Advanced Control Systems

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Module No. # 01
Model Based Controller Design
Lecture No. # 07
Limitations of PID Controller

Welcome to the lecture on Limitations of PID controllers. PID controller is found to be work house in process industries. Is there any limitation associated with a PID controller? That we shall see in this lecture.

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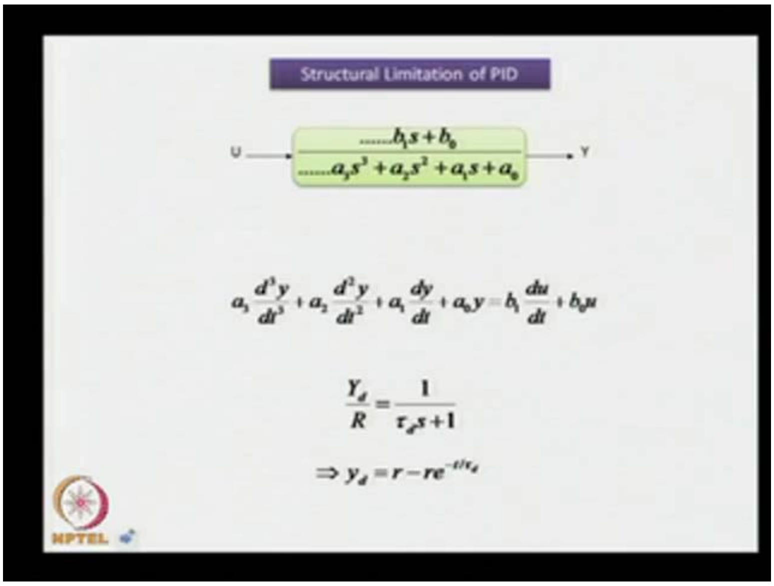

PID controller can be given in various forms, in series, in parallel form. When it is expressed in series form or in parallel form, it might be subjected to structural limitation.

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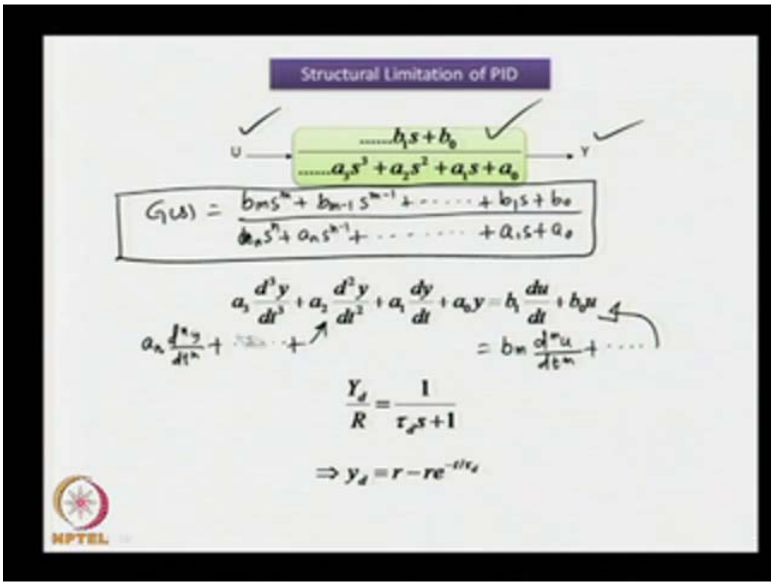

Let us consider, first a plant dynamics given in the form of G s equal to b m s m plus b m minus 1 s m minus 1 plus and so on terms, with the last two terms given as b 1 s plus b 0 upon a n s n plus a n s n minus 1 plus and so terms giving us the last two terms as a 1 s plus a 0. So, we have described the dynamics of a process in this polynomial form, in this transfer function form.

When the dynamics is expressed in this form, since the input to the process is U and the output from the process is Y, the dynamics of the process same process can be also expressed in differential equation form. As an equation giving us a n d n y upon d t n plus and so on with the last three terms as shown over here, equal to b m d m u upon d t m plus the last two terms shown over here. Why we are trying to get the dynamics expressed in the differential equation form? That helps us in analyzing the dynamics of a process and the limitations of a controller employed for controlling the process.

Let us first assume that, we have got a third-order process which dynamics is given by a numerator as b 1 s plus b 0 upon a 3 s cube plus a 2 a square plus a 1 s plus a 0 in the denominator, cross multiplying and taking the inverse Laplace transform gives us the differential equation of the third-order system as a 3 d cube y upon d t cube plus a 2 d square y upon d t square plus a 1 d y upon d t plus a 0 equal to b1 d u upon d t plus b 0 u. So, this differential equation gives us the dynamics of a third-order system in time domain.

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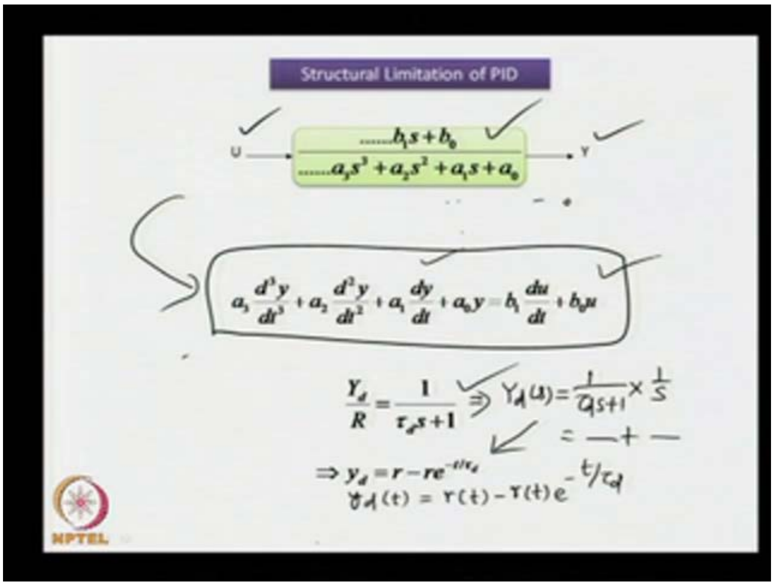

Next, let us make some assumption. Let us assume that, we wish to have a desired performance from the system, a first-order performance from the system. What is a first-order performance? Suppose, the reference input is like this and the desired output has to be of this form, a first-order response form, as so as possible with high rise time, less settling time and to know our (( )). So, this set of response often can be represented by a first-order filter given as 1 upon tau d s plus 1. This is the time constant one can find from here, when you have got 0.63 percent of the reference value. So, that gives us the time constant tau d over here.

So, assuming that, the closed loop system has to have a suitable response of this form, Y d upon R equal to 1 upon tau d s plus 1, we get the expression Y d s equal to 1 upon tau d s plus 1 times R s. So, when we employ you need set point inputs to the system or you need step reference inputs to the system, at that time R s becomes 1 upon s. Thus, we get Y d s is equal to 1 upon tau d s plus 1 times 1 upon s, which again using the partial fraction expansion and inverse Laplace transform gives us an expression in the time domain as Y d t, the desired response in time domain Y d t equal to r t minus r t e to the power minus t upon tau d.

Now, with small tau d, what we will get? We will get high rise time for the system; with large tau d, we will get large rise time. So, often it is desirable to have small tau d time constant for the closed loop system. Thus, we have got the desired response for a closed loop system with the process dynamics given as this one as y d is equal to r minus r e to the power minus t upon tau d.

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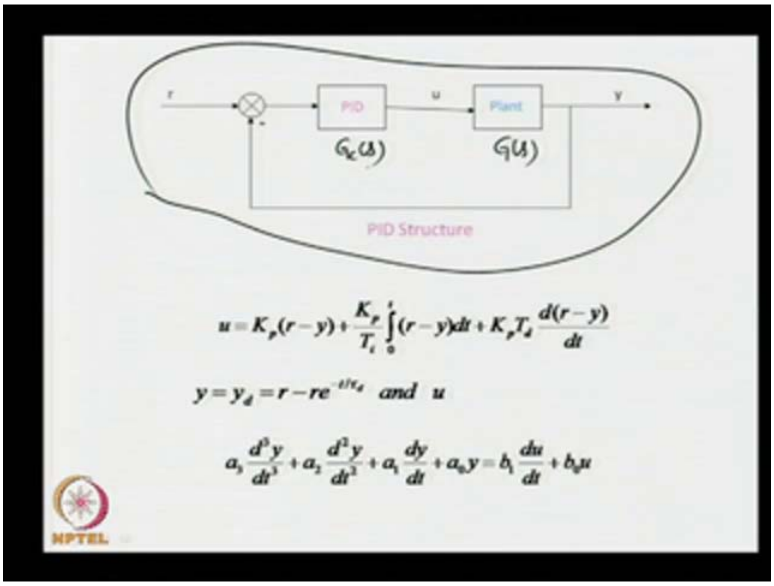

Now, and we have got the closed loop control system shown in this fashion. So, we have a process G s and the controller G c s a PID controller; let the PID controller be a parallel, PID controller this giving us its dynamics in the form of G c s is equal to K p 1 plus one upon T i s plus T ds.

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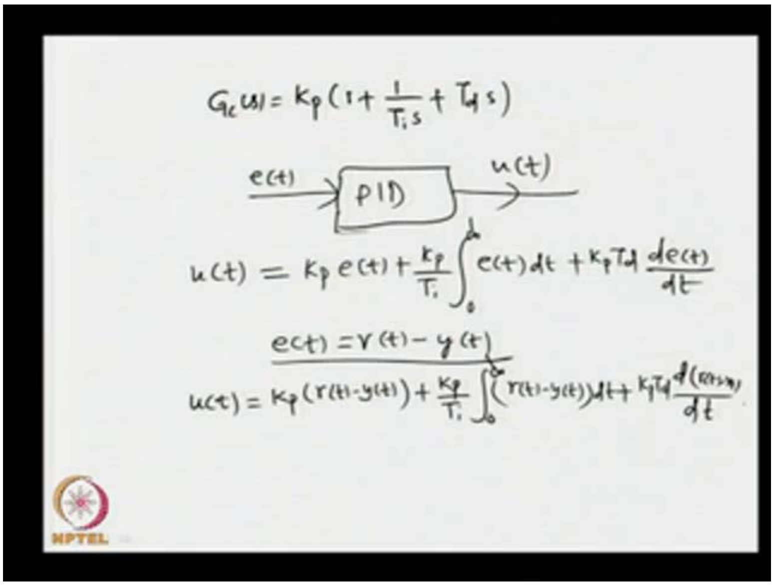

Then, since the PID controller has got inputs e t and output u t and here we have got the PID dynamics, therefore, expression for the PID controller output u t in time domain can be obtained as K p e t plus K p upon T i, integration from 0 to infinity e t d t plus K p T d d e t upon d t. What is e t? It is nothing but, the signal we get from the difference of the desired output and the actual output, desired output and the actual output. So, substituting e t over there, we get the expression for u t as k p r t minus y t plus k p upon T i integration from 0 to infinity, r t minus y t d t plus K p T d d r t minus y t upon d t, that has been shown over here (Refer Slide Time: 09:57).

So, the conventional PID controller, the parallel PID controller output can be expressed in this form, but we wish to have some desired output from the closed loop system, which is nothing but, y equal to y d t y t equal to y d t equal to r t minus r t, e to the power minus t upon tau d.

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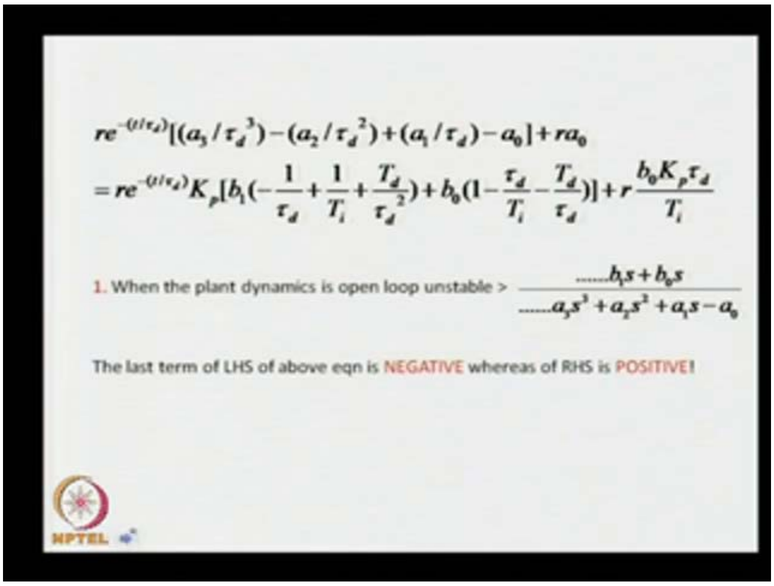

So, substituting y equal to y d equal to r minus r e to the power minus t upon tau d and the above u in the differential equation dynamics for the process, we obtain a 3 d 3 y upon d t 3 plus a 2 d square y upon d t square plus a 1 d y upon d t plus a 0 y is equal to b 1 d u upon d t plus b 0 u expressed in the form of r e to the power minus t upon tau d times a 3 upon tau d cubed minus a 2 upon tau d square plus a1 upon tau d minus a 0 plus r a 0; which is equal to r e to the power minus T upon tau d k p times b 1 minus 1 upon tau d plus 1 upon tau T i plus T d upon tau d square plus b 0 times 1 minus tau d upon T i minus tau d upon t tau d t d upon t d plus r times b 0 k p tau d upon T i.

How do we obtain this expression? It is very important to get this expression, expressed in correct form because, the structural limitation of a PID controller can be evaluated properly provided this expression has been obtained accurately. So, substitution of Y d and u over here will definitely the equation (Refer Slide Time: 12:30).

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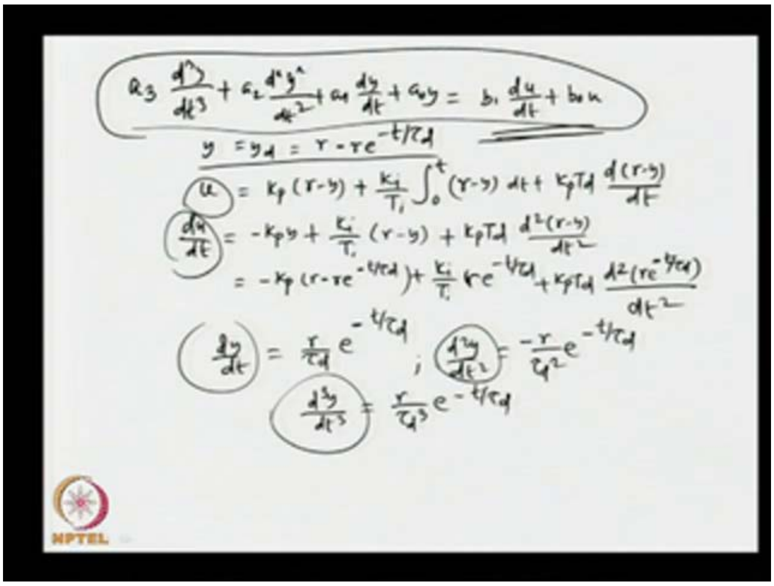

So, let us write down, how we obtain that one a 3 d 3 y upon d t 3 plus a 2 d 2 y 2 upon d t square plus a 1 d y upon d t plus a 0 y is equal to b 1 d u upon d t plus b 0 u. Now, y will be equal to y d, which is equal to r minus r e to the power of minus t upon tau d and u is equal to k p r minus y plus k p upon t i integration of 0 to t, r minus y d t plus k p t d d r minus y upon d t. Then, d u upon d t will give us minus k p y plus k i upon T i r minus y plus k p t d times d square r minus y upon d t square, substitute y equal to y d that gives us minus k p r minus r e to the power minus t upon tau d plus k i by T i r minus y.

So, r minus y will give you r e to the power minus t upon tau d plus k p t d double differentiation of r minus y will give us d square of r e to the power of minus t upon tau d upon d t square, which can further be simplified and put in the right half. Similarly, we have got d y upon d t as r upon tau d, e to the power minus t upon tau d then, d square y upon d t square will give us minus r tau d square e to the power minus t upon tau d and third derivative of y will give us r upon tau d cube e to the power minus t upon tau d. So, substituting all these expressions in the left and right half of the dynamic equation, enables us finally to get this expression.

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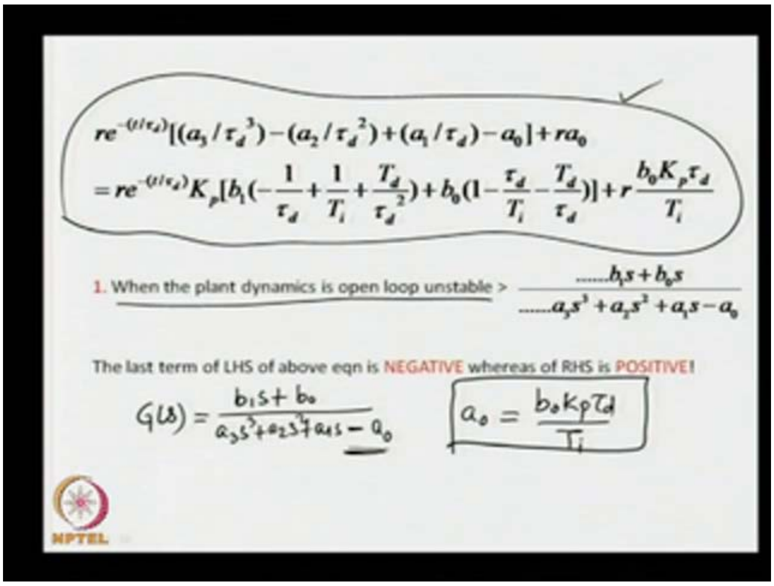

So, collecting the term enables us to get the final expression, expressed in the form of r e to the power minus t upon tau d time, a 3 upon tau d cubed minus a 2 upon tau d square plus a 1 upon tau d minus a 0 with the delay term like this plus a constant value plus r a, 0 a positive value keep in mind. And in the right half we get, r e to the power minus t upon tau d K p times b 1 times minus 1 upon tau d plus 1 upon T i plus t d upon tau d square plus b 0 1 minus tau d upon T i minus t d upon tau d plus r times b 0 k p tau d upon T i.

So, carefully look at the last two terms, why we concentrate on the last two terms? If you look at the expressions or the terms even in the left hand and right hand side of the equation, the term associated with the exponential term we will die down with time after sometime as time elapses, that will die down and finally, what will remain in steady state? We will get an expression of the form r a 0 is equal r b 0 K p tau d upon T i, which is again a 0 is equal to b 0 K p tau d upon T i.

This is very important (Refer Slide Time: 18:16); this expression carries much meaning as far as analysis of a closed loop control system is concerned. Let us consider few cases, when the plant is open loop unstable, when the plant is open loop unstable, how do we get the dynamics of the plant expressed in the transfer function from? As far as the third-order dynamics is concerned, G s will be given as b 1 s plus b 0 upon a 3 s cubed plus a 2 s square plus a 1s minus a 0. Earlier, we have seen this was plus a 0 and with plus a 0, we have obtained the expression given as this (Refer Slide Time: 19:10).

Now, when the plant dynamics possesses instability or plant dynamics is open loop unstable, at that time we have got minus a 0 or anyone of the coefficient go negative. For simplicity let us assume that, a 0 is assuming a negative value, in that case what happens? When a 0 is assuming a negative value, is it possible to provide similar value in the right half of the equation with the help of control parameters? No, you see b 0 is positive as I assumed; now, what are the other parameters we have? Tau d cannot be negative; otherwise, the closed loop response will be unstable.

Now, we have freedom to choose K p and T i which are nothing but, the controller parameters. We have freedom to set any values for the controllers, not the other parameter. So, that way can we make K p or T i or both negative? No, we cannot make, if K p is met negative, then we have got unstable controller; if T i is met negative, again we have got an unstable controller. So, when the process is unstable and the controller become unstable, then it will the output will explode simply.

It will be very difficult to get any set of desire output from that control system. That is why when the plant dynamics is open loop unstable irrespective of any setting of the controller parameters. It is not possible to get a 0 negative a minus a 0 value with the settings of controller parameters and therefore, the PID controller cannot successfully control unstable processes, that is the logical we can put forward. Simply the logical is that, when the process is unstable, this right half this term, the term of the right half cannot be make negative with the help of the controller parameters. Therefore, the PID controller has got limitation in controlling unstable processes.

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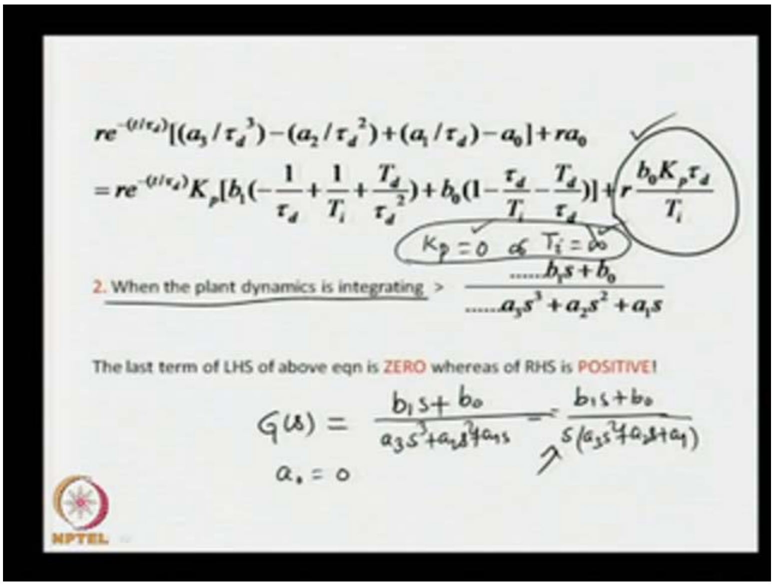

Let us see another case, when the plant dynamics is integrating. What do you mean by integrating? When the plant dynamics or the plant transfer function has got a pole located at the origin, then in that case, we tell that the process is integrating. For the integrating process, the dynamics in transfer function form can be given as b 1 s plus b 0 upon a 3 s cubed plus a 2 s square plus a 1 s, which can further be written in the form of b 1 s plus b 0, upon s times a 3 s square plus a 2 s plus a 1.

So, we see that there is a pole located at the origin of the s plane. Thus giving us the process as an integrating process; in this case, what happens? a 0 becomes 0 a 0 has become 0 is it possible to get the last term of the right half as 0 with the setting of the controller parameters? No, unless K p is 0 or T i is infinity we cannot make this last term to be 0 when a 0 equal to 0. That means, when the process is integrating in nature at that time, one needs to set P p to 0 or T i to infinity to get effective control of the closed loop system.

When K p is set to 0, what will happen? We will get no controller in the loop, when T i is set to 0, we have got no integral control in the system, when there is no integral control in the system, no integral control action in the system as expected, although we may get satisfactory set point response disturbance rejection cannot happen. So, for that case to have overall satisfactory closed loop performance of a system T i or integral action cannot be neglected.

So, this is how the PID controller had got limitations; the controller parameters in spite setting of any values of controller parameter, we are not able to get the last term as 0. Thus the PID controller is found to have limitations in controlling unstable or integrating processes. Let us see through simulation results, whether that is really happening or not.

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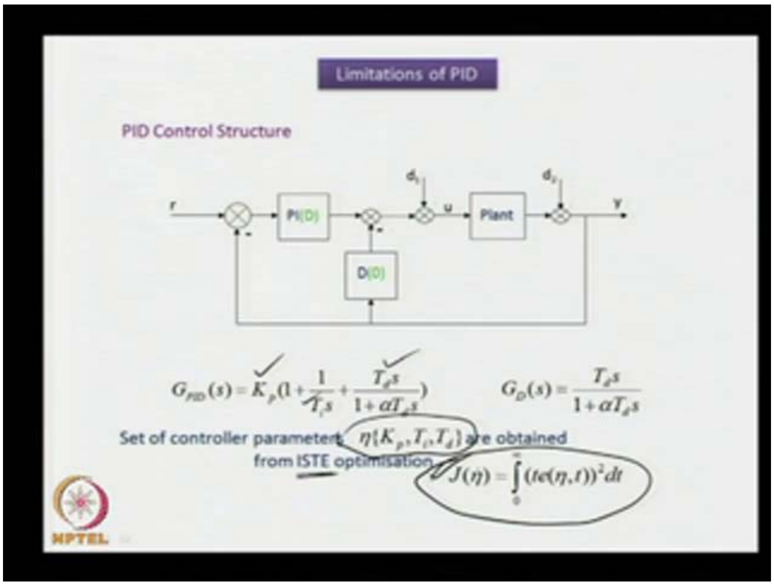

We shall consider a PID of this form (Refer Slide Time: 25:20), the PID controller may be PI and PID and its variant like PI in the feed forward path, with D in the feedback path, thus giving us also a PID controller or the PID in the feed forward path, with no inner feedback control or no feedback control at that time. So, we will consider different types of PID controller or PID and its variants in the simulation study.

So, let us assume that the form of the PID controller in the feed forward path be of this parallel PID controller form. And the derivative control in the feedback path is given by G d s equal to T d s upon one plus alpha T d s, where again alpha is nothing but, the derivative filter constant. As far as this simulation study is concerned, to study the limitation of a PID controller, what we have done? We have tried to find optimum values for the PID gains, the K p, T i and T d are estimated by minimization of the performance index known as is T e performance index. A set of parameters controller parameters can be employed and using the lustrum refer ship algorithm, that is not difficult to minimize the ISTE performance index and to obtain optimum controller parameters for PID control structure.

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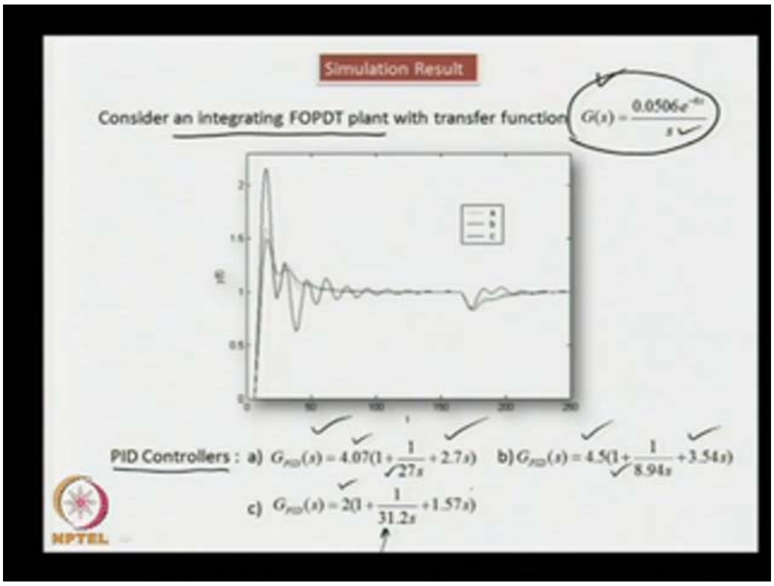

In the first simulation study, let us consider an integrating first-order plus time delay plant, given as G s is equal to 0.0506 e to the power minus 6 s upon s. So, this gives us the process dynamics of an integrating first-order plus dead time plant. This e to the power minus 6 s gives us the time delay and we have got a very small gain, static gain for the system, in this case the gain is given as K equal to 0.0506, why this is first-order? It is because the denominator polynomial is of first-order.

For this process, using minimization of the performance index ISTE performance index, controllers have been obtained by various methods. Those controllers are given over here, a PID controller is designed where the proportional gain is 4.07 and the integral time constant is given as 27 seconds and the derivative time constant is given as 2.7 seconds. Using another technique, PID controller has been designed and in which case, the proportional gain is found to be 4.5, the integral time constant is found to be 8.94 seconds and the derivative time constant is of 3.54 seconds.

Another method, control design method has yielded a PID controller, with the proportional gain as 2 integral time constant as 31.1 seconds and derivative time constant as 1.57 seconds. Now, simulation results for the three situations, for the 3 PID controllers are shown over here (Refer Slide Time: 30:00). For the first PID controller, we obtain a response like this, which shows not only over shoot of more than 50 percent, but also a high settling time, although the rise time is satisfactory, a high settling time and the disturbance rejection is also not smooth.

So, we do not get smooth step input responses, why that is not the case, why we are not getting smooth responses because the PID controller has got limitation in controlling integrating processes. Now, if I look at the performance given by the second controller, it is further inferior and the over shoot is more than 100 percent with very high settling time and oscillatory response.

Similarly, the third controller is giving little bit of satisfactory performance no doubt but, it is also having over shoot and under shoot of about 50 percent. Now, not only the time responses are quite unsatisfactory because, the responses are having multiple cycles and bonds, we do not get good settling time as well, which is one of the most desired things in closed loop control system.

The disturbances are rejected no doubts, static load disturbances had been rejected successfully, but the responses are not smooth. So, one can conclude from this plot that, the PID controllers have got limitations in controlling integrating processes. In spite of employing controller designed by various techniques, it has been found that, the time responses are quite oscillatory with very high over shoot and settling time.

Let us go to another example, in this example we consider a resonating second-order plus dead time plant (Refer Slide Time: 32:54). The plant dynamics or the process dynamics is given by a transfer function, which has got a time delay term e to the power minus 0.1 second in the numerator and s square plus 0.02 s plus 1 in the denominator.

Why this process dynamics is known as a resonating second-order plus dead time dynamics? If I calculate the damping ratio, damping ratio can be calculated from this term, which gives us 0.02 upon 2 times root of 1, this is our omega n square and this is given by 2 zaye omega n s. Therefore, zaye can be obtained as 0.002 upon 2 into root of 1 which gives us equal to 0.01.

So, we have got a very small damping in the system almost 0, therefore the system is oscillatory in nature. So, the system itself is oscillatory in nature. So, obviously, one needs to design carefully a controller for the resonating second-order plus dead time dynamic. So, let us consider one more example, where we have got a resonating second-order plus dead time plant, with the transfer function given as G s equal to e to the power minus 0.1 second upon a square plus 0.02 s plus 1.

Why this process is known as a resonating process? A process having resonance characteristics, if one look at carefully the denominator term, when it is compared with the standard denominator of a second-order transfer function form, s square plus 2 zaye omega n s plus 1, we obtain the damping ratio to be of the value 0.02 divided by 2 times root of 1 which gives us 0.1, so 0.01. So, the damping ratio is found to be of value 0.01, which is almost negligible; as if the system has got no damping therefore, the dynamics of the system will be resonating or oscillating.

So, we have got an oscillatory system dynamics for which, one need to design carefully, proper PID controller for satisfactory closed loop performances. Various methods have been employed to design PID controller for this process dynamics, one such PID controller, where d is d control inner feedback controller is there, feedback derivative controller is there, is designed as PI controller in the feed forward path way 0.144 gain and 0.595 seconds time constant and the derivative field controller has got the parameters T d equal to 0.61 second.

Thus, we have got a PID controller for the resonating process, employing another technique; a series PID controller has been designed, where the proportional gain is of same value 0.144 with integral time constant of 0.372 seconds and derivative filter time constant, derivative time constant of magnitude 5.613 seconds. **A** third PID controller designed has got the proportional gain as 0.144 integral, time constant of 4.4 seconds and derivative time constant of 1.1 seconds. Thus, we have got 3 different types of PID controllers designed by various techniques available in the literature.

Let us see the responses given by all those PID controllers. The PI with D feedback controller is given as this one, gives us a response shown by the solid line (Refer Slide Time: 38:28). So, this is the one we have got with the PI D controller. Let us analyze the time response of the controller, although it has got no over shoot, the response is sluggish and the settling time is very high. There are under shoots associated with the response and the response is not smooth.

Similarly, the PID controller given by b is giving us an oscillatory response and the third PID controller is giving us very poor time response, even the settling time is almost infinite in this case. Thus we see that, the PID controller has got limitation in controlling certain type of processes, if the process is unstable, if the process is integrating, if the process is resonating, in those cases the PID controller fails, because it has got structural limitations in controlling unstable, integrating or resonating processes.

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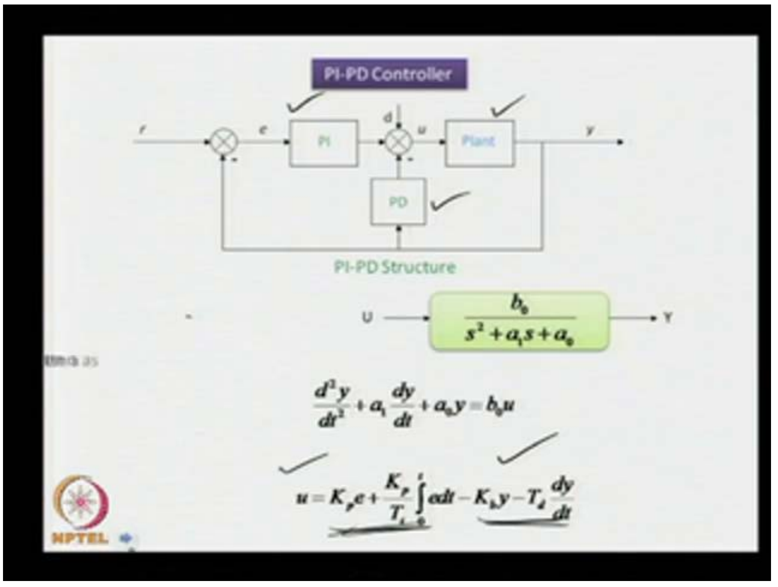

Let us consider a PI PD controller to elevate the problems associated with a PID controller. One can employ a PI PD controller, in which case a PI controller is there in the feed forward path and a PD controller in the feedback path. Let us assume the process dynamics to have the transfer function expressed as b 0 upon s square plus a 1 s plus a 0. We are assuming a simple transfer function for the plant, for each in analysis although one is not ((divide)) to take any form of process dynamics in the analysis.

That will give us the dynamics of the process expressed in time domain as d square y upon d t square plus a 1 d y upon d t plus a 0 y equal to b 0 u. Now, the PI PD controller dynamics will give us the control signal expressed in the form of u t equal to K p e t plus K p upon T i, integration from 0 to t e T d t minus K b y t minus T d d Y t upon d t. We have got a feed forward PI action and feedback, PD action given by the control signal.

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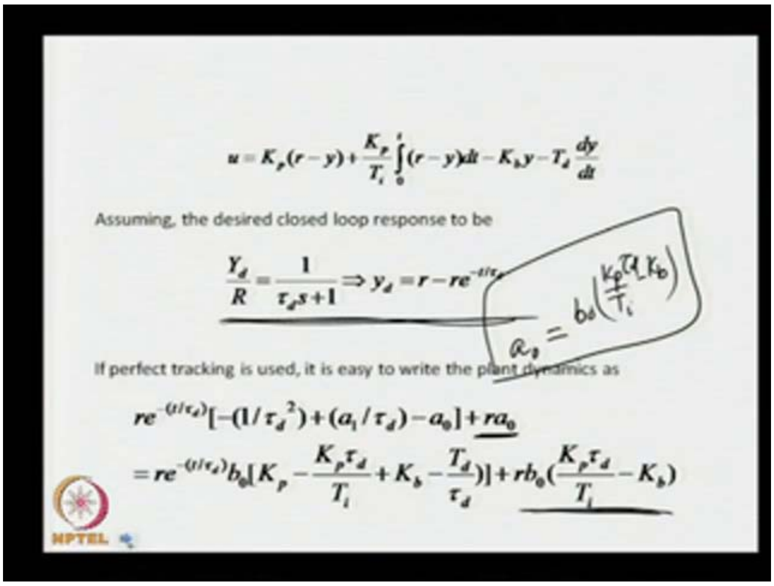

So, the control signal is made up of two components. With this control signal, we can find the expressions we have obtained earlier for the case of a PID controller, a series PID controller. A similar expression can be obtained with the exception that, the last term in the left half side of the equation is r a 0, we had got the same expression in earlier case also. Let me show we have got r a 0, whereas the right half had the last term expressed as r b 0 K p tau d upon T i but, when a PD control is employed, the last term becomes r b 0 times K p tau d upon T i minus K b. This minus K b is given by the PD controller; we assume the same desired response for the closed loop system as we had for the earlier case. This last term will enable us to compare with the left hand sides and get an expression of the form a 0 equal to b 0 K p tau d upon T i minus K b.

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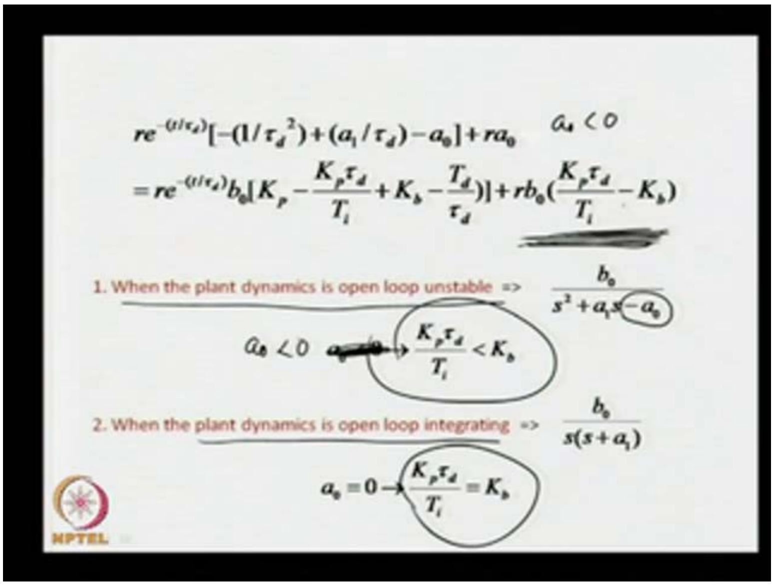

Let us see what we get from these last terms, when the plant dynamics is open loop stable, then a 0 becomes minus a 0. For open loop unstable processes, its transfer function in simple form can be given as b 0 upon s square plus a 1 s minus a 0. Then a 0 is… then is it possible to get a negative value for this term when a 0 is negative. So, when a 0 is less than 0, what happens? Can we set this to less than 0? Yes, one can set with the help of the requirement that K p tau d upon T i has to be less than K b.

So, when K p tau d upon T i is less than K b, this term becomes negative and how we have got a 0 as negative? Equating the two, one can get the design parameters as well. So, the limitations with PID controller can be overcome in this fashion, if one uses a PI PD controller, then it is possible to get the last term as negative. Similarly, when the plant dynamics or the process is open loop integrating, in which case a 0 becomes 0 when K p tau d upon T i becomes K b, then we get this term to be 0 and since a 0 is 0 this two can be compared.

When a 0 is when a 0 is less than 0, then this first part has to be less than K b, K p tau d upon T i has to be less than K b. So, what we have seen from this analysis that, employing a PI PD controller, it is possible to design controller parameters in such a way that, the closed loop performances can be obtained as per our wish, the closed loop performances can be satisfactory.

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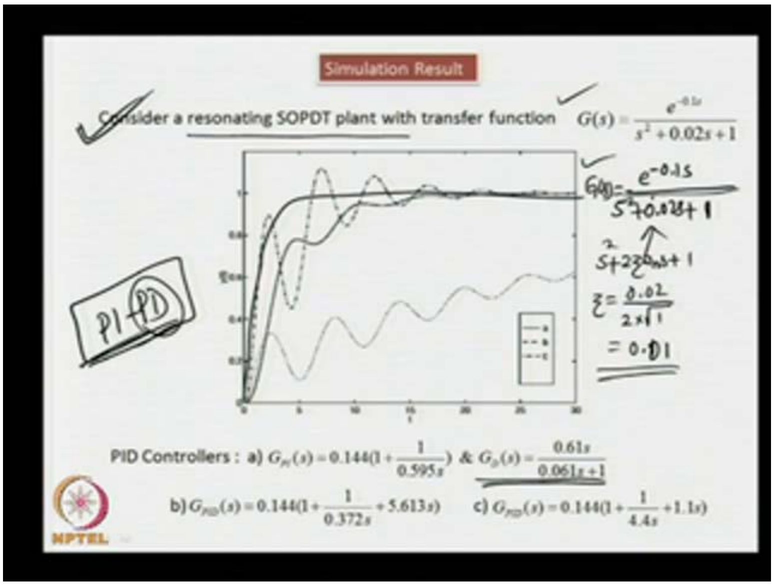

Let us see some example studies, simulation studies. If we look at, although we have got let us go back to the example, we had already consider during our analysis, we have got a D controller, although we do not have PI PD controller, we have got D controller in the feedback path, that is why we are getting a quite satisfactory; although it may not be so satisfactory, a quite satisfactory performance compared to the other two performance given by the PID controller.

A PID controller is giving us inferior responses, but a PID with D controller in the feedback is giving a little superior performance compared to the other two, had they been a PI PD controller, this response can further be improved and obtained in some convenient form. This response would become like this, what we wish to have from the closed loop system (Refer Slide Time: 47:24).

So, if one employs a PI PD controller then definitely, it is possible to obtain a response of this form from the closed loop control systems. Let us summarize what we have learned from this lesson, PID controller have got limitations in controlling certain processes like unstable process, integrating process and resonating process. PI PD controllers can be employed to improve upon the performance and the limitations associated with a PID controller.

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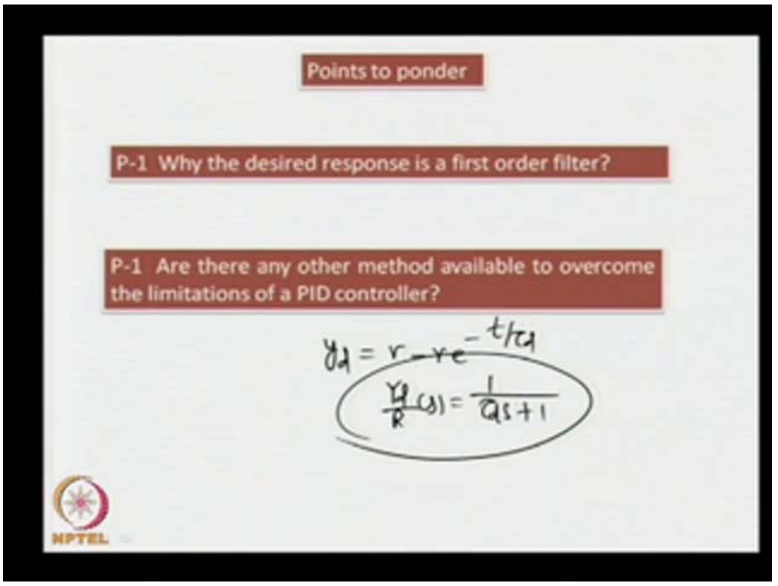

Let us go to the points to ponder; one may ask why the desired response is a first-order filter? As you have seen, we have assumed the desired response to be of the form r minus r e to the power minus t upon tau d within transfer function form gives us y d upon r s at 1 upon tau d s plus 1. So, we have got a first-order filter response from the closed loop system, not necessarily it has to be of first-order filter form; one can employ third-ordered transfer closed-loop transfer function form or standard transfer function also in the analysis, only that will complicate the analysis, for (( )) analysis we have assumed the desired response to be a first-order filter.

Second point might be, are there any other controller or control method available to overcome the limitations of a PID controller? Yes, we have got variants of PID controllers; smith predictor controller, internal model controller, to name a few those can be used to overcome the limitations of a PID controller, thank you.