Computer Vision and Image Processing - Fundamentals and Applications

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Lecture 13

Image Transform: Introduction to Wavelet Transform

Welcome to the NPTEL MOOC course in Computer Vision and Image Processing – Fundamentals and Applications. In my last classes I discussed the concept of image transformation; I discussed the concept of the Fourier transformation, discreet cosine transformation, and the KL transformation. Fourier transform gives the frequency information present in a signal, but there is a drawback.

The major drawback of the Fourier transform, it does not give the time information that means, at what time a particular event took place, and that information is missing. So, to consider that issue, we are now considering another transformation that transformation is the Wavelet transformation.

So, today I am going to discuss the fundamental concept of Wavelet transformation, it is not possible to discuss all the mathematical concepts of the Wavelet transformation. So, that is why I will briefly discuss the fundamental concept of the Wavelet transformation. So, let us see what is the Wavelet information?

So, first I will discuss the concept of the Fourier series and after this, I will discuss the Fourier transformation and I will highlight the disadvantages, the drawbacks of the Fourier transformation and after this, I will discuss the STFT, the Short Time Fourier Transformation and finally, I will discuss the discrete wavelet transformation.

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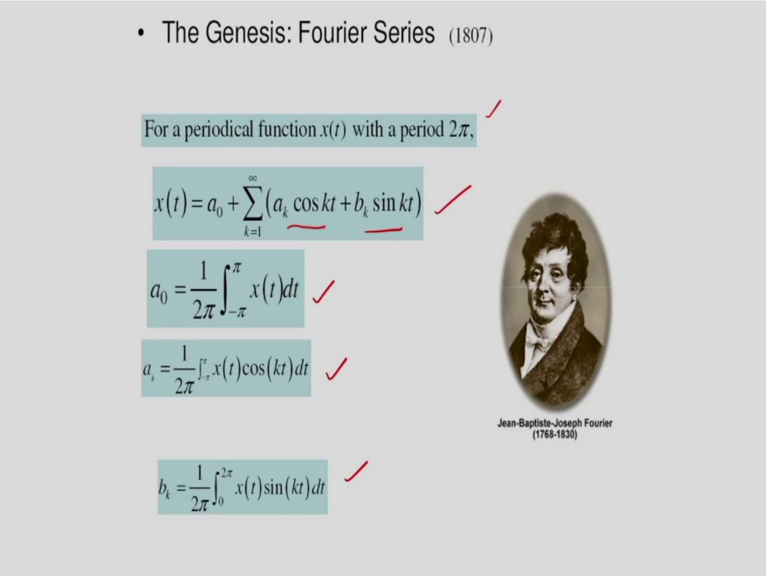

So, first one is the Fourier series you can see here I am considering a periodic function. The periodic function is x t and, in this case, I am considering a period twice pi then in this case, this x (t) can be represented by the Fourier series, the Fourier series is a naught plus the summation k is equal to 1 to infinity and I have two components one is the cosine component another one is the sine component.

So, due to this Fourier series, I will be getting the fundamental component of the signal and also the harmonic components present in the signal. The coefficient a naught is given by this, the coefficient a (k) and the b (k) is given by this expression. So, I think already you know about the Fourier series.

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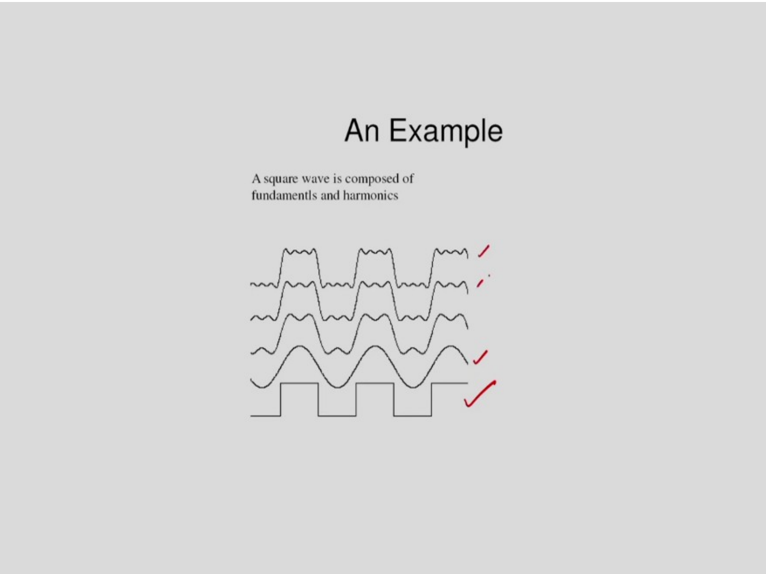

I can give you one example of the Fourier series. A square wave is composed of fundamentals and the harmonics. So, if I considered a square wave it is composed up the fundamentals and the harmonics. So, you can see the fundamental components and all the harmonic components present in the signal.

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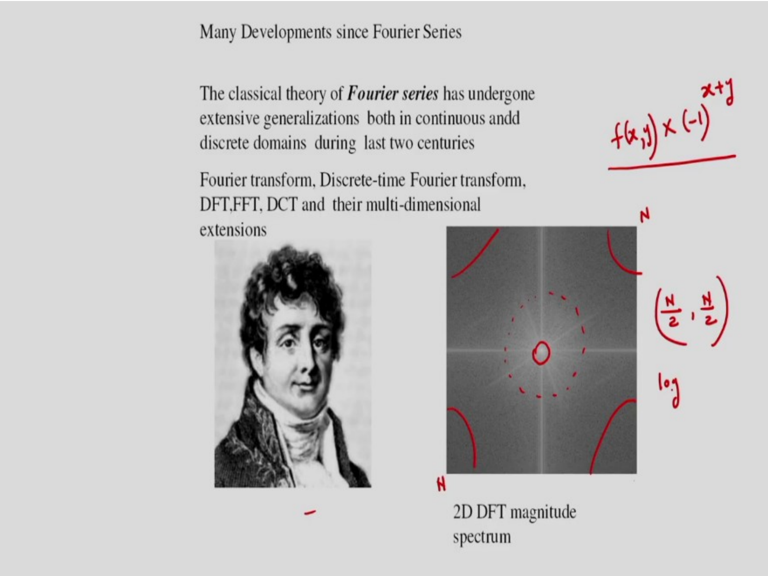

After this the next development is the development of the Fourier transformation. So, already I have discussed about the discrete time Fourier transformation DFT, the DFT can be implemented by FFT and also, last class I discussed about DCT the discrete cosine transformation. The DFT gives the frequency information present in an image or a signal. So, here I have shown the image of the Fourier and this is the 2D-DFT magnitude spectrum of the Fourier image.

So, these are Fourier transform of Fourier and in this case the Fourier spectrum. So, this is the center point of the Fourier spectrum and if I consider, so this portion, the central portion corresponds to the low frequency part and if I consider the outside part, that part, that corresponds to the high frequency part. Now, before the Fourier transform, I have to multiply the image if I consider one image is f (x, y), the image is multiplied by - 1 to the power x plus y that is the pre-processing I have to do.

The image is multiplied by minus 1 to the power x + y. So, because of this the Fourier transform, if the size of the image is, suppose N cross N, then the Fourier transform will be cantered at the point, the point will be N / 2, N / 2. So, that means, the pre-processing is the image is multiplied by - 1 to the power x + y and corresponding to this the Fourier transform, the center of the Fourier transform spectrum will be N / 2, N / 2.

And in this case, last class I have shown that log transformation is used for displaying the Fourier transformation. The log transformation is used to compress the dynamic range of the pixels for better visualization.

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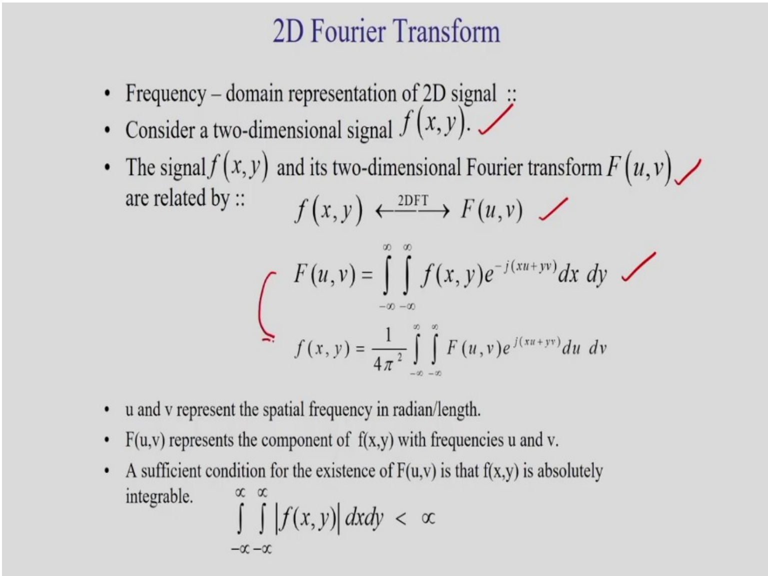

And in my last classes, I discussed about the 2D Fourier transformation. So, if the input image is f (x, y), the corresponding Fourier transform is F (u, v). So, f (x, y) the 2D transformation pair that is a 2D-DFT, f (x, y) that is transformed to F (u, v), then in this case u and v means the spatial frequency along the x direction and v is the spatial frequency along the y direction. If I apply this one that is the Fourier transform, I am getting, F (u, v) I am getting and from this I can determine the inverse Fourier transformation.

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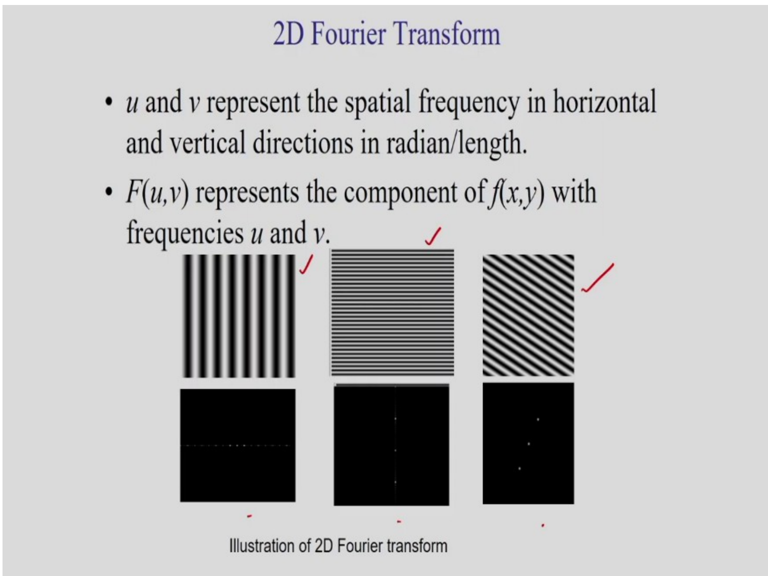

And in this example, I have considered three images. And corresponding to these images I have shown the Fourier spectrum one is the first images, the vertical lines, the horizontal lines, and the diagonal lines and corresponding to this I am getting the Fourier spectrum like this. So, u and v means the spatial frequency in the horizontal and the vertical directions in radian per link respectively.

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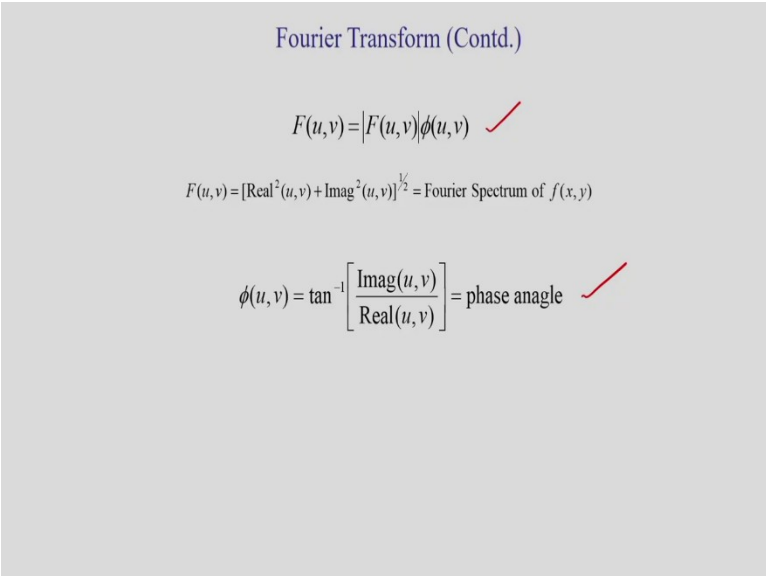

And already, I have mentioned that this Fourier transform can be represented in the Polar form. So, I have the magnitude part and the phase angle part, the phase angle I can determine like this. So, I have two components, one is the real part another one is the imaginary part of the Fourier spectrum.

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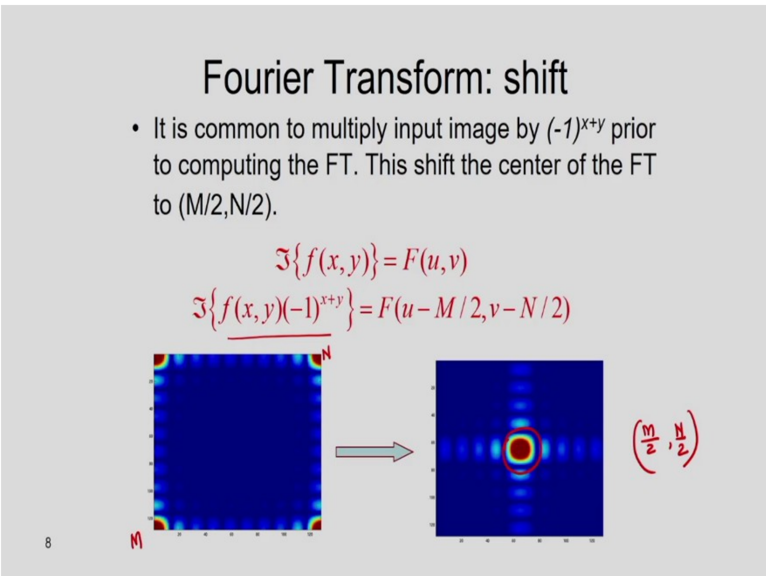

And this pre-processing I have shown here that what I have mentioned earlier. So, the image is multiplied by minus 1 to the power x plus y and after this the Fourier transform is applied, then the Fourier transform will be cantered at the point M by 2 and N by 2.

The size of the image is, suppose M cross N, M number of rows and N number of columns, then this will be the center of the Fourier transformation. The centre will be M by 2 and N by 2, this is the center of the Fourier transform that is the Fourier transform spectrum. So, this pre-processing I have to do before the Fourier transformation.

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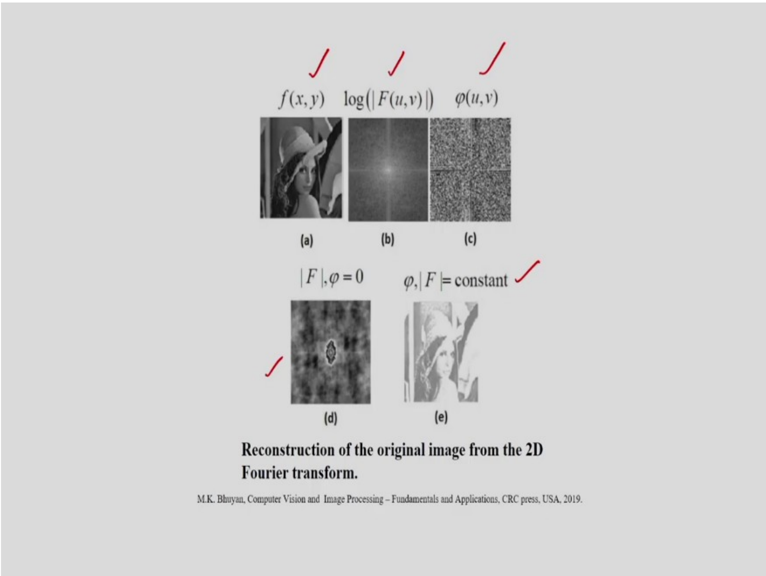

And in my last class, I have shown that image reconstruction from the magnitude information and the phase information. So, you have seen here the input image is f x y and this is the magnitude spectrum. So, I am using the log transformation to compress the dynamic range and after this, I am considering the face spectrum, the face spectrum is this and, in this case, I am only considering the magnitude information, I am not considering the phase information and this is my reconstructed image.

In the second case, I am considering the phase information, the magnitude information is not considered because the magnitude is constant, then corresponding to this, this is the reconstructed image. So, perfect reconstruction is not possible, if I only consider the phase information or the magnitude information. So, for perfect reconstruction I need both phase information and magnitude information.

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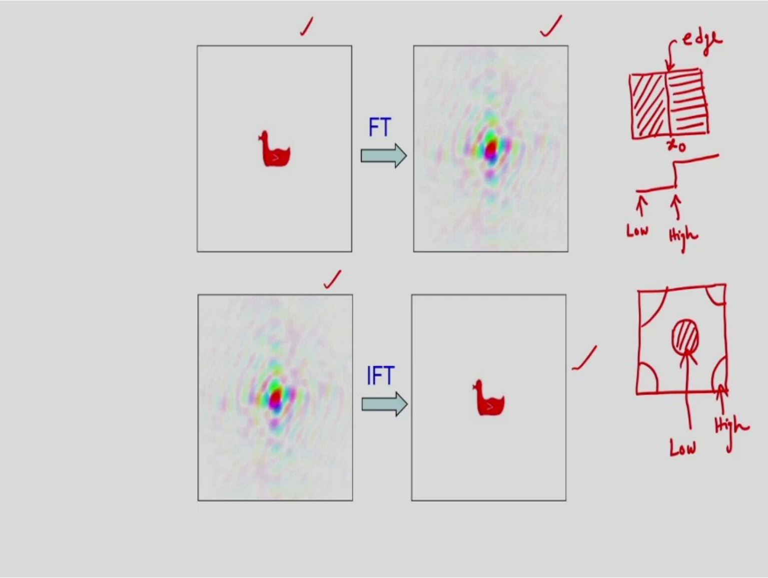

Now, in an image, what do you mean by the frequency? So, suppose if I consider one image, suppose if I consider edges here, one edge is and this is one intensity portion and that is another intensity portion. And, in this case, I am considering the edge. So, in this case, if I draw the profile here, suppose intensity is something like this and the edge is present at the location at this, at the edge there is a sudden change of the grayscale intensity value.

So, that means, if I consider the edges that corresponds to the high frequency information and if I consider the constant intensity portion or the homogeneous portion, that portion corresponds to the low frequency information. So, an edge means the high frequency information because there is an abrupt change of grayscale intensity value.

And in the Fourier transform, if I apply the Fourier transform, so already I have explained. So, the central portion corresponds to the low frequency information, so this is my low frequency information and if I consider the outside portion that corresponds to the high frequency information.

In this example, you can see this example here, I am considering one image and I am applying the Fourier transformation and I am getting the Fourier spectrum, the spectrum is this. Next what I am doing, I am considering the Fourier spectrum, and I am applying the inverse Fourier transformation to reconstruct the original image. So, I am getting the reconstructed image.

So, this is about the Fourier transformation and inverse Fourier transformation and the perfect reconstruction is possible because I am considering all the frequency components present in the signal that means, all the high frequency components and the low frequency components, I am considering for a reconstruction.

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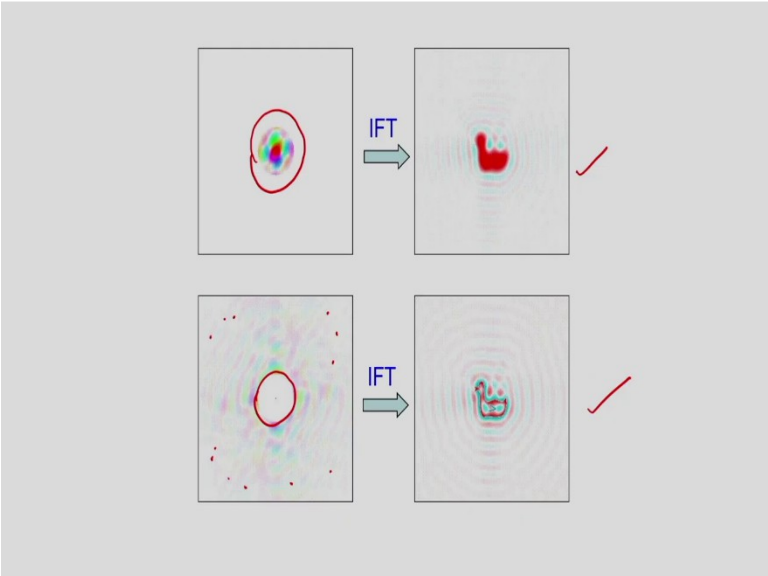

Now, in this example, I have shown here, I am considering the inverse Fourier transformation, but in this case, I am only considering the central portion of the Fourier transformation. The central portion of the Fourier transformation or the spectrum corresponds to the low frequency.

And in this case, if I want to reconstruct the original image, the perfect reconstruction is not possible because I am only considering the low frequency information and corresponding to this if I apply the inverse Fourier transformation, then I will be getting this image.

In the second case, I am not considering the central portion of the Fourier spectrum, that is the low frequency information I am not considering, only I am considering the high frequency information of the Fourier spectrum and corresponding to this I am reconstructing the image by using the inverse Fourier transformation.

So, I am getting the reconstructed image that means, I am getting the high frequency information. So, this is about the low frequency information and the high frequency information present in the Fourier spectrum.

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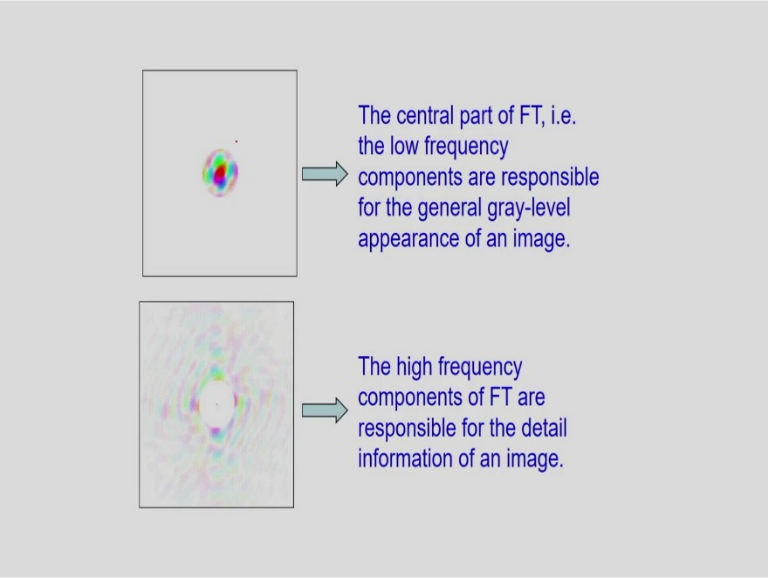

So, that is why I can say, the central part of the Fourier transformation that is the low frequency component are responsible for the general gray-level appearance of an image. So, that means, this low frequency information gives the general appearance of an image and if I considered a high frequency component or the Fourier transformation, these are responsible for the detail information of an image. So, if I consider the high frequency information, they give the detail information of an image.

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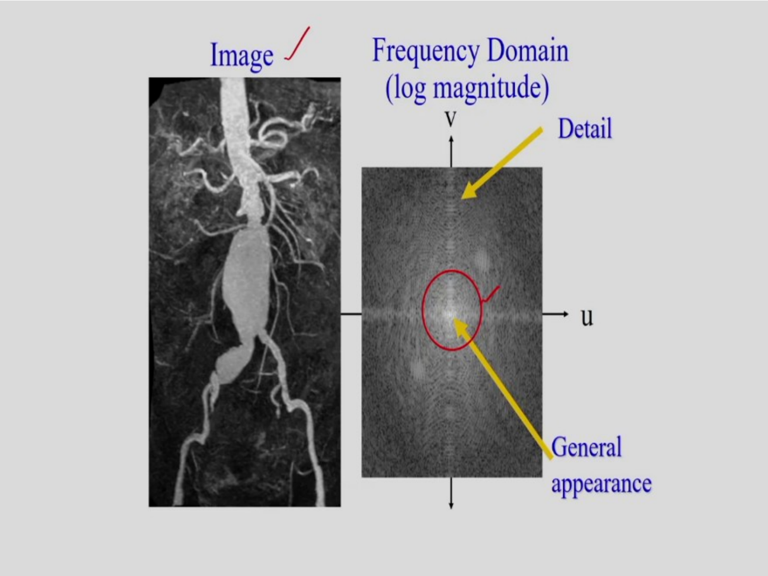

So, in this example, also I have shown here and the input image I have shown here and corresponding to this I am considering the Fourier spectrum and I am using the log magnitude spectrum, because I have to compress the dynamic range for better visualization.

So, you can see the lower frequency component present in the Fourier spectrum and the high frequency that is the detail information present in the Fourier spectrum. So, central portion corresponds to the low frequency information and if I consider the outside portion, the outer portion of the spectrum that correspond to the high frequency or the detail information.

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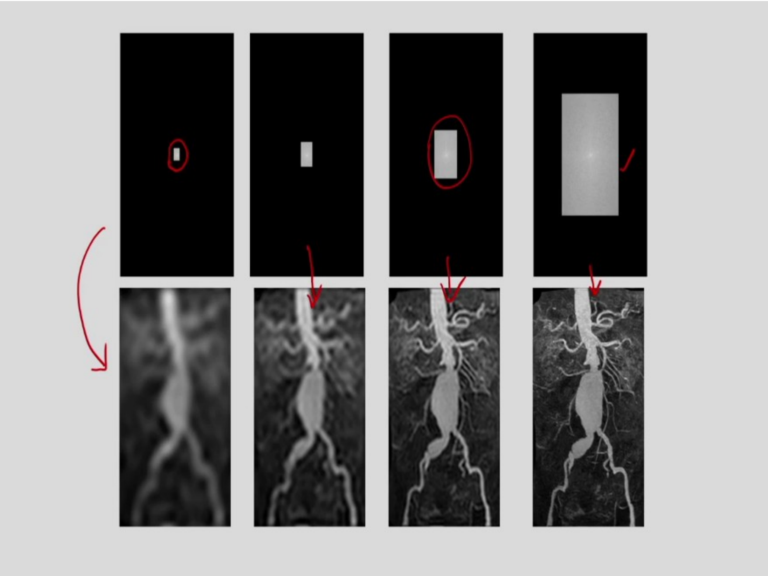

And in this case, what I am doing the reconstruction of the image from the Fourier spectrum. So, in this case, I am applying the inverse Fourier transformation. So, you can see here, only this portion that is this information is considered that means only the low frequency information is considered and corresponding to this, this is my reconstructed image. In the second case, thus, I am considering more information, low frequency information or maybe some high frequency information and corresponding to this, this is my reconstructed image.

Again, I am considering more information that is more important than I am considering that may contain low frequency as well as some high frequency information and corresponding to this, this is my reconstructed image. And similarly, if I consider this one, that this information then in this case, I have both low frequency information and also high frequency information and corresponding to this, this is my reconstructed image. So, you can understand the concept of the Fourier transformation.

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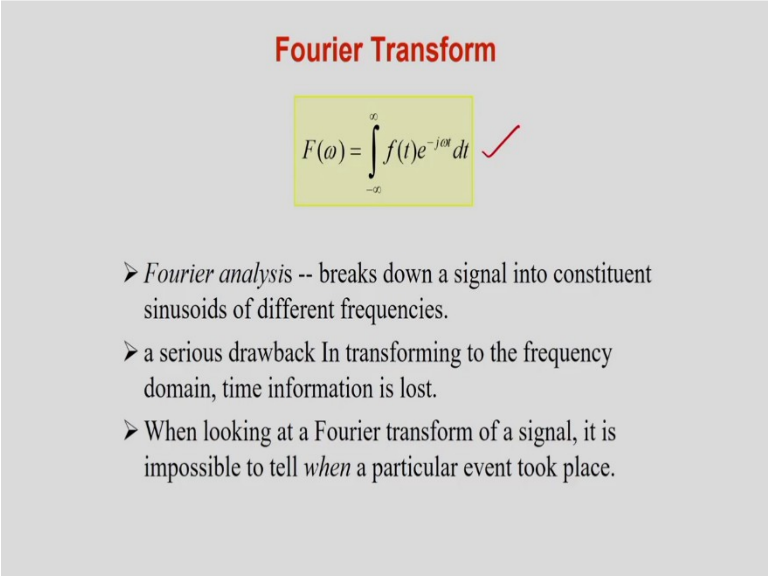

So, Fourier transformation breaks down a signal into constituent sinusoid of different frequencies. So, this is my Fourier transformation expression, but one drawback in transforming to the frequency domain is that, the time information is lost. So, when looking at the Fourier transformation of a signal, it is impossible to tell when a particular event took place. So, that is the main drawback of the Fourier transformation.

So, up till now, I have discussed the concept of the Fourier transformation in an image, I have explained the central portion of the Fourier spectrum, it gives the low frequency information and if I consider the outer portion of the Fourier spectrum, it gives the high frequency information. So, for perfect reconstruction, I need both low frequency information and the high frequency information.

Now, after this I have considered the Fourier transformation expression. And already I have explained it one main drawback of the Fourier transformation is that, the time information is not available, yet what time a particular event took place and that information is not available in the Fourier transformation. So, for this we have to consider another transformation that STFT first I will discuss. And after this, we will discuss the DWT the discrete wavelet transformation.

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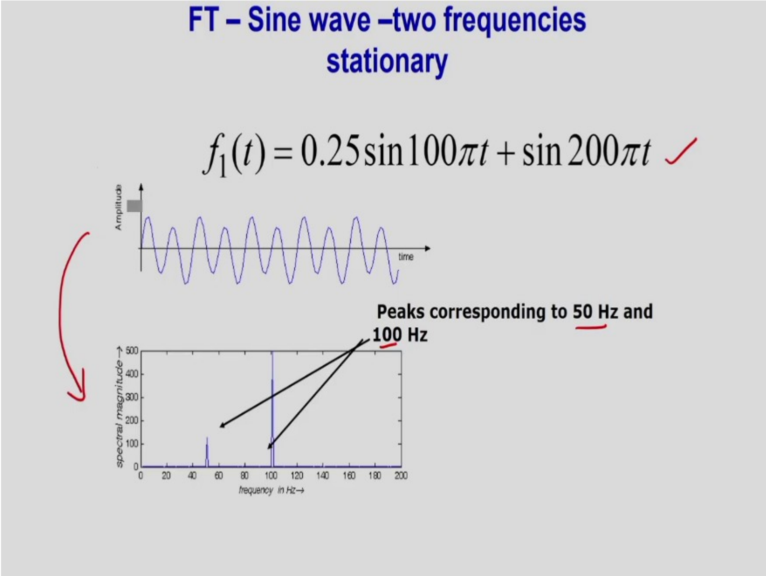

So, corresponding to the Fourier transformation I can give one example. So, if I consider this signal, this signal has two frequency components. One frequency is at the 50 hertz, another is at 100 hertz. So, corresponding to this signal, I have the spectrum, the Fourier spectrum is this. So, you have seen that there are two frequency components one is the 50 hertz frequency component and under only the 100 hertz frequency component corresponding to that signal.

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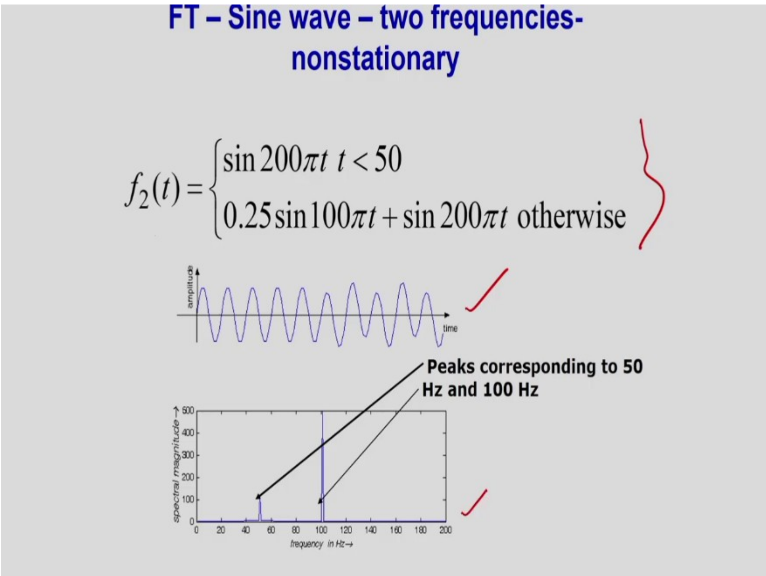

In the second case, I am considering one non-stationary signal; the signal is represented by this. So, it is f 2 t and I am considering the non-stationary signal and a signal will be something like this and corresponding to this signal also I am getting the peaks corresponding to 50 hertz and 100 hertz, so this is my spectrum.

So, if you compare the spectrum and the previous spectrum, the previous spectrum was this for the stationary signal, then in this case I am getting identical spectrum one for the stationary signal and the second one is for the non-stationary signal. So, that means, the time information is not available in the Fourier transformation.

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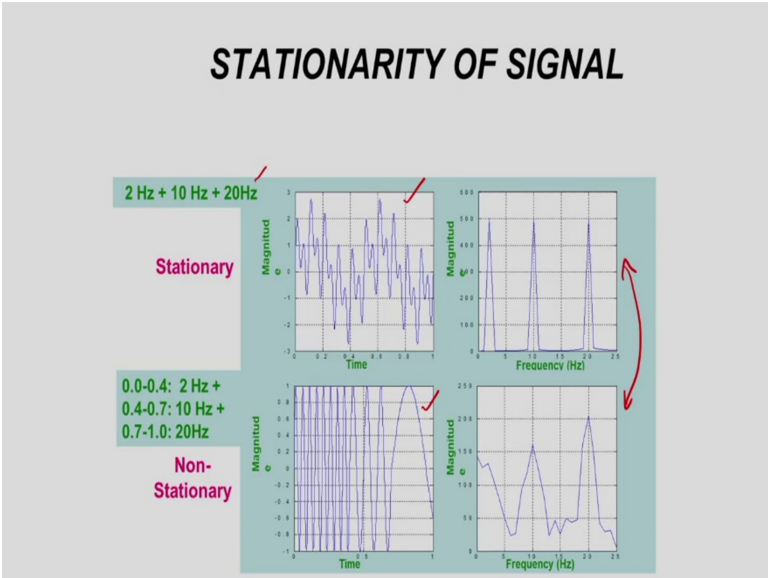

I can give another example you can see there are two signals one is the stationary signal. The first one is the stationary signal and second one is the non-stationary signal and you can see the frequency components present in the signal 2 hertz, 10 hertz, 20 hertz like this. In the nonstationary also you can see the frequency components present in the signal and corresponding to this, you can see the spectrum, the magnitude spectrum and if you see these two spectrum, they are almost identical. So, that means the time information is not available in the Fourier spectrum.

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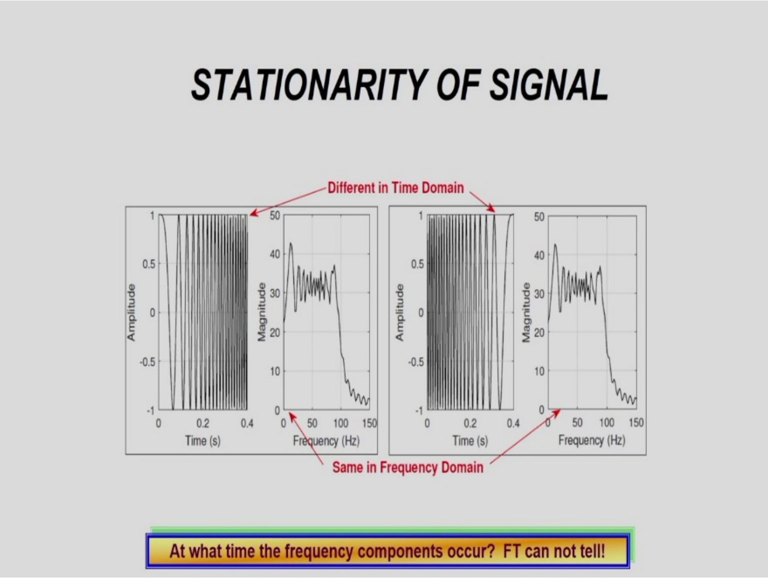

Again, I am considering the same explanation. You can see I am considering two signals deferent in time domain, but same in the frequency domain. That means, at what time the particular frequency component occur, that information is not available in the spectrum.

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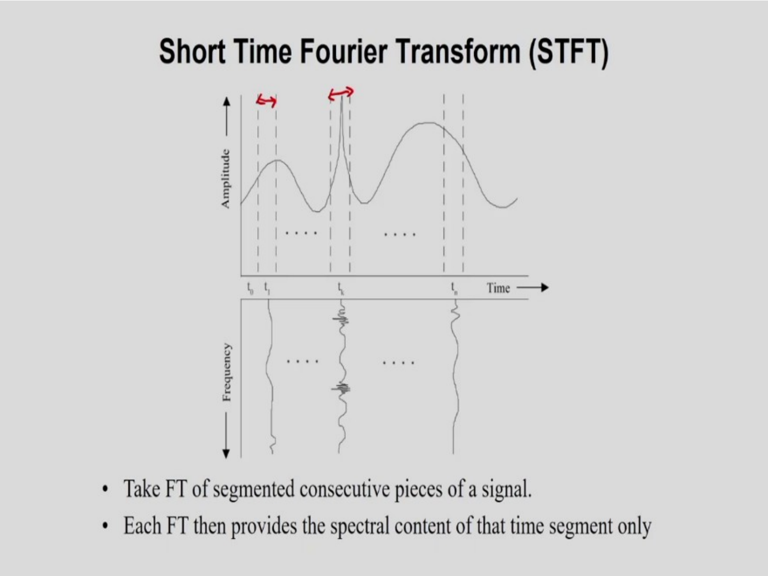

So, for this we will consider another transformation. So, that transformation is the short time Fourier transformation STFT. So, for this we consider the window, particular window I am considering. So, suppose this window time window I am considering and corresponding to this time window, I want to see what are the frequency components present in the signal.

So, that means, take Fourier transformation of segmented conjugative pieces of a signal and each Fourier transformation then provides test spectral content of that time segment only. So, corresponding to that time segment, so if I consider this time segment or the time window, I can see what are the frequency components present in the signal that is the short time for your transformation.

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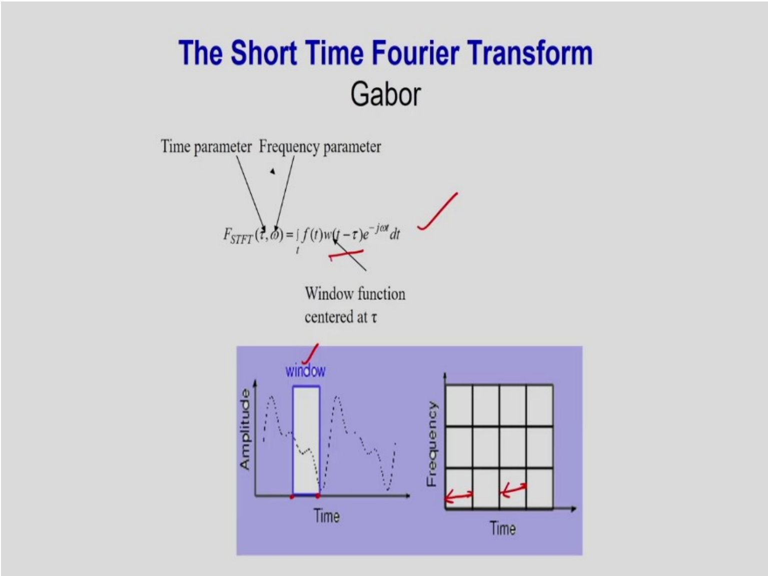

This short time Fourier transformation is called Gabor. So, this is the Gabor, so for this I am considering one window function, the window function is cantered at tau. So, corresponding to this function I am considering this window and corresponding to this window I want to see what are the frequency components present in the signal.

So, this window is considered in the time domain. So, corresponding to this time interval, so what are the frequency components present a signal that I want to see? So, that means, corresponding to this time window, I can see what are the frequency component present in the signal and like this, I can see the frequency components present in the signal, this is the short time Fourier transformation and this is also called a Gabor.

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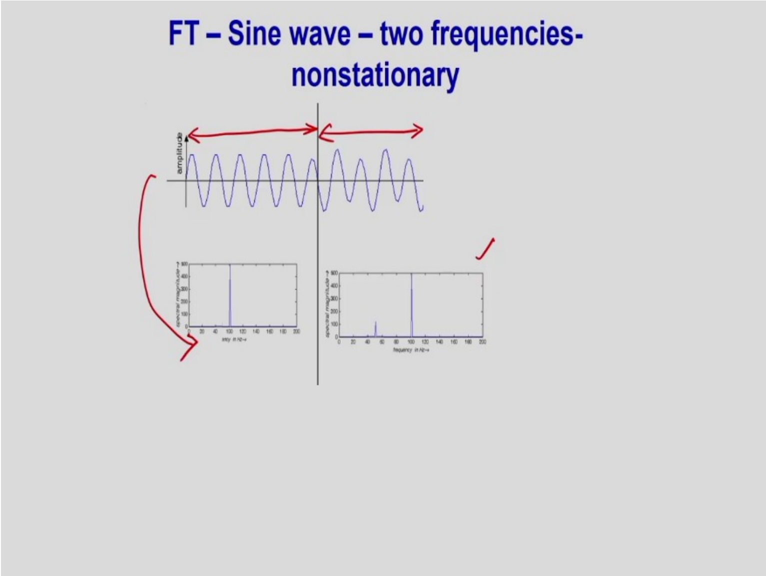

So, in this example, I have shown the STFT. So, I am considering a non-stationary signal. So, you can see the frequency components. So, from this point to this point, if you see from this to this time window, only one frequency component is present that is available in the Fourier spectrum and corresponding to this time interval two frequency components are present in the signal. So, I am having the two frequency components in the Fourier spectrum.

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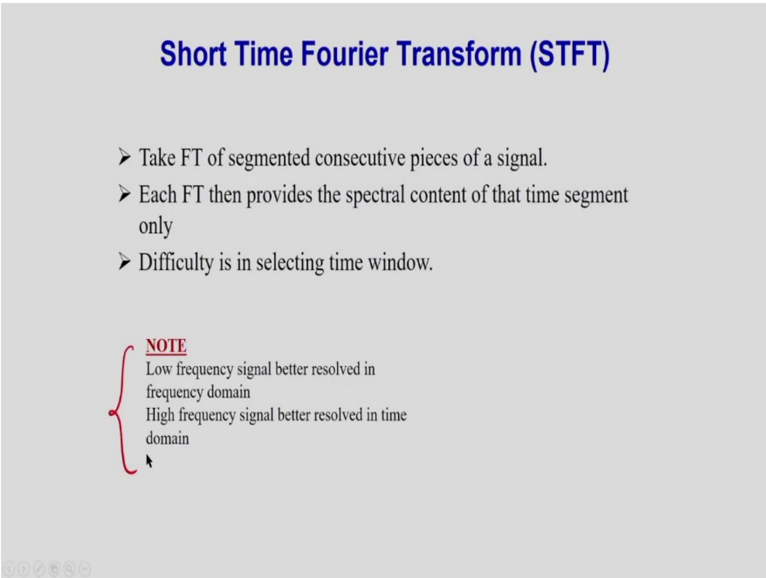

So, what is the STFT, take Fourier transform of segmented consecutive pieces of a signal? Each Fourier transformation then provides the spectral content of their time segment only. But one problem is how to select the time window? That is the one main problem of the STFT. So, how to select the time window that is the problem?

Now, I want to consider this case that is the low frequency signal, better resolved in the frequency domain and the high frequency signal better resolved in the time domain. This concept I can explain in the next slide. What is the meaning of this? This is a very important concept. You can see here.

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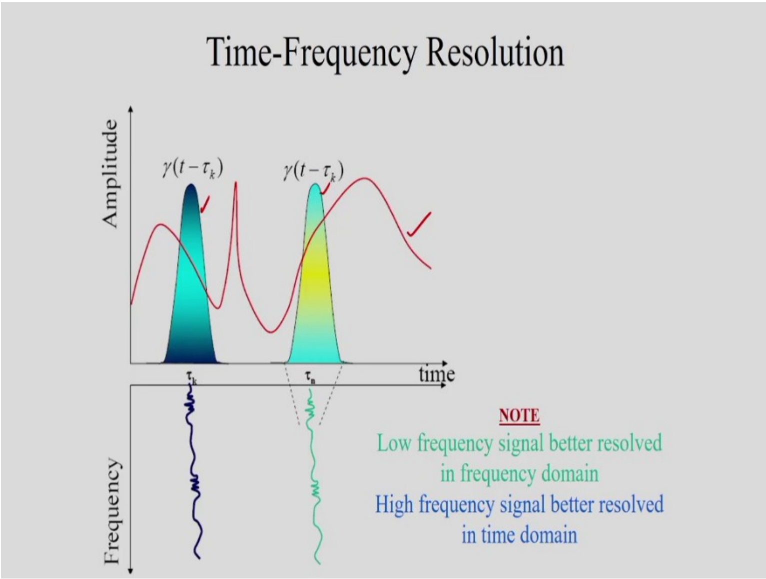

Here, I have shown two signals, one is the low frequency signal. The first one is a low frequency signal and another one is the high frequency signal. So, if I consider low frequency signal, that signal can be better resolved in the frequency domain. That means, in the frequency domain I can see the information present in the signal and in this case if I consider a high frequency signal that can be better resolved in the time domain.

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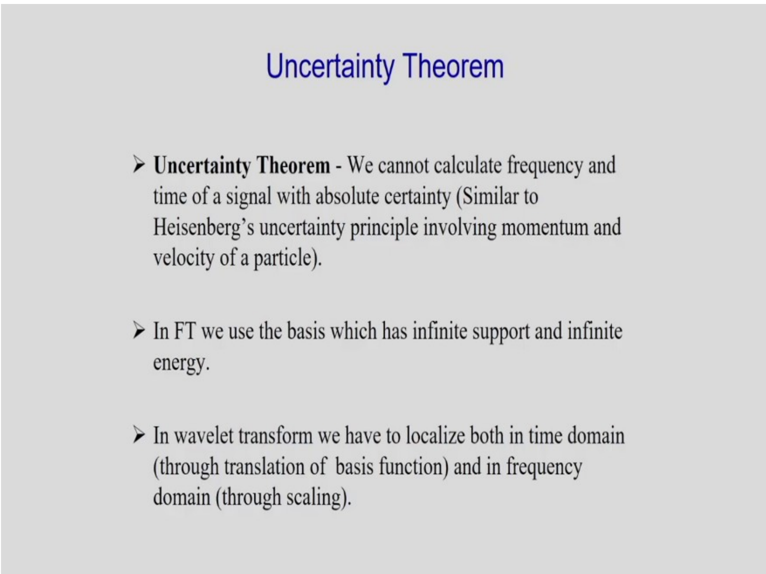

Based on this concept I can show the uncertainty theorem. So, uncertainty theorem is we cannot calculate frequency and the time information of a signal with absolute certainty. This is similar to Heisenberg uncertainty principle involving momentum and the velocity of a particle. In Fourier transform, we use the basis which has infinite support and infinite energy.

In wavelet transformation, we have to localize both in time domain and in the frequency domain. So, that means, in the time domain, we can do some translation of the basis function and in frequency domain we can do scaling. So, this is the uncertainty theorem.

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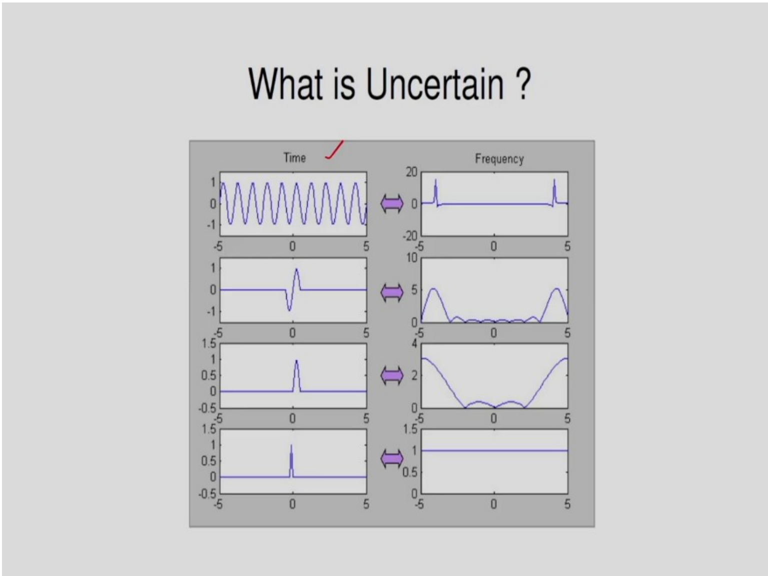

So, in this example, you can see, I am considering the signals in the time domain and the in the frequency domain you can see the signals, but one problem is that, at what time particular event took place, at what time a particular frequency is present in the signal that information is missing in these examples, that is uncertain.

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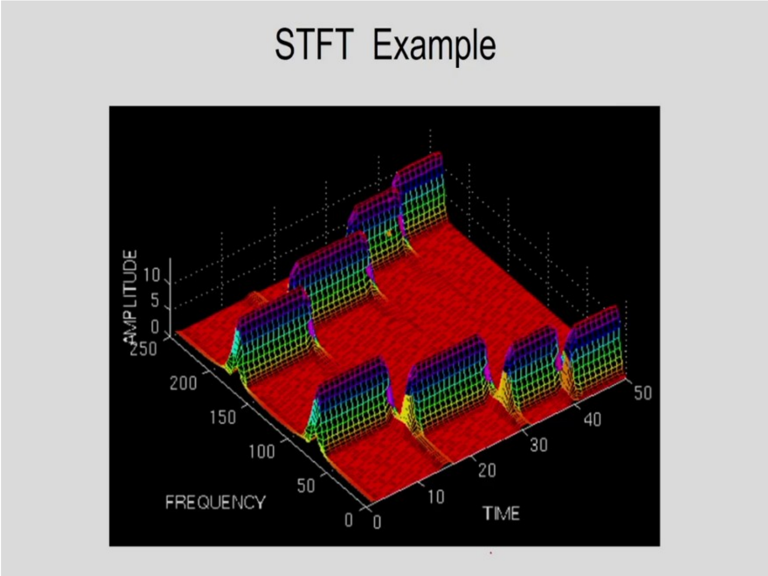

So, in this example, I have shown the STFT of a signal, you can see the frequency information, the time information, and the amplitude information that we can obtain by using STFT.

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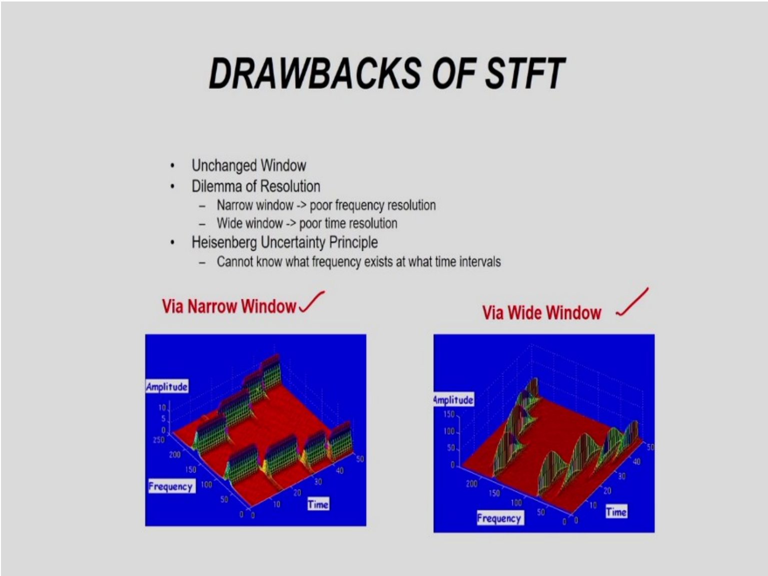

Now, this uncertainty principle I can explain here. So, if I consider the narrow window that gives poor frequency resolution and if I consider the wide window that gives poor time resolution, so first example, I am considering the narrow window and in a second case I am considering the wide window and, in this case, this is very similar to the Heisenberg uncertainty principle.

Cannot know what frequency exist at what time intervals that information is not available. So, in these two examples, I have shown that cases one is for the narrow window another one is for the wide window.

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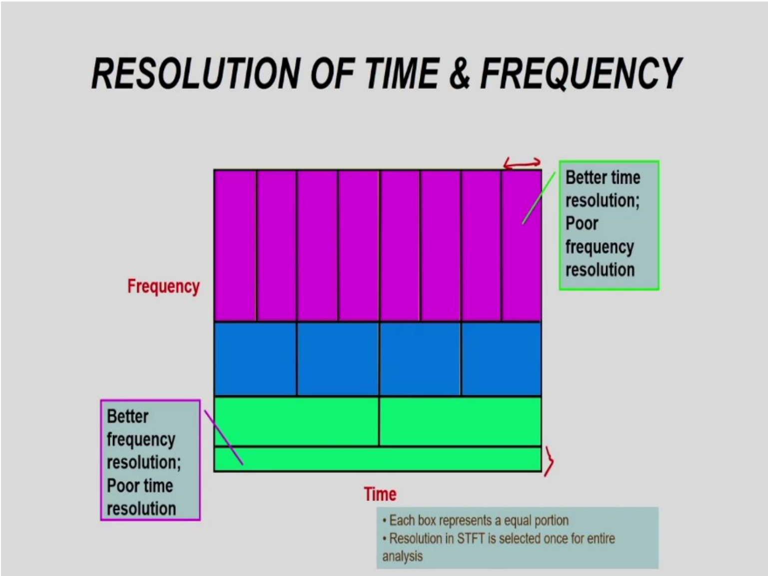

The next I am considering the resolution of the time and the frequency. So, if you see these windows, I am considering different windows here. So, if I consider this window, this gives better frequency resolution, but it is poor time resolution. But if I consider this window, this window gives better time resolution, but poor frequency resolution. So, you can see the concept of the resolution, one is the time resolution another one is the frequency resolution. And in this case, I have shown, this is the time axis and this is the frequency axis.

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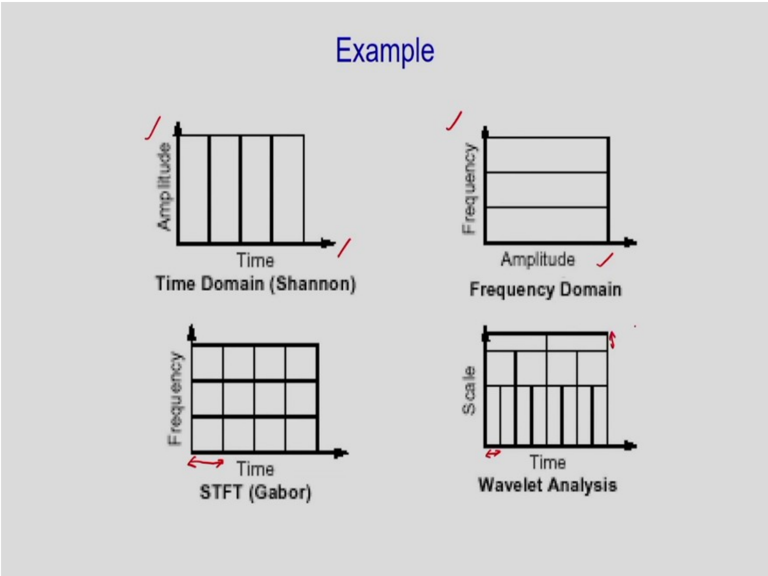

And, in this example, I have shown that cases like the time domain, you know that Shannon. So, in the time domain information of a signal that means we have the time information and the amplitude information of a signal. And in the frequency domain that we have explained the Fourier transformation, we have the amplitude information and the frequency information, but timing permission is not available.

And if I consider the STFT, that is the Short Time Fourier Transformation. So, we can see that corresponding to a particular time window, I have the frequency information present in the signal. In case of the wavelet analysis, we are considering variable size windows, I can consider the narrow window or maybe something like the window something like this.

And in this case, I can see in a particular time interval, what are the frequency component present in the signal. So here, I have the time information and scale information gives the frequency information that is the wavelet analysis. So, you can see the distinction between the signal representation in the time domain, frequency domain, and the STFT, and the wavelet analysis.

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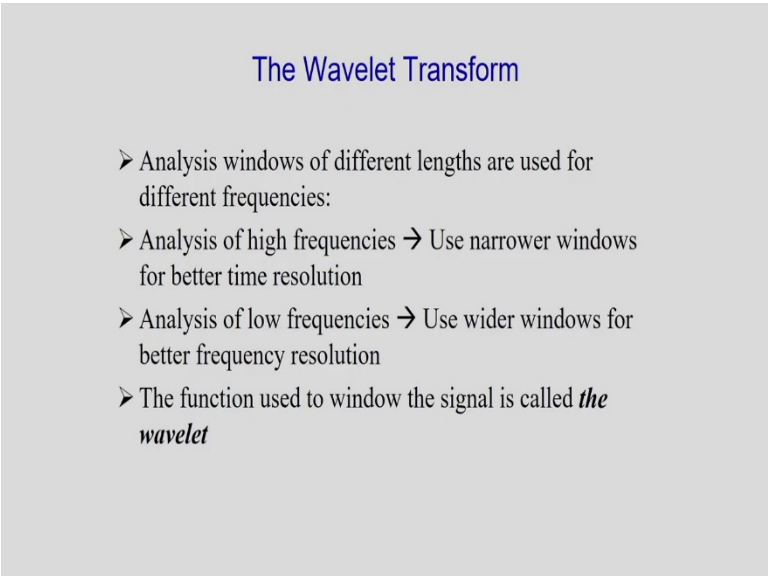

So finally, you can see, for analysis of the high frequencies, we can consider narrow windows for better time resolution. And for the analysis of the low frequency signal, we can use wider windows for better frequency resolution. The function used to window the signal is called a wavelet. So, this is the main concept of the wavelet transformation, the window size is not fixed in the wavelet transformation. In STFT, the window size is fixed.

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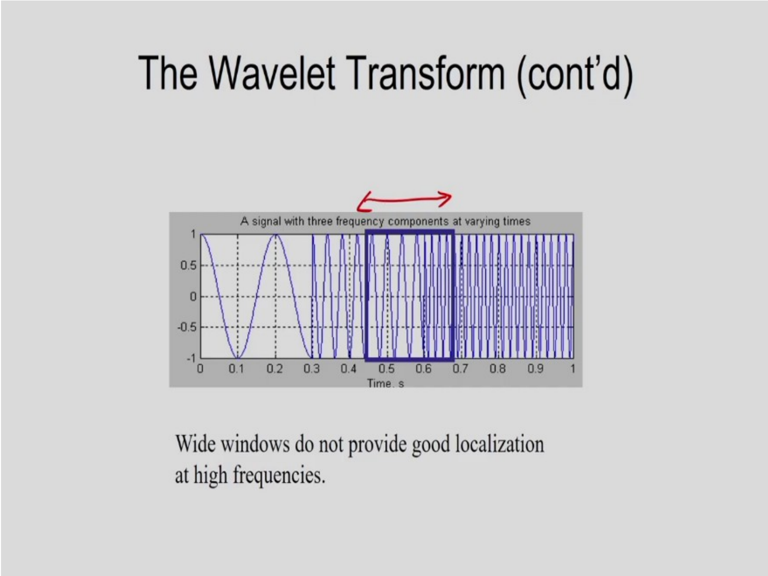

So, you can see this example, I am considering one nonstationary signal, A signal with three frequency components at varying times. So, in this case, you can see the wide windows do not provide good localization at high frequency. So, if I consider this window first this wide window, this wide window do not provide good localization at high frequency.

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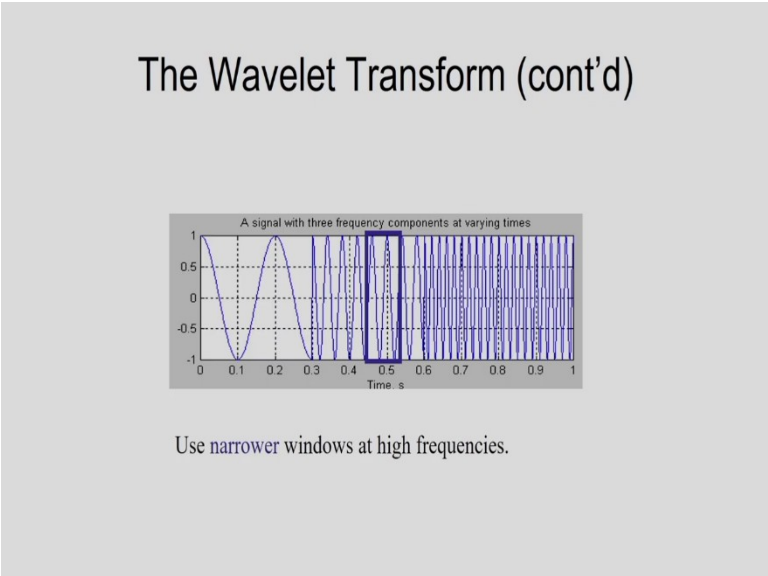

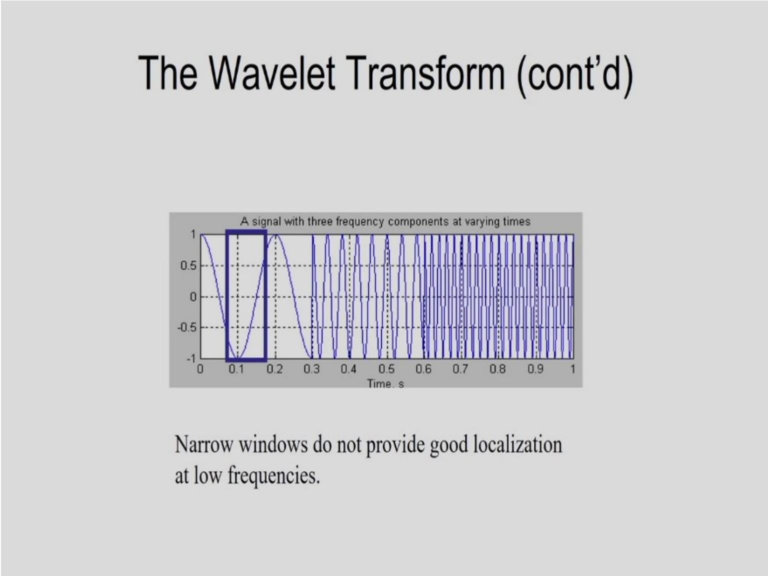

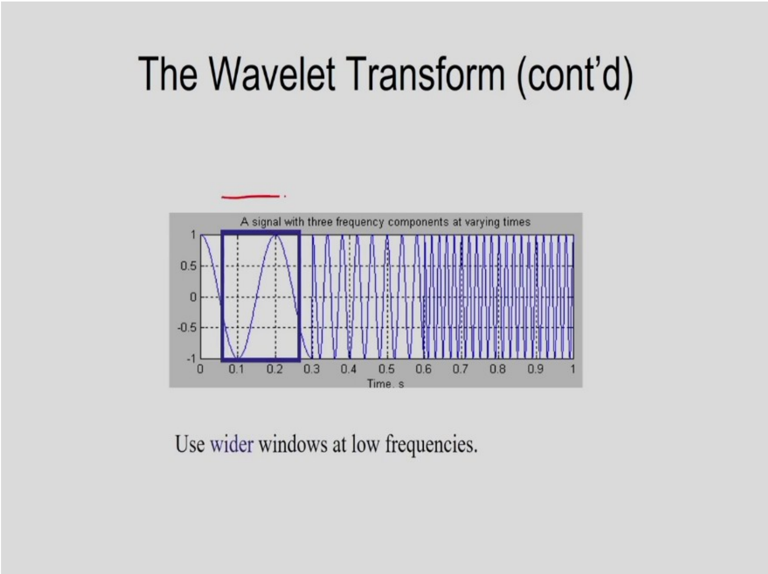

And if I considered a narrow window that is good for the high frequency, use narrow windows at high frequency. And corresponding to the low frequency component, if I consider low frequency part, the narrow windows do not provide good localization at low frequencies. So, for this we have to consider wide windows. So, like this we have to consider the wide window. So, this is my wide window that I am considering. This is the concept of the variable size windows.

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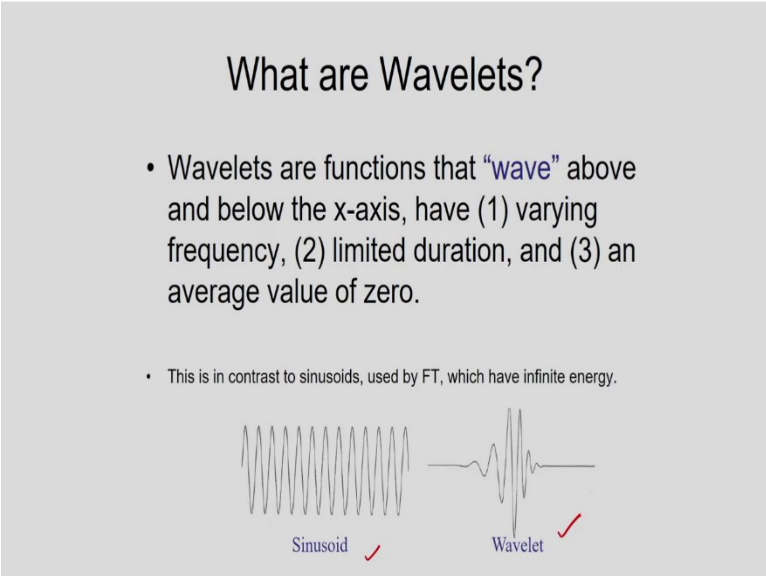

So, what is wavelet? Wavelets are functions that wave above and below the x axis. The first one is the varying frequency, limited duration, and average is value will be zero. So, here you can see the one example is the wavelet another one is the sinusoid. In case of the wavelet, we have the varying frequency, limited durations, and the average is value will be zero.

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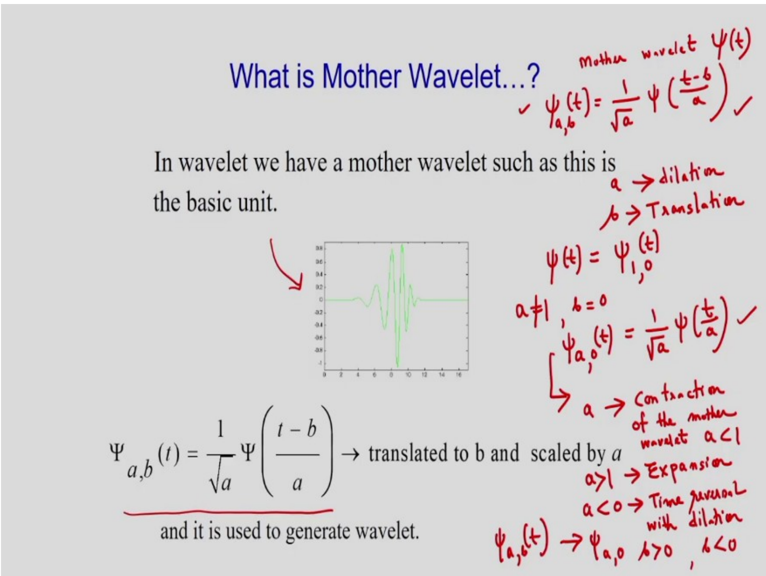

So, what is a mother wavelet? In wavelet we have a mother wavelet and in this case the mother wavelet will be something like this. And mathematically, it can be shown like this, this is the definition of the wavelet. So, I am considering the mother wavelet, the mother wavelet is denoted by psi t, this is the mother wavelet.

And other wavelets, the other wavelet already I have defined, the other wavelet is defined like this, psi a, b t, 1 by root a, psi t minus b divided by a. So, in this case, I have two parameters a and b, these are two arbitrary real numbers. Now, this a is the dilution parameter and b is the translation parameter. So, now, how to represent the mother wavelet?

The mother wavelet psi t is nothing but psi a is equal to 1, b is equal to 0. So, if I put a is equal to 1 here and b is equal to 0 then in this case, I will be getting the mother wavelet and, in this case, if I consider a is not equal to 1 and suppose b is equal to 0 then in this case, I can get psi a 0, t it is 1 by root a psi t by a. So, that means I am considering a is not equal to 1 and b is equal to 0.

So, what is the meaning of this? The meaning of this is, this is obtained from the mother wavelet indeed in this case the time is scale by a and the amplitude is scale by root a, that this parameter a is it causes the contraction of the mother wavelet, if a is less than 1, or otherwise if a is greater than 1 that corresponds to expansion or the stressing, expansion of the mother wavelet or distressing of the mother wavelet, if a is greater than 1. This parameter a is called the dilution parameter or the scaling parameter.

And if I consider a is less than 0. So, what will happen if a is less than zero? It corresponds to the time reversal with dilation. Now, this function psi a, b, it is a shift of psi a, 0 by an amount of b when b is greater than 0 that means in this case, there is a sifting along the right direction in the time axis by an amount b when b is greater than 0. On the other hand, this function is shift in the left along the time axis by an amount b when b is less than 0.

So, that means shifting in the right direction and shifting in the left direction in the time axis, if b is greater than 0 that means it is the shifting right along the time axis by an amount b and in this case, if the b is less than 0, then in this case shifting in the left along the time axis by an amount b. So, this is the interpretation of the parameters a and b and I have defined the mother wavelet and from the mother wavelet, you can see I can get the other wavelets.

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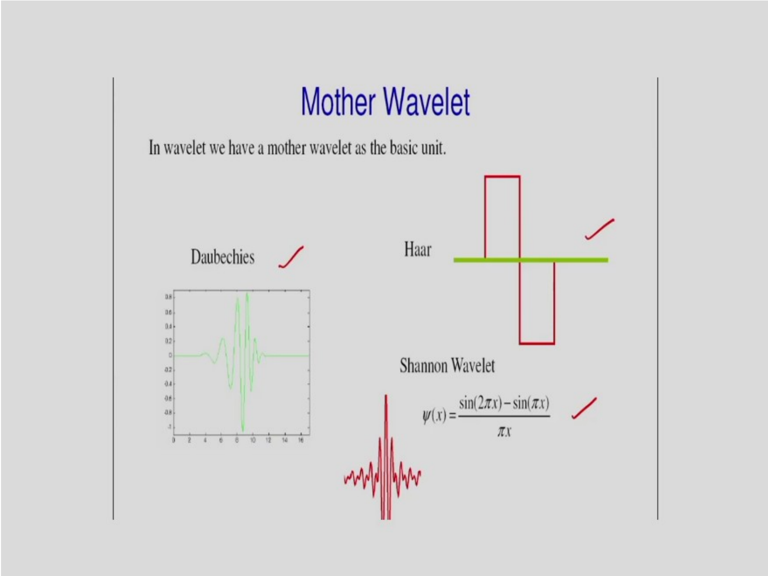

So, in this case, I had given some examples of the mother wavelet. So, first one is the Daubechies wavelet, one is the Haar wavelet, and another one is the Shannon wavelet. I have not mathematically defined, but you can see the shape of this wavelets, one is the Daubechies wavelet, Haar wavelet, and the Shannon wavelet.

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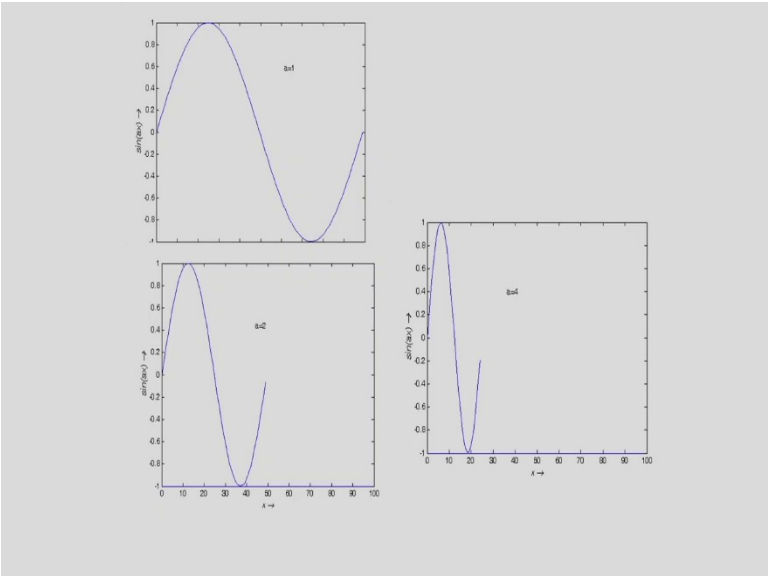

So, in this example, I can show you the mother wavelet and, in this case, I am considering a is equal to 1, a is equal to 2, a is equal to 4, that means what is the a? a is the dilation parameter and b is the translation parameter.

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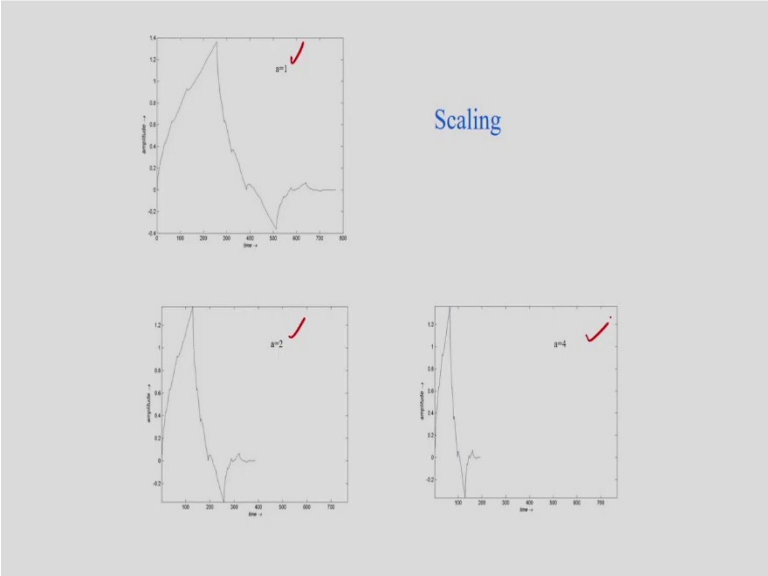

So, you can see I am doing the scaling because I am changing the parameter, the parameter is a, so a is equal to 1, a is equal to 2, a is equal to 4, like this. So, I am doing the scaling.

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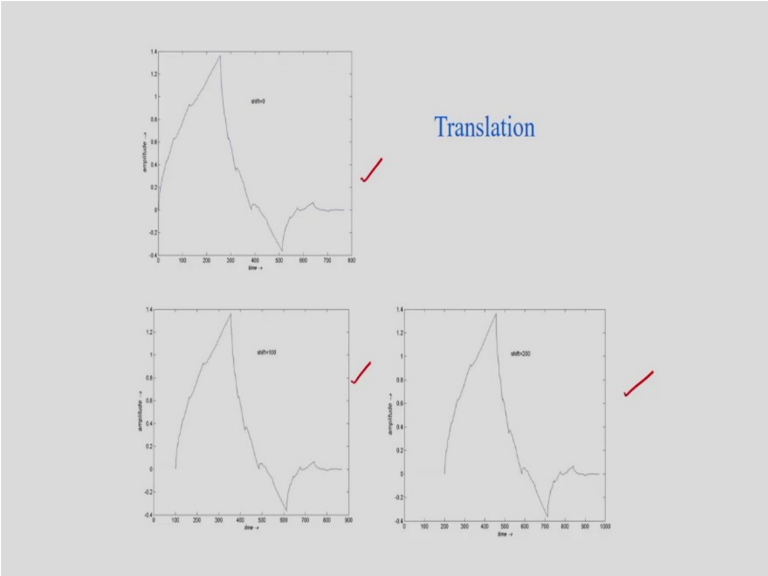

And also, I can do the translation with the help of the parameter, the parameter is b. So, b is the translation parameter. So, in this case, I am considering the shift, shift is equal to 0, shift is equal to 100, shift is equal to 200. So, I can do the translation and the dilation.

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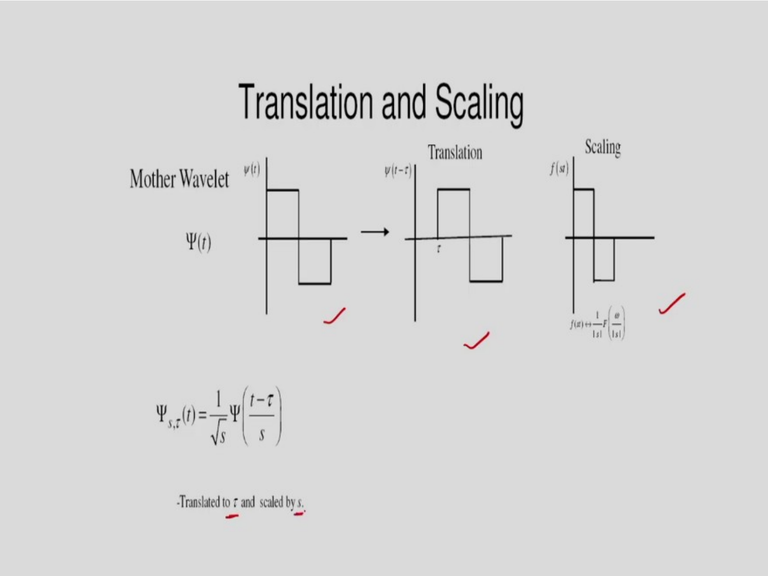

In this case, I have shown here, I have the mother wavelet, that is the Haar wavelet, and I can do the translation and also, I can do the scaling. So, in this case, instead of considering the parameters a and b, I am considering the parameter s and the tau. So, the tau is for the translation and s is for the scaling.

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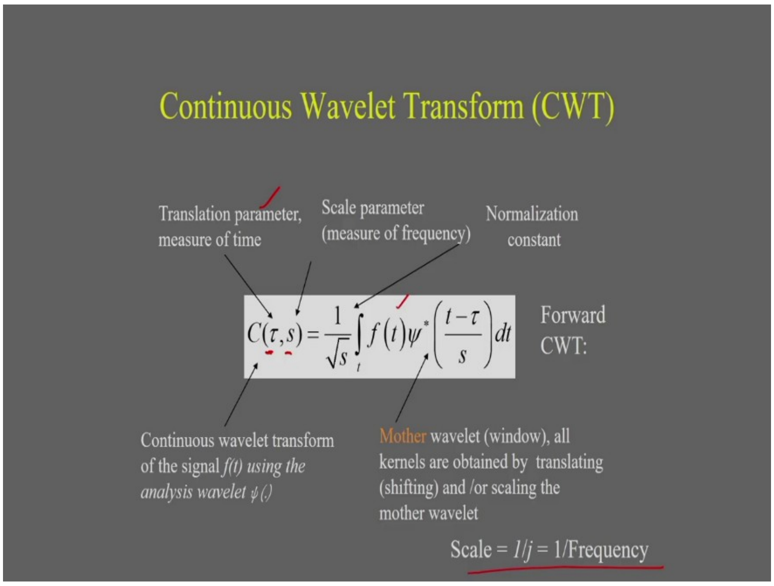

Now, let us consider the definition of the continuous wavelet transformation. So, C tau s is the continuous wavelet transformation. And in this case, I am considering the input signal, the input signal is f t and I am considering the mother wavelet, the mother wavelet is psi t minus tau divided by s. And in this case, the mother wavelet can be translated and it can be scaled. And in this case, what is the meaning of the scaling?

Scaling means it gives frequency information. So, scale is equal to 1 by frequency. So, in this case, you can see these parameters the tau is the translation parameter, and s is the scaling parameter that is nothing but the measurement of the frequency. So, this is the definition of the continuous wavelet transformation that is the forward continuous wavelet transformation of the signal, the signal is FT.

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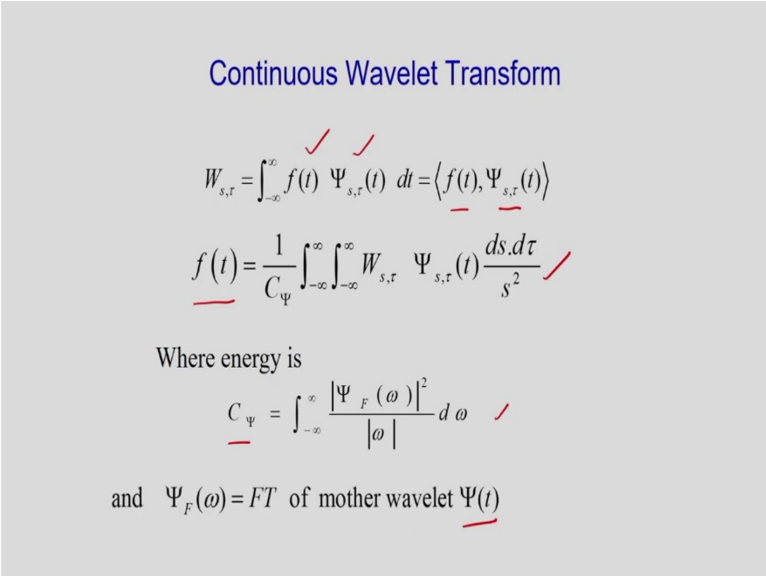

And you can see, this W s tau is the continuous wavelet transformation, f t I am considering and the psi s tau is the wavelet I am considering, wavelet function that is nothing but the inner for the between the f t and the wavelet function. And from this you can reconstruct the original signal by the inverse CWT the continuous wavelet transformation. So, inverse continuous wavelet transformation is obtained like this.

And in this case, C psi that corresponds to energy, the energy is given by this. And in this case, this psi F is the Fourier transform of the mother wavelet, the mother wavelet is the psi t. So this is the definition of continuous wavelet transformation and also we can determine the reconstructed signal that is the inverse continuous wavelet transformation we can determine.

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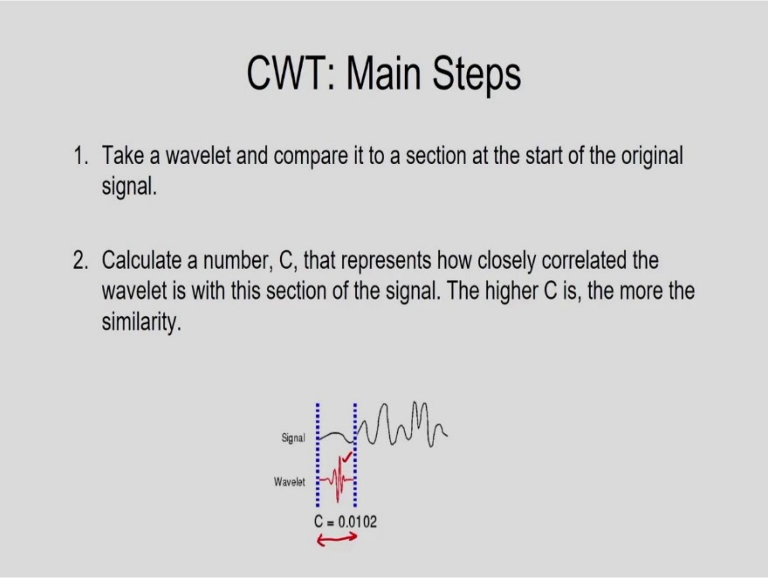

So, for CWT what are the main steps? You can see, take a wavelet and compare it to the section at the start of the original signal. So I am considering one wavelet, the wavelet is this. And one section I am considering and wavelet is this. Calculate a number C that represents how closely correlated the wavelet is with the section of the signal. The higher C is, the more the similarity.

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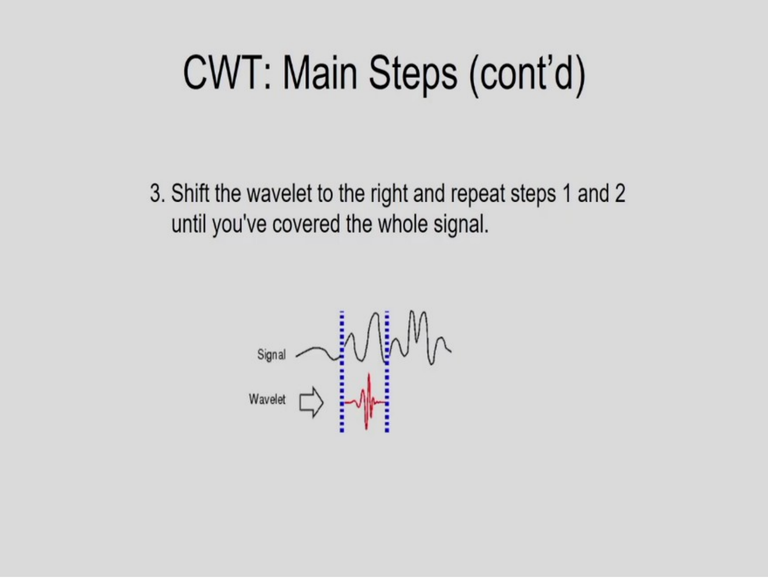

And after this, shift the wavelet to the right and repeat steps one and two, until you have covered the whole signal. So, these steps, I have to repeat for the whole signal.

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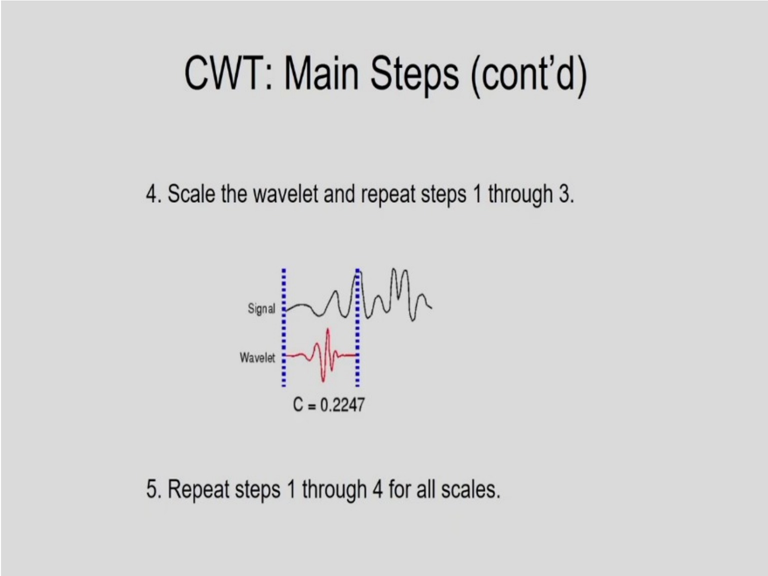

And after this, I have to do the scaling, the wavelet and repeat the steps 1 to 3. And finally, repeat steps 1 to 4 for all the scales. That means, I want to find a similarity between the wavelet and the signal at different scales. So this example I have given, corresponding to CWT, the continuous wavelet transformation.

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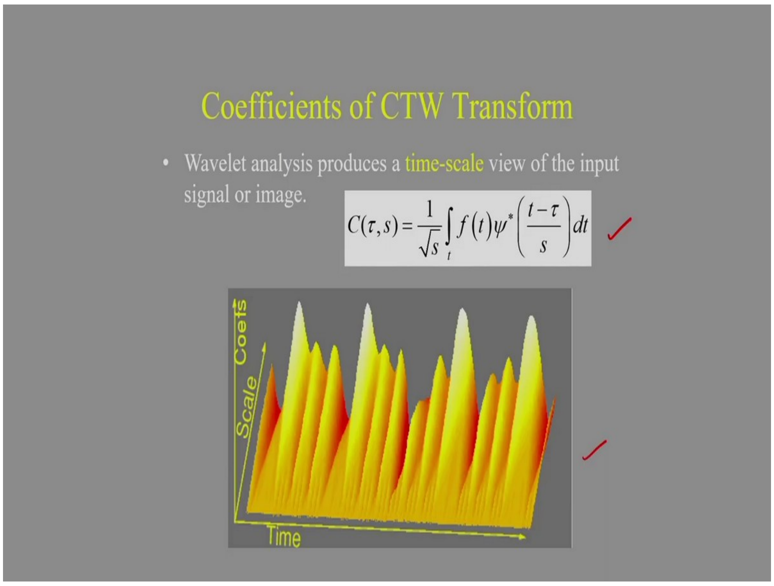

This CWT you can display like this. Already, I have defined the continuous wavelet transformation and CWT you can display like this. So we have the time information and CWT coefficients. You will be getting the CWT coefficients and the scale information is also available, this should be CWT.

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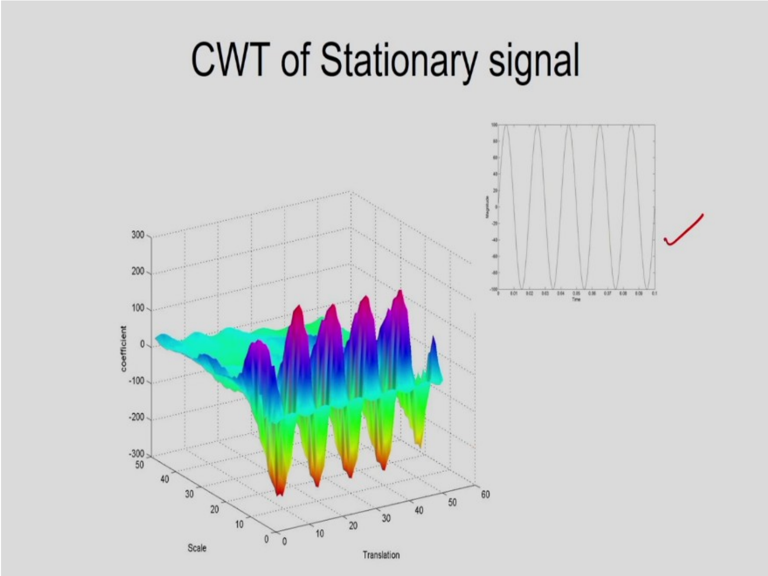

And in this case, it is a CWT of a stationary signal. So, if I consider this signal and this is the CWT of the stationary signal, so you can see here I have this information scaling information, the translation information, and the coefficients. Coefficients means, the continuous wavelet transform coefficients. The scale gives the frequency information.

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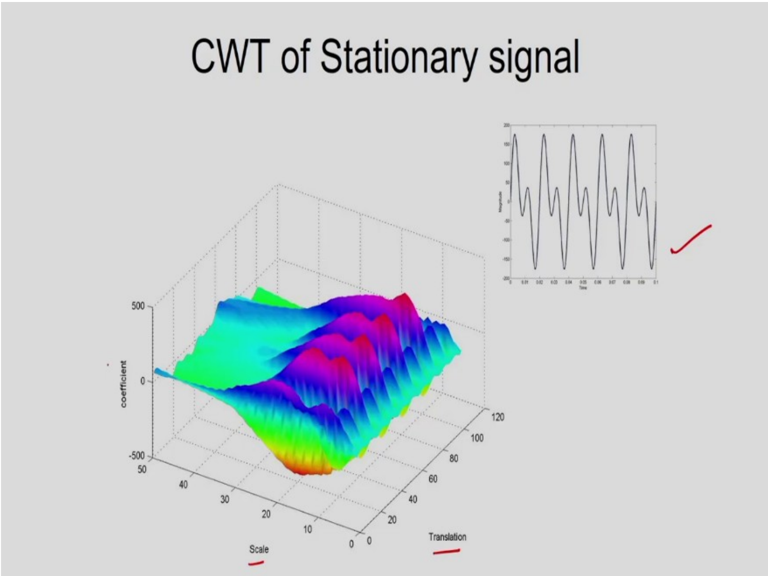

Similarly, I am considering the CWT of another signal and signal is this. And in this case, you can see I have the scale information, the translation information, and the coefficients of the CWT.

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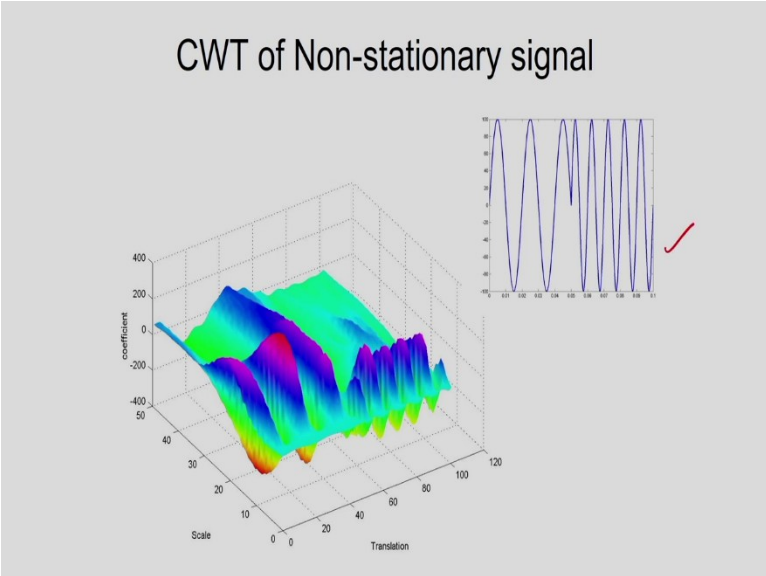

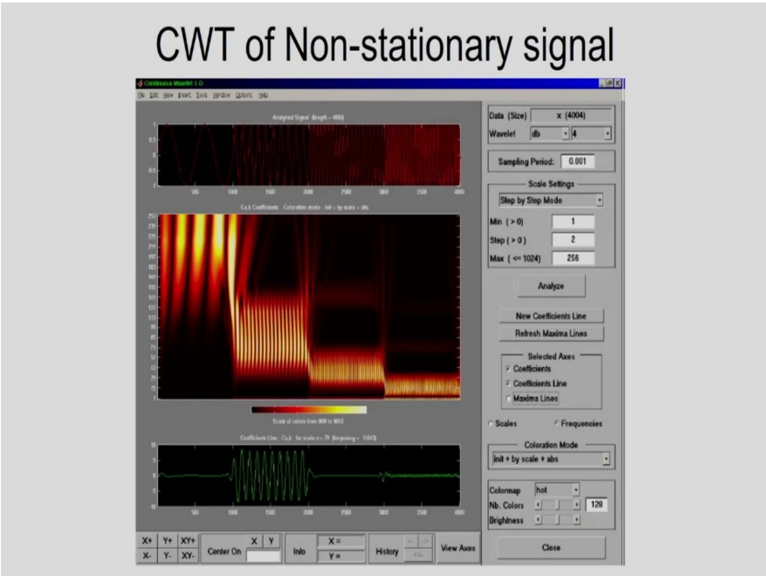

CWT of non-stationary signal, so I am considering one non-stationary signal and corresponding to this you can see the CWT. And in the MATLAB also, you can show the CWT of the non-stationary signal or maybe the stationary signal.

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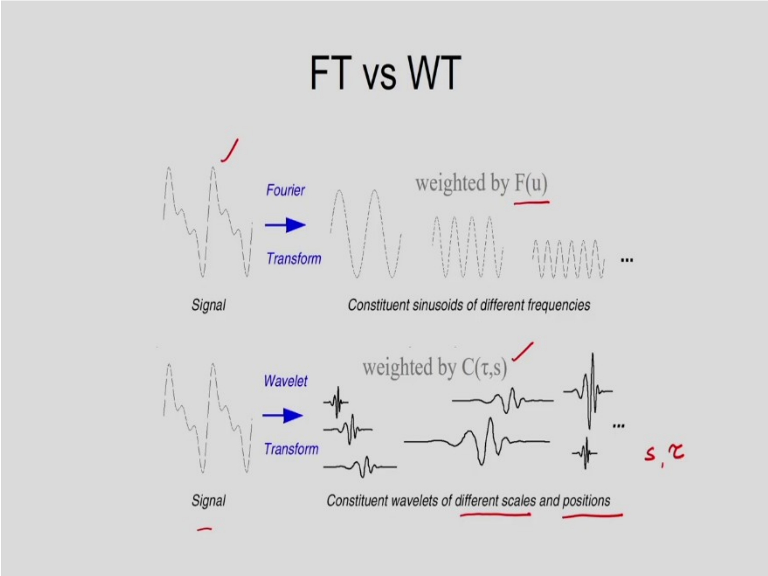

So, you can see the difference between the Fourier transform and the wavelet transformation. So, I have shown the signal, the signal is this and if I apply the Fourier transformation, that is nothing but the signal is weighted by F u, that is the Fourier transform of the signal, the signal is weighted by F u.

And in case of the wavelet transformation, I am considering this signal that means the signal is weighted by C tau s that means I am considering the continuous wavelet transformation. So, you can see the similarity between the Fourier transform the wavelet transformation. But in this case, in the case of the continuous wavelet transformation, we are considering different scales and positions, because I have two parameters, one is s another one is tau, one is the scaling parameter, another one is the translation parameter.

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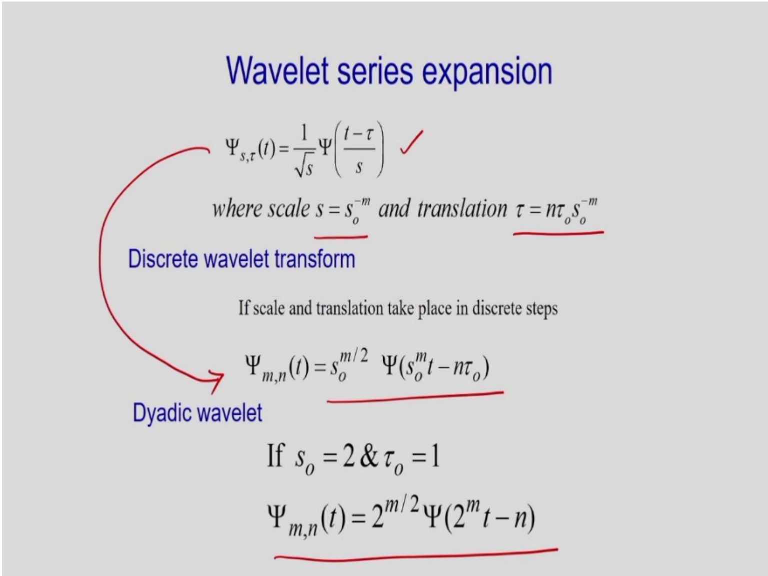

And for wavelet series expansion, so already, I have defined that wavelet like this and if I consider scale is equal to, s is equal to s naught to the power minus m and translation tau is equal to n tau naught s naught to the power minus m, so I am considering this one. And in this case, if I considered a discrete wavelet transformation, if the scale and the translation take place in discrete steps, then in this case, I can consider this expression as like this.

Because I am considering the discrete wavelet transformation, if the scale and the translation take place in discrete steps, then in this case, just I can write like this, if I consider s naught is equal to 2 and tau naught is equal to 1, then corresponding to this, I have this expression and this is called the dyadic wavelet. So, this is about the wavelet series expansion.

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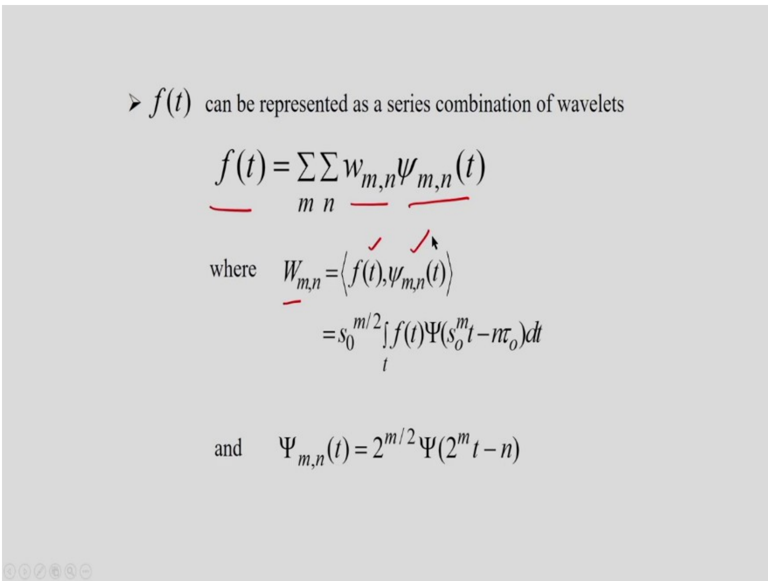

And in this case, the signal f t can be represented as a series combination of the wavelet. So, you can see the signal f t is represented as a series combination of the wavelets. So, I have the wavelet coefficients and this is my wavelet. So, the wavelet coefficients is nothing but the inner for the between f t and f t is the signal and psi m n is the tau wavelet, I am considering.

Next, I am considering multi resolution analysis. So, in this case, I am giving one example of an image. So, an image may contain the big objects or maybe the small-small objects or maybe the low contrast region or maybe the high contrast region, the good contrast region. So, if I want to see or if I want to analyze a particular image and suppose the small objects or presents, then in this case I have to consider high resolution.

Because in the high resolution, I can see small objects and also the regions of the bad contrast and if I want to consider the big objects or maybe the good contrast regions, then in this case I can consider low resolutions, that means, an image can be analyzed in different resolutions, maybe in the high resolution or maybe in the lower resolutions.

For different cases, I have I have given this example, for the big objects, I can consider low resolutions. For the good contrast region, I can consider the low resolution and if I consider small objects, then I have to consider the high resolution, if I consider low contrast region, then in this case also I have to consider high resolution. So, that means, at different resolution, I can analyze a particular image. And based on this concept, you can see here.

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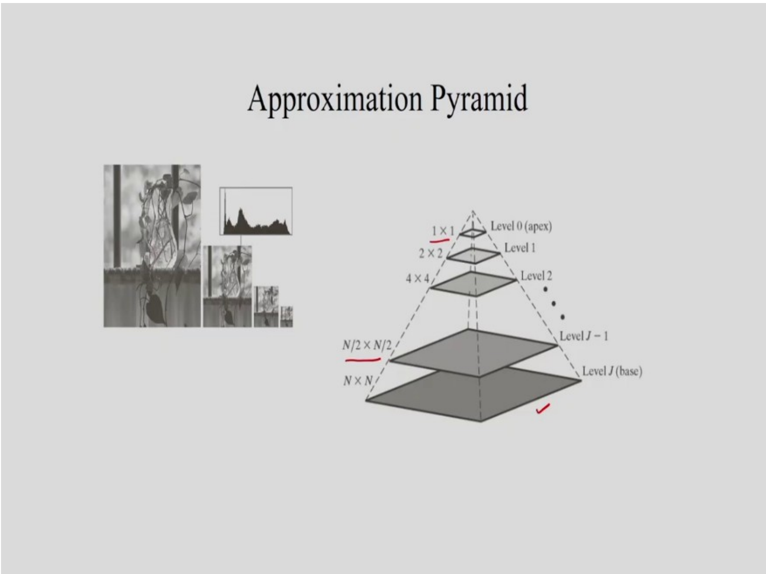

I am considering different resolutions for different image regions. So, you can see the N by N image I am considering that corresponds to the highest resolution and after this if you see this pyramid, the resolution is decreased. The next resolution is N by 2 cross N by 2 and finally, I have that 1 by 1 resolution. So, this is the approximation pyramid I am considering. So, an image I am considering or maybe the signal I am considering, and in this case, the signal or the image can be analyzed at different resolutions.

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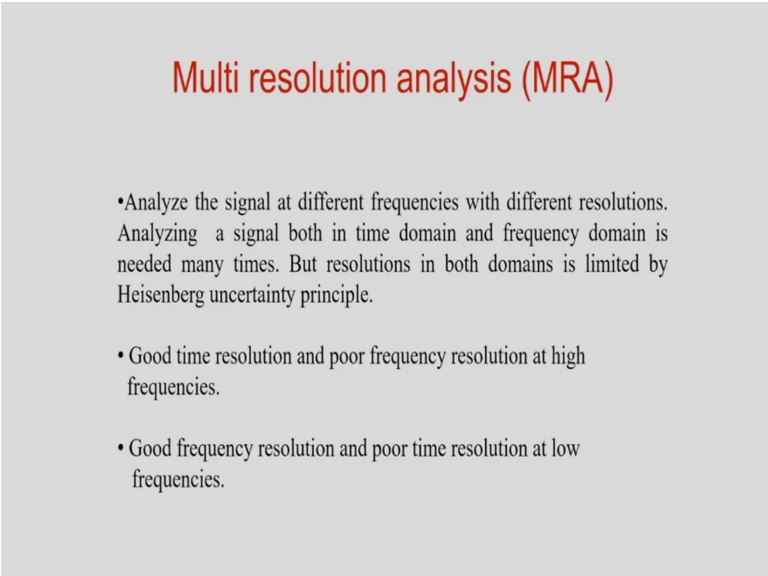

So, that is the objective of the multi resolution analysis. So, analyze the signal at different frequencies with different resolutions. Analyzing a signal both in time domain and frequency domain is needed in many times. But resolution in both domains is limited by the uncertainty principle. And already I have explained good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.

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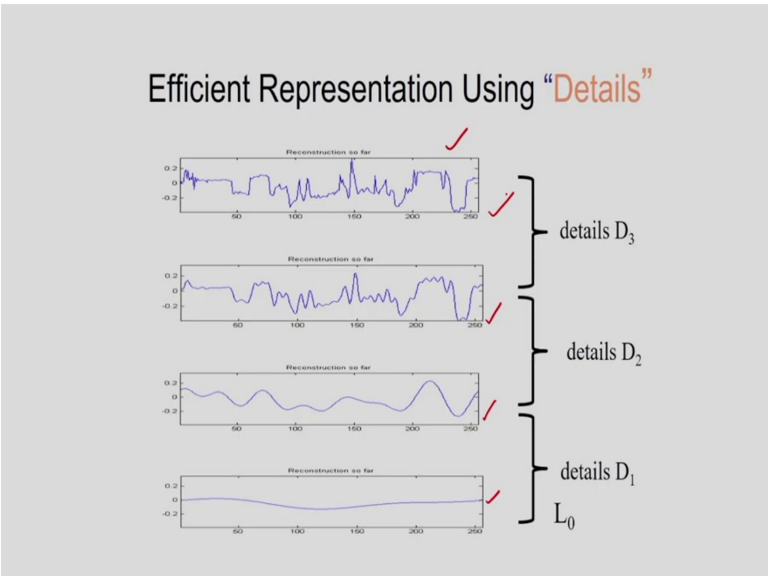

So, you can see this example here, I am considering one signal here and I am doing the reconstruction of the signal. So, this is my original signal I am doing the reconstruction from the low frequency component and the high frequency component. So, this is my low frequency information of the signal, this low frequency information is combined with the detail information.

The detail information is D1 and based on this I am doing the reconstruction. So, these are reconstruction so far. After this, I am again considering the detail information that is the high frequency information and this is the reconstruction I am doing. And after this, again I am considering more detailed information, then after this I am getting the reconstructed signal.

So, that means, with the low frequency signal, I am considering the high frequency information for reconstruction. So, a signal can be also decomposed into low frequency information and the high frequency information. In this example, I have shown the synthesis of the signal that is the reconstruction of the signal, but for analysis, I can decompose a particular signal into low frequency signal and the high frequency components. So, in this case I have shown the synthesis.

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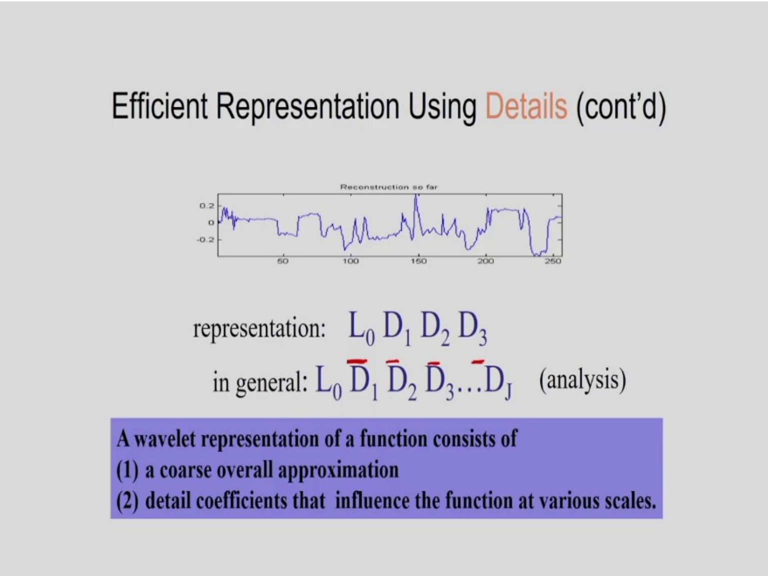

So, that means, a particular signal can be represented like this, the first one is the low frequency component. And after this, the D1, D2, D, so that is the detailed information. So, that means, in wavelet representation of a function consist of a course overall approximation that means the low frequency information. Next, I have the detail coefficients that influence the function at various scales.

So, the meaning of this discussion is the signal is represented by this, that is I have the low frequency information and after this we have the all the detail information of that signal that is for the analysis. For synthesis, for the reconstruction, we have to use the low frequency information and the detailed information we have to use.

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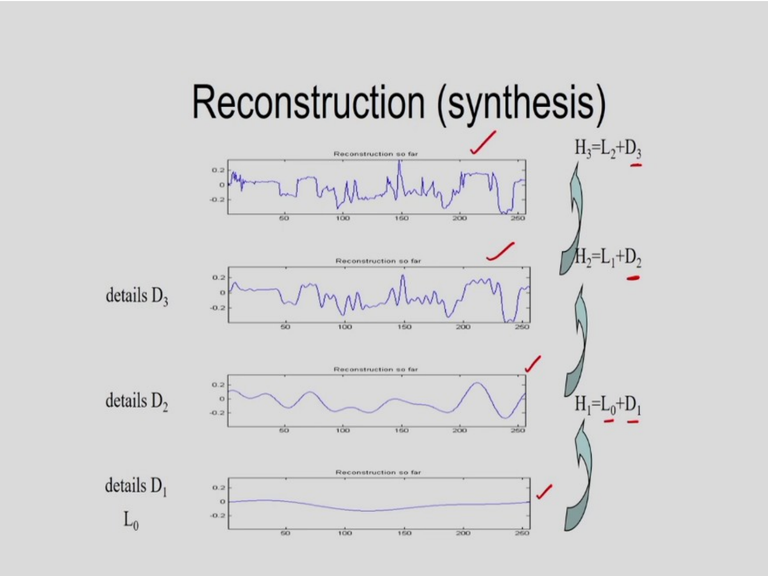

In this case, you can see here the same thing I am showing here. This is I am showing the low frequency information and the low frequency information is added with the detail information. So, I am getting the reconstructed signal and after it is the next I am considering the reconstructed signal and the detail information I am considering D2. So, I can do the reconstruction.

After this, again I am considering the detail information D3 and this is the reconstruction I am doing. So, already I have shown this one and that is the synthesis of the signal, based on the low frequency signal and the detailed information the detailed information is nothing but the high frequency information. So, I can give one example here. Suppose, how to consider the image? Suppose, if I consider one input.

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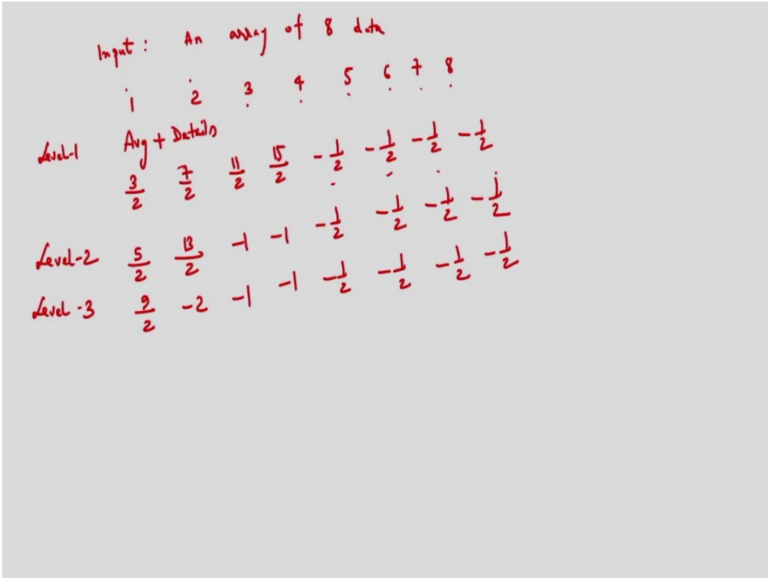

And I am considering an array of 8 data, so 1,2,3,4,5,6,7,8 like this and if I consider the level one approximation, then in this case, I can determine the average plus detail information I can determine. So, average will be 3 by 2, 7 by 2, 11 by 2, 15 by 2. And the details will be minus 1 by 2, minus 1 by 2, minus 1 by 2, and minus 1 by 2. So, you can see how to get the average. So, between this sample and this, I am determining the average. So, average I am getting 3 by 2, 3 and 4 I am considering.

So, average I am getting 7 by 2, 5 and 6 I am considering, the average I am getting 11 by 2 and 7 and 8 I am considering, the average is 15 by 2. And after this, I want to find a difference that is the details. So, 1 minus 2 divided by 2, so I am getting minus half, 3 minus 4 divided by 2, I am getting minus half like this, I am considering the detail information, so this is level 1.

Similarly, level 2, I can also determine the average component and the detail component. So, it is 5 by 2, 13 by 2, minus 1, minus 1, minus half, minus half, minus half and minus half. So, in level 2 also, I am determining the average, average between this and the this I am determining and also I am determining the difference between 3 by 2 and 7 by 2, 11 by 2 minus 15 by 2, so I am getting minus 1, minus 1.

And similarly, level 3 I am having this one average minus 2, minus 1, minus 1, minus half, minus half, minus half, minus half. So, in this case you can see, I am determining the average value and the detail information. In this class, I discuss the concept of the Fourier transformation and also I have highlighted the drawbacks of the Fourier transformation and after this I discussed the concept of the STFT and also I discussed the concept of multi resolution analysis.

So, in my next class, I will continue the same discussion, the multi resolution analysis and finally, I will discuss the discrete wavelet transformation. How to apply the DWT in an image, that concept I will explain in my next class. So, that is all for today. Let me stop here today. Thank you.