Advanced Control Systems

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Module No. # 04
Design of Controllers
Lecture No. # 03
Model-Free Controller Design

Welcome to the lecture titled model-free controller design. In this lecture, we will discuss about the tuning of PID controllers, without obtaining a transfer function model.

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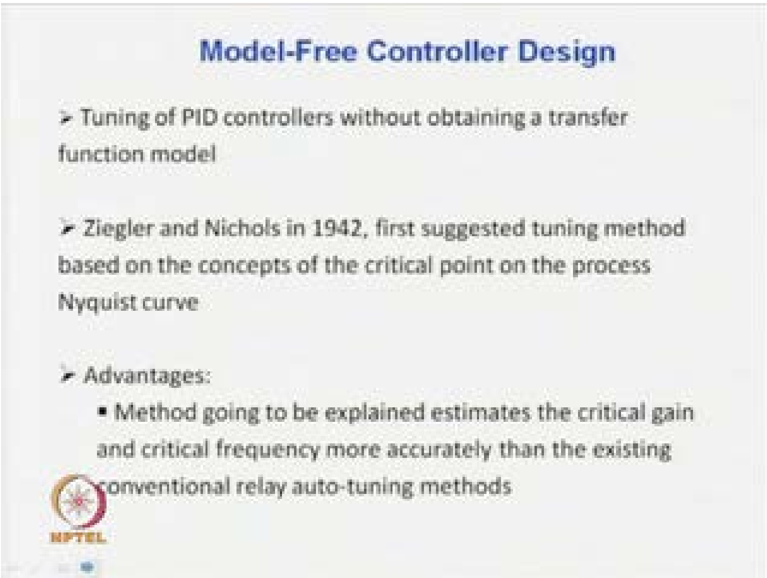

So, the parameters of the controller will be explicitly expressed in terms of the limit cycle output parameters. Ziegler and Nichols in 1942, suggested tuning method based on the concepts of the critical point on the process Nyquist curve, and since then many more methods have been proposed and suggested to design controllers without finding the transfer function models, one such method will be discussed in this lecture. And the advantages of the method, we are going to discuss in this lecture estimates the critical gain and critical frequency more accurately than the existing conventional relay auto tuning methods, because the relay will not be an ideal relay alone, it will be connected in series with some integral controller thus giving us certain advantages.

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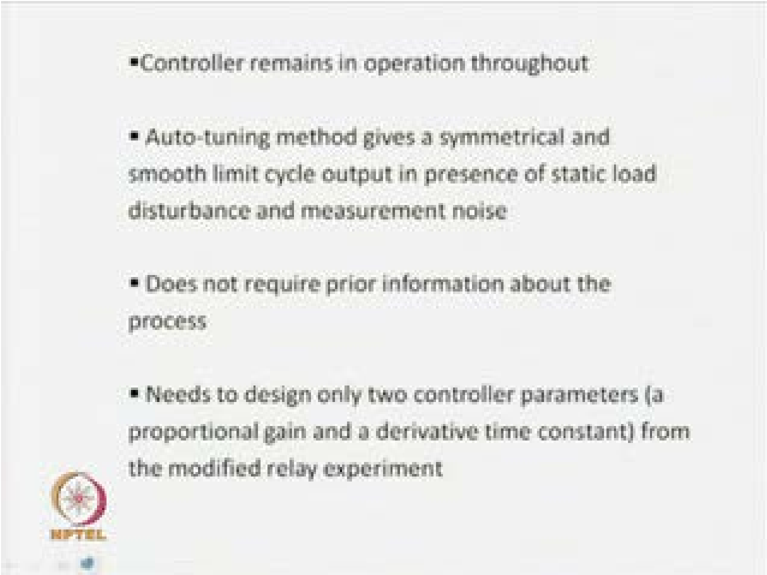

Again, the controller will remain in operation throughout the operation of the closed loop control system. Auto tuning method gives a symmetrical and smooth limit cycle output in the presence of static load disturbance and measurement noise, because of the integral controller with along with the relay or the modified relay, because of the presence of the modified relay, what we will have? We will have clean not only clean limit cycle output, but also the limit cycle output not disturbed by any static load disturbance. Now also the method does not require prior information about the process so, we need not find the transfer function model rather the parameters of the controllers will be functions of the transfer function model parameters or indirectly speaking directly using the limit cycle parameters we shall find explicit expressions for the parameters of a PID controller. So, no need of finding the transfer function model and no more process information is required for tuning the parameters of a controller.

Needs to design only two controller parameters by this method we are going to discuss, we need to design only two controller parameters; namely a proportional gain and a derivative time constant from the modified relay experiment. So when a controller of the form G c (s) is equal to K p 1 plus 1 upon T i s times q plus T d s is considered a series form of PID controller is considered, what I mean by the two controller two controller parameters, we are going to find explicit expressions for K p and T d of course, T i will be also found ultimately, but we need to design only two controller parameters such as K p and T d and we need not go for T i, because T i will be set during the relay experiment. Let us see in detail what type of auto tunings scheme we have; so initially what is done? A modified relay is employed to induce limit cycle output.

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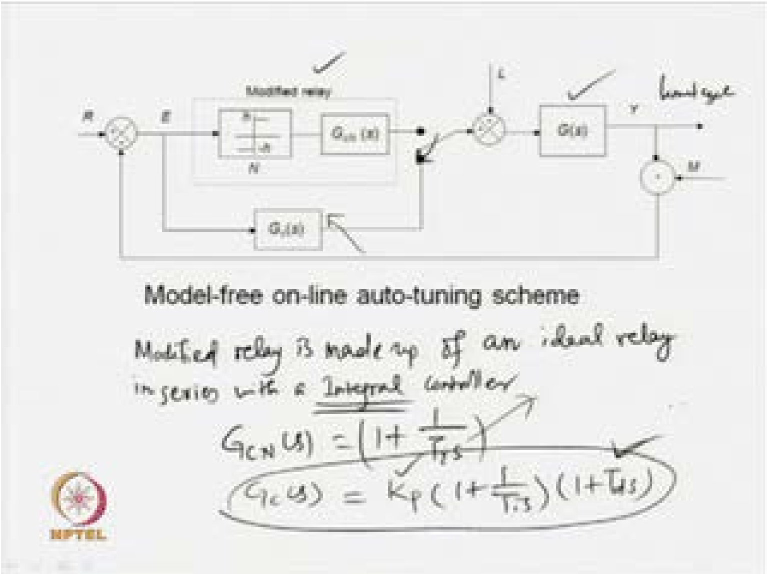

So, the switch is connected to this point and limit cycle oscillation or relay test is carried out; now, the modified relay is made up of an ideal relay so, the modified relay is made up of an ideal relay in series with in series with an integral controller integral controller. So, what is the form of the integral controller G C N (s) will be up the form 1 plus 1 upon T i s so when this integral controller is connected in series with an ideal relay we do get the modified relay and when the modified relay is connected with the process in closed loop feedback then limit cycle output is obtained so, limit cycle output is obtained.

So, after obtaining the limit symmetrical limit cycle output for symmetrical relay we will make measurements of the peak amplitude A p and the period T u which will give you the angular frequency omega equal to 2 pi upon T u. So, the frequency and peak amplitude of the limit cycle output will be measured then based on the information of frequency and peak amplitude we shall set the parameters of the controller and then the switch will be moved to this sub point for normal operation of the closed loop system so that is what we are going to do now, we have to find explicit expressions for parameters of not only the i controller, but also the remaining P D controller which will be added to this giving us a controller PID controller of the form G C (s) is equal to K p 1 plus 1 upon T i s times 1 plus T d s.

So, initially what will be done when you are starting a process or commissioning a process then you have no information about the process so, during that time to initiate the relay test what will be done the T i the integral time constant can be set to certain values like choose from 10 to 20 when of course, the process G (s) is stable so, when G (s) is a stable process then at that time please choose or set the value for T i from 10 to 20 choose any value from 10 to 20 and then you conduct the relay test then after conducting the relay test you will get the limit cycle output measure the frequency of the limit cycle output and then you reset update the value of the integral parameter and the how it will be updated that we shall discuss after sometime.

Now, then that updated value of T i will be put in the integral controller and again a relay test will be conducted to finally, find the peak amplitude and frequency of limit cycle output and that peak amplitude and frequency will be employed to find the parameters of the PID controller or basically the to find the parameter K p and T D and T i has already been found thus we will be able to get a PID controller and then we can resume the normal operation of a process that is how the model free tuning is done model free online tuning or model free design of controller is done.

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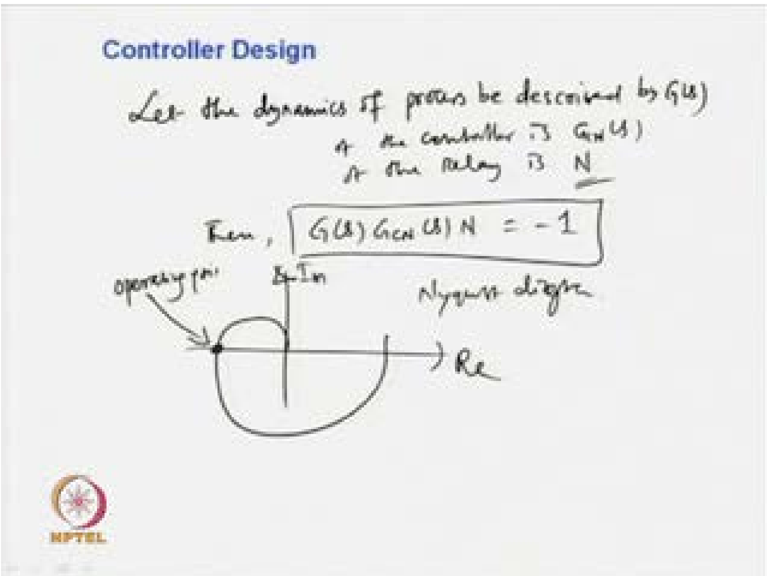

Now, I will go to the mathematics of the way we can derive explicit expressions for parameters of the PID controller. Now, when the modified relay is in the loop when the modified relay in the loop let the dynamics of dynamics of process is described by described by a transfer function G (s) and that of the controller is of the controller is G C N (s) and of the relay is N so we shall use the describing function for the relay.

So, I am using the N for that I am using the N for that then under limit cycle condition or to obtain a limit cycle condition the loop gain has to be minus 1 then G (s) G C N (s) N will be equal to minus 1, why that is so? So if you draw the nyquist diagram; so, this is your nyquist diagram nyquist diagram where we have the real and imaginary then the limit cycle condition occurs this is the operating point operating point when the relay test is conducted so the net gain will be minus 1 plus j 0 so the net gain loop gain will be minus 1 so this point corresponds to minus 1 plus j 0. Now, therefore, the loop gain will be equal to G (s) times G C N (s) times N is equal to minus 1.

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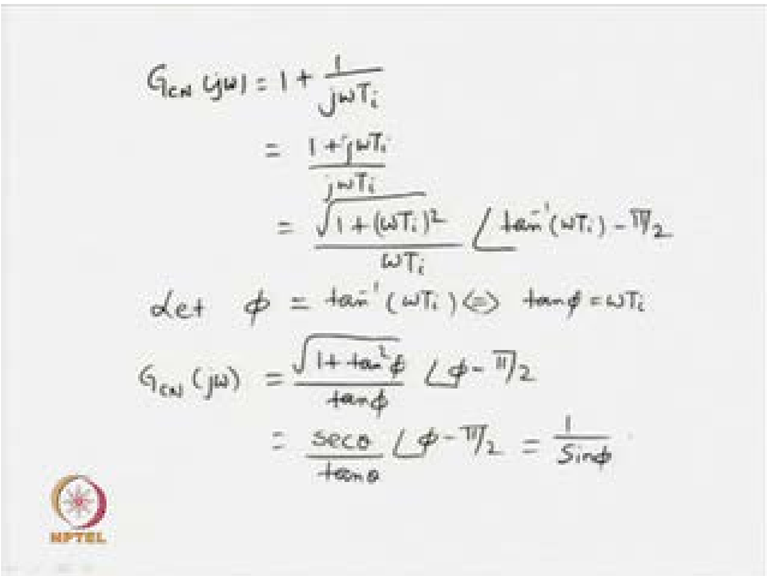

So, we know the form of G C N (s) what is G C N (s) it is given as one plus 1 upon T i s then G C N in frequency domain, now G C N (j omega) will be equal to 1 plus 1 upon j omega T i which can ultimately be expressed in magnitude and phase angle form as 1 plus j omega T i upon j omega T i giving us 1 plus omega T i square root upon omega T i with phase angle tan inverse of omega T i minus pi by 2 so this is how I get the frequency domain representation for G C N (j omega) or the integral controller. Now let me use some function phi let phi is equal to tan inverse omega T i also that implies tan phi is equal to omega T i then now that will enable us to get expression for G C N (j omega) as 1 plus tan square phi root upon tan phi with angle tan inverse omega T i is of course, phi So with angle phi minus pi by 2 so further simplification gives in the numerator sec theta divided by tan theta with angle phi minus pi by 2 which can be simplified as 1 upon sin theta with angle phi minus pi by 2.

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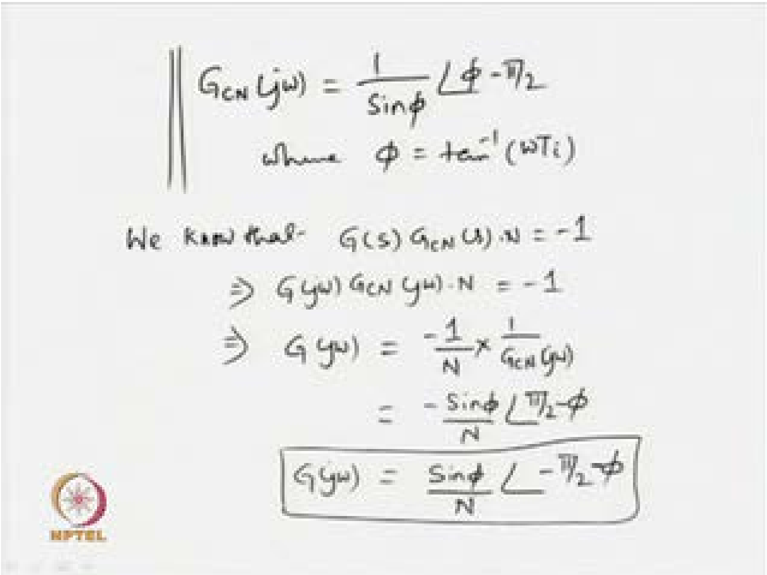

So, what we have got, the G C N G C N (j omega) gives us 1 upon sin phi with angle phi minus pi by 2 where phi equal to tan inverse omega T i so please keep in mind the in frequency domain G C N can be expressed by this form. Now, we know that we know that during the relay experiment the loop gain is minus 1 so, I have already written therefore, the loop gain G (s) G C N (s) times N is equal to minus 1 implies, in frequency domain now G (j omega) G C N (j omega) times N is equal to minus 1 implies G (j omega) the dynamics of the process can be expressed in the form of minus 1 upon N into 1 upon G C N (j omega) then what you get substitute G C N (j omega) over here, that will give us now minus sin phi upon N with angle of course pi by 2 minus phi, which again can be expressed as sin phi divided by N with angle due to this minus 1 minus pi will appear. So I will I can write this as minus pi plus pi by 2 minus phi and upon simplification you get minus pi by 2 minus phi. So, what we have got G (j omega) is equal to sin phi divided by N with angle minus pi by 2 minus phi.

Now, this is what you get during the relay experiment, now when the PID controller is injected or put in the loop when the PID controller comes into picture during normal operation of the system, that time what happens we will get a new Nyquist diagram.

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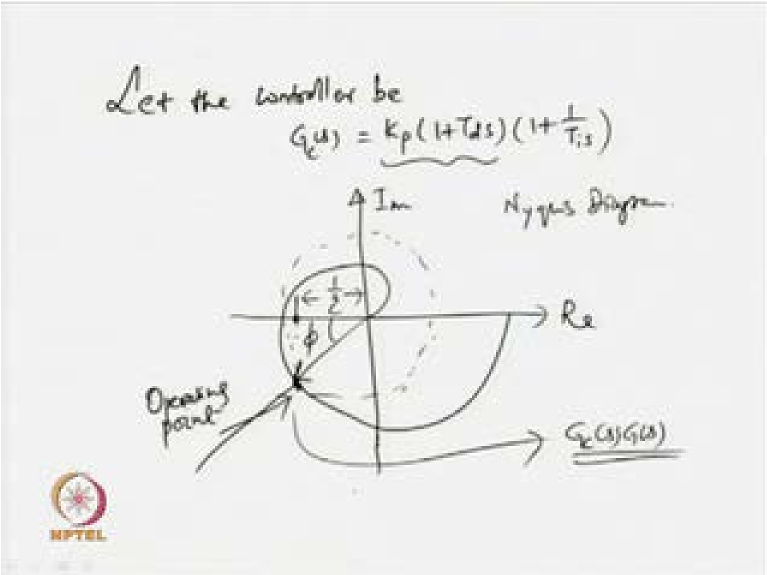

So, let the controller controlling the process be G (s) G C (s) is equal to K p times 1 plus T d s times 1 plus 1 upon T i s, now this dynamics this P D controller will push the operating point from minus 1 plus j 0.2 another operating point which will be the operating point during normal operation of the closed loop system. So, the Nyquist diagram for the new operating point will be shown now, so, let this be the nyquist diagram having real and imaginary parts and let the plot be given like this so we are going from the minus 1 plus j 0.2 some normal operating point of the closed loop system.

So, let that operating point shifted operating point be this one, because you are operating the system with a controller now or and the job of the controller is to get a new operating point such that, the phase and gain margins of a systems are improved that is the objective of posing the operating point from minus 1 plus j 0 to some new point where you will get improved phase and gain margins. So, let this be the new operating point at which you will have a an unit circle of the form it is not to the scale this the unit circle suppose then I will have a phase margin of phi for this one and corresponding gain margin gives us the magnitude 1 upon psi over here. So, this is the new operating point operating point due to the controller G C (s). So, what you have got the loop gain here in this case is, how much now, it is nothing but simply your G C (s) G (s) So, the loop gain at this is G C (s) G (s) and the phase margin is having phi and the gain margin is having psi giving us the magnitude 1 upon psi minus 1 upon psi at this point corresponding to this. So, little bit of analysis will give you the magnitude for this vector and the phase angle for this vector.

So, how this point can be represented, I can represent this point by this operating point is giving us a magnitude of suppose G C (s) G (s) will have in a magnitude with phase angle of course, how much this is minus pi plus phi, how to find M how to find M now, so, M can be found if you take the cosine of phi so cosine of phi will give you minus 1 upon psi cosine of phi times M; this is the M. So, divided by M or M can be obtained as minus 1 upon psi cost phi so, this is how the magnitude and phase of that point is found.

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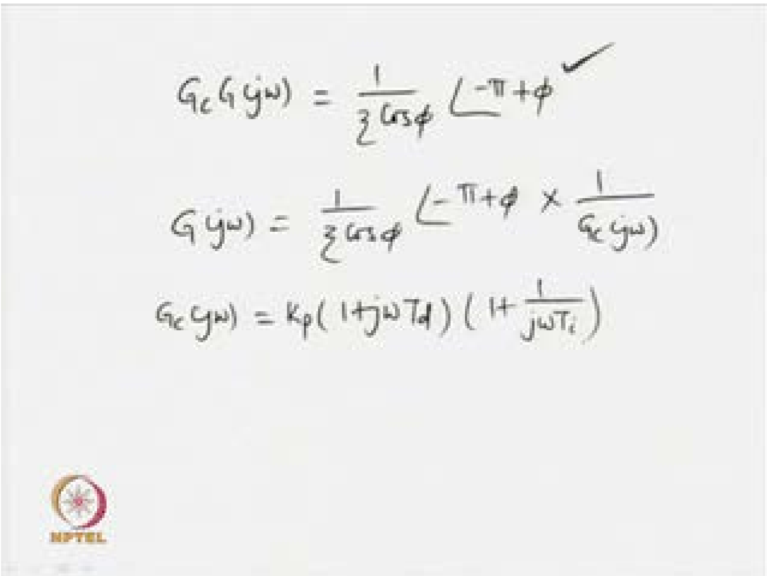

Now, this magnitude as far as magnitude is concerned I will have 1 upon psi cost phi so the new operating point will give us G C G(j omega) can be written as 1 upon xi cost phi this is the magnitude of the vector you have got and of course, with phase angle minus pi plus phi. So, once you have correctly tracked the new operating point with the phase angle of minus pi plus phi and the magnitude of 1 upon xi cos phi then further analysis can be carried out easily why how you have got this operating point I do believe, when you have got earlier the operating point here, at that time you had a relay in the loop and you were on the verge of instability or I mean to say the system oscillates or the output of the system becomes oscillatory when the when the phase is minus 180 degree and at that time you have got oscillation in the system.

So, this point has been pushed to some new operating point new operating point with the help of a controller so, when the controller dynamics is added you go to this new operating point and the new operating point results in results in the phase and gain margin so, you can easily find the phase and gain margins associated with these with the help of what the vector this vector with the help of this vector. So, as I have said if the if the gain margin is psi then 1 upon psi will be the span over here and if the phase margin is phi then you have got the angle minus pi plus phi over here so this vector the new operating point is denoted by a magnitude of 1 upon xi cos phi with angle minus pi plus phi this is very important to get correct expression for the new operating point.

So, after describing all these things let me proceed with the analysis now, the analysis will be very simple now, because you know the magnitude and phase angle of the new operating point or when the controller is in the loop. Now how much will be G (j omega) from, here G (j omega) will be equal to 1 upon xi cos phi with angle minus pi plus phi into 1 upon G C (j omega) and what is G C (j omega)? G C (j omega) the controller dynamics in frequency domain is now K p times 1 plus j omega T d times 1 plus 1 upon j omega T i this is what you have got so, G (j omega) is this much.

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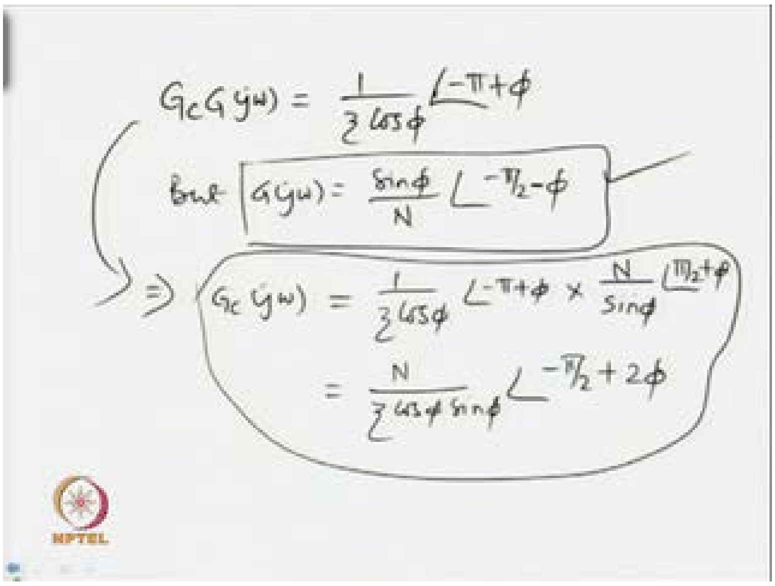

Now, I will go to little bit of analysis of this system like G C G (j omega) is equal to 1 upon xi cos phi times sorry with angle minus pi plus phi, but G (j omega) is equal to sin phi upon N with angle minus pi by 2 plus phi how I have got this one during the limit cycle we had put the loop gain to minus 1 and that gave us the expression for G (j omega) as G (j omega) is equal to sin phi upon N with angle minus pi by 2 plus phi let me show you we have already derived that so G j omega G j omega is equal to sin phi divided by N with angle minus pi by 2 plus phi minus pi so so G j omega is equal to minus pi by 2 minus phi that implies G C G (j omega) from here using that expression G C G (j omega) will be equal to 1 upon xi cos phi with angle minus pi plus phi into 1 upon G (j omega).

So substitute the value for G (j omega) here, which is nothing but sin phi divided by N so N will appear here with angle of course, it will go to the numerator, so pi by 2 plus phi. So, thus giving us G C (j omega) is equal to N divided by xi times cos phi times sin phi with net angle minus pi by 2 plus 2 phi so G C (j omega) is found to be this much.

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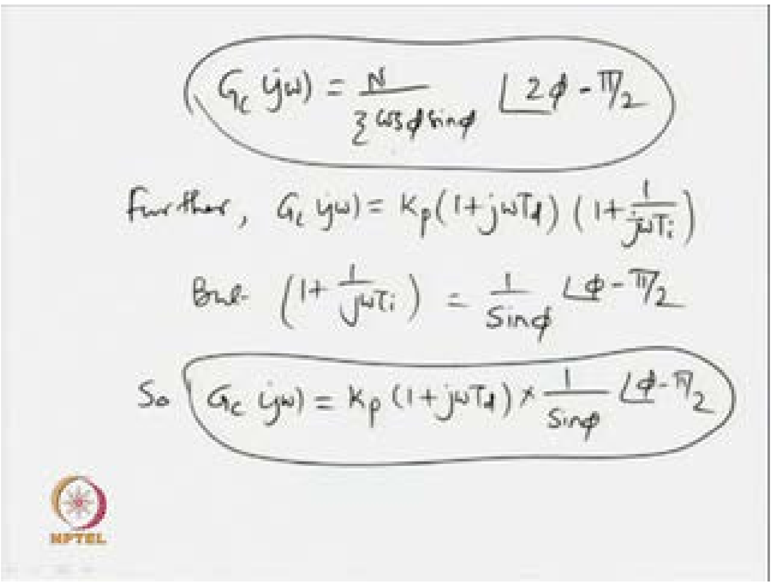

So, let me rewrite once more G C (j omega) G C j omega is equal to N by xi cos phi sin phi N by xi cos phi sin phi with angle 2 phi minus pi by 2 or the angle is minus pi by 2 plus 2 phi, further G C (j omega) is equal to K p times 1 plus j omega T d times 1 plus 1 by j omega T i, but 1 plus 1 upon j omega T i is how much that already we have found let me show you that, how much you have found so that is equal to 1 upon sin phi with angle phi minus pi by 2 so 1 upon sin phi upon with angle phi minus pi by 2 1 upon sin phi with angle phi minus pi by 2 so G C (j omega) is also equal to 1 plus j omega T d times 1 upon sin phi with angle phi minus pi by 2. So, equate the expression for G C (j omega) because you have got two expressions for G C (j omega).

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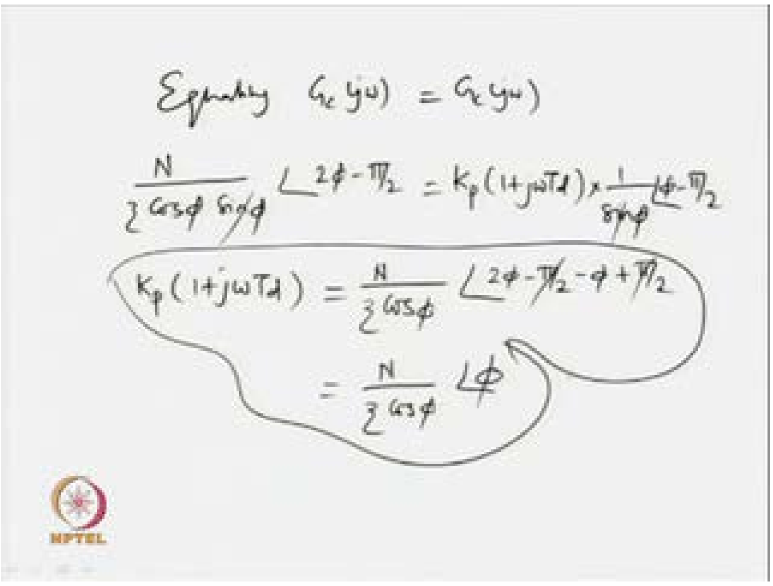

So, when you equate the two what you get equating the two will result in equating G C (j omega) as G C (j omega) and writing the both sides, now will give us N upon xi time cos phi sin phi with angle 2 phi minus pi by 2 this is equal to K p times 1 plus j omega T d with 1 by sin phi of course, with angle phi minus pi by 2. So, what I have done I am equating this with this so, equating with the right half of this so equating with the right half of this so that will give you now further simplification giving us K p times 1 plus j omega T d is equal to N divided by xi cost phi xi cost phi with angle of course, 2 phi minus pi by 2 and then minus phi plus pi by 2 so this is equal to N divided by xi cost phi with net angle of phi. So, finally, what we have obtained for the P D controller; the P D controller has to have this much in frequency domain.

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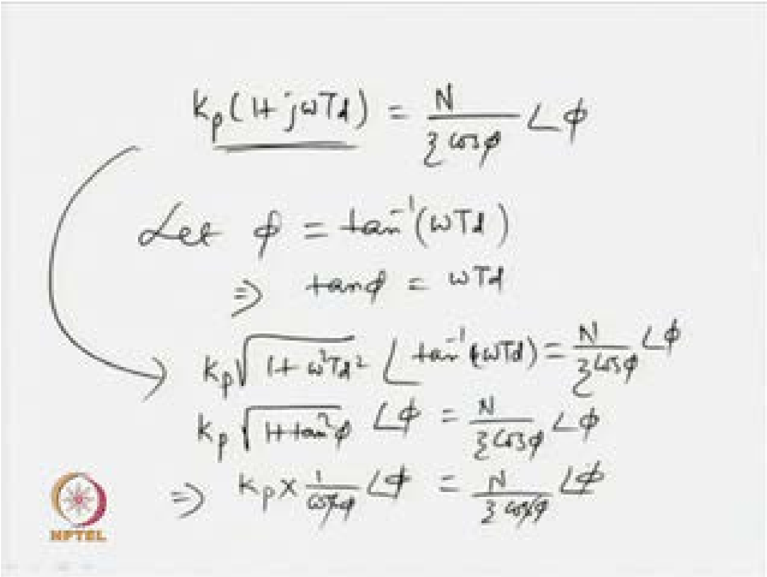

So, K p times 1 plus j omega T d has to be equal to N divided by xi cos phi with angle phi so, K p times 1 plus j omega T d is equal to N divided by xi cost phi with angle phi. So, please allow me again to find the phase angle of the left hand side in the form of let phi is equal to tan inverse omega T d now when phi is equal to tan inverse omega T d implies tan phi is equal to omega T d. Then this expression in magnitude and phase angle form because the right half is expressed in the magnitude and h form will give us K p times root of 1 plus omega square T d square with angle tan inverse omega T d is equal to N by xi cos phi with angle phi.

Now, substitution of omega T d over here will give you K p root of 1 plus tan square phi is equal to sorry with angle with angle phi is equal to N divided by xi cos phi with angle phi so, that implies K p into 1 by cos phi so root of 1 plus tan square phi will be sec phi so giving us 1 upon cos phi with angle phi is equal to N by xi cos phi with angle phi.

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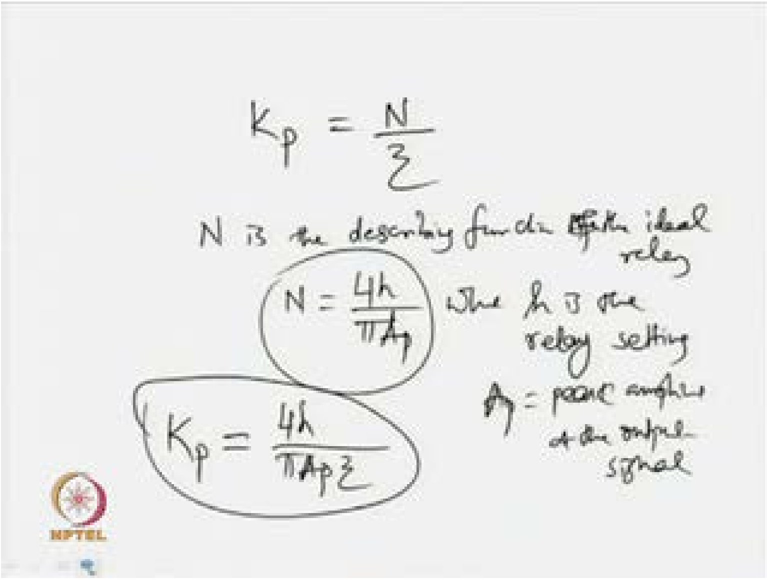

So, this will cancel out this will cancel out angle will cancel out angle will cancel out and giving us K p is equal to N by xi so K p is equal to finally, we have got K p equal to N by ziti, what is N? N is the describing function describing function for the ideal relay ideal relay what is that? So, N is equal to 4 h by pi A p where, h is the relay setting and A p is the peak amplitude of the output signal, which output signal limit cycle output signal. So, you get the final expression for K p one of the important parameter of the PID controller as K p is equal to 4 h upon pi A p times ziti.

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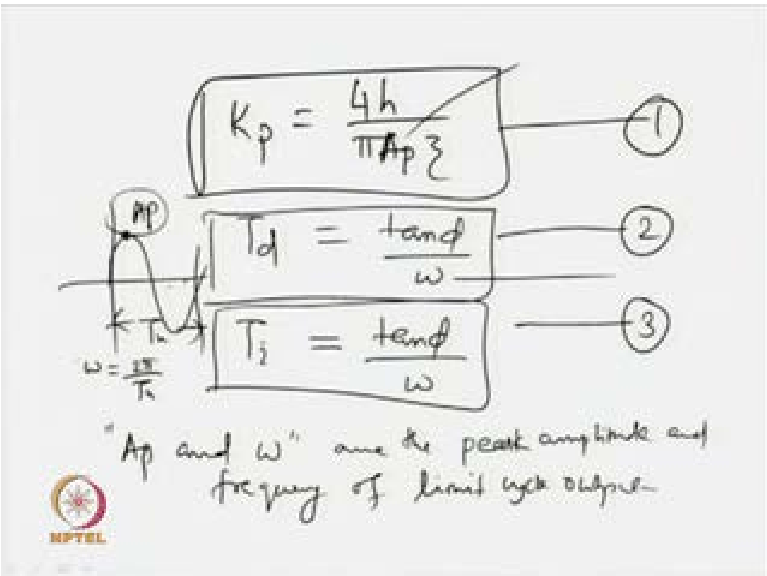

So, this is how you have got expression for the explicit expression for the parameter of the PID controller. So, k p is equal to 4 h upon pi A p xi this the final expression for the parameter of the PID control whatever do the remaining parameters T d T d is equal to if you look at, what is T d by definition tan phi is equal to omega T d. So, T d is equal to tan phi by omega so, T d equal to tan phi by omega this the explicit expression for the second parameter of the PID controller again what is the remaining controller parameter T i; so, the T i has got also the expression tan phi upon omega because we know that tan phi is equal to omega T i.

So, these are the three explicit expressions we have for the parameters of the PID controller and what are the unknowns we have in the right half of the all these three explicit parameters of the PID controller we have got A p and we have got omega so, A p and omega are the peak amplitude and frequency of frequency of limit cycle output.

So, conduct the relay test obtain the symmetrical output measure this peak amplitude A p and measure the period T u which will give you omega equal to 2 pi by T u this is how you obtain the peak amplitude and frequency from the limit cycle output substitute over here, substitute in the one, two and three and estimate the parameters of the PID controller this is how tuning of PID controller, model free tuning of PID controller is done.

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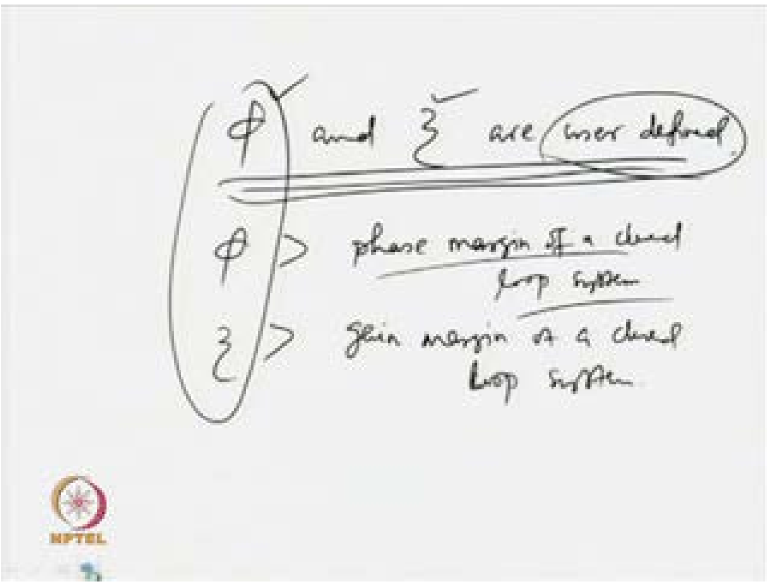

So, what is phi here and xi here let me explain so phi and xi or user or operator defined so the planned operator or system operator he decides, what will be this phi and ziti. So, phi is basically the a value with is greater than the phase margin of a closed loop system so analysis will show that phi is greater than phase margin of a closed loop system and similarly xi is greater than the gain margin gain margin of a closed loop control system. So, phi and xi is user defined so, these are also known thus we obtain the three parameters of the PID controller using formula the given over here, which uses the user defined parameters psi and xi and which also uses the parameters obtain from the limit cycle output such as peak amplitude and frequency.

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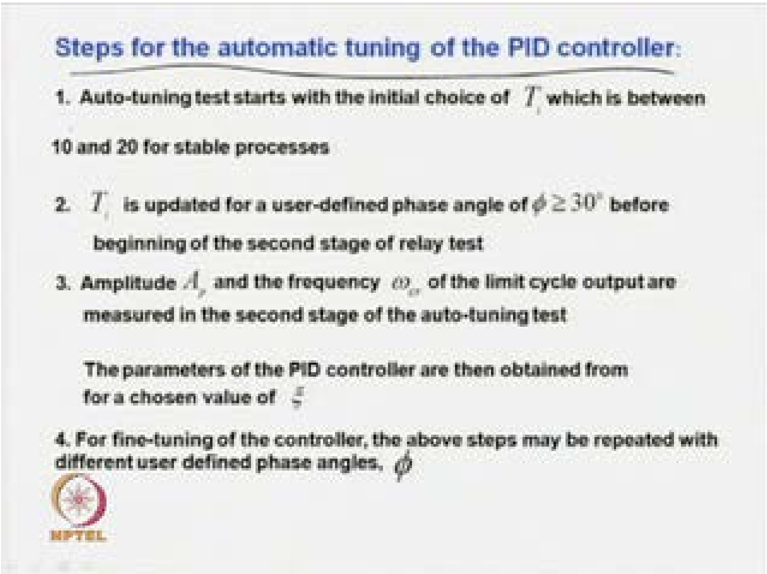

So, we will go to simulation study now and before going to the simulation studies let me once more explain the steps for the automatic tuning of the PID controller without finding transfer function models for the dynamics of a system or process or plant.

So, auto tuning test starts with the initial choice of T i which is between 10 to 20 for stable processes for unstable processes you have check, what suitable values you have to choose for that when you have no information available for the process dynamics, but once the process is in operation this parameter can be easily obtained, because you can use the default value. Now T i is updated for a user defined phase angle of phi greater than equal to 30 before beginning the second stage of relay test using the updating formula of course, T i is equal to tan of 30 degree or greater than equal to 30 degree upon omega. So, omega is the frequency you obtain from the fastest substitute over here then find T i substitute T i while conducting the second stage of relay test amplitude A p and frequency omega of the limit cycle output are output are measured in the second stage of the auto tuning test. So, those values will be used in the formula to find the PID parameters and the parameters of the PID controller are then obtained from set of the formula for a chosen value of psi and phi find tuning of the controller if necessary can be done with different user defined phase angles phi.

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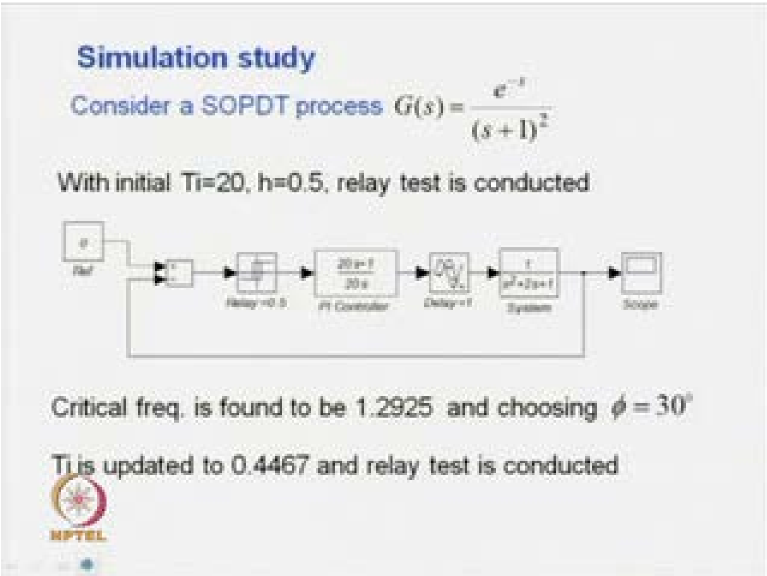

We will go to the simulation study, now consider a second order plus dead time process dynamics given by G (s) is equal to e to the power minus s upon s plus 1 square with the initial choice of T i is 20 seconds and as the relay setting of h equal to 0.5 relay test is conducted, how this is the simulation diagram is given.

So, the relay setting is plus minus 0.5 then a p i a not p i, i controller as 1 plus 1 by 20 s which gives you 20 plus 1 divided by 20 is employed for the process and relay test is conducted then the critical frequency found or the frequency of oscillation is found to be 1.2925 radiant per second and choosing a phase of 30 degree T i is now updated to 0.4467, how do you get this T i as I have said T i is equal to T i is equal to tan 30 degree divided by omega.

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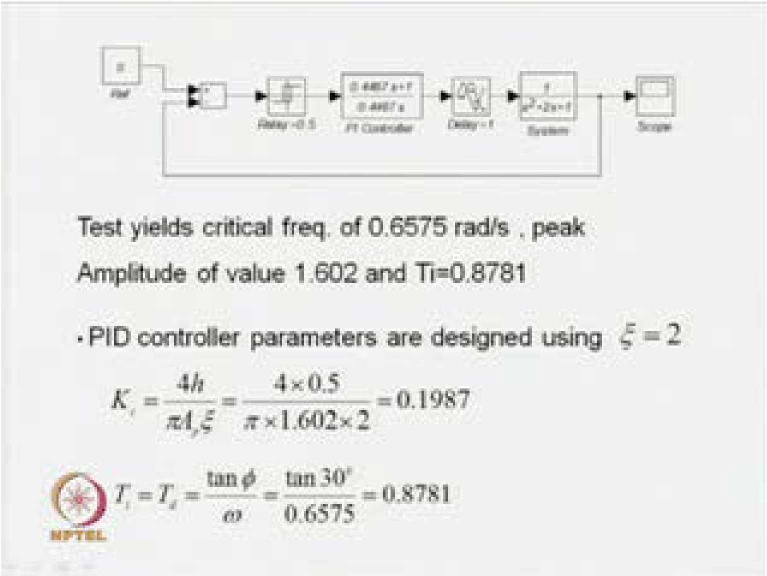

So, 1.2925 this will give you T i as 0.4467 and then another relay test is conducted so another relay test is conducted further where the i controller is now 0.4467 is plus 1 divided by 0.4467.

Then the test yields a critical frequency of omega is equal to 0.675 radiant per second so this is the final value we will be using in the formula for PID parameters amplitude of the peak amplitude of the output is of the magnitude 1.602. So, omega is 0.6575 and A p is equal to 1.602 then the T i is calculated using the formula T i is equal to tan 30 degree tan 30 degree divided by omega so, 0.6575 giving us T i as T i is equal to 0.8781 thus the T i one parameter of the PID controller is finally estimated then the remaining parameters are calculated h, K c is equal to 4 h divided by pi A p ziti, where xi is the user defined value we have chosen a gain margin of xi is equal to 2.

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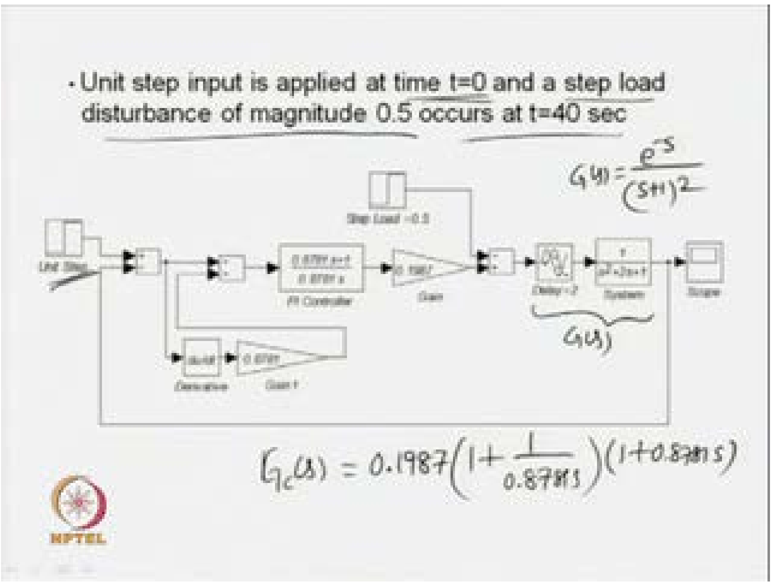

So, K c is equal to 4 h divided by pi A p xi is equal to 4 into h is 0 divided by pi into A p is 1.602 and xi is 2 giving us K p sorry not K c K p as K p is equal to 0.1987 and the other parameters T i which is equal to T d as the expression for both are same so T i is equal to T d is equal to tan phi divided by omega is equal to tan 30 degree divided by 0.6567 is equal to 0.8781. So, T i is equal to T d is equal to 0.8781 thus all the parameters of the PID controller are estimated.

Let us see what sought of result we get from the PID controller, how the PID controller performs? So, a simulation diagram is employed to see the performance of the PID controller this is the process your G (s). So, the process dynamics is now G (s) is equal to e to the power minus s divided by s plus 1 square this is what we have considered please keep in mind we had using a process G (s) is equal to e to the power minus s divided by s plus 1 square. So, the process is there this is the process.

Now, the controllers are present here, now we have got the controller 1 plus 1 divided by 0.8781 which is given in this form is presented and gain up K p is found to be 0.1987 and the P D controller is realized here, if you carefully see observe then what you have got from here it is nothing but one plus T d s this is realized using the derivate block with a gain and you have of course 1 plus T d s realized by this.

So, the series form of PID controller is employed now for the system G (s) given by G (s) is equal to e to the power minus s upon s plus 1 square. So, the PID controller is now

G C (s) is equal to 0.1987 times 1 plus 1 divided by 0.8781 times 1 plus 0.8781 so this PID controller is employed now. So, whole PID controller is from here to here please keep in mind this is the PID controller G C (s) can be realized in this form then you need a applied at time t equal to zero unit step is the reference input and a step load disturbance of magnitude 0.5 occurs at time t equal to 40 second.

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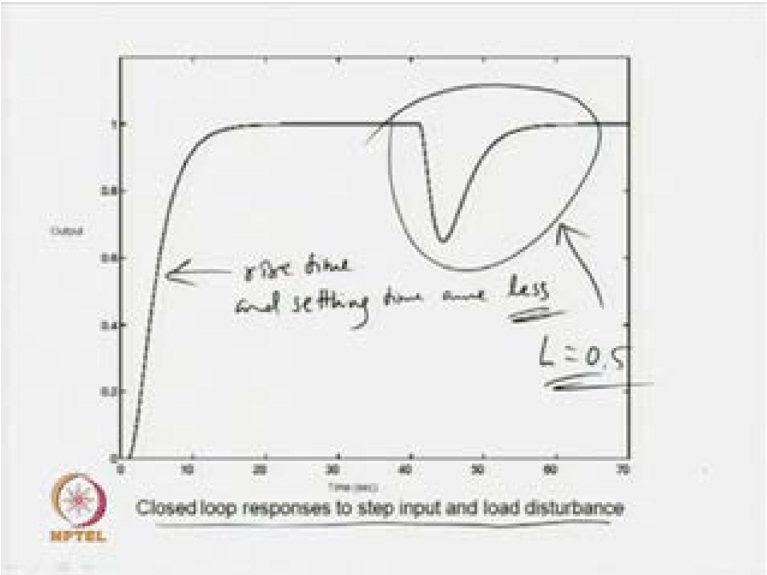

So, the system is subjected to a static load disturbance of magnitude t equal to 40 seconds then what sort of performance we get this is the response we are getting so the closed loop response to step input and load disturbance is shown over here so we have got quite satisfactory time response for the closed loop system of course, with the PID controller using the model free controller design technique, why I am telling we have got quite successful or quite useful response time response, because the rise time rise time and settling time the two important parameters associated with the time response after process are found to be satisfactory.

So, rise time and settling time are less and the disturbance response is also satisfactory because the disturbance static load disturbance magnitude l is equal to 0.5 so we have got satisfactory reference input as well as load disturbance rejection from the closed loop system.

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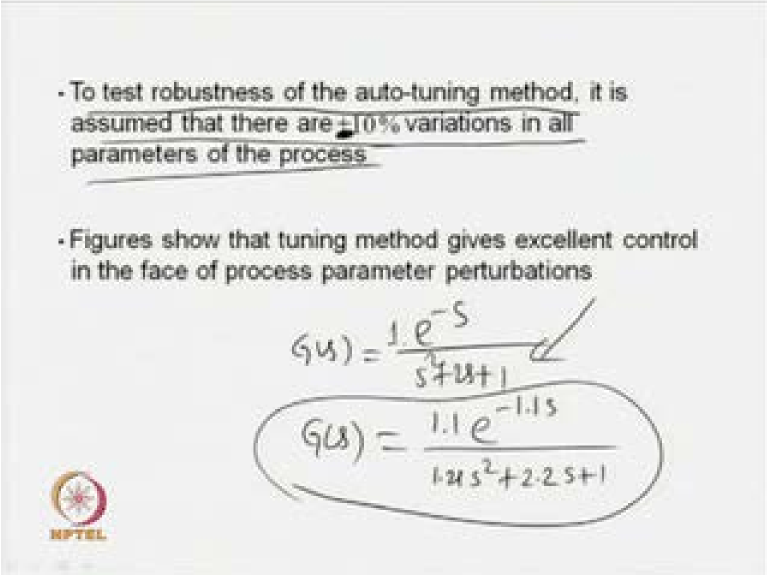

Now, to test the robustness of the method it is assumed that there are plus 10 percent variations in all parameters of the process; that means, the process was G (s) was e to the power minus s upon s square plus 2 plus 1, I have varied the different parameters the steady state gain was 1, I have made it to 1.1 the delay is made to 1.1 seconds then the time constants due to the variation in the time constant I have got 1.21 is square plus 2.2 s plus 1 so this is the perturbed plant.

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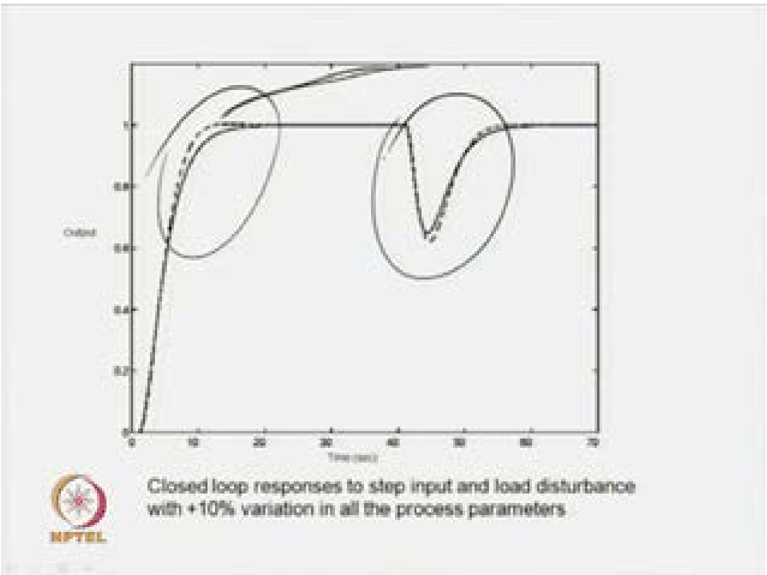

So, assuming plus 10 percent variations in all parameters of the process, now we will see the response we get, we get a performance given by the dashed line. So, this is the response we get from the perturbed plant so the two responses are not different not significantly different from each other giving us or informing us that we do get robust responses provided by the PID controller.

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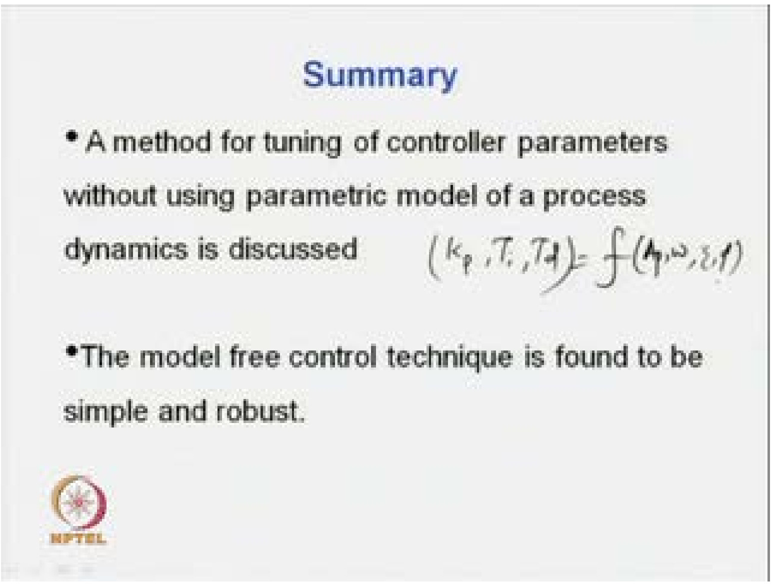

So, let me summarize the lecture now we have discussed a method for tuning of PID controllers without using parametric model of a process dynamics directly we have found formulated some tuning rules based on the limit cycle parameter. So, K p, T i and T d are now functions of K p, T i, T d all the three parameters of the PID controller are functions of peak amplitude and frequency of the limit cycle output and user defined xi and phi the phase and gain of a system gain margin of a system. The model free control technique is also found to be simple and robust it does give robust performances because the design is based on phase and gain margins.

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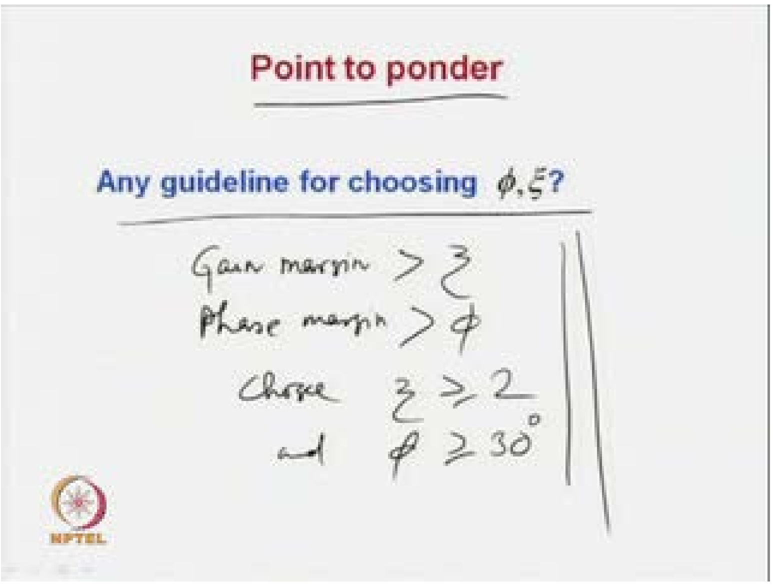

Let us come to the point to ponder any guideline we have for choosing the phase and the gain values, generally it has been found from analysis that the gain margin of the loop gain margin is found to be greater than the user defined xi and the phase margin phase margin is similarly found to be greater than phi. Therefore, one can choose xi to be greater than equal to 2 and phi to be greater than equal to 30 degree that will give not only robust performances rather satisfactory time domain specific performances also for the closed loop system. Thanks.