

**Computer Vision and Image Processing - Fundamentals and Applications**  
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**Lecture: 15**  
**Image Enhancement**

Welcome to NPTEL MOOCs course on Computer Vision and Image Processing: Fundamentals and Applications. In my last class I discussed the concept of image transformation. From now, I will discuss some image processing concepts.

The first concept is Image Enhancement. The objective of image enhancement is to improve the visual quality of an image for better visual perception or for better machine processing of the image. Image enhancement, it is subjective. There is no general mathematical theory for the image enhancement. It is application specific.

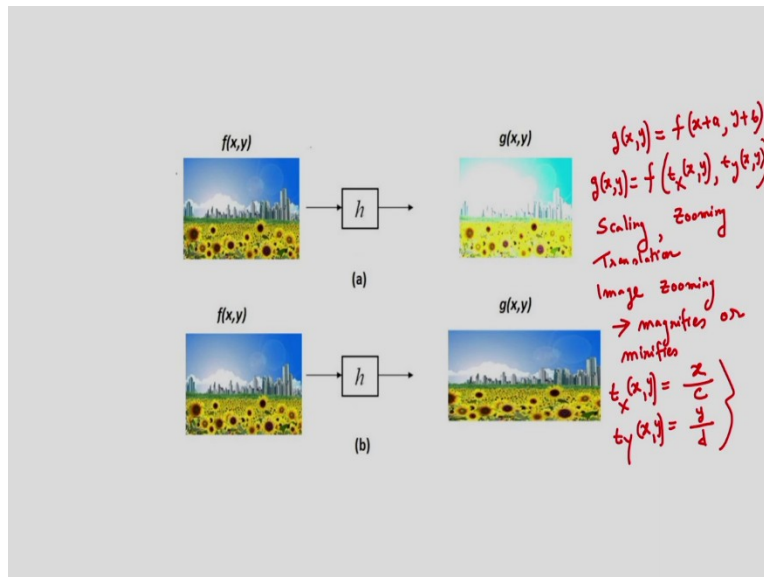
I can give some examples of image enhancement. The one example is the removal of the noise, another one is to improve the contrast of the image, I can change the brightness of an image. I can highlight the edges, highlight a particular region of interest. So these are some examples of image enhancement.

The image enhancement can be implemented in spatial domain or maybe in the frequency domain. In spatial domain, I can manipulate the pixel values directly. In the frequency domain, what I can do, from the input image, I can apply the Fourier Transformation, I will be getting the image in the frequency domain. And after this I can do the processing in the frequency domain, and after this I will apply the inverse Fourier Transformation to reconstruct the processed image.

And in my last class, I discussed one concept. So before applying the Fourier Transformation, I have to multiply the image by minus 1 to the power  $x + y$ . That concept, already, I have explained in my last classes. So image enhancement, already I have defined, that is important for better visual perception.

Now, already I have explained this concept, the image is represented by  $f(x, y)$  and I can change the range of an image and also I can change the domain of an image. Range means the pixel intensity values and domain means the spatial coordinates. In my next slide, I will show this concept.

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That is, the first one as you can see, I am changing the intensity value, the pixel value. And in the second case, I am changing the spatial coordinates. So in the first case that is nothing but changing the range of an image. In the second case, I am changing the domain of an image.

So suppose if I write like this,  $g(x,y)$ , that is, the output image,  $g(x,y)$ , equals to  $f(x,y)$  plus  $a$  and  $y$  plus  $b$ , suppose. This operation, I am considering.  $g(x,y)$  is the output image and  $f(x,y)$  is the input image. And I can also write another expression,  $g(x,y)$  is equal to  $f(t_x(x,y), t_y(y,y))$ , and I am applying some transformation. This is the transformation for the  $x$  coordinate.  $t_x$  means the transformation for the  $x$  coordinate; and  $t_y$ , it is the transformation for the  $y$  coordinate.

So I can consider this expression. Now, based on this expression, I can do these operations. One is the scaling operation, I can do. I can do zooming operation. And also, I can do the translation. The translation operation, I can do. The first, the  $g(x,y)$  is equal to  $f(x+a, y+b)$ , that is nothing but the translation.

Also, I can do the scaling. The scaling along the  $x$  direction, scaling along the  $y$  direction. Image zooming is nothing but it either magnifies, image zooming means the magnifies or I can consider the minifies. Magnifications, I can do or it can minifies the input, the input image.

So I can give one expression for the zooming,  $t_x$  is equal to  $\frac{x}{c}$ . So  $x$  coordinate is scaled by  $c$ . And if I consider this transformation, transformation for the  $y$  coordinate,  $t_y$  is the transformation for the  $y$  coordinate, so  $y$  divided by  $d$ . So that means it is the expression for the

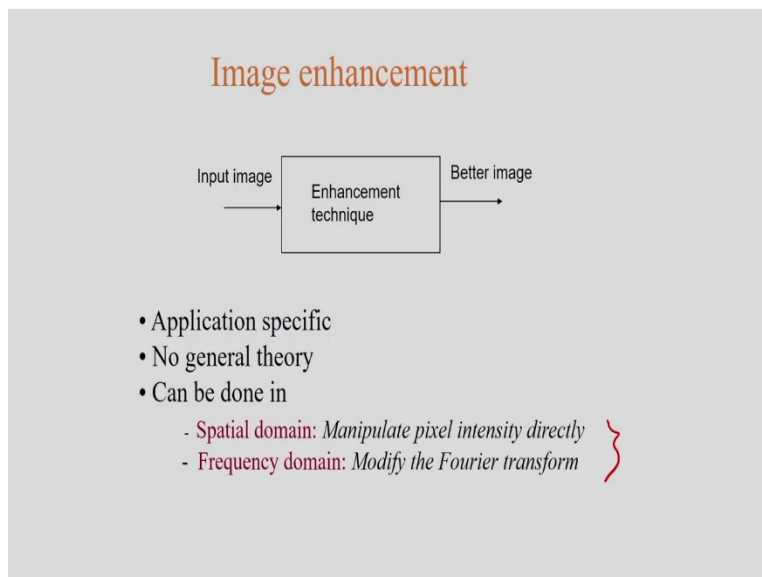
zooming. For the x coordinate, I am doing the scaling and for the y coordinate, I am doing the scaling.

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Now, let us consider what is image enhancement. Image enhancement means, to improve the visual quality of an image for better human perception or for better interpretation by machines. Now, in this case, you can see I am considering one input image and corresponding to this I have the output image. So I am getting the better quality image in the output.

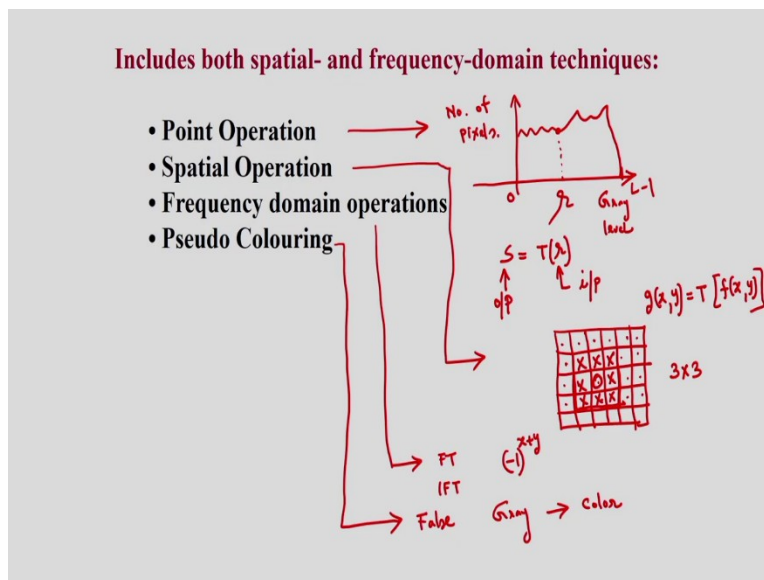
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Now, in this block diagram you can see, I have the input image and another one is the better quality image in the output, and I can apply image enhancement techniques. And already I have explained that it is application specific. Image enhancement is application specific. There is no general theory for the image enhancement. And image enhancement, I can implement it in the spatial domain and also in the frequency domain.

In the spatial domain, I can manipulate the pixel intensity values directly, and in the frequency domain I can modify the Fourier Transform of the image. So these two techniques I can apply. One is the spatial domain technique, another one is the frequency domain technique.

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Now, some of the operations for image enhancement, maybe something like the point operation, spatial operation, frequency domain operations and the pseudo coloring.

What is the point operation? So in the point operation, suppose if I consider the histogram of an image. So I will define the histogram later on. But for the time being just I am explaining like this. So suppose this is the histogram of an image, so  $r$  is the input gray levels. It is from 0 to  $L$  minus 1. And I am considering the number of pixels. So this, I can consider as the gray level histogram. So later, I will define mathematically what is the gray level histogram.

So number of pixels corresponding to a particular gray level, I can see. So corresponding to this particular gray level, I can see how many pixels are available in this particular gray level. And in this case, if I apply this transformation,  $S$  is equal to  $T r$ ,  $r$  is the input gray level. This is my

input. And  $S$  is the output gray value, so output is this. So the point function I am considering, that operates on the gray level value, the gray level is  $r$ , the input value is  $r$ .

So this operation is called the point operation. That means, by using some transformation I am considering the input, the input is  $r$ , and I am getting the output, the output is  $S$ . And  $T$  is the transformation, the transformation I am considering. So this is the point operation.  $S$  is equal to  $T$   $r$ .

And in the spatial operation, what I can consider, I can consider suppose this image. And these are pixels of the image. And in this case, I am getting the output image, the output image is  $g \times y$ . And in this case, I am applying some transformation, the transformation is  $T$ , and it is  $f \times y$ ,  $f \times y$  is the input image.  $T$  is the transformation, I am getting the output image, the output image is  $g \times y$ .

Then in this case, what I am considering, I may consider a particular window, one window I can consider. A window, something, I can consider like this. And this window, suppose the 3 by 3 window, I am considering. Then in this case, I have to consider the neighborhood pixel. Suppose this pixel I am considering, then I have to consider the neighborhood pixel, the neighborhood pixel, these are the neighborhood pixels, I have to consider.

So I am getting the output image, the output image is  $g \times y$  and my input image is  $f \times y$  and I am doing some transformation. Then in this case, I have to consider the neighborhood pixels. In this case, I am considering the 3 by 3 marks. And corresponding to the center pixels, I am considering the neighborhood pixels for this operation. The operation is the spatial operation.

In frequency domain operation, already I have explained that in this case first we have to determine the Fourier Transform of the image. And for this, I have to multiply the image by minus 1 to the power  $x$  plus  $y$ . And after this, I can do the processing. And finally, I can apply the inverse Fourier Transformation to reconstruct the processed image.

And one example of this enhancement technique is the pseudo coloring. Pseudo means the false coloring. Pseudo means the false coloring, that concept I am going to explain in color image processing. False coloring means, I can convert the gray scale image. Gray scale image I can convert to color image. That is the false coloring, that is the pseudo coloring. The gray scale image can be converted into color image.

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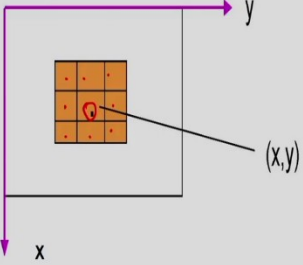
### Spatial Domain technique

$g[x,y] = T(f[x,y])$   
or  
 $s = T(r)$

Point operations

-Simplest case

- $g[.]$  depends only on the value of  $f$  at  $[x,y]$ ; does not depend on the position of the pixel in the image.
- *brightness transform or point processing*

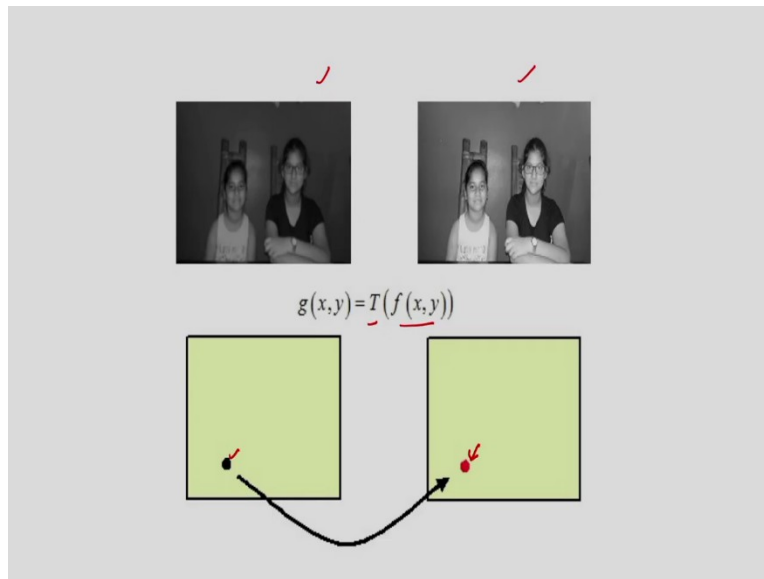


The diagram shows a 3x3 grid of pixels. The central pixel is highlighted in red and labeled with the coordinates (x,y). The grid is positioned within a coordinate system where the horizontal axis is labeled 'y' and the vertical axis is labeled 'x'.

In this case, I have already shown, I have already explained this concept. So I have the  $g \times y$ , my input image is  $f \times y$ . And corresponding to this center pixel, the center pixel is this, I have the neighborhood pixels. I can do some transformation, and corresponding image  $f \times y$ , I am getting the output image, the output image is  $g \times y$ . That means this point operation is nothing but  $s$  is equal to  $T r$ . So  $r$  is my input and  $s$  is my output.

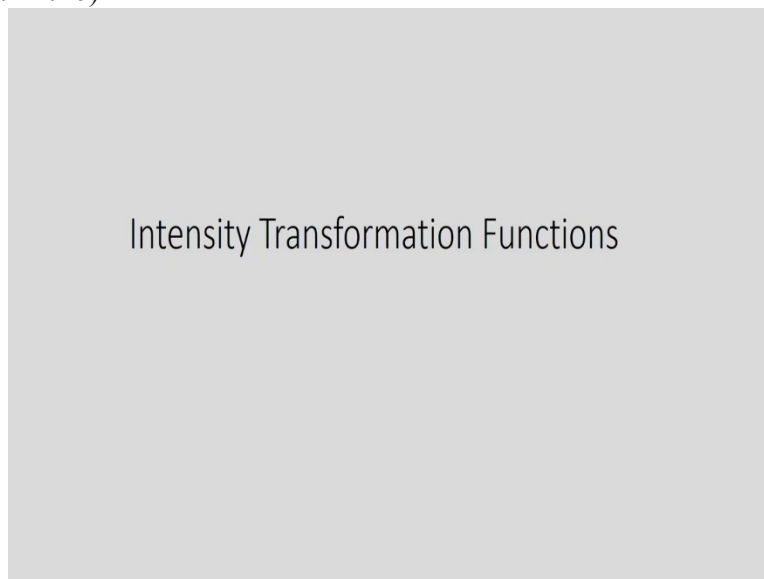
Now, in this case, that is the definition of the point operation. This operation, that means, output depends only on the value of the  $f$  at a particular point,  $x$  comma  $y$ , does not depend on the position of the pixel in the image. So this output depends on the pixel value at a particular point, but does not depend on the position of the pixel, that is, the position of the pixel is  $x$  comma  $y$ . This is the spatial domain technique.

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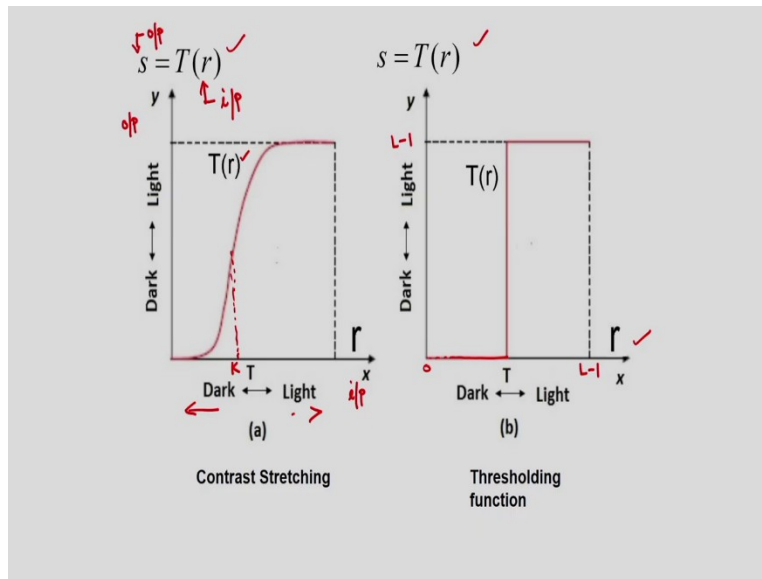
In this case, I have given one example. You can see the two images, the first image you can see. The second image, I am having this image. I am doing some transformation. The input image is  $f(x,y)$ , and I am getting the output image, the output image is  $g(x,y)$ . So that means that this pixel is modified and this modified pixel is here. This is the modified pixel based on the transformation.

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Now, I will discuss some intensity transformation functions.

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So what is the intensity transformation function? You can see. The first function is, that is the contrast stretching function. So if you see the first diagram, that is the contrast stretching function. So in the contrast stretching function, what is the transformation? The transformation I am considering  $s$  is equal to  $T r$ , that transformation I am considering.  $r$  is my input and  $s$  is my output.

So this is, this axis is my input and this is my output. So  $r$  is my input and  $s$  is my output. Now, if I consider this transformation, this transformation if I consider,  $T r$ , so suppose if I consider this point suppose,  $K$  suppose. This, I am considering. Now, in this case, you can see the values of  $r$  lower than the  $K$  are compressed by the transformation functions into a narrow range of  $S$  towards the black. So if I consider dark side is this and the light side is this.

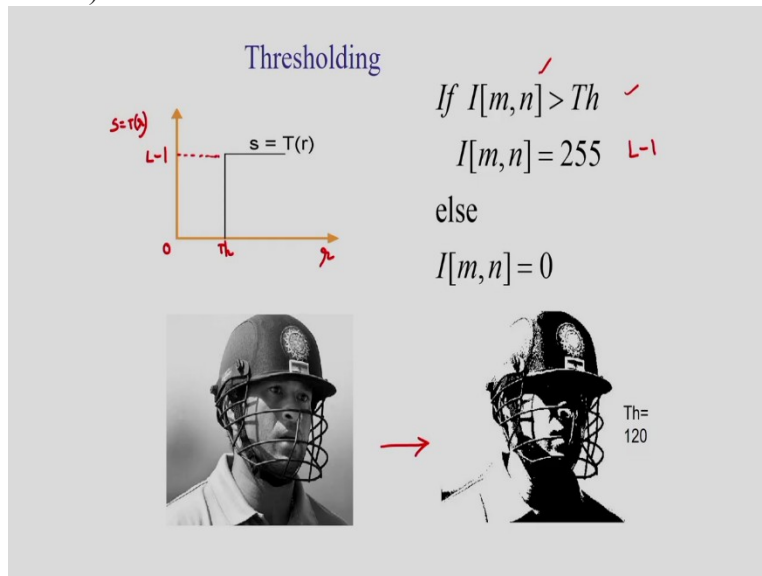
So what is the function of the transformation function? That is, the values of  $r$ ,  $r$  means the input values, lower than the threshold value, the value is  $K$ . The values of  $r$  lower than  $K$  are compressed by the transformation function into a narrow range of  $s$  towards dark. And the opposite is true for values of  $r$  higher than  $K$ . So this is the function of the contrast stretching.

In the second case, I am considering a threshold function. So again the same concept. In the  $x$  direction, my input is there. In the  $y$  direction, the output is there. And corresponding to, if you see, up to this, from  $0$  to  $T$ , the threshold, the gray scale value, my output is zero. This red, my output is zero. This output is zero. And if the input value is greater than  $T$ , my output will be  $L$  minus  $1$  because it is from  $0$  to  $L$  minus  $1$ , and this is also  $0$  to  $L$  minus  $1$ .



So if I apply the thresholding function, then in this case I will be getting the binary image. I can give one example in the next slide.

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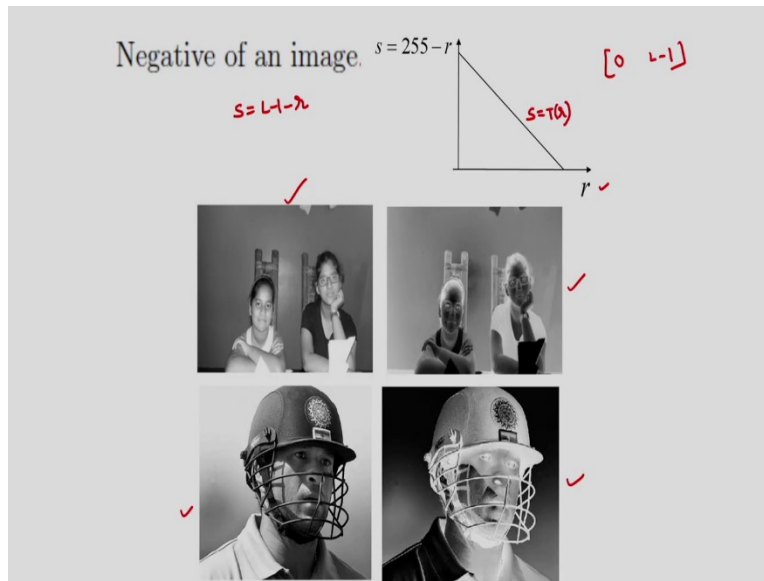


You can see, so, I am considering the thresholding function. So the thresholding, I am considering, suppose this is the threshold, my,  $Th$  is the threshold. My input is this and output is  $S$  is equal to  $T r$ .

And this value, if you consider, this value is nothing but  $L$  minus 1, and this is 0. So if I consider here, if the input image,  $I m n$  is greater than the threshold, then what will be the value of  $I m n$ ? The value of the output image will be 255. 255 means,  $L$  minus 1. Else,  $I m n$  is equal to 0.

This is the output of the thresholding operation, you can see. My input is this and output I am getting the binary image, because I have only two levels, one is zero another one is  $L$  minus 1, so  $L$  minus 1 is 255.

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Next, I am considering another operation. That operation is the image negative. Intensity range already I have considered, it is from 0 to  $L$  minus 1. This is my intensity range. And if I apply this transformation, the transformation is  $S$  is equal to  $T r$ , if I apply this transformation, my input is  $r$ , output is  $S$ .  $S$  is nothing but, I can write here also,  $S$  is equal to  $L$  minus 1 minus  $r$ . I am considering this.

Then in this case, I will be getting the negative image. So this is my input image, I am getting the negative image. This is my input image, I am getting the negative image. Why the negative image is important? To enhance white details embedded in the dark regions of an image.

That is why the negative image is important, to enhance white details embedded in the dark regions of an image. So that is why I have to consider the negative of an image. So this is the negative of an image.

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Log transformation

- Compresses the dynamic range

$$s = c \log(|r| + 1) \quad \gamma > 0$$

where  $c$  is the scaling factor.

*Example : Used to display the 2D Fourier Spectrum*

Next important transformation. So in my some classes, I discussed about the log transformation. That is used to compress the dynamic range. So in this case, I am considering this transformation,  $s$  is equal to  $c \log r$  plus 1. So in this case,  $c$  is the scaling factor,  $r$  is my input gray scale value,  $s$  is the output gray scale value, I am considering. And in this case  $r$  is greater than equal to 0, that I am considering.

And in this case, it is used to compress the dynamic range of an image. It is used to display the 2D Fourier Spectrum that already I have explained in one of my class.

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Some useful transformations

The diagram illustrates various image transformations on a coordinate system where both axes range from 0 to  $L-1$ . The vertical axis is labeled  $y = c \cdot x^\gamma$  and the horizontal axis is labeled  $x$ . Several curves are plotted for different values of  $\gamma$ :  $\gamma = 0.2$ ,  $\gamma = 0.4$ ,  $\gamma = 0.67$ ,  $\gamma = 1$ ,  $\gamma = 1.5$ ,  $\gamma = 2.5$ , and  $\gamma = 5$ . A diagonal line is labeled "log". To the right of the main plot, a legend lists: "log", "Identity / Log gamma man", "Inverse log", and "Image negative". Above the main plot, a smaller diagram shows a square with axes from 0 to  $L-1$ , with a curve labeled "log" and "s = (r+1)". Below it, a square with an "X" is shown.

So, I can show this transformation, what is the log transformation here? This is the log transformation. If I consider this curve, log transformation is nothing but, so this is  $L$  minus 1, this is 0, this is  $L$  minus 1, this is my  $r$ , this is  $S$  is equal to  $T r$ . And if I consider this transformation, that is the log transformation. This is the log transformation.

So in this case, what will happen in this case of the log transformation, transformation maps a narrow range of low intensity input, if I consider the narrow range of the low, this is the dark side and this is the bright side. So the transformation maps a narrow range of low intensity input values into a wider range of output values.

The opposite is true for higher values of input gray scale values. That means expanding the values of the dark pixels in the image and compressing the higher-level values. That is the application of the log transformation. I am repeating this. Expanding the values of the dark pixels in the image and compressing the higher-level values. That means, in this case if I consider a high-level value, high level value is from this to this and low-level pixel values are this.

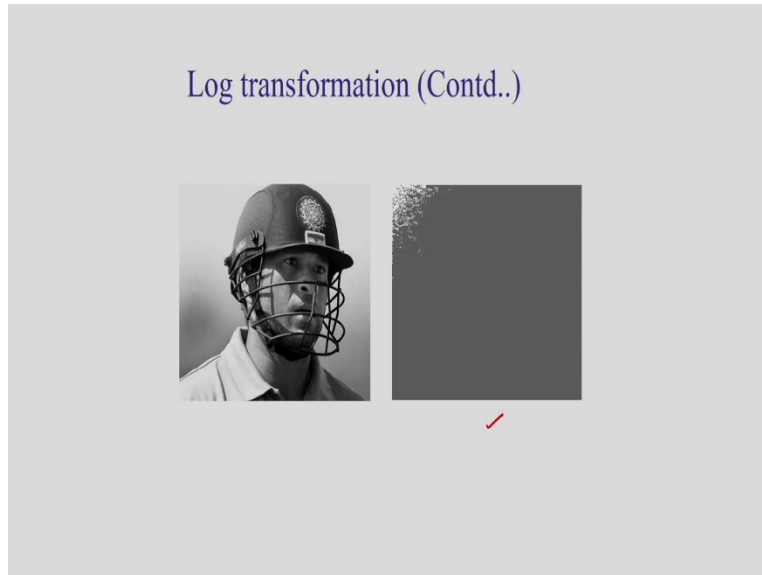
So for the low level pixel value, I am expanding. And for the high-level pixel values, I am doing the compression because output is only this, if you see. But corresponding to dark level, the output will be this.

So this transformation is used to display the Fourier Transform of an image. So if you see, here I am considering some useful transformation. The one transformation, I have shown the log transformation.

If you see this transformation, the transformation is this transformation. And this transformation is this. In this case, if I apply this transformation, then in this case no influence on the visual quality. There will be no change in the visual quality. And this is called the identity and sometimes it is called lazy man. It is called the lazy man operation. The lazy man transformation.

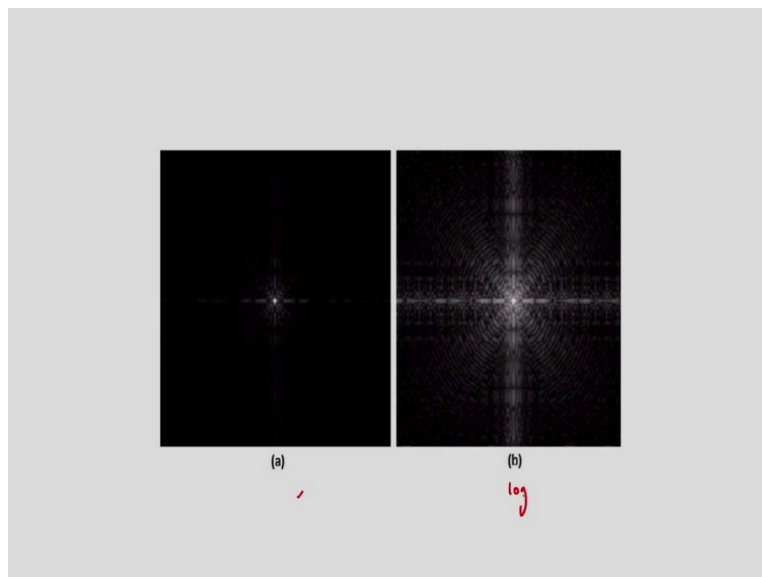
Because if I apply this transformation, no influence on visual quality. And if you see this transformation, this transformation, that is nothing but the image negative. So, I have shown some transformations. One transformation I have explained, one is the log transformation. And you can see here, this transformation is the anti-log, that is the inverse log transformation. One is the image negative, one is the identity.

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I can show this example. If I consider this image, then in this case, suppose this is the Fourier Transform of the image, then in this case, to visualize the Fourier Transform Spectrum, I have to apply the log transformation, because it compresses the dynamic range.

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So in this case you can see, first, you can see the Fourier Transform of the image that is, the Fourier Spectrum, that is not clearly visible. If you see the second image, that is clearly visible because I am applying the log transformation. So first one is without log transformation and the


second one is, I am applying the log transformation. In the log transformation, it compresses the dynamic range.

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Power law transformation (Gamma Transformation)

- Expands dynamic range  $s = cr^\gamma$  ✓  $s = c(r + \epsilon)^\gamma$   
where  $\gamma$  and  $c$  are positive constants
- Often referred to as gamma-correction

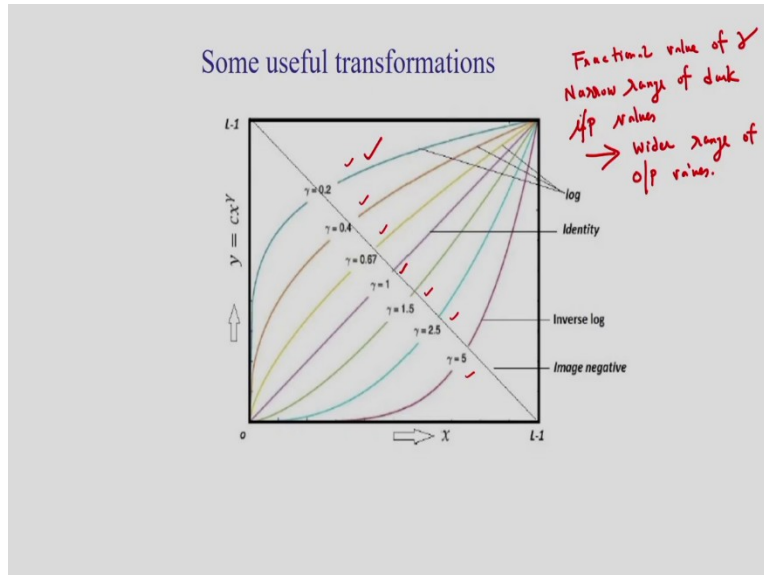
Example :  $\gamma = 1$ , Image scaling, same effect as adjusting camera exposure time.



Another very important transformation is the power law transformation. And in this case I am considering this transformation, the power law is  $s$  is equal to  $c r$  to the power gamma. This is also called the gamma transformation, sometimes it is called the gamma transformation. So the transformation is  $s, c r$  to the power gamma. And sometimes you can write also this one,  $s$  is equal to  $c r$  plus offset, one offset is considered, epsilon, to the power gamma. So I am considering this offset because I am getting the measurable output when the input is zero.

Now, in this case, if you see this transformation, based on the gamma is equal to 1, suppose if I consider gamma is equal to 1 in this expression, then it is nothing but the image scaling. It is the same effect as adjusting the camera exposure time. If I consider gamma is equal to 1. Because if I consider gamma is equal to 1, that means  $s$  is equal to  $c r$ .

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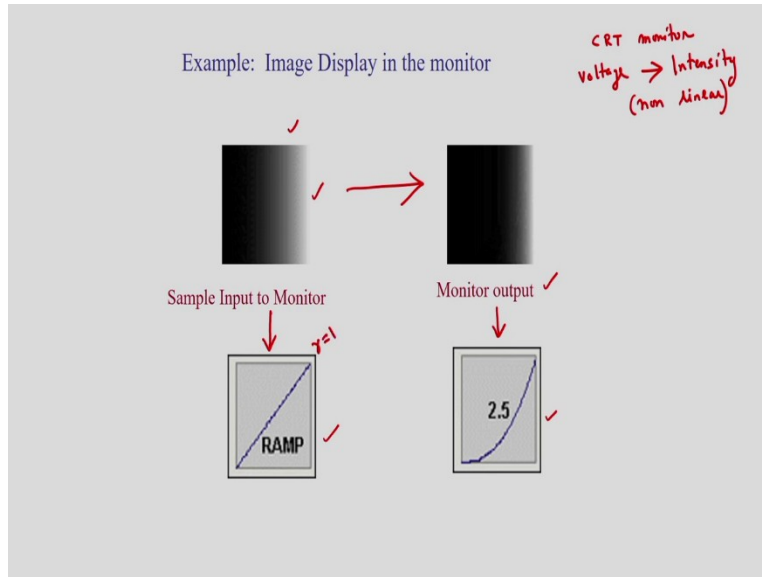
And based on this gamma, I have this transformation. So you can see for gamma is equal to 1, I have the identity. And if you see gamma is equal to 0.2, gamma is equal to 0.4, gamma is equal to 0.6, 0.7, that is, I am considering the fractional value of gamma. And you can see, if I consider, it is greater than 1. The gamma is 1.5, gamma is 2.5, gamma is 5, like this.

So if I consider the fractional value of gamma, what is the meaning of the fractional value of gamma? That means if I consider a fractional value of gamma, narrow range of dark input values, dark input values, that is converted into the wider range of output values. So fractional value of gamma, that means the narrow range of dark input values, that is converted into wider range of output values.

You can see here, this transformation, gamma is equal to 0.2. So that is very similar to the log transformation. And if I consider the gamma is 1.5, then in this case it will be opposite. Gamma is suppose 1.5, gamma is 2.5, then the outcome will be opposite.

So I have the number of transformation corresponding to different values of gamma.

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Now, one application of gamma transformation, I can give one example. Suppose in the CRT monitor, the cathode ray tube monitor, what is happening? It is nothing but the voltage is converted into light intensity. That means, I am getting the output in the monitor. Then in this case, it is basically the non-linear. This operation is the non-linear operation. This is non-linear operation. That is the non-linear power function.

So in this case what will happen, if I consider a simple monitor, and corresponding to this input, if I consider this input, suppose my input is this, something like the ramp input and this is my image, the first one is the image. Corresponding to this, what you can see in the output, because it is a non-linear device. The monitor output will be something like this, and corresponding to this output the gamma will be something like this, gamma will be 2.5.

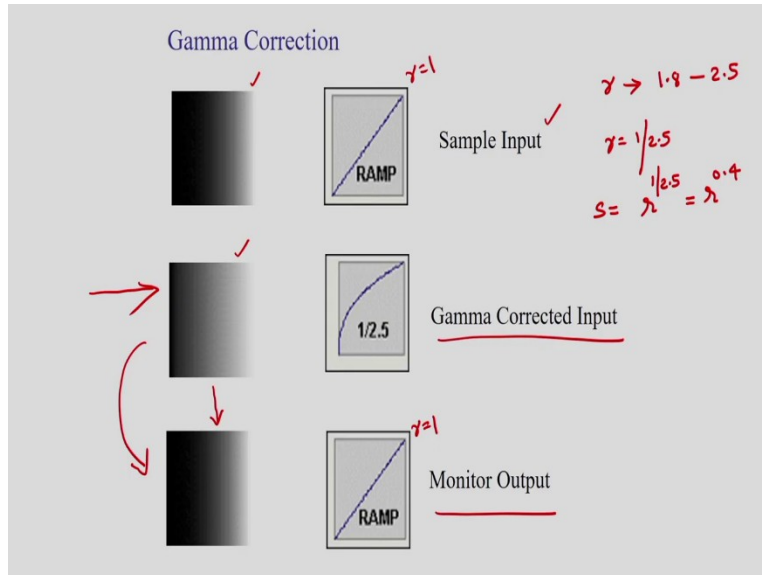
That means, the display system would tend to produce images that are darker than the intended input. So I am repeating this. That means if I consider this case, gamma is equal to 2.5, the display systems will produce images that are darker than the intended input.

So that means, in this case if I give this input, corresponding to this input I am getting the output, this output. And corresponding to this output, the gamma is 2.5. Corresponding to the input image the gamma is 1. In this case, the gamma is equal to 1.

Now, in this case, how to avoid this condition?



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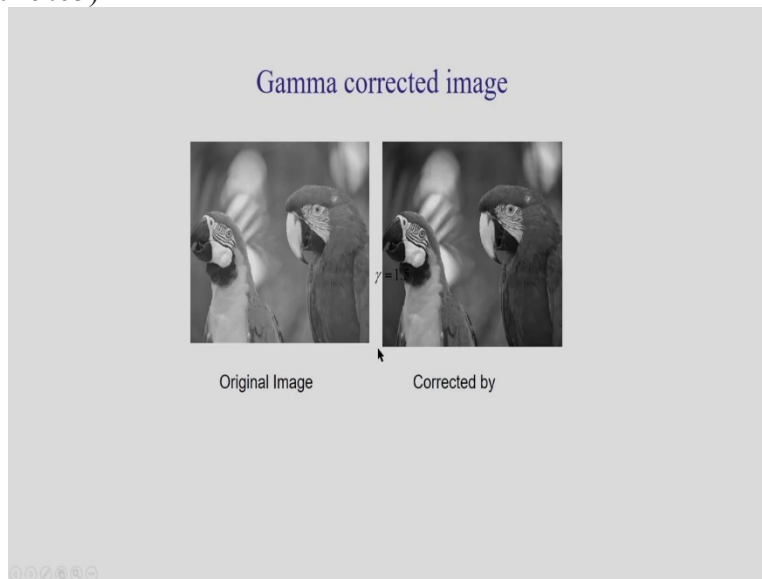
So I have to do some corrections, that is called the gamma corrections. So suppose my input is this, that means this image. So, corresponding to this image my gamma is, gamma is equal to 1. Now, I have to do some gamma correction. Generally in the TV monitor, gamma lies between 1.8 to 2.5. And in this case, if I want to do some gamma corrections, so what I have to do? So suppose gamma is equal to 1 divided by 2.5. So suppose if I do the processing  $S$  is equal to  $r$  divided by 1 by 2.5, then it is nothing but  $r$  is equal to 0.4.

So first, I am doing the gamma correction. If I do this correction, then in this case my input will be this, input to the monitor will be this. And in this case, corresponding to this input, my output will be this. So then monitor output will be the ramp output I am getting, that is the gamma will be 1 in this case.

So that means, you can see what I am doing. So before applying to the monitor, I am doing the gamma correction. So the second step, you can see, this step is nothing but the gamma corrected step. So this input is actually inputted to the monitor. And corresponding to this I have the output, the output is this output. And the output is nothing but the gamma is equal to 1, corresponding to this.

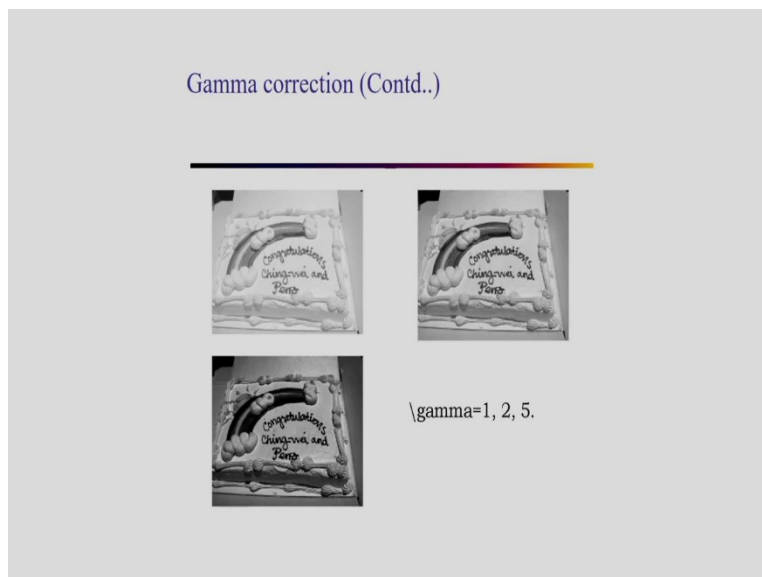
And one thing is important, that different monitors have different settings, that means gamma will be different for different monitors. So this is the concept of gamma correction. I can give some examples of gamma corrections.

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You can see here, the original image and corrected by, that is by, gamma I am considering, gamma is equal to 1, that I am considering. Gamma is equal to 1.2 in this case, I think. So it is a gamma corrected image.

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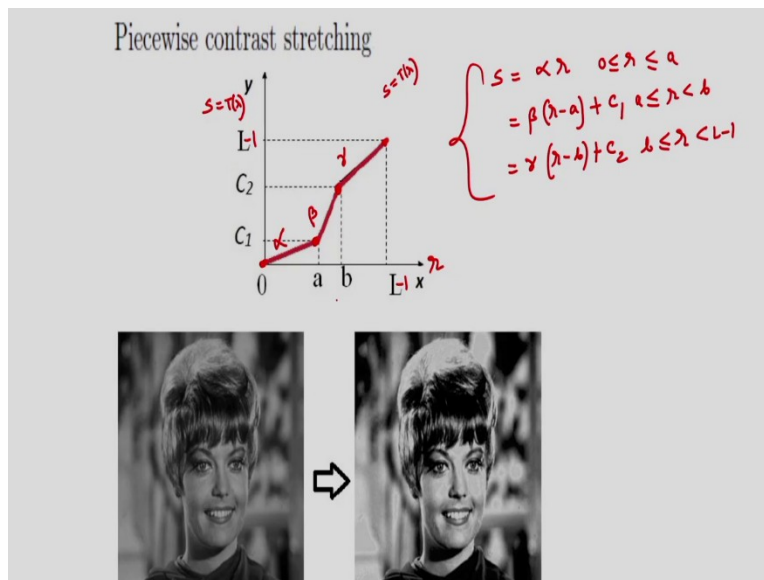
Similarly, I can give some examples, gamma is equal to 1, 2, 5. So for gamma is equal to 1, gamma is equal to 2, gamma is equal to 5, I have these images. So this is about the gamma corrections.

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## Piecewise Linear Intensity Transformation Function

Next point I want to discuss, the piecewise linear intensity function.

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So what is the meaning of the piecewise linear intensity function? I will show some transformation. The first transformation is piecewise contrast stretching operation. In this case, I am considering, the transformation is this. This transformation is  $S$  is equal to  $T r$ . My input is this, so I can consider it is 0 to  $L$  minus 1. And also here I am considering 0 to  $L$  minus 1.

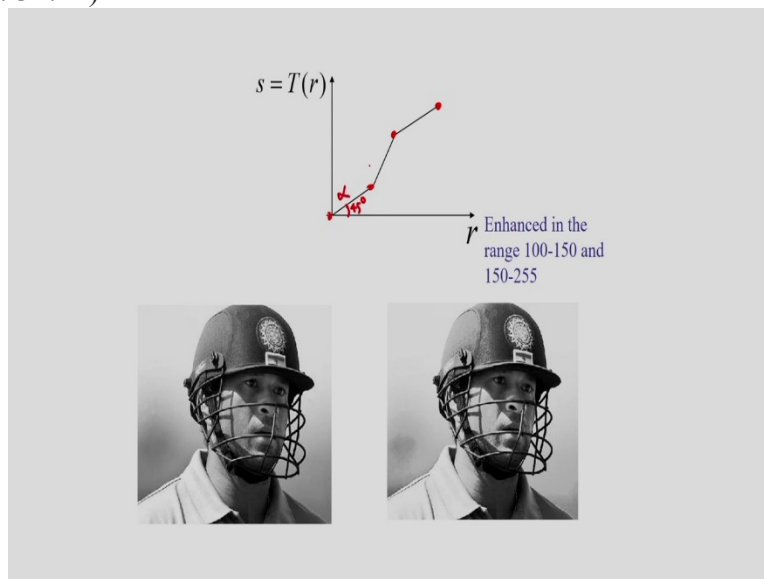
And suppose the slope of this is, the slope of this section. So I have multiple sections here, if you see the transformation. Corresponding to this section, if I consider this section, my slope is alpha.

Corresponding to the second section my slope is beta. And corresponding to the third section, my slope is gamma.

Then in this case, if I consider suppose this case, my input is this side is  $r$  and this side is  $S$ . This side is  $S$  is equal to  $T r$ . So that means if I consider mathematically what is the transformation,  $S$  is equal to, the first one is  $\alpha r$ , if the  $r$  lies between  $a$  and  $0$ ; is equal to  $\beta r - a + c_1$ , if  $r$  lies between, if  $r$  lies between  $a$  and, if  $r$  lies between  $a$  and  $b$ . And is equal to  $\gamma r - b + c_2$ , if  $r$  lies between  $L - 1$  and  $b$ . So mathematically this transformation I can show like this.

Now, in this case the slope  $\alpha$ ,  $\beta$  and  $\gamma$  determine the relative contrast stretch. So based on the value of  $\alpha$ ,  $\beta$  and  $\gamma$ , I can adjust the contrast stretching. So this is the piecewise contrast stretching operation. And corresponding to this example, I have the input image and you can see the output image, the output image is this.

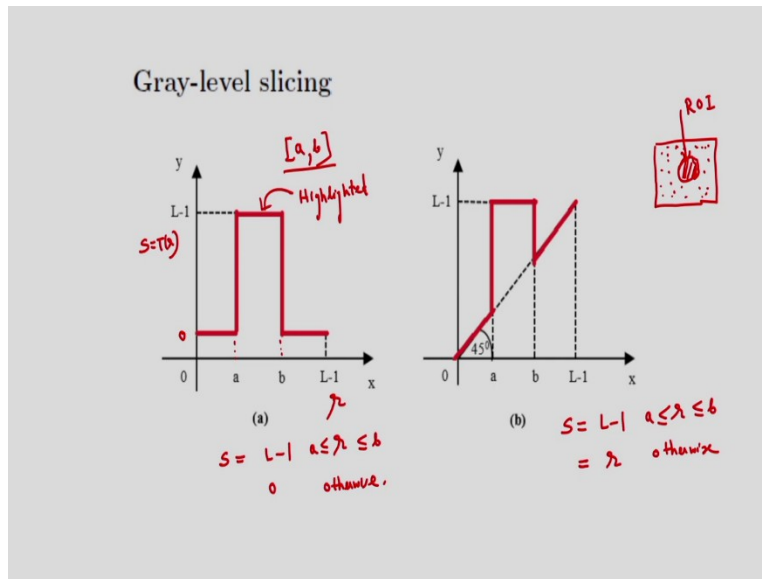
(Refer Slide Time: 32:21)



Again, I am showing another example, the contrast stretching. So enhanced in the range, the range is 100 to 150, that is enhanced the range, and also 150 to 255. So I am doing the enhancement of this. So from this point to this point nothing is there because it is a linear function, 45 degree, this angle is 45 degree. If it is 45 degree then in this case it is identity.

But corresponding to this portion, I am doing the contrast stretching and corresponding to this point to this point also I am doing the contrast stretching. So the slope is  $\alpha$ ,  $\beta$  and  $\gamma$ .

(Refer Slide Time: 33:02)



The next one is the gray-level slicing. So in this case you can see, I have shown two transformation. The first you can see, the first one is, this side already I have told you, this is  $r$  and this is  $s$ . In the first case you can see, mathematically I can show is,  $S$  is equal to  $L$  minus 1, corresponding to  $r$  lies between  $b$  and  $a$ . That means, this is my  $a$  and this is  $b$ . Otherwise, it will be 0. So this I can consider as 0. This value I can consider as 0. So otherwise it will be 0.

So what will be the output for this transformation? That means if I want to consider this range ranges from  $a$  to  $b$ , this range,  $a$   $b$ , this portion is highlighted. That means now this range is highlighted, not portion. This range is highlighted, highlighting the intensity range, the intensity range is  $a$  and  $b$ , and reduces all other intensity to some lower value, lower value may be 0. That I am considering. So this intensity range  $a$   $b$ , from  $a$  to  $b$ , that is highlighted, so this is highlighted. This is highlighted and if you consider the rest of the intensity values, the rest will be 0.

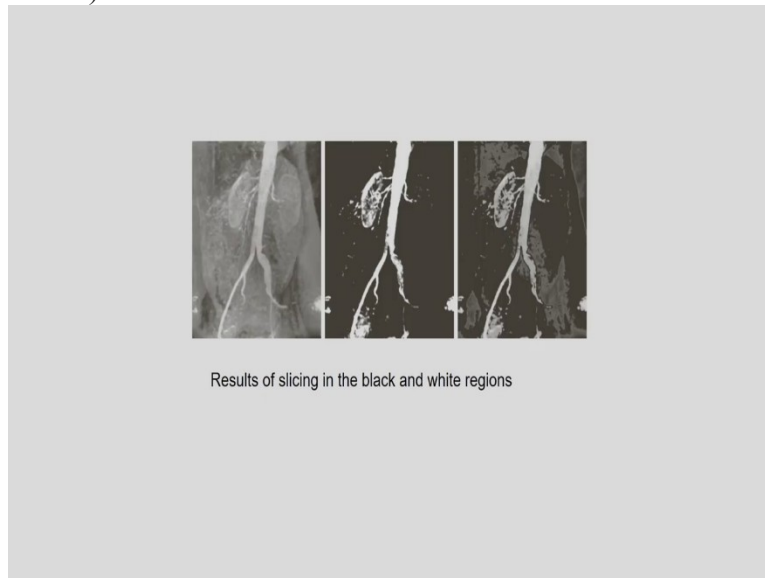
In the second case, I am doing the same thing. In the first case you can see, if I have a image, a particular image. And suppose I have one object here, particular range, intensity range. Suppose this intensity range I am highlighting. Then in this case, if I consider the first transformation, the rest of the things, the rest of the pixel values, that will be 0. Corresponding to the first transformation.

In the second case, I am showing another transformation. In this case what I am considering, from  $a$  to  $b$ , that range, that is highlighted but remaining intensity, they are not changed. So that

means, in this case I can write  $S$  is equal to  $L - 1$  corresponding to the range, corresponding to this range the output I am having, the  $L - 1$ , that portion is highlighted. But the remaining is not changed, otherwise,  $S$  is equal to  $r$ .

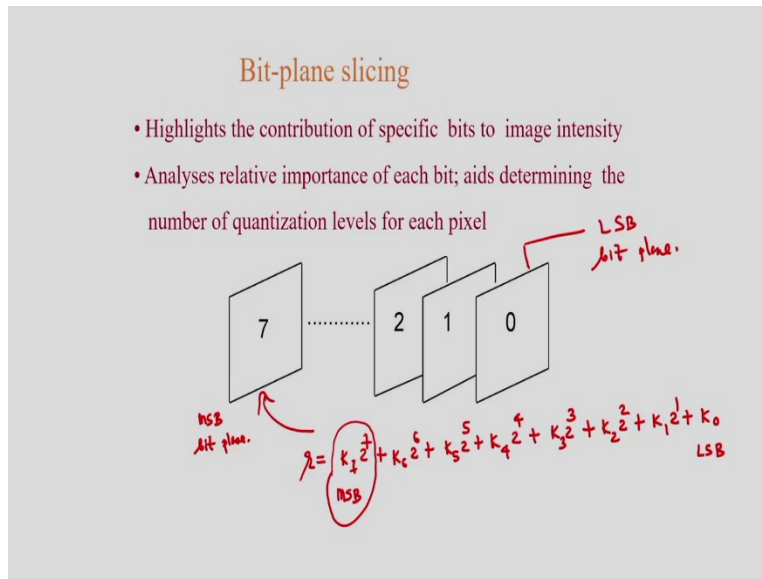
So that means this background, I am not making zero. But if I consider this portion, the region of this, so this is the region of interest, so that portion is highlighted. So that portion will be  $L - 1$  but the rest of the portion, I am not changing any pixel values.

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So I can show some of the slicing techniques. Result of slicing in the black and white regions. So I am doing the slicing in the black and white regions. This is one example.

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Another important slicing technique, that is the bit-plane slicing. That is one slicing technique. In this case, in a particular gray scale value can be represented like this,  $k_7$  suppose,  $2$  to the power  $7$ . Because I am considering the 8 bit image,  $k_6$   $2$  to the power  $6$   $k_5$   $2$  to the power  $5$ ,  $k_4$   $2$  to the power  $4$ ,  $k_3$   $2$  to the power  $3$ ,  $k_2$   $2$  to the power  $2$ ,  $k_1$   $2$  to the power  $1$ , plus  $k_0$ . Because each pixel is quantized by 8 bits. That means, I am considering 8 bit image. So that means, how many intensity levels, I have the 256 number of intensity levels.

So in this case, if I consider this representation, this corresponds to the MSB bit, the most significant bit, and this corresponds to the LSB. One is the LSB, another one is the MSB. And corresponding to this MSB, suppose I will be getting one plane, that is, one bit-plane I am getting and that bit-plane I can consider as the MSB bit-plane. The most significant bit, MSB bit-plane.

So I will be getting 8 numbers of bit planes. And corresponding to this, this is nothing but the LSB bit plane. So I will be getting the bit planes like this. This representation is quite important, because in this case I can highlight the contribution of specific bit to image intensity. And in this case, I can analyze relative importance of each bit. So which one is important, the bit 7 is important or bit 2 is important or bit 1 is important, that I can see.

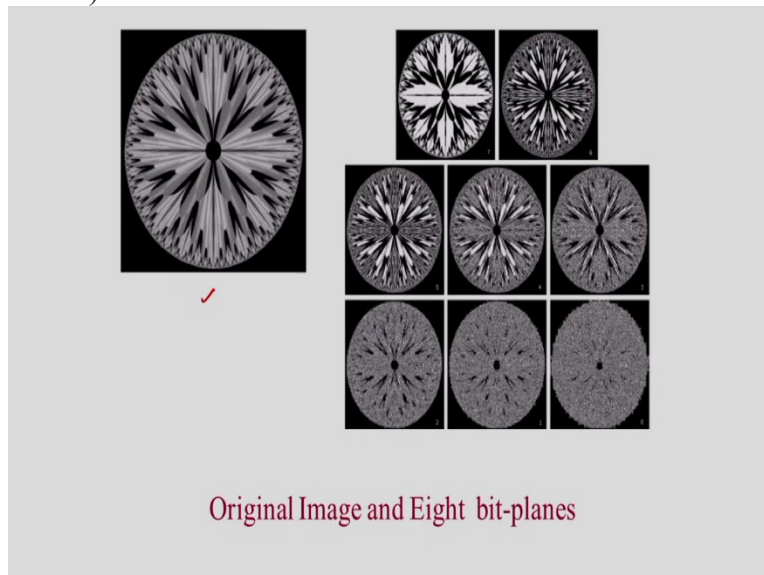
So one is to highlight the contribution of specific bits to image intensity, that I can see, and also I can analyze relative importance of each bit. In this case, I have shown the bit planes, the LSB bit-plane, MSB bit-planes, like this. And in this case, the MSB bit-plane can be obtained by thresholding at 128.

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So you can see here. This is my original image, and in this case MSB plane is obtained by thresholding at 128. So I am having this MSB bit-plane I am having. This is the MSB bit-plane.

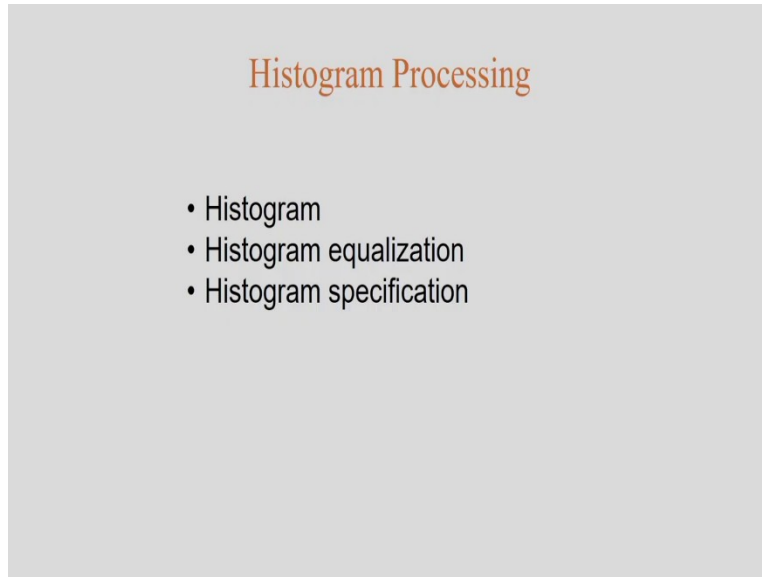
(Refer Slide Time: 39:20)



So in this case, corresponding to this image, input image, you can see, I am considering the 8 bit-planes. The LSB, MSB, all the bit-planes I am getting. And I can see the importance of the bit. Which bit is more important for visual information? So visually which bit-plane is important, that I can analyze. So that is the importance of the bit-plane slicing.



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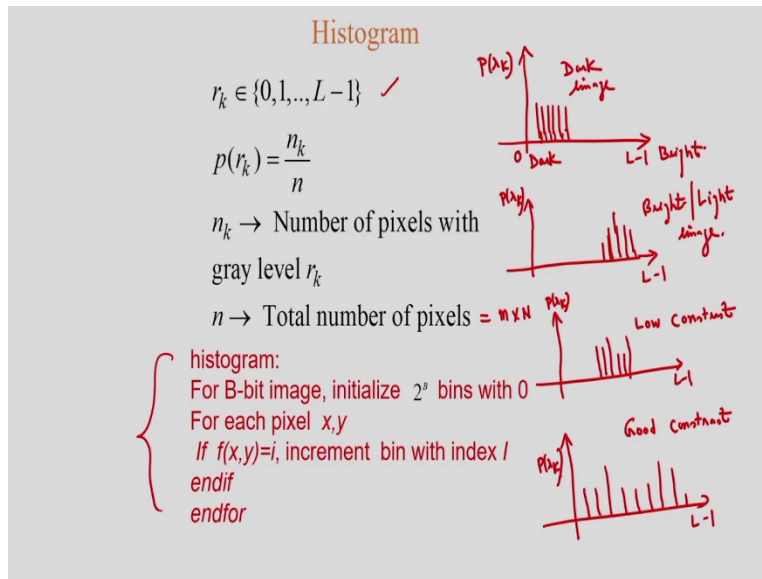


After this the next important concept is the Histogram Processing. So first, I will discuss what is the meaning of the image histogram. And after this I will discuss two important algorithms, one is the histogram equalization technique, another one is the histogram specification technique. And in this case, by histogram equalization technique, I can improve the contrast of an image. And in case of the histogram specification, I can generate a particular image based on the specified histogram.

So these two techniques are important, one is the histogram equalization, another one is the histogram specification.

So first, going to this concept, first I have to define the histogram of an image. And based on the histogram of an image, I can define the contrast of an image. Low contrast image, good contrast image, bad contrast image, I can define. And also, the assumptions for histogram equalization technique, I have to consider some assumptions and based on these assumptions, I have to apply the histogram equalization technique.

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So let us first define what is the histogram of an image. So in this case the histogram of an image, I can consider, suppose,  $r_k$  is the input intensity levels, that is the number of gray levels I am considering, from 0 to  $L$  minus 1. Now, I am determining the probability of  $r_k$ , what is the probability of the  $r_k$ ?  $n_k$  divided by  $n$ . What is  $n_k$ ? Number of pixels with gray level  $r_k$ , and  $n$  is the total number of pixels. The total number of pixels is nothing but the  $m$  cross  $n$ , the size of the image.

So based on this, I can define the histogram of an image. And I am showing one program segment. So by using this program segment, I can generate the histogram of an image.

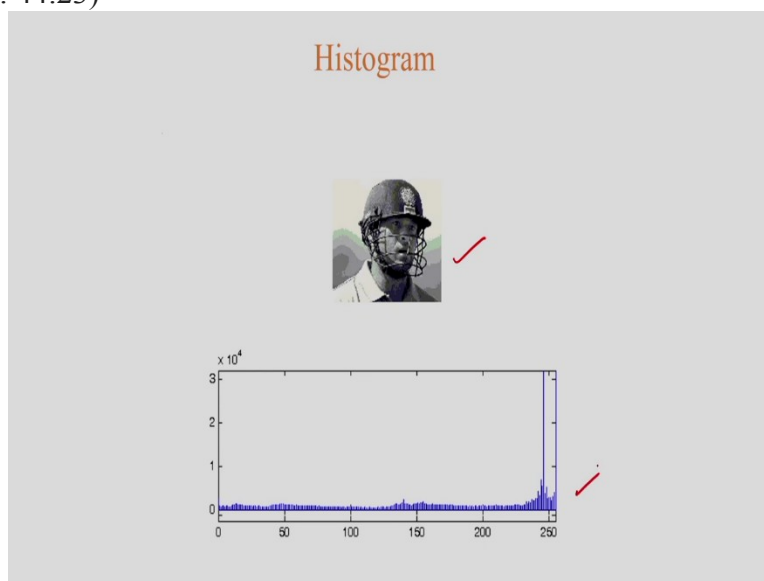
And in this case, I can show some of the cases of the histogram. Suppose if I consider histogram, is something like this. This is 0 and this is  $L$  minus 1, and I am considering the probability of  $r_k$ , that I am considering. Another histogram I can consider, suppose this is probability of  $r_k$ ,  $L$  minus 1, my histogram is something like this. Another histogram, I am considering. And this is  $L$  minus 1. And another histogram I am considering.

So if you see the first histogram, this is my dark side, this is the dark side and this is the bright side. So corresponding to the first histogram, I will be getting the dark image. Dark image I will be getting. Corresponding to the second histogram, I will be getting the bright image or maybe the light image, I will be getting. Bright, or the light image I will be getting.

And in my first class I discussed about the contrast of an image. The contrast of an image depends on the dynamic range. Dynamic range means the highest pixel value I have to consider in an image and the lowest pixel value I have to consider. The difference between these two is called the dynamic range. In the third case, the histogram is not uniformly distributed. Then in this case, I will be getting a low contrast image. And in the final case, the fourth case, the histogram is uniformly distributed. Then in this case I will be getting the good contrast image.

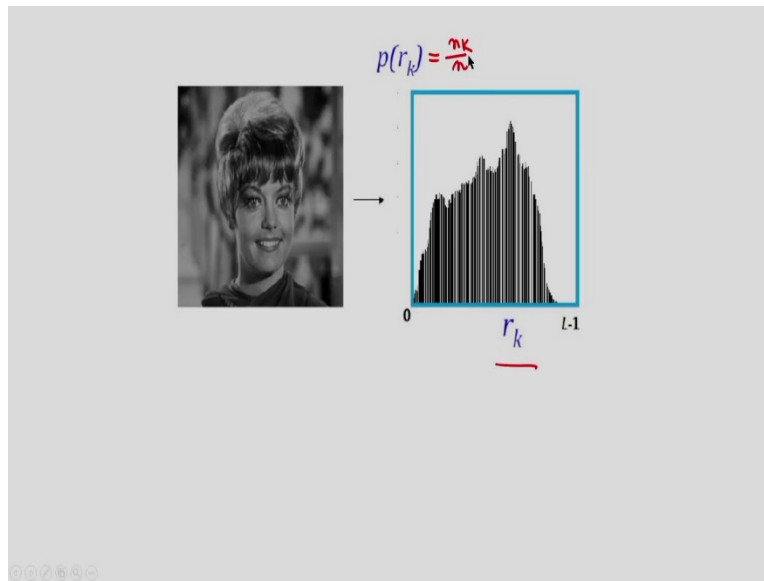
So you can see the histogram concept and based on the histogram I can define the dark image, the bright image, low contrast image and the good contrast image.

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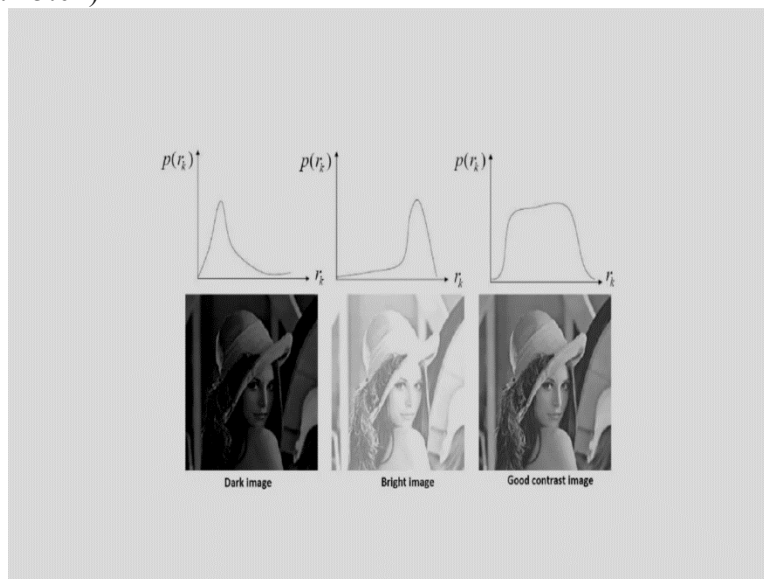
In this case, I have shown the histogram of an image. So this is my input image, and I have shown the histogram of the image. The histogram of the image, you can plot.

(Refer Slide Time: 44:33)



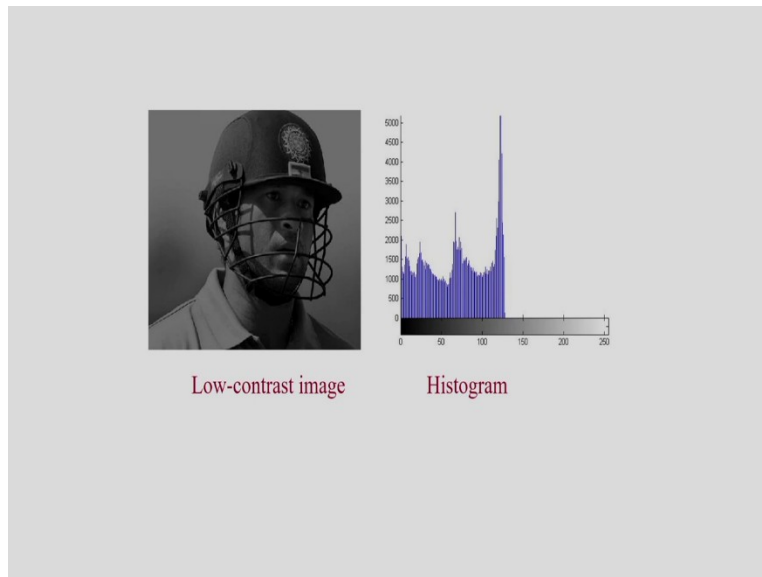
And again in this case, I have shown the histogram of the image. In the x-axis, I am considering  $r_k$ , that means the input pixel values, the pixel values of the image. And in the y-axis, I am considering the probability of  $r_k$ , that is nothing but  $n_k$  divided by  $n$ . That means, what is  $n_k$ ? Number of pixels with gray level  $r_k$ . And  $n$  means the total number of pixels.

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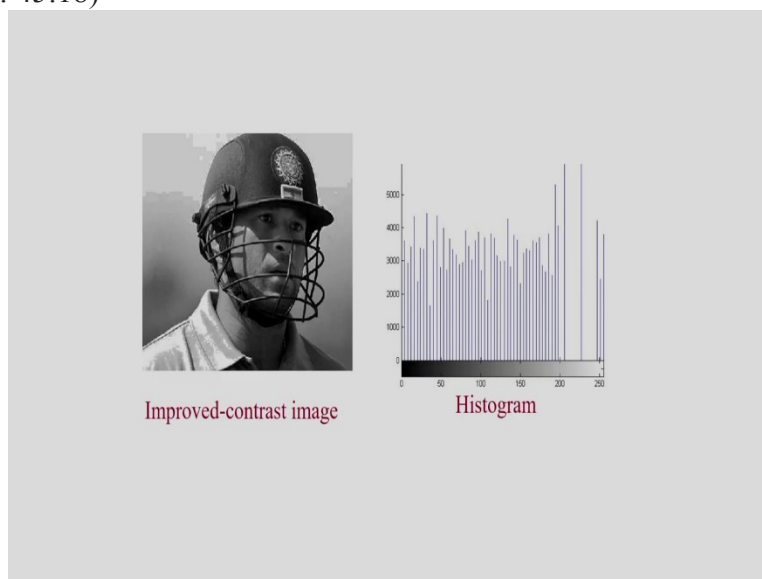
And in this example, I have shown, the one is the dark image, one is the bright image, another one is the good contrast image. So that already, I have explained.

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And you can see one example of the low contrast image and corresponding histogram, you can see.

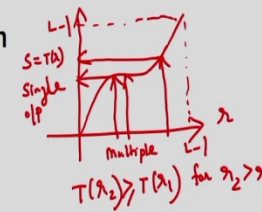
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This is the improved contrast image. You can see the histogram of the image. This histogram is uniformly distributed so the contrast is better as compared to the previous one.

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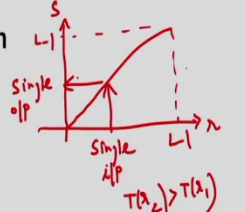
### Histogram Equalization

$$s = T(r) \quad \underline{0 \leq r \leq L-1}$$


**Assumptions**

- (a)  $s = T(r)$  is a monotonically increasing function in the interval  $0 \leq r \leq L-1$   $\Rightarrow$  o/p intensity value will never be less than the corresponding input intensity value.
- (b)  $0 \leq T(r) \leq L-1$  for  $0 \leq r \leq L-1$   $\Rightarrow$  range of o/p intensity is same as the i/p intensity
- (c)  $r = T^{-1}(s)$ ,  
 $T(r)$  is a strictly monotonically increasing function in the interval  $0 \leq r \leq L-1$   $\Rightarrow$  mapping from  $s$  back to  $r$  will be one to one

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 $T(r)$  is a strictly monotonically increasing function in the interval  $0 \leq r \leq L-1$   $\Rightarrow$  mapping from  $s$  back to  $r$  will be one to one

Now for histogram equalization, what is the histogram equalization? I will explain now. So I am considering this point transformation  $s$  is equal to  $T r$ . So that already, I have explained. So  $r$  is the input image and  $s$  is the output image. And the input range is from 0 to  $L$  minus  $L$ ,  $L$  minus 1.

The first assumption is  $s$  is equal to  $T r$ , that is monotonically increasing functions in the interval, the interval is  $r$  is equal to from 0 to  $L$  minus 1. So first assumption is the transformation function, the transformation function, that is the transformation is  $T$ , is monotonically increasing function. So what is the interpretation of this?

The interpretation is this, output intensity value will never be less than the corresponding input intensity value. So this is the first case. That is, the transformation is monotonically increasing function, the  $T$  is a monotonically increasing function. The interpretation is, the output intensity value will never be less than the corresponding input intensity value.

The second one is, I am considering the second condition is this, the  $T r$  lies between 0 to  $L$  minus 1, corresponding to input  $r$  lies between 0 to  $L$  minus 1. So what is the meaning of this? The meaning is, the range of output intensity is same as the input intensity. So this is assumption, the second assumption is this, range of output intensity is same as that of the input intensity.

And finally, I am considering another assumption. So what is the  $r$ ?  $r$  is the  $T$  inverse  $s$ , because I have  $s$  is equal to  $T r$ . So in this case I am considering  $r$  is equal to  $T$  inverse  $s$ . So that means, the meaning is the  $T r$  is a strictly monotonically increasing functions in the interval,  $r$  lies between 0 to  $L$  minus 1. So what is the interpretation of this? That  $T r$  is a strictly monotonically increasing function in the interval 0 to  $L$  minus 1, corresponding to the input value is  $r$ . That is, mapping from  $s$  back to  $r$  will be one to one. That is the meaning of this. The mapping from  $s$  back to  $r$  will be one to one.

So these three assumptions I am considering for histogram equalization.

Now, in this case, what is the meaning of the monotonically increasing function? I can show one example. I can interpret. Suppose I am considering this and suppose this I am considering. So this is  $L$  minus 1 and also, this is  $L$  minus 1. So you can see, if I consider this multiple input value. It will give a single output value. This side is input and this side is output,  $s$  is equal to  $T r$ .

So this is multiple input and I will be getting the single output. And if I consider a single input then in this case also, I will be getting the single output. The single output I will be getting. That means monotonically increasing function means  $T r_2$ , suppose is greater than equal to  $T r_1$ , for  $r_2$  greater than  $r_1$ . So this is the definition of the monotonically increasing function. And what is the strictly monotonically function, I can explain.

The strictly monotonically function, I can explain here. So in the strictly monotonically increasing function, so for the strictly monotonically increasing function, something like this. So this is  $L$  minus 1 and this is  $L$  minus 1. This is my  $s$  and this is  $r$ . So in this case the single input

that will give the single output. Single input and single output. This is nothing but one to one mapping. Single output. Single input, single output. That is one to one mapping.

That means  $T(r)$ , mathematically I can show you. The  $T(r_2)$  is greater than  $T(r_1)$ , for  $r_2$  greater than  $r_1$ . So that is the definition of the strictly monotonically increasing function. So these assumptions are very important.

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Histogram Equalization (contd.)

$r$  can be treated as a random variable in  $[0, 1]$  with pdf  $p_r(r)$ . The pdf of  $s=T(r)$  is

$$p_s(s) = \frac{p_r(r)}{\left| \frac{ds}{dr} \right|_{r=T^{-1}(s)}}$$

Suppose  $s=T(r) = \int_0^r p_r(u) du, 0 \leq r \leq 1$

then,  $\frac{ds}{dr} = p_r(r)$

$$\therefore p_s(s) = \frac{p_r(r)}{p_r(r)} = 1, 0 \leq s \leq 1$$

$p_r(x) \Rightarrow$  pdf of  $r$   
 $p_s(s) \Rightarrow$  pdf of  $s$

The next one is the histogram equalization technique. So in this case you can see, the  $r$  can be treated as a random variable and corresponding to the interval from 0 to 1, that I can consider. And what is the  $P_r$ ? That is, the  $P_r$  is nothing but the pdf of  $r$ . So that is the pdf. So  $P_r r$  is the nothing but the pdf, the density function, the probability density function of  $r$ . And I am considering  $P_s s$ , that is the pdf of the output, the output is  $s$ .

So in this case, we have the distance from  $s$  is equal to  $T(r)$ . Then in this case, you can determine the  $P_s s$ , that is the pdf of  $s$ , you can determine by this expression. And we have this transformation  $s$  is equal to  $T(r)$ , then in this case, you can determine  $s$  by this integration, and in this case I am considering  $u$  is the dummy variable, then in this case the  $ds, dr$  will be this, and corresponding to this I can determine the pdf of  $s$ , I can determine. Then in this case I will be getting 1.

What is the meaning of this, pdf of the output variable? The output variable is  $s$ . That means I am getting the uniform distribution. So I can show this one. My  $P_r r$  maybe something like this.



Distribution is maybe something like this. This is  $L - 1$ . But after the histogram equalization, I will be getting a pdf, this is  $L - 1, 1$  by  $L - 1$ . So I am having this one.

So you can see, after histogram equalization, I am getting the uniform pdf.

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Histogram Equalization

$$r_k \in \{0, 1, \dots, L-1\}$$
$$p(r_k) = \frac{n_k}{n}$$
$$g_k = \sum_{i=0}^k p(r_i)$$

The resulting  $g_k$  needs to be scaled and rounded.

And for the discrete case, thus, you can implement the histogram equalization technique. This is my  $r_k$  and I can define the probability of  $r_k$  and after this, I am getting the output, the  $g_k$  I can determine. So for the discrete case also you can apply the histogram equalization technique.

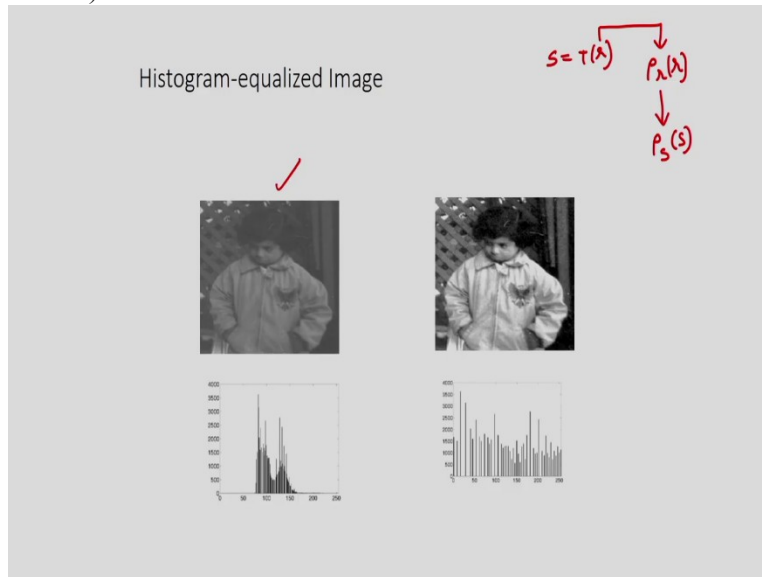
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Histogram-equalized image image      Histogram

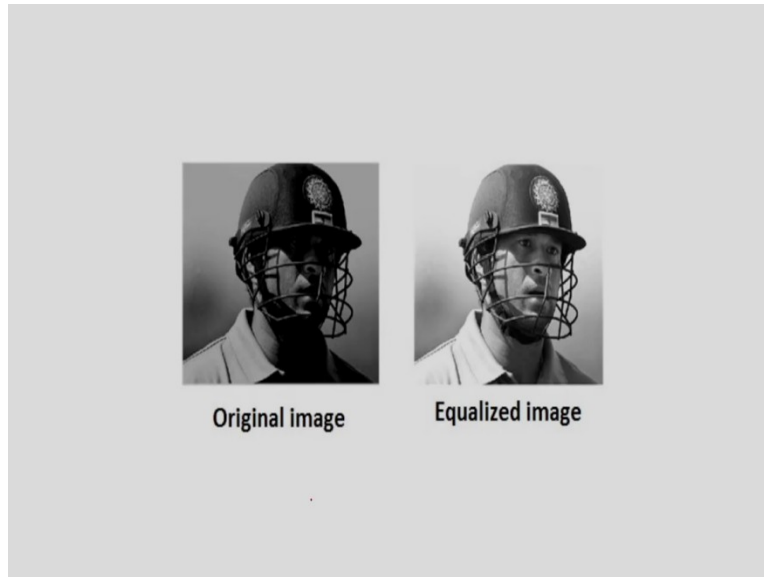
And I can show some examples of the histogram equalized image. So here you can see, it is the histogram equalized image. And you can see the corresponding histogram, that is, uniformly distributed.

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Here also I have shown one example, that is the histogram equalized image I am having. So first one is the low contrast image, the second one is the histogram equalized image. So this is about the histogram. So that means in this case what I have done, so  $s$  is equal to  $T r$ ,  $r$  is the input image. And in this case, from the input image I can determine this pdf. And from this, I have to determine  $P s s$ , I have to determine. That is the concept of the histogram equalization technique. So from the input image, I am determining the pdf of  $r$ , and from the pdf of  $r$  I am determining the pdf of the variable  $s$ , that I am determining.

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And in this case also I am showing the example, the histogram equalization techniques, I am applying and you can see the first one is the low contrast image, the second one is the equalized image.

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**Histogram specification**

Given an image with a particular histogram, another image which has a specified histogram can be generated and this process is called histogram specification or histogram matching.

$p_r(r) \rightarrow$  original histogram  
 $p_z(z) \rightarrow$  desired histogram

$$s = \int_0^r p_r(u) du$$

$$r = \int_0^z p_z(w) dw$$

$$z = G^{-1}(s) = G^{-1}T(r)$$

*Handwritten notes:*  
 input  $r$   $\downarrow$   $p_r(r)$   
 o/p image  $z$   $\uparrow$   $p_z(z) \rightarrow$  specified histogram  
 $s = T(r) = \int_0^r p_r(u) du$   
 Transformation function  $G(z) = \int_0^z p_z(t) dt = s$   
 $G(z) = T(z)$   
 $\therefore z = G^{-1}[T(r)] = G^{-1}(s)$   
 Equalized image

The another thing is the histogram specification. So what is the histogram specification? Given an image with a particular histogram, another image which has a specified histogram can be generated and this process is called the histogram specification or histogram matching.

So what I am considering, just suppose I have the input image. Input image is suppose  $r$ . From the input image, I can calculate this one, this pdf I can calculate. And suppose, I have the output, image is  $z$ . This is my output image. Now, the specified histogram is given. The specified histogram is this. This is my specified histogram. From the specified histogram, I have to get the output image.

That means the specified pdf, it is given. The output image should have this pdf. So I can consider, suppose  $s$  is equal to  $T r$ , you know this. That is nothing but  $L$  minus 1, 0 to  $r$ , and this is the  $P r W$ ,  $W$  is the dummy variable for the integration. And in this case,  $z$  is the random variable, so  $G z$  is equal to  $L$  minus 1, 0 to  $z t$ ,  $t$  is the dummy variable for the integration. Then in this case, I will be getting the equalized image,  $S$  is the equalized image.

So specified histogram is given and I have to generate the image. So  $s$  is equal to  $T r$ ,  $L$  minus  $r$ , so first I am considering this one. And after this, the transformation function,  $G z$  is the transformation function. Transformation function I am determining. So  $G z$  is equal to  $T r$ , and from this you can determine  $z$ .  $z$  is the output image you can determine. The output image is  $G$  inverse  $T r$ . This, you can determine.

So you can see the difference between histogram specification and the histogram equalization.

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**Algorithm 1 HISTOGRAM SPECIFICATION**

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- INPUT: Input image, target image
- OUTPUT: Output image which has the same characteristic as that of the target image
- Steps:
  - Read the input image and the target image.
  - Determine the histogram of the input image and the target image.
  - Do histogram equalization of the input and the target image.
  - Calculate the transformation function  $G(z)$  of the target image.
  - Obtain the inverse transformation  $z = G^{-1}(s)$ .
  - Map the original image gray level  $r_k$  to the output gray level  $z_k$ .  
So, pdf of the output image will be equal to the specified pdf.

---

So in this algorithm, I have shown the algorithm for the histogram specification. So you can just see the algorithm for the histogram specification, that already, I have applied.

So up till now, I have discussed these two important techniques, one is histogram equalization technique, another one is the histogram specification technique. In histogram equalization technique, what we have done, we have considered the three assumptions that already I have explained, and based on the three assumptions I am getting the equalized image. So after equalization, I am getting the  $n$  from pdf. In case of the histogram specification, the specified histogram is given and from this, I have to generate the image and it should be equalized, the equalized output image.

So you can see the difference between the histogram equalization and the histogram specification. The histogram equalization technique you can apply for the entire image, for the whole image, or maybe you can apply in a particular region. So one is the local histogram, another one is the global histogram. Global histogram means the histogram equalization technique is applied for the whole image, and if I consider only the region of interest, in the region if I apply the histogram equalization technique, that is the called the local histogram. This histogram equalization technique, histogram specification technique and also the point operations, very important operations for image enhancements.

So next class, I will continue the same concept. And I will discuss in the next class the concept image filtering. So let me stop here today. Thank you.