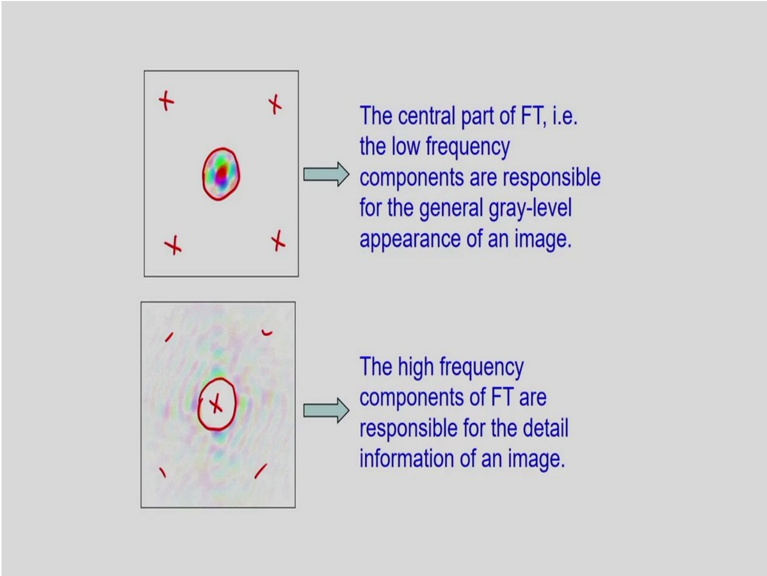

So, here I am explaining this one. The central part of the Fourier Transform, that is the low frequency components are responsible for the general gray level appearance of the image. On the other hand, the high frequency components of the Fourier Transform are responsible for the detail information of an image. So, this concept already I have explained.

Now in case of the frequency domain filtering I can consider the filters like this, suppose if I consider a low pass filter, for low pass filter I have to select this portion only. I have to neglect the outer portion of the spectrum. Then in this case I will be getting the low pass filter image. And if I consider high pass filter, then I have to neglect the central portion of the Fourier Transformation, so this portion I am neglecting and I am considering the outer portion of the Fourier Transformation.

So, that means I can consider the high pass filter and the low pass filter based on this concept.

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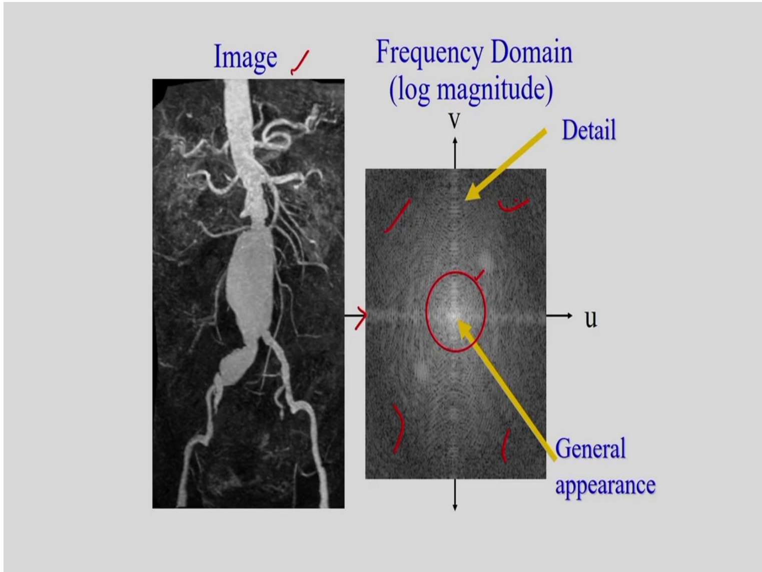

Here, I am showing one example. I have shown one image, that is the image is this, input image. And corresponding to this image I have the Fourier Transform, this is my Fourier Transform. And if I consider this circle, I have shown a circle, the red circle. And if I consider this portion only that means I am only considering the low frequency information, that gives general appearance of the image.

And if I consider the outer portion, the outside portion like this, if I consider this portion or this portion or this portion, it gives the fine details of the image. Like the edges boundaries or a fine information of the image.

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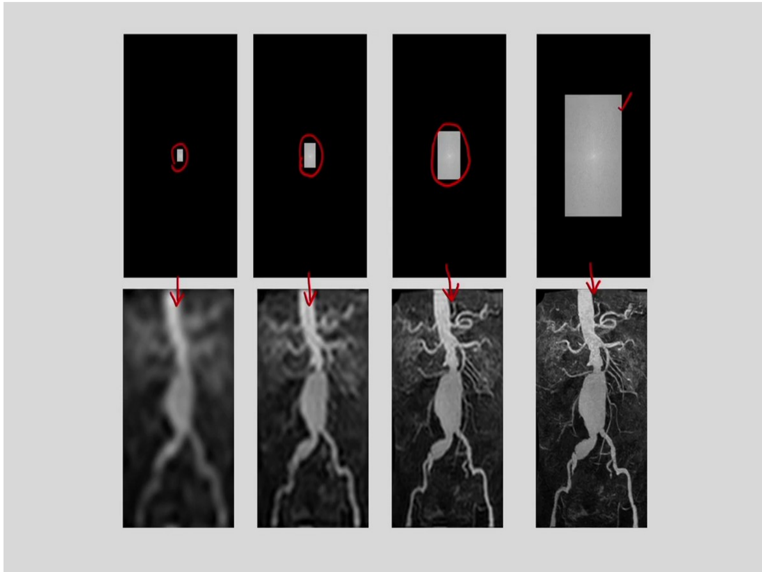

And in this case, again I am showing here what I am considering. I am considering the Fourier Transformation and I have to reconstruct the image by using the inverse Fourier Transformation. In the first case you can see only I am considering this portion of the Fourier Transform. That means I am only considering the low frequency information and corresponding to this, this is my reconstructed image.

After this if you see the second case, I am considering this portion of the Fourier Transformation, that means I am increasing the size. That means I am only considering the low frequency information or maybe some high frequency information also I am considering maybe. And corresponding to this one my reconstructed image will be like this.

In the third case I am increasing this portion, if you see, that means I am considering low frequency information and maybe some high frequency information. And considering this, if I determine the inverse Fourier Transformation, then I will be getting the reconstructed image like this. And in the final case, you can see I am considering the big portion here.

So, this portion I am considering that means I am considering the low frequency information and the high frequency information. Not all the high frequency information but significant high frequency information I am considering. And corresponding to this, this is my reconstructed image. So, you can see the concept of the filtering, the frequency domain filtering.

So, based on selection of this, the portion of the Fourier Transformation you can reconstruct the image. You can select the low frequency information, you can select the high frequency information, like this, based on your requirement you can select the portion of the Fourier Transformation. This is the Fourier spectrum.

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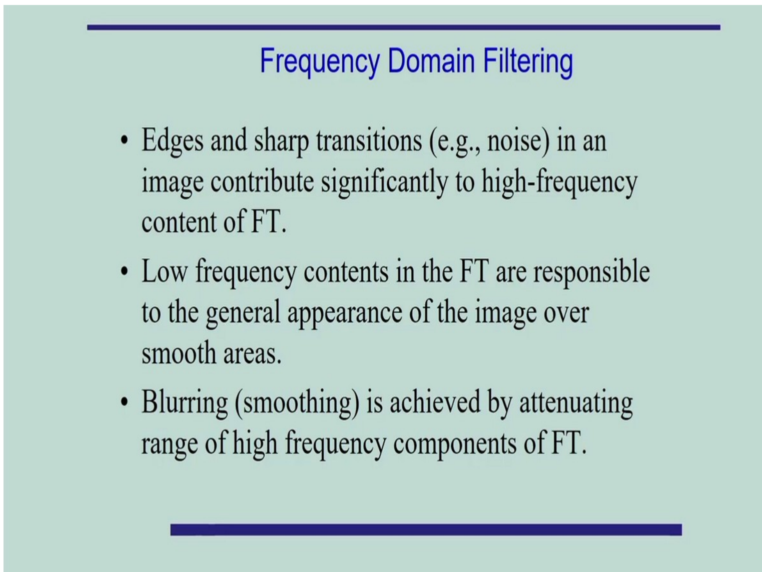

So, in the frequency domain filtering edges and the sharp transitions is mainly the high frequency content of the Fourier Transformation. Low frequency content in the Fourier Transform are responsible to the general appearance of the image, that means it gives the information of the general appearance of the image. And in this case the blurring or the smoothing is achieved by attenuating range of high frequency components of Fourier Transformation. That means if I only consider the low frequency information, I can do the blurring, I can do the smoothing.

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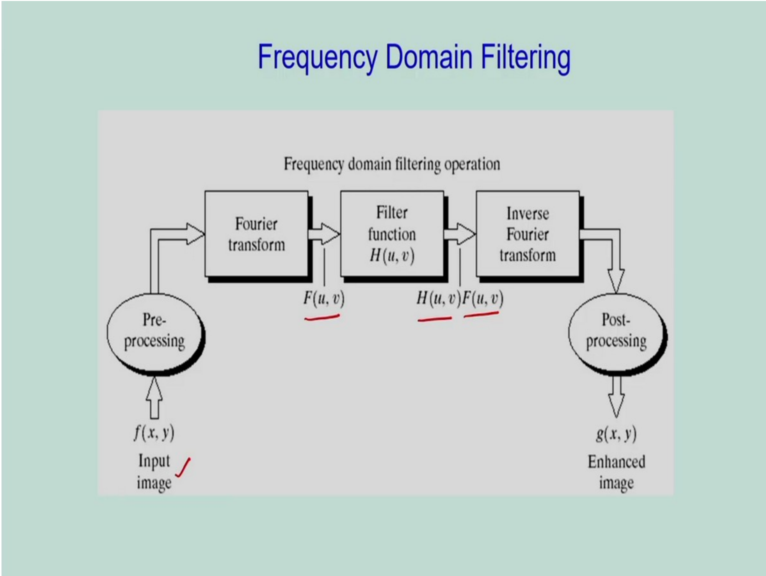

And this is the typical frequency domain filtering. So, first one is the input image I am considering. After this I am doing the preprocessing, the preprocessing is nothing but the image is multiplied by minus 1 to the power x plus y. And after this I am taking the Fourier Transform of the image, so the F u v is the Fourier Transform of the input image. And after this I am considering the filter function. The function is H u v. The filter function maybe low pass filter, maybe the high pass filter, I am considering the filter function, the function is H u v.

The filter function is multiplied with the image, that is the F u v. F u v is the Fourier Transform of the image. That is multiplied with the filter function. After this, I am taking the inverse Fourier Transformation to reconstruct the image after the processing. That means the processing means the frequency domain filtering. And in the post-processing, again I have to multiply this by minus 1 to the power u plus v. So, I will be getting the output image, the output image is g x y. That is the enhanced image.

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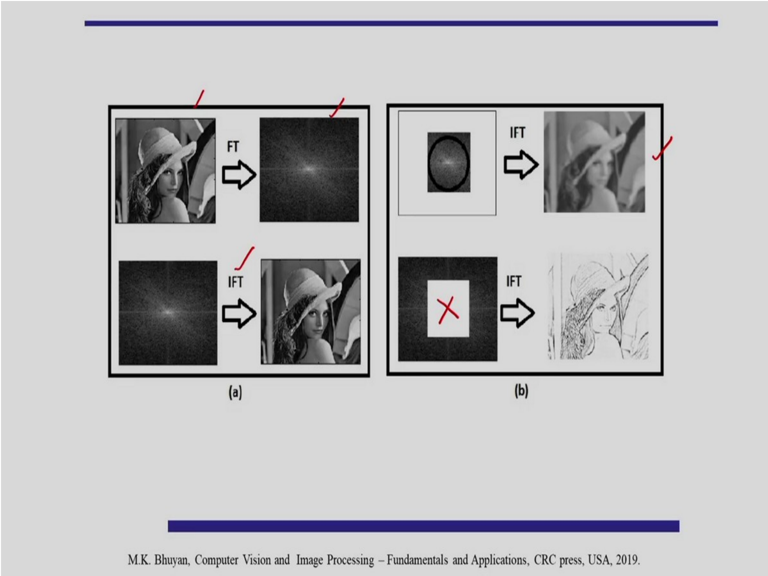

And this concept I am showing here. So, I have the input image, in the first diagram if you see in the first figure, I have the input image. I am taking the Fourier Transform of the image, corresponding to this I am getting the Fourier Spectrum. And I have shown the reconstructed image by considering the inverse Fourier Transformation. So, here I am showing the inverse Fourier Transformation.

And in the second figure if you see here, I am only considering the central portion of the Fourier Transformation and I am reconstructing the image based on the central portion of the Fourier Transformation. Then in this case I will be getting the blurred image. That is nothing but the low pass filtered image.

In the second case I am neglecting the central portion of the Fourier Transformation. This portion I am neglecting. I am only considering the outside portion of the Fourier Transformation and I am applying the inverse Fourier Transformation for reconstruction. Then in this case I am getting the edges and the boundary, the fine details of the image. So, from this you can understand the concept of the low pass filtering and the high pass filtering.

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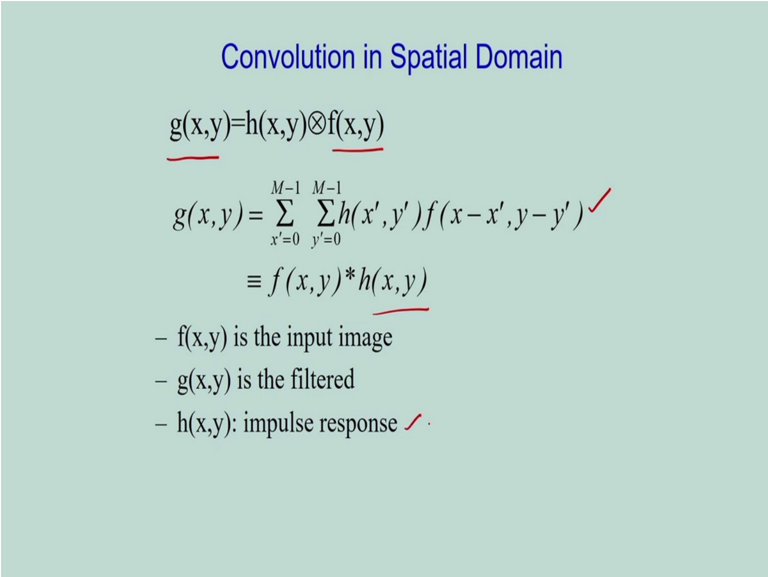

Now, you can see the convolution operation in spatial domain can be represented like this. So, g (x, y) is the output image suppose, my input image is f (x, y), I am doing the convolution of f (x, y) with h (x, y). In this case I am considering g (x, y) is the output image, f (x, y) is the input image and I am considering the convolution of the input image with h (x, y).

So, this is the convolution operation. You can see. So, g (x, y) is equal to f (x, y) convolution h (x, y). So, f (x, y) is the input image, g (x, y) is the output filtered image, and I am considering the impulse response, the impulse response is h (x, y). So, this is the definition of the convolution in spatial domain.

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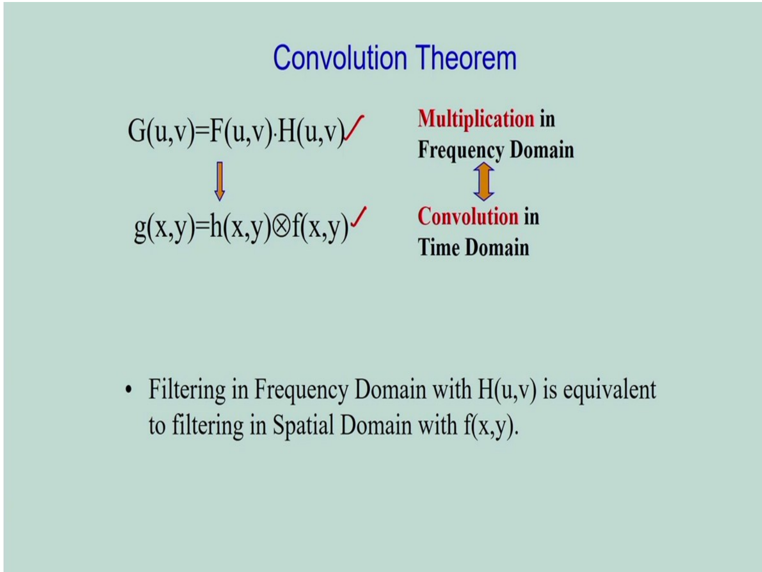

And already you know in signal processing the convolution theorem. The convolution in the spatial domain or in the time domain is equivalent to multiplication in the frequency domain. That is the convolution theorem. So, here you see a convolution in the spatial domain is equivalent to multiplication in the frequency domain.

So, that means in the frequency domain filtering what I have to do, I have to do the multiplication instead of convolution. So, in this case F (u, v) is the input image, the Fourier Transform of the input image, and H (u, v) is the filtered transform function. So, only I have to do the multiplication between F (u, v) and H (u, v).

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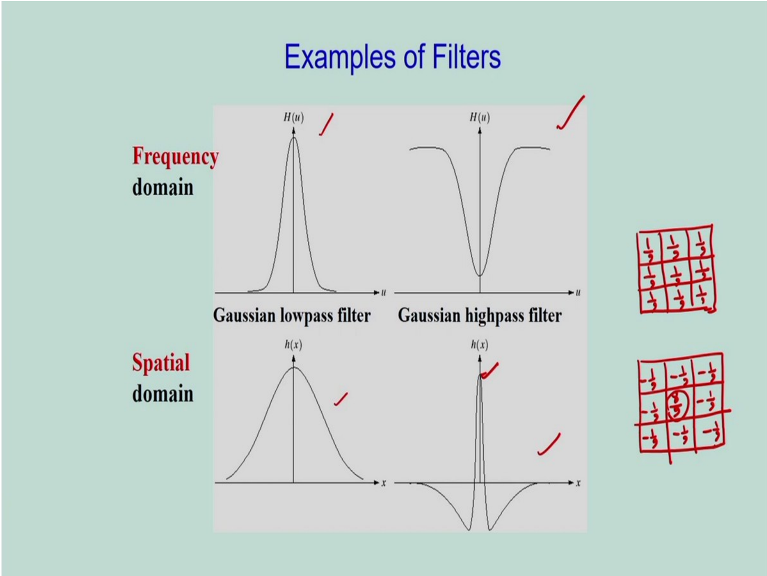

And in this case I have shown one example of the Gaussian low pass filter. So, in the frequency domain I have shown. And corresponding to this I have shown the function in the spatial domain. In spatial domain it is something like this. That is nothing but the averaging. Because if you see the averaging filter in the spatial domain, if you see the filter coefficients of a filter widths, the filter widths are something like this, 1 by 9, 1 by 9, 1 by 9, 1 by 9, 1 by 9, 1 by 9, so these are my filter coefficients for a low pass filter.

And if you see this Gaussian high pass filter, this is in the frequency domain. And corresponding to spatial domain this is my function, that is h (x), corresponding to the high pass filter. If you see the central portion the high value, because in the high pass filtering if you know what is the mask corresponding to the high pass filter in the spatial domain.

If you see the central pixel it is 8 by 9. The remaining is 1 by 9, minus 1 by 9, minus 1 by 9, like this. This is the mask corresponding to the high pass filter in spatial domain. So, that is why corresponding to this 8 by 9 I am having a peak here in the spatial domain. You can see the response of the filter in frequency domain and in the spatial domain.

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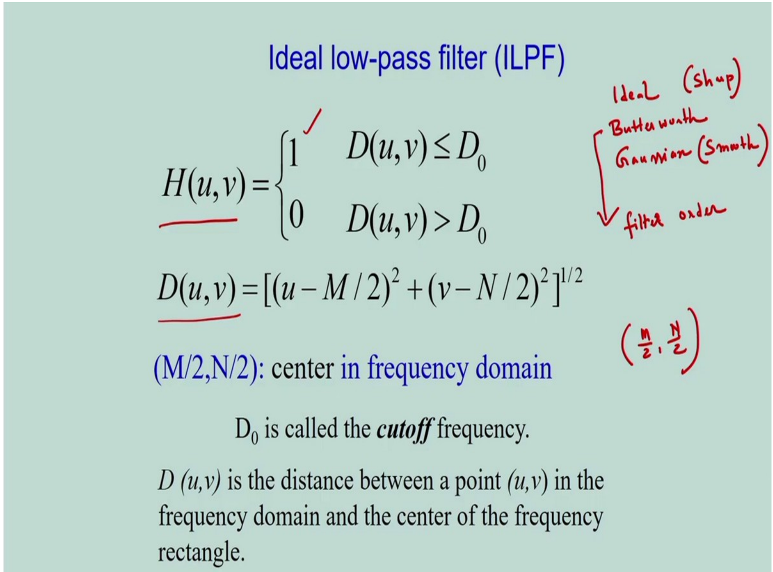

Now, first I am considering the ideal low pass filter. So, before going to the ideal low pass filter in my discussion I will be considering the ideal low pass filter. So, I will be considering the ideal filter, I will be considering the Butterworth filter, also I will be considering the Gaussian filter. In case of the ideal filter that is mainly the sharp filtering, you can consider as a sharp filtering.

And in case of a Gaussian filter it is something like the smooth filtering. In case of the ideal filter the transition from the pass band to the stop band is very sharp. I am repeating this. For the ideal low pass filter or maybe the ideal filter the transition from the pass band to the stop band is very sharp. So, that is why I am getting the sharp filtering. But in case of the Gaussian it gives smooth filtering. In case of the Butterworth filter we have a parameter, the parameter is the filter order. In Butterworth filter we have a parameter that parameter is filter order.

If I considered the high order values, the filter order is very high. The Butterworth filter approaches to the ideal filter, for lower order values the Butterworth filter is more like a Gaussian filter. So, that means in this case Butterworth filter maybe viewed us providing a transition between two extremes, one is ideal low pass filter, another one is the Gaussian low pass filter. Now corresponding to the ideal low pass filter in the frequency domain, I am defining the filter function, the filter function is H (u, v).

Already I have explained what is u? u is the special frequency in the x direction and v is special frequency in the y direction. And corresponding to this I am considering D naught is a positive constant. D naught is called as cutoff frequency, it is a positive constant. And in this case if D u v is less than equal to D naught, the response will be 1. If D u v is greater than D naught, the response will be 0.

Now, what is D u v? What is the meaning of the D u v? D u v is the distance between a point u v in the frequency domain and the center of the frequency rectangle. The center of the Fourier Transform is M by 2 comma N by 2. So, D u v, that is the distance between a point u v in frequency domain and the center of the frequency rectangle, that is D u v. And D naught is a cutoff frequency, D naught is a positive constant. So, this is a definition of the ideal low pass filter.

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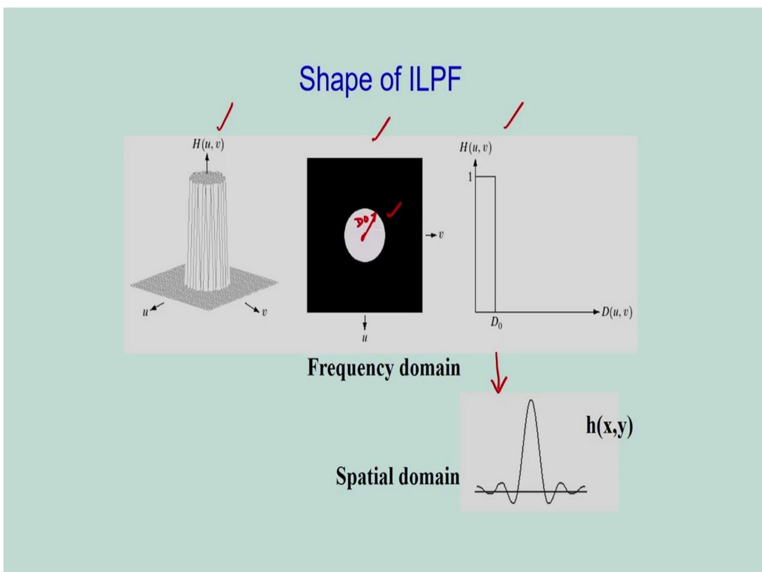

Corresponding to this filter transfer function I am showing the ideal low pass filter. So, if you see the H u v, that is the filter function, that is very similar to the box function. And in the second case I am showing the filter as an image. The filter as an image I am showing. And in the final case, if you see the last case, I am showing the filtered response, that is the low pass filter. And D naught is the cutoff frequency. And already I have explained that D naught is a positive constant.

So, all the frequencies on or inside the circle of radius D naught, so in this case I am considering this circle. And radius is D naught here. So, all the frequencies on or inside the circle of radius D naught are passed without attenuation. Whereas, all frequencies outside the circle are completely attenuated. And in this case you can see the ideal low pass filter is symmetric about origin. So, this is the meaning of the D naught.

So, if you see the previous expression for the filter function, you can see D u v is less than equal to D naught, if this condition is satisfied then H u v will be 1. D u v is greater than D naught, then H u v will be 0. That means all the frequencies on or inside the circle of radius D naught are passed without attenuation. Whereas, all frequencies outside the circle are completely attenuated. That is the meaning of this expression.

And in this case of you see, corresponding to this rectangular function, in spatial domain I have the same function in spatial domain. So, this is the concept of the ideal low pass filter in frequency domain and in the spatial domain.

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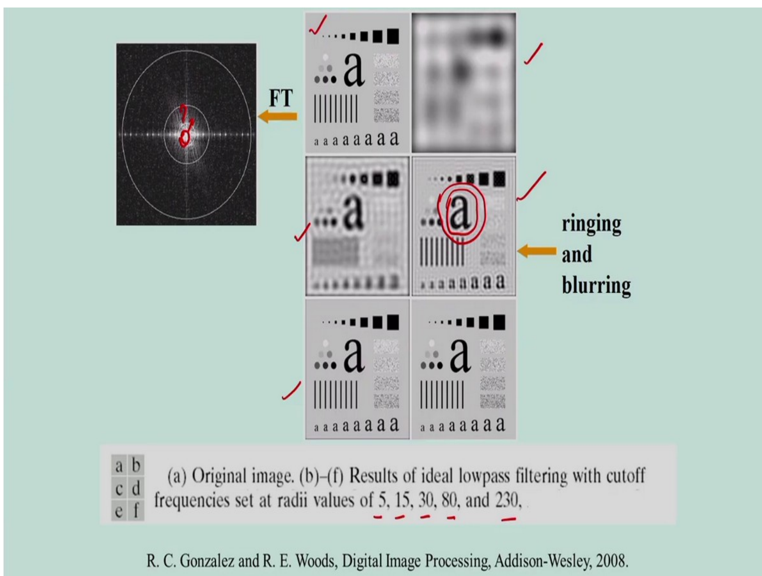

Now, in this case I am showing the example of the ideal low pass filter. And in this case you see my input image is this, the first one is the input image. After this what I am considering? I am considering the ideal low pass filtering with cutoff frequency, that is the cutoff frequency is the D naught and the radii values, something like the 5, 15, 30, 80, and 230. If the radii value is 5, that means I am considering a very small circle, radius value is 5.

Corresponding to this, this is my output image. That means this is blurred image. Because I am considering only the low frequency information. And suppose if I consider suppose the cutoff frequency, the radius is 15, corresponding to 15 my reconstructed output image will be like this.

Corresponding to 30, that means my output will be something like this. That means I am continuously selecting the low frequency information and also some high frequency information. That means I am increasing the size of the circle. So, this is the circle here, I am increasing the size of the circle like this. And in this case you can see, if I consider the radius is 5 only the image will be completely blurred.

And if I consider suppose the radius is something like 80, then in this case the blurring will be less. But if I consider this case, suppose the radius is 30, you can see some effects the rings like this. This is called the ringing artifacts. You can see the ringing and the blurring. The blurring depends on the size of the radius. That means it depends on the cutoff frequency. The cutoff frequency is D naught.

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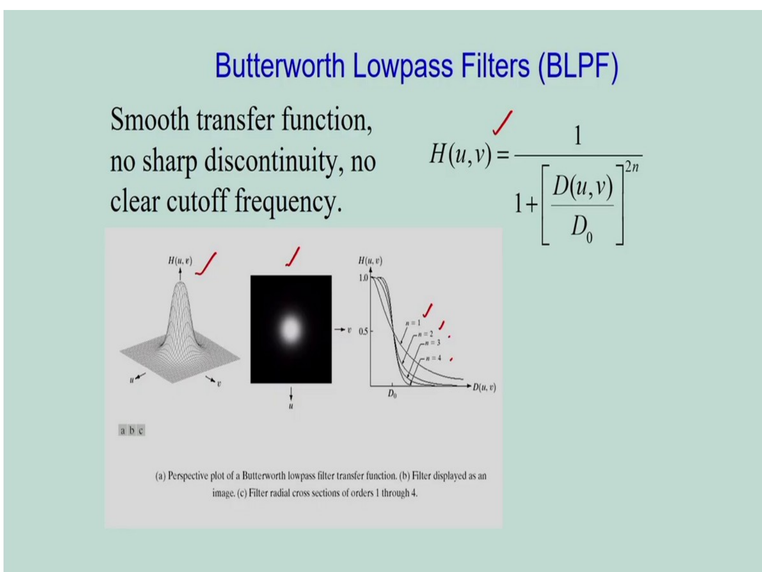

Now, I will consider the concept of the Butterworth low pass filter. And corresponding to the Butterworth low pass filter you can see the function, the transfer function in H u v is equal to 1 divided by 1 plus D u v divided by D naught to the power twice n. And in this case I have shown the response for different orders of the Butterworth filter. The first one is n is equal to 1, n is equal to 2, 3, 4 like this.

That means in this case is I increase the order of the Butterworth filter, it approaches the ideal low pass filter. In the first case I have shown the Butterworth filter transfer function. So, this is the Butterworth filter transfer function. The second one is the filter displayed as an image. So, this is the Butterworth filter displayed as an image. And in this case I have shown the filter radial cross sections of order 1 to 4.

And in case of the Butterworth filter, if I considered suppose low order, then in this case there is no sharp discontinuity, no clear cutoff frequency for the low order Butterworth filter. That means if I consider a high order Butterworth filter, it approaches the ideal low pass filter.

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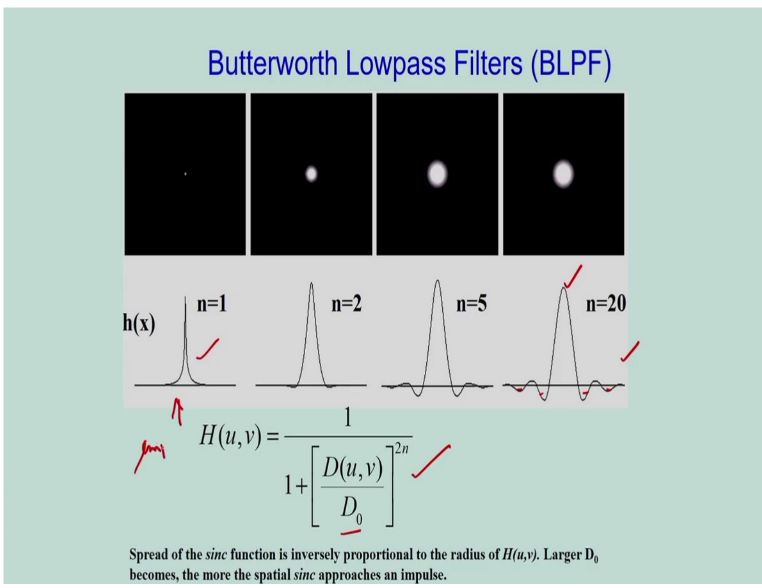

So, in this case you can see I have shown the Butterworth low pass filter that is displayed as an image. So in this case you can see, H u v is the filter transfer function of the Butterworth filter. Now, corresponding to frequency domain, my response in the spatial domain will be sinc function. So, in the case of the Butterworth low pass filter the spread of the sinc function is inversely proportional to the radius of H u v.

That means, in this case if I consider the radius is D naught, so the spread of the sinc function that is in the spatial domain, sinc function in the spatial domain is inversely proportional to the radius of H u v. So, if I consider D naught is high suppose, then in this case the sinc function approaches the impulse function. So, this is my impulse.

I repeat this. The cross section of the low pass filter, the ideal low pass filter in the frequency domain looks like a box filter. But in the spatial domain it would be sinc function. So, here I have shown the sinc function. And in case of the filtering in the spatial domain, is done by convolving h x y with the image. And in this case, each pixel in the image is like a discrete impulse.

So, if I considered a image, each pixel in the image is like a discrete impulse and convolving a sinc function with an impulse copies the sinc at that location of the impulse. I am repeating this, that means convolving a sinc function with an impulse, impulse means the pixel value, copies the sinc at that location of the impulse.

Now, in case of a sinc function, the central lobe of the sinc function is responsible for blurring. So, I have a central lobe, this is a central lobe, that is responsible for blurring. And in this case if I consider outer lobes, small lobes, they are responsible for ringing artifacts. So, this central portion, central lobe is responsible for the blurring and the side lobes, the small lobes are responsible for the ringing artifacts.

And in this case already I have explained the spread of the sinc function is inversely proportional to the radius of H u v, that means the larger D naught becomes the more the spatial sinc function, approaches an impulse. So, that means if I considered the high value of D naught, then the sinc function approaches to the impulse, the impulse is this.

Then in this case if I considered the impulse in spatial domain, that means there will be no blurring. Corresponding to this case I have the blurring because of the central lobe of the sinc function. Also, I have the ringing artifacts.

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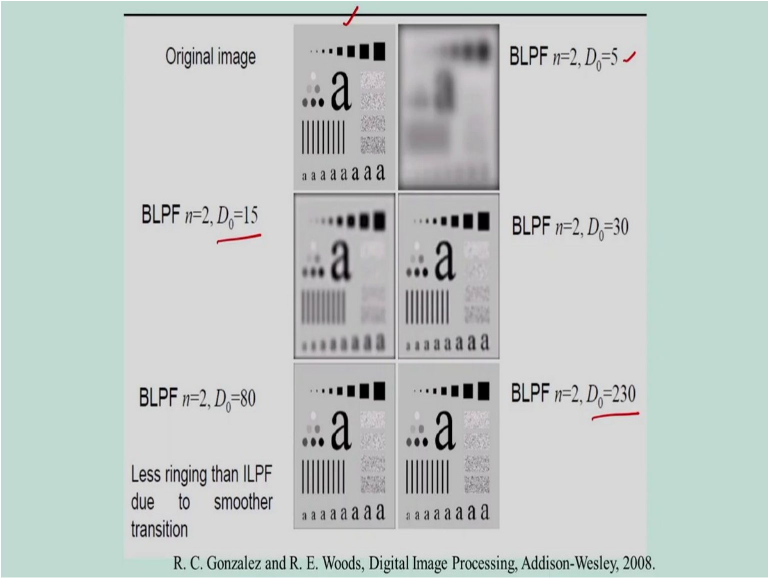

So, in this case I have shown the example here. Original image I have shown, and I am considering the Butterworth low pass filter. The order I am considering n is equal to 2, and I am changing the cutoff frequency, the cutoff frequency is D naught I am changing. The first one is 5, next one is 15, 30, 80, 230 like this. And in this case if you see, if a cutoff frequency is less the blurring will be more. Because of the central lobe of the sinc function.

And in this case if I considered the high value of D naught, suppose D naught is 15, corresponding to this I have the ringing artifacts because of the side lobes, the side lobes of the sinc function. And if I considered the high value, D naught is very high, 230, then corresponding to this I have the impulse, that is the sinc function to be converted into impulse, then I do not have the blurring effect. Then ringing will be also less because of the smoother transition from the pass band to the stop band. So, you can understand that concept of the ringing artifacts and also the concept of blurring.

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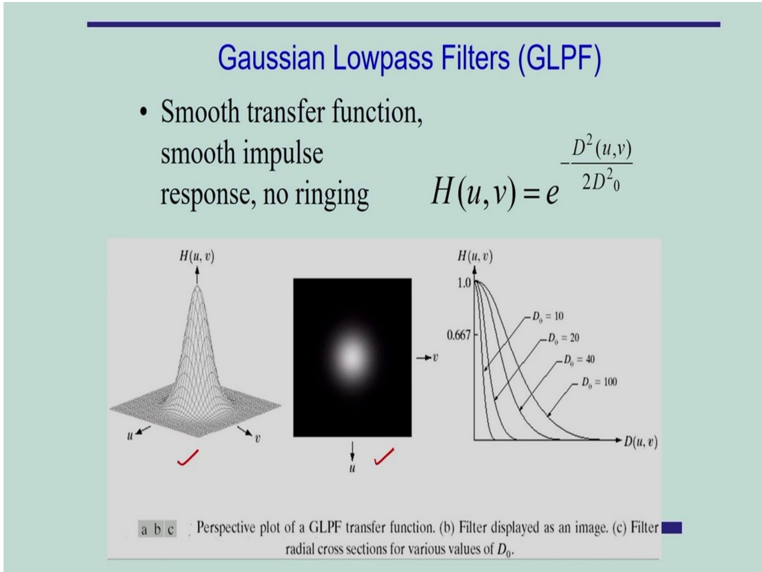

And after this I will discuss the concept of the Gaussian low pass filter. And in case of this, I am getting the smooth transition from the pass band to the stop band. And in this case, smooth impulse response and the ringing artifacts will be less in case of the Gaussian filter. So, first one is, I have shown the Gaussian low pass filter transfer function, the second one is the filter displayed as an image. And after this I am showing the Gaussian low pass filter for different values of D naught. D naught is equal to 10, 20, 30,40, 100 like this. For different values of D naught I am showing the filter radial cross sections for values of D naught.

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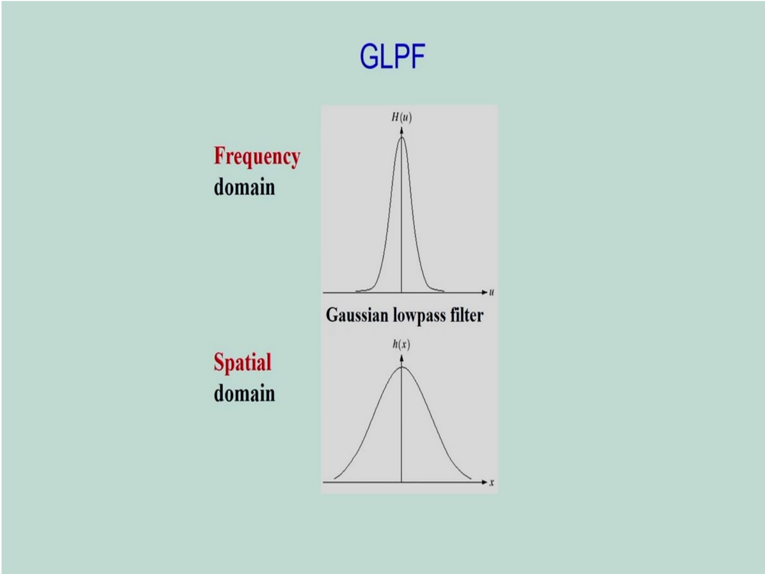

And in this case I have shown the Gaussian low pass filter in frequency domain and after this I have shown the corresponding to spatial domain representation of the Gaussian low pass filter.

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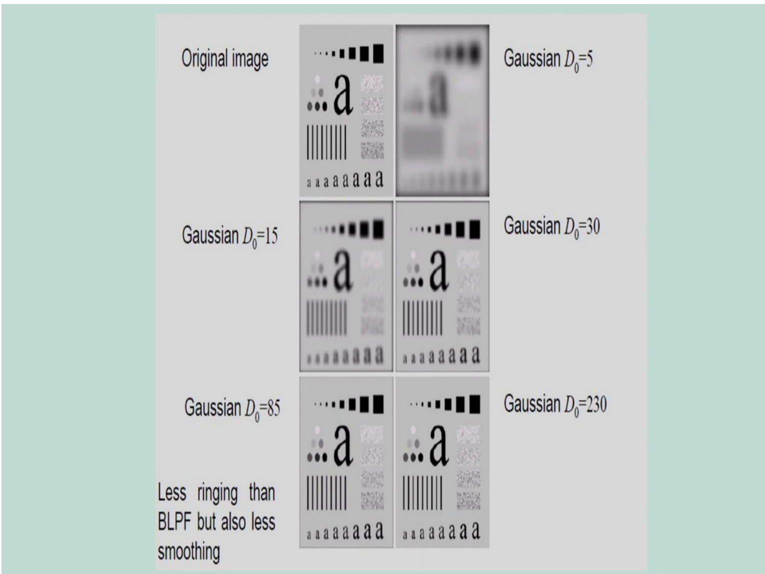

And corresponding to the Gaussian low pass filter I have the outputs the first I am considering the original input image and I am considering the D naught, something like the 5, 15, 30, 85, 230. And you can see the blurring and the ringing artifacts. Then in this case, the less ringing than the Butterworth low pass filter, but also less smoothing. The blurring will be less. Ringing artifacts will be less but also the less smoothing.

So, I have shown the comparison between the ideal low pass filter, the Butterworth filter and also the Gaussian low pass filter. And in this case the one important concept that you have to understand, the concept of ringing artifacts and the blurring.

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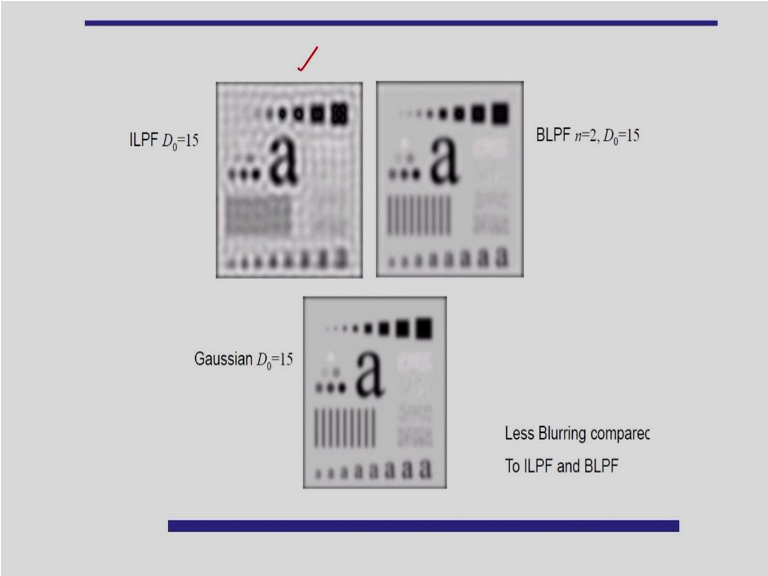

And in this case I have shown the ideal low pass filter response corresponding to D naught is equal to 15, you can see the blurring and the ringing artifacts. And in the next case I am considering the Butterworth low pass filter, the order is 2, n is equal to 2; and D naught is 15, again 15. You can see the ringing artifacts will be less. And in this case of the Gaussian, D naught is again 15, less blurring but ringing artifacts will be less. So, you can see the comparison between the ideal low pass filter, the Butterworth low pass filter and also the Gaussian low pass filter.

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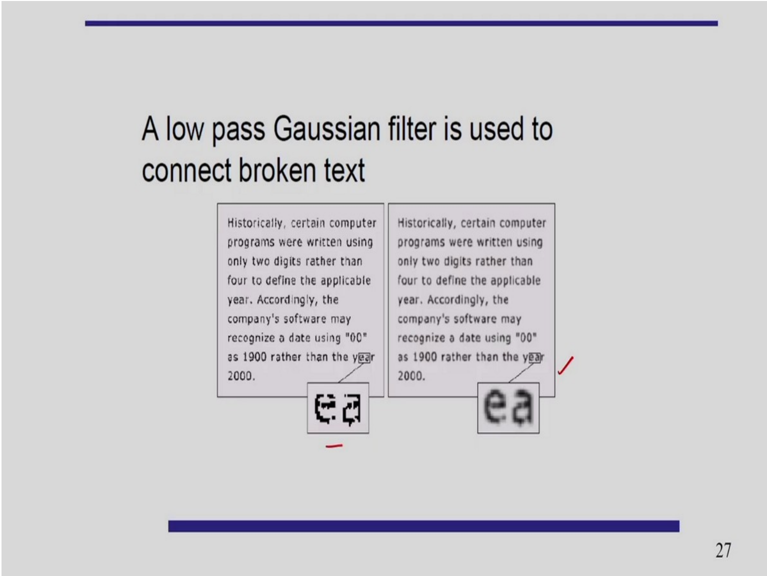

Next I am showing one example, one practical example. A low pass Gaussian filter is used to connect broken text. So, in the first figure if you see, I have shown the broken text here and if I apply the Gaussian filter, the low pass filter, then you can see the output that in this case we can connect the broken text because of the blurring. Because the Gaussian filter produces the blurring of the image. So, that is why the broken text is connected.

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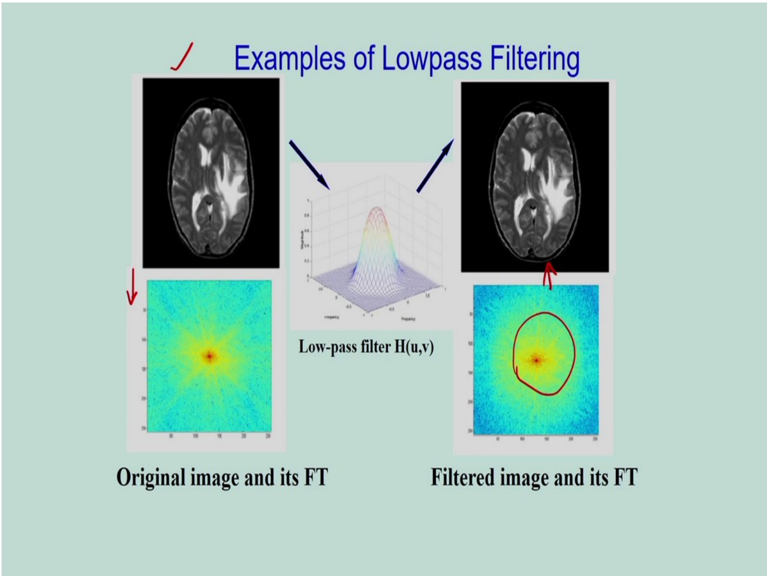

Again I am showing the concept of the low pass filtering. My input image is this and corresponding to this my Fourier Transform is this. The original image and its Fourier Transformation. In the next case I am showing the reconstructed image, in the second case what I am considering, I am only considering the central portion of the Fourier Transformation. And this is the reconstructed image. That is the filtered image and corresponding Fourier Transformation. So, you can see the output image and the input image. And you can do the competition between the input image and the output image visually.

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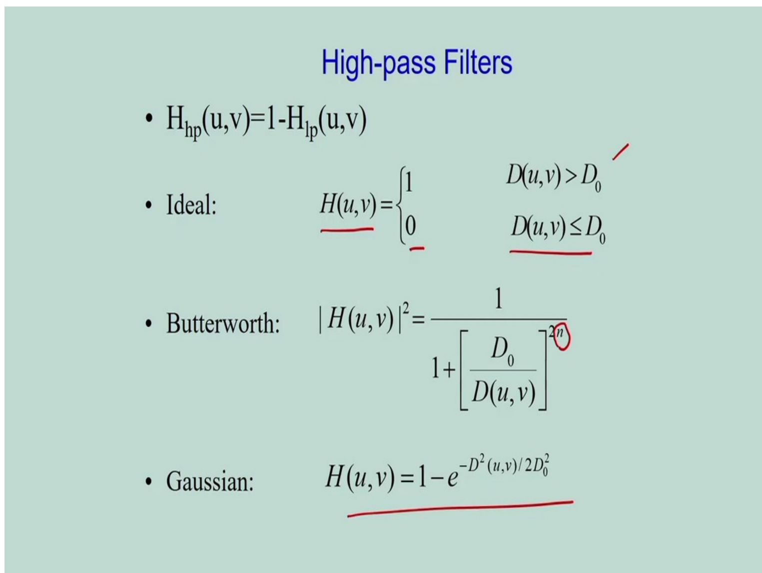

Next one is the concept of the high pass filter. So, what is high pass filter? The high pass filter is nothing but 1 minus low pass filter. So, in spatial domain filtering also I have explained the how to get the high pass filter. High pass filter is nothing but 1 minus low pass filter. So, in this case also I have three cases, one is the ideal high pass filter Butterworth filter and the Gaussian high pass filter.

And corresponding to this you can see the filter function, that filter function is H u v and it is equal to 1, corresponding to D u v greater than D naught, D naught is the cutoff frequency, that is the positive constant. And it is equal to 0 if this condition is satisfied. For the Butterworth filter also, I have to considered the parameter. The parameter is the order of the filter. The order of the filter is n. And Gaussian transfer function for the high pass filter is H u v you can see this expression. So, this is a Gaussian high pass filter.

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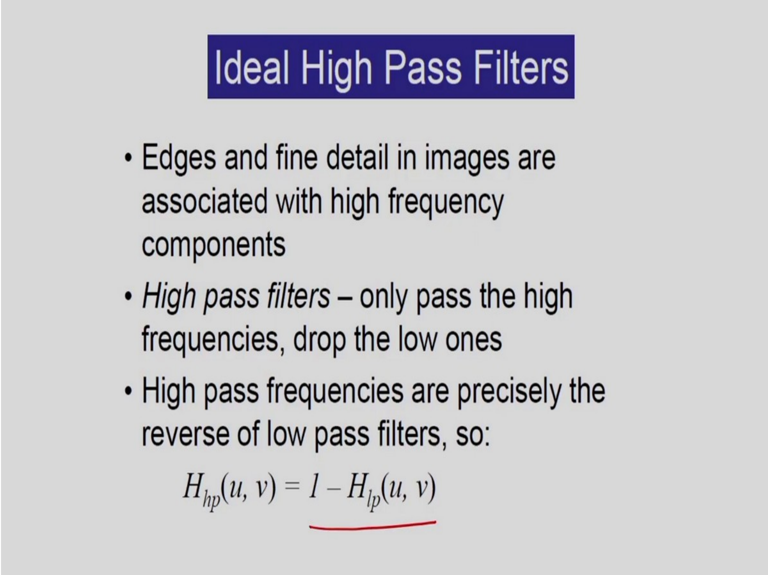

So, for finding the fine details of the image like edges and the boundary I can consider the ideal high pass filter, so high pass filter is nothing but 1 minus low pass filter. So, by using the high pass filter we can see the fine details of the image like the edges in the boundary.

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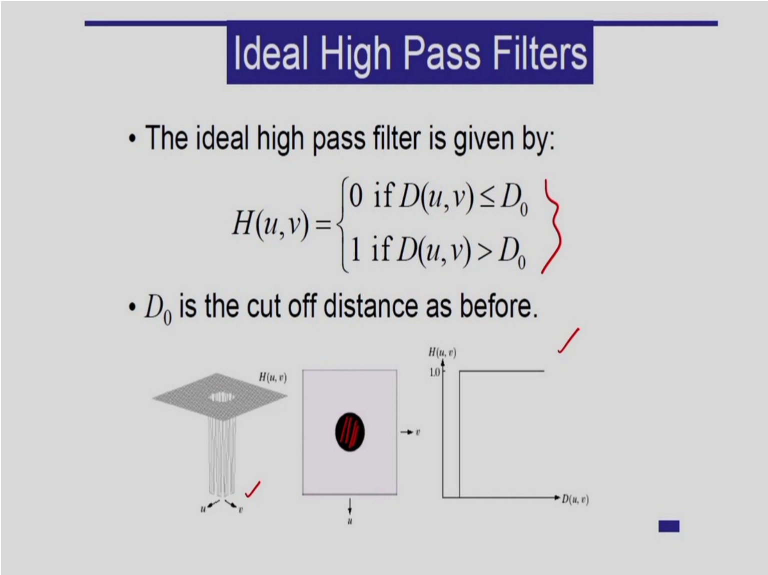

So, like the ideal low pass filter again I am considering the ideal high pass filter, and in this case this is the condition for the filter transfer function. So, I have shown the H u v. So in case of the high pass filter we have only considered the outer portion of the Fourier Transformation. This black portion, if you see this black portion, that is not considered. So, only I am considering the outer portion of the Fourier Transformation. And corresponding to this you can see the response of the high pass filter, and D naught is the cutoff frequency in this case.

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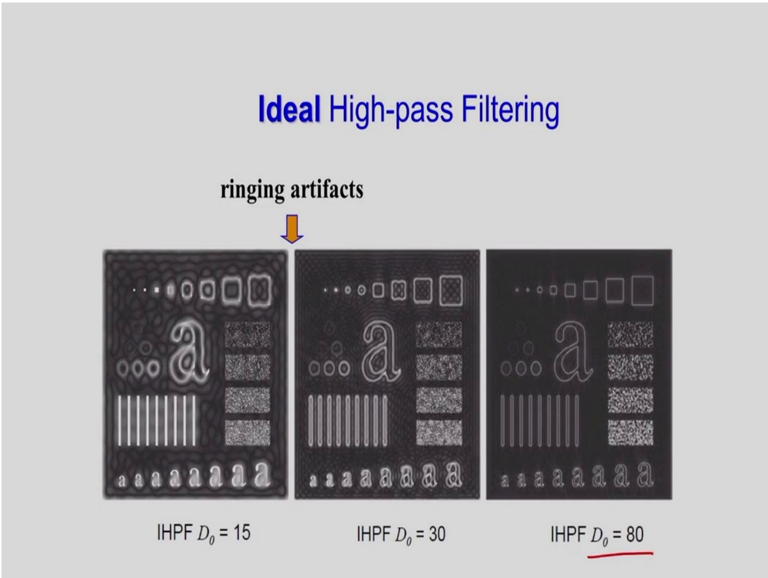

So, in this case you can see the ringing artifacts corresponding to the ideal high pass filter as the transition from the pass band to the stop band is very sharp. So, that is why I have the ringing artifacts. And if you increase the cutoff frequency, the cutoff is D naught, then in this case you can see the ringing artifacts will be less. This ringing artifact concept already I have explained.

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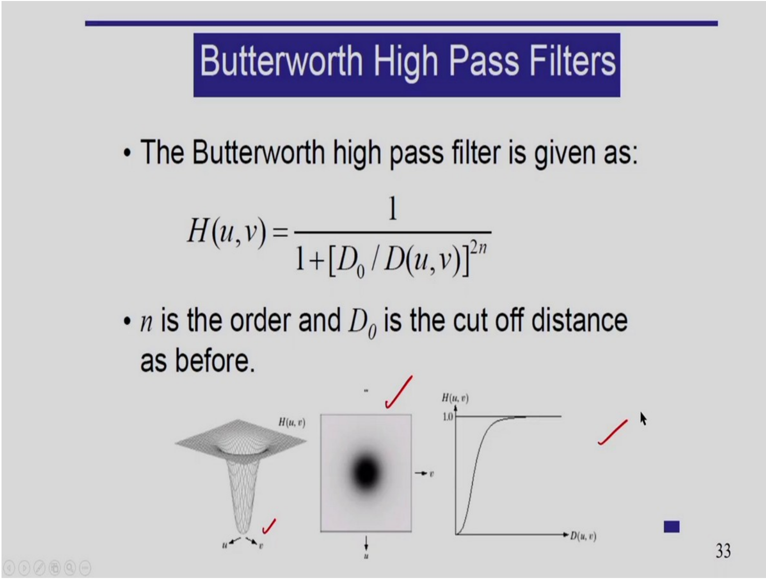

And after this the concept of the Butterworth filter, so you can change the order of the Butterworth filter, n is the order of the Butterworth filter and D naught is the cutoff distance as explained earlier. So, corresponding to this Butterworth filter, you can see the filter function H u v. The filter is displayed as an image and corresponding to this the response of the filter you can see. So, this is the Butterworth high pass filter.

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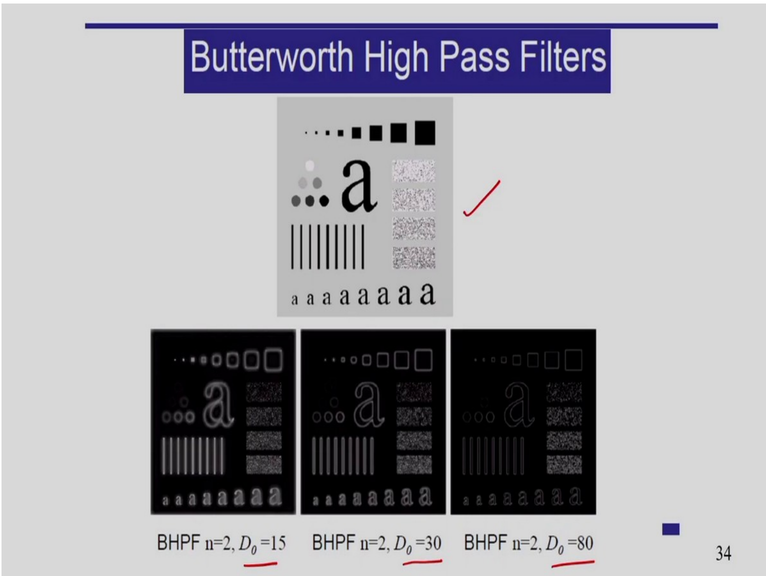

And corresponding to the Butterworth high pass filter, you can see the output, first one is the input image, the next one is the Butterworth filter, order is 2, but D naught is 15. So, corresponding to D naught 15 you can see ringing artifacts. But if I increase D naught to 30 or D naught to 80, then the ringing artifacts will be less.

But, if I consider D naught is very high, then in this case I will be getting the blurred image. Because that means I am considering the high frequency component as well as low frequency component. That means I am considering the high frequency information as well as some low frequency information. That is why the image will be blurred corresponding to D naught is equal to 80.

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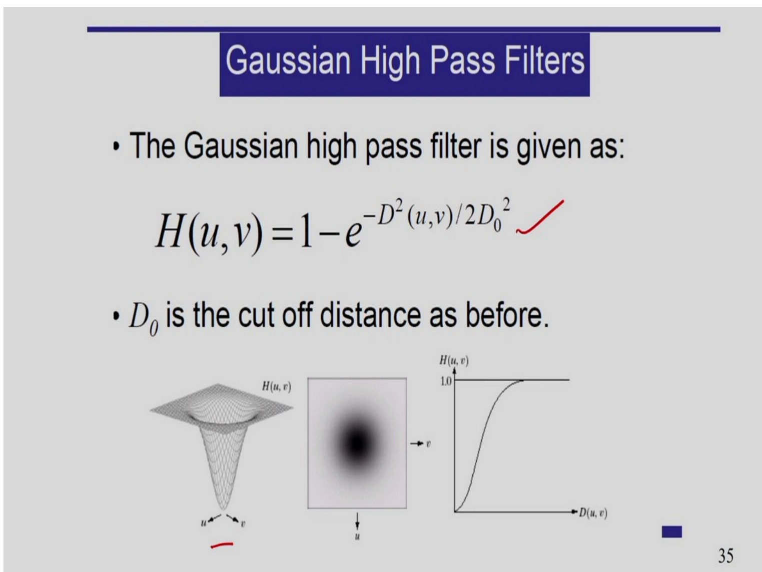

And finally I want to show the Gaussian high pass filter. So, the Gaussian high pass filter is defined like this and D naught is the cutoff distance as discussed earlier. So, corresponding to the Gaussian high pass filter I have the filter function and the filter is displayed as an image and I have the response of the Gaussian high pass filter. Like this.

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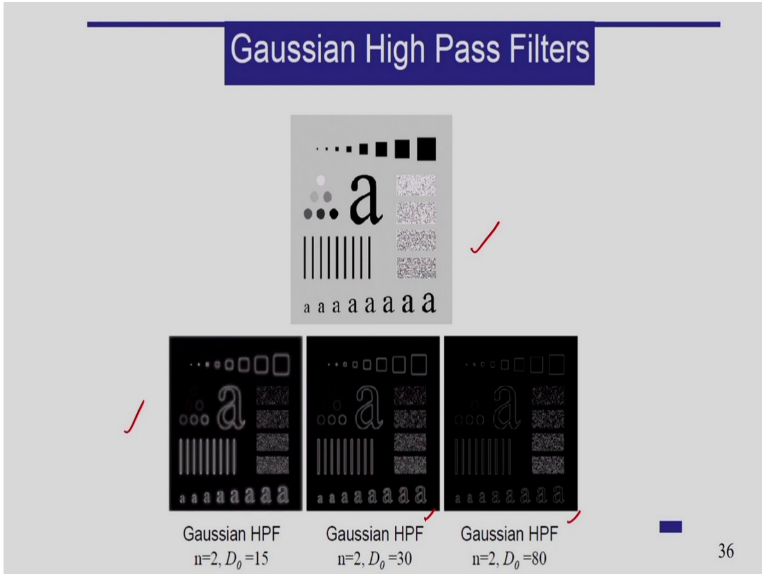

And corresponding to the Gaussian high pass filter you can see the outputs. The first one is the input image, and for different values of D naught you can see the outputs. The order is, I am considering, n is equal to 2 here. So, if you see corresponding to D naught is equal to 15, the first is this. D naught is equal to 30, you can see the output. D naught is equal to 80, you can see the output. If I increase D naught, what will happen, that means I am considering high frequency information also some low frequency information. So, that is why the image will be blurred.

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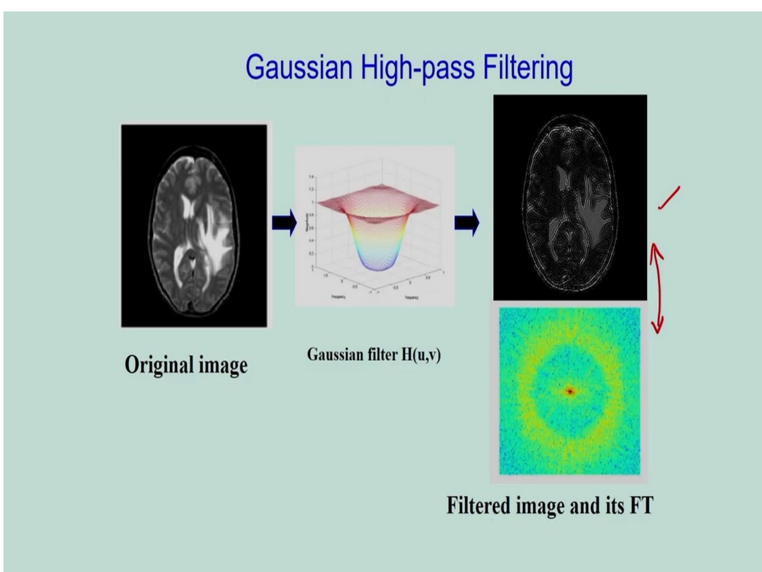

So, here I have shown one example of the Gaussian high pass filter. I have shown the original image after this I am considering the Gaussian filter. Corresponding to the Gaussian filter you can see the output, that is high pass filter. So, I am having the edges. You can see the fine details of the image and corresponding Fourier Transform you can see. Corresponding to this image this is the Fourier Transformation. So, this is the Gaussian high pass filter.

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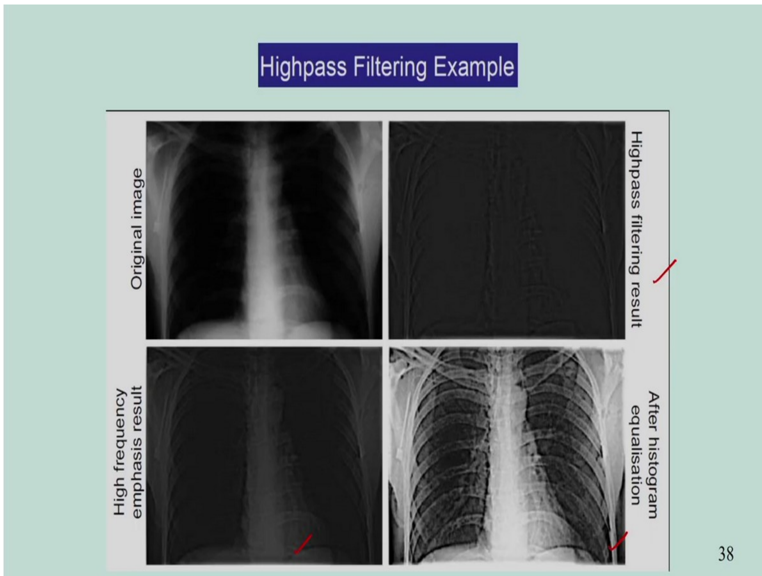

And in this case I have given one example of the high pass filtering. The original image I have shown first. The next one is the high pass filtering output you can see next one. And after this what I am doing, high frequency emphasis result I am showing here. And after histogram equalization I will getting the output something like this. So, after this what I doing, finally I am applying the histogram equalization technique to improve the contrast of an image. So, this is one example of the high pass filtering.

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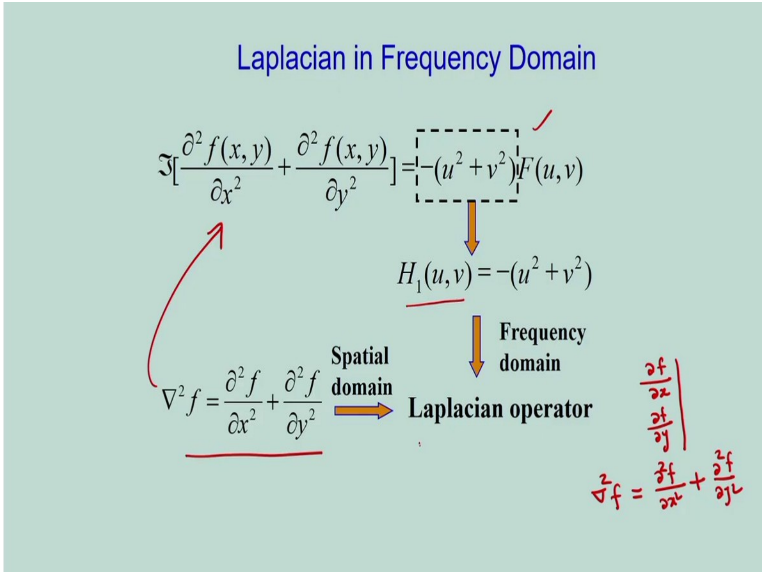

The next one is the Laplacian in frequency domain. This Laplacian I will explain when I will discuss the concept of the edge detection. So, Laplacian is spatial domain is represented like this, that is the second order derivative, f is the image. So, the first order derivative is del f divided by del x. That is the gradient of the image along the x direction and gradient along the y direction. This is the gradient.

And what about the second order derivative? The second order derivative is del 2 f, del x 2 and another one is del 2 f, del y 2. And if I consider this one, that is nothing but the Laplacian of the image. So, this Laplacian I will explain later on. So, corresponding to this Laplacian, if I take the Fourier Transform of the Laplacian, then in this case you will be getting this one.

Minus u square plus v square, F u v. F u v is the Fourier Transform of the input image. Then in this case what is H 1 u v, that is nothing but minus u square plus v square. So, you can see in spatial domain how to represent the Laplacian, in frequency domain how to represent the Laplacian. So, this is the Laplacian operator.

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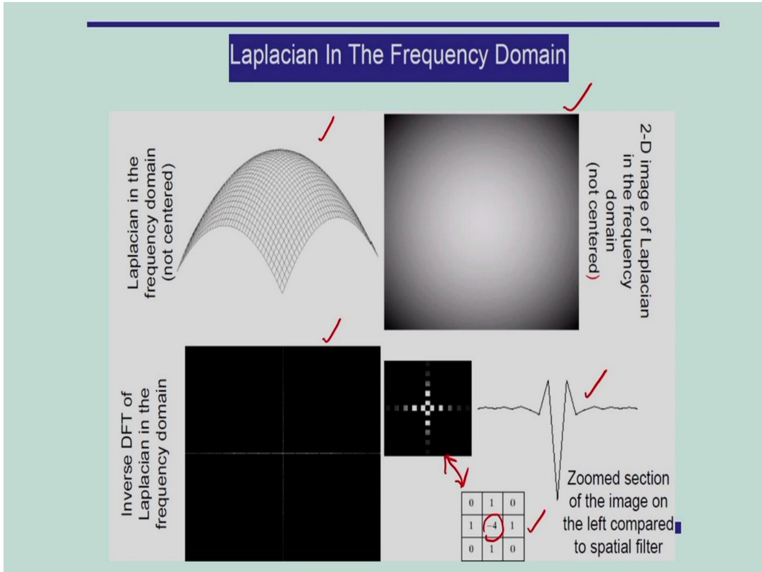

And corresponding to the Laplacian operator, the first figure if you see, Laplacian in the frequency domain, you can see this, the Laplacian will be something like this. So, by using the Laplacian you can determine the location of the edge pixel. I will explain this concept later on how to determine the location of the edge pixel by using the Laplacian.

And next figure if you see, the 2D image of Laplacian in the frequency domain. So, in the frequency domain the Laplacian will be something like this. And the third figure if you see, inverse DFT of Laplacian in the frequency domain. So in the frequency domain you can see the inverse DFT of the Laplacian in the frequency domain. In the spatial domain, corresponding to the Laplacian operator, the mask will be something like this 0, 1, 0, 1, - 4, 1, 0, 1, 0.

So, if you see the central pixel the central pixel, the central position of the mask, the mask value is minus 4. So, corresponding to this you can see here the mask will be something like this. So, in spatial domain the mask response will be something like this because of the central point of the mask is minus 4 of the Laplacian operator.

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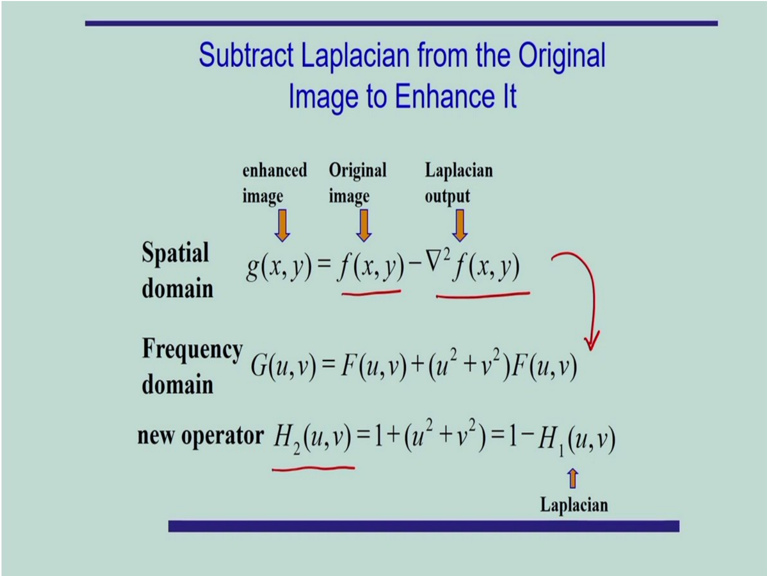

So, how to do the image enhancement by using the Laplacian operator? In spatial domain what you can do suppose I have the original image. The original image is f x y and if I take the Laplacian of the image that means I can select the high frequency component. So, high frequency information is subtracted from the original image, that I am doing.

And in frequency domain, this can be represented like this, so G u v is equal to F u v plus u square plus v square F u v. That is the Laplacian I am considering in the frequency domain. And I have the new operator, the new operator is H 2 u v, that is nothing but 1 plus u square plus v square is equal to 1 minus H 1 u comma v. That is the Laplacian.

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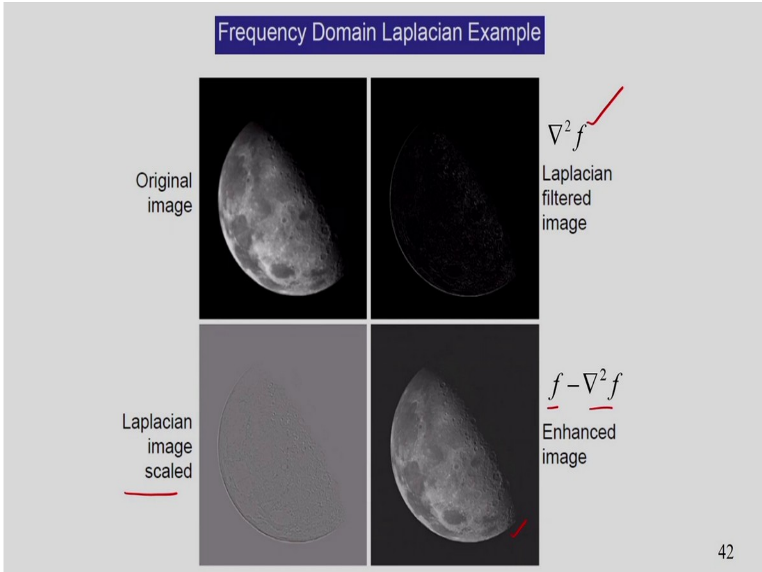

So in this case I am showing one example of the Laplacian, so how to apply the Laplacian of the image. In this case you can see the first one is the original image, the second one is I am considering the Laplacian filtered image. That means I am determining the edges by using the Laplacian. The next one is I am considering the Laplacian image scale, and after this I am subtracting the Laplacian from the original image.

I am getting the enhanced image, so you can see the enhanced image here, this is enhanced image. So, by using the Laplacian you can enhance the image. The Laplacian can be implemented in frequency domain or in spatial domain.

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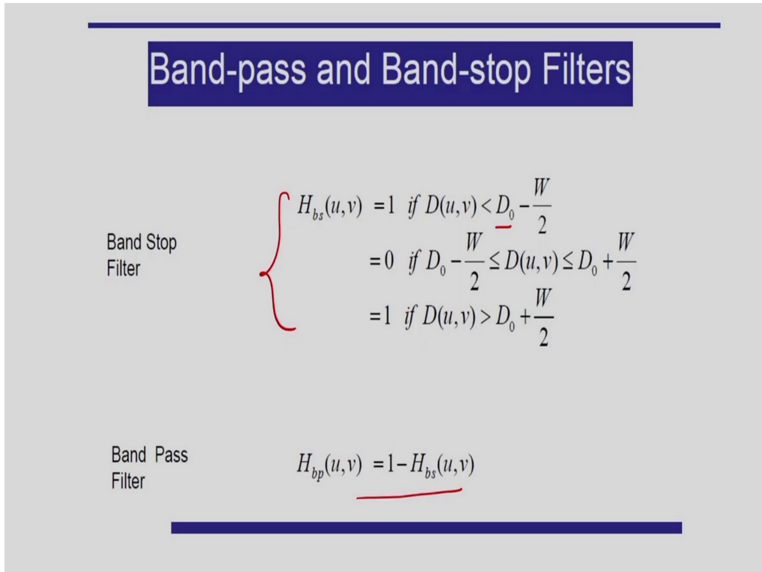

And after this, just I am considering the band stop filter and the band pass filter. And in this case the band stop filter, the transfer function you can represent like this, and again the concept of the cutoff frequency, the cutoff frequency is D naught and W is the width of the band. W is the width of the band. And you can see the band pass filter, what is the band pass filter, 1 minus band stop filter. That is the band pass filter.

And in this case, later I will explain if I take the difference between 2 Gaussians then in this case I will be getting the band pass filter. Difference of two Gaussian functions that will give the band pass operation.

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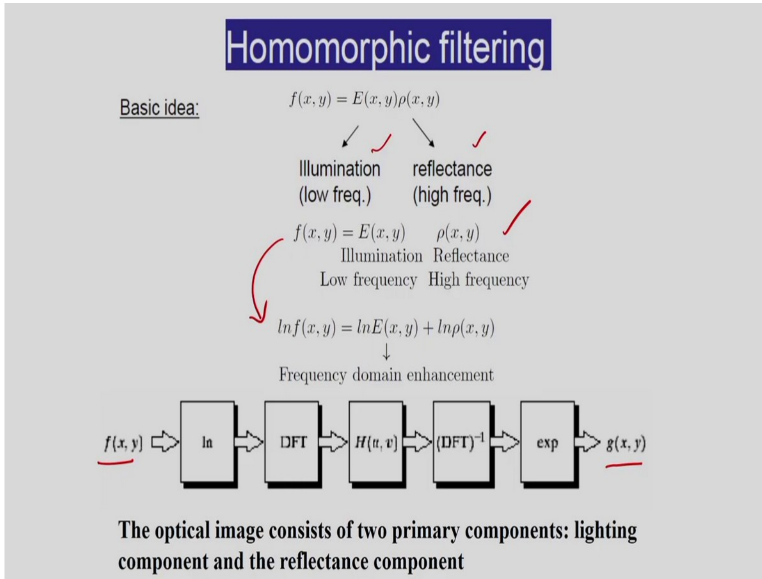

After this I am discussing another filter that is Homomorphic filtering. This filtering technique can be used to improve the appearance of an image by simultaneous intensity range compression and contrast enhancement. Then in this case the main concept is the optical image consists of 2 primary components, what are the primary components? One is the lightening component and another is the reflectance component.

So, I can consider these two components, here you can see one is the lightening component that is nothing but the illumination, and f x y is the image. And another component is the reflectance. The reflectance is nothing but the albedo, the albedo I have explained in my, I think, third or fourth classes about the albedo, that is the reflectance. So, f x y is the image and I am considering 2 components, one is the illumination component another one is the reflectance component.

The lightening component corresponds to the lightening condition of a scene. That means this component changes with the lightening conditions. That is the incident illumination. The reflectance component results from the way the objects in the image reflect light. That is the albedo of the surface. And it is the intrinsic property of object itself. And normally it does not change. In many image processing applications it is useful to enhance the reflectance component while suppressing the contribution from the lightening component.

That means for some applications it is used to enhance the reflectance component while suppressing the contribution from the lightening component. So, homomorphic filtering is a frequency domain filtering technique. So, in the filtering technique what we have to do, we have to compress the brightness, the brightness means from the lightening condition while enhancing the contrast. The contrast means it is coming from the surface reflectance property of the object.

So, in this expression you can see I have two components, one is the illumination component another one is the reflectance component. So, f x y is equal to E x y and rho x y. And after this what I can consider, this E x y is mainly the low frequency component. This illumination component is a low frequency component because the lightening condition varies slowly over the surface of the object.

This component is responsible for the overall range of the brightness of the image, that is the overall contrast. And if I considered the this component, the reflectance component, so this reflectance component is a high frequency component because it varies quickly at the surface boundaries, edges, due to varying phase angle. And this component is responsible for the local contrast. These assumptions are valid for many real images. So, what I am doing in this case, if you see this expression, after this I am taking the logarithm, the log I am taking so that the multiplication is converted into addition.

And after this, I am applying the frequency domain enhancement technique. So, what is a frequency domain enhancement technique? From the input image I am taking the log because, why I am taking the log, the multiplication is converted into addition. After this I am doing the frequency domain filtering, for this I am doing the DFT and H u v is the filter function. And after this I am applying the inverse DFT, and after this exponential function I am considering, that is nothing but the inverse log function I am considering. And after this I am getting the output image, the output image is g x y I am getting. So, in the frequency domain I am doing this operation. This Homomorphic filtering is a frequency domain filtering technique.

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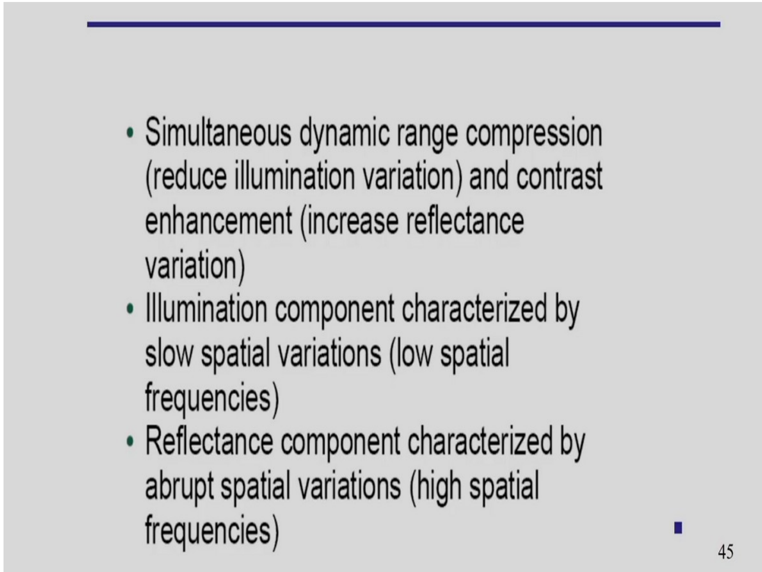

So, already I have explained this concept, the simultaneous dynamic range compression and contrast enhancement. And the illumination component is a low frequency component, that is a slow spatial variation, and a reflectance component is a high frequency information, it is characterized by abrupt spatial variations because it varies quickly at the surface boundaries, edges due to varying phase angles.

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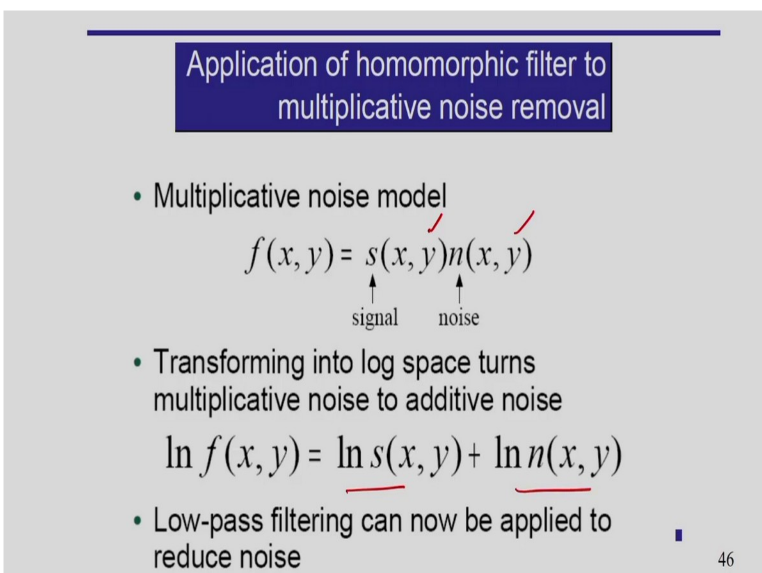

And in this case I can give one example of the Homomorphic filtering to remove multiplicative noise. So, what is this noise model, you can see the multiplicative noise model. I have the signal and I have the noise, both are multiplied. And after this I am doing what, the log I am considering. So that the multiplication is converted into addition this multiplication is converted into addition. After this I can apply the low pass filtering to remove the noise.

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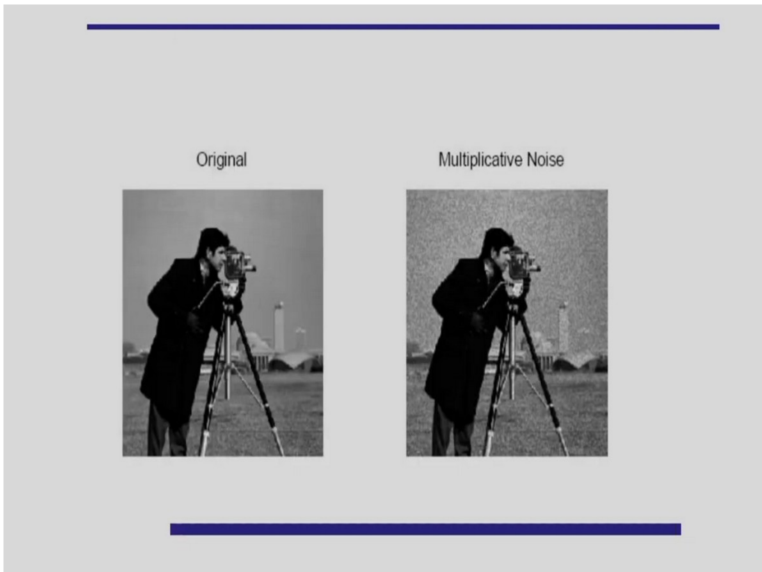

In this case you can see I am considering the original image and I am considering the multiplicative noise. I am considering. After this what I am considering?

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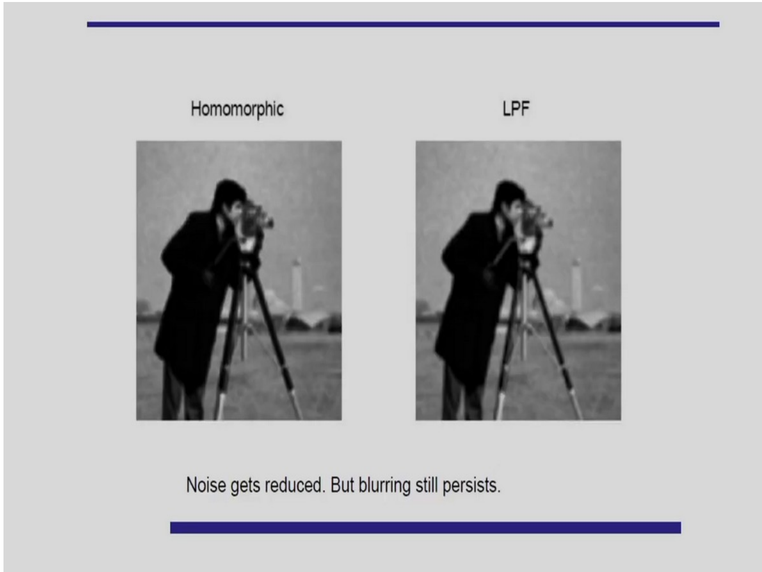

The Homomorphic filtering, I am considering you can see the output of the Homomorphic filtering and another one is the low pass filtering I am considering.

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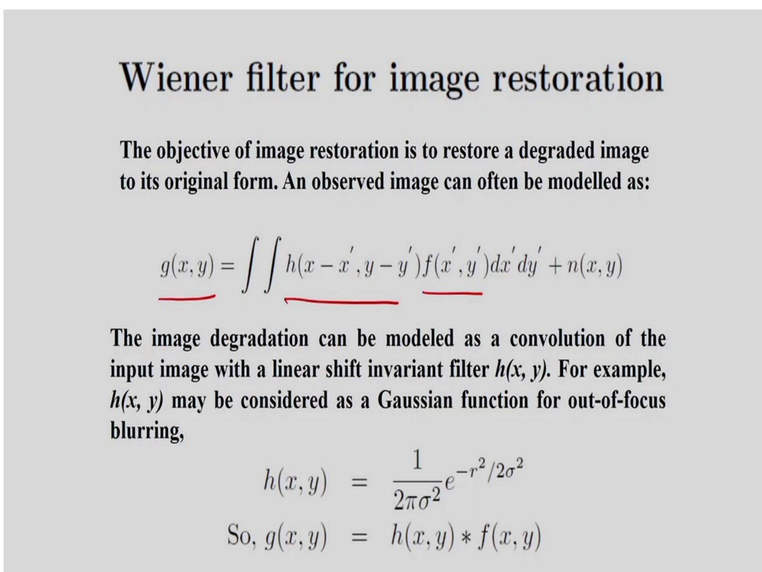

Next, I am considering the Wiener filter for image restoration. This image restoration already I have explained, in this case it is objective, enhancement is subjective. In case of the image restoration model we can develop a mathematical model and based on this mathematical model we can remove the noise or we can improve the visual quality of an image. The objective of the image restoration is to restore the degraded image to its original form. Then in this case I can consider some examples like motion blur or maybe the optical blur.

So, in this case I can develop a mathematical model, and based on this mathematical model I can improve the visual quality of an image. And in this case I am showing one example, an observed image can be modelled as, so g x y is the observed image and I am considering the image, the image is f, x dash y dash. And in this h is nothing but the point spread function, the PSF of the imaging function and n is the additive noise.

Now, the objective of the image restoration in this case is to estimate the original image, the original image is f, from the observed degraded image, the degraded image is g. So, that is the objective of the image restoration. So, I am repeating this, the objective of the image restoration is to estimate the original image f from the observed degraded image g. Now, this image degradation can be modeled as a convolution of the input image with a linear shift invariant filter, h x y. And in this case h x y may be considered as a Gaussian function for out of focus blurring. That I can consider.

Then in this case what will be the g x y, g x y will be h x y convolution f x y that I can consider.

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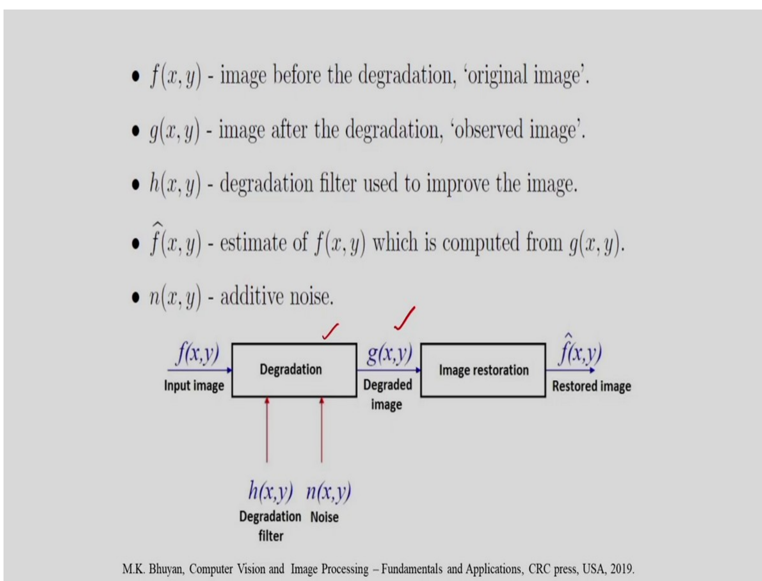

So, the definitions are like this f x y input before the degradation that is the original image. What is g x year? Image after the degradation that is the observed image, h x y that is the degradation filter used to improve the image. And I am considering the approximate f x y, that is the estimate of x y which is computed from g x y, so I can do the estimation of f x y and n x y is the additive noise.

So, you can see the degradation model that is how to do the image restoration. The first one is the input image, I am considering the degradation. So for this I am considering the degradation filter, that is used to improve the image. And I am considering the noise, and after this I am getting the degraded image, the degraded image is g x y. And after applying the image restoration technique I am getting the restored image. That is the better quality image I am getting.

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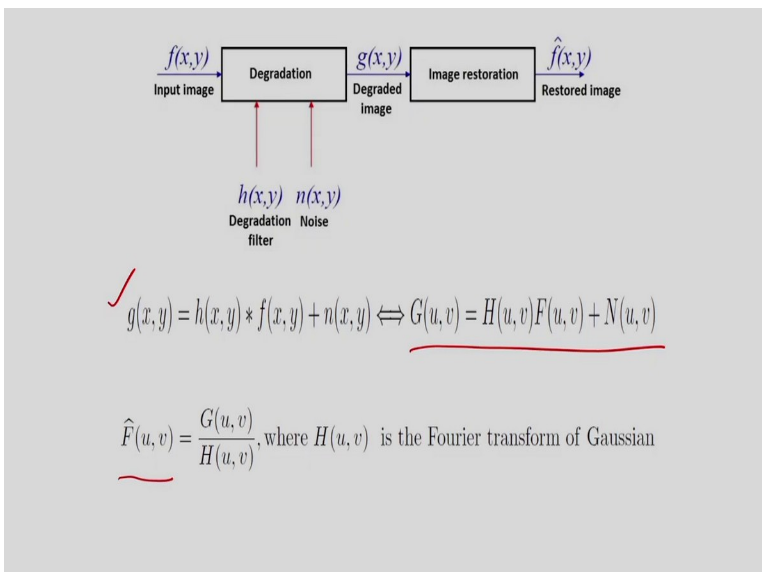

Now, in this case the same thing I am showing here, that is the degradation filter I have shown, the noise I have shown, the input image I have shown, and finally I am getting the restored image. Then in this case this model can be mathematically represented in the spatial domain, and in the frequency domain as follows. In spatial domain I can represent like this the g x y is nothing but h x y, that is the degradation filter, convolution with f x y plus the noise. That is the additive noise.

In frequency domain I can represent like this, because convolution in the spatial domain is equivalent to multiplication in frequency domain. So, G u v is equal to H u v into F u v plus N u

v. And in this case what is this approximate F u v, that is the g u v divided by H u v. What is H u v, H u v is the Fourier Transform of the Gaussian.

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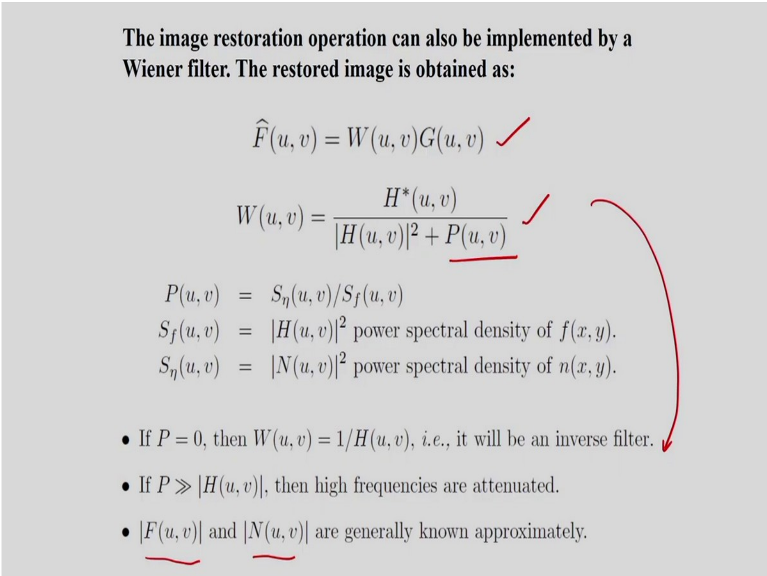

The image restoration can also be implemented by a Wiener filter. So, what is the Wiener filter? The restored image is obtained as W u v, G u v. W u v is nothing but the Wiener filter. And that you can represent like this, so later I will show how to get this expression, H complex conjugate u v, and it is divided by H u v magnitude whole square plus P u v. What is P u v? That is the ratio of S eta u v divided by S f u v.

What is S f u v? That is the power spectral density of f x y. S eta u v, that is the power spectral density of the noise, additive noise. And if I considered the P is equal to 0, so in this case suppose P is equal to 0, then W u v, in the expression if you see this expression then W u v is equal to 1 by H u v that is nothing but the inverse filter. If the P is greater than H u v then the high frequency components, the high frequency information will be attenuated.

And in this case the F u v, the magnitude of this F u v and N u v are known approximately. This information is available. F u v means that is the Fourier Transform of the image and N u v means the Fourier Transform of the noise.

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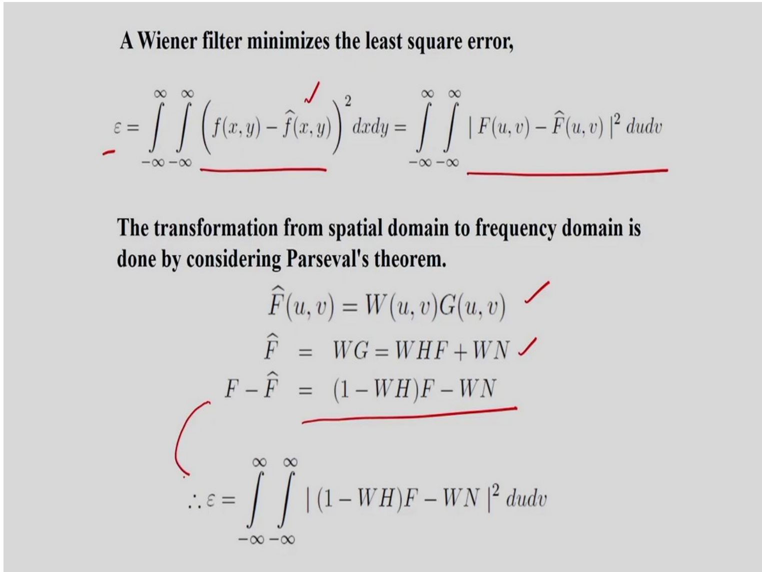

So, a Wiener filter minimizes the least square error. So, you can see I am computing the error here, so here is defined like this f x y is the original image and I am considering the reconstructed image the approximate image f approximate x y whole square. And in frequency domain I can write like this, d u v. The transformation from spatial domain to frequency domain is done by considering Parseval's theorem.

That means energy in the data domain is equal to energy in the frequency domain. Then in this case you can see this one, F approximate u v is equal to W u v, G u v and just you can represent like this, this expression. And after this, the F minus F approximate you will be getting this one, and from this if you put this value in the previous equation, then in this case you can determine the least square error you can determine.

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Since $f(x, y)$ and $n(x, y)$ are uncorrelated, so

$$\varepsilon = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{ |(1-WH)F|^2 + |WN|^2 \right\} du dv$$

Here, integrand is the sum of two squares. We need to minimize the integral, *i.e.*, integrand should be minimum for all (u, v). Since, $\frac{\partial}{\partial z}(zz^*) = 2z^*$, hence the condition for minimum integrand is:

$$2\left(-(1-W^*H^*)H|F|^2 + W^*|N|^2\right) = 0$$

$$\therefore W^* = \frac{H|F|^2}{|H|^2|F|^2 + |N|^2}$$

$$\text{or, } W = \frac{H^*}{|H|^2 + |N|^2/|F|^2}$$

And since f x y and n x y are uncorrelated, this input image and the noise they are uncorrelated, then this error cam be represented like this, if you see the previous expression it can be represented like this. Then in this case the integrand is the sum of two squares. So, you can see the integrand, the integrand of the sum of two squares. We need to minimize the integrand, that is integrand should be minimum for all the values of u and v, u is the spatial frequency in x direction and v is the spatial frequency in the y direction.

So, that means we have to consider this condition. And the condition for minimum integrand you can determine like this from this expression. And finally, if you do all this mathematics, then in this case you will be getting that W is equal to H complex conjugate, the magnitude of H square plus magnitude of N square divided by F square. That you will be getting, that is nothing but the Wiener filter.

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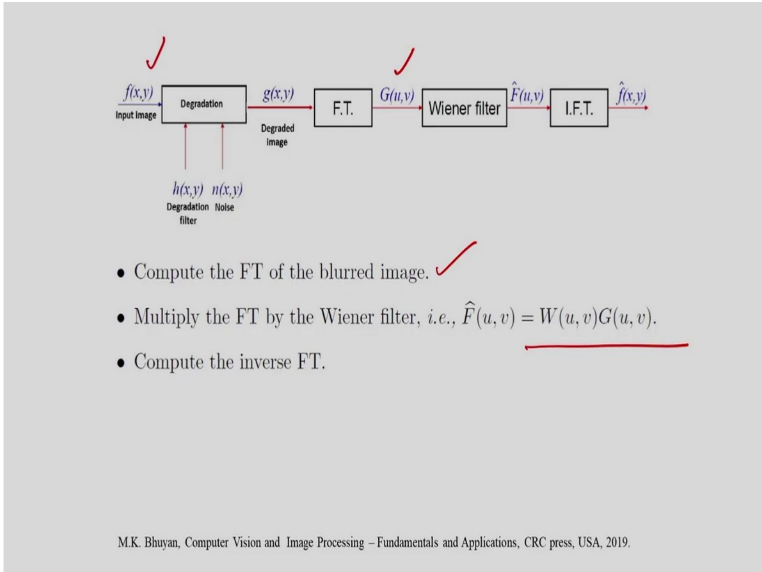

So, finally I can show how to do the denoising by using the Wiener filter. First the input image is this, after this I am considering the degradation filter, noise I am considering, that is the additive noise I am considering. After this I am getting the degraded image, I am considering the Fourier Transform of the degraded image. I am getting G u v, after this I am applying Wiener filter. So I am determining the approximate value of F u v, and after this I am taking the inverse Fourier Transform to get the restored image.

So, steps are like this compute the Fourier Transform of the blurred image. Multiply the Fourier Transform by the Wiener filter, that Weiner filter already I have explained. And after this finally we have to take the inverse Fourier Transformation to get the restored image. This is about the brief discussion about the Weiner filter.

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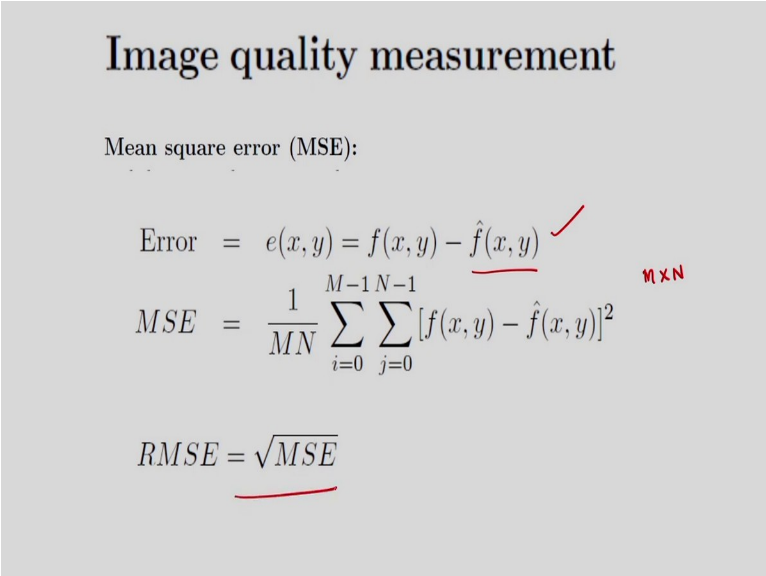

The next concept what I want to discuss, the image quality measurement, so how to measure the image quality. So for this I can calculate the mean square error, so error can be calculated like this, f x y is the original image and I am considering the approximate, that is the reconstructed image. The difference between this that will give the error. And from this you can determine the mean square error. Then in this case the image size I am considering M cross N, M number of rows and N number of columns.

So, from this you can determine the mean square error between the input image and the reconstructed image. So, that is one measure for image quality measurement. And from the MSE you can determine the root mean square error, that you can determine. That is the root mean square error you can determine from the MSE.

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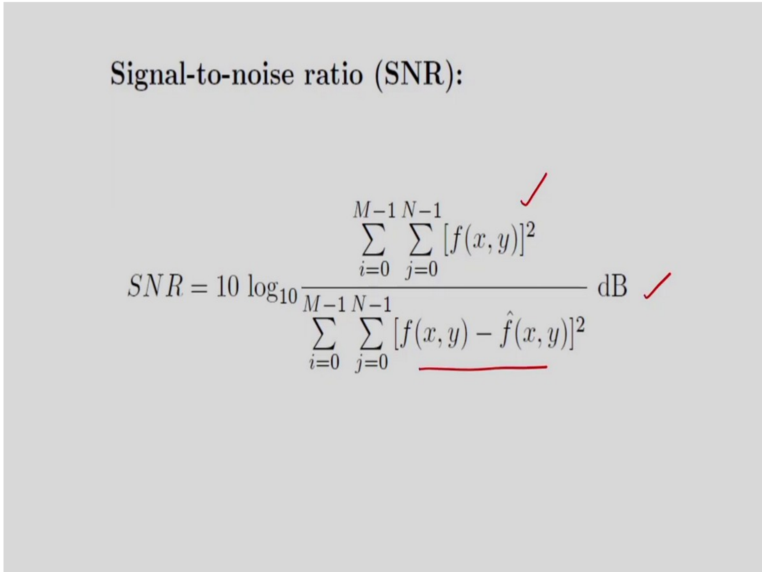

Another measure is the signal to noise ratio. That you can determine by this expression. So my input image is f x y, so I am getting the signal power here. And also the difference between this two is nothing but the noise. So, first I am considering the signal divided by noise that will give the signal to noise ratio.

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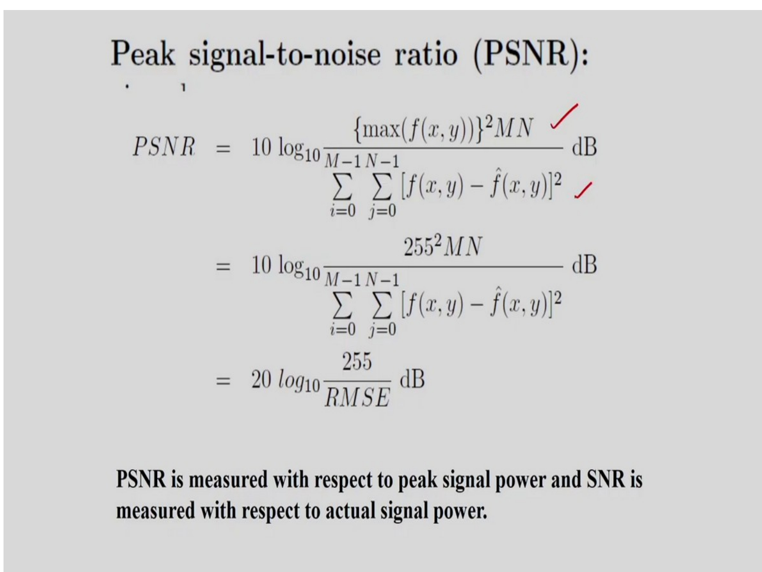

And another one is the peak signal to noise ratio, PSNR. That also you can determine, you can see I am considering the signal power and also the peak signal power and also the noise power I can determine. And maximum pixel value I am considering 255, so you can determine the PSNR. So, PSNR is measured with respect to the peak signal power, but in case of the SNR, SNR is measured with respect to the actual signal power. That is the difference between the PSNR and the SNR. So, by using this measure, so one is the mean square error another one is the signal to noise ratio, another one is the PSNR, we can judge the quality of the reconstructed image.

In this class I discussed the concept of the frequency domain filtering. For frequency domain filtering the concept is, I have to modify the Fourier Transform of that image. And I have discussed the concept of the low pass filter. For this I discussed about the concept of the ideal low pass filter, Butterworth low pass filter and the Gaussian low pass filter. And also two important properties, one is the blurring effect another one is the ringing artifacts.

So, how to consider these two cases, and it depends on the cutoff frequency, the cutoff frequency is D naught. After this I explained the concept of the high pass filter, again I am considering the ideal high pass filter, Butterworth high pass filter and the Gaussian high pass filter. After this I have discussed the concept of the Laplacian. So, by using the Laplacian how to improve the visual quality of an image. So, by Laplacian you can detect the edges.

And after this I discussed the concept of the Homomorphic filtering. And in this case I have two components, one is the illumination component another one is the reflectance component. Illumination component is the low frequency component and another the reflectance component, that is the albedo, is a high frequency component. And how to do the filtering in frequency domain that concept I have explained.

After this finally I have discussed the concept of the image restoration by Wiener filter. And after this I discussed the image quality measurement, like this one is the mean square error, one is the signal to noise ratio, and another one is the peak signal to noise ratio. This is about the frequency domain filtering. So, let me stop here today. Thank you.