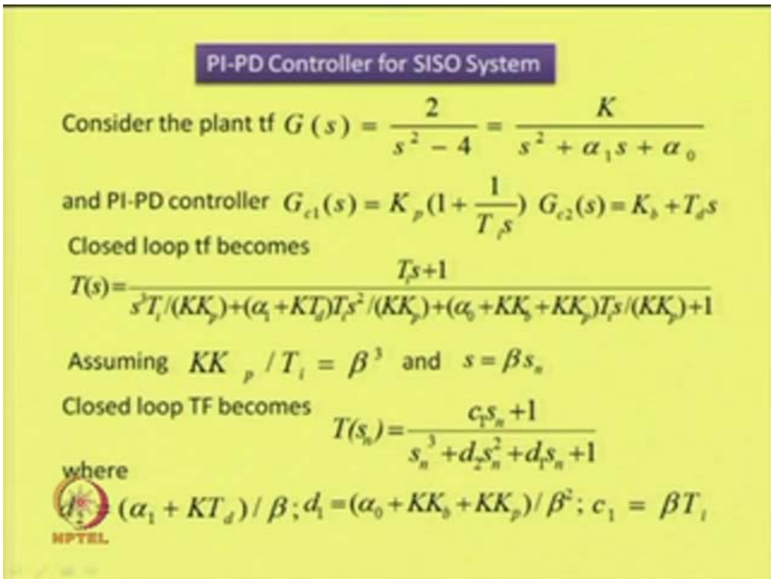
Advanced Control Systems
Prof. Somanth Majhi
Department Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati

Module No. # 01
Model Based Controller Design
Lecture No. # 08
PI-PD Controller for SISO System

Welcome to the lecture on PI PD controller for SISO system. Earlier in our earlier lecture, we have seen the limitation of PID controller, attempt will be made to design PI PD controller by two methods in this lecture.

(Refer Slide Time: 00:43)

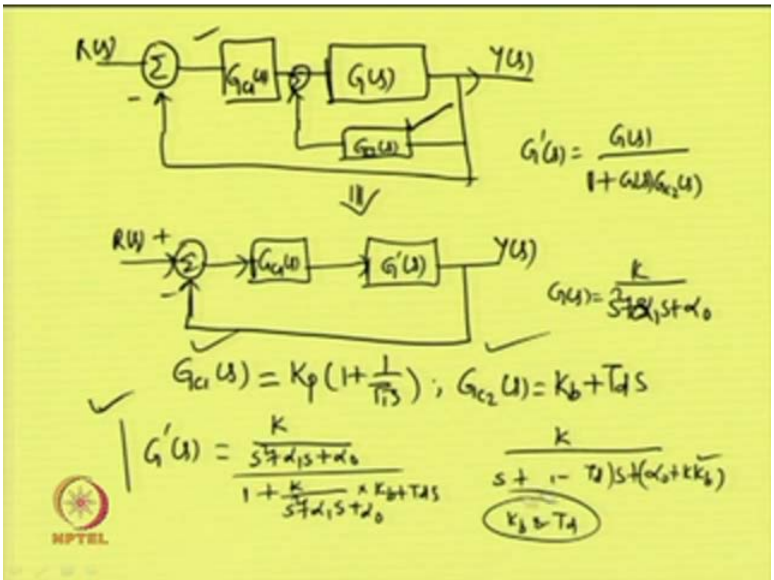


Initially, we shall consider a plant transfer function of the form shown over here, which is having a numerator K and denominator s square plus alpha 1 s plus alpha 0. This all pole transfer function, can assume different form depending on the values of alpha 1 and alpha 0. Now, when alpha 1 becomes 0, we get a second order integrating process, when alpha 1 equal to 0 and alpha 0 equal to 0, we get second order integrating process.

Now, depending on the values of K also, we get the plant dynamics to be stable or unstable, but we shall assume a positive K of for all the cases. Now, we shall design a PI

PD controller, which in block diagram form can be given as a PI controller having G c1 s and the process dynamics given by G s output denoted by Y s and reference input by R s.

(Refer Slide Time: 01:49)



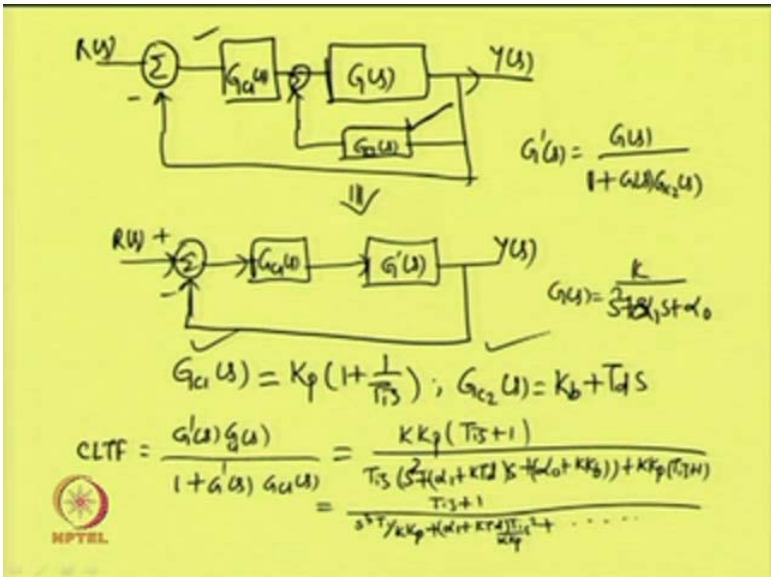
And we have the feedback path, a negative feedback over here; again a controller in the inner feedback path given as G c2s is there. Now, this block diagram representation shows us the representation for a series feedback control scheme, this can equivalently be represented in the form of R s Gc1s with a modified process given by G dash s and Y s as the output. Now, G dash s is given by G s upon 1 plus G s G c 2 s, this modified process is been now controlled by a feed forward controller G c1 s. Let us assume the form of the controllers to be of G c1 s a PI controller given by K p1 plus 1 upon T i s and G c2s a PD controller given by K b plus T d s. Then, we have got a PI PD control structure, where we have got the feed forward path as well as inner feedback path controllers G c1s and G c2 s, respectively.

Now, for the all pole system or plant transfer function K upon s square plus alpha 1 s plus alpha 0, when the PI and PD controller are employed, then the closed loop transfer function becomes T s equal to T i s plus 1 in the numerator divided by s cubed T i upon K Kp plus alpha 1 plus K Td times T i s square upon K Kp plus alpha 0 plus K Kb plus K Kp times T i s upon K Kp plus 1.

How do we get that closed loop transfer function? If we substitute G s equal to K upon s square plus alpha 1 s plus alpha 0 and the form of the G c1 s and G c2 s, then the closed loop transfer function can be obtained as shown over here. Now, G dash s, G dash s can be obtained as K upon s square plus alpha 1 s plus alpha 0 divided by 1 plus K upon s square plus alpha 1 s plus alpha 0 times K b plus T d s, which can be simplified to the form of K upon s square plus alpha 1 plus K T d s plus alpha 0 plus K Kb, the role of the inner feedback controller can be apparent from this G ds.

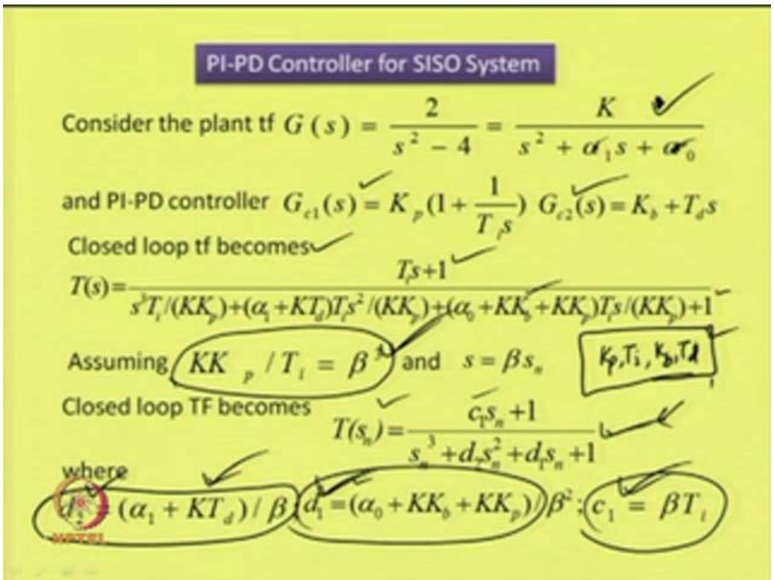
Now, we see that with suitable choice of K b or design value of K b, it is possible to locate the poles of the modified transfer function at suitable locations. That is the benefit one gets by employing the inner controller G c2s. Had there been no G c2s, there is no scope for placing the poles of the original process at desired locations. So, K b and T d now enables one enables one to locate the poles of the modified process or the original process at suitable locations with the help of G c 2s.

(Refer Slide Time: 07:18)



Now, the inner loop transfer function is to locate the poles of the modified process at desired locations, where as the closed loop transfer function will give us in the form of closed loop transfer function. Now, as G dash s G s G c1 s upon 1 plus G dash s G c 1 s, so which upon substitution will give us in the form of K Kp T i s plus 1 over T i s times s square plus alpha 1 plus k T d s plus alpha 0 plus K Kb plus K Kp T is plus 1. Now, this can further be simplified as T i s plus 1 in the numerator upon s square s cubed T i by K Kp plus alpha 1 plus k T d T i upon K Kp s square plus other terms, that we have got here.

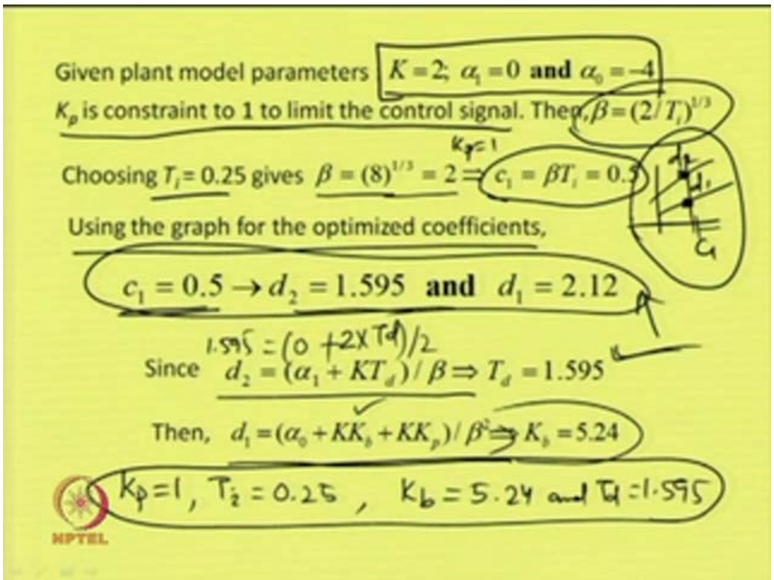
(Refer Slide Time: 08:42)



So, the closed loop transfer function using the modified dynamics, process dynamics G dash s gives us a transfer function with numerator T i s plus 1 and denominator as shown over here. Now, assuming K Kp upon T i to be as beta cubed and s equal to beta s n, the closed loop transfer function can be expressed in the standard form given as T s n equal to c 1 s n plus 1 divided by s cubed plus d 2 s n square plus d 1 s n plus 1, where d 2 is alpha 1 plus K t d upon beta d 1 is equal to alpha 0 plus K Kb plus K Kp upon beta square and c 1 equal to beta T i. Now, the closed loop transfer function has been obtained in the form of a standard transfer function of third order.

Now, if c 1 is given or for a given c 1, we have got definite values of d 2 and d 1 as far as the third order transfer function is concerned. Optimization of ISTE criterion results in optimum values of c 1 d 2 and d 1 for the third order transfer function. Now, using this we have got now four inequalities, now d 2, d 1, c 1 and K Kp upon T i equal to beta cubed. These 4, the 4 expressions, the four expressions 1, 2, 3and 4, the 4 encircled expressions can be used to estimate the controller parameters K, Kp, T i and T d; k p, T i k b, T d are the 4 unknowns in the controllers. So, both controllers G c 1 s and G c 2 s has has got the unknowns K p, T I, K b, T d with the help of the four expressions or four inequalities, it is possible to estimate the four unknowns k p, T I, k b, T d either by solving a state set of linear equations or by employing some convenient technique. Now, let us try to design the controller parameters for a given process model.

(Refer Slide Time: 11:58)



Given plant model parameters k equal to 2, alpha 1 equal to 0 and alpha 0 equal to minus 4, attempt will be made now to design the 4 controller parameters K p, T I, K b and T d. Now, how the process model looks like? Now, G s is equal to 2 upon s square minus 4, so we have got a very difficult process, difficult in the sense the process has got one pole in right half of the s plain. Now, I can write this in the form of s minus 2 and s plus 2 plus 2, so we have a pole located in the right half s plain, therefore we have got an unstable process.

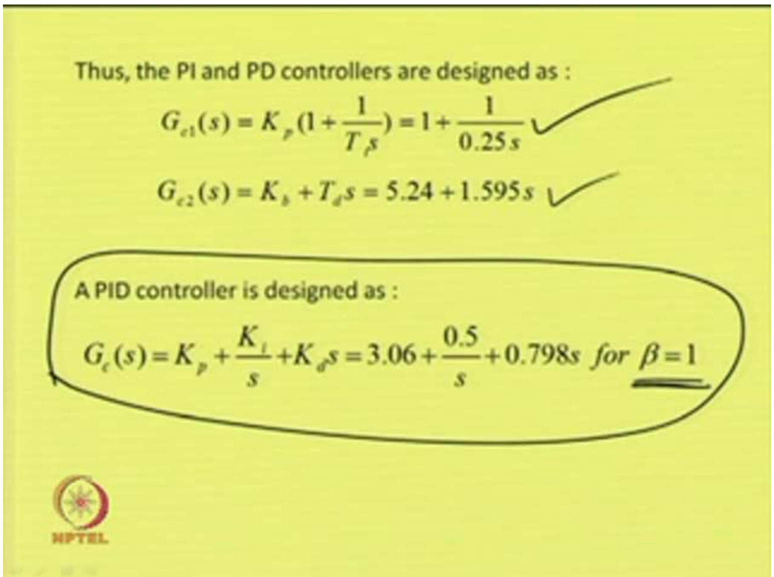
Now, for these processes to design the PI PD controller, let us assume K p to be 1 to limit the control signal, when the proportional controller magnitude is 1, that time the control signal will have less excursion that is why we are assuming K p to be 1. With that assumption of K p equal to 1, as we see when K p equal to 1, this expression will give us when K p equal to one beta cube tool equal to K upon T i. Therefore, beta is equal to 1 upon T i to the power one upon 3, choosing T i equal to 0.25 gives us beta equal to cube root of 8 that is equal to 2.

Now, when beta equal to 2 and T i equal to 0.25, c 1 is equal to beta times T i is equal to 0.5, once we open c 1, obtain c 1, then using the graph for the optimized coefficients we have got the plots for c 1 versus d 2 and d 1 as we have seen earlier. So, using the value of c 1, it is possible to obtain the optimum values of d 2 and d 1 for optimum scare response performance of a closed loop system. Now, c 1 equal to 0.05 gives us d 2 equal to 1.595 and d 1 equal 2.12, these values are obtained from minimization of the ISDE criterion.

Now, d 2 is further expressed as alpha 1 plus K T d upon beta , which gives us T d equal to 1.595, because alpha 1 is known , which is equal to 0 and d 2 is 1.595, so this is equal to plus K t d, k is 2 2 times T d upon beta, so beta beta is equal to 2, so thus we get T d equal to 1.595. Similarly, d 1 is estimated from the expression alpha 0 plus K Kb plus K Kp upon beta square is equal to that gives us K b equal to 5.24.

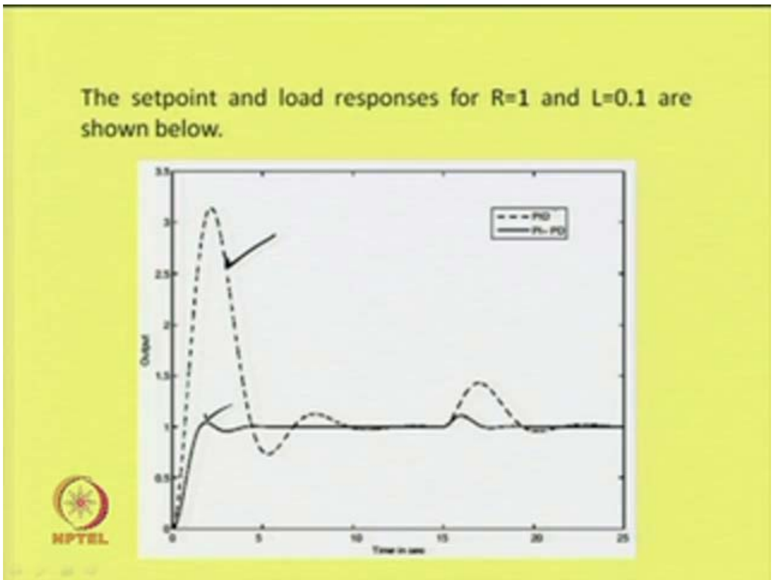
Thus the design values of the PI PD controller are obtained as K p equal to 1, T i equal to 0.25, K b equal to 5.24 and T d equal to 1.595. So, applying some intuition, it is possible to estimate or design the parameters of a PI PD controller, one can reset to many techniques to obtain the PI PD parameters as we have discussed earlier, one can solve the set of equations to obtain the four unknown from the four expressions.

(Refer Slide Time: 16:38)



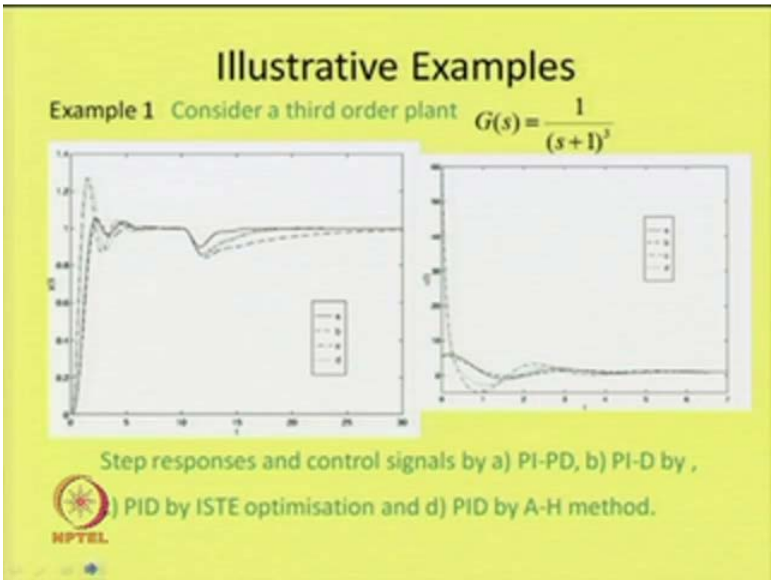
Now, with these values of the PI PD controller, let us see the performance we get from the closed loop system. Thus the PI controller is of this form and the PD controller of this form, for comparison we have also designed a PID controller for beta equal to o1. We can have the same beta, it does not matter respective of any value of beta, it is possible to design a PID controller using standard form.

(Refer Slide Time: 17:15)



Now, the comparison of results are given here, when a unit step input is applied and a load disturbance of magnitude 0.1 is applied to the closed loop system, the output of the closed loop system is found to be of this form in the case of the PI PD controller and of this form for the PID controller. As evident from the responses, the PI PD controller is giving a quite satisfactory closed loop performance for both set point as well as load disturbance inputs, as the over shoot settling time when compared with the PID controller are found to be very very small and the overall performance of the closed loop system as far as the PI PD controller is concerned is found to be extremely improved one and desirable one as far as the PID controller is concerned.

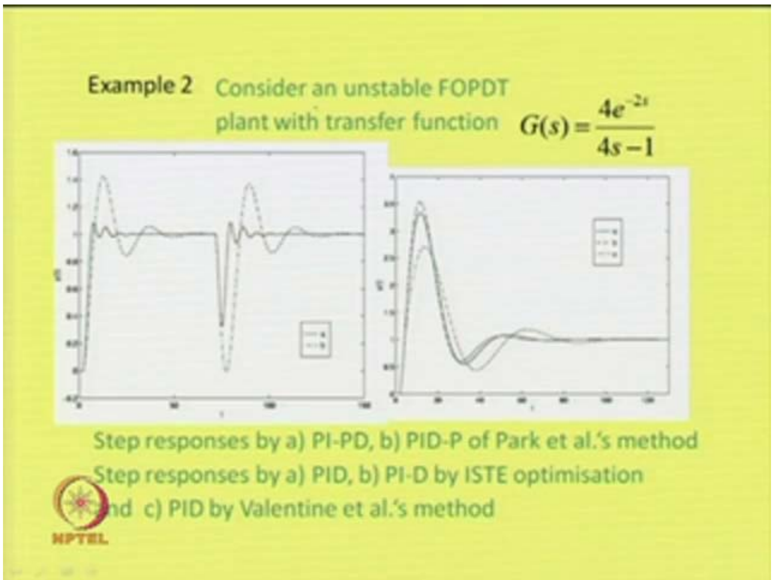
(Refer Slide Time: 18:31)



Now, we shall go through some more illustrative examples, let us consider one more example, where the process is assumed to have a third order dynamics of this form, G s is equal to one upon s plus 1 to the power 3. So, for this third order stable process, PI PD PI with d in the feedback path and PID controllers are designed by ISTEISTE optimization, not necessarily one has to reset to the standard form technique to design the PI PD, PID and PID parameters. So, by ISTE optimization using some routine, it is possible to design the parameters of a PI PD controller, PI with d controller and PID controller. Similarly, a PID controller designed by some other techniques Astram and Hoagland method is also considered in this simulation study.

Now, the simulation results are showed here, where again it shows that the PI PD controller response given by the solid line in this plot, out performs the responses or performances given by other method. The control signals given by different methods are also shown over here and the plot shows that the PI PD controller gives us improved control variations. So, less control effort is employed to obtain superior performance compared to other controller that is the benefit one get from the PI PD controllers.

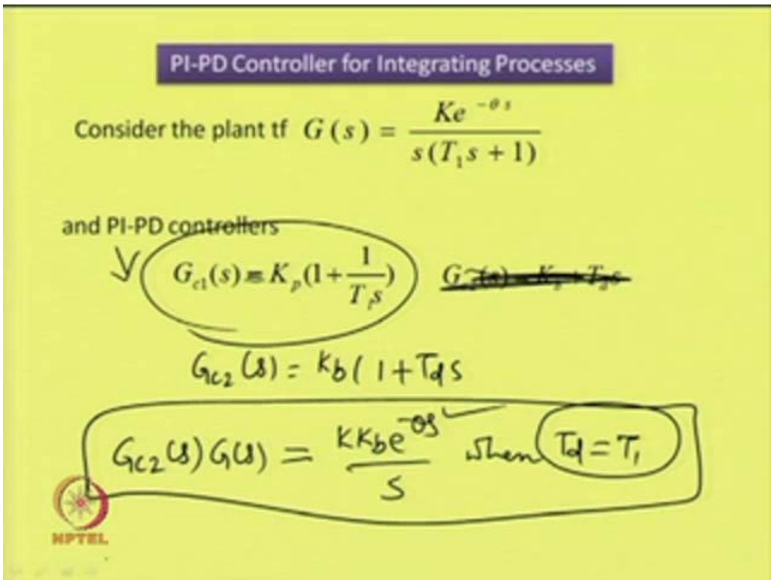
(Refer Slide Time: 20:42)



In this example, an unstable first order plus time delay plant with transfer function given by 4 e to the power minus 2 s upon 4 s minus 1 is considered. So, for this unstable process, with time delay also, PI PD controller is designed and a PID P controller is designed by some other technique. This is also a four parameter controller, a PI PD controller is a four parameter controller and we have got four parameters in the controller. Similarly, the PID P controller of park at all is also having four parameters, so for fair comparison, the results given by park et al's method and the PI PD controller designed by our method is shown in the figure given in the left side. Then as I expected, the results obtained by the four parameter PI PD controller designed by standard form is better than that is obtainable by park et al's method.

Next, the second figure shows the responses given by various PID controllers, a PID controller, a PI with d in the feedback path controller are designed by ISTE optimization and another PID controller is designed by valentine et al's method. All the results are included in the second figure, so the second figure shows the results given, the state responses performances given by the different PID controller. So, suddenly if we compare the responses given in the left hand side and the right hand figures as expected, the PI PD controllers outperform the performances given by PID controller. Because, not only we have got an extra parameter in the PI PD controller, the PI PD controller is not subjected to structural limitations, that is why we have got better or superior responses compared to the PID controllers.

(Refer Slide Time: 23:18)

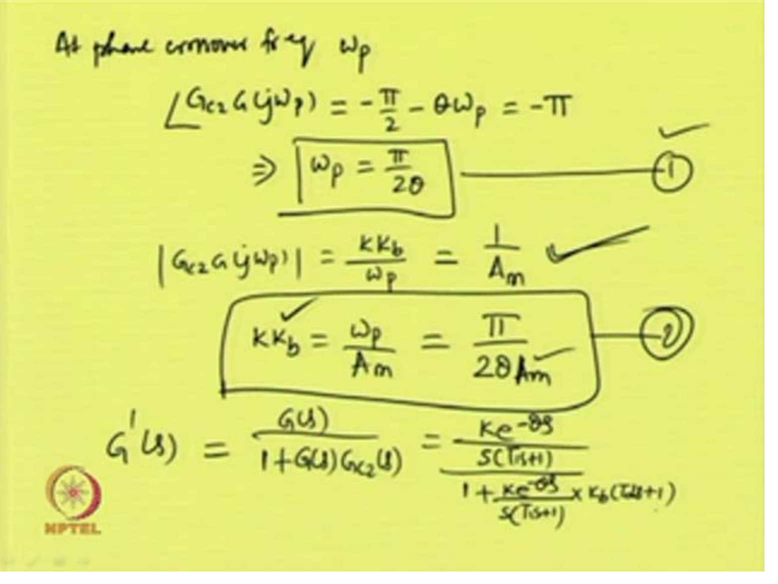


Next, we shall attempt to design a PI PD controller for integrating processes, earlier we have seen how PI PD controllers are useful for not only stable, but also for unstable processes with or without time delay. Now, attempt will be made to design a PI PD controller for integrating processes, so consider the process model transfer function to be of the form K e to the power minus theta s upon s times T 1s plus 1. So, this gives us the transfer function model of an integrating second order integrating process, an integrating not only a second order, integrating process. And let us assume the form of the PI PD controllers to be of the form of G c 1 s given by K p times 1 plus 1 upon T i s, the standard PI controller, where as the PD controller in the feedback path is assumed to have the form K b plus T d s, but for is in analysis of closed loop system, often it is useful to assume the form of this controller G c 2sh K b times 1 plus T d s.

So, without the loss of generality, one can get the inner feedback controller in this form, because if you multiply this, we get it is K b plus K b T d s, which can ultimately be expressed in the form of K b plus T d dash s. So, it is all about expressing the derivative value in some different form, so that way without loss of generality, let us assume the form of the PD controller to be of the form of G c 2 s is equal to K b 1 plus T d s.

So, with this choice of the controllers G c 1 s and G c 2s and the assumed form of the process dynamics, the loop gain as far as the inner loop gain is concerned, the loop gain can be written as G c 2 s G s, which is equal to K Kb e to the power minus theta s upon s, when T d is equal to T 1. So, that is why we are choosing a a convenient form of the controller G c 2 s, the G c 2 s is defined in this form for the sake of cancellation of the 0 of the controller with a pole of the process. Then the loop gain, this is the inner loop gain; we can say the inner loop gain of the system is given by K Kb e 2 the power minus theta s upon s, when T d is equal to T 1.

(Refer Slide Time: 27:33)

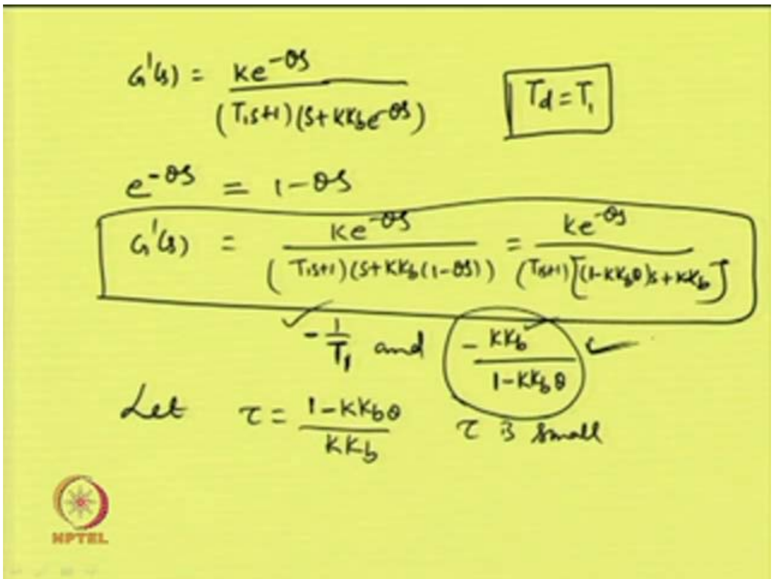


Then, at phase cross over frequency, at phase cross over frequency, let the frequency be denoted by omega P. The loop gain G c 2 G j omega P will give us an angle of minus PI by 2 minus theta omega P and which is to be equal to minus pi, because we are considering the phase cross over point at which the loop phase is minus 180 degree or minus pi. So, this expression gives us omega Pi s equal to pi upon 2 theta, so this is one relation we get using the loop gain.

Similarly, at the same phase cross over frequency, magnitude of the loop gain G c 2 G j omega P magnitude is equal to K Kb by omega P, which is nothing but inverse of the gain margin is equal to 1 upon a m. This is by definition we get the definition of gain margin for the loop gain gives us this expression, so from here, we get K Kb is equal to omega p upon A m, but omega PIs equal to pi upon 2 theta, therefore, this can be written as pi upon 2 theta pi upon 2 theta A m.

So, thus we have got a relation, which is relating the unknown K b with the gain margin of the loop, then using this it is possible to design K b. Next, the stabilized process, G dash s is equal to G s upon 1 plus G s G c 2 s, which can be written as K e to the power minus theta s upon s times t 1 s plus 1 divided by 1 plus K e to the power minus theta s upon s times t 1 s plus 1 times K b T d s plus 1.

(Refer Slide Time: 30:42)

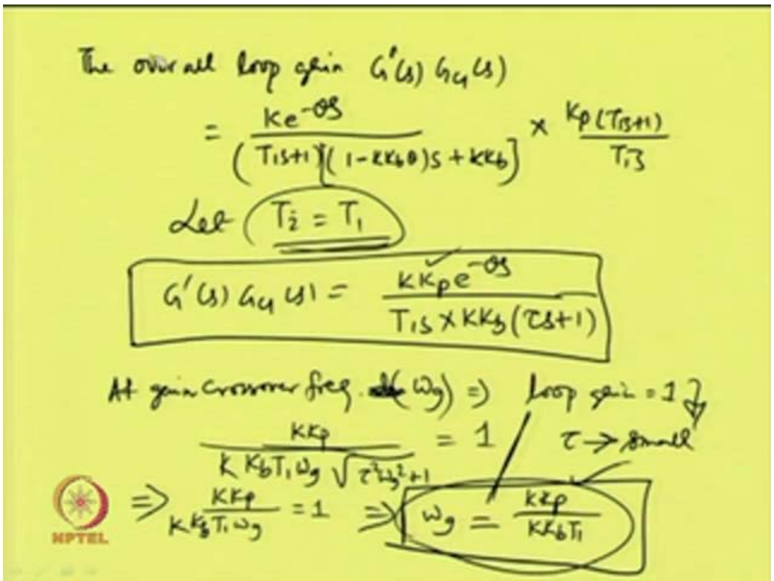


G dash s further can be simplified and written in the form of K e to the power minus theta s K e to the power minus theta s upon T 1 s plus 1 s plus K Kb e to the power minus theta s. Please keep in mind that T d is equal to T 1, with that assumption only we get this reduced form of the modified transfer function for the plant.

Now, this, with the assumption of e to the power minus theta s is equal to 1 minus theta s, G dash s further can be written as K e to the power minus theta s times T 1 s plus 1 s plus K Kb 1 minus theta s, which further can be simplified and written in the form of K e to the power minus theta s upon T 1 s plus 1 times 1 minus K Kb theta s plus K Kb K Kb K Kb. So, what we see from here, the modified process has got poles located at minus 1 upon T 1 and minus K Kb upon 1 minus K Kb theta.

Now, T 1 is the process parameter, we have no hold over that and T 1 is assumed to be 0, but we do not know about the magnitude of T 1, value of T 1, but any how the pole is located in the left half of the s plain. Now, with the design, suitable design of K b, it is possible to locate the other pole of the modified process at suitable locations, that is at far left in the s plain, far left in the s plain, that is possible when we choose a large value for the number or when this becomes a large number. That means, now let tau equal to 1 minus K Kb theta upon K Kb, so when tau is small, then this is going to be a large negative number and thus the pole is pushed far left of the s plain that is what we wish, so the tau has to be a small number. Now, we shall consider the loop gain of the overall loop now for designing the remaining parameters of the PI PD controller.

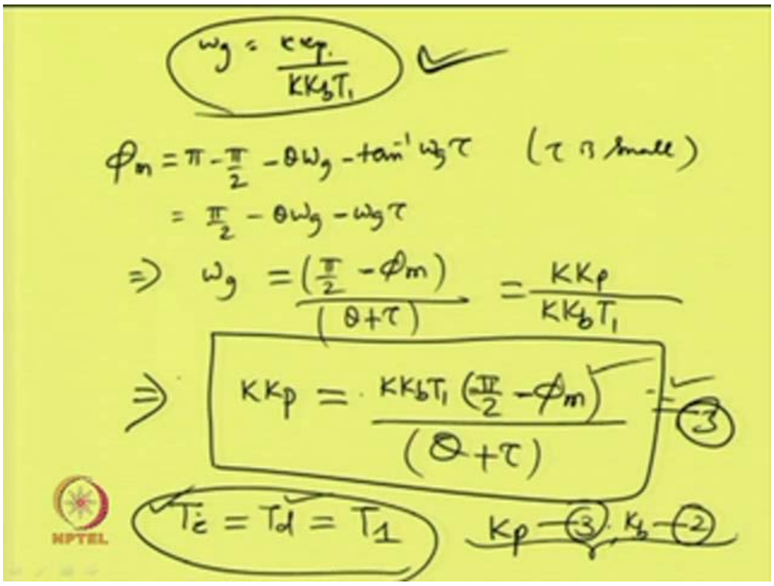
(Refer Slide Time: 34:23)



Now, the overall loop gain the overall loop gain is given by G dash s G c 1 s is equal to K e to the power minus theta s times T 1 s plus 1 1 minus K Kb theta s plus K Kb times K b K p T i s plus 1 upon T i s. Again, let T i equal to T 1, why do we take such assumptions for each and analysis of the overall loop gain, this is desirable to make this assumption. So, with this assumption, the loop gain becomes G dash s G c 1 s is equal to K Kp e to the power minus theta s times T 1 s times K Kb times tau s plus 1.

So, with this loop gain, using the phase margin condition, it is possible to design the unknown K p. At gain cross over frequency, at gain cross over frequency which is denoted by omega g, the loop gain becomes K Kp upon T 1 omega g root of tau square omega g square plus 1 is equal to 1, because at this frequency, the loop gain is equal to 1 that is what we have expressed. So, this loop gain equal to 1, this condition gives us K Kp upon T 1 omega g root of tau square omega g square plus 1 is equal to 1, but earlier, we have seen that to push the pole of the closed loop system to far left of the s plain, tau has to be a small number. So, tau is a small number, so with that assumption, this expression can be simplified and written as K Kp upon T 1 omega g is equal to 1. This k b is missing here, so that way that will give us omega g is equal to K Kp upon K Kb and T 1. Here, k is also missing, so that way this will give us K Kb K Kp upon K Kb T 1 omega g, yes, root of tau square omega g square plus 1 is equal to 1. So, thus we are getting the gain cross over frequency expressed in the form of K Kp upon K Kb T 1.

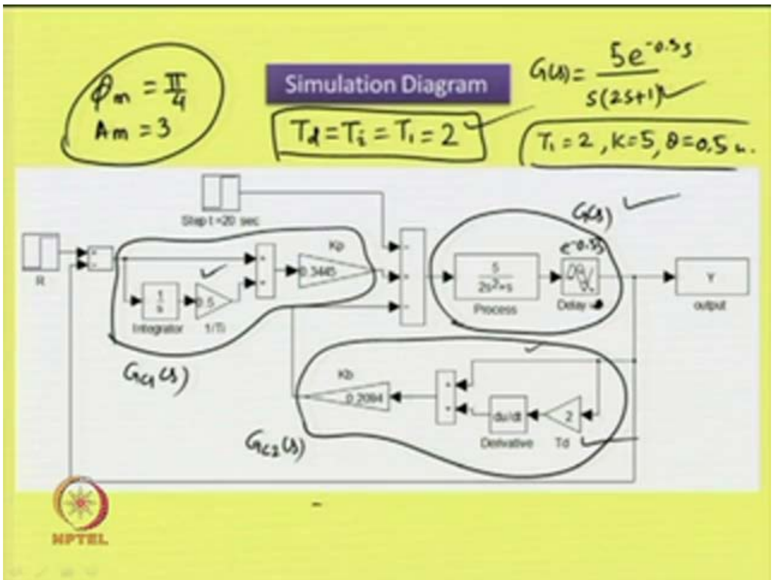
(Refer Slide Time: 38:34)



So, let us rewrite that once more, the loop gain at gain cross over frequency gives us the expression omega g is equal to K Kp upon K Kb T 1. Now, the phase margin, the phase margin at the gain cross over frequency further results in an expression, which can be expressed as phase margin is equal to pi minus pi by 2 minus theta omega g minus 10 inverse omega g tau, but because tau is small, we know that tau is small, therefore this can be expressed as pi by 2 minus theta omega g minus omega g tau. So, after simplification, one will get the omega g in the form of pi by 2 minus phase margin divided by theta plus tau, but omega g already we have get got in this form. So, upon comparison it is possible to get this is equal to again K Kp upon K Kb T 1, which gives us the expressions for K Kp as K Kb T 1 pi by 2 minus phase margin upon theta plus tau.

So, we have got one more expression for the K Kp, earlier we have made the assumption, please keep in mind that those assumptions are T i is equal to T d is equal T 1. So, already we know two parameters of the PI PD controllers, those parameters are T i and T d. So, rest we need to estimate the remaining two unknowns K p and K b, so K p is estimated using this expression and with the choice for a phase margin, and the K b is estimated using this expression, K b is estimated using this expression and with a choice of gain margin. So, given gain and choosing certain gain and phase margins and using equation 2 and 3, it is possible to design the remaining two unknowns of the PI PD controller. So, using 2 and 3, it is possible to design the parameters K p using expression 3 and K b using the expression 2 of course with the choice of certain phase and gain margins.

(Refer Slide Time: 42:11)



Let us design a PI PD controller for a second order integrating process, let us assume that the second order integrating process is given by 5 upon s times 2 s plus 1 with a time delay of e to the power minus 0.5 s. So, we have a time delay of 0.5 second and the time constant as T 1 equal to 2 seconds and K equal to 5 and theta equal to 0.5 seconds. So, these are the given parameters, let us choose the phase and gain margins to be phase margin is equal to 45 degree, that means pi by 4 and gain margin A m is equal to 3.

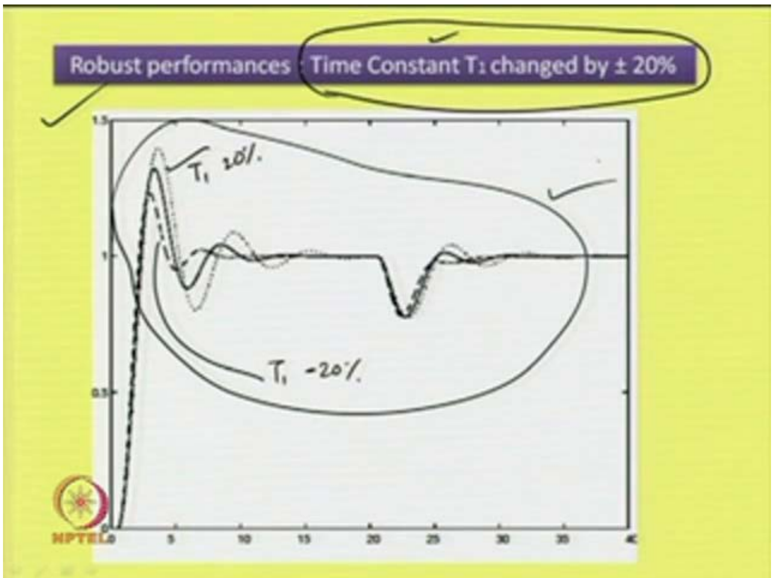
So, with these choices of phase and gain margins, using the expressions 2 and 3, we will be able to estimate the unknown parameters of the controller. Now, K b K b is equal to, K b is equal to pi upon 2 theta A m K, so in our case, now it is pi upon 2 into 0.5 into 3 into 5, that gives us K b as 5 upon 15 is equal to K b as 0.20. So, that will come out to be 0.2094, so thus we have got the design value of the K b shown over here 0.2094.

Similarly, K p is estimated using expression 3, so upon substitution of K T 1 Pi m, which is nothing but pi upon 4 theta 0.5 and tau, expression for the tau is given over here. So, we have assumed the tau to be of this form, so substituting all those values in 3, we get K p as 0.3445. So, it is not difficult to design the unknown parameters for the PI PD controllers using the phase and gain margin conditions.

Now, as we know T d is equal to T i is equal to T 1, which is nothing but 2, so those values are also shown over here. So, K p T i is equal to 2, therefore one upon T i is equal to 0.5 shown over here. Now, T d is 2, T d is equal to T i is equal to T 1, therefore T d is equal to 2. So, thus all the parameters associated with the PI PD controllers are estimated and put in this simulation diagram. The simulation diagram shows the process model given as 5 e to the power minus 0.5 s, delay is 0.5 s upon 2 s square 2 s square plus s, this is the process.

Now, the PD controller has got the parameters K b times, please keep in mind it is in the form of the G c 2 s, G c 2 s is K b 1 plus t d s. So, in the diagram, it is 0.2094 times 1 plus 2 s, which can be shown in the semolina in this form. This is one, then plus 2 plus 2 s coming over here is multiplied with the gain 0.2094, thus we get the G c 2 s given by this. So, this is what we have got the G c 2 s and the upper one is G s, now the PI controller parameters are given here and the PI controller is implemented here, in this form G c 1 s, thus we have got the PI PD controller G c 1 s, G c 2 s and the process.

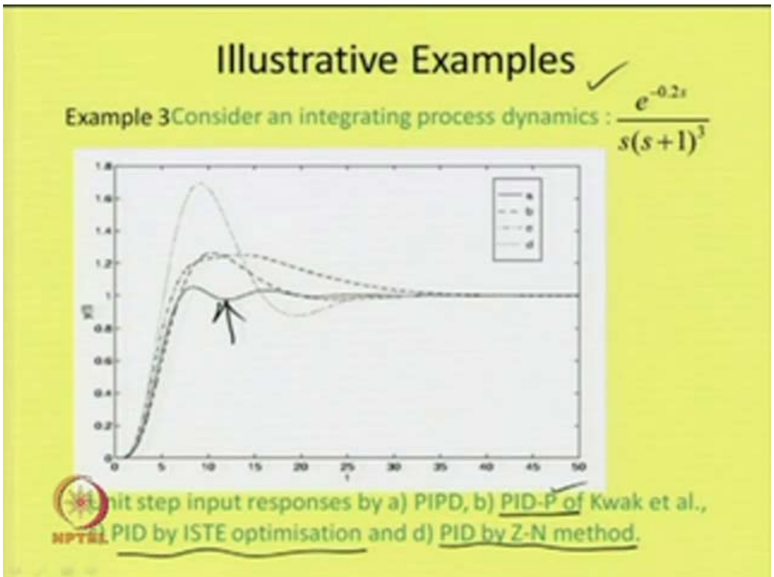
(Refer Slide Time: 47:44)



And the simulation diagram now gives us the results, the state responses and the load disturbances responses are shown over here, is very nice responses as far as the phase and gain margins are concerned. Now, since we have chosen suitable phase and gain margin, it is expected that we shall get robust performances from the closed loop system that is shown by this diagram. When the time constant T 1 is changed by plus minus 20 percentages also, the responses do not vary by very much, this is what we get when the time constant is changing by 20 percent and this is the response we get when the time constant is changing by minus 20 percent.

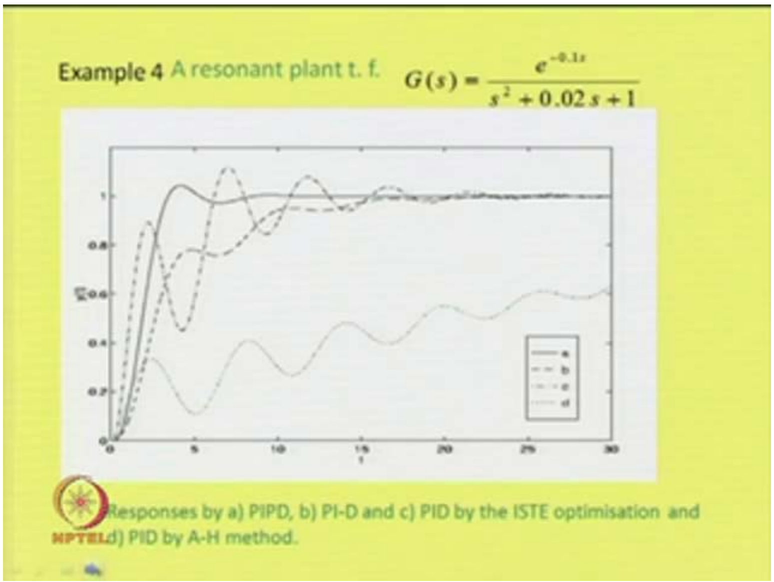
So, the responses show us that we have got robust performances given by the PI PD controllers. Now, when the time, not only the time constant, when the steady state gain of the process is also changed, in spite of the changes, the closed loop system is expected to get is expected to be subjected to better performances since the PI PD controller has been designed based on suitable phase and gain margins.

(Refer Slide Time: 49:26)



Now, there are some illustrative examples also. Consider an integrating process dynamics, a fourth order integrating process dynamics, for that also the PI PD controller responses given by the solid line is found to be improved one compared to the four parameter PID P control of Kwak et al, PID controller by ISTE optimization and PID controller designed by Ziegler Nichols method.

(Refer Slide Time: 50:00)



So, we get improved performances not only for intergrading processes, also we get improved performances of PI PD controller for resonant processes also. Let us consider this resonating process, for this also the PI PD controller response given by the solid line is found to be superior compared to the PID and PID designed by ISTE optimization and PID controller is designed by Astrom Hoagland method. Now, Astrom Hoagland method is giving a very poor set point response, where as the PI PD controller is excepted, as excepted is giving quite superior responses.

In summary, I can say that the PID controller has got structural limitations in controlling on unstable and integrating the processes, where as PI PD controller can successfully control resonating unstable integrating processes. The extra control parameter is leading to stupendous superior performances by splitting the proportional controller that is there in the feed forward path. Derivative controller in the feedback path not only provides compensation but also over comes derivative kick, often found in many practical control systems. So, the PI PD controller is superior to PID controller in controlling, particularly unstable integrating and resonating processes.

Now, some points to ponder. One may ask can the standard form based PI PD controller design be extended to processes with time delay. Yes, the PI PD controller can be extended to processes with time delay, but when a standard form based PI PD controller design is made, at that time the time delay need to be approximated and one can make use of either stellar series expression or paid peered approximation for the time delay turn. But, when the time delay is quite large, smith predictor controller may be used, in which case, there is no need for approximations to time delay and the PI PD controllers can be designed conveniently using the standard form.

Second point may be raised are the performances given by PI PD controller improved always, then that obtainable with a PID controller. Yes, due to the additional controller parameter in the PI PD controller, then the PID controller, the PI PD controller results in better performances, splitting of the proportional controller results in significantly improved closed loop performance; that is all in this lecture.