

Advanced Control Systems

Prof. Somanath Majhi

Department of Electronics and Electrical Engineering

Indian Institute of Technology, Guwahati

Module No. # 04

Design of Controllers

Lecture No. # 05

Model Based PI-PD Controller Design

Welcome to the lecture titled model based PI-PD controller design. In this lecture, we will study the design of a proportional integral proportional derivative controller. So the control, there will be two controllers in the loop to control either stable or unstable or integrating processes.


(Refer Slide Time: 00:42)

Model based PI-PD controller design

- PI-PD structure is the commonly used configuration which is a two degree of freedom and four parameter controller.
- PI-PD controller parameters are designed for a FOPDT transfer function model

$$G(s) = \frac{K_p e^{-\theta s}}{T_1 s + 1}$$

$T_1 \rightarrow \text{Time constant}$
 $\frac{K_p}{T_1} \rightarrow \text{Slope}$

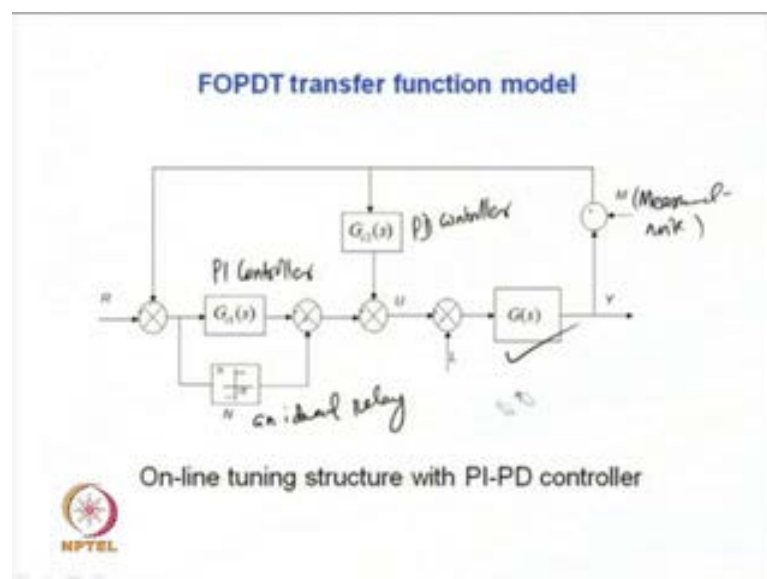


The benefit of using PI-PD controller has been described in one of our previous lectures, now PI-PD structure is the commonly used configuration, which is a two degree of

freedom, and four parameter controller. Unlike the SISO controller - many SISO controllers, we have discussed in our previous lectures, PI-PD control structure is a two degree of freedom control structure.

PI-PD controller parameters will be designed for a first order plus dead time transfer function model, where will assume the transfer function model to be of the form $G(S)$ is equal to $K e^{-\theta S} / (T_1 S + 1)$. And to find design parameters or controller parameters for an integrating process limiting values for T_1 can be applied, when T_1 tends to infinity such that K / T_1 is finite; it can be used to find integrating process model or the formulae will derive for the PI-PD controllers will be made applicable for integrating processes as well.

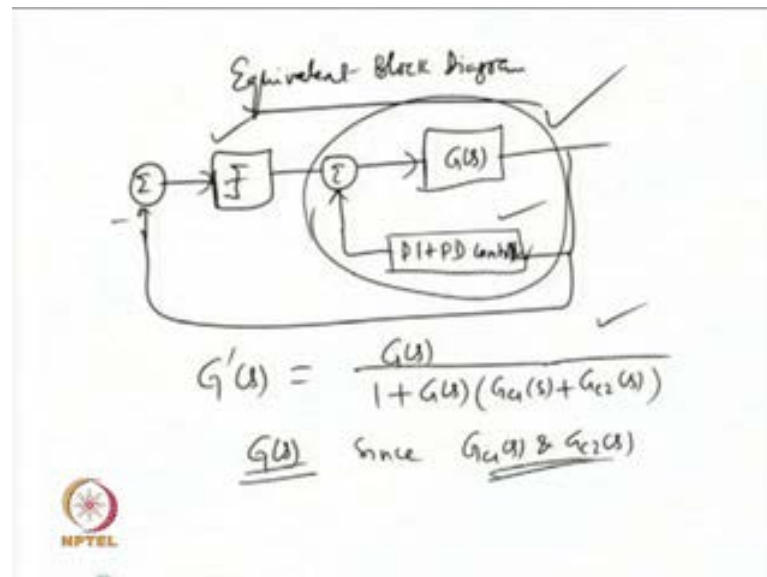
(Refer Slide Time: 01:58)



Now, how we will identify the transfer function model of a dynamics; first we will **we will** employ a relay - an ideal relay in parallel with the PI controller. And of course, the PD controller will be present during the relay experiments. So, this is the scheme that is going to be used for identification for estimation of first order plus dead time transfer function model for the dynamics of the real time process. M stands for the measurement noise - **measurement noise**, and L stands for the static load disturbance - **static load disturbance**.

Now, how the online tuning will be done, initially an ideal relay will be connected in parallel with a PI controller, and limit cycle will be induced; limit cycle output will be obtained. And from the measurements made on the limit cycle output parameters of the transfer function model basically K θ and T_1 will be estimated.

(Refer Slide Time: 03:38)



Now, I will describe a little bit of the equivalent block diagram, we will have for the online tuning structure, fact the relay sees basically. So, we will go for the equivalent block diagram, where we can represent the earlier block diagram in the form of a block diagram having relay in the feed forward path. And the real time process dynamics available also in the forward path, whereas both the controller dynamics will be present in the inner feedback path. Now, we will have $G_{c1}(s)$ plus $G_{c2}(s)$ here, where this is the dynamics of PI controller; therefore, we will have PI plus PD controller. PI, let me write the PI plus PD controller over here, So we will have PI plus PD controller in the inner feedback path.

So, this is what the relay sees during relay test. Now, the relay will be subjected to some modified process given by $G'(s)$ is equal to $G(s)$ divided by $1 + G(s)$ times the dynamics of the PI plus PD controller, and that will be nothing but your $G_{c1}(s)$ plus $G_{c2}(s)$. So, this is what? A relay sees.

So, the identification will be carried out for this modified process from where, again it will be possible to estimate the parameters of the transfer function model for a dynamic system $G(s)$. Since, we know $G_{c1}(s)$ and $G_{c2}(s)$; how you know $G_{c1}(s)$, and $G_{c2}(s)$ when you are commissioning a process. Initially, you may have no information about the controller parameters that requires us to go for the choice of start controllers to initiate relay test at the beginning.

(Refer Slide Time: 05:40)

The forms of the PI and PD controllers are

$$G_{c1}(s) = K_p \left(1 + \frac{1}{T_i s} \right), \quad G_{c2}(s) = K_f \left(1 + \frac{T_d s}{1 + \beta T_d s} \right)$$

$\beta = 0.01$

$$= K_f (1 + T_d s)$$

The relay sees $G_{c1}(s) + G_{c2}(s)$ in the inner feedback path

Choice of start controllers : $(K_p = 0.01 \text{ to } 0.1, T_i = 1)$

PI controller $G_{c1}(s) = (0.01 - 0.1)(1 + 1/s)$

PD controller $G_{c2}(s) = 0$ $K_f = 0$

Then, PI-PD controller parameters are updated before the second stage of relay test to estimate the FOPDT model parameters using DF technique.

Let, the forms of the PI and PD controllers be $G_{c1}(s)$ equal to K_p times $1 + 1$ upon $T_i s$, and $G_{c2}(s)$ is equal to K_f times $1 + T_d s$ upon $1 + \beta T_d s$. Normally, β is chosen a very small value, when β is equal to 0.01, we can neglect the derivative filter term from mass of sequent analysis. So, $G_{c2}(s)$ can be written as K_f times $1 + T_d s$ for analysis for ease in analysis in our subsequent analytical - subsequent dealing of the analytical expressions.

Now, the relays sees actually $G_{c1}(s)$ plus $G_{c2}(s)$ in the inner feedback path; that I have described. Of course, assuming the R is equal to 0, when R is not **not** equal to 0 also you can draw an equivalent block diagram, and from there also you can find that the relay sees basically the PI plus PD controller present in the inner feedback path. Now, to initiate the relay test, when no information about the process dynamics is available, We

have to make some choices for the controller parameters. Now, the PI controller parameters can be said as K_p as a value from 0.01 to 0.1; so choose a value from 0.01 to 0.1 for the proportional gain of the PI controller, and the T_i is chosen as one. And at that time the PD controller is said to be 0, that means the gain of the PD controller K_f is assumed to be 0. This is what you are **you are** required to choose initially, when you have no information about the process dynamics, and you have no default values for the PI-PD controller.

Then, with these initial choices a relay test is conducted, and process model parameters are estimated. Then the PI-PD controller parameters are updated **updated** based on the process model parameters, we will develop the PI-PD tuning formulae which can be used to update its parameters, **its parameters** using the **using the** information we get from the process model. And then, PI-PD controller parameters are updated before the second stage of relay test. So, another relay test is required to estimate the parameters of the first order plus dead time model using describing function, then you will get some accurate estimation for the first order plus dead time transfer function model. So, two stages of relay tests are conducted to find the model parameters.


(Refer Slide Time: 09:08)

Design of PD controller

$$G(s) = \frac{ke^{-\theta s}}{T_1 s \pm 1}; \quad G_{c2}(s) = k_f(1 + T_d s)$$

$$G'(s) = \text{Modified proc} = \frac{G(s)}{1 + G(s)G_{c2}(s)}$$

$$= \frac{\frac{ke^{-\theta s}}{T_1 s \pm 1}}{1 + \frac{ke^{-\theta s}}{T_1 s \pm 1} \times k_f(1 + T_d s)}$$

$$\checkmark \quad G'(s) = \frac{ke^{-\theta s}}{T_1 s \pm 1 + kK_f e^{-\theta s}(1 + T_d s)}$$


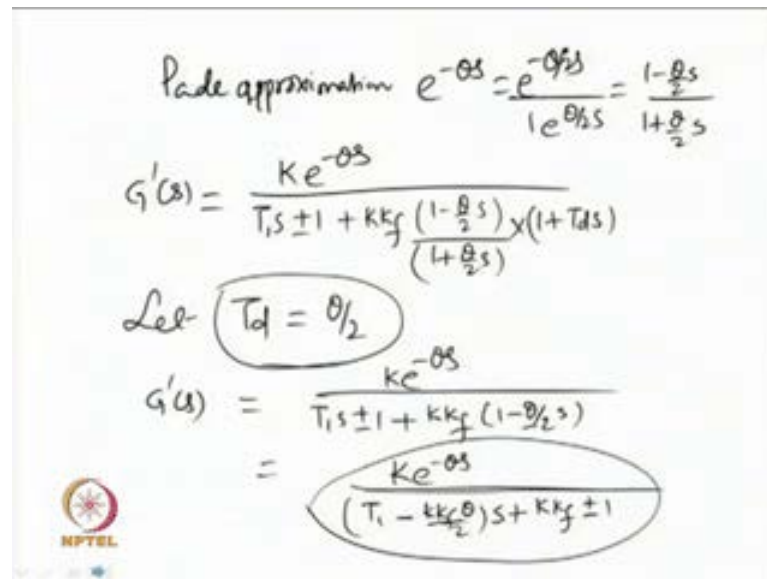
Now, I will begin with the design of PD controllers, we will now discuss about the way

the PI-PD controller parameters can be tuned or we will now develop analytical expressions for the parameters of the PI, and PD controllers. So to begin with I will choose again the process dynamics given by $G(s)$ is equal to $K e^{-\theta s}$ upon $T s + 1$ **plus minus 1**. Now, the controller $G_c(s)$, this is the PD controller is described by $K_f (1 + T_d s)$. Therefore, we will get some due to the inner feedback controller $G_c(s)$ in the PI-PD control structure, please look at the PI-PD control structure when there is no relay in the loop. Then the process is subjected to a PD controller, and due to that the **the the** PD controller will form some modified process given by $G_d(s)$, as already I have told.

Now, that $G_d(s)$, the modified process - **the modified process** dynamics can be expressed as $G(s)$ divided by $1 + G(s) G_c(s)$. Giving us $K e^{-\theta s}$ divided by $T s + 1$ plus minus 1 divided by $1 + K e^{-\theta s}$ divided by $T s + 1$; so 1 minus is here, plus minus 1 times $K_f (1 + T_d s)$.

So, let us simplify this expression for designing the PD controller; so $G_d(s)$ is now, $K e^{-\theta s}$ divided by $T s + 1$ plus minus 1 plus $K K_f e^{-\theta s}$ times $1 + T_d s$. Now, I will use **pade estimation** or approximation for the time delay term for further simplification of the modified process. What is further approximation?

(Refer Slide Time: 11:49)



Handwritten derivation showing the Padé approximation for time delay and the resulting transfer function:

$$\text{Padé approximation } e^{-\theta s} = \frac{e^{-\theta/2 s}}{e^{\theta/2 s}} = \frac{1 - \frac{\theta}{2}s}{1 + \frac{\theta}{2}s}$$

$$G'(s) = \frac{K e^{-\theta s}}{T_1 s + 1 + K K_f \frac{(1 - \frac{\theta}{2}s)}{(1 + \frac{\theta}{2}s)} (1 + T_d s)}$$

Let $T_d = \theta/2$

$$G'(s) = \frac{K e^{-\theta s}}{T_1 s + 1 + K K_f (1 - \frac{\theta}{2}s)}$$

$$= \frac{K e^{-\theta s}}{(T_1 - \frac{K K_f \theta}{2})s + K K_f + 1}$$

The NPTEL logo is visible in the bottom left corner of the slide.

We know that padé approximation for time delay is found in the form of $e^{-\theta s}$ to the power minus θ by $2s$ divided by $1 + e$ to the power θ by $2s$, Which can be further extended in the form of $1 - \theta$ by $2s$ divided by $1 + \theta$ by $2s$. So, using that the modified process dynamics becomes $K e^{-\theta s}$ divided by $T_1 s + 1 + K K_f$ times $1 - \theta$ by $2s$ in the numerator, and $1 + \theta$ by $2s$ in the denominator. So, I have substituted the time delay term by the padé approximation giving us an expression of this form $1 + T_d s$.


So, when T_d is equated to $\theta/2$ with the choice of the derivative time constant for the PD controller, we have in the PI-PD control structure; T_d as $\theta/2$ $G'(s)$ reduces to a form $K e^{-\theta s}$ divided by $T_1 s + 1 + K K_f$ times $1 - \theta$ by $2s$. Which can further be written in the form of $K e^{-\theta s}$ divided by $T_1 - K K_f \theta/2$ times $s + K K_f + 1$. So, the modified process is given by this expression, this transfer function. Now, what information you get from this modified transfer function, if you carefully look the denominator of this transfer function.

(Refer Slide Time: 14:00)

For stability of the modified process

$$\begin{cases} K K_f \geq 1 \Rightarrow K_f \geq \frac{1}{K} \\ K K_f \leq \frac{2T_1}{\theta} \Rightarrow K_f \leq \frac{2T_1}{K\theta} \end{cases}$$

$$\boxed{\frac{1}{K} \leq K_f \leq \frac{2T_1}{K\theta}}$$

$$K_f = \sqrt{\frac{1}{K} \times \frac{2T_1}{K\theta}} = \frac{1}{K} \sqrt{\frac{2T_1}{\theta}}$$


The modified process will be stable; for stability of the modified process **for stability of the modified process**. We need to have $K K_f$ is greater than 1, because minus 1 is there; so unless $K K_f$ is greater than 1, you **you** do not get a positive constant over here or coefficient over here. So, $K K_f$ has to be greater than equal to 1. Similarly, if you look at the first term the coefficient of the first term decides to be a positive value, and for that what has to happen? $K K_f$ has to be less than $2 T_1$ by θ . So, when $K K_f$ is less than $2 T_1$ by θ , then we will get a **get a** positive coefficient in the denominator for the first term. And when all the coefficients are positive, we ensure that the modified process is stable or the dynamics of the modified process will be a stable dynamics.

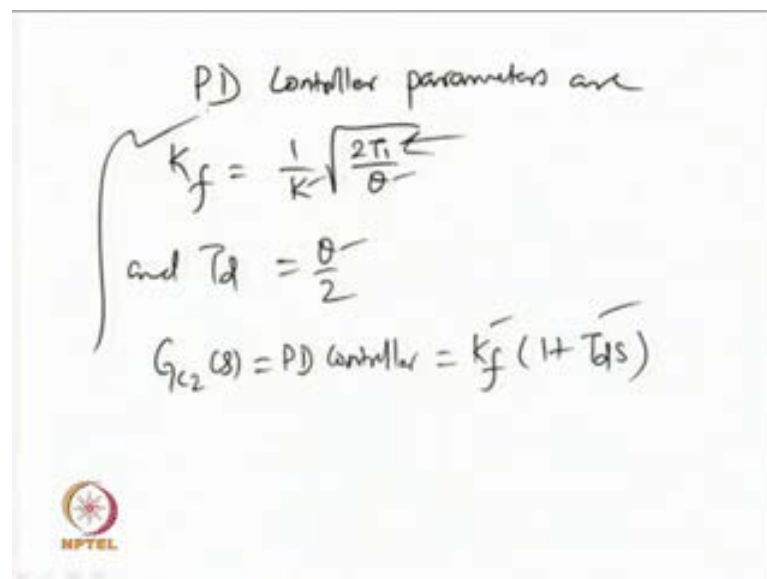
So, the second condition is that $K K_f$ has to be less than equal to **$K K_f$ has to be less than equal to** $2 T_1$ by θ . So, $2 T_1$ by θ ; when I club together these two inequalities or requirements, what I get $K K_f$ is less than equal to K_f is so $K K_f$ is less than equal to 1 is less than equal to $2 T_1$ by θ . So, this is the requirement; since $K K_f$ no, we have to have **sorry** $K K_f$ has to be greater than equal to 1, and $K K_f$ has to be less than... So, when I put the two conditions together $K K_f$ has to be greater than equal to... So, the two conditions $K K_f$ will be greater than equal to 1, definitely $K K_f$; so this is the inequalities you will get corresponding to the **to the** two requirement. So, when this is satisfied or when the two conditions are put together, we need that $K K_f$ has

to be a value less than equal to 1 not less than equal to 1 rather.

Let me, put it in an explicit form. It is not clear, so basically this requires K_f has to be greater than equal to 1 by K , and another we have got $K K_f$ **sorry** $K K_f$ here. So, K_f is to be less than equal to $2 T_1$ by $K \theta$. Now, I will put these two requirements in the form of 1 upon K is less than equal to K_f is less than equal to $2 T_1$ by $K \theta$. Now, it is correct. So, if you look at carefully, when this is made when $K K_f$ is greater than 1 indirectly speaking $K K_f$ is greater than equal to 1 by K or 1 by K is less than equal to K_f ; it is correct now.

Is again K_f is less than equal to $2 T_1$ by $K \theta$, then we get an equality inequality of this form. Now, how to find the K_f value, if I take the geometric **geometric** progression of the 2; so one upon K times $2 T_1$ upon $K \theta$ root. So, if I use that; when I use the geometric the mean for this one, I can find a value for K_f which can ultimately be given by 1 upon K , because K square is their root of $2 T_1$ by θ . So, this is how, we do estimate one parameter of the controller $G_c(s)$. So both the parameters of the PID controller have been designed now.

(Refer Slide Time: 18:26)



PID Controller parameters are

$$K_f = \frac{1}{K} \sqrt{\frac{2T_1}{\theta}}$$

and $T_d = \frac{\theta}{2}$

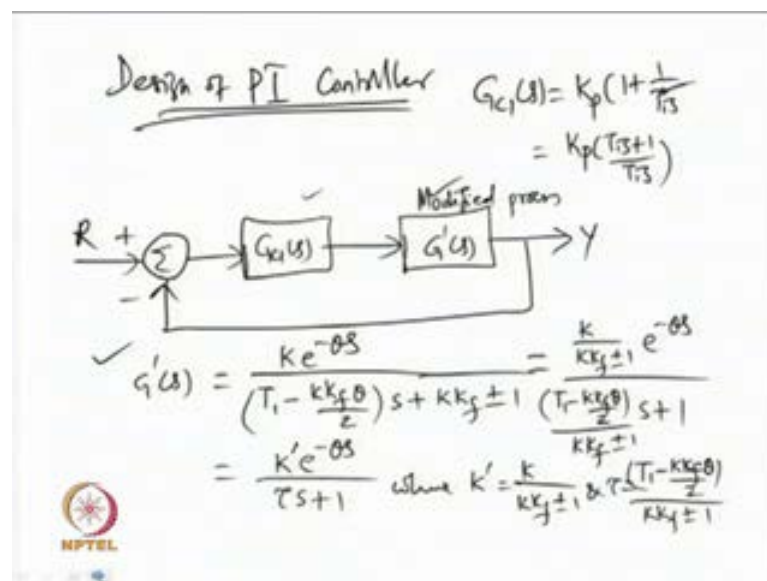
$$G_c(s) = \text{PID Controller} = K_f (1 + T_d s)$$

NPTEL

Let me rewrite again the PD controller parameters, so the PD controller parameters are K

f is equal to 1 upon K root of $2 T_1$ by θ , and T_d is equal to θ by 2 , because we know that the PD controller $G_c(s)$ which is nothing but a PD controller has got the dynamics K_f times 1 plus $T_d s$. Thus both the parameters of the PD controller are designed using of course, the parameters of a transfer function model. What are the parameters of a transfer function model? $T_1 K \theta$ **theta**; so therefore, in terms of the model parameters, we will **we will** be able to design the PD controller.

(Refer Slide Time: 19:27)



Now, we will go to the design of a PI controller. So, design of **of** PI controller. How the PI controller looks like for us $G_c(s)$ is equal to K_p times 1 plus 1 upon $T_i s$, which can ultimately be written as K_p times $T_i s$ plus 1 divided by $T_i s$. So, this is the PI controller; so design of PI controller means, now we will derive explicit expressions for the K_p , and T_i . The proportional gain, and the time constant associated with the PI controller in terms of the model parameters; that means, $T_1 \theta$ and K or $K \theta$ and T_1 .

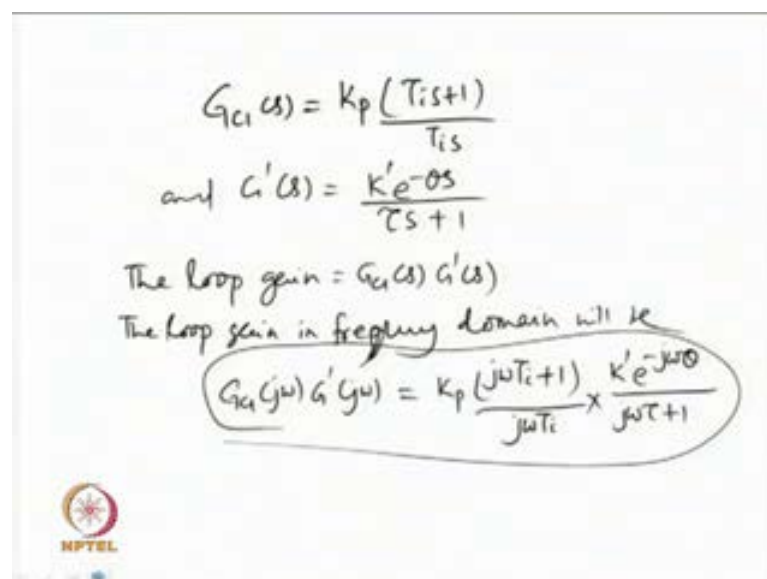
Now, to design to proceed with the design of PI controller, let me rewrite the modified process with respect to which, we will be designing the PI controller. How the loop looks like now. So, we have got reference input here, and a PI controller in the loop $G_c(s)$ subjected to the modified process, please keep in mind we have the modified process in

the loop now. So, designing controller for this will need us will require us to consider the loop gain using the phase margin, and gain margin criteria; it is possible to design the parameters of the PI controller.

The way phase, and gain margin criteria can be applied is been has been discussed already in our previous lecture, I will extend it the same to design PI controller for the PI-PD control structure. Now, I will consider the G d S modified process which is given as now G d S is now $K e^{-\theta s} \frac{1}{T_1 s + 1} \frac{K_f s + 1}{K_f s + 1}$. So, I am rewriting the modified process, please keep in mind the PI controller will be desired designed with respect to the modified process, and the modified process is derived as this.

Now, this can further be simplified, and written in the form of K divided by $K_f s + 1$ minus $1 e^{-\theta s}$ divided by $T_1 s + 1$ minus $K_f s + 1$ divided by $K_f s + 1$. Giving us a transfer function of the form $K e^{-\theta s} \frac{1}{\tau s + 1}$, where K dash is how much K divided by $K_f s + 1$ plus minus 1, and τ is equal to $T_1 s + 1$ divided by $K_f s + 1$ plus minus 1. Why I am doing so, I can get rid of the plus minus sign associated with the modified process.

(Refer Slide Time: 23:06)



Handwritten mathematical derivation of the loop gain and its frequency domain representation:

$$G_c(s) = K_p \frac{(T_i s + 1)}{T_i s}$$

and $G'(s) = \frac{K' e^{-\theta s}}{\tau s + 1}$

The loop gain = $G_c(s) G'(s)$

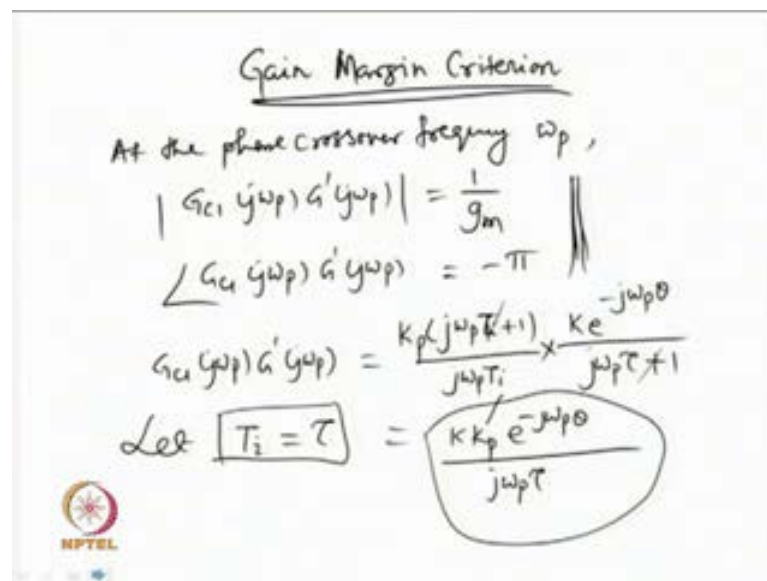
The loop gain in frequency domain will be

$$G_c(j\omega) G'(j\omega) = K_p \frac{(j\omega T_i + 1)}{j\omega T_i} \times \frac{K' e^{-j\omega \theta}}{j\omega \tau + 1}$$

NPTL logo is visible in the bottom left corner of the slide.

So, we have found the PI controller to be of the form $G_c(s) = \frac{K_p T_i s + 1}{T_i s}$, and the modified process in the form of $K e^{-\theta s}$ to the power minus θs upon $\tau s + 1$. Now, the loop gain **the loop gain** of the control loop is $G_c(s)$ times $G(s)$. In the frequency domain, the loop gain will be the loop gain in frequency domain - **in frequency domain** will be $G_c(j\omega) G(j\omega)$ **sorry**; in the frequency domain, it will be $G_c(j\omega) G(j\omega)$ substitute s by $j\omega$ simply, then you get it the frequency domain representation of the loop gain. So, $G_c(j\omega) G(j\omega)$ will be equal to $K_p j\omega T_i + 1$ divided by $j\omega T_i$ times $K e^{-j\omega\theta}$ divided by $j\omega\tau + 1$. So, these loop gain in frequency domain will be used for designing the parameters of the PI controller.

(Refer Slide Time: 24:54)



Gain Margin Criterion


At the phase crossover frequency ω_p ,

$$|G_c(j\omega_p) G(j\omega_p)| = \frac{1}{g_m}$$

$$\angle G_c(j\omega_p) G(j\omega_p) = -\pi$$

$$G_c(j\omega_p) G(j\omega_p) = \frac{K_p(j\omega_p T_i + 1)}{j\omega_p T_i} \times \frac{K e^{-j\omega_p \theta}}{j\omega_p \tau + 1}$$

Let $T_i = \tau$ = $\frac{K K_p e^{-j\omega_p \theta}}{j\omega_p \tau}$



Now, let us make the gain margin criterion for designing the parameters of the PI controller. So, using the gain margin criterion, what is the gain margin criterion, We know that at the phase cross over frequency at the phase cross over frequency ω_p loop gain will have a magnitude of inverse of gain **gain** margin. So, $G_c(j\omega_p) G(j\omega_p)$ **G dash j omega p** magnitude will be one by gain margin. And the phase angle of the loop gain at that frequency will be minus 180 degree or minus π , so using the two I will be designing the parameters S - the parameters of the PI controller.

So, what is the loop gain now, $G_c = \frac{1}{j\omega_p} \frac{K_p}{j\omega_p \tau} + 1$ by $\frac{K_e}{j\omega_p \tau} \frac{1}{j\omega_p \tau + 1}$. Now, if I look at carefully the loop gain at this phase cross over frequency, we have scope for cancelling one pole with one zero; so this pole can be canceled with G this zero with the choice of T_i is equal to τ .

So, when the PI controller time constant is equated to τ , when T_i equal to τ , then we will have this cancellation giving us the loop gain as $K \frac{K_p}{j\omega_p \tau} \frac{1}{j\omega_p \tau + 1}$ because T_i is now equal to τ . So, I will use this loop gain now with the choice of T_i is equal to τ , and apply the condition gain margin criterion to find the controller parameter K_p .

(Refer Slide Time: 27:44)

$$\frac{K_p}{\omega_p \tau} = \frac{1}{g_m} \Rightarrow \omega_p = \frac{K_p g_m}{\tau}$$

$$\pm \pi/2 \pm \omega_p \theta = \pm \pi \Rightarrow \omega_p = \frac{\pi}{2\theta}$$

$$\frac{K_p g_m}{\tau} = \frac{\pi}{2\theta}$$

$$\Rightarrow K_p = \frac{\pi \tau}{2\theta g_m} = \frac{\pi}{2\theta g_m} \cdot \frac{(T_i - K_f \theta/2)}{K}$$

$$T_i = \tau$$

So, using the two conditions: I can write the gain condition will result in $K \frac{K_p}{\omega_p \tau} \frac{1}{j\omega_p \tau + 1}$ will be equal to 1 by gain margin. And using the phase condition $\pm \pi/2 \pm \omega_p \theta = \pm \pi$. So, the first one gives us ω_p is equal to $\frac{K_p g_m}{\tau}$ sorry $\frac{K_p g_m}{\tau}$. So, it will give ω_p is equal to $\frac{K_p g_m}{\tau}$; and the second one will give you ω_p is equal to $\frac{\pi}{2\theta}$. So, equating the two ω_p 's now, you get $\frac{K_p g_m}{\tau}$ is equal to $\frac{\pi}{2\theta}$, yes so you will get K_p is equal to $\frac{\pi \tau}{2\theta g_m}$.

tau.

So, final expression for the K_p is the gain of the PI controller - the proportional gain of the PI controller K_p as $\pi \tau$ by $2 \theta K_g m$. So, when you have got information or when user define the gain margin is used, it is possible to estimate the proportional gain of the PI controller. And what about the remaining parameter of the PI controller, we have already taken T_i as tau; so in terms of the planned parameters, the PI controller parameters are estimated. What are the planned parameters? Tau is the plant parameter, now theta is the plant parameter, K is the plant parameter. Now, if you substitute tau now, here then what you what you will get the tau tau by K will give you pi by two theta g m. Please keep in mind, I have got K dash; I am making some mistake, because the loop gain has got $K_p K_p K$ dash. Please keep in mind, here we have got K dash. So, I have to make use of K dash not K; so it will give K dash K dash. Please make this correction; K will be K dash K dash.

So that way, we will have K dash over here. So, in place of K, we will have K dash, but we know that tau upon K dash will be simply here $T \frac{1 - K K_f \theta}{2}$ divided by K dash is k by $K K_f$ plus minus 1. So $K K_f$ plus minus 1 is there in the denominator of both terms. So finally, K_p expression for K_p will be this one which is in terms of the planned model parameters.

(Refer Slide Time: 31:12)

Phase margin criterion

$$\text{Loop gain} = \frac{K_p(j\omega T_i + 1)}{j\omega T_i} \times \frac{K' e^{-j\omega\theta}}{j\omega\tau + 1}$$

Choose $T_i = \tau$

$$\Rightarrow \text{Loop gain} = G_c(j\omega) G'(j\omega) = \frac{K' K_p e^{-j\omega\theta}}{j\omega\tau}$$

At the gain crossover frequency ω_g

$$|G_c(j\omega_g) G'(j\omega_g)| = 1 \Rightarrow \frac{K' K_p}{\omega_g \tau} = 1$$

$$\pi + \angle G_c(j\omega_g) G'(j\omega_g) = \phi_m$$

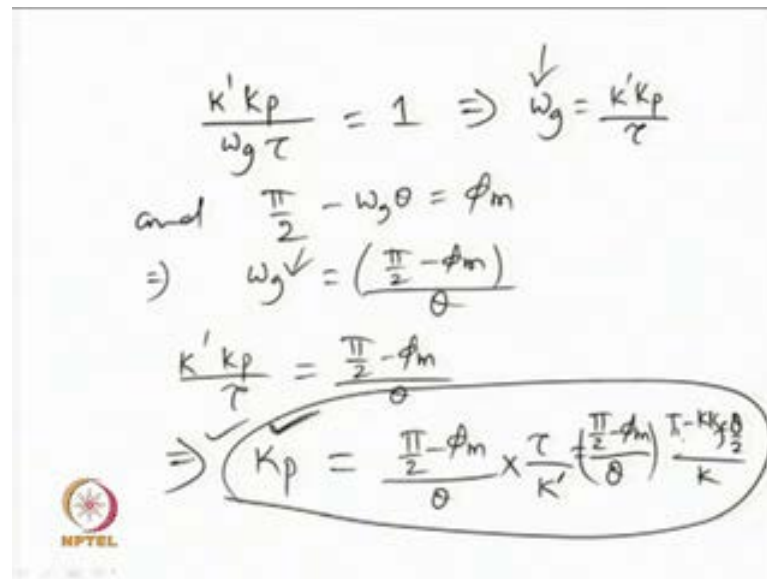
$$\pi + \left(-\frac{\pi}{2} - \omega_g \theta\right) = \phi_m$$

Now, I will use the phase margin criterion **phase margin criterion** to find the PI controller parameters. Again the loop gain in frequency domain is found to be $K_p(j\omega T_i + 1)$ divided by $j\omega T_i$ into K' , please keep in a mind. We have got K' minus $j\omega\theta$ divided by $j\omega\tau + 1$. So, choose like the earlier case choose T_i is equal to τ giving the loop gain as loop gain, now it becomes the loop gain becomes $G_c(j\omega) G'(j\omega)$ is equal to with the cancellation of this choice.

You have got $K' K_p$ to the power minus $j\omega\theta$ divided by $j\omega\tau$. So, what is the phase margin give gives us at the gain cross over frequency, **at the gain cross over frequency at the gain cross over frequency** ω_g . We get $G_c(j\omega_g) G'(j\omega_g)$ magnitude is equal to 1, and $\pi + \angle G_c(j\omega_g) G'(j\omega_g)$ is equal to the phase margin.

So, using that I will get the two as now, this will give you I will let me write here the magnitude of this function at frequency ω_g will give us $K' K_p$ by $\omega_g \tau$ is equal to 1. And the second one will give you $\pi + \text{phase angle of this one}$ will be minus 90 degree due to this j ; so minus $\pi/2$ minus $\omega_g \theta$ is equal to ϕ_m .

(Refer Slide Time: 33:51)



The image shows a handwritten derivation on a whiteboard. It starts with the equation $\frac{K' K_p}{\omega_g \tau} = 1 \Rightarrow \omega_g = \frac{K' K_p}{\tau}$. Below this, it states 'and' followed by $\frac{\pi}{2} - \omega_g \theta = \phi_m$. This is rearranged to $\omega_g = \frac{(\frac{\pi}{2} - \phi_m)}{\theta}$. Then, the two expressions for ω_g are equated: $\frac{K' K_p}{\tau} = \frac{\pi - \phi_m}{\theta}$. Finally, K_p is solved for, resulting in $K_p = \frac{\pi - \phi_m}{\theta} \times \frac{\tau}{K'} \left(\frac{\pi - \phi_m}{\theta} \right)^{\frac{1 - K_f \theta}{2}}$. The final expression for K_p is circled. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

$$\frac{K' K_p}{\omega_g \tau} = 1 \Rightarrow \omega_g = \frac{K' K_p}{\tau}$$

and $\frac{\pi}{2} - \omega_g \theta = \phi_m$

$$\Rightarrow \omega_g = \frac{(\frac{\pi}{2} - \phi_m)}{\theta}$$

$$\frac{K' K_p}{\tau} = \frac{\pi - \phi_m}{\theta}$$

$$\Rightarrow K_p = \frac{\pi - \phi_m}{\theta} \times \frac{\tau}{K'} \left(\frac{\pi - \phi_m}{\theta} \right)^{\frac{1 - K_f \theta}{2}}$$

So, what we have got $K' K_p$ by ω_g $K' K_p$ by $\omega_g \tau$ is equal to 1, and $\frac{\pi}{2} - \omega_g \theta$ $\frac{\pi}{2} - \omega_g \theta$ is equal to the phase margin. Now, from the first one again, we get ω_g is equal to $K' K_p$ by τ , and the second one will get ω_g as $\frac{\pi}{2} - \phi_m$ by θ . So, $\frac{\pi}{2} - \phi_m$ by θ . So, now equate the two ω_g ; so then you will get $K' K_p$ by τ is equal to $\frac{\pi}{2} - \phi_m$ by θ or K_p as $\frac{\pi}{2} - \phi_m$ by θ not θ by θ into τ by K' .

Now, τ by K' can be substituted, and we will get the final expression for the proportional gain of the PI controller as $\frac{\pi}{2} - \phi_m$ times T $1 - K_f \theta$ by 2 divided by K . So, this is how the proportional gain of the PI controller can be designed using the phase margin criterion. So, we have developed two formulae: one for the same gain controller of the PI controller using the phase margin condition, and using the gain margin condition. When the two expressions for K_p are observed carefully.


(Refer Slide Time: 36:01)

When K_p of ~~the~~ both the design methods are considered,

$$\frac{\frac{\pi}{2} - \phi_m}{\theta} = \frac{\pi}{2\theta g_m}$$
$$\Rightarrow \boxed{\phi_m = \frac{\pi}{2} \left(1 - \frac{1}{g_m}\right)}$$

Choose

$g_m = 2$	$\Rightarrow \phi_m = \frac{\pi}{4} = 45^\circ$
$g_m = 3$	$\Rightarrow \phi_m = \frac{\pi}{3} = 60^\circ$
$g_m = 4$	$\Rightarrow \phi_m = \frac{3\pi}{8} = 67.5^\circ$



Then what do you find? You find interestingly when K_p of the both K_p of both the design both the design methods methods are considered, then what we get? We get a relationship of the form π by 2 minus π by 2 minus phase margin divided by theta is equal to π by 2 theta g_m , cancel the two. So you get ultimately an relation of the form phase margin is equal to π by 2 will be common; so 1 minus 1 by gain margin. So, this is what you get, so there is direct relationship between the phase and gain margins given by this.

So, when you choose when you choose different gain margins; gain margin equal to 2 that time phase margin will be equal to π by 4, which is equal to 45 degree. When you choose gain margin is equal to 3, then the phase margin will be equal to 2, so π by 3; π by 3 means 60 degree. So, when you choose gain margin of a value 4, that time phase margin will be equal to π by, so 4, 1 by 4 3 by 4, so 3 by 8 π 3 π by 8 is giving roughly here 67.5 degree. So, what we observe from here with the choice of a gain margin of a value greater than 2; so, when a gain margin of a value greater than 2 is used ultimately you design a PI controller having a phase margin greater than 45 degree.

So, either you choose gain margin or phase margin, and design the PI controller parameters, this is how the PI controller parameters can be designed. Now, we will go to

the design of the PI-PD controllers using the phase, and gain margins together. Instead of using either the phase margin condition or the gain margin condition, I will use both criteria at a time. And try to design the parameters of the PI controller; PD controller will of course, be designed using the technique, we have already discussed earlier where. The derivative control parameter T_d will be equated to $\theta/2$, and your the remaining parameter K_f is found from the expression one upon K root of $2 T/1$ by θ .

(Refer Slide Time: 39:16)


Design of PI controller using both phase and gain margin criteria

The phase and gain margin criteria give

$$\begin{aligned} |G_c(j\omega_g)G'(j\omega_g)| &= 1; \angle G_c(j\omega_g)G'(j\omega_g) = -\pi + \phi_m \\ |G_c(j\omega_p)G'(j\omega_p)| &= \frac{1}{g_m}; \angle G_c(j\omega_p)G'(j\omega_p) = -\pi \end{aligned}$$

Assuming that $\omega_g T_i \gg 1; \omega_p T_i \gg 1$
 $\omega_g \tau \gg 1; \omega_p \tau \gg 1$

and using $\tan^{-1}(x) = \frac{\pi}{2} - \frac{\pi}{4x} \quad \forall x > 1$



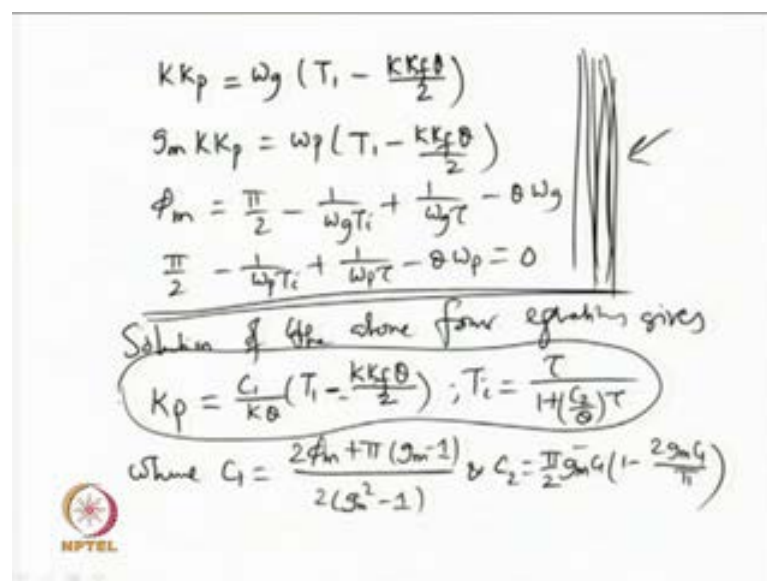
So, PD controller will be designed as usual using the earlier technique, where as the PI controller will be designed using some other technique now. So, design of the PI controller using both phase, and gain margin margins criteria at a time. So, we know that given a loop gain, the expressions for the phase and gain margins can be expressed as the phase, and gain margin criteria give us the loop gain $G_c(j\omega_g)G'(j\omega_g)$ magnitude will be one at the gain **gain** cross over frequency, at which the phase of the loop will be angle $\angle G_c(j\omega_g)G'(j\omega_g)$ will be equal to minus π plus ϕ_m .

Similarly, when the loop gain $G_c(j\omega_p)G'(j\omega_p)$ magnitude is equal to $1/g_m$ by the gain margin, at the time the phase angle will be minus π . So, $\angle G_c(j\omega_p)G'(j\omega_p)$ will be equal to minus π , so keep in mind the two frequencies - gain

cross over frequency **gain cross over frequency**, phase cross over frequency **phase cross over frequency**. So at different frequencies you have got different magnitude, and phase angle for the loop gain. So, we have got four equations now; **now** assuming **assuming** that $\omega_g T_i$ is larger than one; $\omega_p T_i$ is greater **greater** than 1, and $\omega_g \tau$ is large compare to 1, $\omega_p \tau$ is large compare to one, the 4 conditions can give us simpler expressions.

And also using the formula for the R tan function $\tan^{-1} x$ is equal to $\pi/2$ minus $\pi/4x$, for all x greater than; 1 it will be possible to find simpler expression, now given by the four equations.

(Refer Slide Time: 42:46)



Handwritten mathematical derivations for control system equations:

$$K K_p = \omega_g \left(T_i - \frac{K K_f \theta}{2} \right)$$

$$g_m K K_p = \omega_p \left(T_i - \frac{K K_f \theta}{2} \right)$$


$$\phi_m = \frac{\pi}{2} - \frac{1}{\omega_g T_i} + \frac{1}{\omega_g \tau} - \theta \omega_g$$

$$\frac{\pi}{2} - \frac{1}{\omega_p T_i} + \frac{1}{\omega_p \tau} - \theta \omega_p = 0$$

Solution of the above four equations gives

$$K_p = \frac{C_1}{K \theta} \left(T_i - \frac{K K_f \theta}{2} \right); T_i = \frac{T}{1 + \left(\frac{C_2}{\theta} \right) \tau}$$

Where $C_1 = \frac{2\phi_m + \pi(2m-1)}{2(g^2-1)}$ & $C_2 = \frac{\pi g_m}{2} \left(1 - \frac{2g_m}{\pi} \right)$



We have got four equations when $G < 1$ G dash are substituted they will yield us equations in the form of $K K_p$ is equal to $\omega_g T_i$ minus $K K_f \theta$ by 2; $g_m K K_p$ is equal to $\omega_p T_i$ minus $K K_f \theta$ by 2 ϕ_m ; the phase margin is equal to $\pi/2$ minus $1/\omega_g T_i$ plus $1/\omega_g \tau$ minus $\theta \omega_g$. And $\pi/2$ minus $1/\omega_p T_i$ plus $1/\omega_p \tau$ minus $\theta \omega_p$ is equal to 0.

So, these four equations can be obtained from the phase, and gain margin criteria using these assumptions. So, I am not going to derive all these things in this lecture, because it

takes much time to find the final expressions beyond the scope of this lecture. So that way I will write the **the the the** final expression for the PI-PD parameter, when the four analytical expressions we have got are solved simultaneously. So, when you solve these 4 equations simultaneously solution of the above 4 equations gives K_p is equal to C_1 by k_θ times T_1 minus K_f theta by 2. And T_i is equal to τ by 1 plus C_2 by theta times τ , where C_1 is equal to $2\phi_m$ plus πg_m minus 1 divided by 2 times g_m square minus 1, and C_2 is equal to π by 2 $g_m C_1$ times 1 minus 2 $g_m C_1$ by π .

So, this is how the parameters of the PI controller are designed using the phase, and gain margin criteria. So, you get some horrible expressions, because you have got four analytical expressions which are highly non-linear in nature.


(Refer Slide Time: 45:40)

Simulation study 1

Considers a higher order transfer function $G(s) = \frac{2}{(s+1)^4}$

The process model parameters and the PI-PD controller parameters obtained using $g_m = 3.6$ and $\phi_m = 30^\circ$ are tabulated in the following

Process	Process model parameters			PI-PD Controller parameters			
	K	T	θ	K_p	T_i	K_f	T_d
$G(s) = \frac{2}{(s+1)^4}$	2.0	4.613	1.773	0.282	0.707	1.140	0.886

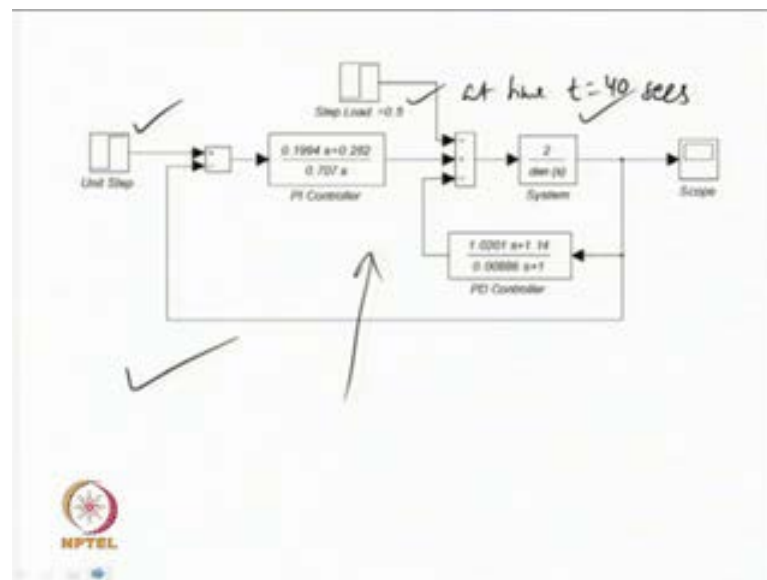
 NPTEL

Let us go the simulation studies, we will have two stimulation studies: In the first one, consider a higher order transfer function $G(S)$ given by 2 upon S plus 1 to the power 4. So, the process dynamics is of higher order. The process model parameters, and the PI-PD controller parameters obtained using the choice of gain margin of 3.6, and phase margin of 30 degree are tabulated in the following table. So, in the following table what we see for this process - the process model parameters, the first order plus dead time transfer function model parameter we obtain, I have got the steady state gain K as two

time constant T as 4.613 seconds, and time delay θ as 1.773.

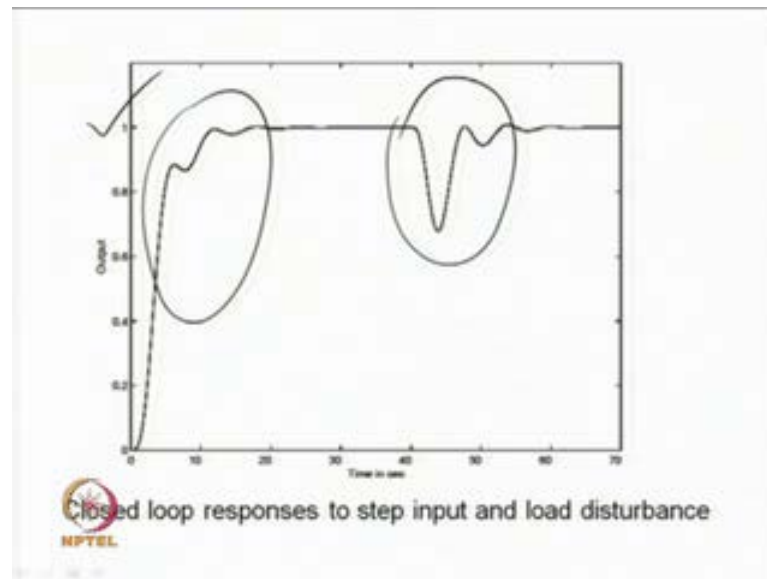
And based on the K , T , and θ the PI-PD controller parameters are estimated; using either this formula - this formula or we have got a number of formula means for K_p , T_i we have got this formula or using this formula K_f , T_d ; so we have got a number of formula for the PI-PD controllers. Where the PI-PD controllers are estimated using the process model parameters - K , T , and θ . So, the estimated values for the PI-PD controllers are given here for the parameters of the PI-PD controller are given here.

(Refer Slide Time: 47:21)



Now, we will we have simulated the PI-PD controller along with the plant in closed loop using unit step input as the reference input, and a static load disturbance of magnitude 0.5 occurring at time T is equal to 40 seconds.

(Refer Slide Time: 47:50)



So, when the PI-PD controller are put (()) in the loop for the system, what sort of result we get will see; so we get a time response of this form. So, please see this is the load response you have got, and this is the time response you have got, and the responses are found to be satisfactory.

(Refer Slide Time: 48:05)

Simulation study 2

Considers an unstable transfer function $G(s) = \frac{4e^{-2s}}{(4s-1)}$

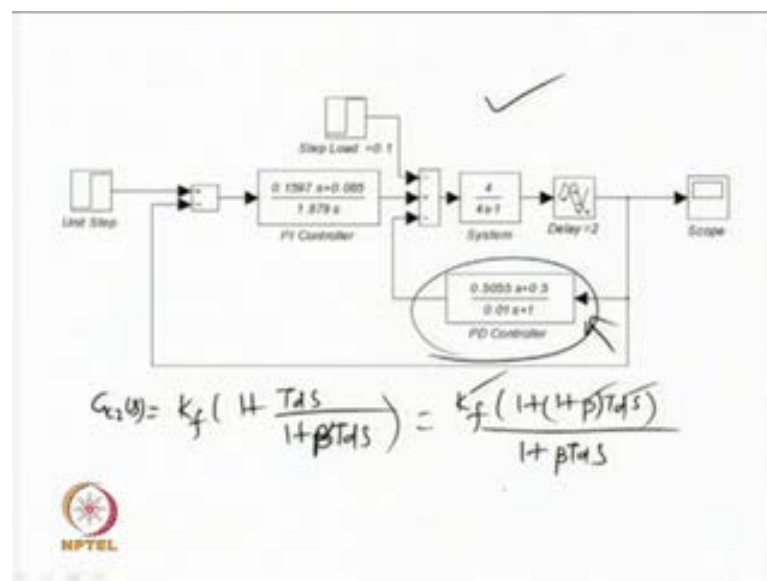
Controller parameters obtained by the method for the process model using $g_m = 4.5$ and $\phi_m = 60^\circ$ are tabulated below

Process	Process model parameters			PI-PD Controller parameters			
	K	T	θ	K_p	T_i	K_f	T_d
$G(s) = \frac{4e^{-2s}}{(4s-1)}$	4.0	3.999	2.001	0.085	1.879	0.5	1.001

NPTEL

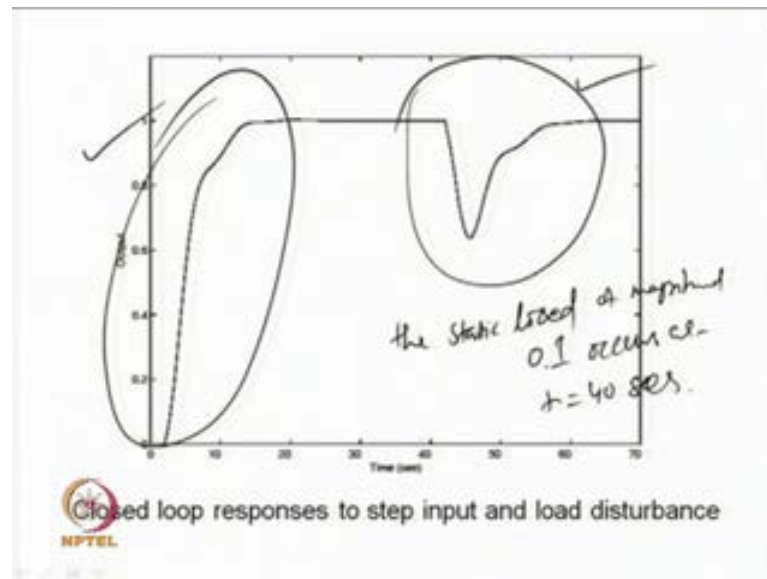
Now similarly, in the simulation to we have considered **considered** an unstable transfer function; unlike the first simulation study, here we have an unstable first ordered plus dead time transfer function model given by $G(S)$ is equal to $K e^{4s} e^{-2s}$ to the power minus 2 S upon $4S$ minus 1. And with the choice of gain margin of 4.5, and phase margin of 60 degree; the PID controller **sorry** PI-PD controller parameters are estimated as K_p is equal to 0.085, T_i is equal to 1.879, K_f as 0.5, and T_d as 1.001.

(Refer Slide Time: 48:51)



Then a simulation model has been developed. The PI controller parameters out write you can write, because you have got K_p value as this, and T_i as this; whereas the PD controller parameters you see carefully how I have found, the PD controller parameters $G_{C2}(S)$ is equal to $K_f (1 + T_d S)$ divided by $1 + \beta T_d S$ **beta $T_d S$** . That gives you $K_f (1 + \text{1 plus } \beta T_d S)$; $1 + \beta$ will remain here, $T_d S$ divided by $1 + \beta T_d S$ or that has been. So, substitute the value **value** of $K_f \beta T_d$ here, and you get this PD controller in this form. To find the correct value for the PD control PD controller, you have to use this expression.

(Refer Slide Time: 49:58)



Now, the simulation results obtained from the PID, **P** PI-PD controller are shown here. The time response for the unit step input is shown in the initial part, whereas the load response is shown in the later part. Again the static load disturbance - static load of magnitude 0.1, please keep in mind of magnitude of magnitude 0.1 occurs at time t equal to 40 seconds. So, the responses are found to be satisfactory.

(Refer Slide Time: 50:44)

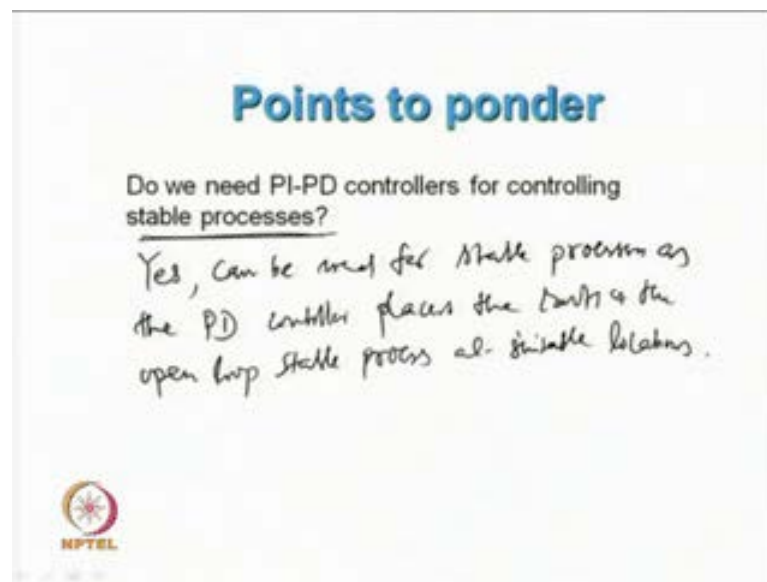
Summary

- A first order plus delay process model is estimated from a relay feedback test.
- Using the gain and phase margins based tuning formulae parameters of the PI-PD controllers are obtained.
- The proposed control technique enables one to re-tune controller settings on demand to achieve enhanced performance.
- The method can easily be extended to control a varieties of integrating and unstable processes.

NPTEL

Let us, summarize the lecture a first order plus delay process model is estimated initially from two stages of relay tests. Using the gain, and phase margins based tuning formula parameters of the PI-PD controllers are obtained. The proposed control technique enables one to re-tune controller settings, as and when necessary to achieve enhanced performance of the closed loop. The method can easily be extended to control a varieties of processes especially for controlling for effective control of integrating, and unstable processes. So, the method can find extensive use for integrating, and unstable processes.

(Refer Slide Time: 51:37)



Coming to the points to ponder: Do we need PI-PD controllers for controlling stable processes, **yes** the PI-PD controllers can be used for stable processes, **can be used for stable processes** as the PD controller - **PD controller** basically places the roots of the open loop stable process **process**, at suitable locations. Also the PI-PD controllers find extensive use in controlling integrating, and unstable processes. It can also be used for controlling stable processes. Thanks.