Advanced Control Systems

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Module No. #03
Time Domain Based Identification
Lecture No. #19
Improved Identification using Fourier Series and Wavelet Transform

Welcome to the lecture titled improved identification using Fourier series, and wavelet transform. When the limit cycle parameters are obtained accurately, then the transfer function model parameters can be estimated accurately. So, it is very essential to have accuracy of the measurements or it is essential to make measurements accurately. So, that the transfer function model parameters can be estimated accurately. In today’s lecture, we shall see how one make use of Fourier series and wavelet transform for obtaining accurate limit cycle parameters, that will give you again accurate critical gain, and critical frequency, those are used in analytical expressions to find the parameters of a transfer function model.

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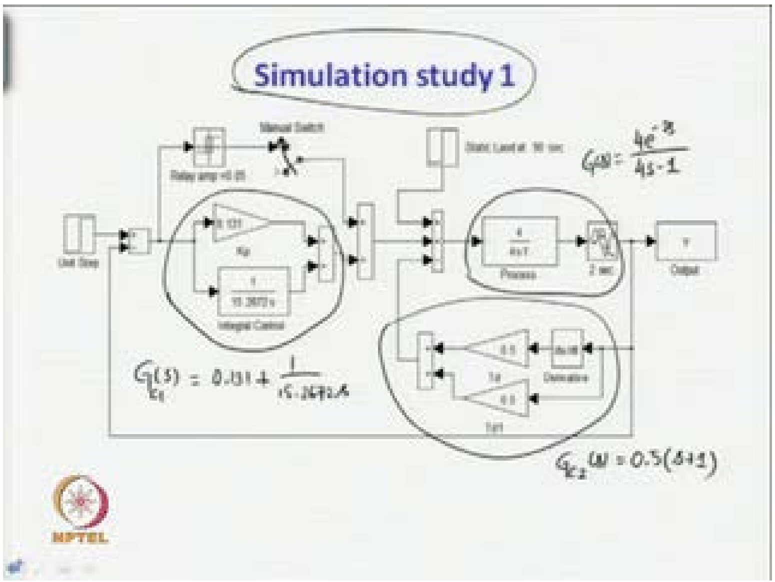

Let us, go through this simulation study number one, where we have got a process given by G(s) is equal to k e to the power minus 2 s upon 4 s minus 1. So, we consider an unstable first order plus dead time process. Now, the process is controlled by the P I, P D controller, where the P I controller is in the feed forward path. So, the dynamics of the P I controller is given as now, 0.131 plus 1upon 15.2672 s. Similarly, the unstable process is subjected to an inner feedback controller given by G c sorry this is a G c 1(s), this is G c 2(s) with can be written as 0.5 times (s plus 1).

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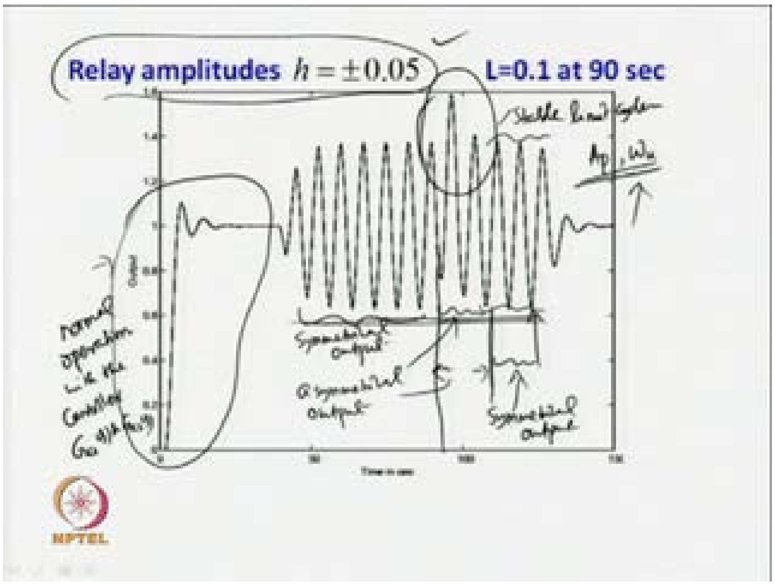

So, when the unstable process is subjected to a pair of controllers a feed forward and an inner feedback controller then it results in satisfactory output. So, this is the first part of this plot is for normal operation of the system, normal operation of the unstable system or unstable process of course, with the controllers controllers G c 1(s) and G c 2(s). Then when the relay auto tuning test is initiated with the relay amplitude of h is equal to plus minus 0.05, we do obtain sustained oscillatory output of this form, superimposed upon the set point value or the reference input.

Now,… So, a relay with amplitude point zero (( )) is connected now. Once it is connected then the output takes this form. Now, when the system is subjected to a static load disturbance at 90 seconds then what happens, what type of output you get, you please see, if this is the 90 second approximately then we see changes to the sustained oscillatory output and the sustained oscillatory output for certain duration does not give us symmetrical sustained oscillatory output or we do not get symmetricity in the output.

But of course, after few cycles almost after three cycles, the normal c is restore or the sustained oscillatory output becomes symmetrical. So, for this point of time we have got asymmetrical output output when the relay is symmetrical with the amplitudes plus minus zero point zero five whereas, after few cycles again symmetrical output is obtain. So, we do get symmetrical output which is exactly of the form that you have got preceding the asymmetrical output. So, this is the symmetrical output you get.

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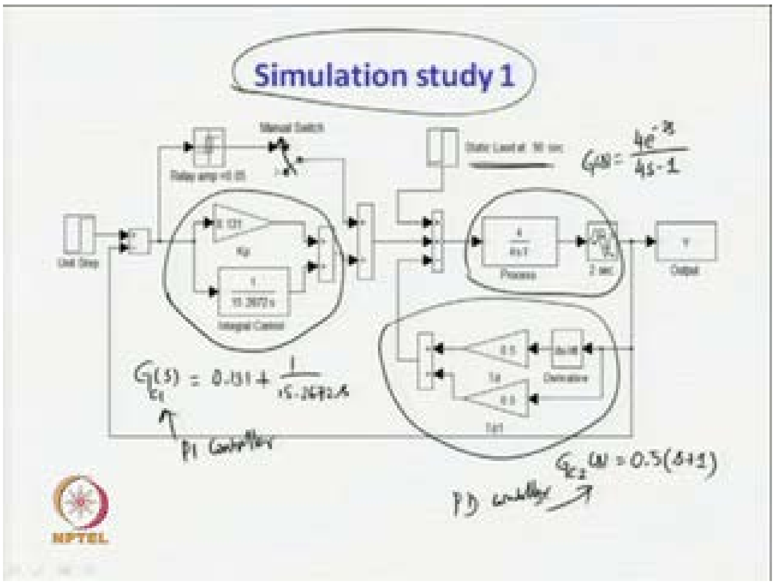

So, the effect of static load disturbance is not there after few cycles of the output signal; that means, now, we with the help of this P I and P D controller, this is a P I controller and this is our P D controller. We have been able to successfully eliminate the ill effect of static load disturbance during the auto tuning test and when there is no P I controller during the auto tuning test, we may get asymmetrical limit cycle output, please keep in mind, only during the presence of the P I controller in the loop and when the auto tuning test is carried out then the limit cycle signal is restored or we do get correct form of limit cycle output.

Now, the measurements can be made from the stable limit cycle output. So, this is what I mean by the stable limit cycle output. So, the measurements on the stable limit cycle output will result in correct parameters of the limit cycle output. So, correct values for the peak amplitude and the period of the limit cycle output can be obtained or the frequency of the limit cycle output can be obtained when the measurements are made from the stable limit cycle output not from here. So, if you make measurement from this, then you will not only get incorrect peak amplitude, but also you may get incorrect critical frequency or ultimate frequency from the limit cycle output. So, this is very important for us.

So, what is this static load disturbance that is given to a process or system during operation, we have got many type of disturbances for a process or system, we have got static and dynamic load disturbances, how static load disturbance is different from that of a dynamic load disturbance, I will give one simple example. Suppose, the weight of a bridge, the weight of a bridge and weight of a truck, moving truck, they are different from each other, the weight of a bridge gives us static load whereas, the weight of a moving truck gives us dynamic load.

Similarly, a structure consider one house or so, that also gives us static load disturbance at any point of time, but during earthquake, the magnitude of the static load changes and the effective load, you get from the structure or building during earthquake is different and that qualify as a dynamic load disturbance. So, it is very important to consider the effect of static loads for many systems because the effects of static loads are there for many practical systems and effort must be made to nullify the effect of the static load disturbance at the output during the auto tuning test.

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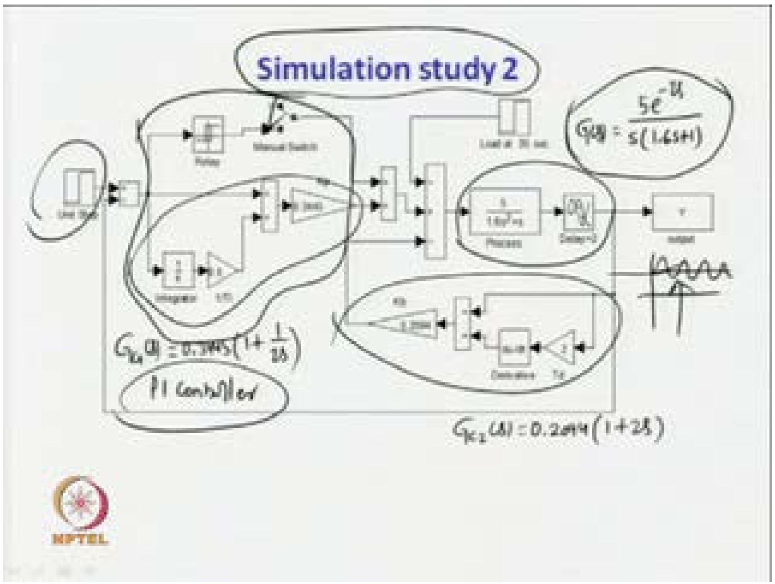

We shall go to another example or the simulation steady, where this time, the process is assume to have dynamics G (s) is equal to 5 e to the power minus 2 s upon s times (1.6 s plus 1). So, thus we are considering an integrating process now, like the earlier case, the integrating process is subjected to a P I controller given by G c 1 s as 0.3 4 4 5 times 1 plus (1 upon 2 s) and a P D controller of course, of the transfer function G c 2 s is equal to 0.2 0 9 4 times (1 plus 2 s). Now, the basic objective of having two controllers are that, the integrating processes are difficult to control and for getting desired output, the inner feedback controllers controller is employed. So, with help of the inner feedback controller, first the dynamics of open loop integrating process is stabilized and thereafter, the P I controller is applied to get desired response from the system.

Now, when the relay is connected then we do get sustained oscillatory output from the system. So, that sustained oscillatory output will be available with that unit step reference input or it will basically get superimposed with the unit step reference input now the relay has been connected in parallel with the P I controller. So, this is our P I controller and the relay has been connected in parallel with the P I controller, as you know when the relay is connected in parallel with the P I controller, the upside in the output can be nullified or thus it is possible to get sustained oscillatory output, symmetrical sustained oscillatory output with no effects of static load disturbance.

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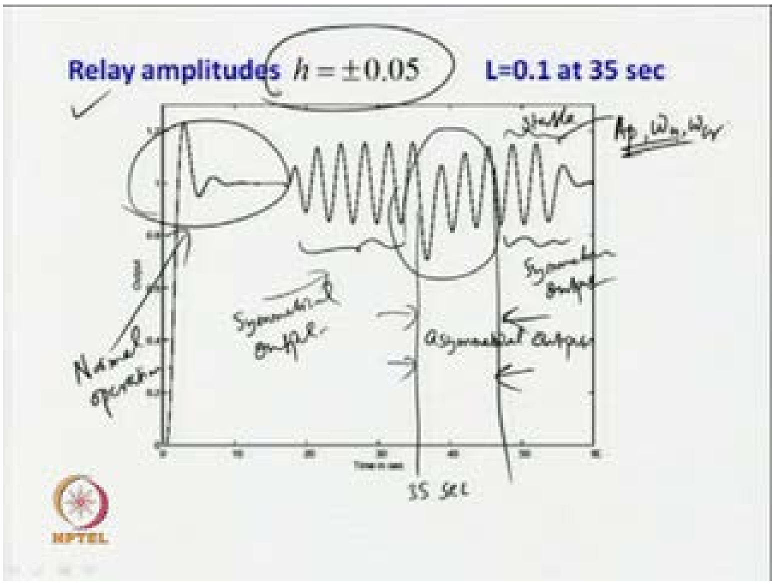

Now, for this unstable sorry for this integrating process, the static load is injected at time T equal to thirty five seconds and let us see the type of waveform will expect during the auto tuning test. So, when the static load disturbance occurs at thirty five seconds then the output is having asymmetrical output for a certain duration. So, this is the duration during which we have got asymmetrical output and apart from this duration, we do have symmetrical output from the process.

So, this is the symmetrical output (No audio from 12:37 to 12:45) and this is the output you get during normal operation of the system, normal operation of course, with the relay amplitude of plus minus 0.0 5. So, the relay is symmetrical, we do get symmetrical output when there is no static load disturbance. Now, please do not make measurement for this duration or during this period of the output and when the measurements are taken, when the limit cycle is stable then we do get correct parameters of the limit cycle output and those parameters have nothing, but the peak amplitudes and the ultimate frequency or the critical frequency omega u or omega c r.

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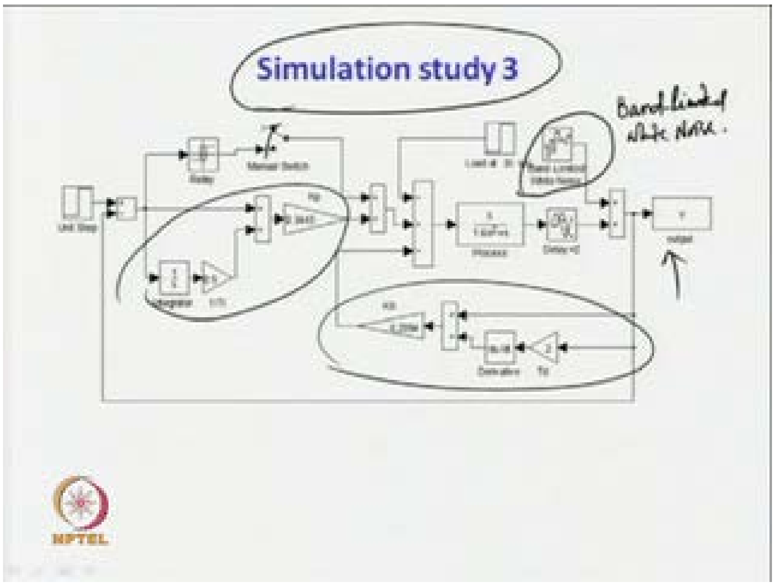

Now, we will go to one more example, one more simulation study. In this study again, we do consider the same process, the same second order integrating process which dynamics is given by G (s) is equal to 5 e to the power minus 2 s upon s times(1.6 s plus 1). Now, same P I and P D controllers are used, what is the difference in the simulation study or block diagram we have, basically we have injected or added at the output of the process, please carefully look at this the output of the process is now corrupted with band-limited white noise.

So, I have added one more block over here, and this block represents the band-limited white noise band-limited white noise then we do expect corrupted output from the closed loop system. Now, when the relay is connected, we do get further corrupted output due due to the auto tuning test and the final result, we will see that the type of output, you will get will be like this.

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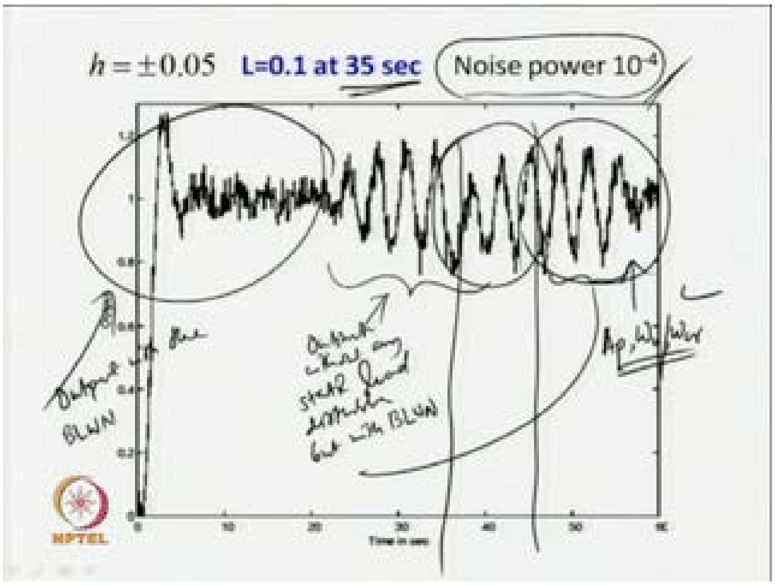

So, when there is no relay, you see the type of operation, you will get or the type of output with the band-limited white noise output with the band-limited white noise when the relay is connected in parallel with the P I controller that times, we do get this output without any static load disturbance, but with but with B L W W N. Now, this is also the same output you get, but when the static load disturbance is applied at time 35 seconds, then we obtain this limits cycle output. Now, what is B L W W N band-limited white noise is normally distributed random numbers, those can be added to see the effects of or to implement realize the effects of sensor noise in a system.

And the random numbers with height of power spectral density of 10 to the power minus 4 that is defined as the noise power. Noise power means power spectral density of the noise with height of 10 to the power minus 4 when this typical noise power is added to the system then we do get the type of output signal which is quite corrupted with noise then how to make measurements for the peak amplitude and the ultimate frequency or the critical frequency accurately that possess a challenge because many sensors may not have accuracy when the sensors are inaccurate or when the measurements are subjected to measurement errors or noise that time, we do get this type of sustained oscillatory output and it is very difficult to extract correct limit cycle parameters or accurate limit cycle parameters and unless accurate limit cycle parameters are extracted then we will get erroneous peak amplitudes and critical frequency which will result in erroneous ultimate gain and ultimate frequency those are used in estimating the transfer function parameters. Thus, the estimated parameters also will be subjected to errors.

Now, is there any way we can avoid this often, it is found that, we do get corrupted output signal from many real time systems, in spite of many effects employing accurate or reliable sensors, it is not possible to obtain clean output signal thereby giving us faulty or erroneous model parameters. So, we have to go for some technique those can be used to obtain clean or smooth or denoised signal from that of a corrupted output signal.

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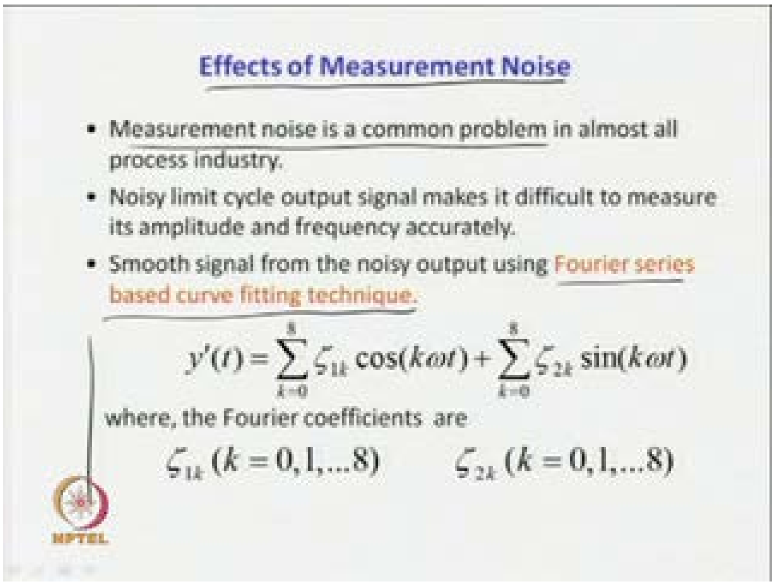

We shall discuss one such technique known as the Fourier series based curve fitting technique. So, what is there in this Fourier series based curve fitting technique, we will see in details later on. Now, let me discuss about the effects of measurement noise. So, measurement noise is a common problem in almost all process industries irrespective of presence of accurate sensors, the output is often found to be corrupted. So, that way, it is a very common problem in almost all industries Now, noisy limit cycle output signal makes it difficult to measure its amplitude and frequency accurately.

It is essential to measure accurately amplitude and frequency of the limit cycle output or the sustained oscillatory output signal because based on that only, we do have analytical expressions or explicit expressions to find parameters of a dynamic model. So, we have to go for some smoothening technique using Fourier series and how can you describe a Fourier series, the Fourier series expression for a function y prime t can be given in the form of, sum of k equal to 0 to 8 xi 1 k cos (k omega t) plus sum of k equal to 0 to 8 xi 2 k sin (k omega t) where the Fourier coefficients are xi 1 k, k varying from 1 0 to 8 and xi 2 k, k varying from 0 to 8.

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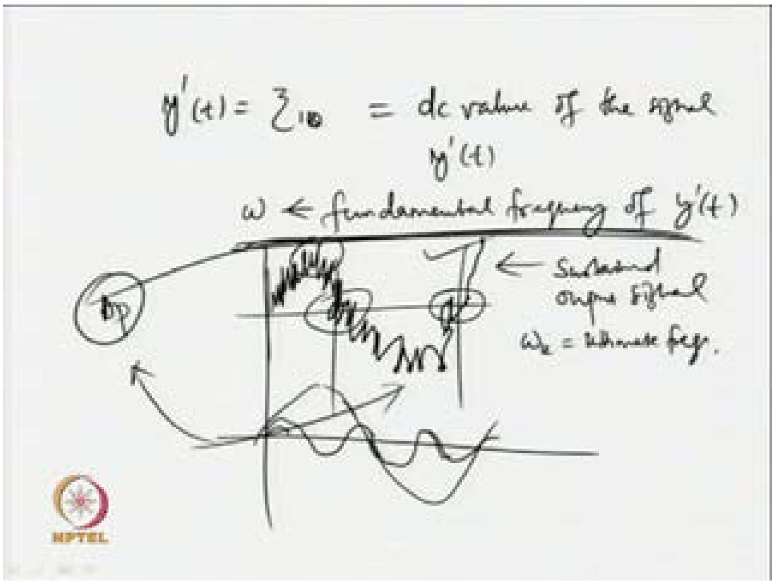

Now, when k equal to zero when k equal to zero what will happen, y prime t will be equal to simply your xi 1 k and k is 0. So, will get a constant of the form xi 1 0. So, this gives the d c value of the signal y prime t and rest of the components or terms will have will have certain frequency where the signal is assume to have a frequency of magnitude omega. So, omega is the fundamental frequency of y dash t.

So, it is assumed that, we can represent any type of practical system with the help of a Fourier series and why we have limited our discussion to eight, ideally it should be infinite, if you considered more number of terms then you have got appropriate representation of any practical signal by it is equivalent Fourier series, but practically what happens you have to have computational complexities to reduce the computational complexities often it is require to limit the number to certain value and that is why I have put eight over here.

So, I do consider seven harmonics sorry eight harmonics basically, because I go from k equal to 0 to 8 thereby we have got nine terms and that way you have got eight harmonic components will be considered what are those harmonic components starting with frequency omega, I will get two omega like this up to eight omega, so, that that way, we will get eight harmonic components of the signal.

So, how can this be used basically the noisy signal, the noisy signal will be represented by its equivalent Fourier series representation. So, the noisy signal is suppose like this, then now, it is very difficult to obtain correct information about its period or frequency because at the zero crossings also you can have noise. So, let me draw this, in a more realistic manner. So, I will have noise like this. So, when this is the output signal sustained oscillatory output signal then how to obtain the information about its period or fundamental frequency, it is very difficult because the zero crossing, I have got hell out of zero crossings over here and so, also I have a number of zero crossings here.

So, how to measure the distance between the two exact zero crossings, I do not know where the zero crossings exist exist. So, that way we may not be able to get accurate information about the ultimate frequency from this signal. Similarly, what about the peak amplitude of this signal, you see which which can qualify as the peak, I may get erroneous value, if measure this value. Similarly, I can have multiple peaks, look at the signal here, how many negative peaks you have, this can qualify as one negative, peak this can qualify as one negative, peak this can qualify as, so, also this one.

So, that way we have got multiple points or multiple measurements then which one to choose. So, certainly this is going to give us erroneous peak amplitude that, but when this signal can be represented by a number of signals having multiples of fundamental frequency then what will happen, I will get one clean signal corresponding to this noisy signal. So, one fundamental, one signal with fundamental frequency must exist then we will have so many harmonic components. So, if this is the signal with fundamental frequency that with the higher order frequency then I will get like this, is not up to the scale or exact to the scale.

So, you go and getting a number of components now like this, when you adopt all these then you get the practical signal that you have obtained from the system, but the analysis gives us a signal with fundamental frequency. Now, I can make measurements on this signal and that is a clean signal which can give us accurate values for the peak amplitude and so, also for the zero instants of zero crossings correctly from where we can measure the frequency or period of the signal period of the signal.

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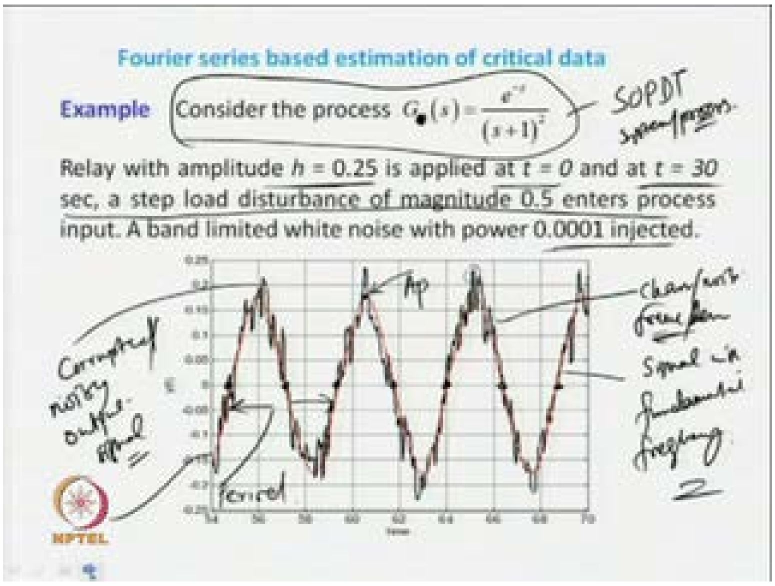

So, this is the concept basically the way, a curve fitting technique is use to extract a signal with fundamental frequency from a practical signal. I will consider an example to illustrate that point. So, consider the process with the transfer function of the form, G p s is equal to e to the power minus s upon, let us take G s because you have been taking G s in our studies. So, G s is equal to e to the power minus s upon (s plus 1) square, this second order plus dead time (( )) system or process is now, subjected to a relay test with the relay amplitude of h equal to 0.25.

So, we are employing asymmetrical relay and when the relay is switched on at time T equal to 0 then from time T equal to 0, we get some output signal then at time T equal to 30 seconds, we do apply also a step load disturbance of magnitude 0.5. Now, this plot is not showing the plot from the time T equal to 0 to time T equal to fifty four seconds why I have neglected that, the main reason for that is that as I have said earlier, we have to look for the signal that we get after the static load disturbance. So, you look at we have to collect information from last few cycles of the output signal not from the from this period where you have got normal operation, but once you have got static load disturbance the output gets distorted.

So, we have to avoid that just prior to stopping the relay auto tuning test you collect signal and that signal is of interest and that is why I have collected the signal from time T equal to fifty four seconds to T equal to seventeen seconds. So, we have obtained some corrupted signal and that is shown our here. Now, band-limited white noise with power 0.0001 is injected, then we get this corrupted output signal this is the corrupted or noisy output signal.

Now, when the curve fitting is applied when the curve fitting is applied now, what do we do get, we get the fundamental component of the signal given by the red curve or the plot in red color. So, the red one if you observe carefully is the fundamental or the signal with fundamental component signal with fundamental frequency. So, the red one if you look carefully is no more corrupted and it has got 0 crossings, no multiple 0 crossings unlike the corrupted signal here, it is touching over here and here. So, I have got multiple zero crossings, the red one is not having multiple zero crossings.

So, that way you can easily identify the zero crossings and using the zero crossings you can obtain or measure the period of the signal, this the period of the signal and now similarly, if I look at the plot in red, the peak amplitude is unique, unlike this plot which one we used to take as the peak amplitude, if you take this one, this might be the contribution from the p s d of the noise power spectral density or the random values we have added. So, that way the correct value for the peak amplitude must be obtained from this plot in red. So, when the Fourier series based for curve fitting is employed then we are able to get the clean waveform or the clean plot the clean one or the noise free I would say noise free one or the denoised one with the help of the Fourier series analysis.

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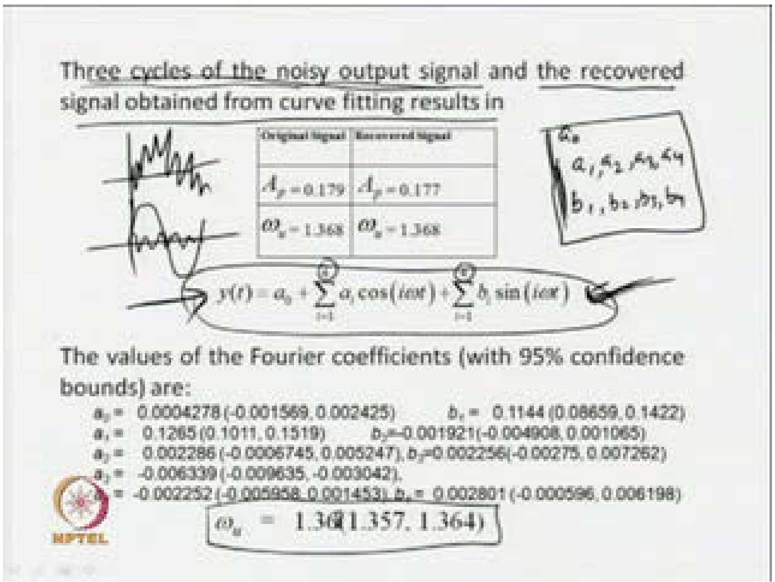

Now, what has been done, how we have obtain three cycles of the noisy output signal is plotted over here, if you see almost three cycles of noisy output signal is plotted along with that, the recoverd signal obtained from curve fitting also provided, how do you recover this one, it is all about simply data fitting. So, given a set of data, you try to feed it with the expression, we have obtained for the curve fitting.

So, this is the data available left hand side, we have a set of datas; that means, what are available y 1 prime, y 2 prime, y three prime and so on and we have got another data said of y 1 prime prime, So, corresponding to the time. So, I will write t 1, t 2, t 3. So, at different time instants, you have got different data and when you collect the data set t and y and fit with that of the actual signal y prime t then you do get various coefficients Fourier coefficients. So, when a corrupted signal y t is fitted with the Fourier series given in the right side of this expression with the terms, y t is y t is equal to a 0 plus sum of i equal to 1 to 4 a i cos (i omega t) plus sum of i equal to 1 to 4 b i sin (i omega t) then we get different coefficients like a 1, a 2, a 3 and a 4 and similarly, b 1, b 2, b 3 and b 4 of course, the d c value of the signal is given by the coefficient a 0.

So, we will get these estimated values for these coefficients with certain confidence levels are bounds, those are given over here, now please keep in mind, this output data is collected and fitted with a Fourier series which has got nine number of coefficients keep in mind, I have limited my simulation or program to up to four; that means, how many harmonic components we will have, we will have four harmonic components for the corrupted signal.

So, the corrupted signal is being met with the corrupted signal is being met with that of a signal with fundamental frequency and then you have got harmonics different harmonics. So, we have got four three harmonics additionally apart from the fundamental frequency. So, basically the signal is represented by a signal which has got four frequency components then we get the ultimate frequency from the mapping as omega u is equal to 1.36 one digit is missing here 1.368.

So, thus the ultimate frequency is estimated as 1.368 and from the clean signal from the fundamental component of the signal, we get the peak measured as A p is equal to 0.177 the had there been no noise added to the system, we do get the peak as A p is equal to 0.179 and the ultimate frequency as1.368 that source that using this Fourier series based the curve fitting, it is possible to obtain correct information of the signal correct correct limit cycle parameters from the corrupted signal because the critical frequency or ultimate frequency is obtained accurately and there is minor error in in in in the peak amplitude.

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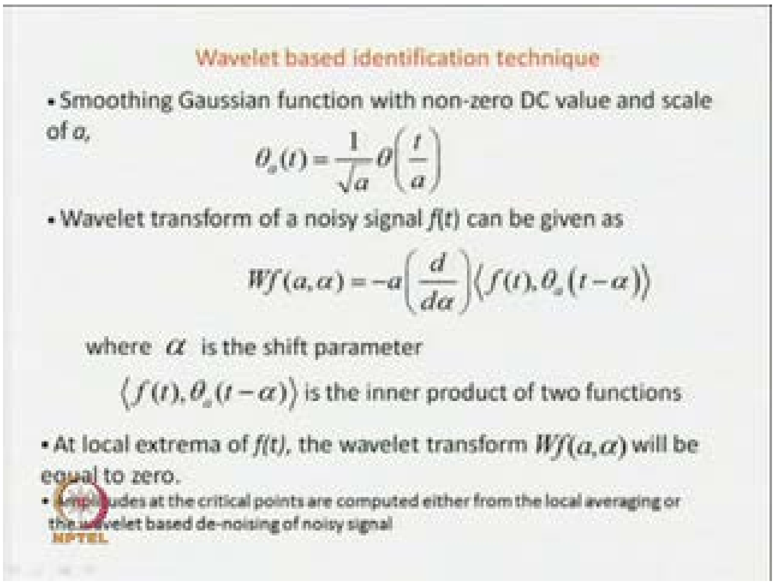

Next, we shall go to some other technique that is known as wavelets wavelet based identification technique.

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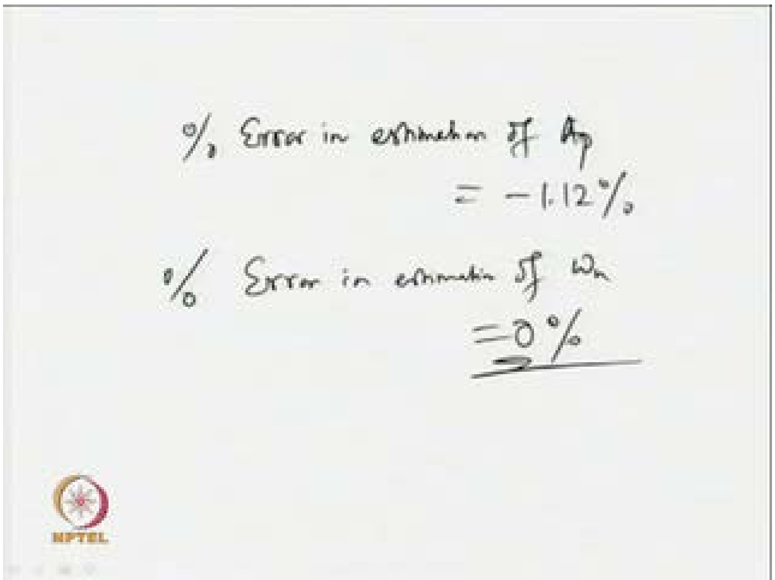

So, why we are not happy with this technique or what are the limitations of the this technique actually, if you see the tabular values the percentage error in estimation error in estimation of the peak amplitude is almost minus 1.12 percentage and percentage error in estimation of the ultimate frequency is almost 0 is 0. So, still the method is subjected to some estimation error although, we do obtain demised or clean signal using the Fourier series based curve fitting still the method is not free from errors therefore, effort is made to discuss another technique known as wavelet based identification technique, that can be used to obtain or estimate the parameters associated with a noisy limit cycle accurately.

Now, I will introduce some Gaussian function some smoothing Gaussian function with non-zero D C value and a scale of a, as theta a t is equal to one upon square root of a theta (t upon a) when you take the plot of this function then you can see the way, it will appears now, what is theta, theta is not time delay in this case, this is a function. Now, theta is the basis function basis function of a wavelet and a is the scale that controls dilation of the signal. So, I do believe that, you have some basic of wavelet transforms and wavelets. So, this smoothening function can be used in our study, now, when this wavelet transform of this function smoothing Gaussian function is found.

So, wavelet transform of the noisy signal can be now given as W f (a, alpha) W f wavelet transform is now a function of the scale and some shift parameter, alpha is a shift parameter then the wavelet transform can be given by the expression W f (a, alpha) is equal to minus a times first order differentiation with respect to the shift parameter of inner product of the function and the basis function, the function means what, the signal, the corrupted signal with that of the basis function.

So, the inner product of the two functions, what are the two function, the the signal shown by the function f t and the basis function given by theta a in our case. So, the inner products of the two and first order differentiation is taken, that results in interestingly a situation where at local extremum of the signal local extremum of the signal, the wavelet transform will result in coefficients with zero.

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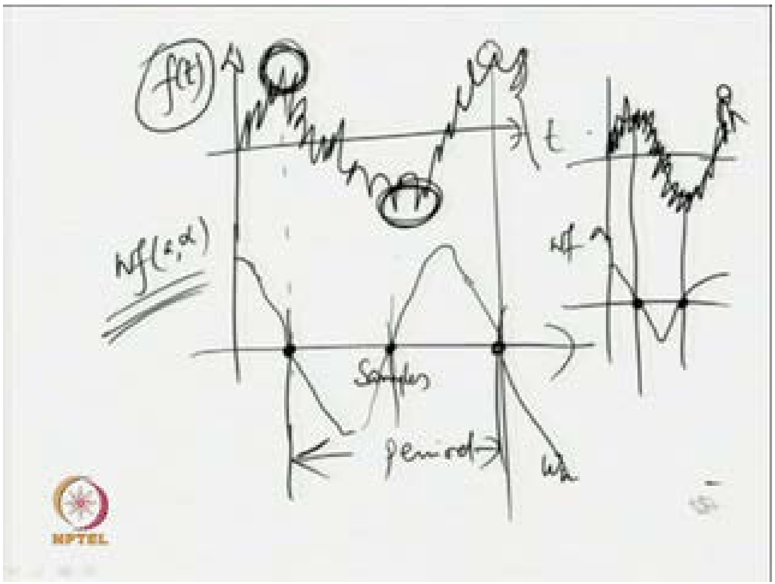

So, coefficients with values zero, What I mean by that, suppose the signal is original signal is corrupted like this, when its wavelet transform found, this is the signal f t upon versus t then for the wavelet transform W f a and alpha will give us samples will give us samples, where the samples will have zero magnitude at the extremum where are the extremam of the signal, we do have extremum here, we do have extremism here. So, at the exact extremism only due to the effect of basis function will get the coefficient as 0 and at other at any other location suppose, this is the correct then I will have a wavelet transform with coefficient zero.

So, how this, we have got two extremums now, what about the remaining coefficients those will be nonzero. So, I do not mind what are these values sorry what these values you have. So, I can have plot like this, only at the extremism here another extremum is here, suppose then you will have zero. So, actually the form or nature of this wavelet transform plot is not important, what is very important for us is that the extremism of the signal is tracked accurately by the wavelet transform technique.

So, using the wavelet transform, the peaks associated with the, the peaks of the corrupted signal can accurately be found, how can you find, as I have done here. So, the peaks results in zero values therefore, we can accurately find the zero crossings. So, what is the benefit of getting zero crossings, we know that the zero crossings of the this wavelet transform transform corresponds to the peak of the corrupted signal means, actual signal that is present present in the corrupted signal. So, if the actual signal, let me again give this examples, suppose, this is the actual signal or the real signal or uncorrupted signal.

Now, it will be corrupted like this, then the wavelet transform of this one, this is the peak from the clean signal and this is the peak from the clean signal. So, the wavelet transform will be having some value like this. So, you will have a plot like this, the peaks have been located correctly. So, from the location of the peaks basically, we do get correct information about the zero crossings of the signal and from the zero crossings, you do get correct information about the period of the signal or ultimately the ultimate frequency of the signal. So, omega u can be obtained can be obtained accurately that is very important, what other information you can obtain.

Now, you have been able to locate where your peak remains then take the averaging of this signal. So, average find the average of this signal, around that peak then you can locate or you can find correct information about the peak amplitude. So, the averaging, first of all, it is very important to locate first of all it is very important to locate the zero crossings zero crossings sorry this is the zero crossings and next, is it is very important to locate, where your actual peak amplitude remains. So, once you have located, if the noisy signal is average around that, then the incorrect information about or correct measurement of peak amplitude can be obtained or can be made. So, that is the benefit you get from wavelet based identification technique.

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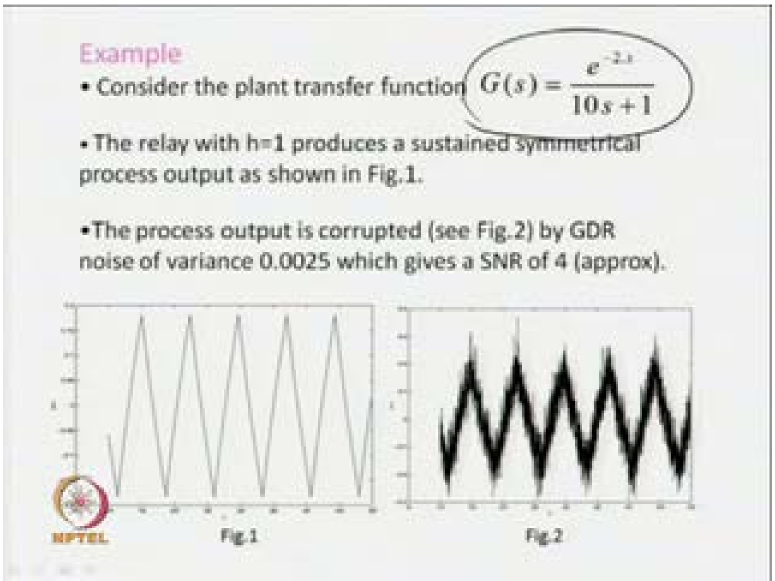

Let us, see some example for the first order plus dead time transfer function model, a relay with h equal to 1, parameter 1 produces a sustained symmetrical process output as shown over here, when the process output is corrupted by the noise with variance of 0.0025, which gives a signal to noise ratio of 4 approximately, I get the signal corrupted with this much of a S N R in this form. So, basically this signal as resulted in this one, then how to get back this signal from the noisy one, that is what, we have been discussing.

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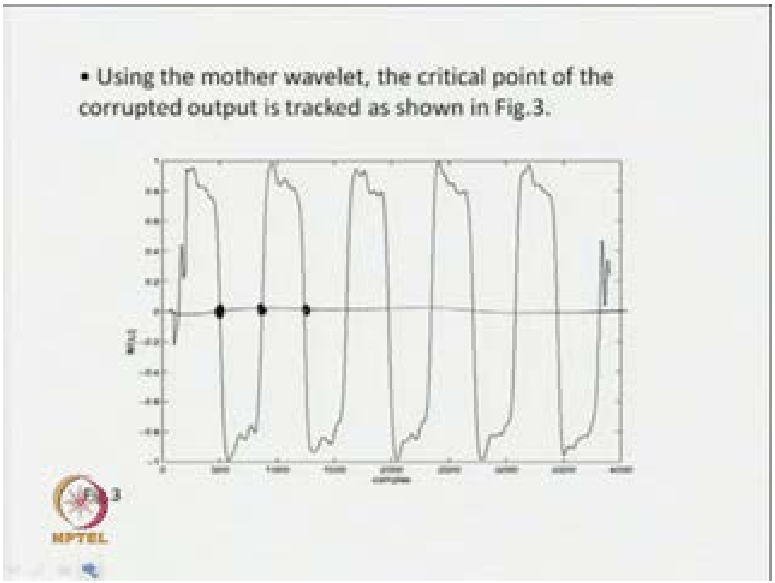

So, given a noisy signal like this, how to retrieve the correct limit cycle output signal. Now, when the wavelet transform of this is taken, what type of output we get, this is what we get. So, if you look at this, then the peak of the output signal are here. So, basically the peak of the basically the peak corresponding to these peaks will have these zero crossings. So, the period can be measured easily because the period is nothing, but if you measured from any peak to peak, you do get information about the period of the signal and from the period, you can get correct information about the ultimate frequency. Now, how to find the magnitude, now, once I have located where the where the peaks remains, suppose, at this time, the peak remains then I will take the average of this signal around that and I will get correct information about the peak amplitude.

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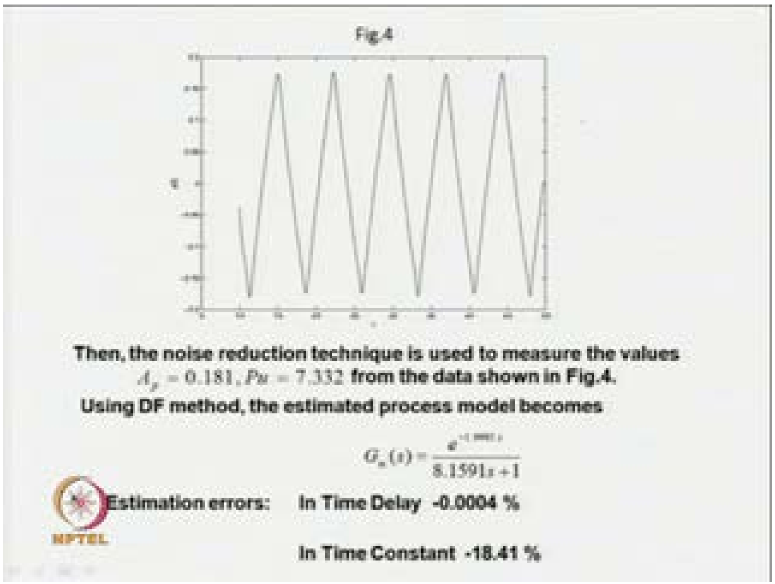

Thus, it is possible to obtain or retrieve the signal from the corrupted or noisy signal. Then the noise reduction technique is used to measure the values A p of this and of peak amplitude of this from shown data and then the describing function method is employed to estimate process model parameter. So, when you see this one, the process model parameters now calculated from this noise free or recover signal have got estimation errors of 0.0004 percent for the time delay and about 18.41 percent for the time constant. So, these are the underestimated values. So, this way although, it it is also not free from errors keep in mind, but wavelet transform provides or some cleaner signal which can be used for measurement of limit cycle parameters.

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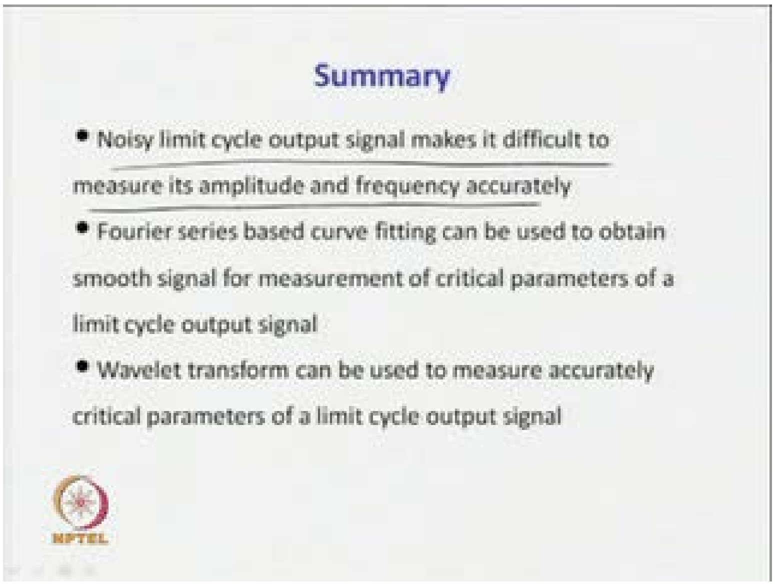

So, in summary we have, noisy limit cycle output signal makes it difficult to measure its amplitude and frequency necessary for estimating parameters of a transfer function model. Now, Fourier series based curve fitting can be used to obtain smooth signal and there were critical parameters of the limit cycle output signal effectively. Now, also wavelet transform can be used to measure accurately critical parameters of the limit cycle output signal. So, thus we have studied two techniques using which we can retrieve limit cycle output signal from the noisy one.

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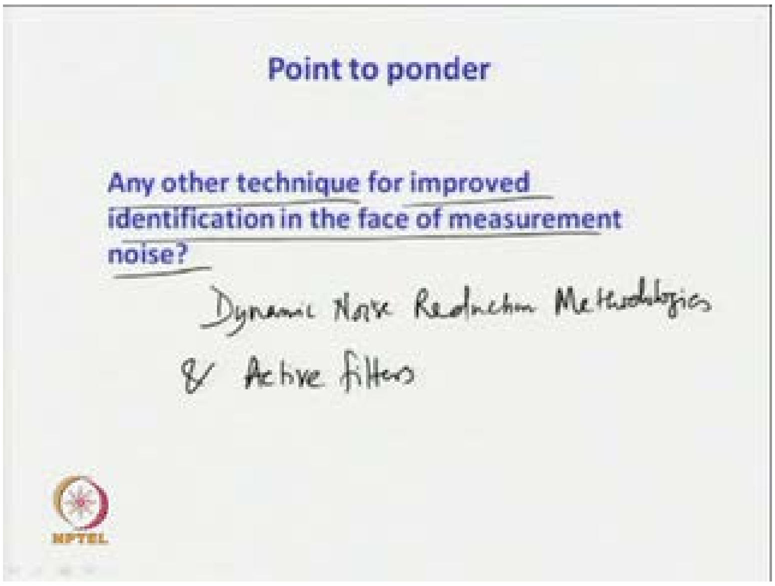

Now, any point to ponder do we have any other technique for improved identification in the face of measurement noise or when the limit cycle output is corrupted with noise. Apart from, the wavelet based smoothening technique or curve fitting technique or and the Fourier series based denosing technique or curve fitting technique. We do have dynamic noise reduction methodologies, and active filters; those can be used to improve upon the identifications or to make correct measurements from noisy signals, Thanks.