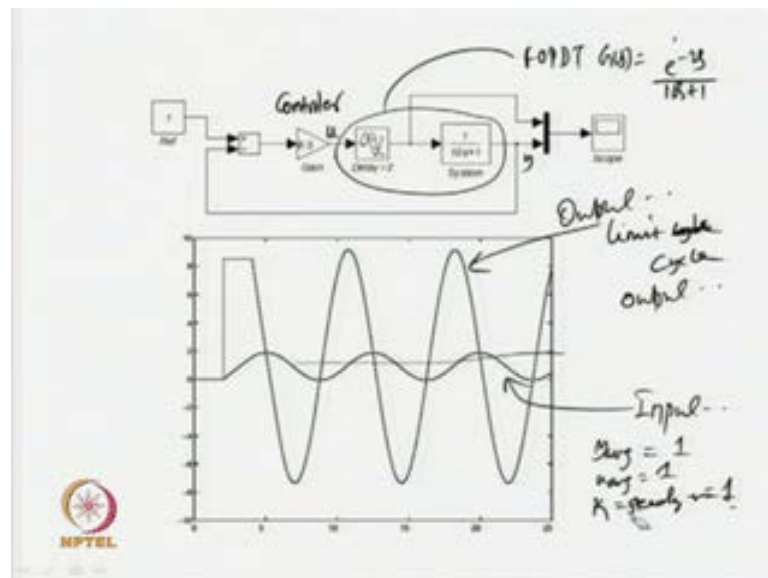


**Advanced Control Systems**  
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**Module No. #03**  
**Time Domain Based Identification**  
**Lecture No. #20**  
**Reviews of DF Based Identification**

Welcome to the lecture titled reviews of DF based identification. We shall revisit DF based identification techniques, and see what difficulties we have or what benefits we have from the DF based identification techniques.

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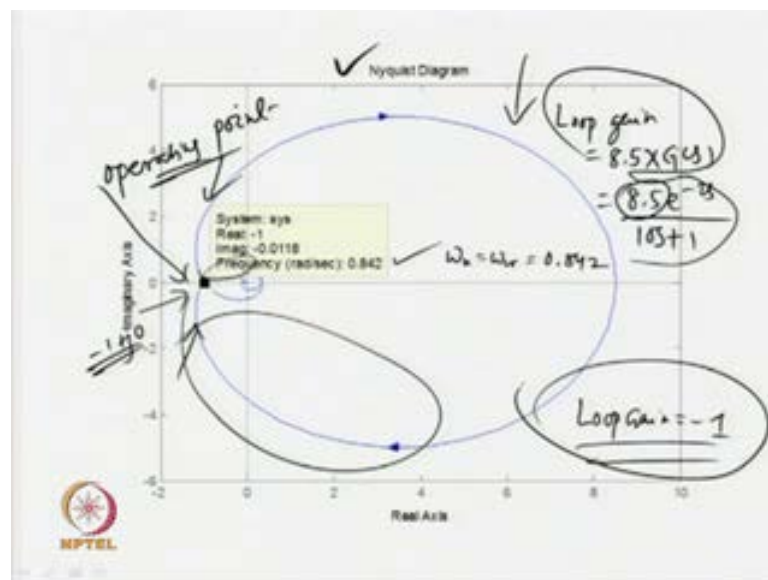


Now, consider the closed loop system subjected to a gain controller, the gain of the controller is 8.5, and the process is the first order plus dead time process with transfer function  $e^{-2s} / (s+1)$ . When this first order plus dead time process is subjected to a gain controller with gain of 8.5, what type of output is expected from that system closed loop system, the output becomes oscillatory. When the gain is lower, then the output become stable, and when the gain is increased further, then we expect unstable or unbounded output from the first order plus

dead time system. But at exactly the gain of 8.5, with a gain of 8.5 we do get sustained oscillatory output of this form.

So, we do get a limit cycle output limit cycle output of this form. Now, also the plot is showing the input signal of the system. So, this is the input and this is the output, we have from this. Now, the reference is one. So, when you take the average value of the input signal, the average will be one. So, average value of input Avg or input, if the input signal to the process is denoted by  $u$  and the output by  $y$ , then  $y$  average will be 1 and  $u$  average will be 1; thus giving us the steady state gain of the process as 1. So, I will not discuss about the steady state gain; rather, the point of considering this case or case study is that, the when the controller gain is 8.5, we do get a limit cycle output.

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Then, when the Nyquist diagram of the loop gain or loop transfer function is taken, what is the loop transfer function now? The loop gain is now 8.5 times  $G(s)$ ; that means  $8.5 e^{-2s} / (10s + 1)$ . So, when the Nyquist plot of the loop gain is considered, then the Nyquist diagram assumes this form. From here, what information we do get? We do have the negative real axis crossing at a frequency of  $\omega_u$  or  $\omega_c$ , critical frequency or ultimate frequency of 0.842 radian per second. So, the Nyquist diagram shows that, the operating point has been pushed to  $-1 + j0$ ; with the help of the gain controller, which is having a gain of 8.5. Now, when that value is changed, then you will have different type of Nyquist diagram.

From this, the objective of showing this Nyquist diagram is that, we do get sustained oscillatory output of this form; when the loop gain is **loop gain is** equal to minus 1. Why that is so? because the operating point is now the one shown by this rectangle. So, this is our operating point **this is our operating point**. So, with other values of 8.5, the operating point will change; they will go to either stable operation or unstable operation or stable operating region given by this or unstable operating region given by this. **You please see** that part of the Nyquist diagram is not shown, you can make out easily. So, what we have found? When the loop gain is equal to minus 1, that time we do get sustained oscillatory output from the closed loop system.

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Let's assume a transfer function model  $G(s) = \frac{Ke^{-\theta s}}{(T_1 s + 1)}$

and the controller gain is  $G_c(s) = 8.5$

Plant output oscillates at critical frequency  $\omega_c = 0.844 \text{ rad/s}$

Further,  $|G_c(j\omega_c)G(j\omega_c)| = 1$   
 $\Rightarrow (\omega_c T_1)^2 + 1 = 8.5^2 \Rightarrow T_1 = 10$

Next,  $\angle G_c(j\omega_c)G(j\omega_c) = -\omega_c \theta - \tan^{-1}(\omega_c T_1) = -\pi$   
 $\Rightarrow \theta = (\pi - \tan^{-1} 8.44) / 0.844 = 2$

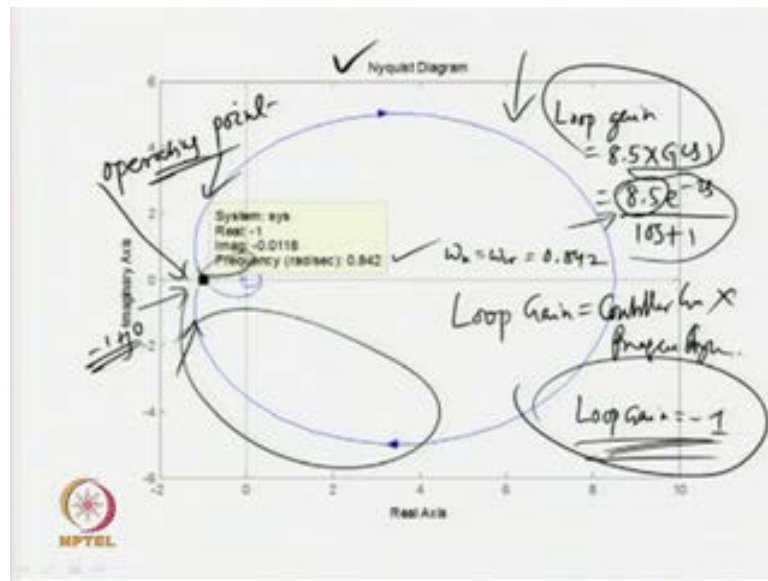
The transfer function model for the plant dynamics becomes

$G(s) = \frac{e^{-2s}}{(10s + 1)}$

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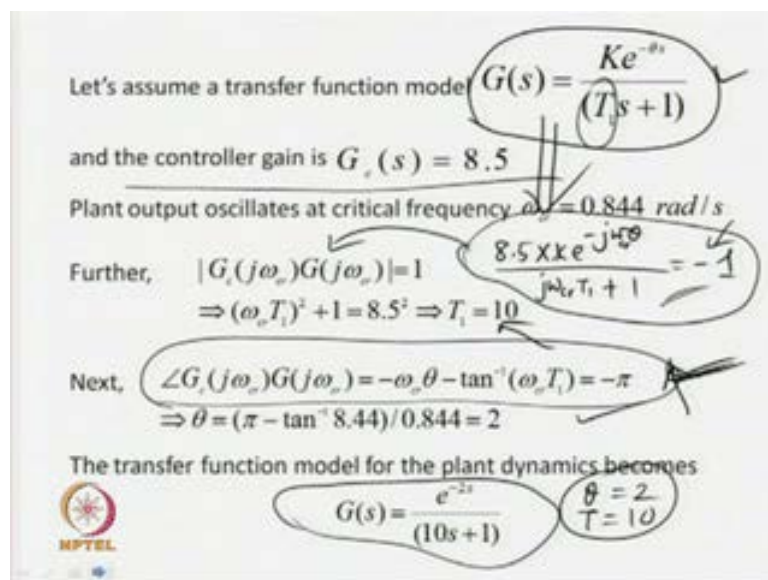
Now, how to identify the parametric model of a transfer function model for the process dynamics or plant dynamics? Initially, we assume a transfer function model for the dynamics of the process. Let that transfer function model be defined by  $G(s)$  is equal to  $K e^{-\theta s} / (T_1 s + 1)$  and we know that, the controller gain  $G_c(s)$  is equal to 8.5. Now, when the controller is put, then we have found sustained oscillatory output with the oscillating frequency of  $\omega_c$  is equal to 0.844 radian per second. Now, as I have told the Nyquist diagram gives us the information that, the loop gain is equal to minus 1 or **the** at the operating point, when the gain is equal to 8.5.

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The controller gain is 8.5. The operating point is pushed to a point of minus 1 plus  $j0$  and the loop gain is equal to minus 1. What is that loop gain now? Loop gain is nothing but, the controller gain times the process dynamics.

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So, that way we have got the expression 8.5 times  $K e^{-\theta s}$  upon  $T_1 s + 1$ , as the loop gain and the loop gain has to be minus 1, to obtain sustained oscillatory output. So, using this expression when the magnitudes of these expressions are equated; magnitudes means, when the magnitudes of both sides of these expressions

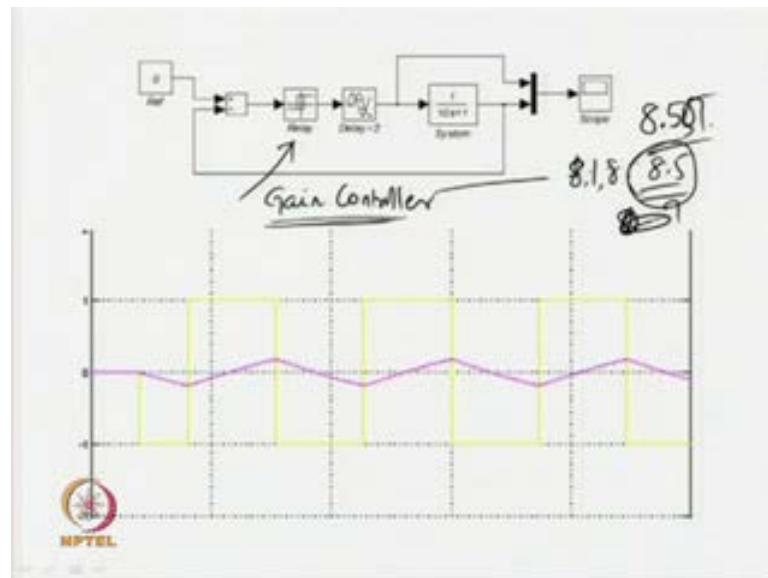
this **this** equation are equated; we do get an expression of the form in frequency domain,  $|G_c(j\omega) G_r(j\omega)| = 1$ . This is what; you are getting from equating magnitudes of the loop gain.

Now, when the phase angles of the both sides of again the loop gain is considered, then we do get an expression of the form;  $\angle G_c(j\omega) G_r(j\omega) = -\omega T_1 \theta - \tan^{-1}(\omega T_1) = -\pi$ . So, where from you are getting all these things? Basically, first you try to find the loop gain in frequency domain. How that will look like?  $8.5 \times K e^{-\omega T_1} \theta$  divided by  $j\omega T_1 + 1 = -1$ . Now, you take the magnitude of this expression; magnitudes means, magnitudes of both sides of this expression.

When you equate that, you get this; which can be further simplified and obtained in the form of  $\omega T_1 \theta = 8.5$  giving ultimately,  $T_1 = 10$ . Thus, one parameter of this transfer function model  $T_1$  can be estimated using the loop gain. Similarly, when the phase angles of both sides of this loop gain or the condition that gives results in limit cycle output are considered; phase angles of both sides are equated then, we get this analytical expression. Simplification of this expression results in the explicit expression  $\theta = \pi - \tan^{-1}(8.44)$  divided by  $0.844 = 2$ .

Why I am horridly explaining this one? Already we have discussed this example in one of our lecture earlier. Thus, we have been able to identify the parametric model or of the transfer function model of the process dynamics as  $G(s) = \frac{e^{-s}}{s^2 + 10s + 1}$  or the  $\theta$  has been estimated as  $2$ ;  $T$  has been estimated as  $10$ . So, we have been able to estimate accurately the parameters of the transfer function model; because the Nyquist diagram has been considered appropriately. How that is so? That will be evident after sometime, when I consider the same example on the different situations and when the describing function is used.

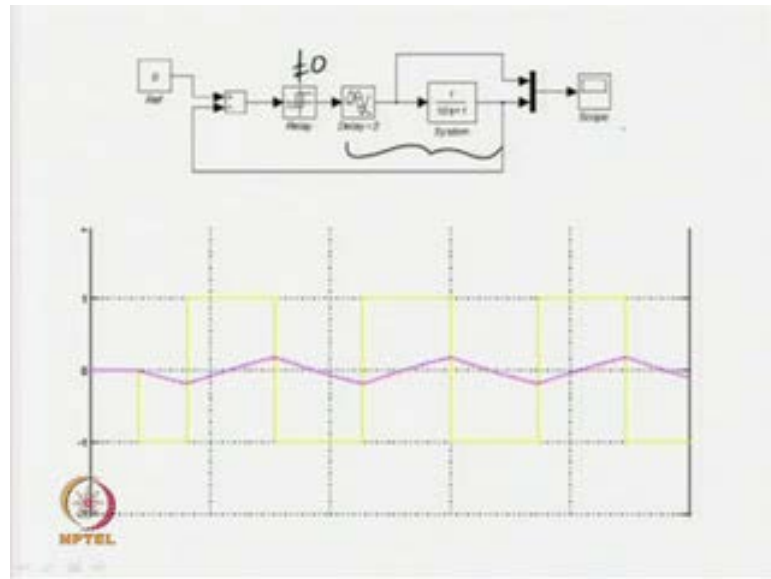
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Now, **So** in this case, in place of a controller with a gain or **a gain controller** in place of a gain controller, we have used a relay. Earlier, we were using some gain controller. Why we are using a relay now? The main reason for that is, it is very difficult to obtain limit cycle output with a gain controller; because you need to go on increasing or decreasing the gain of the gain controller heuristically. So, you start with some nonzero value 1. So, you go on increasing the gain value 1, 2, 3, 4, then you slowly increase; when you come to **8**, 8.1, 8.2, at 8.5 you will get sustained oscillatory output.

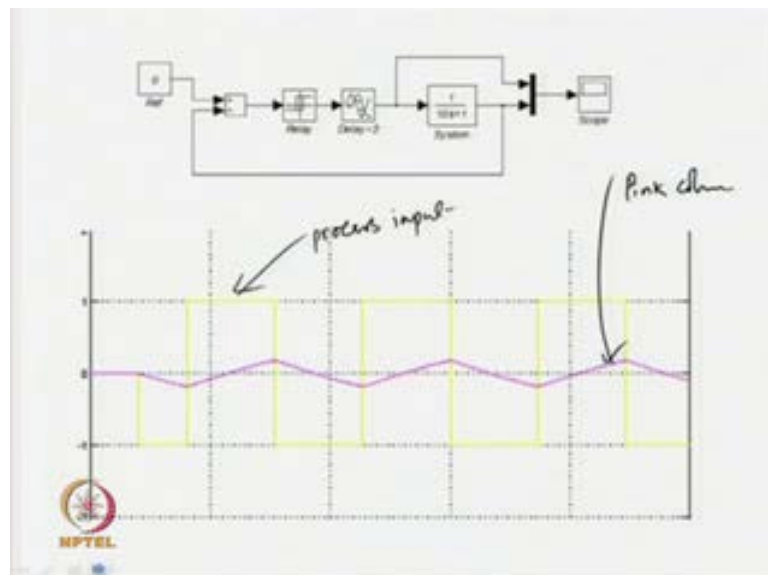
But you will take much time; you will spend much time to obtain this correct value of the gain. If you by mistake, exit that value by a small magnitude also 8.501 now; in place of 8.5, if the magnitude become 8.501 or 5 5, then the output will be unstable or unbounded. So, there is risk involved with the gain controller. How to find correct magnitude or value for the gain controller? To avoid that, a relay is used in the closed loop, which guarantees you sustained oscillatory output or limit cycle output irrespective of any value of relay setting other than the setting of 0.

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So, when the relay setting is not equal to 0, then this process definitely results in sustained oscillatory output. So, it is very easy. **I do not mind**. What value or what **what** parameter of relay is to be there. So, that is why, the gain controllers are normally avoided or the limit cycle using gain controller concept is normally avoided.

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Now, with the use of a relay ofcourse, I have been able to get this sustained oscillatory output shown by the pink color and the yellow one shows us, the output from the relay or process input **process input** during the auto tuning test **now** or identification test.




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A symmetrical relay with height  $h = 1$  produces a sustained symmetrical process output

Measurement of peak amplitude and the ultimate period gives:

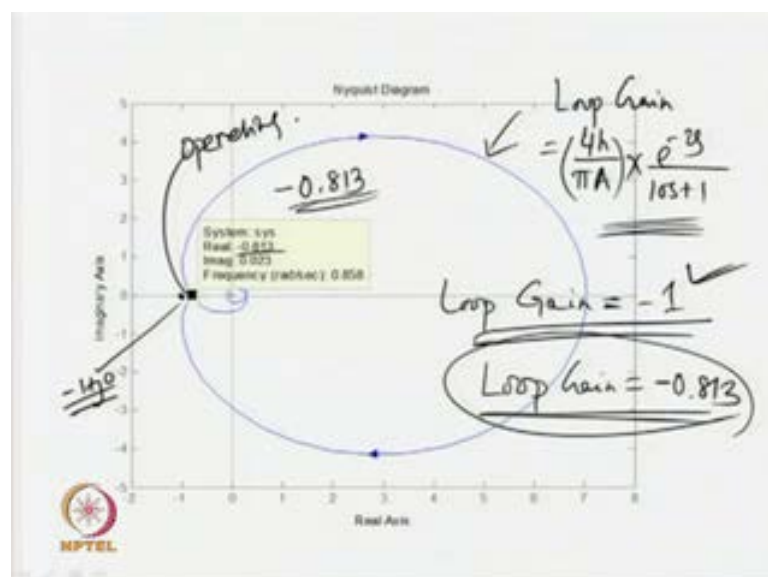
$A = 0.181$  and  $P_u = 7.332$

$\omega_c = \frac{2\pi}{P_u} = \frac{2\pi}{7.332} = 0.857 \text{ rad/s}$



Now, what type of output you do get? When a symmetrical relay with parameter  $h$  is equal to 1 is used, a sustained symmetrical process output is guaranteed; in the absence of ofcourse, static load disturbances. Then, when the measurement of peak amplitude and the ultimate frequencies are taken, we do obtained for this particular case; a peak amplitude of  $A$  is equal to 0.181 and ultimate frequency of magnitude 7.332 seconds, which giving us the critical frequency as  $2\pi$  upon  $P_u$  is equal to 0.857 radian per second. So, this is how, we do obtain the peak amplitude and ultimate frequency or critical frequency from the measurements.

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Now, let me go to the Nyquist diagram **diagram** of the closed loop process now. The loop gain is the gain of the controller. In place of a controller, I have used a relay. Therefore, the gain of the relay is approximated as  $4h$  by  $\pi A$ . **keep in mind**. The gain of the device relay is now  $4h$  by  $\pi A$ . We have already derived; how we obtained the gain of the relay using describing function? So, when this value of the gain is used along with that of the dynamics of the process, which is in this case  $e^{-2s}$  **e to the power minus 2 s** upon  $10s + 1$ .

I do get a Nyquist plot or diagram of this form. Now, there is one interesting observation. This point in rate particularly, this point corresponds to the point  $-1 + j0$  or the negative real axis with real value of minus 1; whereas, the Nyquist diagram of the loop gain is crossing the **negative relay** negative real axis at some other point. The operating point has got shifted now. Unlike the earlier Nyquist diagram, where the operating point was at minus 1. This is not so and the operating point has shifted to some value of minus 0.813.

So, when the equivalent gain of relay is considered for identification of plant model parameters or transfer function model parameters, when you write the condition for limit cycle as, the loop gain equal to minus 1. That is not true. When the loop gain is equal to minus 1, this is considered; then, we may get inaccurate estimation for the time constant and time delay associated with the transfer function model. So, when this loop gain would have been given as in place of minus 1 as minus 0.813.

That must result in correct estimation or the estimation errors associated with the estimation of plant model parameters will be significantly low. So, this loop gain concept is very important; outright, we write that a limit cycle is possible or is obtained, when the loop gain is 1. And with this loop gain is minus 1 and with that assumption, we do carryout our analysis and find explicit expressions for the parameters of transfer function model. So, let me see really, we are getting erroneous results are not; when the shifted point of the Nyquist diagram is used in place of the correct point. So, what is done basically?

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$$\frac{4h}{\pi A} \times \frac{Ke^{-\theta s}}{Ts + 1} = -1$$

$$\frac{4h}{\pi A} \times \frac{Ke^{-j\omega\theta}}{j\omega T + 1} = -1$$

Equating magnitudes

$$\frac{4h}{\pi A} \times \frac{K}{\sqrt{(\omega T)^2 + 1}} = 1 \Rightarrow (\omega T)^2 + 1 = \left(\frac{4Kh}{\pi A}\right)^2$$

$$\Rightarrow T = \frac{\sqrt{\left(\frac{4Kh}{\pi A}\right)^2 - 1}}{\omega}$$

$$-\omega\theta - \tan^{-1}(\omega T) = -\pi$$

$$\Rightarrow \theta = \frac{\pi - \tan^{-1}(\omega T)}{\omega}$$

Now, the relay is having a gain of  $\frac{4h}{\pi A}$ , and the process has got the dynamics  $\frac{Ke^{-\theta s}}{Ts + 1}$ . I am using the symbol  $T$  for the time constant. So, this is equated to minus 1, when this is written in the frequency domain, we get  $\frac{4h}{\pi A} \times \frac{Ke^{-j\omega\theta}}{j\omega T + 1} = -1$ . Now, the magnitude of the left hand side becomes  $\frac{4h}{\pi A} \times \frac{K}{\sqrt{(\omega T)^2 + 1}}$  is equal to 1.

So, equating the magnitudes of both sides, **equating the magnitudes of both sides** we do get this expression; which can ultimately be simplified in the form of  $(\omega T)^2 + 1 = \left(\frac{4Kh}{\pi A}\right)^2$ . I can write outright  $\frac{4Kh}{\pi A} = \sqrt{(\omega T)^2 + 1}$ . Further, giving us  $T = \frac{\sqrt{\left(\frac{4Kh}{\pi A}\right)^2 - 1}}{\omega}$ . Similarly, equating the phase angles of both sides of this expression will give us,  $-\omega\theta - \tan^{-1}(\omega T) = -\pi$  implies  $\theta = \frac{\pi - \tan^{-1}(\omega T)}{\omega}$ . directly I will bring this  $\pi - \tan^{-1}(\omega T)$  upon  $\omega$ .

So, these are the two explicit expressions, we do obtain from the analysis of the loop gain. But this loop gain **this this** inequality is necessary for generating limit cycle output. Now, when I substitute the measurements of  $\omega_c$ ; here,  $\omega_{cr}$  is used; please keep in mind; in place of  $\omega$ , please write  $\omega_{cr}$ . **c r** So,  $\omega_{cr}$  will

come in place of omega c r. So, simply substitute omega by omega c r, anywhere you have omega.

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Assuming the steady state gain to be  $K = 1$

$$T = \frac{\sqrt{\left(\frac{4hK}{\pi I}\right)^2 - 1}}{\omega_c} = \frac{\sqrt{\left(\frac{4 \times 1 \times 1}{\pi \times 0.181}\right)^2 - 1}}{0.857} = 8.1249$$

$T = 10$   
 $\theta = 2$

$$\theta = \frac{\pi - \tan^{-1}(\omega_c T)}{\omega_c} = \frac{\pi - \tan^{-1}(0.857 \times 8.1249)}{0.857} = 1.9993$$

The model parameter  $T$  is underestimated by 18.75 % and  
 $\theta$  is underestimated by 0.035 %



Then, you get the expression, T is this one; already I have derived and theta is equal to this one. When I substitute the major values of A as and omega c r, then I estimate the model parameters as T is equal to 8.1249 and theta is equal to 1.9999. But the process has got T of 10 and theta of 2. Therefore, the model parameter is underestimated; which parameter? The model parameter time constant is underestimated by 18.75 percentage and theta is underestimated by 0.035 percentage. So, this is the estimation errors, we have. When you use the condition that, loop gain is equal to minus 1 to generate limit cycle oscillation.

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Assuming the steady state gain to be  $K = 1$

$$T = \frac{\sqrt{\left(\frac{4hK}{\pi A \omega_s}\right)^2 - 1}}{\omega_s} = \frac{\sqrt{\left(\frac{4 \times 1 \times 1}{\pi \times 0.181 \times 0.813}\right)^2 - 1}}{0.857} = 10.029$$

$$\theta = \frac{\pi - \tan^{-1}(\omega_s T)}{\omega_s} = \frac{\pi - \tan^{-1}(0.857 \times 10.029)}{0.857} = 1.968$$

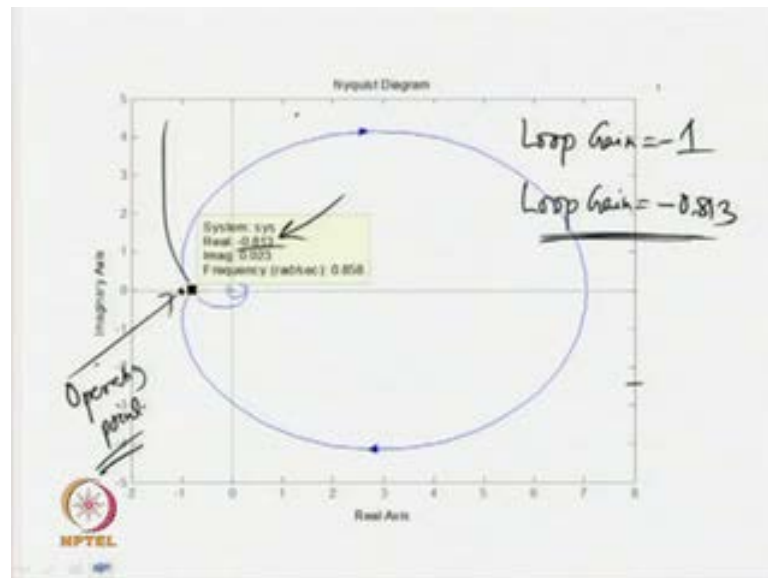
The model parameter  $T$  is underestimated by 0.3% and  $\theta$  is underestimated by 1.6%

$\frac{4h}{\pi A} \times \frac{k e^{-sT}}{Ts+1} = -0.813$

Now, I will do. So, when **when that** that condition is not used; rather, when I will use the condition that, actually your loop gain  $4h$  by  $\pi A$  times  $K e$  to the power minus  $\theta$  s upon  $T s$  plus  $1$  is equal to minus  $0.8$ ; as you had seen  $0.813$ . Because actually, the system is operating at the point minus  $0.813$ , then certainly the loop equation will result in correct estimation for the parameters of the transfer function model. So, when this **this** expression is analyzed now, in place of  $4h$  upon  $\pi A$  into  $K e$  to the power minus  $\theta$  s upon  $T s$  plus  $1$  is equal to minus  $1$ ; in place of minus  $1$ , when I use the real operating point, which is at minus  $0.813$ . Then the parameters of the transfer function models are estimated now, as you see the presence of  $0.813$ . So now, the formula will get modified.

So, I will have here  $0.813$  divided and similarly, this expression I have no effect. Then, the  $T$  is the time constant is estimated as  $T$  is equal to  $10.029$  and  $\theta$  is equal to  $1.968$ . Therefore, the estimation error has come down to a value of  $.3$  percent for the time constant and  $1.6$  percent for the time delay. Unlike the earlier case which had got the estimation error of around  $18.75$  percent; now, that has come down to  $.3$  percent. Therefore, it is very important to consider this point that, when the describing function analysis is **analysis is** used for the relay, then you have to look at the operating point. In place of the operating point of  $\text{minus } 1 \text{ plus } j 0$ , the operating point has shifted somewhere else.

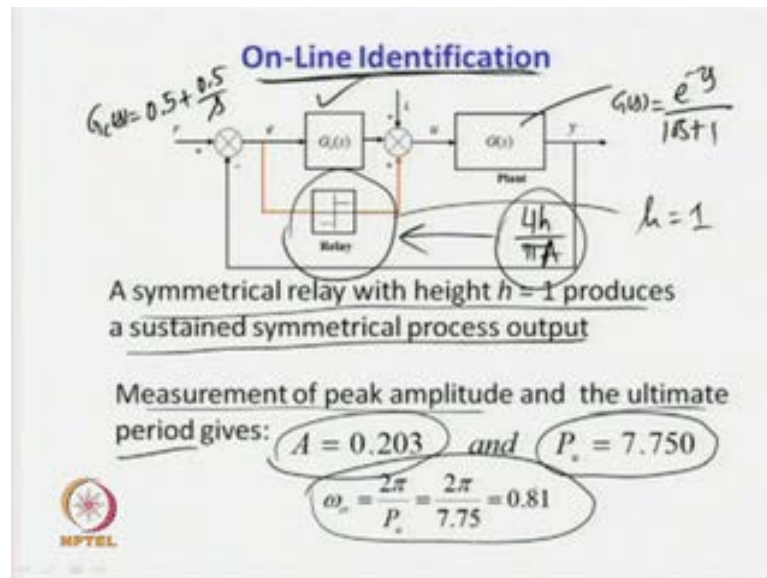
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Unless that information is used correctly, this operating point has shifted from this one. This should have been the operating point ideally, as per the analytical expression; we have been using during our lectures. On DF based identification, the operating point is somewhere else, when you use the approximation of relay by describing function. So, this point is very important as I am telling, unless correct operating point is used or correct analytical expression like the loop gain outright; instead of writing loop gain is equal to minus 1, you should write loop gain is equal to minus 0.813.

Please look at the point, the negative real axis crossing point. So, when this loop gain is equated to minus 0.813, you will be able to estimate accurate parameters for the models of the transfer function; for the models of the dynamics of the system.

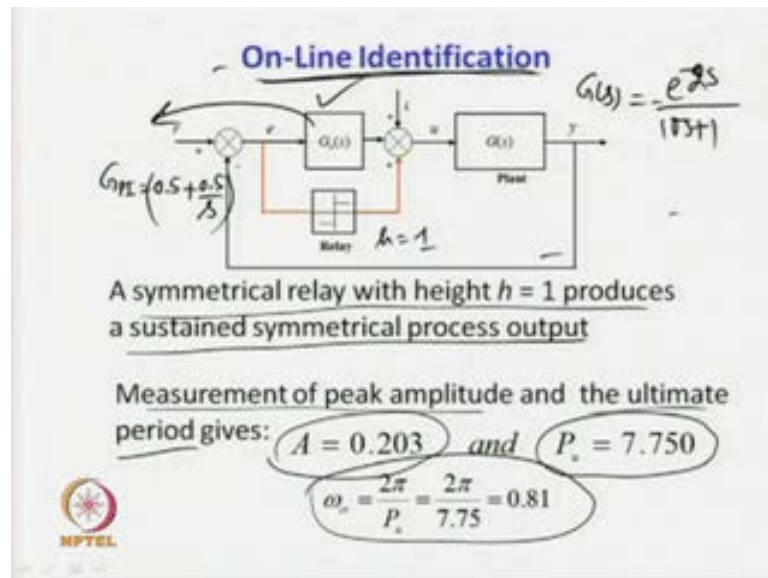
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Now, I will go to the on-line identification. Whatever we discussed so far were related to off-line identification. Is the on-line identification now is subjected to estimation error or not? That we shall see. So, in the on-line identification, the relay is connected in parallel with a controller. Now again, the relay is approximated by its describing function or the relay is approximated by an equivalent gain of the form  $4h$  upon  $\pi A$ ; where  $A$  is the peak amplitude of the fundamental component of the output signal. So, when a symmetrical relay test is conducted, a sustained symmetrical process output is obtained.

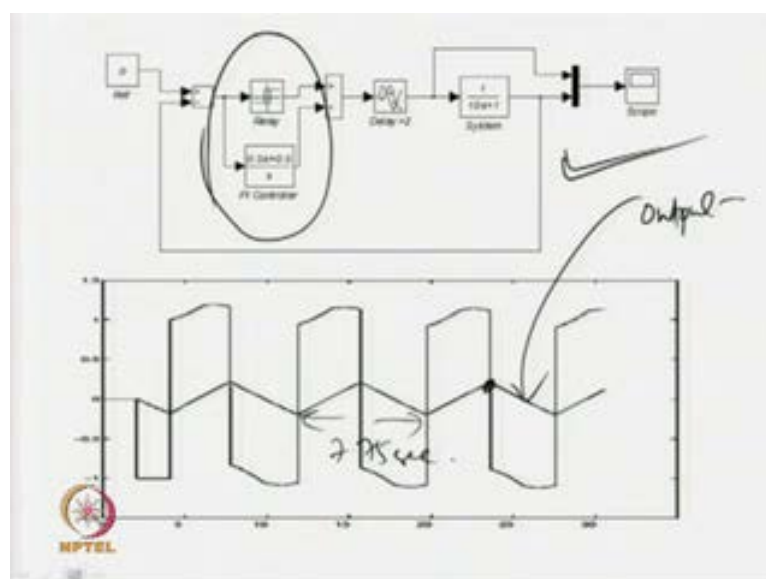
From that, when the measurements are made, then we can get typical values for the same process. Now, I will when I take the process as  $G(s)$  is equal to  $e^{-3s}$  upon  $15s + 1$ . And use the same value of the relay setting,  $h$  equal to 1 and whatever the controller; when the controller is used as  $G_c(s)$  is equal to  $0.5 + \frac{0.5}{s}$ . Then, I do get sustained oscillatory output and measurement of the peak amplitude and the ultimate frequency ultimate period gives us the peak amplitude as  $A$  is equal to 0.203 and ultimate period as 7.750 giving us critical frequency as  $\omega_c$  is equal to 0.81. How do obtain all these things? you know

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Only thing, you need to do is when you are stimulating this on-line identification scheme, the controller again I am saying use the controller, a PI controller; GPI of magnitude .5 plus 0.5 by s. Let me repeat, because this is very important and relay setting is height equal to 1. Relay height or parameter is set to 1 and the process is the same process, we have been considering in our earlier example;  $e$  to the power minus 2 s upon 10 s plus 1.

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Now, for this case already the stimulation diagram is given over here. What type of limit cycle output is obtained? We do get the sustained oscillatory output of this form. This is the output I get and when the peak is measured, it is found to be of the magnitude 0.203 and the period; if you start from here to here, the period is approximately 7.75 seconds. It has been given earlier 7.75 seconds. So, I will do the analysis of this system now. How to estimate the model parameters? Now, for the on-line system, we do have the PI controller connected in **in** parallel with the relay.

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$$K_u = \frac{4h}{\pi A}; G_c(s) = k_p + \frac{k_i}{s}$$

$$\left(k_u + k_p + \frac{k_i}{s}\right) \left(\frac{ke^{-\theta s}}{Ts+1}\right) = -1$$

$\Rightarrow$  freq domain

$$\left(k_u + k_p + \frac{k_i}{j\omega cr}\right) \left(\frac{ke^{-j\omega cr \theta}}{j\omega cr T + 1}\right) = -1$$

Equating the magnitudes of both sides.  
 $\rightarrow$  the above equation.

$$\sqrt{(k_u + k_p)^2 + \left(\frac{k_i}{\omega cr}\right)^2} \frac{k}{\sqrt{(\omega cr T)^2 + 1}} = 1$$

Therefore, the loop gain will be now given by, 4 h upon pi A or let me write K u ultimate gain as the 4 h upon pi A. Then, also the PI controller is given by Gc(s) is given by, suppose K p plus K i. Let us use the same type of symbol K p plus K i upon s. Then the loop gain will be now K u plus K p plus K i upon s times **K e to the power** K e to the power minus theta s upon T s plus 1. So, K is 1. So, here I have not taken K; K is this one; K is equal to 1 here. It does not matter.

So, I will give the general expression for the loop gain, when on-line identification is carried on. Now, I will **I will** write now in frequency domain, the loop gain as K u plus K p plus K i upon j omega c r K e to the power minus j omega c r theta divided by j omega c r T plus 1 is equal to minus 1. Now, equating the magnitudes of both sides of the above equation; what do we get? We get K u plus K p square plus K i upon omega c r square root times K upon omega c r T square plus 1 root is equal to 1.

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$$K \sqrt{(K_u + K_p)^2 + \left(\frac{K_i}{\omega_{cr}}\right)^2} = \sqrt{(\omega_{cr} T)^2 + 1}$$

$$\Rightarrow (\omega_{cr} T)^2 = K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{\omega_{cr}}\right)^2 \right] - 1$$

$$\Rightarrow T = \frac{\sqrt{K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{\omega_{cr}}\right)^2 \right] - 1}}{\omega_{cr}}$$

$A = 0.203, P_u = 7.75, \omega_{cr} = 0.8107$   
 $K_u = \frac{4h}{\pi A} = 6.272$   
 $T = 8.125$

Then, giving us  $K_u + K_p^2 + \frac{K_i}{\omega_{cr}^2}$  is equal to  $\sqrt{(\omega_{cr} T)^2 + 1}$ . So, when I simplify this one, again I will get  $\omega_{cr} T$  is equal to  $\sqrt{K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{\omega_{cr}}\right)^2 \right] - 1}$ . So,  $K^2$  now  $K_u + K_p^2 + \frac{K_i}{\omega_{cr}^2}$  minus 1. Giving ultimately an expression for the time constant of the transfer function model as  $T$  is equal to  $\sqrt{K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{\omega_{cr}}\right)^2 \right] - 1}$  divided by  $\omega_{cr}$ . So, this is the explicit expression for time constant of the process model parameter  $T$ , when an on-line identification scheme is used.

Now, using the measured values of  $A_p$ , we have  $A$  as 0.203 and  $P_u$  as 7.75 or  $\omega_{cr}$  is equal to 0.8107. Similarly,  $K_u$  is equal to  $\frac{4h}{\pi A}$  giving us 6.272. When these values are substituted, then we do obtain the  $T$  and the  $T$  will be now  $T$ ; when these values are substituted in the above expression,  $T$  is calculated as 8.125. It should have been 10. So, how much error is estimation or a error is associated due to the approximation of the relay, by some describing function. It is having almost 20 percentage of error. It should have been 10. Now, we have got a value of 8.125.

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Equating the phase angles on both sides.

$$\tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \omega\theta - \tan^{-1}\omega T = -\pi$$

$$\Rightarrow \omega\theta = \pi + \tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \tan^{-1}\omega T$$

$$\Rightarrow \theta = \frac{\pi + \tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \tan^{-1}\omega T}{\omega_c r}$$

$T=10$   
 $\theta=2$

$\theta$  is calculated as  $\theta = 1.977 \neq (2)$

Estimation Error =  $\frac{1.977 - 2}{2} \times 100 = -0.05\%$

Similarly, for the time delay equating the phase angles of both sides, **equating the phase angles angles of both sides** we get an expression of the form  $\tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \omega\theta - \tan^{-1}\omega T = -\pi$ ; implies  $\omega\theta = \pi + \tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \tan^{-1}\omega T$ ; implies  $\theta = \frac{\pi + \tan^{-1}\left(\frac{-K_i}{\omega_c r (K_u + K_p)}\right) - \tan^{-1}\omega T}{\omega_c r}$ . So, this is the explicit expression we have obtained for the time delay of the transfer function model, when an on-line identification scheme is considered.

Now, when I again substitute the values, necessary values  $A$  is equal to 0.203 or indirectly when I put  $K_u$  is equal to 6.272,  $\omega_c r$  is equal to .8107 in the expression, **theta is calculated** theta is calculated as, theta is equal to 1.977. How much it should have been ideally, it should have been 2. As you know, in the stimulation we have considered a transfer function model with the time constant,  $T$  is equal to 10 and the time delay of theta equal to 2. So, these are the ideal values; whereas, the estimated values for the time constant is 8.125 and that of the time delay is now 1.977. So, it is also the estimation error is also there, but the estimation error for the time delay is very less.

How much it is? It is accurately, it is minus 0.06 percentage; that means, the value has been underestimated by minus 0.06; underestimated with an estimation error of minus 0.06 percent. How do I find these percentage errors? Suppose it should have been 2, but

it has been estimated as 1.977. So, the estimation error; let me use another slide. So, theta **sorry** we have some plot. So, theta is equal to this much. So, estimation error is estimated value **So, estimated value** exact value divided by exact value times 100. So, in this case if I use this, then in that case how much it would be 1.977 minus 2 divided by 2 into 100. That is how; I get an estimation error of minus 0.06 percentage.

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$$K \sqrt{(K_u + K_p)^2 + \left(\frac{K_i}{w_r}\right)^2} = (w_c T)^2 + 1$$

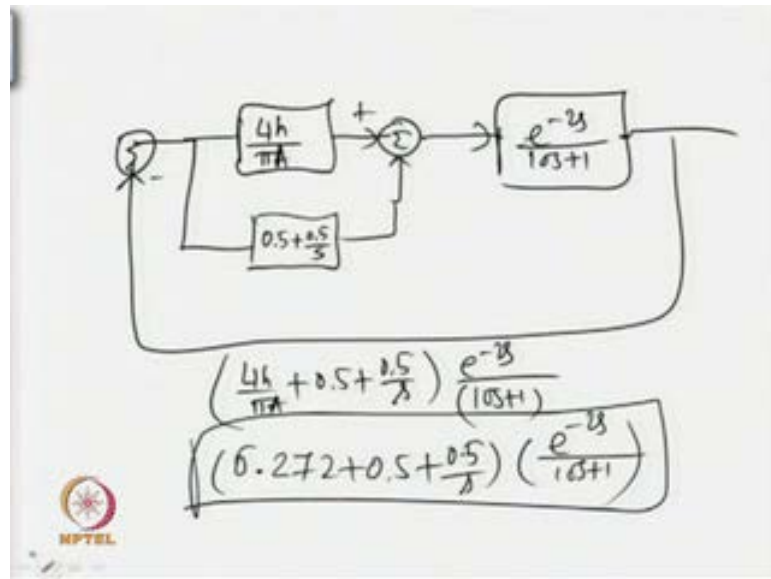
$$\Rightarrow (w_c T)^2 = K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{w_r}\right)^2 \right] - 1$$

$$\Rightarrow T = \frac{\sqrt{K^2 \left[ (K_u + K_p)^2 + \left(\frac{K_i}{w_r}\right)^2 \right] - 1}}{w_c}$$

$A = 0.203, P_n = 7.75, w_r = 0.8102$   
 $K_u = \frac{U_h}{\pi A} = 6.272$   
 $T = 8.125$   
 Estimation Error =  $\frac{8.125 - 10}{10} \times 100 = -20\%$

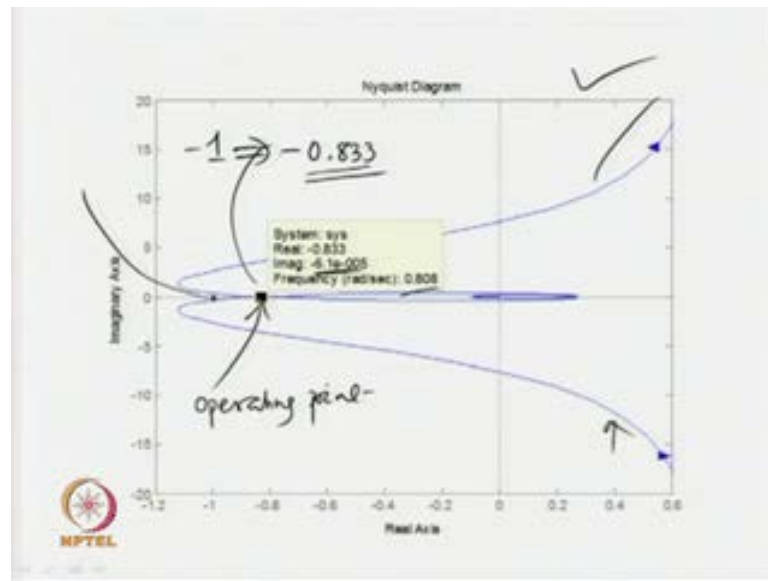
And in the earlier case, estimation error associated with the time constant is, estimation error **error** is 8.125 minus 10 divided by 10 into 100. So, that is giving us a value of minus 20 percent approximately. So, what we have found from this analysis that, because of the use of the describing function for the relay, the time constant and the time delay estimations are subjected to erroneous values and those are sometimes found to be of very high value **value**. So, we have got estimation error up to 20 percentage, which are not tolerable or one should not allow so much of estimation error, while identifying process dynamics. Now, I will see the Nyquist diagram for the closed loop system under relay control. So, what is this? Where from we are getting this Nyquist diagram?

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If you look at, we have got the relay which is given by its equivalent gain  $4h$  upon  $\pi A$ . Then, we have got a controller connected in parallel with this one with the controller is  $.5$  as plus  $.5$  upon  $.5$ . Therefore, we are using the same PI controller. So, it will be  $.5$  plus  $0.5$  upon  $s$ , a PI controller. Ofcourse, we are using  $e$  to the power minus  $2$   $s$  upon  $10s$  plus  $1$  for our analysis. So, the loop gain **loop gain** will be how much it would be?  $4h$  upon  $\pi A$  plus  $0.5$  plus  $0.5$  upon  $s$  times  $e$  to the power minus  $2$   $s$  upon  $10s$  plus  $1$ . Again, we have found  $4h$  upon  $\pi A$  to be of the value **6.67**  $6.272$  giving us an expression for the loop gain as  $6.272$  plus  $0.5$  plus  $0.5$  divided by  $s$  times  $e$  to the power minus  $2$   $s$  upon  $10s$  plus  $1$ .

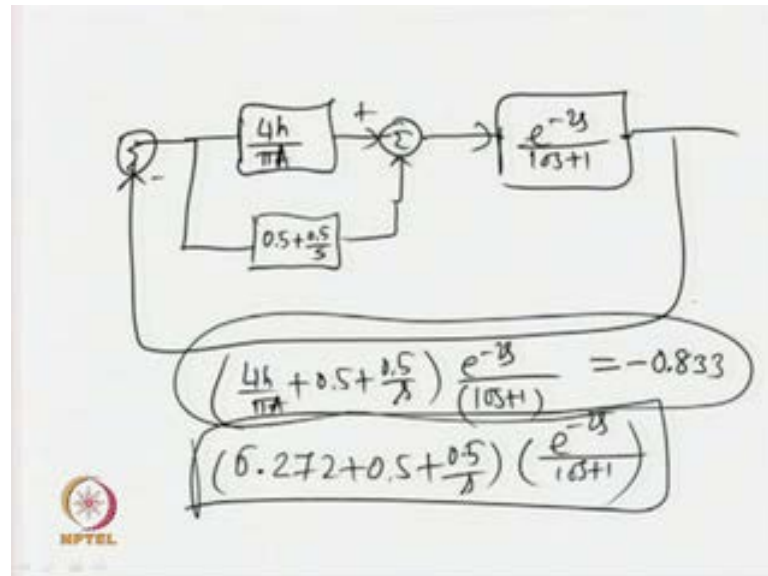
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So, when the Nyquist plot of this loop gain is obtained, then we do get a Nyquist diagram of this form. So, all these are for positive frequencies and the upper plot is for negative frequencies. You are acquainted with the Nyquist diagram, I do believe. Then, the Nyquist diagram shows us, that operating point has shifted substantially. The operating point should have been here, because we are using the analytical expression that, the loop gain is equal to minus 1. So, we take the operating point as minus 1 plus  $j 0$ . So, the operating point in the analysis is used as minus 1 plus  $j 0$ ; whereas, in real life the operating point is somewhere else.

This is the operating point of the closed loop system. So, ideally it should have been here, but practically it has shifted to a value or a point with the coordinates almost given as minus 0.833 plus  $j 0$ ; because it is small number. The complex value is very small. So, in place of minus 1, the operating point has gone to minus 0.833 and the frequency of oscillation is ofcourse, this we have got slightly a different value; because we may not had been able to locate exactly the 0 crossing point or negative real axis crossing point. It does not matter.

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Now, when one uses the loop gain or when I write the loop gain equation for this as, this is equated to minus 0.833 8 3 3 and carry on with the analysis; that means, find explicit expressions for time delay and time constant associated with the transfer function model. Then, the **the the** parameters can be estimated accurately. Let us see, how we can find those T. So, what will be done now in this analysis now?

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$$\left( k_u + k_p + \frac{k_i}{j\omega_{cr}} \right) \left( \frac{k e^{-j\omega_{cr}\theta}}{j\omega_{cr}T + 1} \right) = -0.833$$

$$\left\{ \begin{aligned} T &= \frac{\sqrt{(k/0.833)^2 \left( (k_u + k_p)^2 + \left( \frac{k_i}{k_u} \right)^2 \right) - 1}}{\omega_{cr}} \\ \theta &= \frac{\tan^{-1} \left( \frac{-k_i}{k_u + k_i} \right) - \tan^{-1}(\omega_{cr}T + 1)}{\omega_{cr}} \end{aligned} \right.$$

$k, k_u, k_p, k_i, \omega_{cr}$

Simply, what you have to do?  $K_u + K_p + K_i$  by  $j\omega_{cr}$  times  $K_e$  to the power minus  $j\omega_{cr}\theta$  upon  $j\omega_{cr}T + 1$  is equal to minus



0.833. So, what changes will have in the expression for this one? Now, we will have the expression for time constant can be given now as, the time constant as root of  $K_u$  plus  $K_p$  square plus  $K_i$  upon  $\omega_c r$  square **yes** minus 1 divided by  $\omega_c r$ . And similarly, there will be no changes to the expression for theta. Because **the** when you equate the phase angles of both sides, this 0.833 or 1 is not going to give any or contribute any phase angle.

So, that way, the expression for theta will remain as it is; which can be now  $\tan^{-1} \frac{K_i}{\omega_c K_p + K_i} - \tan^{-1} \frac{\omega_c T}{\omega_c r}$  plus pi divided by  $\omega_c r$ . So, **please** use  $\omega_c r$  in place of  $\omega_c$ . So, this is how, now you get revised expressions for that time delay and time constant of the transfer function model. Now, what changes are there? Earlier, we had 1 here; 1 in place of .833. So, when you use 0.833 and substitute now  $K_u$ ,  $K_p$ ,  $K_i$ ,  $\omega_c r$ ; which are known to us now. Then, we do get estimated value for T.

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$T = 9.9937$  (10)  
 $\theta = 1.9788$  (2)

% Estimation Error in T has gone down from -20% to **-0.06%**

% Estimation Error in  $\theta$  has **gone off** from -0.6% to **-1%**

T is estimated as 9.9937 in place of 10 and theta is estimated as the same value 1.9788, because there are no changes in place of 2. So, the estimation errors, now estimation percentage estimation errors in the time constant has gone **gone** down from **down from** approximately minus 20 percentage to minus 0.06 percentage. Similarly, the percentage estimation error in theta has gone off **gone off** from minus 0.6 percentage to minus 1 percentage. So, the estimation error has increased in this case, the estimation error has

increased not significantly marginally. But overall if you look at the estimation errors, final estimation errors basically, those are of small values. In this case, absolute value of estimation error is .06 percent almost negligible and in the second case, it is 1 percent which is tolerable.

So, the estimation errors have been reduced significantly, when the describing function based analysis uses **we uses** the correct operating point. So, **these operating point** to locate the correct operating point, **you need to do** you need to find the Nyquist diagram. Once you use Nyquist diagram, then only you can overcome the problems associated with the describing function approximation for the dynamics of a relay. So, the relay has been approximated by some equivalent gain and **that** that is an approximation only due to that **the** operating point in the Nyquist diagram gets shifted. So, if the shifted point is considered in the analysis, then accurate estimation for the model parameters can be obtained or achieved.

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**Summary**

$\frac{4h}{\pi A}$

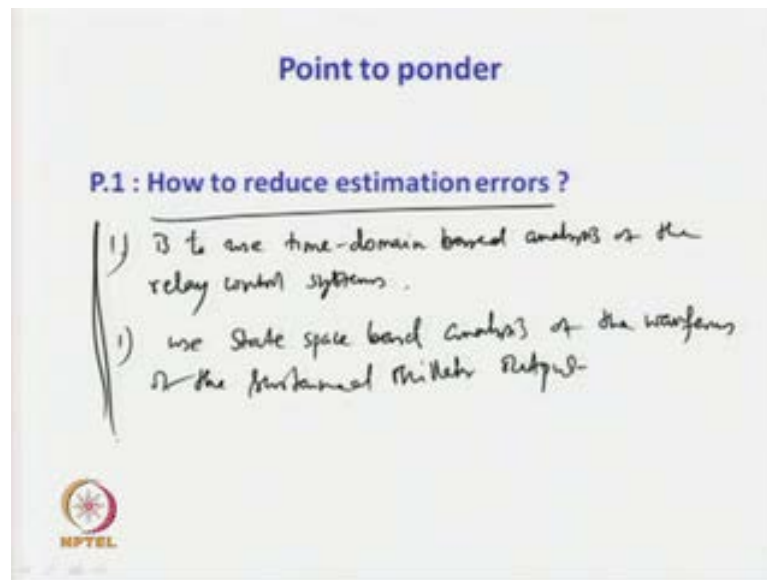
- Estimation of steady state gain by a separate relay test or by some other technique
- Equivalent gain of the relay results in substantial estimation errors in model parameters
- Both the off-line and on-line identifications are subjected to estimation errors

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Now, in the summary, estimation of steady state gain by a separate relay test or by some other techniques. So, we have not discussed here; how the steady state gains are obtained? Rather, we have concentrated or discussion into basically the **estimation in** estimation of time constant and time delay, the two important parameters of the transfer function model. Now, equivalent gain of the relay; basically, the relay dynamics has been approximated by an equivalent gain given by  $4h$  upon  $\pi A$ . So, this is an

approximation and this approximation results in substantial estimation errors in model parameters, unless correct operating point is identified and used in the analytical expressions. Both the off-line and on-line identifications schemes are subjected to estimation errors, because the relay is approximated by some gain.

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How to reduce estimation errors? Estimation errors can be reduced by many ways: One way of reducing the estimation errors is to use time domain based - **time domain based** analysis of the relay control systems, **relay control systems**. Another is to use state space based analysis of the waveforms of the sustained oscillatory output, **oscillatory output**. So, there are many techniques, those can be used to get estimated parameters with reduced estimation errors, and basically the time domain based estimation techniques result in less estimation errors. Thanks