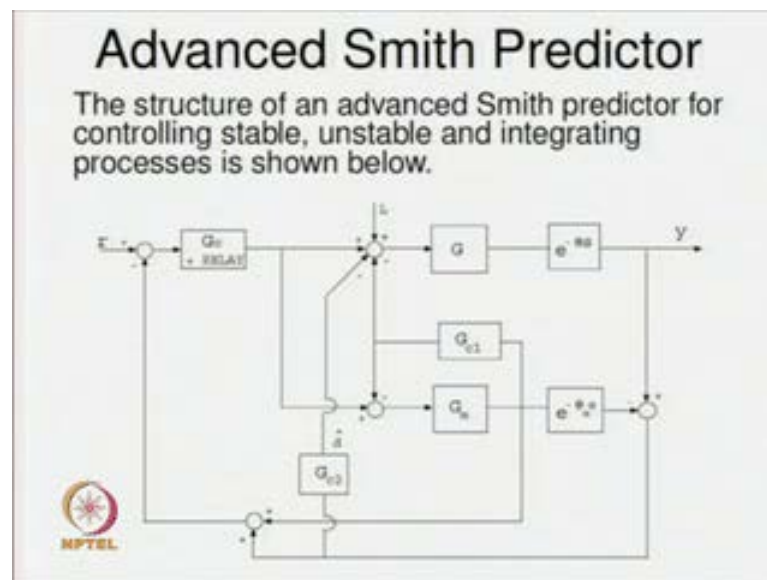


Advanced Control Systems
Prof. Somanath Majhi
Department of Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati

Module No. # 04
Design of Controllers
Lecture No. # 02
Design of Controllers for the Advanced Smith Predictor

Welcome to the lecture titled design of controllers for the advanced smith predictor. In this lecture, we shall discuss about the design technique for the controllers of the advanced smith predictor proposed by Majhi et al. In our last lecture, we have seen the structure for the Majhi et al's modified smith predictor and, controller; there were three controllers in the structure.

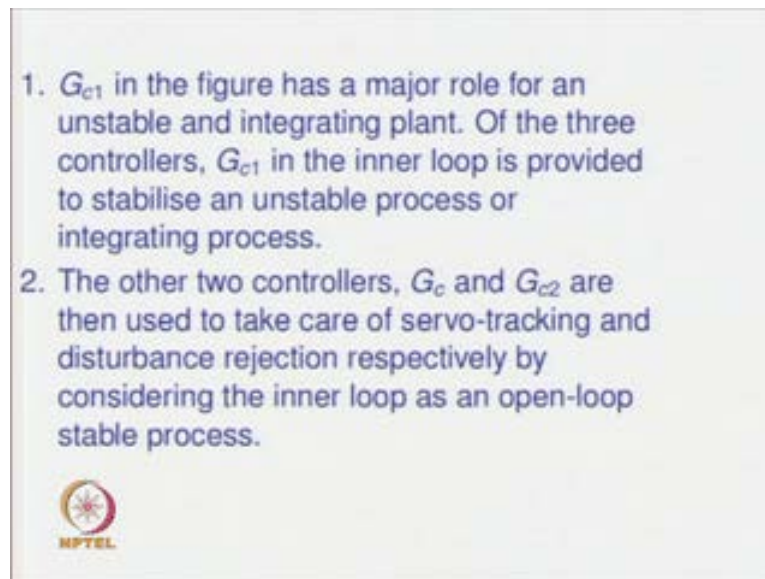
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Now the structure has been reproduced, once more where the servo controller or servo tracking controller G_c is also connected in parallel with a relay. The main objective of the relay is to identify the dynamics of the real time process. So, once the dynamics of the real time process is obtained, then that is now shown by the transfer function model $G_m e^{-\theta s}$.

The advanced smith predictor has got the three controllers: G_c , G_{c1} , and G_{c2} ; initially a relay is connected in parallel with the controller G_c for inducing limit cycle signal, and using the limit cycle data, and the state space based exact analytical expression, we have to obtain an earlier for estimations of parameters of a $(())$ parameters of a transfer function model, for the dynamics of a real time system. The G_m to the power minus $\theta_m s$ or the time delay θ_m , and parameters of delay free part of the transfer function model G_m are obtained.

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Next, G_{c1} in the figure has a major role for an unstable, and integrating plant. The G_{c1} stabilises, open loop unstable integrating plants of the three controllers G_{c1} in the inner loop is provided to stabilise open loop unstable process or integrating process. Also it can relocate the positions of poles of stable processes. The other two controllers as I have said earlier G_c , and G_{c2} are used to take care of servo tracking, and disturbance rejection by considering the inner loop as an open loop stable process. What we mean by that with the help of G_{c1} , we are getting some modified process, and we assume that modified process to be a stable one; assuming that we are as if dealing with a stable process G_c and G_{c2} are designed to improve upon servo tracking, and disturbance performances

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Development of the Tuning Algorithms
 The closed loop response to setpoint and disturbance inputs is given by

$$Y(s) = Y_r(s)R(s) + Y_L(s)L(s) \quad (1)$$

where

$$Y_r(s) = \frac{GG_c e^{-\theta s}}{(1 + G_m[G_c + G_{c1}]) \frac{(1 + G_{c2}G_m e^{-\theta_m s})}{(1 + G_{c2}G e^{-\theta s}) + G_c(G e^{-\theta s} - G_m e^{-\theta_m s})}} \quad (2)$$

$G_m e^{-\theta_m s}$ and $G e^{-\theta s}$ are the transfer functions of the plant model and the plant



Now, how the tuning algorithms or the formulae for or them technique method for designing the three controllers are done will be explained now. The closed loop response to set point, and disturbance inputs is given by $Y(S)$ is upon equal to $Y_r(S) R(S)$ plus $Y_L(s) L(s)$, where $Y_r(S)$ is the transfer function which is given by $GG_c e^{-\theta s}$ upon $1 + G_m[G_c + G_{c1}] \frac{(1 + G_{c2}G_m e^{-\theta_m s})}{(1 + G_{c2}G e^{-\theta s}) + G_c(G e^{-\theta s} - G_m e^{-\theta_m s})}$.

The detail derivation is again avoided here, already we have discussed that with the use of signal flow graph; it is not difficult to obtain this expression $Y_r(S)$. Now, $G_m e^{-\theta_m s}$, and $G e^{-\theta s}$ are the transfer functions of the plant model, and the plant. And please keep in mind G_m is the delay free part of the transfer function model.

As is evident from this denominator under model matching condition, as you see in equation number; one part of the denominator will get eliminated or removed, thus giving us a relatively simpler expression for $Y_r(S)$.

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$$Y_L(s) = \frac{Ge^{-\theta s}}{(1 + G_m[G_c + G_{c1}]) \frac{(1 + G_m[G_c + G_{c1} - G_c e^{-\theta_m s}])}{(1 + G_{c2} G e^{-\theta s}) + G_c(G e^{-\theta s} - G_m e^{-\theta_m s})}} \quad (3)$$

Based on the assumption that $G_m = G$ and $\theta_m = \theta$,
Equations (2) and (3) reduce to

$$Y_r(s) = \frac{GG_c e^{-\theta s}}{1 + G(G_c + G_{c1})} \quad (4)$$

$$Y_L(s) = \frac{Ge^{-\theta s}}{1 + G(G_c + G_{c1})} \frac{1 + G(G_c + G_{c1}) - GG_c e^{-\theta s}}{1 + GG_{c2} e^{-\theta s}} \quad (5)$$

Similarly, $Y_L(s)$ the response or the transfer function between the output to the load disturbance can be given as $Y_L(s)$ is equal to this much, as shown in equation number three. Now, based on the assumption that G_m is equal to G , and θ_m is equal to θ ; equations two and three. What is equation two? You see the equation two here. So, equation two, and three reduce to $Y_r(s)$ is equal to $GG_c e^{-\theta s}$ upon $1 + G(G_c + G_{c1})$, and $Y_L(s)$ is equal to $Ge^{-\theta s}$ upon $1 + G(G_c + G_{c1})$ into $1 + GG_c e^{-\theta s}$ upon $1 + GG_{c2} e^{-\theta s}$.

Please look at equation number 4 carefully, the denominator is now free from any time delay term. And that particularly facilitates in designing controllers for the advanced smith predictor control structure. Also you please see that equation number 4, and five shows us that we have got two different denominators; the denominators of equation 4 is different from that of the equation number five. The equation 4 does not have a term like $1 + GG_{c2} e^{-\theta s}$ in the denominator. Therefore, the two responses; the set **set** point response, and the disturbance response are decoupled from each other. What we mean by decoupling of the **(())** responses; one response does not affect the other or indirectly speaking, the controller you have designed for set point or servo tracking will not get effected by that from the controller, you have designed for disturbance rejection.

Also, when G_{c2} equal to zero. That means, when we do not have the disturbance response controller; particularly for unstable process $Y L(S)$ will give unbounded output. That you can make out provided you substitute G_c , G_{c1} by some transfer functions.

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1. Equations (4) and (5) show that the stability of the advanced Smith predictor depends on the roots of the characteristic equation

$$(1 + G[G_c + G_{c1}])(1 + GG_{c2}e^{-\theta s}) = 0 \quad (6)$$

2. Since the roots of the factor $(1 + G[G_c + G_{c1}])$ can be placed properly by the standard form based design, the optimum phase margin approach for design of a proportional controller for a time delay process that has a single right-half plane pole, can be used to find the parameters of

Equations **equations** 4 and five again show that, the stability of the advanced smith predictor depends on the roots of the characteristic equation, $1 + G$ times G_c plus G_{c1} times $1 + G G_{c2} e^{-\theta s}$ is equal to zero. Particularly, if you look carefully equation number five, it has the denominator having the characteristic polynomial given over here or the characteristic equation given in equation number six.


Earlier we designed controller G_c , and G_{c1} based on equation number 4; since the roots of the factor $1 + G G_c + G_{c1}$ can be placed properly by **by** the standard form based design. Which we are going to discuss, after some time the optimum phase margin approach for design of a proportional controller, for a time delay process that has a single right half plane pole can be used to find the parameter of G_{c2} . So, basically equation number six has got three controllers, and those are G_c , G_{c1} , and G_{c2} ; design of G_c , and G_{c1} are done with the help of standard form. Whereas, G_{c2} is obtained using some optimum phase margin criteria.

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The Case of an Unstable FOPDT Process
For the process

$$G_p(s) = \frac{Ke^{-\theta s}}{(T_1s - 1)} \quad (7)$$

and selecting $G_c = K_p + \frac{K_i}{s}$, $G_{c1} = K_b$ and $G_{c2} = K_l$
yields the delay free part of Equation (4) as

$$Y_r'(s) = \frac{KK_p s + KK_i}{T_1 s^2 + (KK_p + KK_b - 1)s + KK_i} \quad (8)$$


Consider the case of an unstable first order plus dead time process. The process can be given by $G_p(s)$ is equal to $Ke^{-\theta s}$ upon $T_1s - 1$. Now, I will design the controllers: three controllers for this unstable first order plus dead time process.

Let us select the form of the controller G_c , as a pi controller having G_c equal to K_p plus K_i upon s . Now, G_{c1} the stabilizing controller is a proportional controller given by G_{c1} is equal to K_b , and G_{c2} is equal to K_l ; please keep in mind this is not K_i that is G_{c2} is equal to K_l . Now, when you substitute G_c , G_{c1} , G_{c2} , and G_p , then the delay free part of equation 4, the delay free part of equation 4; please look at the delay free part of equation 4 means $e^{-\theta s}$ will not be there. We will have G_c upon $1 + G$ times G_c plus G_{c1} will yield an expression of the form $Y_r'(s)$ is equal to $KK_p s + KK_i$ divided by $T_1 s^2 + KK_p + KK_b - 1$ times s plus KK_i . Now, how to obtain this in the standard form, if you divide the numerator of equation 8, and the denominator of equation 8 by the term KK_i .

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Normalisation with $\beta = \sqrt{\left(\frac{KK_i}{T_1}\right)}$ gives

$$Y_r(s_n) = \frac{c_1 s_n + 1}{s_n^2 + d_1 s_n + 1} \quad (9)$$

where

$$c_1 = \beta(K_p/K_i) \quad (10)$$

and

$$d_1 = (KK_p + KK_b - 1)(T_1\beta)^{-1} \quad (11)$$


Then you will get another expression which can ultimately be expressed in the normalized form as $Y_r(s_n)$ is equal to $C_1 s_n + 1$ divided by $s_n^2 + d_1 s_n + 1$. Where C_1 is equal to βK_p divided by K_i , and d_1 is equal to $KK_p + KK_b - 1$ times $T_1\beta$ inverse. So, this is how, we have obtained a standard form, we have obtained a standard transfer function $Y_r(s_n)$ with the help of the normalization with β equal to square root of KK_i/K_i upon T_1 .

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Normalisation with $\beta = \sqrt{\left(\frac{KK_i}{T_1}\right)}$ gives

$$Y_r(s_n) = \frac{c_1 s_n + 1}{s_n^2 + d_1 s_n + 1} \quad (9)$$

where

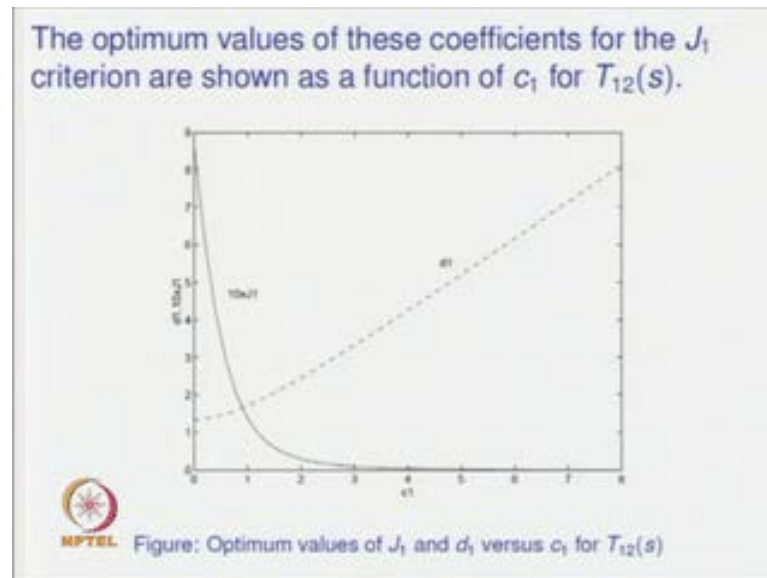
$$c_1 = \beta(K_p/K_i) \quad (10)$$

and

$$d_1 = (KK_p + KK_b - 1)(T_1\beta)^{-1} \quad (11)$$


So, basically the dynamics of equation number 8, and equation number nine are not different from each other. Of course, there is a scaling factor, because you are getting a normalized frequency now in place of the normal frequency of operation. So, it will be nearly scaling of the responses, you will have corresponding to this normalized transfer function.

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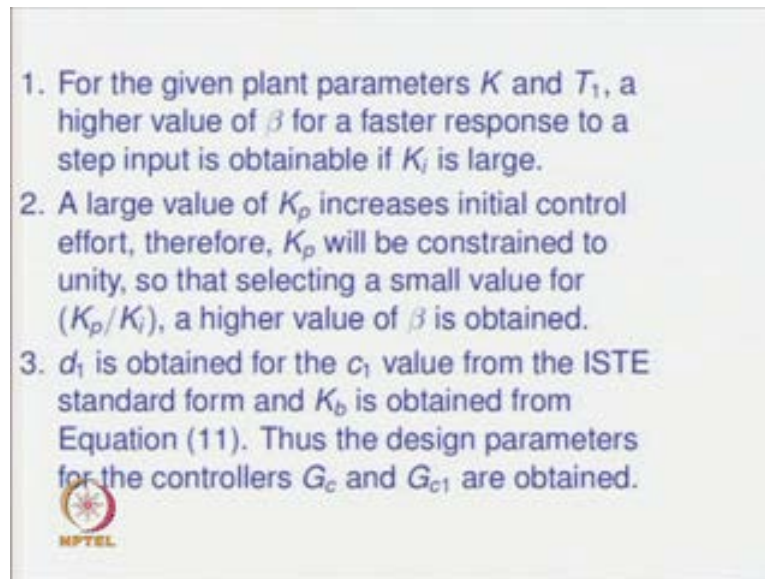
Now, we have got the optimum values for the coefficient of the standard transfer function d_1 and c_1 . So, the standard transfer function given by equation number nine, has got two coefficients. One is c_1 , and another is d_1 ; one can find optimum values of d_1 corresponding to certain c_1 . So, for every c_1 there exists some optimum d_1 , and with the help of some optimization function or with the help of some performance index, particularly the integral square time error criterion that is given by J_1 . It is possible to get optimum values for d_1 for different values of c_1 .

So, these plot shows the optimum values for d_1 given by the dotted line for different values of c_1 given by the x axis. So, c_1 is varying from 0 to 8, and d_1 is having different values when c_1 assumes values from 0 to 8. Also this figure shows the J_1 value, when the ISTE integral square time error criterion optimization is used to find the values for coefficients d_1 and c_1 . So, this figure basically shows the optimum values of J_1 , and d_1 versus c_1 for some transfer function $T_{12}(s)$, what we mean by $T_{12}(s)$, Y

$r \text{ dash}(S) n$ is equal to $T^{-1} \text{ two } S$ is equal to $c^{-1} S^n \text{ plus } 1$ divided by $S^n \text{ square plus } d^{-1} S^n \text{ plus } 1$.

Now, what benefit we get from getting the delay free part expressed in this standard form, then we can find the values d^{-1} corresponding to or for certain c^{-1} , and we can make use of β , c^{-1} , d^{-1} to estimate model parameters **parameters** of the controllers G_c , G_{c1} , and G_{c2} .

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For the given plant parameter K , and T^{-1} ; we have got the plant dynamics given by equation 7, please keep in mind the plant parameters are $k \theta$ and T^{-1} . So, for given $k \theta$, and T^{-1} particularly for the given plant parameters K , and T^{-1} ; a higher value of β results in a faster response to a step input. If K_i is large, **if k_i is large** - a large value of K_p increases initial control effort, as you know higher proportional gain not only increases the control effort rather can leads to instable or unstable operation or instability in the closed loop system, for that we will constraint the K_p value to unity.

So, that selecting a small value for K_p upon K_i or selecting a small value for K_p upon K_i or indirectly speaking one upon K_i , a higher value of β is obtained. Then d^{-1} is obtained for the c^{-1} value from the ISTE standard form, and K_b is obtained from equation 11. So, 11 has got unknowns like K_p and K_b , because all other things are known. And of course, β **beta** is or we are choosing β , that way since K_p has been

constant to 1, the only unknown **unknown** that is in equation number 11 is K_b , which can be estimated easily provided, we have certain value for d_1 - provided d_1 is known.

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The controller G_{c2} , whose prime job is to reject unwanted load disturbances, is designed on the basis of stabilisation of the characteristic equation

$$1 + GG_{c2}e^{-\theta s} = 1 + \frac{KK_l e^{-\theta s}}{T_1 s - 1} = 0 \quad (12)$$

Based on the optimum phase margin criterion for the stabilisation of an unstable FOPDT process, it is easy to obtain

$$K_l = \sqrt{\frac{T_1}{\theta K^2}} \quad (13)$$

with the constraint $\theta/T_1 < 1$.



Now, using these the steps has been explained now the way you should proceed with to design the controller parameters. The controller G_{c2} prior to that what we have done, the steps for designing G_c , and G_{c1} are given in the steps 1, 2, 3 two. The steps for designing G_c , and G_{c1} are given here, whereas for designing the remaining controller G_{c2} , we have to resort to some other technique. The controller G_{c2} which job is to reject load disturbances is designed on the basis of stabilization of the characteristic equation $1 + GG_{c2}e^{-\theta s}$.

Now, substitution of G , and expression for G_{c2} gives us an expression 12 or equation 12 which has got the expression $1 + \frac{KK_l e^{-\theta s}}{T_1 s - 1}$. Please keep in mind in the upper in the numerator of this expression, you have got K_l not K_{i1} plus $\frac{KK_l e^{-\theta s}}{T_1 s - 1}$ is equal to 0. Now, based on the **(C)** optimum phase margin criterion, now how can you find the optimum phase margin criterion; how **how** can you find use the optimum phase margin criterion here.

Again get it this expression expressed in the form of the frequency domain expression, and then you can find the phase angle of this function - whole function $1 + \frac{KK_l e^{-\theta s}}{T_1 s - 1}$ upon $j\omega T_1 - 1$. Then optimized that phase


angle with respect to the frequency, ω , and then you will get a condition, and that condition will be obviously, K_1 is equal to square root of T_1 upon θK square.

So, the equation in 12 will give will have optimum or maximum phase angle provided K_1 is equal to square root of T_1 upon θK square; it is not difficult to obtain this expression 13. So, based on the optimum phase margin criterion for the stabilization of unstable first order plus dead time process, we obtain the parameter of the controller G_c , but this controller is designed with a constraint given by θ upon T_1 less than 1. The normalized dead time of the unstable first order plus dead time process has to be less than one; if it is not so, then the formula given in equation 13, may not be correct.

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Case Study 1

1. Let $K = 1$, $T_1 = 10$ and $\theta = 5$. Since $\beta = \sqrt{0.1 K_i}$, constraining of the value of K_p to unity and choosing $(K_p/K_i) = 0.1$ gives $\beta = 1$ and from Equation (10) $c_1 = 0.1$.
2. For this value of c_1 the corresponding value of d_1 from the standard form for the ISTE criterion is 1.347 (see Figure 1).
3. Equation (11) gives $K_b = 13.468$ and from Equation (13) $K_i = 1.414$.

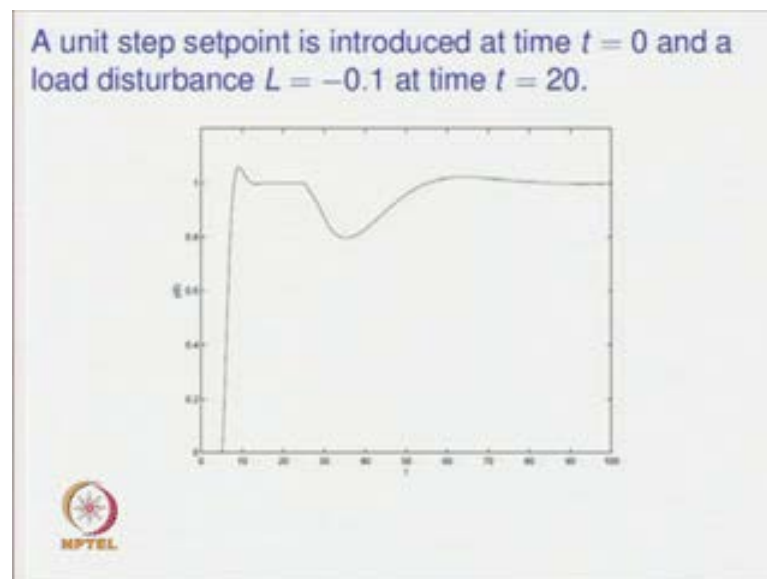


Let us go to case study one, where we will have we consider a an unstable first order plus dead time system with steady state gain K equal to 1, time constant T_1 equal to 10, and time delay θ equal to 5. The normalized dead time θ upon T_1 is 5 upon 10 is equal to 0.5, which is less than 1. Therefore, there will be no difficulties in designing the controller G_c . Since β is equal to given by square root of 0.1 times K_i , how do you find this one; we know β is equal to $K K_i$ divided by T_1 . So, when K equal to 1, T_1 equal to 10, and K_p equal to 1; then, β becomes square root of 0.1 times K_i . Now, constraining K_p to unity, and choosing K_p upon K_i as 0.1 gives β equal to 1, we are choosing the value of K_i indirectly. So, that way the controller G_c , which has got parameters K_p , and K_i has already been designed with proper choices for K_p and K_i .

So, this shows that k_i is equal to 10, which gives us ultimately β is equal to 1, and from equation 10, we get c_1 equal to 0.1, because you look at equation 10; c_1 is β times K_p upon K_i , and since β equal to one K_p upon K_i is 0.1. Therefore, c_1 also will be equal to 0.1. So, thus we get c_1 is equal to 0.1. Now, for these value of c_1 , the corresponding value of d_1 from the figure, which figure from the standard form figure for the standard form from this figure. So, when c_1 equal to 0.1, we do get exact value of d_1 - optimum value of d_1 as the value of d_1 as 1.347. So, d_1 is equal to 1.347.

Now, equation 11 can be used to estimate the unknown parameter of the controller G_{c1} which is nothing but K_b is equal to 13.468, and again from equation 13. We obtain K_l the controller that improves the disturbance rejection is equal to 1.414; thus all the three controllers are designed, where the p_i controller has got the parameters K_p equal to 1, K_i is equal to 10. And now the second controller G_{c1} is equal to having value 13.468, and G_{c3} is having value one 1.414.

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So, when this controller parameters are put, and simulation is carried out simulation of the modified smith predictor structure with the unstable first order plus dead time unstable process is carried out. We do get the response from a unit step set point input, that is introduced at time t equal to zero, and a load disturbance L of magnitude minus 0.1 at time t equal to 20. So, we do get responses corresponding to the set point and disturbance inputs, and the responses are shown over here.


Here, we see that we have got a quite good acceptable time response for the controlled system or the for the closed loop system. Why it is acceptable, if I look at carefully the plot, the over shoot is not beyond 10 percent; it is rather quite low, the over shoot is less, the raise time is high. The settling settling time is also its small. So, we have got all desired time domain performance parameters from the closed loop system. And of course, the disturbance response is not satisfactory, not so satisfactory as it is for the set point input response.

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The Case of a SOPDT Process
For the process

$$G_p(s) = \frac{Ke^{-\theta s}}{(T_1s \pm 1)(T_2s + 1)} \quad (14)$$

with the controllers $G_c = K_p + \frac{K_i}{s}$, $G_{c1} = K_b + K_d s$ and $G_{c2} = K_i(T_1s + 1)$, then the delay free part of Equation (4) is

$$Y_r(s) = \frac{(K_p/K_i)s + 1}{\frac{T_1 T_2}{KK_i} s^3 + \frac{(T_1 \pm T_2 + KK_d)}{KK_i} s^2 + \frac{(KK_p + KK_b \pm 1)}{KK_i} s + 1} \quad (15)$$


Let us go to the case of a second order plus dead time process, we will attempt to design the three controllers G_c , G_{c1} , and G_{c2} for this second order plus dead time process, where we can have stable or unstable poles with with the controllers choice of G_c is equal to K_p plus K_i upon S , G_{c1} is equal to K_b plus $K_d S$, please see the difference. Now, we are using the stabilizing controller as a PD controller.

So, the stabilizing controller G_{c1} has got derivative action as well. So, it assumes the form now, G_{c1} is equal to K_b plus $K_d S$, and the disturbance rejection controller takes the form of G_{c2} is equal to K_i times $T_1 S$ plus 1. Now, the delay free part of equation 4, Y_r dash S can be written as K_p upon $K_i S$ plus one divided by $T_1 T_2$ upon $K K_i S$ cubed plus T_1 plus minus T_2 plus KK_d upon $K K_i S$ square plus $K K_p$ plus $K K_b$ plus minus 1 divided by $K K_i S$ plus 1. How do you get Y_r dash upon S , Y_r dash upon S is given by the delay free part is given by the ratio of GG_c upon $1 + G$ times G_c

plus G_c . When you substitute those G_c , and G_c then the delay free part becomes this one as shown in equation number 15.

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Normalisation with $\beta = \left(\frac{KK_i}{T_1 T_2}\right)^{1/3}$ gives

$$Y_r(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (16)$$

where

$$c_1 = (K_p/K_i)\beta \quad (17)$$

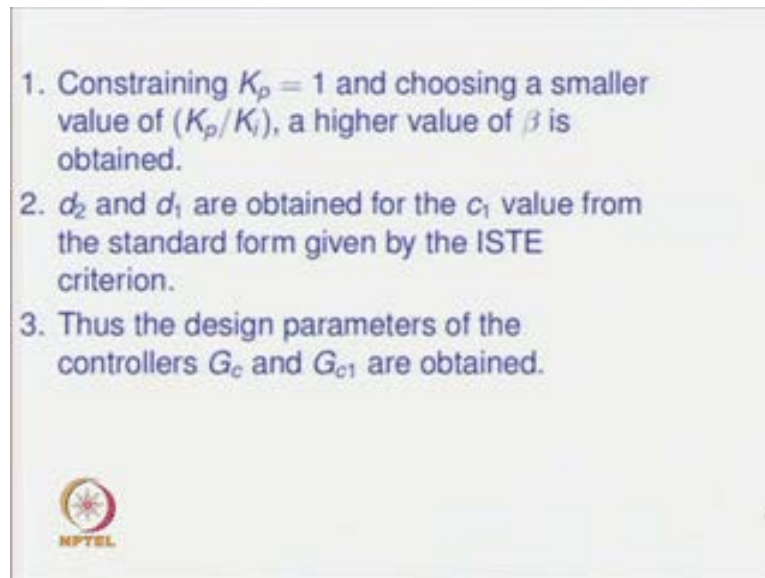
$$d_2 = \frac{(T_1 \pm T_2 + KK_d)}{T_1 T_2 \beta} \quad (18)$$

$$d_1 = \frac{(KK_p + KK_b \pm 1)}{T_1 T_2 \beta^2} \quad (19)$$


Now, normalization with beta is equal to $\sqrt[3]{\frac{KK_i}{T_1 T_2}}$ gives the standard third order transfer function $Y_r(s_n)$ is equal to $C_1 s_n + 1$ divided by $s_n^3 + d_2 s_n^2 + d_1 s_n + 1$. Where C_1 is equal to K_p upon K_i times beta, d_2 is equal to $T_1 \pm T_2 + KK_d$ upon $T_1 T_2 \beta$, and d_1 is equal to $KK_p + KK_b \pm 1$ divided by $T_1 T_2 \beta^2$. So, 16, 17, 18, 19 gives us the normalized third order transfer function involving three coefficients; and the coefficients are C_1 , d_2 , d_1 . Like the earlier case with the optimization of IST/ISTE criterion or with the use of ISTE criterion with the optimization of the function, it is possible to obtain optimum values for d_2 and d_1 coefficients with respect to C_1 or for various C_1 .

How to obtain is how to obtain optimum values for the coefficients for different order of transfer functions have been already discussed in one of our lecture, you need not worry. Simply, you have to transport the final plot, you have for either second order or third order standard transfer function.

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Now, I will explain you the procedure the way we will design the three **three** controllers. So, initially constraining K_p to 1, again do we constraint K_p to 1 to reduce the control effort. And choosing a smaller value of K_p upon K_i **1** K_i , a higher value of beta is obtained. I believe you are getting this concept why we are going for a higher value of beta, now d_2 and d_1 are obtained for the C_1 value from the standard form given by the ISTE criterion.

So, I am not going to show you further one plot. Now, directly the d_2 , and d_1 values for a given c_1 will be used in our further analysis. Thus the design parameters of the controllers G_c , and G_{c1} are obtained using what using of course, 17, 18 and 19, and with the choice of a smaller K_p upon K_i .

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$$1 + GG_{c2}e^{-\theta s} = 1 + \frac{KK_i(T_1s + 1)e^{-\theta s}}{(T_1s + 1)(T_2s + 1)} = 0 \quad (20)$$

Since for a stable SOPDT process, $G_{c2} = 0$, Equation (20) is therefore analysed for an unstable SOPDT process only. Letting $T_1 = T_2$ to cancel a pole with the zero in Equation (20) gives a characteristic equation of the form shown in Equation (12). Again the optimum value of K_i is obtained by the optimum phase margin criterion given as


$$K_i = \sqrt{\frac{T_1}{\theta K^2}} \quad (21)$$


Again, if you look at the denominator of the expression for y upon $1/s$, as I had earlier $1 + GG_{c2}e^{-\theta s}$. This is to be stabilized, when G is having unstable pole; for stabilization what you have to do is simple technique can be employed, when you write the expression in detail which gives us now $1 + KK_i(T_1s + 1)e^{-\theta s} / (T_1s + 1)(T_2s + 1) = 0$; with the choice of $T_1 = T_2$, one term in the numerator and denominator will get cancelled.

So, cancel $T_1s + 1$, and $T_2s + 1$ with the choice of **of course**, $T_2 = T_1$ or $T_1 = T_2$. Then it will render again us a characteristic equation of the form $1 + KK_i e^{-\theta s} / (T_1s + 1)$. For stable processes you need not worry, we do not use the controller G_{c2} . So, the parameters K_i, T_i, T_1 are zero, but for unstable process what will happen, we will get the characteristic equation now in the form of $1 + KK_i / (T_1s - 1) = 0$. Then the way the parameter K_i is obtained is already explained to you earlier, and that has been given in equation number 12. So, the optimum value of K_i given by this expression $K_i = \sqrt{T_1 / \theta K^2}$ will be used in this study as well.

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Case Study 2


1. Let $K = 2$, $T_1 = 10$, $T_2 = 2$ and $\theta = 3.5$.
2. Constraining $K_p = 1$, (K_p/K_i) is chosen with a small value of 0.1. Then $\beta = 1$ and $c_1 = 0.1$.
3. The ISTE criterion minimisation of the normalised third order transfer function with a zero gives $d_2 = 1.487$ and $d_1 = 2.046$.



Now, I will go to a specific transfer function of the process which is having the steady state gain to the time constant T_1 as 10, the second time constant T_2 as 2, and theta equal to 3.5. Now, constraining K_p equal to 1, and K_p upon K_i is equal to 0.1, then beta becomes 1. And consequently C_1 becomes 0.1; then corresponding to this C_1 we do get optimum values for the coefficients d_2 and d_1 which are found to be 1.487, and 2.046 respectively. Then we shall make use of the expression 16, 17, 18, as I have said you **sorry**, 17, 18 and 19 to estimate the unknown parameters of the transfer function model.

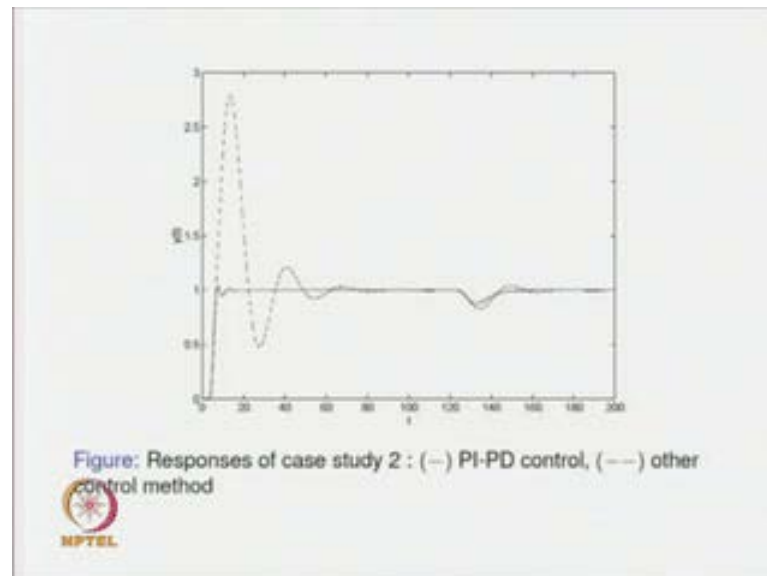
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1. Equation (19) gives $K_b = 19.963$ and Equation (18) gives $K_d = 10.867$. So also from Equation (21) $K_i = 0.845$ with $T_1 = T_2 = 2$.
2. The magnitude of the step load disturbance $L = -0.1$.
3. The response of the controller setting is compared with the response obtained using a popular method suggested in literature and shown in Figure 2.



Now equation 19 gives K_b is equal to 19.96, and equation 18 gives K_d is equal to 10.867. So, also from equation 21, K_I is equal to 0.845 with of course, with the choice of T_1 is equal to T_2 is equal to 2. Then only there will be pole zero cancellations, and ultimately rendering the second order expression into the first order thereby by enabling us to make use of the optimum phase margin criterion to find the parameter K_I . So, the parameters of the controller G_{c1} are K_b and k_d , the parameters of the controller G_{c2} are K_I and T_1 are thus obtained. The magnitude of the step load disturbance l is now minus 0.1, and the response of the controller setting is compared with the response obtained using a popular method suggested in the literature, and shown in the next figure.

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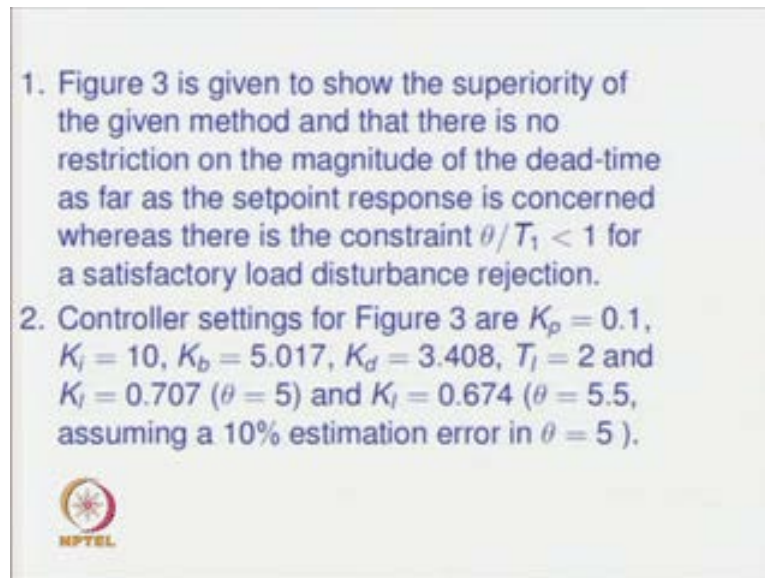


Now, in this case study two the solid line shows the responses obtained from the advanced smith predictor, and controller by Majhi et al. Whereas the dotted one shown in this figure is obtained by some popular smith predictor, and controller method. The responses show that the design method, and the advance smith predictor controller proposed by Majhi et al are far superior to that of the other smith predictor controllers, as far as controlling an unstable second order plus dead time process is concerned.

So, as far as controlling an unstable second order plus dead time process is concerned. This the beauty of the advance smith predictor controller, but the control design

technique is not complicated; it is **it is** quite straight forward. You to have very limited number of analytical expressions, and make use of those simple expressions can yield all the parameters of the controllers in the advance smith predictor controller.

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Now, another figure is shown here, the description about the figure let me give now; another figure is given to show the superiority of the given method. And that there is no restriction on the magnitude of dead time as far as the set point response is concerned, whereas there is the constraint theta divided by T 1 less than one for a satisfactory load disturbance rejection. So, controller settings for the next figure are K p equal to 0.1, K i equal to 10, K b is 5.017, K d is 3. 408, T l is equal to 2, and K l equal to 0.707. When the theta is 5, and K l is equal to 0.674, when theta equal to 5.5. What we have done basically, why there are two thetas here.

Now, we have assumed a 10 percent estimation error in theta, you are using some estimation technique; suppose the time delay is estimated erroneously; there is estimation error of 10 percentage in the estimation of time delay. Then it can go to a value of theta is equal to 4.5 or a value of 5.5 depending on under estimation error or over estimation error. So, assuming that we have got over estimation, then in place of theta equal to five we will use theta equal to 5.5, and simulate and see what type of performances we do get from the advanced smith predictor and controller.

Now, the solid line again shows the response we had obtained earlier; this is the one, we had obtained earlier. Now the dotted line shows the robust performance in the face of variation in parameters of the transfer function model. And the two responses show us that we do get robust performances by the controllers, we do get robust performances provided by the controllers although we do use very simple standard form based controller design technique to design the controllers. So, that is the beauty of the control - advanced control technique, and the tuning algorithm schemes.

Now, we will go to the case of a process with an integrator, and a long dead time. Earlier we have handled the control of unstable process, processes; now we go to the case of a process with an integrator, and large time delay. For the integrating process $G_p(s)$ is equal to K_p / s to the power minus theta, when the controllers chosen are of the form $G_c(s) = K_p + K_i / s$, $G_{c1}(s) = K_b$, and $G_{c2}(s) = K_l$. Then the delay free part of equation 4 results in $Y_r(s)$ is equal to $K K_p / (s^2 + K K_p + K K_b s + K K_i)$.


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$$Y_r(s_n) = \frac{c_1 s_n + 1}{s_n^2 + d_1 s_n + 1} \quad (24)$$

where

$$c_1 = (K_p / K_i) \beta \quad (25)$$

and

$$d_1 = (K K_p + K K_b) \beta^{-1} \quad (26)$$


Now, dividing both numerators and denominators by $K K_i$, we can get a second order standard transfer function. So, assuming beta equal to root of $K K_i$ further results in a second order standard **standard** second order transfer function or second order standard transfer function of the form $Y_r(s_n)$ is equal to $C_1 (s_n + 1) / (s_n^2 + 2 \zeta \omega_n s_n + \omega_n^2)$.

square plus $d_1 s^n$ plus 1. Again, where C_1 is given by the expression C_1 is equal to K_p upon K_i times β , and d_1 is equal to K_p plus K_b times β inverse.


So, relatively simpler expressions, then the earlier cases are obtain in this case, because we do not have any time constants associated with this process **process** dynamics or we do not have any time constant in the denominator of equation number 22. Now, how to design controllers? The three controllers for the integrating processes with long dead time, a higher value of β is obtained again for a smaller value of K_p upon K_i for the chosen C_1 value, d_1 is obtained again from the standard form **for** with minimization of the ISTE criterion. The design value of K_b is obtained from equation 26, k_b **yes** there is the only unknown here, when we assume that K_p is constraint to some value K_p equal to one; and k_i is of course, known and β has been chosen.

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A higher value of β is obtained for a smaller value of (K_p/K_i) . For the chosen c_1 value, d_1 is obtained from the standard form for minimisation of the ISTE criterion. The design value of K_b is obtained from Equation (26). For the typical process, the characteristic equation is

$$1 + GG_{c2}e^{-\theta s} = 1 + \frac{KK_i e^{-\theta s}}{s} = 0 \quad (27)$$

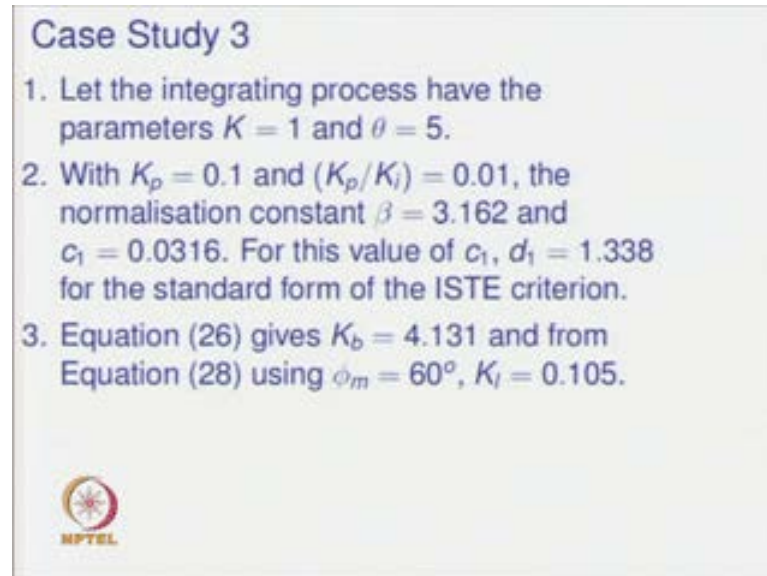
Choosing K_i to give a phase margin, ϕ_m , of 60° gives

$$K_i = \frac{\pi - 2\phi_m}{2K\theta} = \frac{0.5236}{K\theta} \quad (28)$$


For the typical process now the characteristic equation becomes $1 + GG_{c2}e^{-\theta s} = 1 + \frac{KK_i e^{-\theta s}}{s} = 0$. Now, choosing K_i to give a phase margin of some specified value, suppose ϕ_m phase margin is equal to 60 degree; that condition gives K_i or gives an expression for the K_i as K_i is equal to $\pi - 2\phi_m$ upon $2k\theta$ upon substitution of ϕ_m of 60 degree.


That means, pi by 3, you do get in the numerator for K_I as 0.5236 or K_I is now given expressed as K_I is equal to 0.5236 upon K_θ ; K_θ are known to us. As you know K_θ are the parameters of the process or parameters of the process model.

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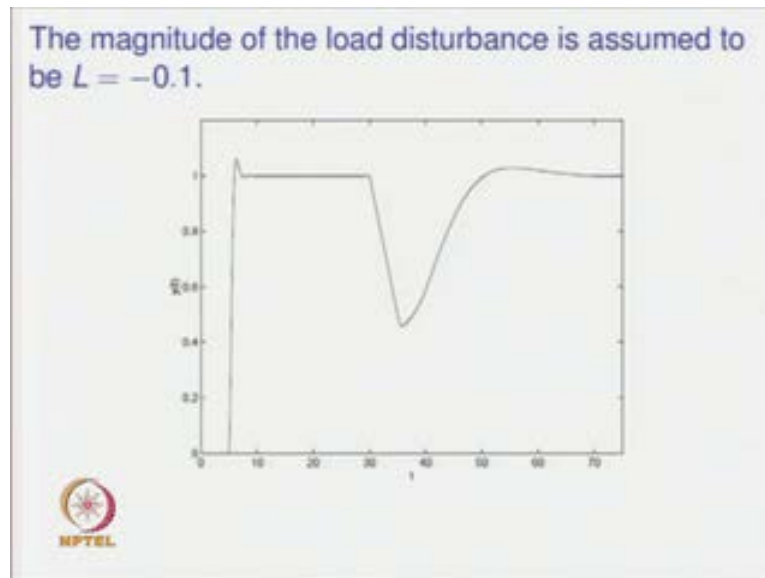
Case Study 3

1. Let the integrating process have the parameters $K = 1$ and $\theta = 5$.
2. With $K_p = 0.1$ and $(K_p/K_i) = 0.01$, the normalisation constant $\beta = 3.162$ and $c_1 = 0.0316$. For this value of c_1 , $d_1 = 1.338$ for the standard form of the ISTE criterion.
3. Equation (26) gives $K_b = 4.131$ and from Equation (28) using $\phi_m = 60^\circ$, $K_I = 0.105$.

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Now, in this case study, we will take the integrating process steady state process gain as K equal to 1, and time delay as θ equal to 5 with K_p equal to 0.1, and K_p upon K_i is 0.01. A further smaller value is used here, the normalization constant β becomes 3.162. So, in this study we are going for a higher value of β . So, higher value this higher value of β results in C_1 equal to 0.0316 for this value of C_1 further using the second order standard transfer function d_1 is obtained as 1.336 for the standard form of the ISTE criterion. Equation 26 gives now K_b equal to 4.131, and from equation 28 using the phase margin of 60 degree, we do obtain K_I as K_I is equal to 0.105. Thus, all the parameters associated with the three controllers for controlling an integrating process are designed.

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Let us see, that sort of responses we do get from the advanced smith predictor that is controlling an integrating process now. When the magnitude of the static load disturbance l is equal to minus 0.1, we do get the set point response that is given in the left hand side of this plot, and the load disturbance response given in the right side of this plot. Again, it is observed from this plot that, we do get very or quite satisfactory set point input response; although the disturbance response can be improved upon further.

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Summary

- Tuning algorithms have been developed for stable, unstable and integrating processes with long dead time
- It is assumed that the transfer function model matches exactly with that of the original process dynamics
- Simple controllers are designed based on the standard form and phase margin criterion

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Let me summarize the lecture now. So, the tuning algorithms have been developed for stable, unstable, and integrating processes with long dead time. The tuning algorithms **algorithms** consider basically the standard transfer function forms, and standard transfer function based controller design techniques. It is assumed that the transfer function model matches exactly with that of the original process dynamics; this is one of the major requirements, we have for our analysis and controller design, and to design a robust controller we need to relax upon this condition. Now, simple controllers are designed based on the standard form, and phase margin criterion.

So, G_c and G_{c1} are designed based on standard forms, and G_{c2} is designed based on phase margin criterion; it can be optimum phase margin criterion, it can be ordinary phase margin criterion. Any point to ponder, can we use other methods to design the controllers, **yes** there exist numerous techniques, such as the direct synthesis method, phase margin methods, optimal controller design methods, loop shaping methods, robust controller design methods and so on; those can be used to design controllers for the advanced Smith predictor control structure. Thanks.