Computer Vision and Image Processing

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Lecture: 14

Fundamentals and Applications

Welcome to NPTEL MOOCs course on Computer Vision and Image Processing: Fundamentals and Applications.

In my last class, I discussed the basic concept of wavelet transformation. I highlighted one important concept that is a signal should be analyzed both in time domain as well as in frequency domain. Also, I discussed the concept of continuous wavelet transformation. So, for this, I considered the mother wavelet and also, I considered two parameters, one is the translation parameter and another one is the scaling parameter.

So, by considering these two parameters, I can get a number of wavelets for analysis of a particular signal. After this, I discussed the concept of multi resolution analysis. A signal should be analyzed in different resolutions, a high resolution and a low resolution. So that concept also I discussed in my last class.

Today, I am going to discuss the concept of discrete wavelet transformation and how to decompose a particular signal into different frequency bands. In my definition of the signal, I am considering either 1D signal or 2D signal. The 2D signal means image. So that concept, I am going to discuss, that is, how to decompose a particular signal by considering the discrete wavelet transformation.

So I can decompose a particular image into number of frequency bands. That means, I can consider a low frequency information, I can consider high frequency information. So that concept I am going to discuss now.

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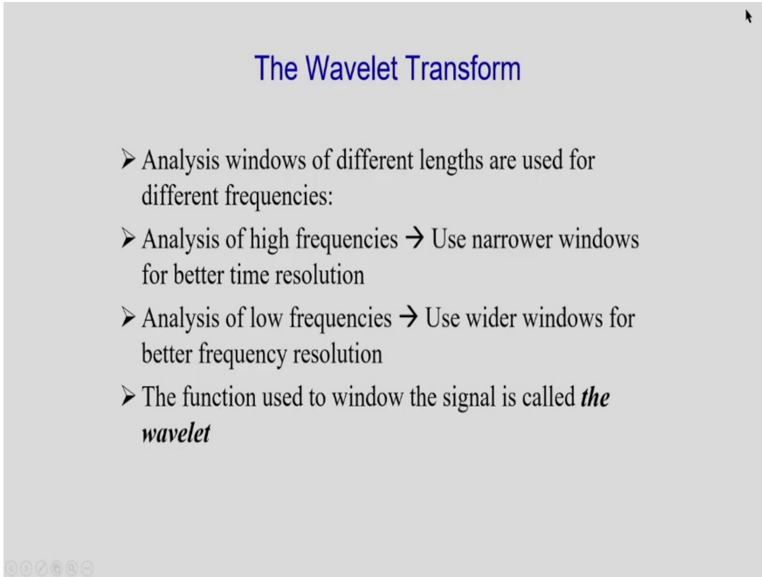

So in my last class, you can see here, the main concept of the wavelet transform is, that is, I am considering analysis windows of different lengths are considered, for different frequencies. So if I consider high frequencies, we have to consider narrow windows. So that is, use narrow windows for better time resolution. And if I consider the analysis or low frequencies, then in this case we have to consider wider windows for better frequency resolution. And the function used to window the signal is call the wavelet. So that concept, I have already explained, that is, the analysis of the high frequency and also the analysis of low frequencies.

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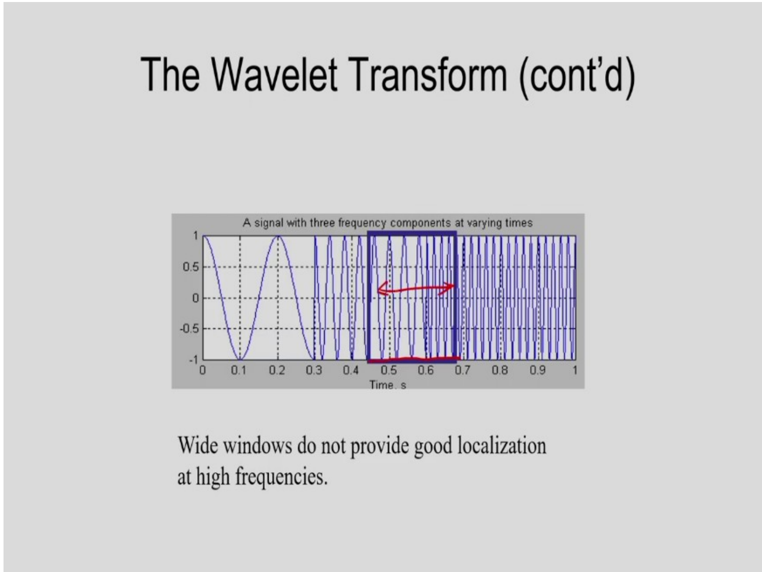

Here, in this example, you can see, I am considering one window. You can see the window here. So this is the window. And in this case, I am considering one wide window. So, wide window do not provide good localization at high frequency. So here you can see, so this portion corresponds to the high frequency. So that is, you can see from this figure.

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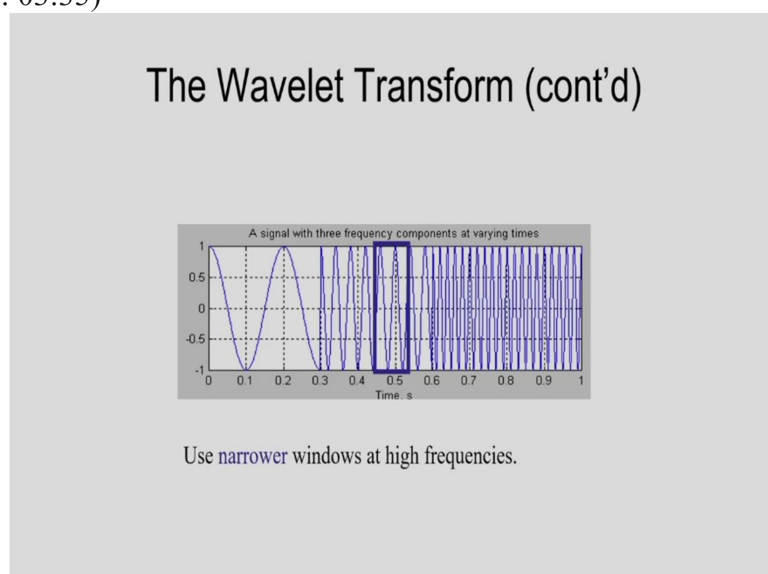

And that is why, if we consider the narrow window, that is better for high frequency. That is, use narrow windows at high frequencies.

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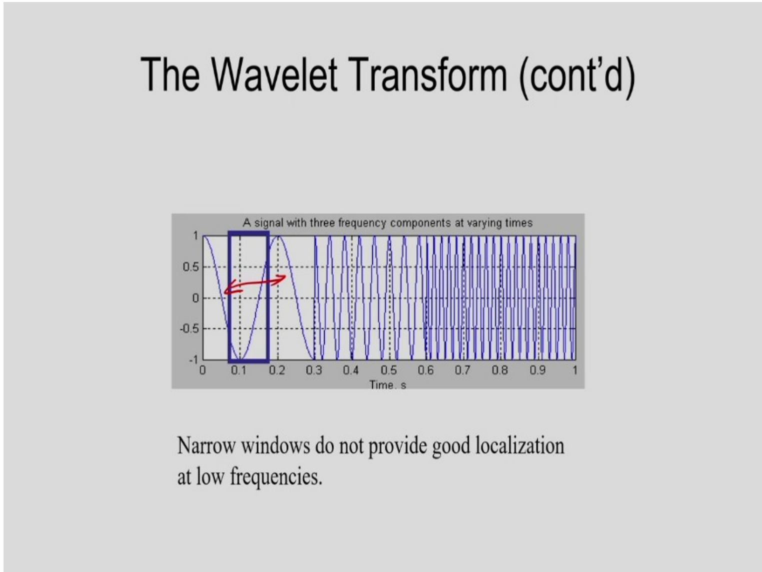

And after this, you can see, I am considering one narrow window. And if I consider this portion, that portion is the low frequency portion of the signal. So if I consider the narrow window, they do not provide good localization at low frequencies. That is why, I have to consider a wide window for analysis of the low frequency components. So that is why, I have to consider one wide window for the analysis of the signal.

So here, in this example, you can see that narrow windows do not provide good localization at low frequencies.

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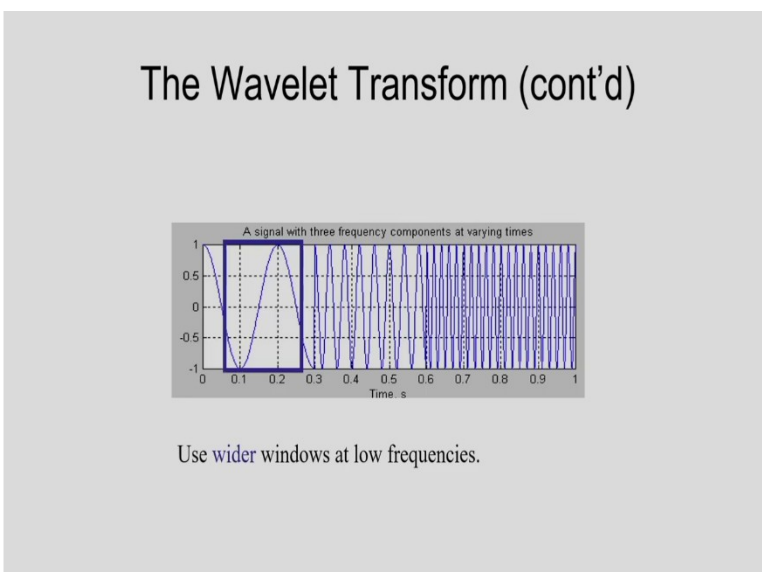

That is why, we are considering one wide window at low frequencies. So that is why the window size is not fixed in case of the wavelet transformation. So I am considering variable size windows. Narrow windows for the high frequency information analysis, and if I consider the wide window, that is good for low frequency signal analysis.

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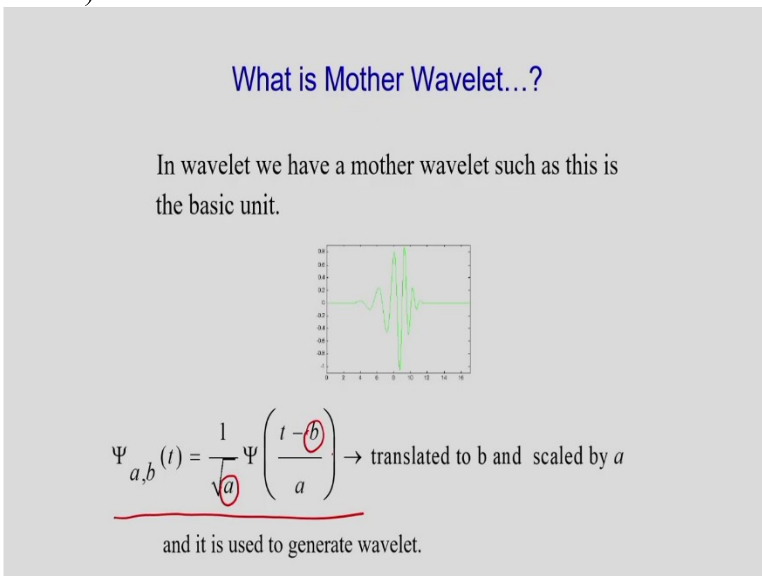

And also, I discussed the concept of the mother wavelet. Here you can see, I am considering the one wavelet function, I am considering. And in this case, you can see, I have two parameters. One is he scaling parameter another one is the translation parameter. So 'a' is the scaling parameter and 'b' is the translation parameter. That concept also I have discussed in my last class.

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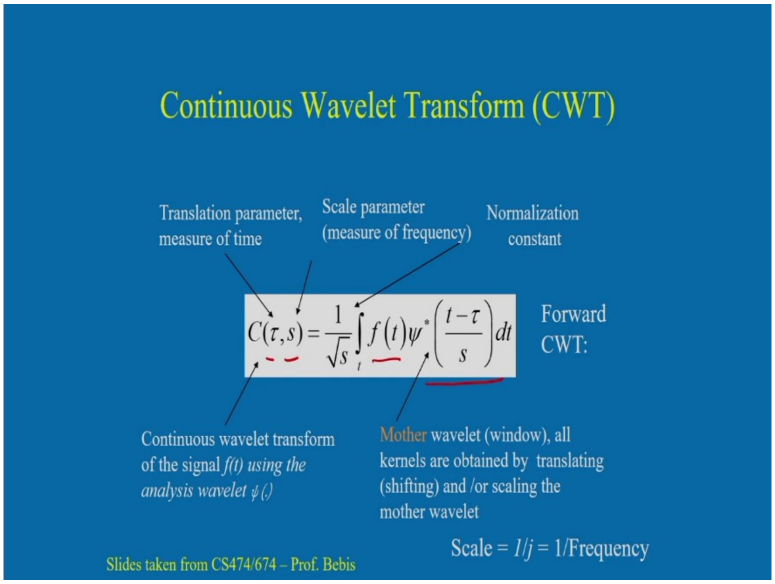

And after this, I discussed the concept of continuous wavelet transformation. So this is the definition of the continuous wavelet transformation. $C(\tau, s)$, for the signal, the signal is f (t). I am considering one 1 dimensional signal. And I have two parameters, one is τ, another one is s. So τ is the translation parameter and s is the scaling parameter which measures frequency. And the translation parameter, that is the measure of time.

And here you can see, so this expression is for the continuous wavelet transformation for the signal. The signal is f (t). And I am considering the kernel, so mother wavelet I am considering, $\Psi(\frac{t-\tau}{s}$¿, I am considering. So, this is the definition of the continuous wavelet transformation.

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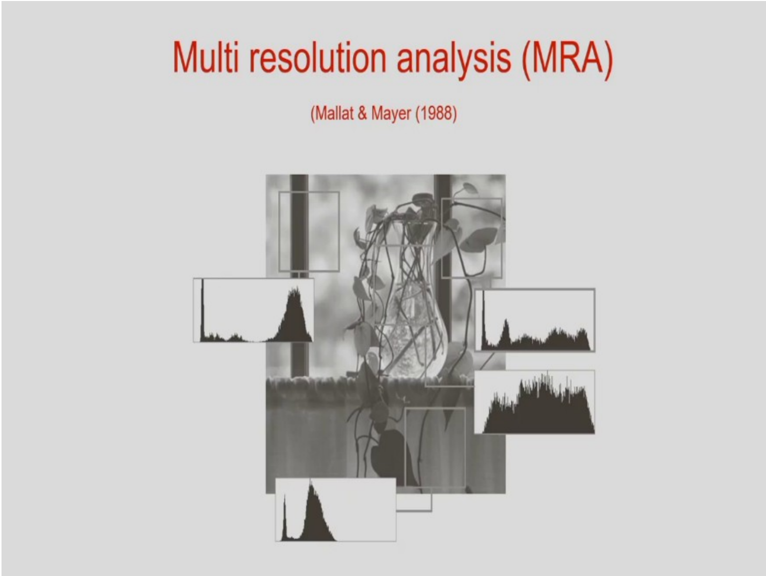

After this I discussed the concept of the multi resolution analysis. So that means, we have to analyze the signal at different frequencies with different resolutions. So I have given the example in case of the image. So suppose if I consider one low contrast region in an image, that I have to analyze at the high resolution. And also, if I consider a small object in an image, that I have to analyze at high resolution.

But if I consider a good contrast region, that I can analyze at low resolution. And if I consider a big object in an image, that I can analyze in low resolution. That is the concept of multi resolution analysis. That means analyze the signal at different frequencies with different resolutions.

And also, it is important to analyze a signal, both in time domain and frequency domain. But the problem is uncertainty principle, that already I have explained the concept of the uncertainty principle.

The point is that, if I consider high frequency, that means the good time resolution and poor frequency resolution at high frequency. On the other hand, good frequency resolution and poor time resolution at low frequencies. So that concept already I have explained.

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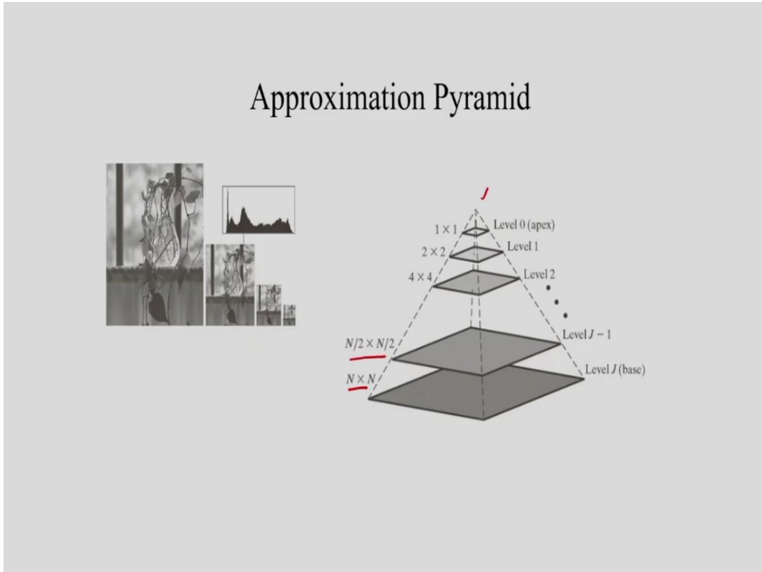

And in this case, you can see I am considering Approximation Pyramid. So if I considered N by N image, that is the highest resolution I am considering. So that is the image. If I go to the top of the pyramid, the next level is N by 2, N by 2, that is the less resolution I am considering. And the lowest resolution is 1 by 1, that is 1 pixel, I am considering.

So I am considering different resolutions for multi resolution analysis. So that means, I want to analyze a particular signal at different frequencies with different resolutions.

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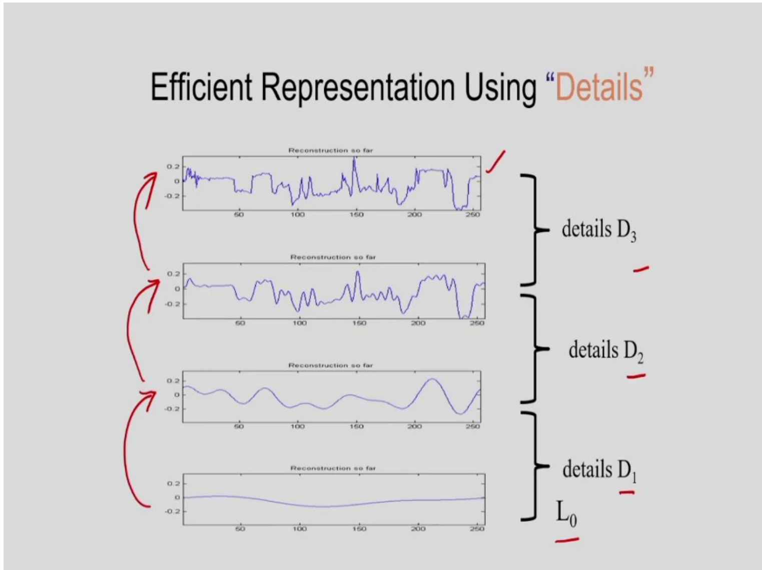

And also, I had explained this concept, that is, how to represent a particular signal by considering low frequency information and the high frequency information. So here you can see, I am considering one signal, the signal is this. And that is approximated by low frequency information and the high frequency information. So L_0 is the low frequency information. And for the high frequency information, that is, the detailed information, I am considering D1, D2, D3.

And suppose if I combine L_0 and D1, L_0 is the low frequency information and D1 is the high frequency information. So by using this, I want to reconstruct the signal, then in this case I can reconstruct the signal like this, that is, by considering L_0 and D1.

After this, I am considering the detail information D2. So if I consider the detail information D2, so the reconstruction can be like this. So I can do the reconstruction like this. And after this, I am considering the detail information D3, so by considering this I can reconstruct the signal.

That means, for reconstruction I am considering low frequency information, also the high frequency information, that is the detail information, D1, D2, D3. That is the efficient representation using the high frequency information, that is the detail information.

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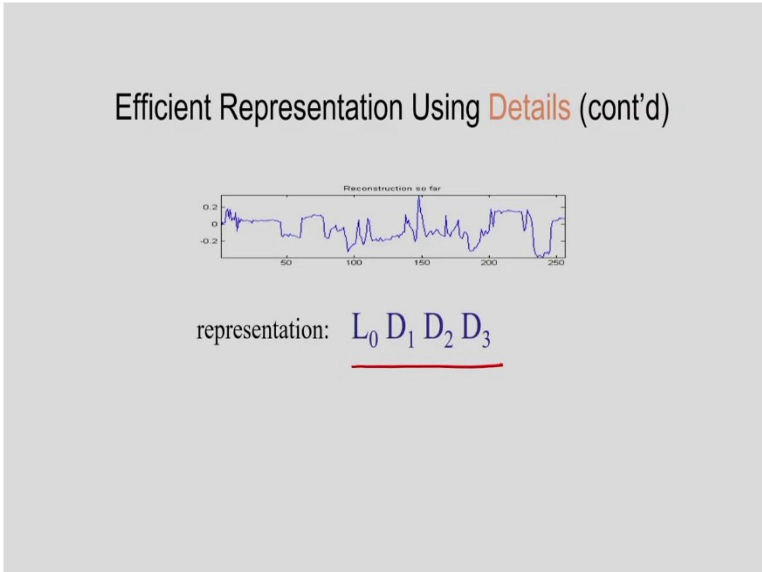

So, you can see, the signal is represented by this one. One is the L_0 is the low frequency information, after this, I have the detail information like D1, D2, D3. So, by using this, I can reconstruct a particular signal or I can represent a particular signal.

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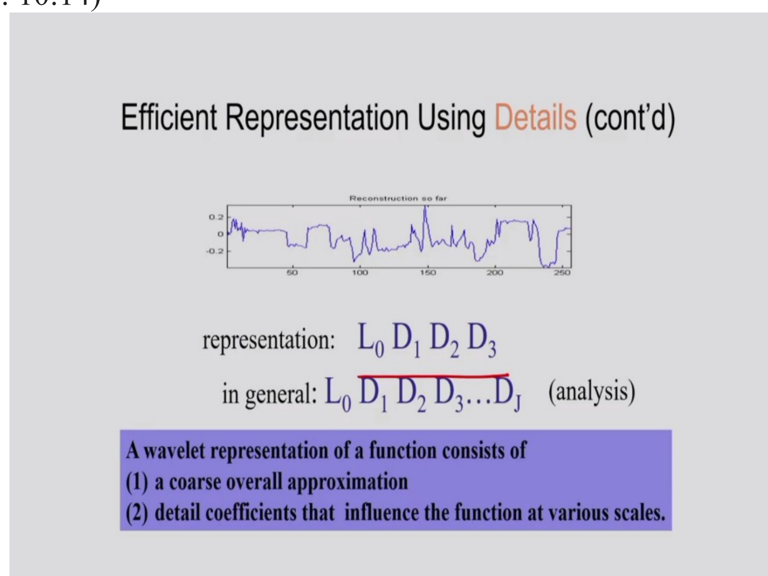

So, for wavelet representation of a function, a coarse overall estimation, that is the low frequency information, and also, I have the detail coefficients that influence the function at various scales. So that means, for wavelet representation of a particular function, so I have to consider coarse overall approximation for the low frequency component. Also, I have to consider detail coefficients that influence the function at various scale.

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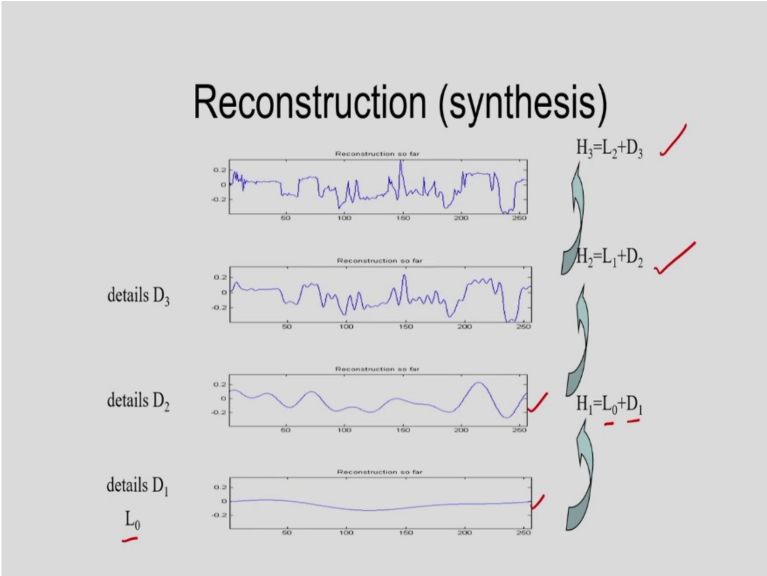

And you can see how to reconstruct the signal. So I am considering the low frequency information. So in the signal you can see this is the low frequency information. And after this, what I am considering. I am just adding the detail information with the low frequency information, that is L_0 + D1. And by using this, I want to reconstruct the signal. So this is the reconstructed signal.

After this what I am considering, the detail information D2 is considered. So that means L1+ D2 I am considering. And like this, I am considering more and more detail information to reconstruct the signal. So you can see, by considering the low frequency information and the detail information D1, D2, D3, I can reconstruct a particular signal. That is called synthesis. In case of the analysis, I can represent a particular signal by the low frequency information and the high frequency information.

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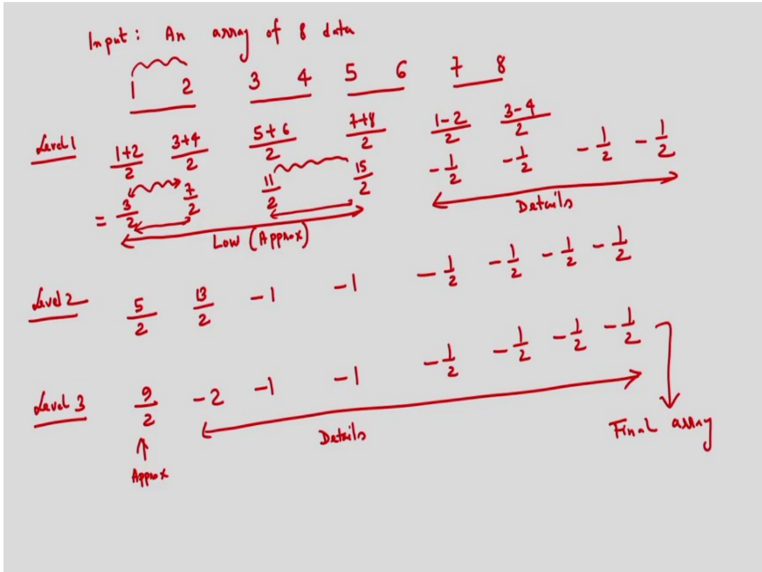

So in my last class also I had given one example. Same example I want to give here.

So suppose I am considering one input, then, suppose the input is an array of 8 data points I am considering. So suppose 1, 2, 3, 4, 5, 6, 7, 8, so eight data points I am considering. And first, I am considering Level 1 decomposition of the signal. That means, I want to decompose this signal into low frequency component and the high frequency component.

So if you see these first two samples, from the first two samples I can determine the approximate component. So the approximate component will be 1 + 2 divided by 2, so that is nothing but 3 / 2. After this, if I consider these two components, 3 and 4, so from this also I can determine the approximate component. So 3 + 4 divided by 2, so it will be 7 / 2.

And after this I am considering next two components, 5 and 6. So 5 and 6, I can consider and I am determining the approximate component. So, approximate component will be 11 / 2. And after this I am considering 7 and 8, so from 7 and 8 I am considering the approximate component. So approximate component will be 15 / 2.

So these are the approximate components. After this, I want to determine the detail components.

So from 1 and 2, that means from 1 and 2, if you see, what I can determine, the detail component. So that is 1 - 2, that is the difference between these two divided by 2, so it will be – 1/ 2. Similarly, for 3 and 4, I can determine the detail information, that is – 1/ 2. So I will be getting – 1/ 2, – 1/ 2, like this.

So in the level one decomposing, you can see I am getting the low frequency information, these are the low frequency information, up to this. And also, I am getting the detail information. Low frequency means it is the approximate. Approximate information I am getting. And in this case, I am getting the detail information. Details, I am getting. So this is Level 1 decomposition.

In Level 2 decomposition, so in the Level 2 decomposition what I am considering, I am considering 5 by 2, I am determining because my sample is 3 by 2 and 7 by 2. So if I consider these two, so I will be getting the approximate component, it is 5 by 2, 13 by 2, 13 by 2 I will be getting if I consider these two components. I will be getting 5 by 2, 13 by 2.

And after this, I can determine the detail components. The detail is - 1, - 1. So from 3 by 2 and 7 by 2, I can determine the detail information, that is, -1. And from 11 by 2 and 15 by 2, I can determine the detail information, the detail information is - 1. And the previous values are - 1 by 2, - 1 by 2, -1 by 2 and -1 by 2. This is Level 2 decomposition.

And if I consider Level 3 decomposition, I will be getting 9 by 2, minus 2, minus 1, minus 1, minus half, minus half, minus half and minus half. So in this case, this corresponds to the approximate component. And if I consider this, all this corresponds to the detail information.

So that is the final array I am getting. And that is nothing but the Haar Transformation of the input data. So that means, this is the concept of the Haar Transformation of the input data. So I considered an array of eight data and if I consider the Level 3 decomposition, that is I have done Level 1 decomposition, Level 2 decomposition and the Level 3 decomposition and I am getting the final array, so here you can see I have one approximate component and you can see the remaining are detail information, the detail components.

So this is one example.

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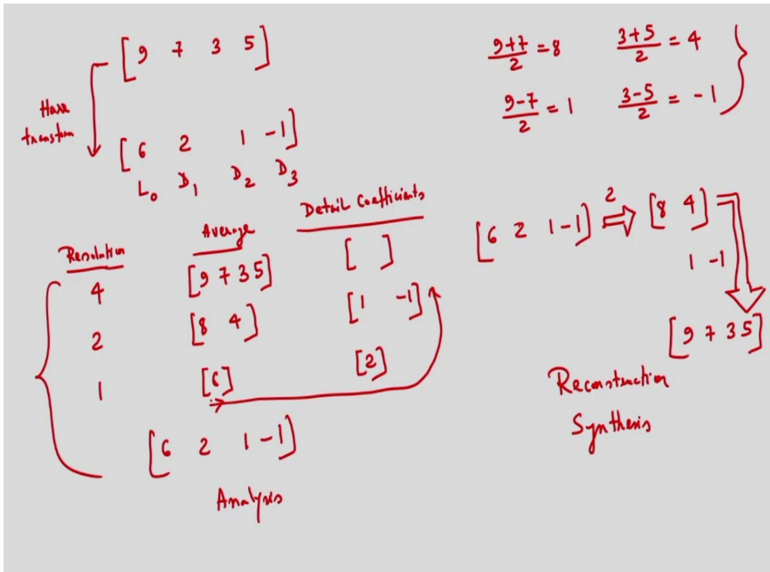

I can give another example. Suppose I have one 1D image, having 4 pixels. The pixels are suppose 9, 7, 3, 5. I am considering one 1D image and resolution, I am considering resolution, 4 pixels I am considering. And if I apply the Haar Wavelet Transformation, then in this case, if I apply the Haar Transformation. Haar Transformation means, I have to determine the approximate component and the detail component.

So, corresponding to this 9, 7, 3, 5, I will be getting 6, 2, 1, - 1. So, the 6 is the low frequency information, that is L_0, D1, detail information, D2, and D3, the detail coefficients. And in this case, suppose if I consider resolution and I am determining the average, also I am determining the detail coefficients.

So first, I am considering the 1D image with resolution of 4 pixels, that means, I am considering 9, 7, 3, 5. And I don't have any detail coefficients in this case. And after this, the resolution is, I am considering the resolution is 2, suppose. So corresponding to this, I am determining the average. Average will be 8 and 4. And what about the detail? The detail will be 1 and minus 1.

How to determine 8 and 4? 9 plus 7 divided by 2, so I will be getting 8. And how to determine 4? So 3 plus 5, divided by 2, I will be getting 4. And how to determine the detail information, that is, detail coefficients, that is, 9 minus 7, divided by 2 so it will be 1. And another one is 3 minus 5 divided by 2, that will be equal to minus 1. So like this, I am calculating the average coefficients and the detail coefficients, that is the approximate coefficient and the detail coefficient.

After this, if I consider resolution 1, then in this case also I can determine the average value. The average is 6. And what about the detail coefficient? The detail coefficient will be 2.

So this is the decomposition of a particular signal. The signal is 9, 7, 3, 5, and I am decomposing that signal. And ultimately what I will be getting? I will be getting 6, 2, 1, - 1. That I will be getting after the decomposition. That is the Haar Transformation I am applying. So you can see, I am getting 6, 2, 1, -1. So if you see here, I will be getting this. First one is 6, after this, 2 and 1, - 1.

And also, we can reconstruct the original image to a resolution by adding or subtracting the detail coefficients from the lower resolution versions. So that means, how to reconstruct the image, because my coefficients are 6, 2, 1, - 1. These are my coefficients.

So, how to reconstruct? So I have to consider the detail information, the detail information is 2. So, if I consider the detail information 2, so I can reconstruct this one, 8 and 4. After this, if I consider detail information, suppose 1, another information is 1 and - 1. So I can reconstruct the signal, the signal is 9, 7, 3, 5. So the signal is reconstructed. 9, 7, 3, 5. So this is the reconstruction procedure.

Reconstruction means synthesis. And if you see the decomposition, the decomposition means analysis. So this is analysis. And this is, reconstruction means synthesis, synthesis of the signal.

So I can reconstruct the signal by considering these detail coefficients. Because I have the approximate information, that is, the average information is available. And by considering the detail coefficients, I can reconstruct the signal. That is the concept of the signal decomposition and the reconstruction.

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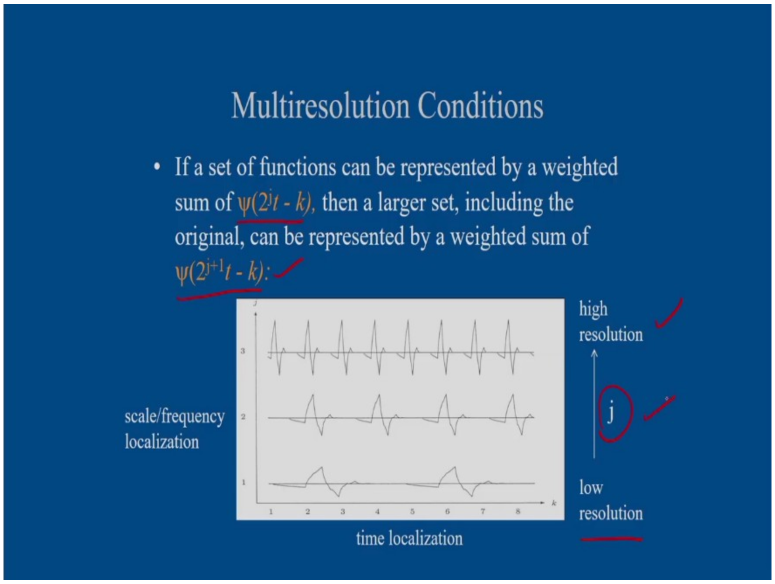

And now, I will discuss the concept of the Multiresolution Conditions, the Multiresolution Analysis, how to analyze a particular signal in different resolutions.

So suppose if a set of functions can be represented by a weighted sum of this basis function, the function is, psi 2 to the power j, t minus k. That means, I am considering a set of functions which can be represented by a weighted sum of psi 2 to the power j t minus k. That, I am considering.

Then, a larger set including the original can be represented by a weighted sum of psi 2 to the power j plus 1 into t minus k. So that means I am considering the higher resolution in this case. So in this figure, you can see, I am considering j, it controls the resolution. So you can see the low resolution signal and the high resolution signal.

So this concept is there. Suppose a set of functions which can be represented by a weighted sum of the basis function. The basis function is, psi 2 to the power j into t minus k. Then a larger set, including the original can be represented by a weighted sum of psi 2 to the power j plus 1 into t minus k.

So you can see in this figure, I am considering a signal at low resolution and the high resolution. And the resolution is controlled by the parameter, the parameters is j.

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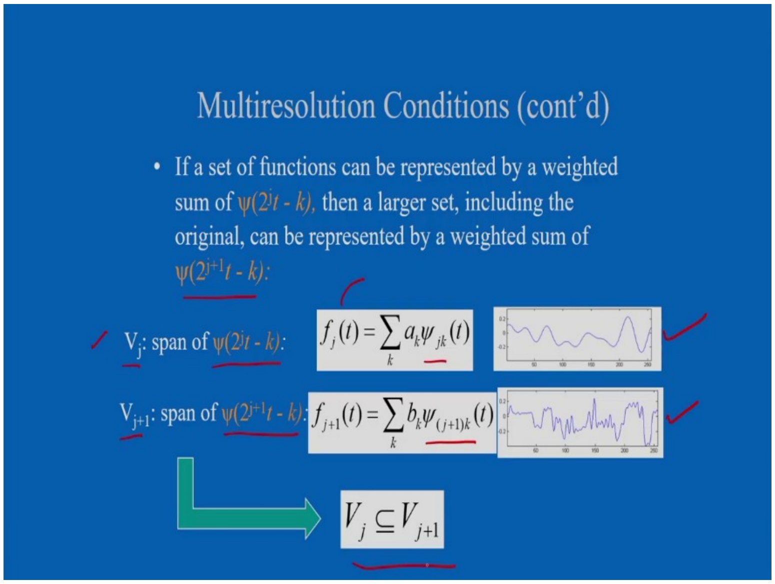

Again, I am repeating this. So if a set of functions can be represented by a weighted sum of psi 2 to the power j, t minus k, then a larger set including the original can be represented by a weighted sum of psi 2 to the power j plus 1 into t minus k.

So what is this concept? You can see, suppose if I consider one set, that is the V j, so that is the span of this function, the basis function is psi 2 to the power j into t minus k. And if I consider another larger set, the another large set is V j plus 1. So larger set I am considering, that is V j plus 1. That should be the span of this basis function, the basis function is psi 2 to the power j plus 1 into t minus k.

So first, I am considering a small set, the small set is V j, and corresponding to this I have to consider the basis function, the basis function is psi 2 to the power j into t minus k. But if I consider one larger set, that is V j plus 1, then in this case, I have to consider the basis function. The basis function is psi 2 to the power j plus 1 into t minus k. That I have to consider.

And in this case you can see that the set V j is a subset of V j plus 1, because the V j plus one is a large set as compared to V j. And you can see, if I consider a particular function, f t, suppose, that is represented by using the basis function, the basis function is psi j k. But if I consider, one larger set, that is, if I consider a function f j plus 1, previously it is f j but now I am considering a large set, that should be represented by or that can be represented by the basis function, the psi j plus 1 into k.

That means I am considering the liner combination of the basis function, that is the weighted sum of basis function, I am considering. So you can see, this function is represented by the weighted sum of psi j plus 1 k. But if I consider f j, that is the small set, I am considering, that can be represented by the weighted sum of the basis function, the basis function is psi j k.

That means, the meaning is this, that is if I consider a small set, suppose V j, that can be represented by a weighted sum of psi 2 to the power j t minus k. But if consider a large set, the large set is V j plus 1, for this I have to consider the basis function, the basis function is psi 2 to the power j plus 1 into t minus k. So that I have to consider, because V j is a subset of V j plus 1.

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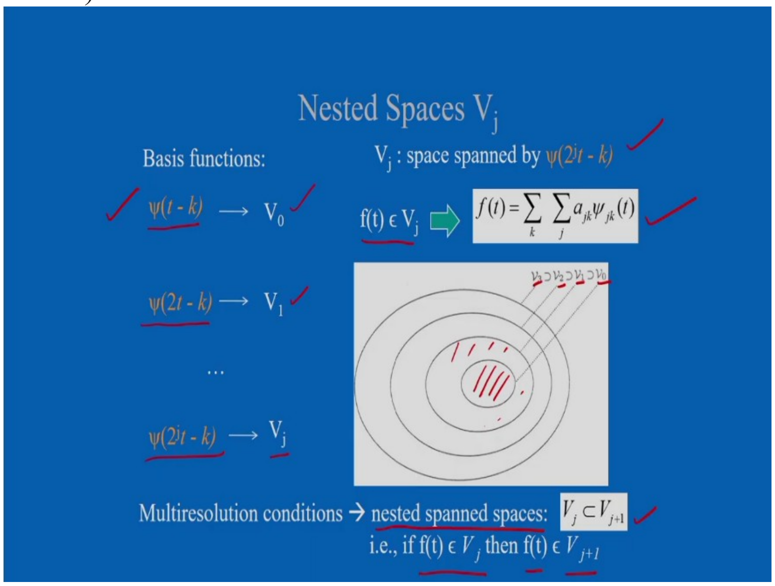

So here, you can see I am considering a set, the set is V naught, that is the small set, I am considering. So for this I am considering the basis function, the basis function is psi t minus k.

Next, I am considering one large set as compared to V naught. So V1 is larger than V naught. So for V1, I have to consider the basis function, the basis function is psi 2 t minus k. Like this, if I consider another set, that is, a large set V j, that should be represented by the basis function, the basis function is psi 2 to the power j t minus k.

So that means, if f t is an element of the set, the set is the V j, then f t can be represented by this. f t is equal to summation over k, summation over j, a j k, that is the coefficients, psi j k, t. That I am considering.

So V j means the space spanned by the basis function, the basis function is psi 2 to the power j t minus k. So that is the sub-space, the V j, that is the space spanned by the basis function psi 2 to the power j t minus k, I am considering.

And in this figure, you can see I am considering the sub-space V naught, V1, V2, V3, like this. So the first small subset is V naught, this is the V naught, and you can see the next one is, you can see the subset, that is the V1 I am considering, the set is V1. So from this, you can see that V j is a subset of V j plus 1, I am considering.

So that means, if f t is an element of V j then in this case f t will be also element of V j plus 1. V j plus 1 means I am considering one large or maybe one big set as compared to V j. So you can see the small set is V naught, the next big set is V1, and compared to V1, the next big set is V2, compared to the set V2 the next big set is V3.

So that means, you can see, I have the nested spanned spaces. So V j is a subset of V j plus 1, I am considering. This is called the nested spaces, that is spaces are V j. So I am considering V naught, V1, V2, like this I am considering. And corresponding to V naught, I need the basis function, the basis function is psi t minus k. Similarly for V1, the basis function is psi 2t minus k. So like this, I am considering.

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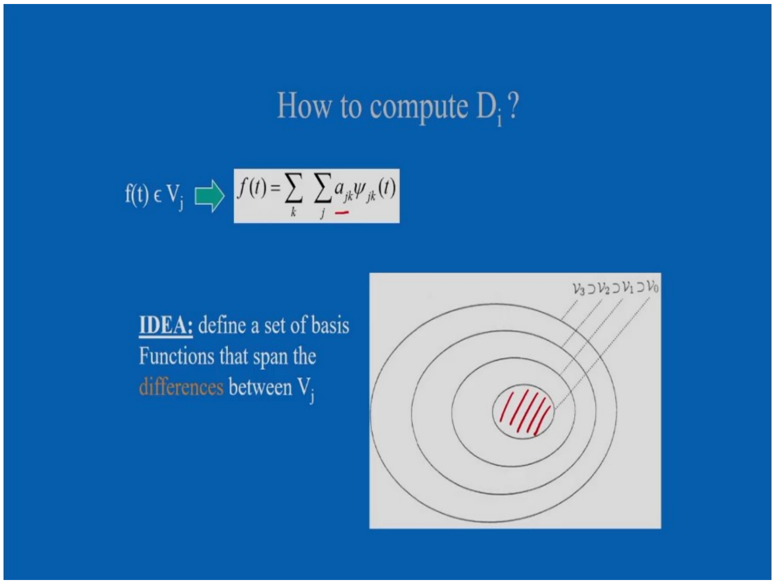

So that means the function is represented like this. f t is equal to the summation over k, summation over j and a j k is the coefficient. And I am considering the basis function, the basis function is psi j k.

Now, in this case you can see, I am considering the space, the space is V naught. Now, how to get V1? That is the big set as compared to V naught. So for this, what you can see, if I can find the difference between V1 and V naught and after this if I consider suppose V naught plus that difference, then I will be getting V1. So that means the idea is, define a set of basis functions that span the difference between V j.

So I am repeating this, suppose I have that information, that is the V naught is available, so how to get V1? So if find the difference between V1 and V naught, after this, if I consider V naught plus that difference, then in this case I will be getting V1. So with the help of this difference I can represent V1.

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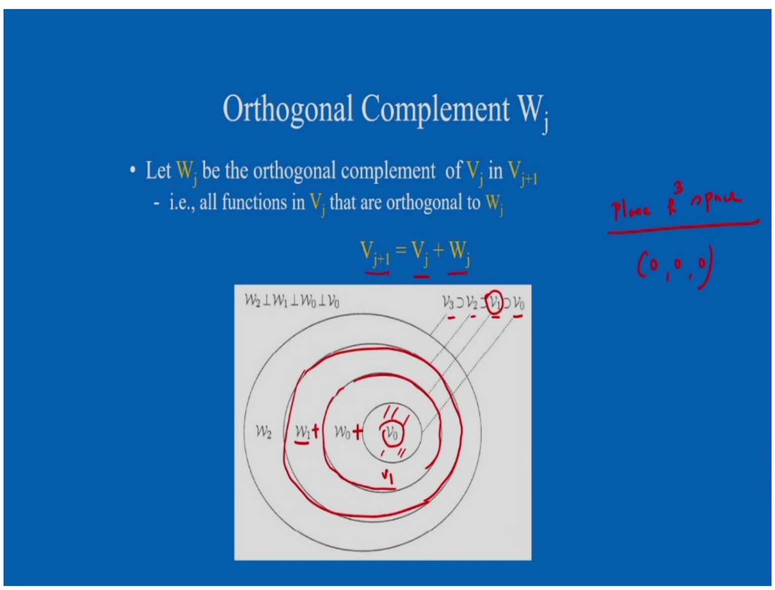

So here you can see, I am considering the set, the set is V naught, V1, V2, V3, like this. And I am considering W j be the orthogonal complement of V j in V j plus 1. So what do you mean by orthogonal complement? The orthogonal complement is a subspace of vectors where all of the vectors in it are orthogonal to all of the vectors in a particular subspace. So that is the meaning of the orthogonal complement.

So I can repeat this one. So that means the orthogonal complement is a subspace of vectors where all of the vectors in it are orthogonal to all of the vectors in a particular subspace. So that is the meaning of the orthogonal complement.

So I can give one example. Suppose for example, if I consider a plane, suppose if I consider a plane in R to the power 3 space, the 3 dimensional space, the 3D space, then the orthogonal complement of that plane is the line that is normal to the plane and passes through the point, the point is 0, 0, 0.

So that means if I consider a plane in 3 dimensional space, that is R to the power 3 space, then the orthogonal complement of that plane is the line that is normal to the plane and passes through the point, the point is 0, 0, 0. So that is the concept of the orthogonal complement.

So, in this case, how to determine V j plus 1? You can see, V j plus 1 is the bigger set as compared to V j. So if I consider V j plus W j, W j is nothing but the difference I am considering. Then in this case, if I consider adding of this, V j plus W j, then I will be getting V j plus 1. So here, in this figure you can see, so if I consider V naught, here it is, the V naught, if I consider V naught as a set. And if I consider the difference, the difference is W naught. If I add V naught plus W naught, I will be getting V1. So I will be getting V1.

Similarly, if I consider V1 plus W1, then I will be getting V2. So this is V1, so if I consider, this is V1, so this is V1. And if I consider V1 plus W1, then I will be getting V2. So this space is V2. So with the help of the difference, I can consider this one, that is, the V j plus 1 is equal to V j plus W j.

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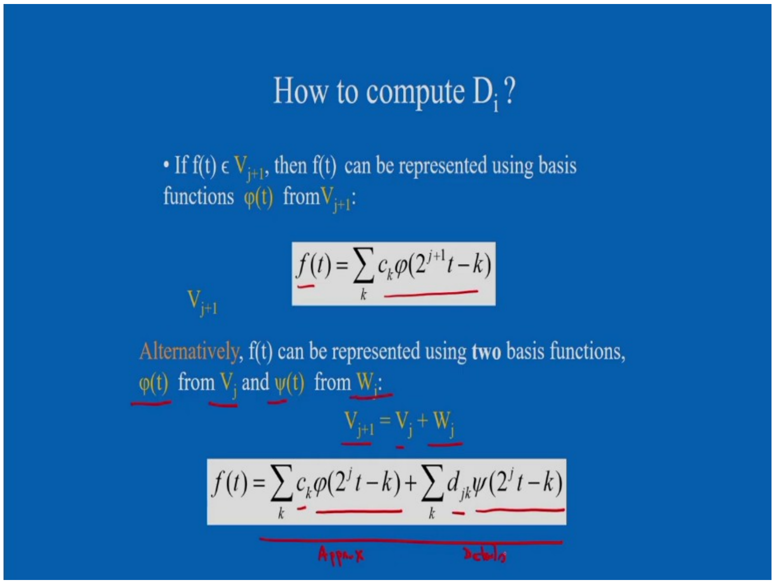

So, how to compute the difference? So, if f t is a element of V j plus 1, then f t can be represented using the basis function phi t from V j plus 1. So that concept I am showing here. That is, f t is represented like this. c k is the coefficient and phi 2 j plus 1 into t minus k, the function is represented like this, using the basis function phi t from V j plus 1.

And alternatively, f t can be represented using two basis functions. One is phi t from the space, the space is V j. And also, I am considering psi t from the space W j. What is W j? W j is the difference between V j and V j plus 1. So, here you can see, the V j plus 1 is represented by V j plus W j.

V j plus 1 is the larger set as compared to the set V j. So that means, the function f t can be represented like this. I am considering two basis functions. One is phi t, I am considering, this basis function, phi 2 to the power j into t minus k. And another one is psi, 2 to the power j t minus k I am considering. And you can see the coefficients, the c k and the d j k are the coefficients I am considering. Because, I am considering the linear combinations.

So f t, can be represented by using these two basis functions. So, for this what I am considering? This component I can consider as approximate component and this difference I can consider as the detail components, the details I can consider.

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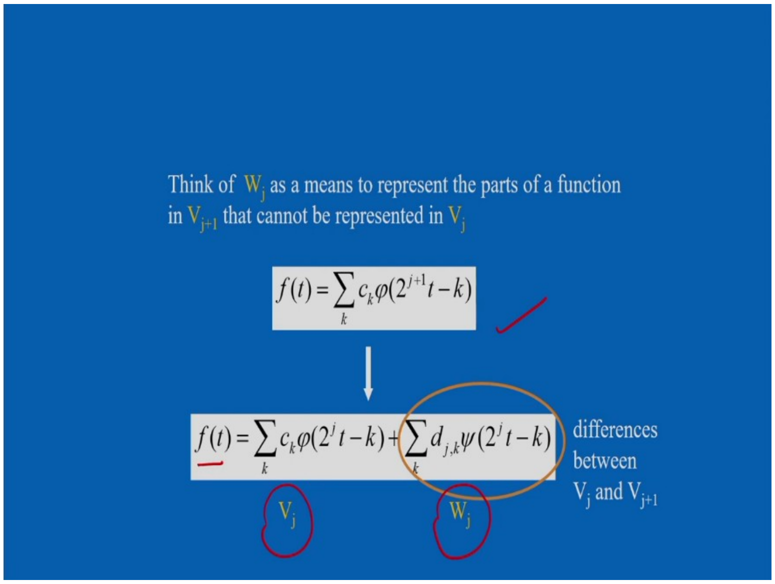

So, that means, you can see these functions, this f t, instead of considering only one basis function, I am considering two basis function, one is phi another one is psi. So, I am getting V j and another one is W j. W j, the difference between V j plus 1 and V j. That difference I am calculating.

So, the signal, the signal or the f t, the function is represented by using these two basis functions. So, what is this component, the second component? The difference between V j and V j plus 1. That is the difference.

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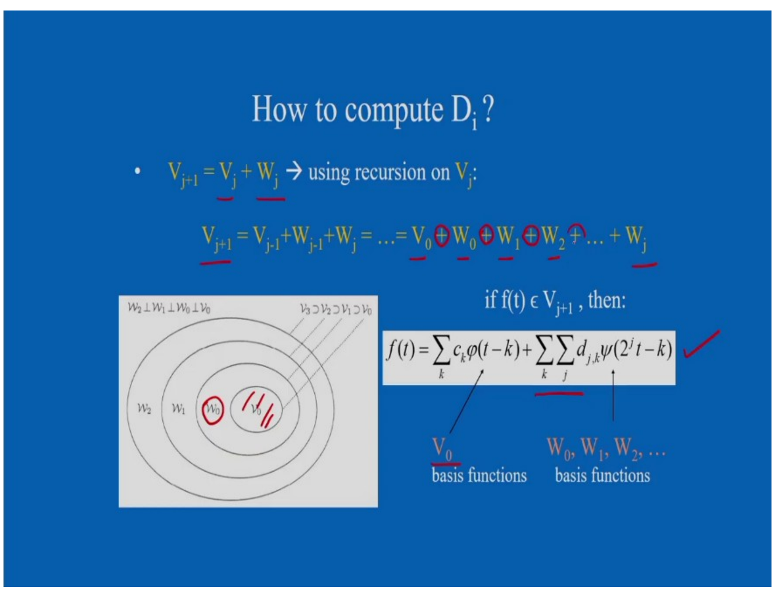

So, here you can see, how to represent V j plus 1. V j plus 1 is nothing but V j plus W j. And recursively I can determine V j plus 1. So V j plus 1 is nothing but V naught plus W naught plus W1 plus W2 plus W3, up to W j I am considering. That means, V naught corresponds to the approximate information and W naught W1, W2, all these are basically the difference, that is nothing but the detail information.

So, here you can see, I am considering the V naught and W naught is what? It is the difference between V1 and V naught. W1 is the difference between V naught and V1. So, by considering these differences, I can represent a particular function.

So, you can see, the function is represented by this, f t is equal to summation over k, c k, phi t minus k, that is for V naught. So this basis function I am considering for V naught. And another one is summation over k, summation over j, that is the second component I am considering.

d j k, that is the coefficients, and phi 2 to the power j, t minus k. Then I am considering the detail component, and this is for W naught, W1, W2, and I am considering the basis function psi. So, that means I am considering two basis functions, one is phi, another one is psi, to represent the function f t.

Since I am considering the vector space, I am considering this summation as direct sum. So, this should be direct sum. Instead of the simple summation, this should be the direct sum because I am considering the vector space.

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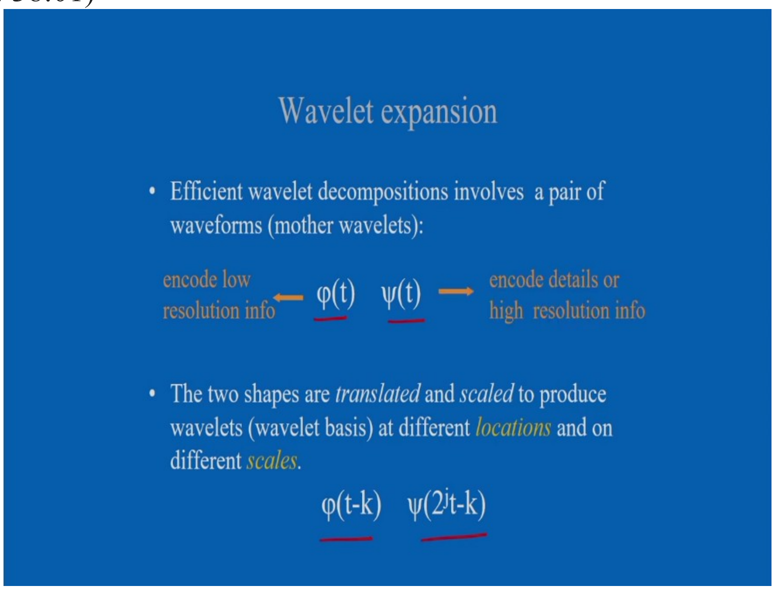

So, here you can see, for wavelet decompositions we consider two basis functions, one is phi t, another one is psi t. And you can see the two shapes are translated and scaled to produce number of wavelets at different locations and on different scales. I can do the translation; I can do the scaling so that I will be getting a number of wavelets.

So, you can see, I am doing the translation, also I am doing the scaling. So, here you can see, this phi t is used to encode low resolution information and this psi t, it is used to encode detailed or high-resolution information.

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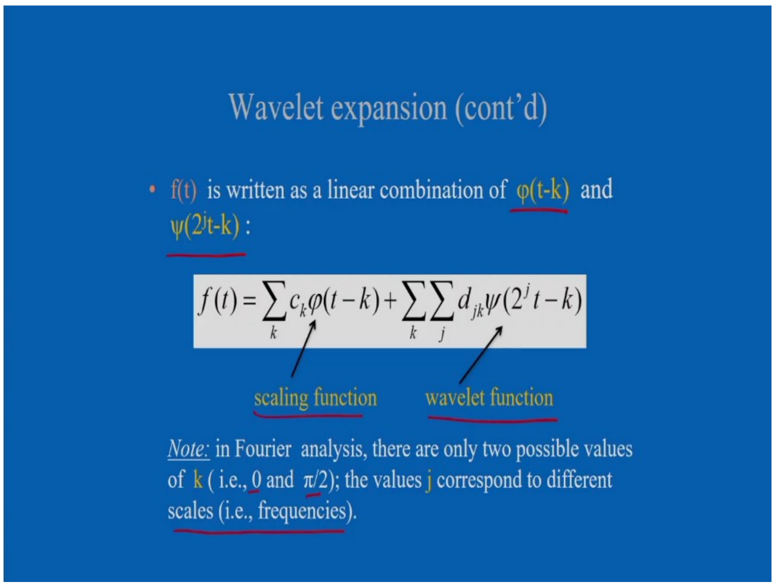

So, I am writing it again, the function f t is represented by these two functions, one is the scaling function I am considering, and another one is the wavelet function. So, I have two basis functions, one is phi t minus k and another one is psi 2 to the power j, t minus k. So, a function is represented as a linear combination of these two basis functions.

In Fourier analysis, there are only two possible values of k. Either it may be 0 or pi by 2. The values of j correspond to different scales. The scale means frequencies. So, you can see the difference between the wavelet expansion and the Fourier analysis.

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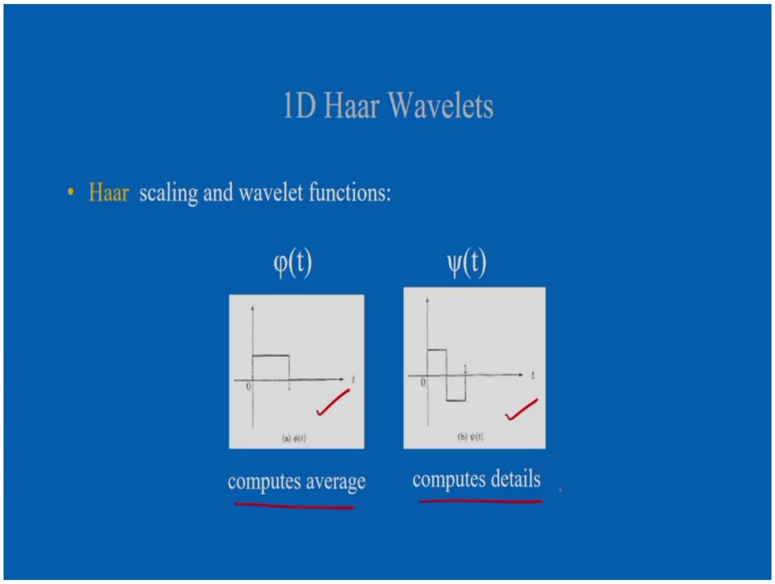

And this is the Haar wavelet I am showing. So, by considering the Haar wavelet, I can determine the approximate component and the detail component. So, by using this wavelet, the Haar wavelet, I can determine the approximate value, approximate means the average value I can determine. And by using these wavelet function, I can determine the detail information. That is, the difference I can determine.

So, the first one is for computing the average, the second one is for computing the details.

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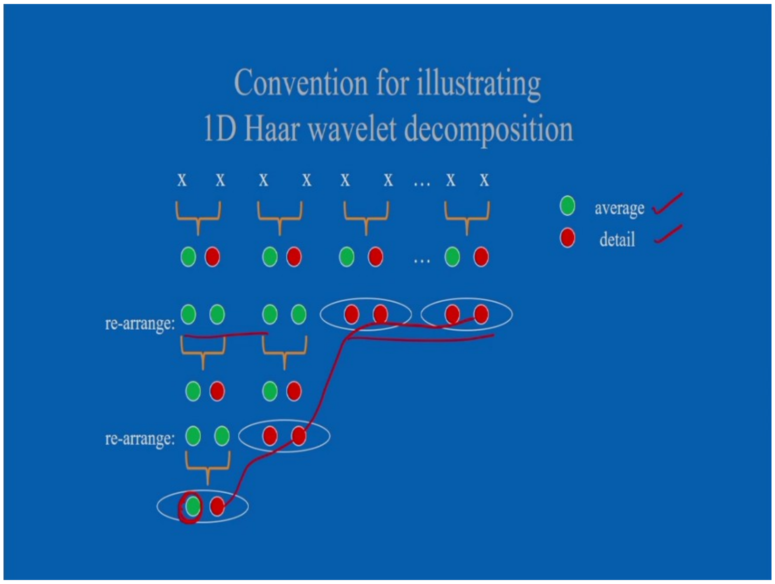

And in this case I am showing one example. I am considering 1D Haar wavelet decomposition, and I want to determine the average and the detail information, you can see. So first, I am calculating the approximate value and the average value. The green one is the average value and the red one is the detail value. So, in my numerical example I have shown this example.

After this what you can do, you can rearrange these values like this. So, the greens you can rearrange and the red you can rearrange like this. After this, again, I can determine the approximate value and the detail value, the average value and the detail value. After this, again, I can rearrange. After this, again, I can determine the approximate value and the detail value.

And finally, I will be getting the signal after decomposition. So, that means, I have the one approximate component, you can see this is the approximate component. And if you see this red one, all these red ones, that is nothing but the detail information.

So, like this I can do the decomposition by using the Haar wavelet. So, Haar wavelet can be used for determining the approximate value and the detail coefficients.

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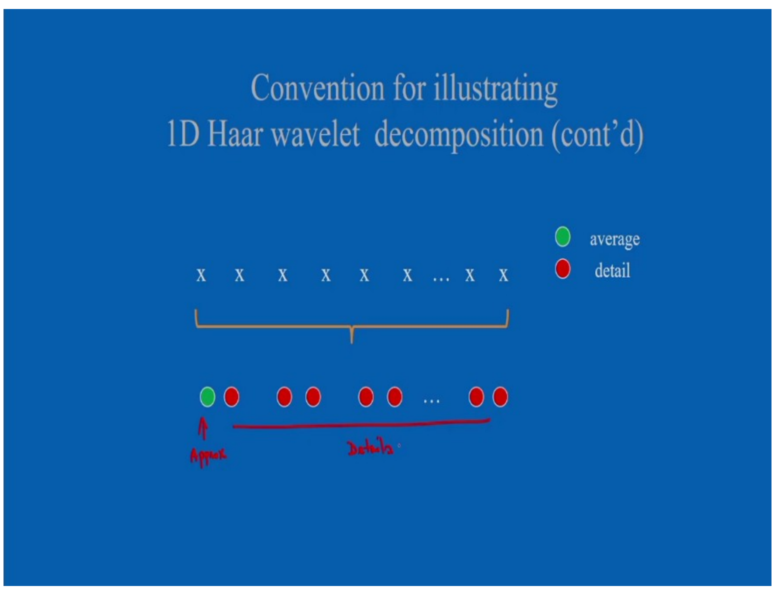

So, here you can see, I am considering 1D Haar wavelet decomposition. So you can see, I will be getting the final array like this. So, this is after decomposition I am getting this one. So this is approximate values, and the remaining red one is the detail, details I am getting.

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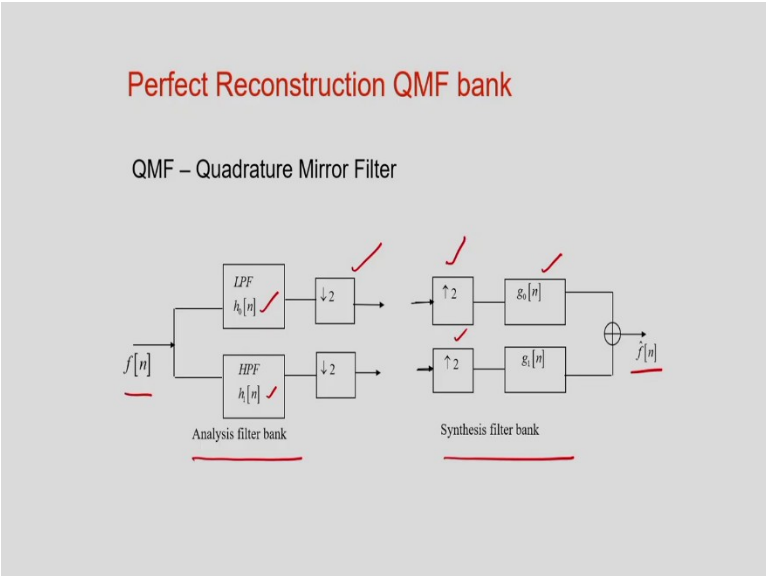

So, this implementation can be done by using two filters, one is the low pass filter, another one is the high pass filter. That means, the decomposition can be done by considering two filters, one is the low pass filter. The low pass filter is h naught n, and high pass filter is h1 n, I am considering.

So, for analysis I am considering this one. So, signal is f n. So, I am considering the low pass filter to get the approximate component and high pass filter to get the detail information. And after this, I can do the down sampling. The down sampling by 2. So, this is for the analysis of the signal. So this is called Analysis Filter Bank. And for the reconstruction, for the synthesis of the signal, what you can do, just opposite.

We have to do the up sampling of the signal, we are doing the up sampling. And after this, again I am considering the synthesis filter bank, that is nothing but, again, the low pass filter and the high pass filter. So g naught n is the low pass filter and g1 n is the high pass filter. So by considering this, I can reconstruct the original signal.

So you can see, this is the approximate reconstructed signal, by considering low pass filter and the high pass filter. And this filter is called the Quadrature Filter, the Quadrature Mirror Filter. The concept of the Quadrature Mirror Filter, I can explain in my next slide.

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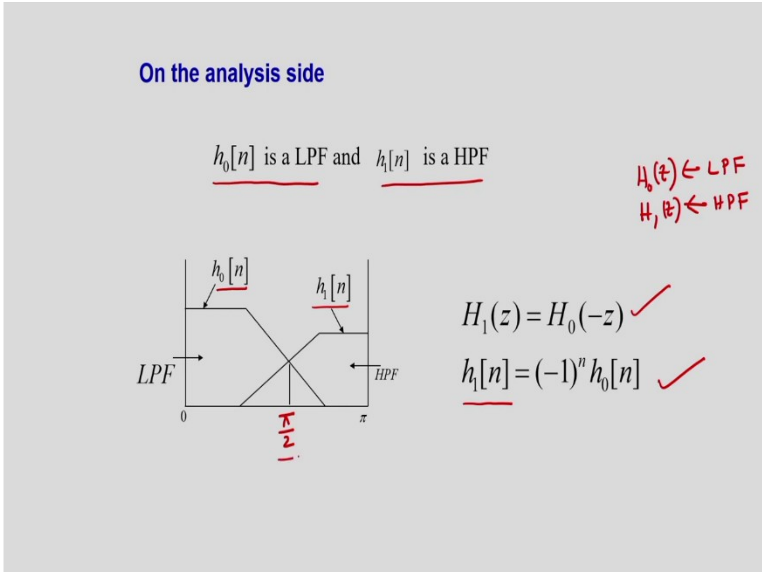

What is a Quadrature Mirror Filter? Here, you can see I am showing the frequency response of the low pass filter and the high pass filter. So, h naught n is the low pass filter and h1 n is the high pass filter. So, how to get the high pass filter? That is nothing but h1 n is the mirror of h naught n. That is, the high pass filter is the mirror of the low pass filter.

So that is why, if I consider the symmetry point, the symmetry point is pi by 2. So this is pi by 2. So suppose if I have only the low pass filter, so high pass filter easily we can obtain by this. So h naught n is available suppose, the low pass filter is available. Already we have designed the low pass filter. So, how to get the high pass filter?

The high pass filter h1 n in the time domain is nothing but it is minus 1 to the power n into h naught n. And corresponding to this, you can see the transfer function. So h naught minus z, I am considering. So, what is h naught z? That is the transfer function of the low pass filter. So, it is the transfer function of the low pass filter, that is in the z domain. z transform I am considering. And h1 z, that is the transfer function for the high pass filter.

So, this is the concept of the Quadrature Mirror Filter. If I know, the low pass filter, from the low pass filter I can determine the high pass filter, because the center of symmetry is pi by 2.

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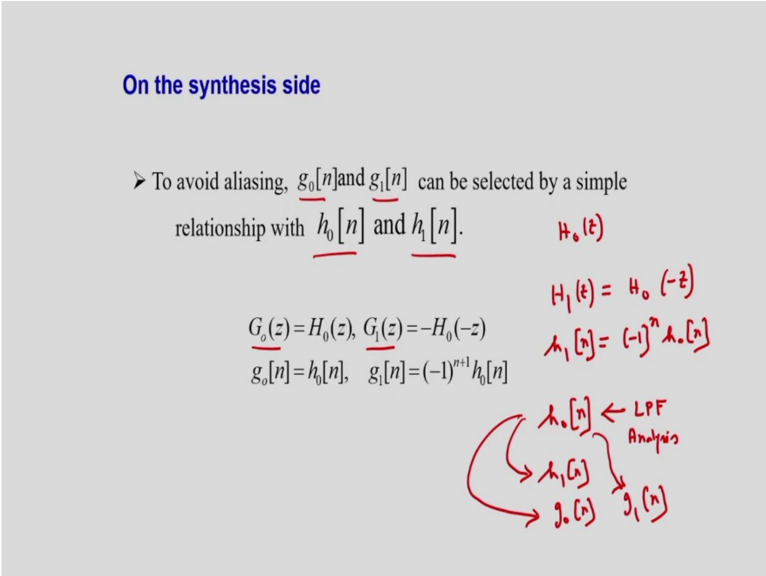

And similarly, for the synthesis side also we need the low pass filter and the high pass filter. So here you can see g naught n, g1 n. g naught n is the low pass filter, that is the synthesis filter. And g1 n is the high pass filter. So from this, you can see, from h naught n, that is, in the time domain, that is, the low pass filter, and h1 n is the high pass filter in the time domain.

You can see. Because, you can determine G not z, that is the synthesis filter, that is the low pass filter, in the synthesis side is nothing but H naught z, because H naught z is the low pass filter in the analysis side, that is the analysis, for the analysis of the signal I am considering the low pass filter. That filter is H naught z.

So, from H naught z you can determine G not z. G not z is nothing but the low pass filter for synthesis. And also, you can determine G1 z. G1 z is nothing but the high pass filter for the synthesis of the signal. So from h naught, you can determine G1 z. And already you know what is H1 z. H1 z is nothing but H naught minus z. You know this. And also you know that in time domain h1 n is equal to minus 1 to the power n, h naught n. You know this.

So that means the concept is, if you only know this one, h naught n, that is the low pass filter of the analysis, the low pass filter for analysis then all other filters, all other filters means h1 n, g naught n, g1 n, you can determine from h naught n. So you can determine h1 n from h naught. You can determine g naught n from h naught, you can determine g1 from h naught. So all the filters you can determine from h naught. So that is the concept.

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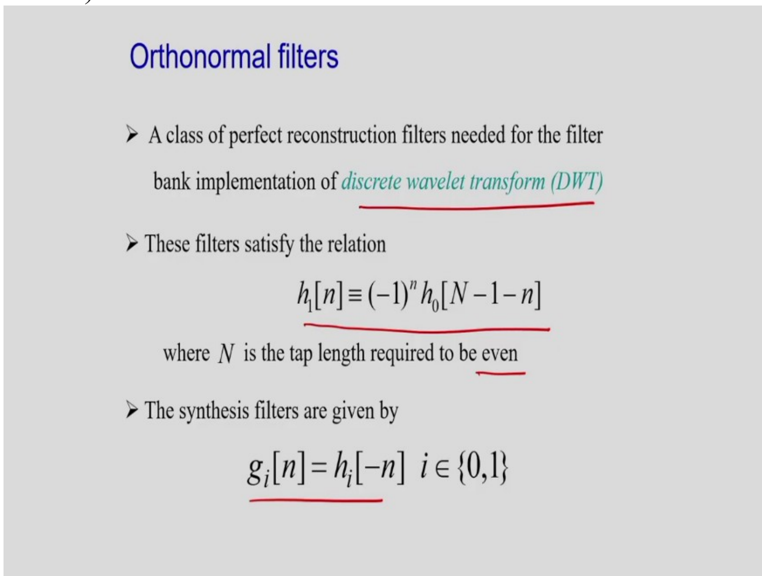

And we are considering a class of perfect reconstruction filters needed for the filter bank implementation. So I have to consider filter banks because I have to decompose the signal for discrete wavelet transformation. For discrete wavelet transformation, we need the filter banks, the low pass filter and the high pass filters.

So in this case, these filters satisfy this condition, h1 n is equal to minus 1 to the power n and, h naught N minus 1 minus n, where N is the tap length required to be even. So it should be even.

And corresponding to this, we can determine the synthesis filters. So that means, for discrete wavelet transformation I have to determine the approximate components, and also the detail components. So for this, I need low pass filters and the high pass filters.

And for analysis we need the low pass filter banks and also the high pass filter banks. And similarly, for the synthesis of the signal, we need filter banks. That is nothing but low pass filters and the high pass filters.

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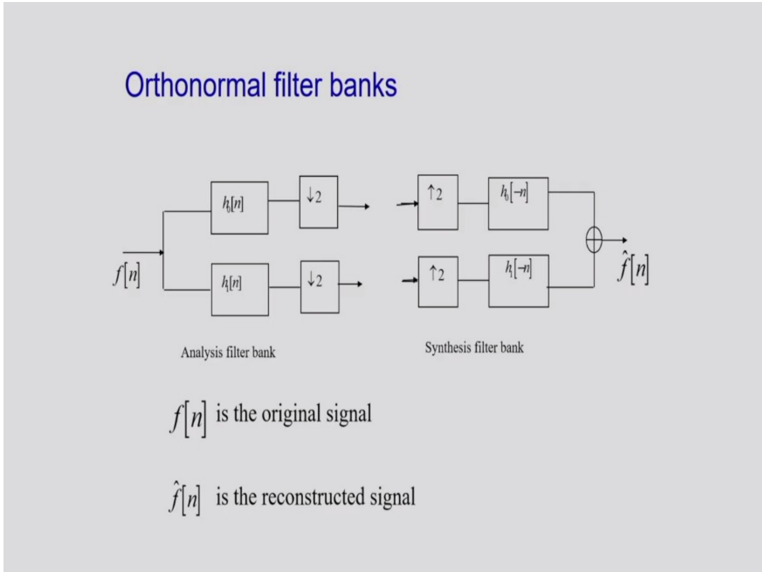

So here, you can see, I am showing the analysis of the signal and also the reconstruction of the signal. So here you can see I am considering the low pass filter h naught n, and high pass filter is h1 n. And I am considering the Quadrature Mirror Filter. So from h naught n you can determine h1 n. So the signal is decomposed by considering this low pass filter and the high pass filter.

Similarly, for the synthesis I am considering h naught minus n, that is the synthesis filter, that is the low pass filter. And h1 minus n, that is the high pass filter for the synthesis, I am considering, which can be obtained from h naught n. So the main filter is h naught n, from h naught n, you can determine h1 n, you can determine h naught minus n and also you can determine h1 minus n, you can determine.

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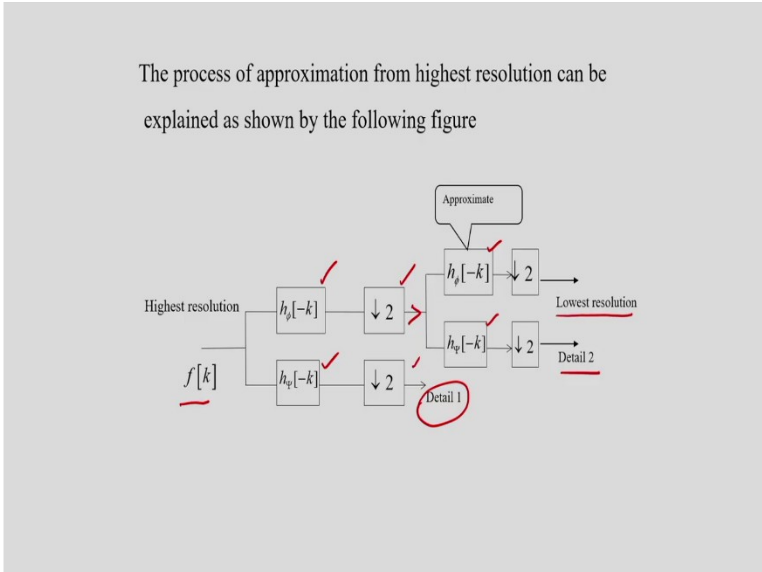

So this multiple level of decomposition we can do by using these filter banks. In this figure, you can see the multiple levels of decompositions I can do. So first, I am considering the signal, the signal is f k, that is of highest resolution. After this, I am considering the low pass filter and the high pass filter. I am determining the approximate value and the detail vale.

After this, I am down sampling. So I am having the detail information, detail here is 1. And from the approximate information, you can see we have the approximate information, from this approximate information I can again do the decomposition. So for this, I am again considering the low pass filter and the high pass filter.

So I will be getting the approximate component and the detail component. So detail information I am getting, the Detail 2. And after this, I can do the down sampling to get the lowest resolution. So like this, I can implement this decomposition. And this is nothing but the multilevel decomposition.

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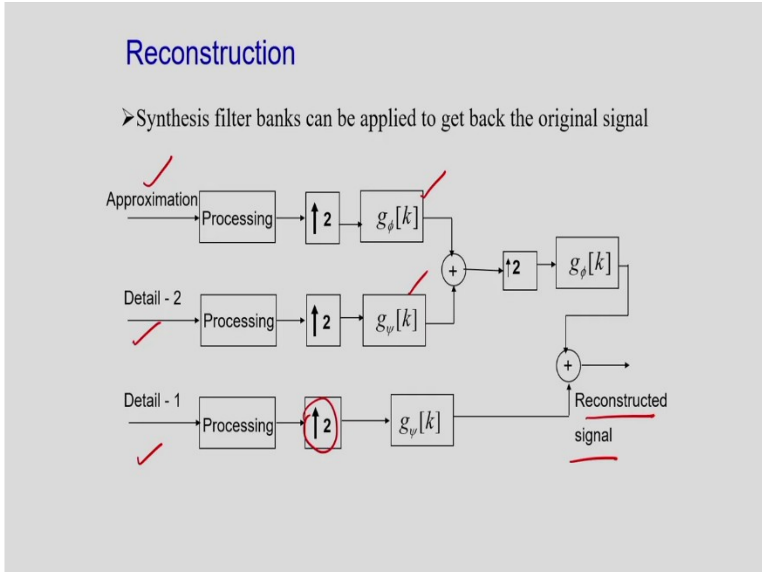

And for the synthesis, for the reconstruction, the opposite is this. Because I have the approximate information, I have the detail information, detail information 1, detail information 2. And you can see, by considering again the filter banks, the low pass filter and the high pass filer, you can reconstruct the signal.

And one thing is important, here I am doing the up sampling. In case of the analysis, we did the down sampling but in this case I am considering the up sampling of the signal. And you can see, I can reconstruct the signal by considering approximate information and the detail information. This is about the reconstruction.

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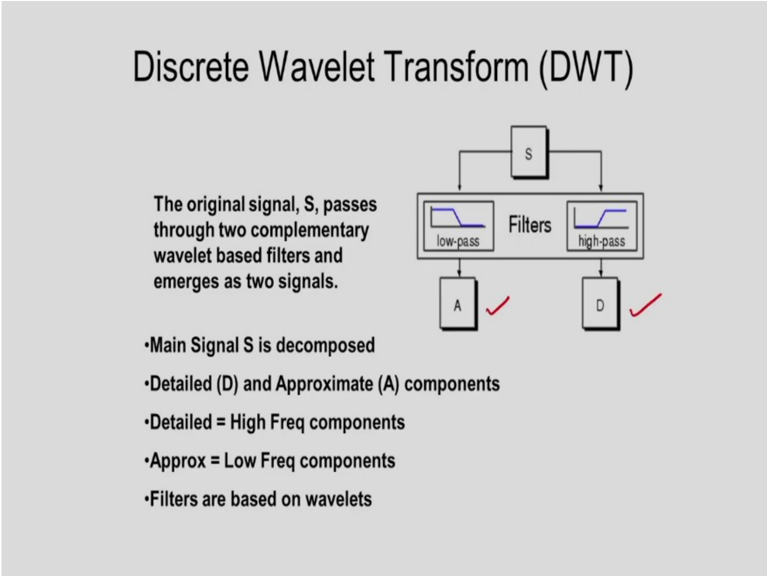

In this figure also I am considering the decomposition of a particular signal, that is the discrete wavelet transformation I am considering. For decomposition of the signal, I am considering the low pass filter and the high pass filter. So I will be getting the approximate component and the detail component. So this is the decomposition of a particular signal.

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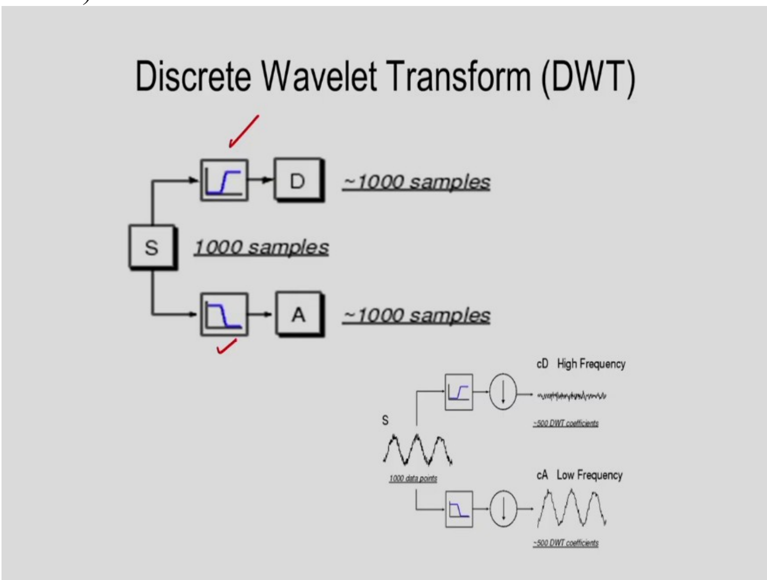

In this case also I am showing one signal, the 1D signal. And I am considering one low pass filter and one high pass filter to get the approximate and the detail components. So, any 1D signal can be decomposed like this. Now, I am discussing the 1D signal that can be extended for 2D signals. Like, in the image we can do the decomposition.

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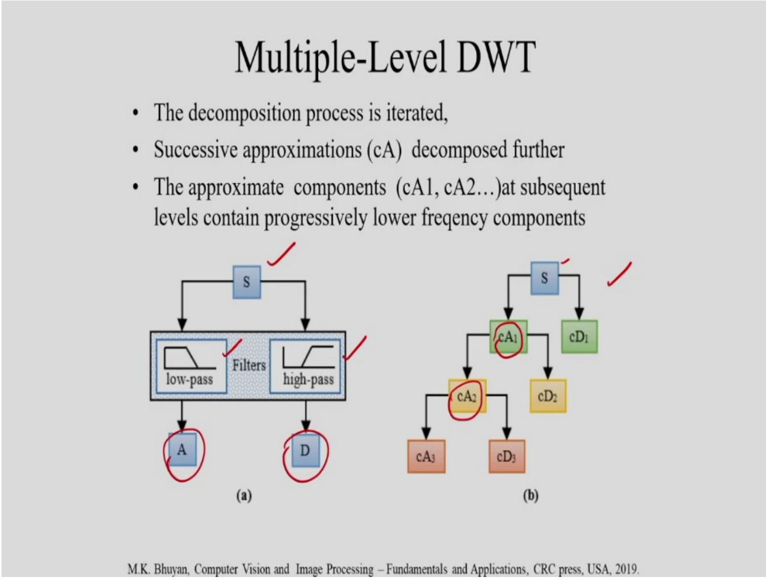

And here you can see the Multiple-Level DWT, the multiple level discrete wavelet transformation. How to implement this one? So the signal is this, I am considering the low pass filter and the high pass filter. So I will be getting the approximate component and the detail component I will be getting.

And you can see in the second figure, what I am doing? I am doing the multiple level decompositions. So signal is S. I will be getting the approximate component c A1, detail component c D1. From this approximate component, again I am getting the approximate component c A2 and the detail component c D2.

And from the approximate component c A2, I am getting the approximate component c A3 and the detail component, c D3. So like this, we can do multiple-level DWT, the discrete wavelet transformation.

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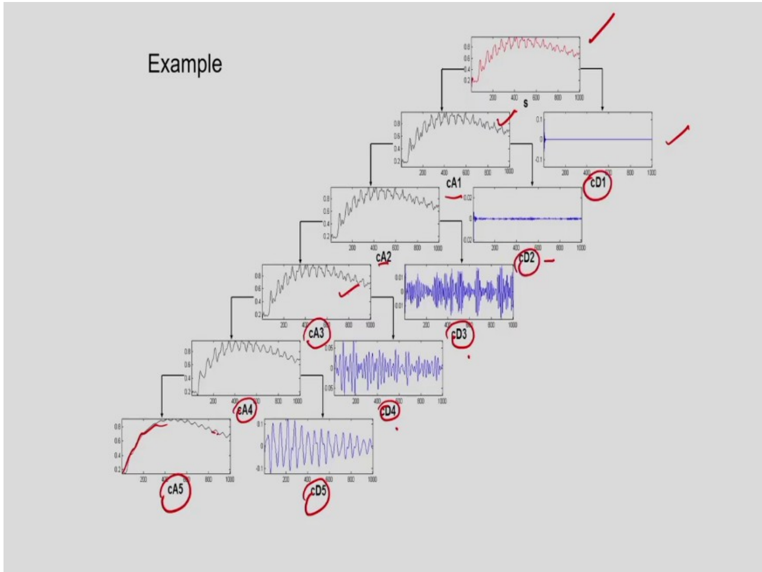

So, in this case I am considering one 1D signal, and you can see how to decompose this particular signal. So corresponding to this signal I am getting the approximate component. The approximate component is c A1 and detail component is c D1. And from the approximate component c A1, I am doing the decomposition again, so I will be getting the approximate component c A2 and detail component I will be getting c D2.

Again, from c A2, again I can do the decomposition. I can get the approximate component, the approximate component is c A3 and the detail component is c D3. And from c A3, that is the approximate component, that is this approximate component, I can again do the decomposition, so I will be getting the approximate component and the detail component.

So like this I can do the multiple level decompositions. So finally, after decomposition what I will be getting? I will be getting one approximate component, that means one approximate signal I will be getting. And if I consider c D5, c D4, c D3, c D2 and c D1, all these are detail information of the signal.

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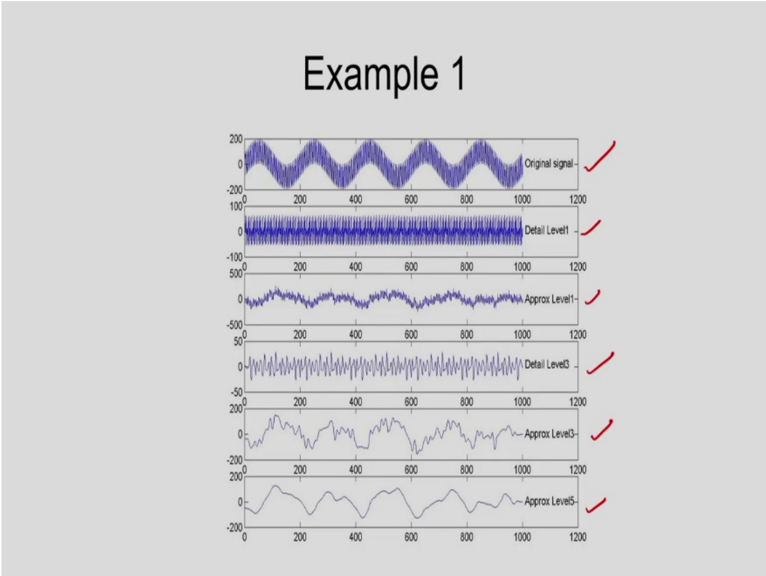

And this is another example you can see. You can see the original signal. And you can see I am getting the approximate signal and approximate level 1 signal, also the detail level 1 signal you can get. This is detail level 1 signal, approximate level 1 signal, detail level 3, approximate level 3, approximate level 5, like this we can do multiple decompositions.

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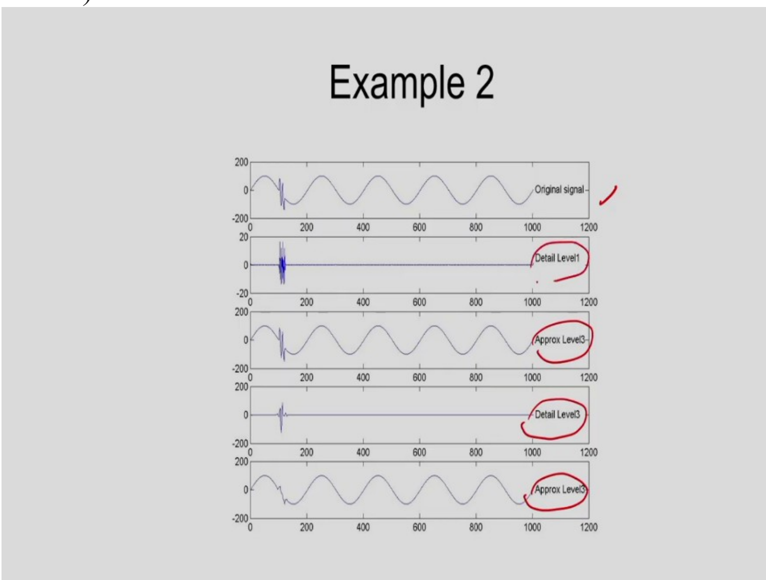

Similarly, again, I am considering another signal here. And corresponding to this, you can get the approximate in level 3 decomposition, detail in level 3 decomposition, approximate level 3 decomposition and detail level 1 decomposition. So like this you can do the decomposition of a particular signal.

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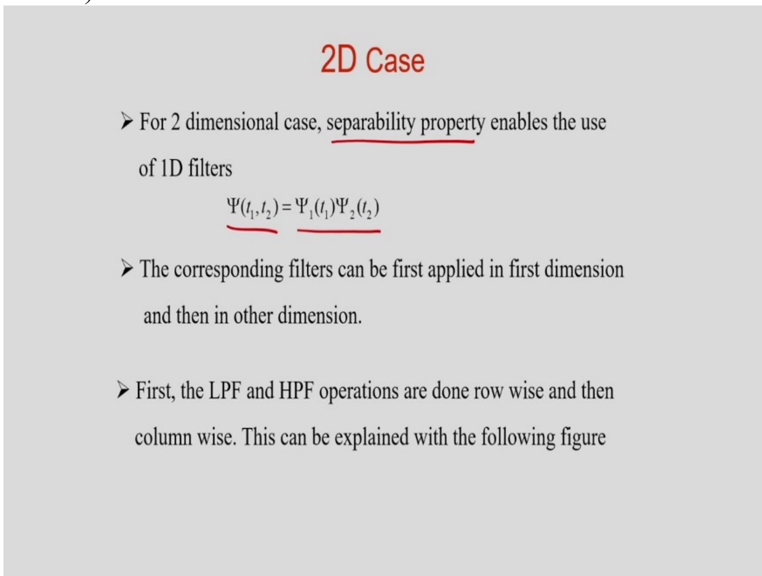

So this can be implemented for 2D signals. The two dimensional signal like image. Because the main property is the separable property of the kernel. So if I consider this kernel which is separable, so based in this property I can implement it for the 2D signal, that is for the image.

For the 2D signal, for the 2D image, what I have to consider. First, I have to apply the transformation along the rows and after this, I have to apply the transformation along the columns. So like this, I have to do.

In case of the DCT, the discrete cosine transformation, I did like this. So first, I applied the transformation along the rows and after this I applied the transformation along the columns. So similarly, I am apply Haar transformation along the rows and after this, I can apply the Haar transformation along the columns.

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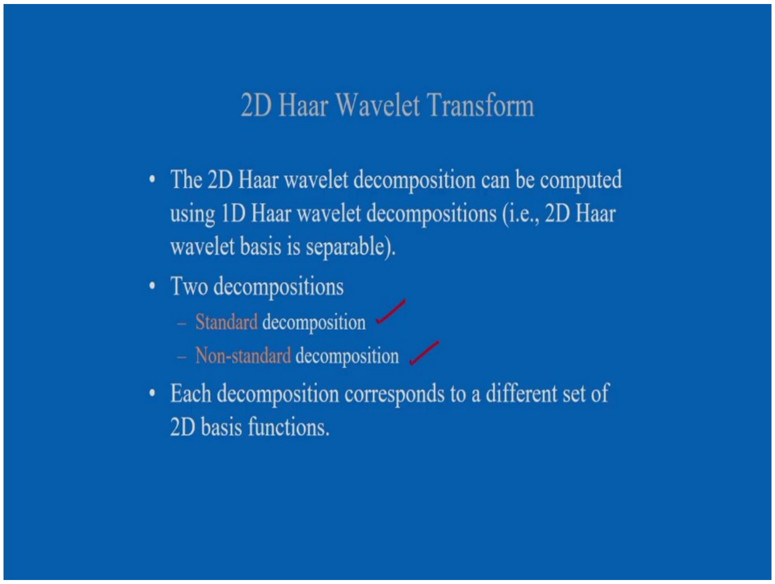

So, there are two techniques for decomposition of a particular signal, that is the image. If I consider the 2D signal, one is the Standard Decomposition, another one is the Non Standard decomposition. That is for 2D Haar wavelet transformation. So I will be explaining these two techniques, one is the Standard Decomposition technique, another one is the Non Standard Decomposition technique.

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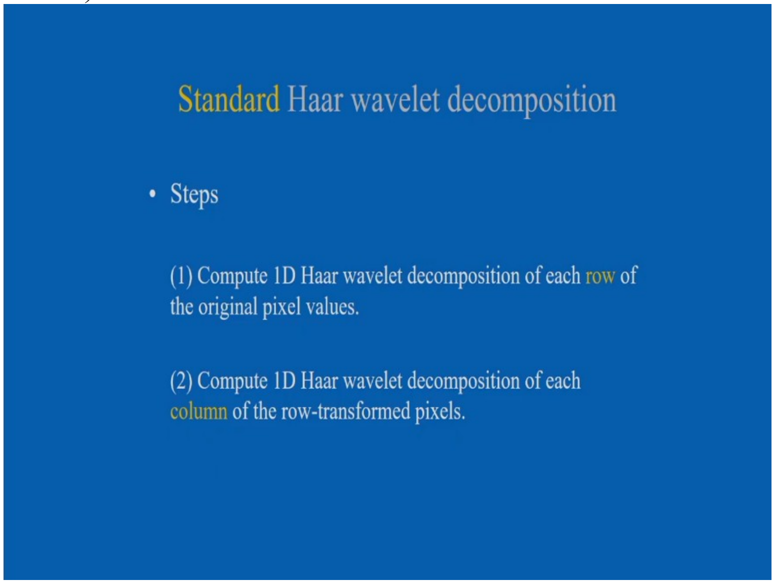

So in case the Standard Decomposition Technique, compute 1D Haar wavelet decomposition of each row of the original pixel values. After this, compute 1D Haar wavelet decomposition of each column of the row-transformed pixels.

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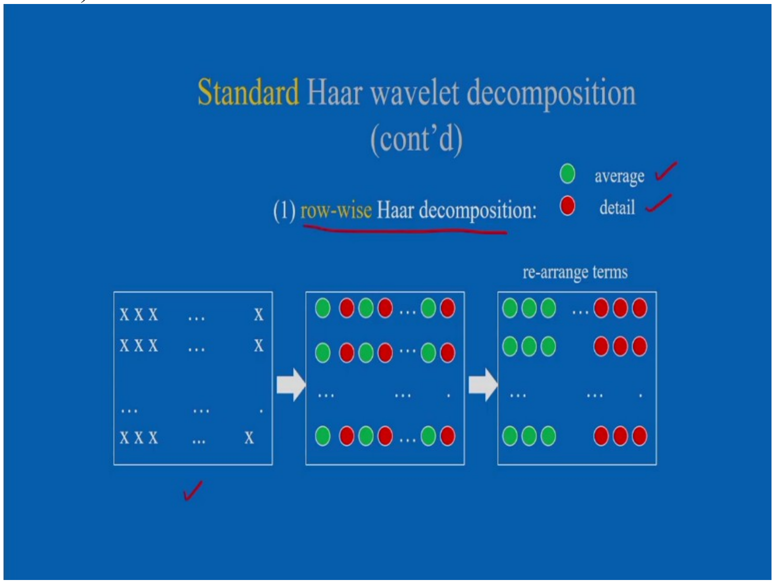

So you can see, I am showing here. So suppose this is the image I am considering. First I have to apply the Haar decomposition row wise. Row wise Haar decomposition. So I will be getting the approximate component, that is the green and detail means the red. So you can see, along the rows I am getting the approximate value and the detail value. And after this, I am rearranging terms.

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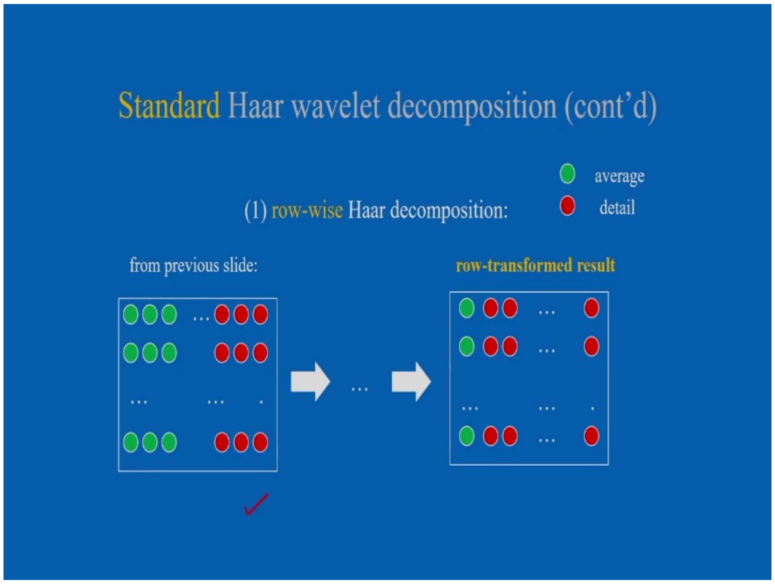

And from the previous slide I am getting this one. So like this we have to do the decompositions. And finally after rearranging I will be getting this, that is the row-transformed result I will be getting. That is, row wise Haar decomposition I am considering.

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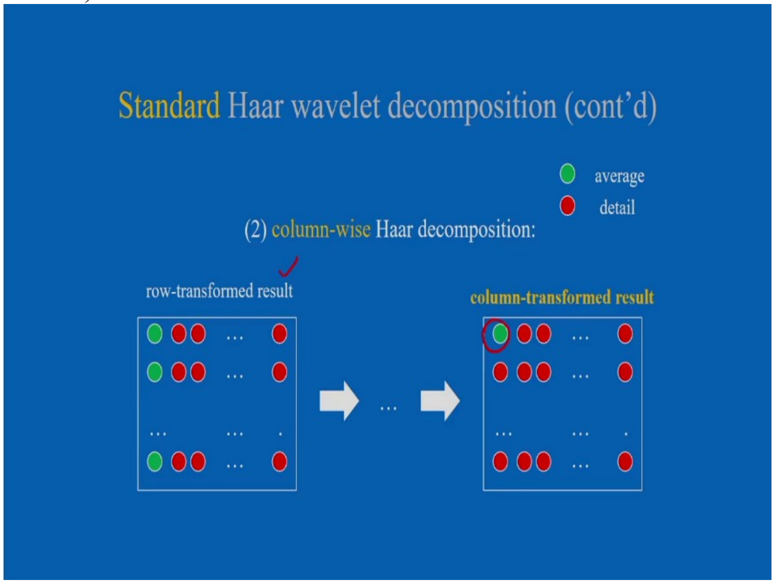

After this, because we have the row-transformed result, this result is available. After this I have to apply the decomposition along the columns. So if I apply the decomposition along the columns, then in this case you will be getting only this as the low frequency component and if I consider others, the red, these are the detail coefficients.

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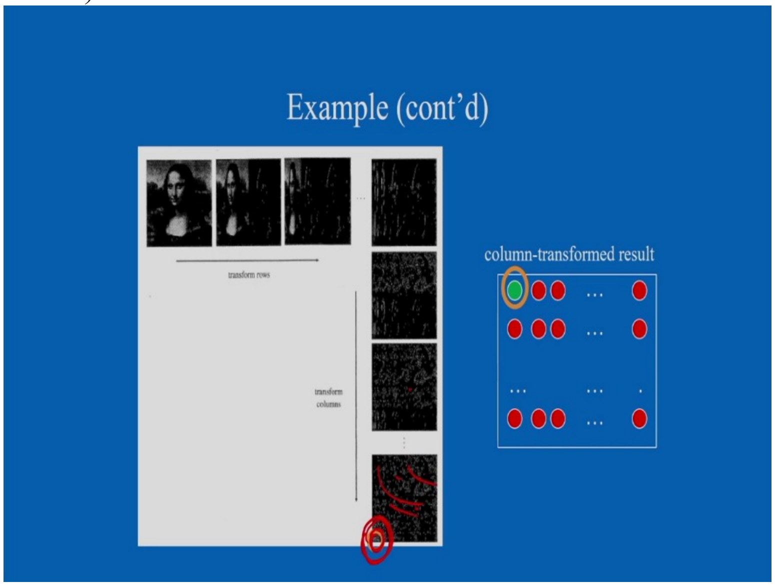

So in this example you can see here. So first I am applying the row wise decomposition. So after row wise decomposition, you will be getting this one. After row wise decomposition, what you can get? Row transform result I will be getting. So after this, I am arranging this one. After arranging this one, so I will be getting this one. That means, this is the output corresponding to row-transform decomposition.

After this what I am getting? I am applying the transformation, that means the decomposition along the columns, and you can see only I have one low frequency information. So that is available here, this is the low frequency information. The rest is the high frequency information. High frequency information means the edges and the boundaries. These are the high frequency information. But if I consider the constant intensity portion, or also the homogenous portion of the image, that corresponds to the low frequency information.

The edges and the boundaries are the high frequency information, because in case of the edges and the boundaries there is an abrupt sense of the grey scale intensity value. So that is why the edges and the boundaries are the high frequency information, the high frequency pixels. And if I consider the homogenous portion of the image, or maybe the constant intensity portion, that corresponds to low frequency information.

So here, you can see after this decomposition, I am getting this one. So I am getting the one low frequency information. If I consider outer side, this one, this is nothing but the high frequency information, I will be getting.

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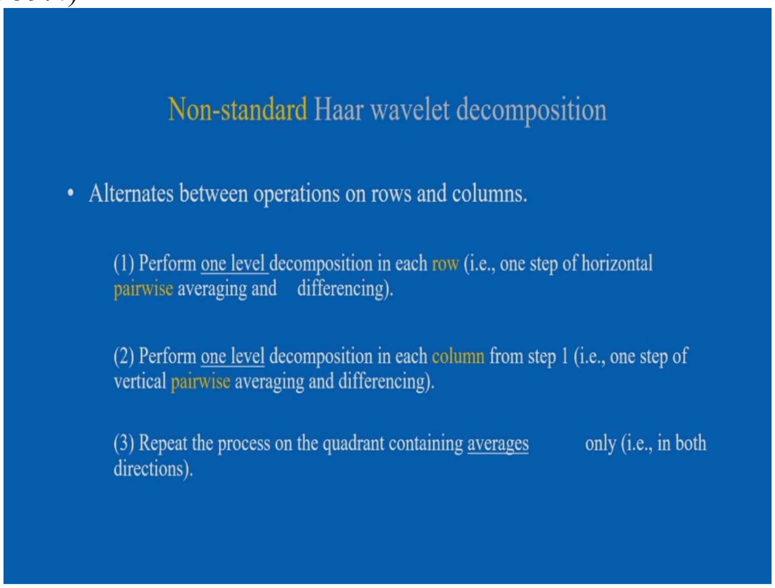

The next one is the Non-standard Haar wavelet decomposition. So for this, perform one level decomposition in each row, that is, one step horizontal pairwise averaging and differencing.

Number 2, perform one level decomposition in each column from Step number 1. After this, repeat the process on the quadrant containing averages only in both the directions.

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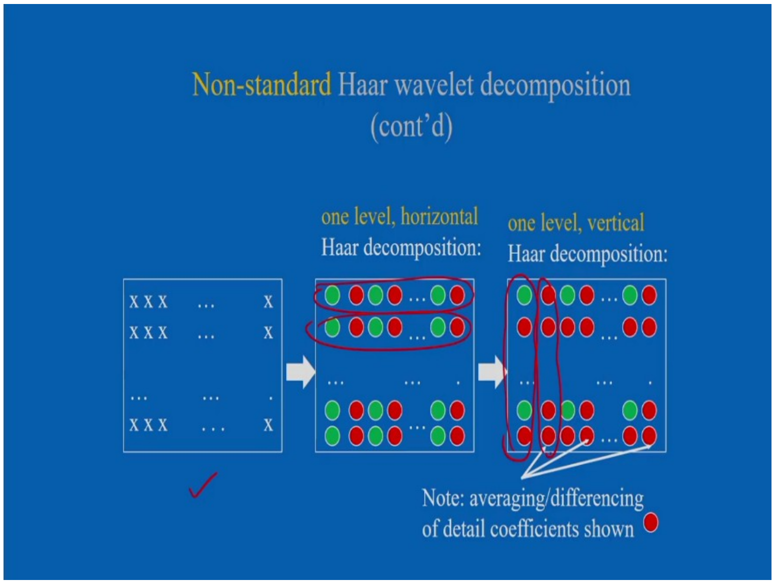

So that concept, I can show in the figure here. So, I am considering the input image. So first I have to do one level horizontal decomposition. So in the horizontal direction, that is along the rows, I am doing the decomposition, I am getting the average and the detail information.

After this, one level vertical Haar decomposition I am doing. You can see, vertically, I am doing. In this case, I am doing the horizontally, the decomposition. In the second case, I am doing vertically the decompositions, I am doing. Like this. So, you can see the averaging and differencing of the detail coefficients, I am doing.

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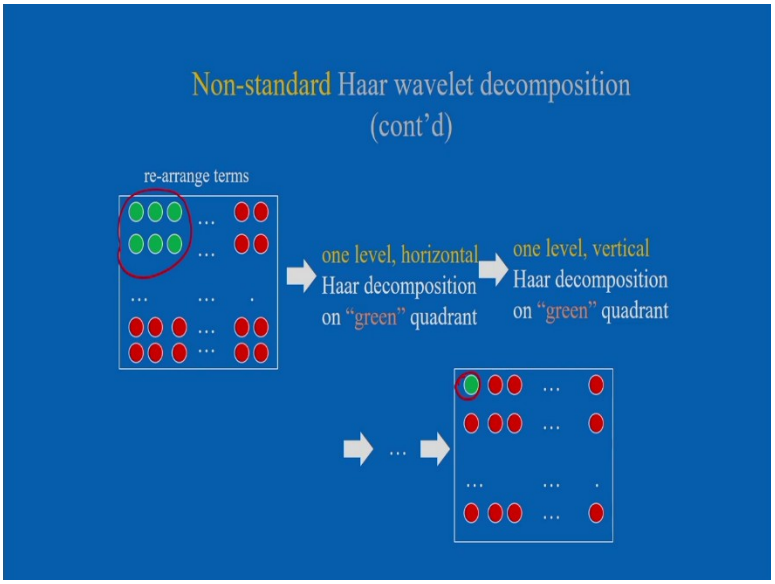

After this, we have to rearrange the terms. After this, I have to consider only the low frequency information, that is the quadrant, and this quadrant I have to consider. That is, I have to consider the green quadrant because it has the low frequency information. After this, again, I have to do the decomposition of this one. The green quadrant, I have to do the decomposition. That is one level horizontal and one level vertical, I have to do.

And finally, I will be getting this one. So here you can see, I have this low frequency information and red means the detail information, the detail coefficients.

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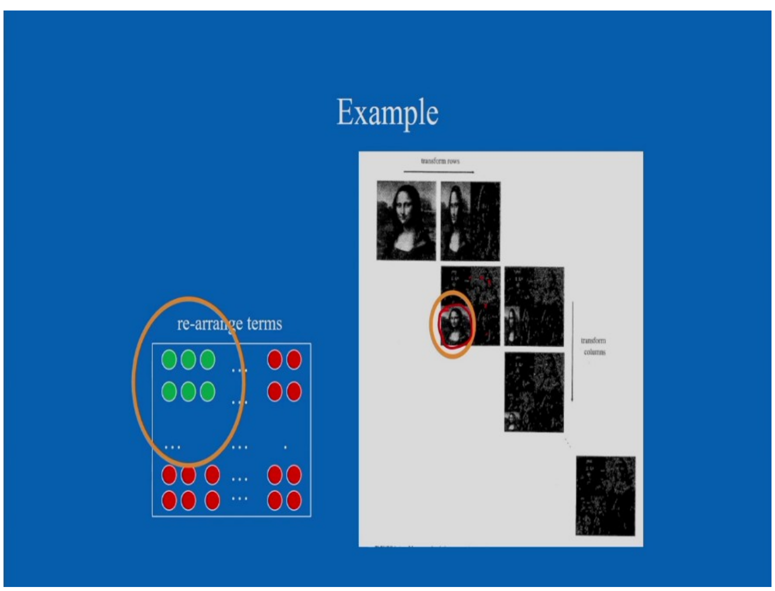

So in this case, you can see, first I am doing the decomposition along the rows, after this, the columns. After this, rearranging the terms and corresponding to this, you can see, I have this low frequency information, that is the average value and you can see, if you consider outside, that is nothing but the detail information. That is the high frequency information.

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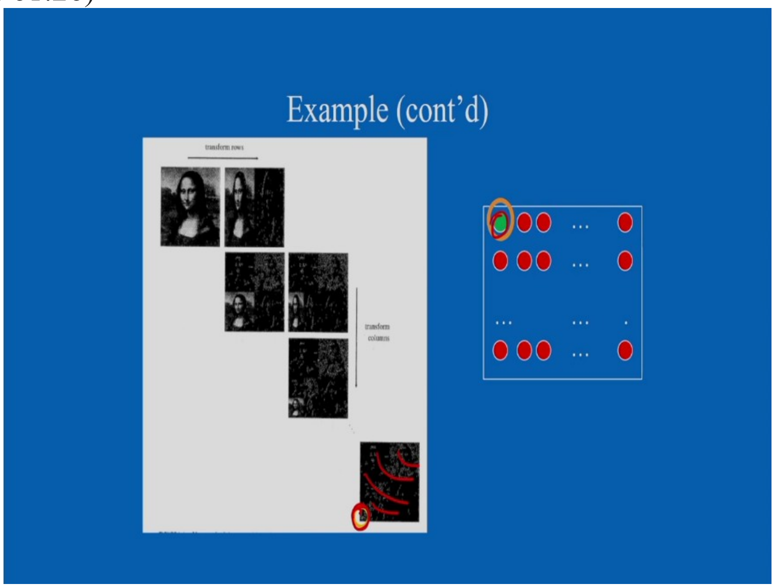

After this, this green quadrant, considering the green quadrant I am doing the decomposition. So if I do the decomposition, I will be getting this one, and you can see only I have this low frequency information. So this low frequency information is available here. And if I consider the rest of the portion, that corresponds to the high frequency information.

So this is the concept of the Non-standard decomposition. So I have two decomposition techniques, one is the standard decomposition technique, another one is the non- standard decomposition technique.

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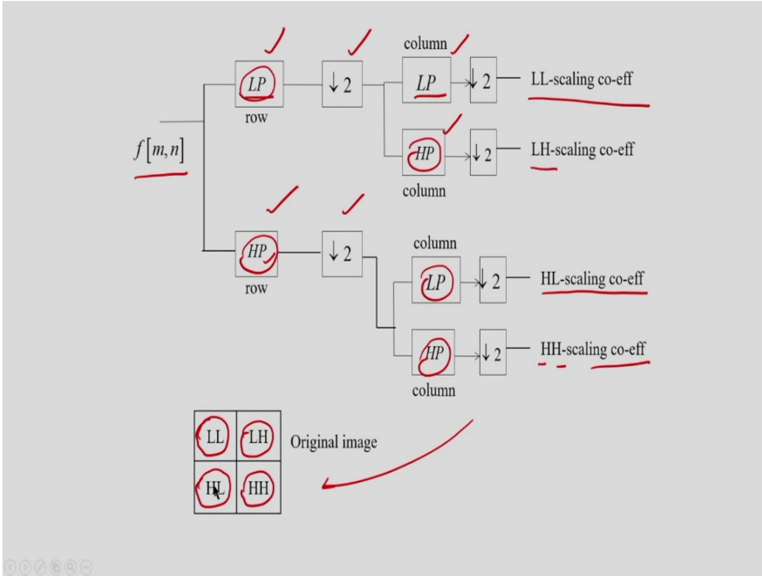

So this concept, I am showing here. So f n is the input image. The two dimensional signal, that is the image I am considering. And I am considering the low pass filter and the high pass filter to get the average value and the detail value. And after this, I am doing the down sampling by 2.

And after this what I am considering, again, I can do the decomposition of the signal, you can see. Again, I am applying the low pass filter and the high pass filter. And after this, I am doing the down sapling by 2. So first component, I am getting LL component because I am applying the low pass filter and the low pass filter here. So I will be getting the LL component. That is, the low frequency low frequency information, I will be getting. LL means the low frequency low frequency information.

And corresponding to the second case, if you see, I am applying the low pass filter and the high pass filter. That means I am getting LH coefficients, that is the low frequency and the high frequency coefficients, I will be getting.

Similarly, in this case I am applying the HP, that is the high pass filter; and after this, the low pass filter. That means I will be getting HL coefficients, the high frequency and low frequency coefficients.

And after this, I am applying the high pass filter and the high pass filter, that means I will be getting the high high coefficients. The high frequency high frequency coefficients, I will be getting. So this is represented like this.

So LL information, that is the low frequency information is available here. LH information is available here, HL information is available here and very high frequency information, that is the high high frequency information is available here.

So that means, after this decomposition I will be getting this transform image.

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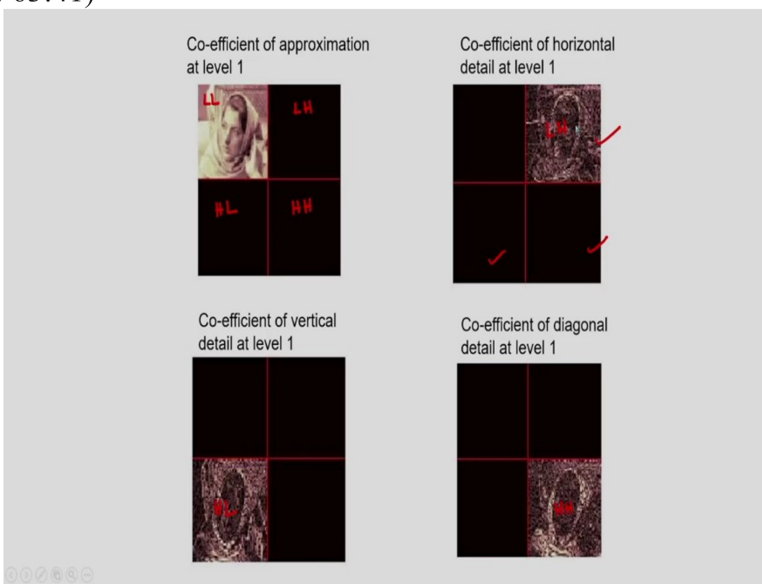

Now, I am considering this example. So suppose one image is decomposed and I am getting these components. So this component is LL component, this is LH component, this is HL component and this is the HH component, I will be getting.

So you can see, I have this component, LH component, HL component, HH component. So this component is the HH component. This component is HL component. This component is LH component. So after the decomposition, I will be getting this one. That is, the level 1 decomposition.

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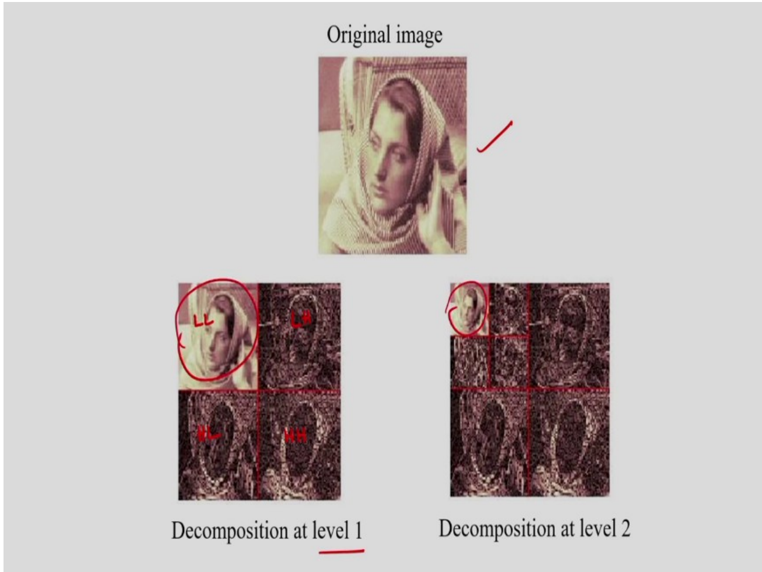

So you can see, I am considering two level decompositions. You can see the original image and after this, I am doing the decomposition at the level 1. So this LL, this is LH, this is HL and this is HH. And after this again, I am doing the decomposition of this image. The second level decomposition. So I can do the decomposition like this.

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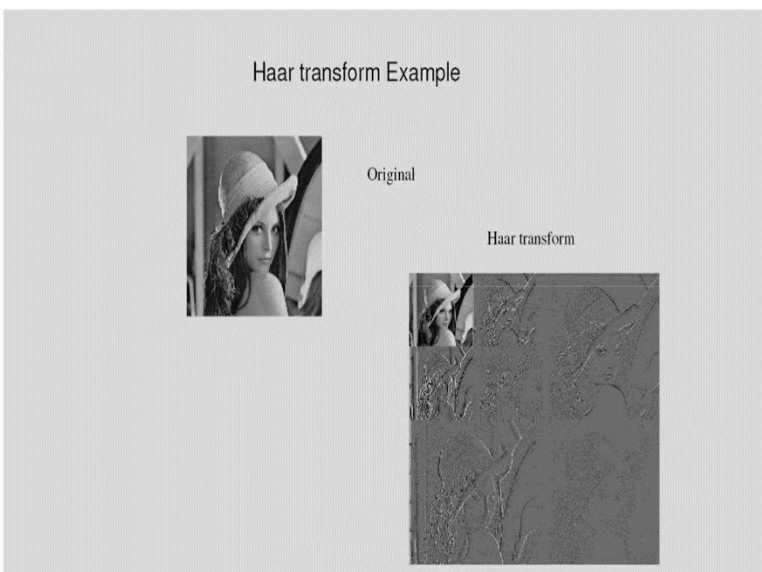

And this is one example of the Haar transformation. So I am applying the Haar transformation for the decomposition of the input image. So like this, we can do the multiple level decompositions. This is briefly about the discrete wavelet transformation.

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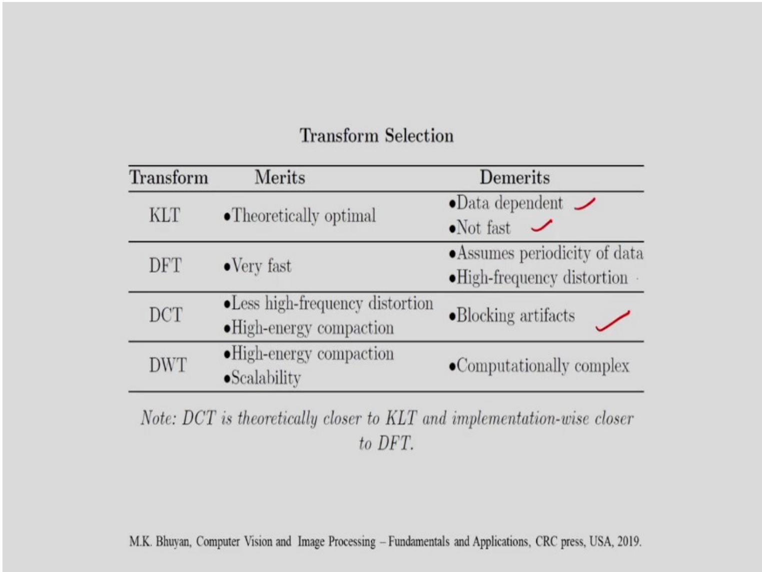

Transform Selection

Transform	Merits	Demerits
KLT	•Theoretically optimal	•Data dependent •Not fast
DFT	•Very fast	•Assumes periodicity of data •High-frequency distortion
DCT	•Less high-frequency distortion •High-energy compaction	•Blocking artifacts
DWT	•High-energy compaction •Scalability	•Computationally complex

Note: DCT is theoretically closer to KLT and implementation-wise closer to DFT.

M.K. Bhuyan, Computer Vision and Image Processing – Fundamentals and Applications, CRC press, USA, 2019.

And already, I discussed different transformations. So what are the advantages and disadvantages of these transformations? So first one is the KLT, that is the KL transformation. So the advantage is, theoretically optimal. What are the disadvantages? It is data dependent and not fast, because the KLT depends on the statistics of the input data. So that is why, it cannot be implemented in real time.

Next one is the DFT. The DFT is very fast but what are the problems? The problem is, it assumes the periodicity of data and also, the high frequency distortion that is nothing but the Gibb's Phenomenon. That concept already, I have explained.

In case of the DCT, what are the advantages? Less high frequency distortion as compared to DFT. And also the high energy compaction. But the problem is the blocking artifacts.

In case of the DWT, the high energy compaction and also the scalable, because I can consider different scales for decompositions, and energy compaction is very high. But the computationally, it is very complex.

So you can see the comparison between these.

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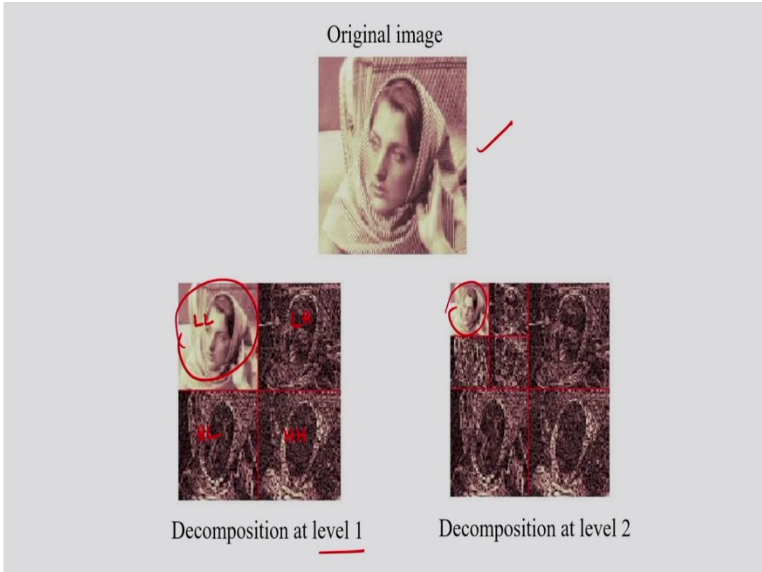

And if you see the previous figures, again I am showing the previous figures here. So suppose in this case I am doing the decomposition. One application I can explain. Suppose image compression, so for image compression I can do this decomposition. And what I can consider for this component, that is the low frequency component, that is the LL component, I can allocate more number of bits because it has more visual information as compared to other components, that is the high frequency components.

So suppose corresponding to this component it has more information, so that is why I have to allocate more number of bits for the LL component as compared to LH component, HL component or HH components. So that means, for image compression I can neglect this component, I can neglect this component, that is the redundant information I can neglect. That means the maximum importance I can give to LL component as compared to other components.

So based on this principle, I can do image compression or video compression. That is, based on DWT. So one compression standard is JPEG 2000. In JPEG 2000 this principle is used. In case on the JPEG, we use the DCT. But for JPEG 2000, we use DWT, the discrete wavelet transformation. This is the concept of image compression by considering discrete wavelet transformation.

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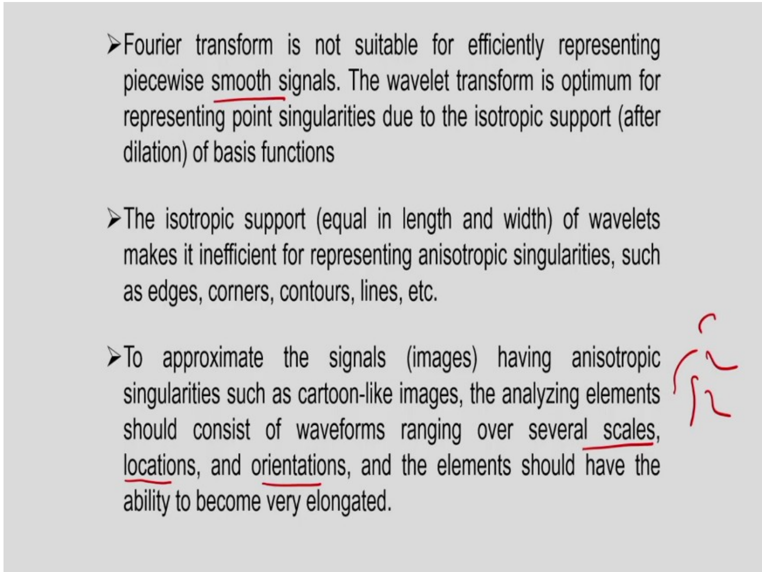

And finally, I want to discuss one point, important point. That is, that Fourier transform is not suitable for efficiently representing piecewise smooth signal. And the wavelet transform is optimum for representing point singularities due to the isotropic support, because we consider dilation of the basis functions. So, that means the Fourier transform is not suitable for representing very smooth signals. For this, we can consider the wavelet transformation.

But there is a problem in the wavelet transformation. What is the problem? Here you can see, the isotropic support of wavelets makes it inefficient for representing anisotropic singularities such as edges, corners, contours, lines, etc. That means, the wavelet is not good for representing anisotropic singularities. I am repeating this, that is, wavelets are not good for representing anisotropic singularities like edges, corners, contours, like this.

So that is why, to approximate the signals having anisotropic singularities, such as cartoon-like images, we have to consider some other transformation. So for this, the analyzing elements should consist of waveforms ranging over several scales. So we have to consider several scales. Several locations we can consider, and orientations. And the elements should have the ability to become very elongated.

So suppose if I consider the representation of a cartoon, suppose a carton image like this, that means it has edges and the corners or the contours may be available. So for this, the wavelet is not good. So for this, we have to consider some other transformation so that we can represent this anisotropic singularities.

For isotropic support we can consider wavelets, but for anisotropic singularities the wavelet representation has a problem. So for this, we have to consider some other transformation. Now, here you can see, we have to consider different scales, we have to consider different locations also we have to consider, and also the orientations also we have to consider.

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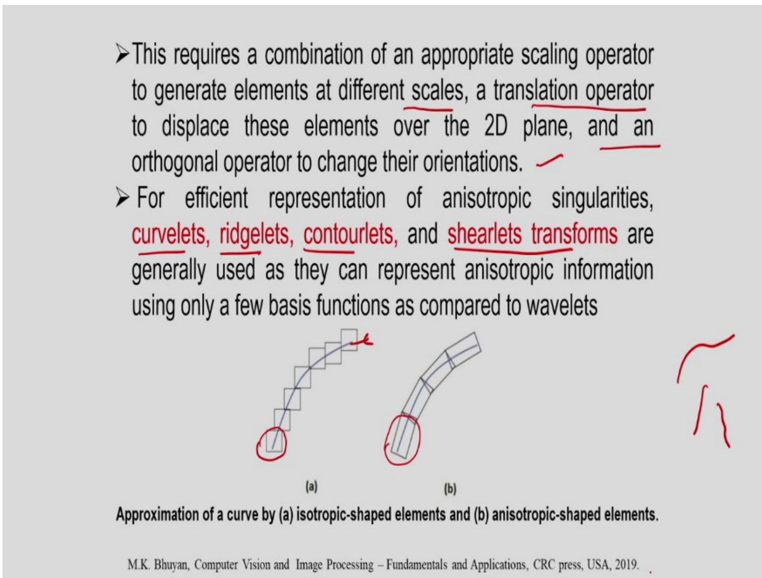

So this requires a combination of an appropriate scaling operator to generate elements at different scales. So we have to consider the elements and in this case we have to consider the different scales, and also we have to consider a translation operator to displace these elements over the 2D plane. And also we have to consider an orthogonal operator to sense its orientations.

So that means, I have to consider the cases like the scaling, the translation and the orientation also we have to consider for representation of cartoon-like characters. Like, suppose if I want to represent smooth edges or maybe the contours or maybe the edges or the lines, I have to consider different scales and also we have to consider the translation parameter and also we have to consider the different orientations.

So that means, in summary, I can say that wavelets are powerful tools in the representation of the signal. The wavelets are good at detecting point discontinuities. However, they are not effective in representing geometrical smoothness of the contours. The natural image consists of edges that are smooth curves, which cannot be efficiently captured by the wavelet transformation.

So that is why we have to consider some other transformation. Like the transformation like the curvelets, ridgelets, contourlets, shearlets, we can consider for representing anisotropic information.

And in this case, you can see, I am showing one example. The approximation of a curve. So I am considering one curve here, you can see, this is the curve. The approximation by isotropic shape elements. So I am considering the isotropic shape elements. But in the second case, I am considering anisotropic shape elements. So you can see I am considering anisotropic shape elements, that is very efficient as compared to the first one.

In the first case, I am considering the isotropic shape elements, that is not efficient for representing smooth contours or maybe the smooth edges or maybe the lines. But in the second case, I am considering anisotropic shape elements which is very good for representation of anisotropic singularities.

So, I am not going to explain the concept of the curvelets transformation, ridgelets transformation, contourlets transformation and the shearlets transformation. So for this transformations you may read books, and in my book also I have discussed about these transformation, the curvelets transformation, ridgelets transformation, contourlets transformation and the shearlets transformation.

So in this class, I discussed the concept of the discrete wavelet transformation. And also, I have explained how to decompose a particular image into different bands. So I can get LL frequency band, that is the low frequency band, LH band, HL band and HH bands. So I can decompose a particular image by using the DWT. So for this, I have to consider the low pass filter and the high pass filter.

So it is not possible to discuss all the mathematical concepts behind wavelet transformation. So if you are interested, then you may see books, the image processing books and you can study yourself about the wavelet transformation.

In my class, I have explained only the basic concepts of the wavelet transformation. So let me stop here today. Thank you.