Advanced Control Systems

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Module No. # 01

Model Based Controller Design

Lecture No. # 09

PID-P Controller for Two Input Two Output System

Today's lecture is on PIP controller for two input, two output system. The control of two inputs, two output processes is difficult, and then the SISO processes mainly due to the interactions between the loops.

Two input, two output systems are from the family of multi input, multi output systems, many methods have been proposed earlier, in the literature for the design of controllers for two input, two output processes. Attempt will made in this lecture to design, simple PID-P controller a 4 parameter controller, for the two inputs, two output system. Now, we have got uses of such processes, in various real time systems in power plants, air craft, and chemical industries and in many other fields.

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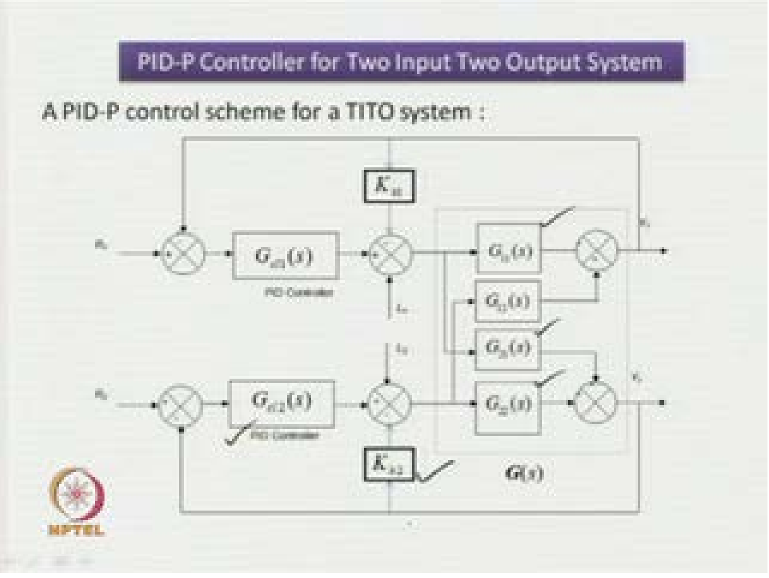

The uses of such processes are very common, now a days in real time scenario, what a two input, two output processes look like, the process is denoted by four sub systems, given as G 1 1 (s), G 1 2 (s), G 2 1 (s), G 2 2 (s). So, a two input, two output process can be repented in this form, now the control of the two input, two output system can be accomplished with the help of a PID controller, in the feed forward path, for the first loop a PID controller in the feed forward path for the second loop. We have inserted proportional controllers in the inner feedback path, so K b 1 is the proportional controller in the first loop, and K b 2 is the proportional controller in the second loop.

This PID-P control is a 4 parameter control, for the TITO system, one can have PI in the feed forward path, and PD in the feedback path as well, that case will not be discussed in today's lecture. We shall concentrate on the design of a PID-P controller rather in this lecture, for each in analysis one can represent this PID-B PID-P controller scheme in some convenient form, first we shall assume a specified form of the PID controllers.

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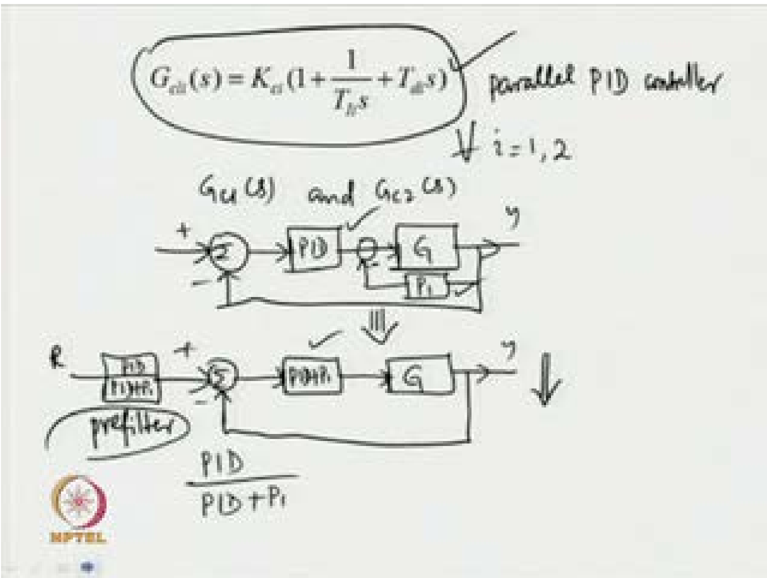

Let the PID controller be represented in the transfer function form, as given over here, this is a parallel PID controller, so depending on various values of I, for all i equal to 1 to 2, we shall have two controllers for the TITO system. So, the two controllers would be G c 1 s and G c 2 s, these are the feed forward path controllers. Now, we know earlier that, for a SISO system, when a PID controller is present in the feed forward path; a PID controller, then we has a single input single output process.

For that case, if we have some inner feedback proportional controller, let us say P 1 then the same structure can be reduced to some convenient form, for each in analysis and design of the controller. Now, let me redraw the block diagram, that will appear as PID plus P 1 in the feed forward path, with the single input single output process G and now, a pre filter is to be added and that pre filter will be given by PID upon PID plus P 1 and we shall have the reference input over here.

So, a single input, single output process with a inner feedback controller P 1 and a PID controller in the feedback path, can equivalently be represented in the single input, single output control scheme, where the PID plus P 1 will be in the feed forward path, with a pre filter in the reference path. And the pre filter dynamics will be given by PID upon PID plus P 1.

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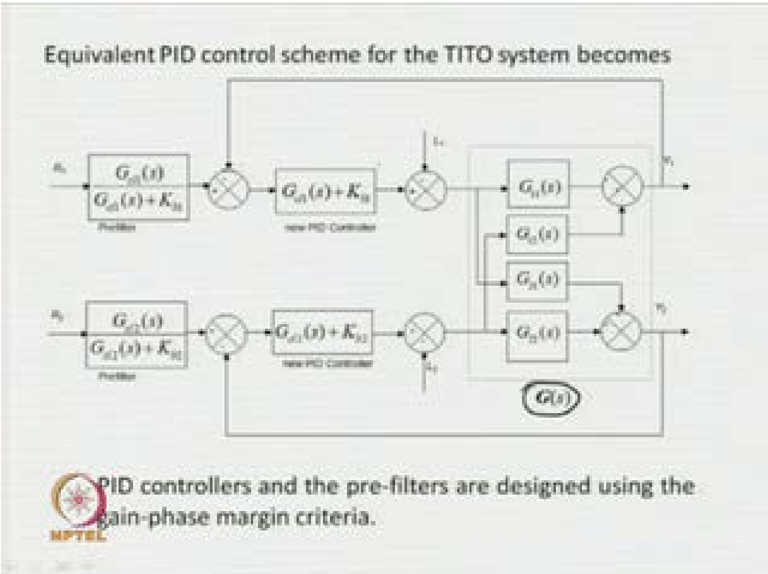

Using this equivalence, now it is possible to convert the TITO system, into some simpler form as shown over here, now G s represents the two input, two output process, which is now subjected to a new PID controller, where the controller PID's, the parallel PID controller is added with the proportional controller K b 1. And for the second loop, the new PID controller has got the proportional controller K b 2; now the scheme has got also pre filters, given by the upper one as G c l 1 s upon G c l 1 s plus K b 1.

And the bottom one pre-filter is given as G c l 2 s upon G c l 2 s plus K b 2, this scheme enables us to design the new PID controller; after designing the new PID controller, the parameters of the pre-filters can be set for controlling the TITO system. Now, effort will be made now to design, the new PID controller.

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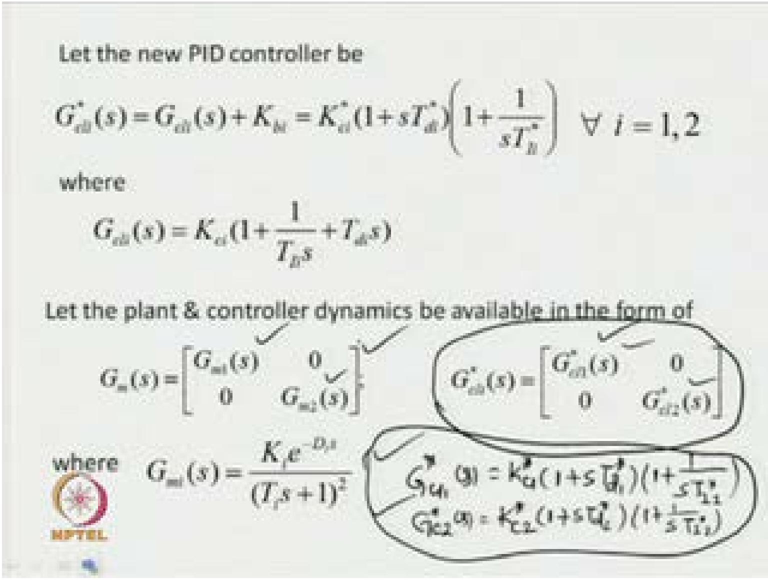

Let us assume the new PID controller, which is now available in the form of a series PID controller be given by G c l i s star is equal to G c l i s plus K b I, please note that K b i the proportional controllers are added thus giving us the new PID controller. G c l i s represents the parallel PID controller, where as the G the K b i are nothing but, the proportional controller. Now, the form of the new PID controller is given over here, which is a series PID controller with proportional gain K c i star, and derivative time constant T d i star and integral time constant T i i star.

So, for all values of i 1 to 2, we shall get two new PID controllers for the two input, two output process. Let the plant dynamics be, assume to be available in the form of G m s is equal to [G m 1 s 0 0 G m 2 s]; that means, the plant dynamics is represented by the diagonal transfer matrix G m s. Where G m i s is given as the elements of the transfer matrix are given by G m i s, which is equal to K i times e to the power minus D i s upon T i s plus 1 square, the plant dynamics is assume to have repeated poles repeated poles at minus 1 upon T i.

So, we have got repeated poles for the plant dynamics, the TITO process can be identified in this transfer matrix form; only when the TITO process, dynamics is available in this transfer matrix form, we shall be able to make use of the design process.

We are going discuss later on, for the design of controller for the TITO process, as we know this represents the time delays associated with the process dynamics. So, D i give us D 1 and D 2, where D 1 is the time delay associated with the G m 1 s, and G 2 is the time delay associated with the dynamics G m 2 s.

And T 1 and T 2 are the repeated poles or the time constants of the upper loop, and the lower loop of the TITO process. Now, after assuming that the dynamics of the TITO process is available, in this diagonal matrix form one needs to also define the dynamics of the controllers, new controllers available in the diagonal transfer matrix form. Now, G c l i star s is having elements G c n 1 c l 1 s star, and G c l 2 s star G c l 1 star s is nothing but, K c 1 star times (1 plus S T d 1 star times) (1 plus 1 upon S T over i 1 star) (Refer Slide Time: 11:59).

Similarly, we can find or write the expression for G c 2 star (s) as K c 2 star (1 upon 1 plus S T d 2 star times) (1 plus 1 upon S T i 2 star). So, the form of the new PID controllers are written over here, with these forms of the new PID controllers we shall now, try to estimate the parameters of the controllers using phase and gain margin criteria.

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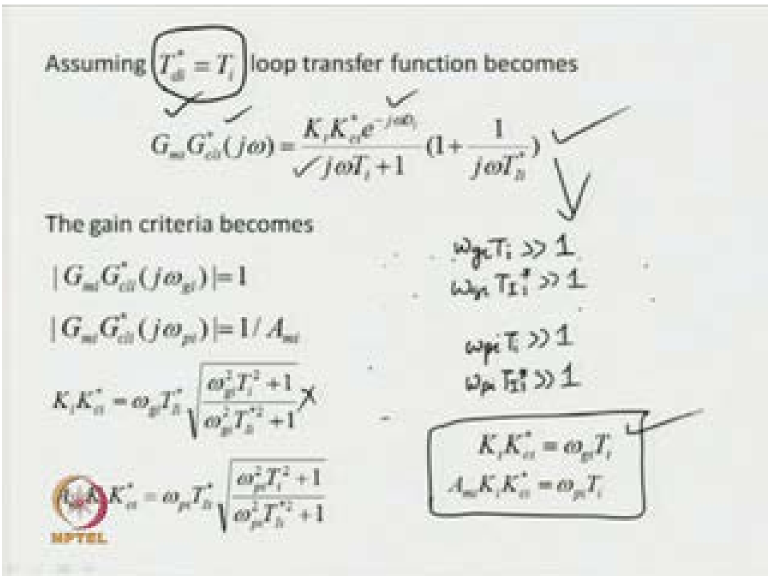

For that for each in analysis, let us assume that T d i star is equal to T i with that assumption, the loop transfer function becomes G m i G c l i star (j omega) is equal to K i K c i star e to the power minus j omega d i upon j omega T i plus 1 times (1 plus 1 upon j omega T I i star). So, the frequency domain representation of the loop transfer function is given in this form, then applying the gain criteria, it is possible to write expressions like, the magnitude of the loop gain is equal to 1 and the magnitude of the loop gain, at the phase crossover frequency is equal to inverse of the gain margin.

Now, omega G i in this expression represents, the gain cross over gain cross over frequencies similarly, omega P i in the second expression, is for the phase cross over frequencies. So, with the gain and phase cross over frequencies, it is possible to write the two equations related to gain of the loop; so loop transfer function gives us the two expressions, as shown over here. Now, the magnitude of the loop transfer function can be further written, in the form of K i K c i star upon (omega T i) square plus 1 root times 1 plus omega T I i star square root upon omega T I i star, so this gives the loop gain in absolute form.

So, the loop gain in absolute form can be represent represented in this form, but when we make use of the first expression, this expression further can be written in the form of K i K c i star is equal to omega g i T I i star root of omega g i square T i square plus 1upon omega g i square t I i star square plus 1. So, it is very easy to get this expression from this condition similarly, the second expression the second expression can be simplified and written in the form of A m i K i K c i star is equal to omega p i t I i star root of omega p i square T i square plus 1 upon omega p i square T I i star square plus 1.

Next, we shall make some assumptions, for each in designing controllers for the TITO process, what are those assumptions, one can easily assume omega g i T i to be large compared to 1. So, when omega g i T i is greater greater than 1, that gives us the numerator h omega g i T i only similarly, when omega g i T I i star is very large compared to 1, then the denominator of the square root becomes omega g i T I i star. Then this expression has got numerator and denominator like this, which can further be written in the form of K i K c i star is equal to omega g i T I, because of the cancellation of the terms.

Similarly, with the assumption of omega p i T i to be very large and omega p i T I i star is greater greater than 1, this assumption when made in this expression, results in the simpler expression given as A m i K i K c i star is equal to omega p i T i. Now, these values of gain and phase cross over frequencies can always be set with proper choices of the controller parameter.

Since, we have been trying to design the parameters therefore, these choices are valid, now with these choices of gain and phase cross over frequencies, it is always possible to get simpler equations like this from the gain criteria. So, the two gain criteria here, will be used later on to estimate, to find simpler expressions for the unknown parameters of the new PID controllers (Refer Slide Time: 19:03).

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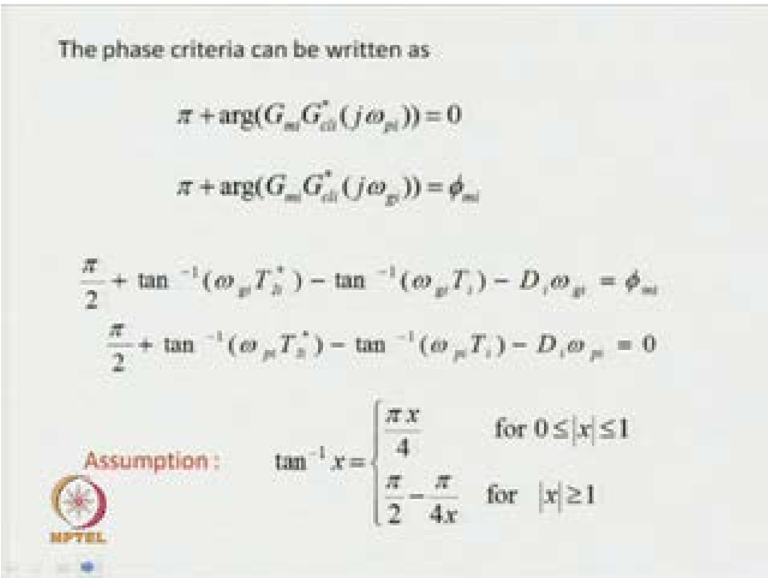

Now, the phase criteria can be written as, at the phase cross over frequency omega p I, when the point lies on the negative real axis, the phase cross over frequency gives us an expression like, pi plus arguments of G m i G c l i star j omega p i is equal to 0; so this is the loop gain in frequency domain. So, angle of the loop gain at phase cross over frequency plus pi must be equal to 0, because the phase cross over frequency points refers to a point on the negative real axis, where the angle is minus pi.

Similarly, the phase margin conditions gives us an expression pi plus argument of the loop argument angle of the loop at gain cross over frequency omega g i is equal to phase margin. So, omega g i is the gain cross over frequencies, and pi m i represent the phase margins phase margins of the loops. So, with the help of the assumption for the inverse tan function tan inverse as x is equal to pi x upon 4, when x is less than 1.

And when tan inverse x is equal to pi upon 2 minus pi upon 4 x for, x is very large than 1 is made use of in the phase criteria; obviously, one can get the simplified expressions like pi upon 2 plus tan inverse omega g i T I i star minus tan inverse omega g i T i minus D i omega g i is equal to phase margin.

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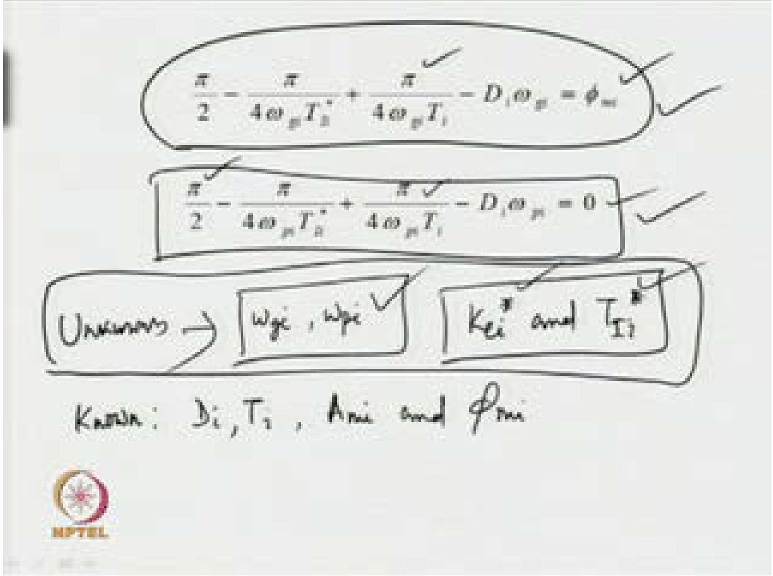

Now, one can substitute the tan inverse function over here, which will result in an expression of this form. So, tan inverse functions are substituted, which results in the expressions pi upon 2 minus pi upon 4 omega g i T I i star plus pi upon 4 omega g i T i minus D i omega g i equal to pi m I, where are we trying to substitute the tan inverse function because, it is very difficult to solve this non-linear equation (Refer Slide Time: 23:35).

So, unless one makes use of this assumption, it is very difficult to solve this non-linear equation similarly, for the second non-linear equation with the use of this assumption, the non-linear equation is obtained in the form of a linear one, not exactly a linear function rather I can say the non-linear function involving tan inverse has been simplified to a very simpler form. Which gives us pi upon 2 minus pi upon 4 omega p i T I i star plus pi upon 4 omega p i T i minus D i omega p i is equal to 0.

So, the two equations, we get from the phase criteria with the help of the of the arctan function, tan inverse function give us these two equation. Earlier, we have got two equations for the gain criteria, using the gain criteria we have obtain two simpler equations, which are shown over here (Refer Slide Time: 25:04). So, making use of the four equations now, four unknowns can be estimated, what are those four unknowns now, for all the unknowns are omega g i omega p I, and we have got the unknowns K c i star K c i star and t I i star, so these are the four unknowns; all others are known in the four equation.

What are known in the equations, the time delays D i's are known, so let us separate out the known and unknown parameters, so known parameters are D I, T i and A m i the gain margins and the phase margins pi m I, what are the unknowns unknowns are omega g i, omega p i the cross over frequencies. And the proportional gain K c i star and the integral time constants T I i star, so with the four equations, four unknowns can be evaluated instead of solving, a set of non-linear equations to evaluate the or to estimate the four unknowns, with little manipulation, it is possible to find explicit expression for the unknowns.

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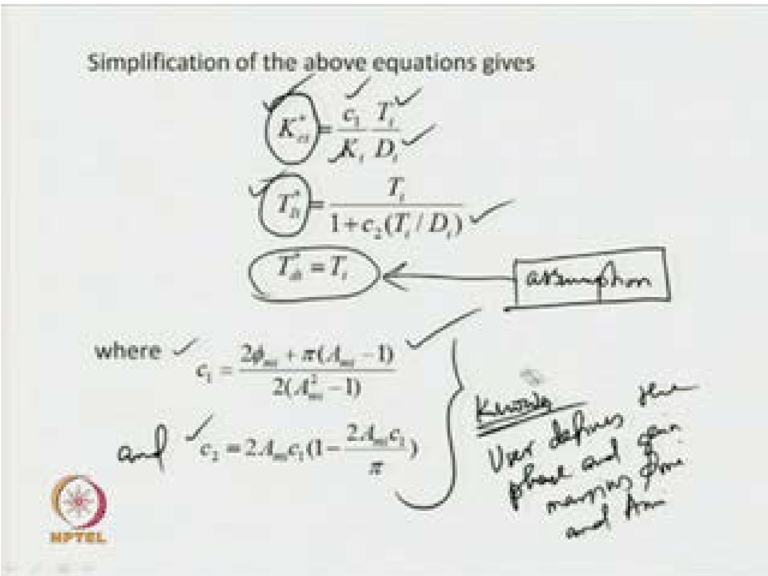

And those explicit expressions are obtained as K c i star is equal to c 1 upon K i T i upon D I, T I i star is equal to T i upon 1 plus c 2 times T i upon D I; and already we know that T d i star is equal to T I, this is the assumption we made at the beginning. So, with this assumption, we proceeded and we have been able to find explicit expressions for the two unknowns of the new PID controllers, K c i star, and T I i star. Now, in the expressions c

1 is equal to 2 phi m i plus pi times (A m i minus 1) upon 2 times (A m i square minus 1), and c 2 is equal to 2 A m i c 1 times (1 minus 2 A m i c one upon pi).

So, these are the two constants used in the expressions explicit expressions for the unknown parameters of the controllers. Now, what are known and unknown in this case, let us see again c 1 and c 2 are known to us, user defines, so one user defines the phase and gain margins phase and gain margins given as phi m i and A m i. So, these are known now, because user has to define this phase and gain margins, then c 1 and c 2 constants are known, c 1 known, K i known, T i known, D i known (Refer Slide Time: 29:09).

So, that way these explicit expressions will enable us to estimate the unknowns of the new PID controller, which are available in the series form; thus the PID controllers parameters are estimated. Now, we have seen that our TITO systems are subjected to a proportional controller, K b 1 for the first loop, and another proportional controller K b 2 for the second loop, so these are the two proportional controllers employed in controlling the TITO system.

Similarly, we have got the feed forward path PID controllers with parameters K c 1, T i 1 and T d 1 for the first loop, and K c 2, T i 2 and T d 2 as the parameters of the PID controller for the second loop (Refer Slide Time: 30:19). So, how to calculate back these parameters, actually our TITO process employs controllers with these parameters only, so one needs to calculate back, the parameters of the parallel PID controllers employed for controlling the TITO process.

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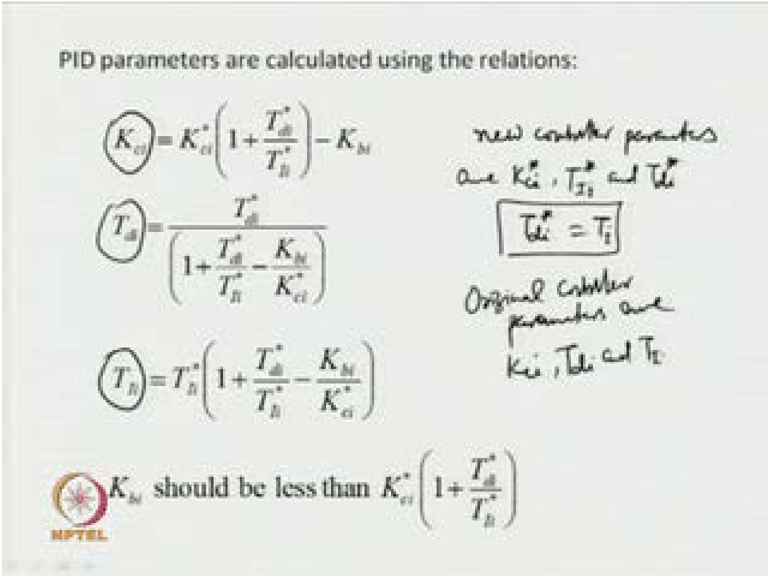

For that using simple calculations, one can find out these relations, now the parallel PID controller parameters K c I, T d i and T I i can be obtained from, the parameters of the new PID controllers, with these expressions, using these expressions.

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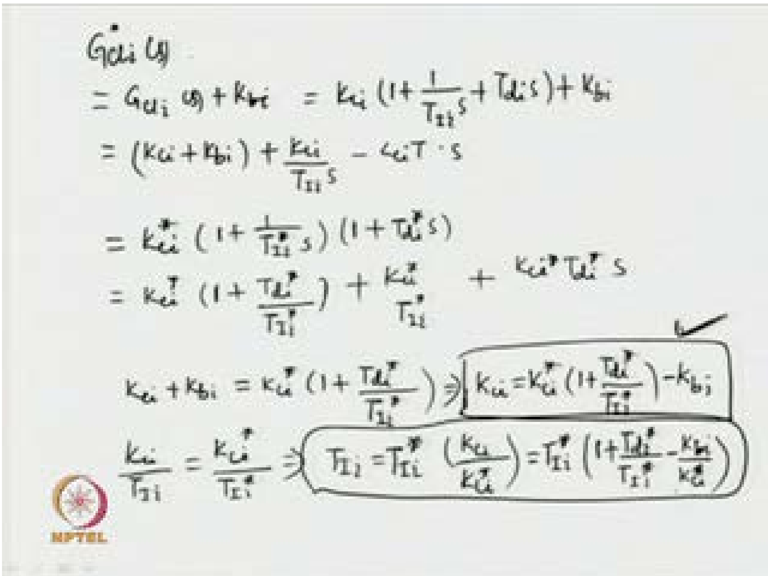

Now, let us not go to the last part, how are we getting these expressions, with each one can find out these relations starting with the new PID controller expression, which is given as G c l i star s is defined as G c l i s plus K b i which is equal to K c i times (1 plus 1 upon T I i s plus T d i s) plus K b i. So, let us simplify this expression which gives us, (K c i plus K b i) the 2 proportional controllers are clubbed together, and the remaining parts can be given as K c i upon T I i s plus K c i T d i s.

But, we have seen that G c l i star s is also given as K c i star times (1 plus 1 upon T I i star s) times (1 plus T d i star s), now this is the form of the new PID controller, which often expansion gives us K c i star times (1 plus T d i star upon T I i star) plus K c i star upon T I i star s plus K c i star T d i star s. So, one can compare these with the final form for the new PID controller, so upon comparison it is not difficult to get K c i plus K b i is equal to K c i star times (1 plus T d i star by T I i star), which again can give us the relation K c i is equal to K c i star times (1 plus T d i star by T I i star) minus K b I, so this is what exactly we have obtained earlier.

The the relation shown over here, the first relation gives us the expression which is K c i is equal to K c i star times 1 plus T d i star upon T I i star minus K b I, that is how we obtain the same relation (Refer Slide Time: 35:20). So, using other terms often comparison, we have to see that this term must be equal to this term, because we are comparing the dynamics of the transfer function form of the new controller, with that of the parallel PID controller with that of the parallel PID controller, and the additional proportional controller in the inner feedback path.

So, obviously, comparison of this term with this term gives us K c i upon T I i is equal to K c i star by T I i star, which again can be written as T I i is equal to 1 upon T I i star times K c i, no this T I i will go to numerator (Refer Slide Time: 36:49). So, T I i is equal to T I i star times K c i by K c i star, now using this expression the upper one, using this expression now I can write down T I i as T I i star times K c i divided by K c i star will give us 1 plus T d i star upon T I i star minus K b i upon K c i star (Refer Slide Time: 37:59).

So, this is what we have got earlier also, if you look at minutely, then in that case the T I i is equal to T I i star time 1 plus T I i T d i star upon T I i star minus K b i upon K c i star. And we have also obtained the exactly same relations from the analysis T I i is equal to T I i star 1 plus T d i star upon T I i star minus K b i upon K c i stat. Similarly, comparing further the remaining terms of the two realizations, it is not difficult to obtain further the relations for the, relations between the parameters derivative, time constant parameter of the two controllers as, as we have seen here this K c i T d i star.

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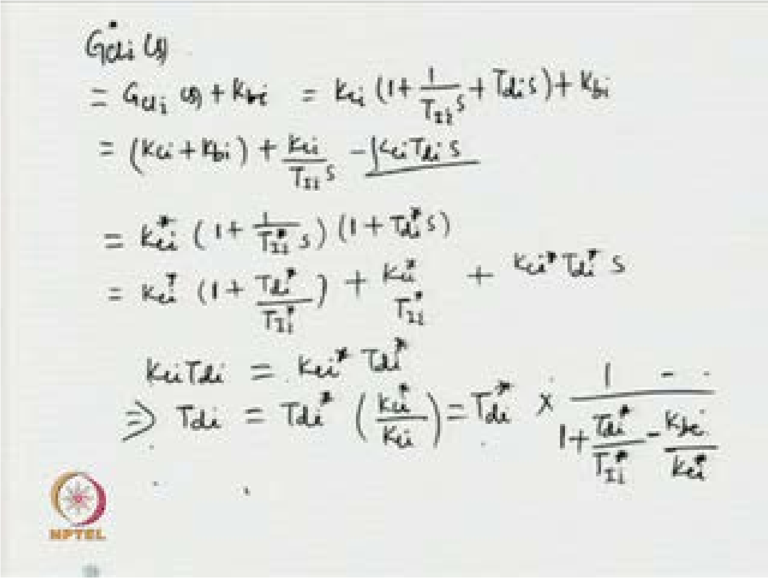

So, K c i T d i is equal to K c i star T d i star, which again gives us T d i is equal to T d i star times, now we will have (K c i star upon K c i) which is same as T d i star times 1 upon 1 plus T d i star upon T I i star minus K b i upon K c i star. So, these relations help us to calculate back the parameters of the original PID controllers that we had employed in the TITO control scheme. These are the original PID controller parameters, which are calculated from the parameters, obtained from the new controllers.

So, new controller parameters new controller parameters are K c i star, T I i star and T d i star, but we know that T d i star is same as T I, where as the original controller parameters are original controller parameters are K c I, T d i and T I I (Refer Slide Time: 40:19). So, using this this relations we calculate back the parameters of the original PID controllers, along with the parameters of the two proportional controllers for both the loops.

Now, what one can observe from these relations that K b i should be less than K b i should be less than this term otherwise, what will happen K c i K c i will be negative, when K c i will be negative similarly, T d i also will be negative, and T i i will also be negative (Refer Slide Time: 41:39). So, all these constants will be negative or unrealizable, which is not allowed in closed loop control systems, will assume negative values unless the first term is greater than the second term, so this factor is very important.

So, design of K b i depends on the values for T d i star T I i star and K c i star, so all though we might have got design values for K c i star T I i star and T d i star, those might not be acceptable to us, unless these condition is satisfied, unless this relation holds unless K c I, T d i and T I i are positive, they must be positive (Refer Slide Time: 42:50). So, K b i is generally designed juristically, after designing the values for the new control parameters, new controller parameters then only the values for K b i are chosen.

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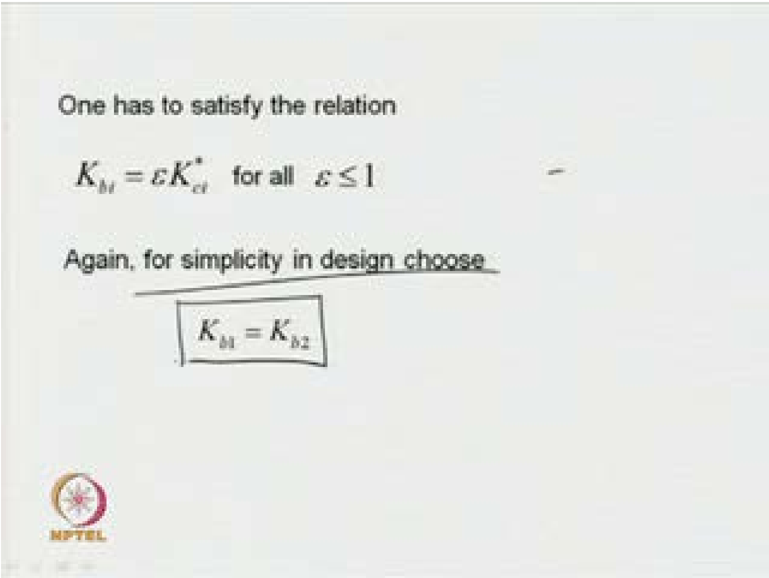

Next, using the same relation I can say K b i is equal to epsilon times K c i star, now epsilon has to be less than 1, and that guarantees that all the parameters of the PID controllers will be positive. So, simply designing the parameters of the new controllers, may not ensure desired control of the TITO system, in that case one must ensure this condition to get all the positive values, all the parameters to be positive.

That means, all the parameters of the new, as well as the original PID controllers to be positive this condition must hold. Again for simplicity in design generally, one chooses K b 1 is equal to K b 2, so with this choice it is easy to design all the parameters of the controllers employed in the TITO system, next we shall go to simulation study.

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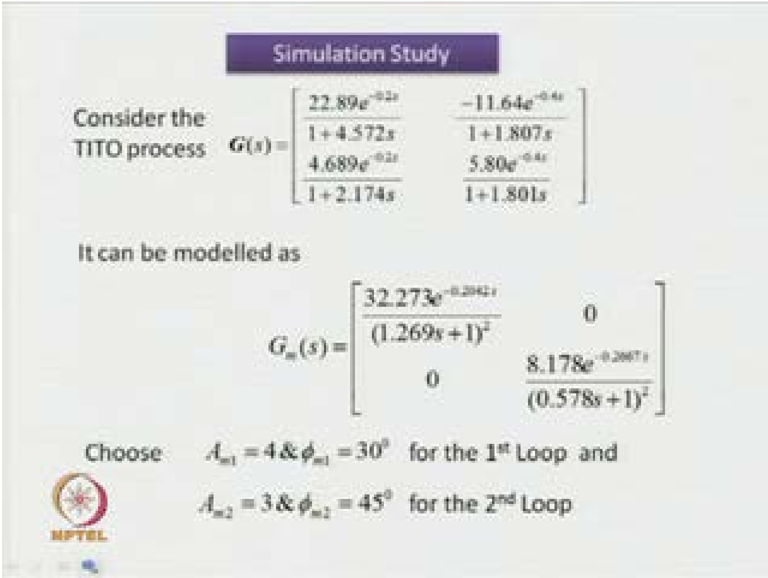

Let us consider a TITO process dynamics, given by this transfer function matrix G (s), which has got four elements and each four elements are nothing but, the first order plus dead time representation. So, we have got enough interaction in the TITO system, the two loops of the TITO systems have got significant interactions; now the dynamics of the TITO process, with the help of proper identification procedures, may be obtained in this form.

So, the model transfer function matrix for the TITO process is obtained in the diagonal form, where the diagonal elements are given as 32.273 e to the power minus 0.2042 s upon 1.269 s plus 1 square. And the second diagonal element is 8.178 e to the power minus 0.2667 s upon 0.578 s plus 1 square; off diagonals elements are assumed to be 0. Next, choosing the gain margin of 4, and phase margin of 30 degree for the first loop, and gain margin of 3 and phase margin of 45 degree for the second loop.

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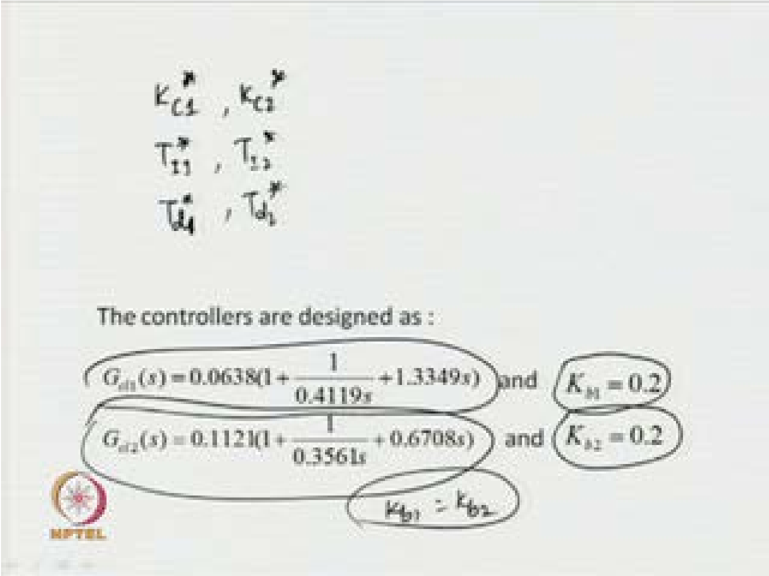

Controller has been designed initially, we get the designed values for K c 1 star, K c 2 star, next T i 1 star, T i 2 star and subsequently T d 1 star, and T d 2 star, then using the relations just, now we had discussed using these relations, the controller parameters of the original controllers are obtained as this, and this. Of course, with the choice of K b 1 is equal to 0.2 and K b 2 is equal to 0.2, as we have said K b 1 is equal to K b 2enables us for each in design.

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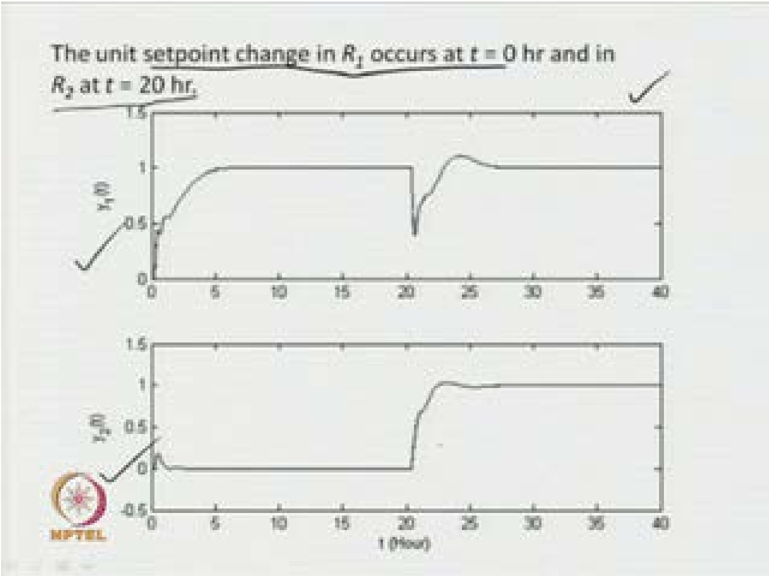

Then with the controllers in the loops, the performances of the TITO system are obtained as shown over here, the first loop has got a set point change, unit set point change at time T equal to 0 hour. And the second loop has got a unit set point change, at time T equal to 20 hours, then the responses of the two loops are shown over here, the upper loop response is given here, and the second loop is giving a time response of this form.

The responses shows us acceptable settling time over shoot and speed of responses, so we have got quite satisfactory, over all responses improved performances from the PID-P scheme. So, splitting the P control helps us in obtaining satisfactory performance of the TITO system.

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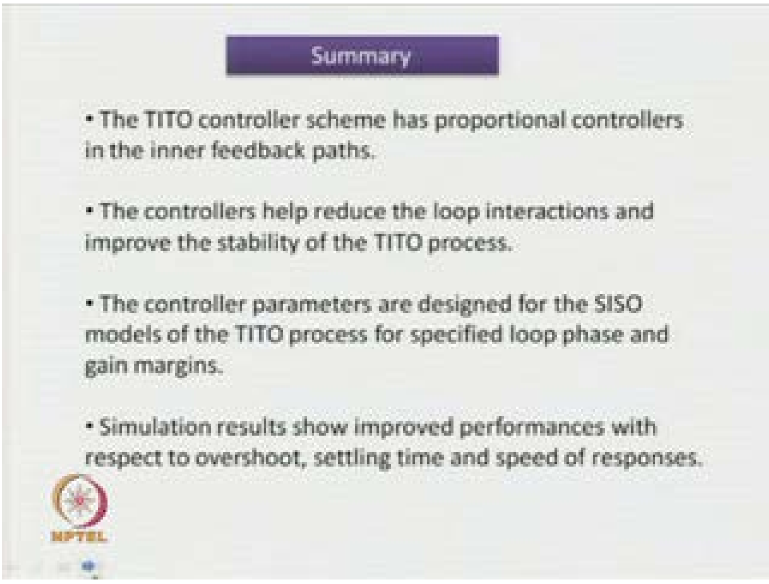

In summary, we can say that the TITO controller has proportional controllers in the inner feedback path, and the proportional controllers have got significant roles in improving the performance of the TITO system. Now, not only the inner feedback proportional controllers, help in improving the performances, rather they help in reducing the loop interactions, and the stability of the TTIO processes; the controller parameters are also designed for the SISO models of the TITO process, for some specified loop phase and gain margin.

Basically, the design technique available for SISO models, have been employed to design controllers for the TITO process, and simulation results of course, show us improved performances with respect to over shoot settling time, and speed of responses.

Although comparison has not been given, often it is found that the PID-P controller results in satisfactory performances, compared to PID controllers alone in the loops.

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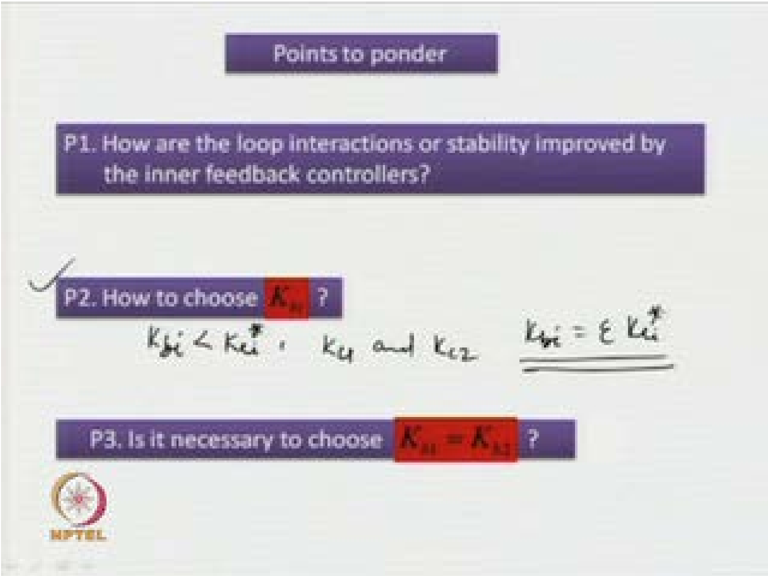

Now, certain points to ponder on this TITO system control are the first point could be how are the loop interactions or stability improved by the inner feedback controllers; inner feedback controllers, enable us to place the poles of the loops, at some desired locations. So, placing the poles of the loops, at some desired locations helps us not only in designing perfect controllers for the system rather that helps also in loop interactions.

Obviously, the controllers play a role, when the poles of the loops are placed at some desired locations; the loop interactions automatically get reduced. And stability of course, can always be improved with the help of providing inner loop controllers or inner controllers in both the loops. Now, second point is how to choose K b I, now as we have seen this K b i has to be less than K c i star star, so first obtain the design values for K c 1 and K c 2. And based on that using the condition that K b i is a function of epsilon times K c i star and epsilon is less than 1, using that condition the K b i values are chosen.

Next point, is is it necessary to choose K b 1 is equal to K b 2 for designing controllers, for the TITO system, when we design PID-P controllers for TITO system often, it has been found from externship simulations that, the choice of K b 1 is equal to K b 2 results in improved performances of the loops. But, suitable choices of choices of K b 1 and k b

2 also can lead to significant reduction in loop interactions, and stability of the lowest loops, that is all in this lecture.