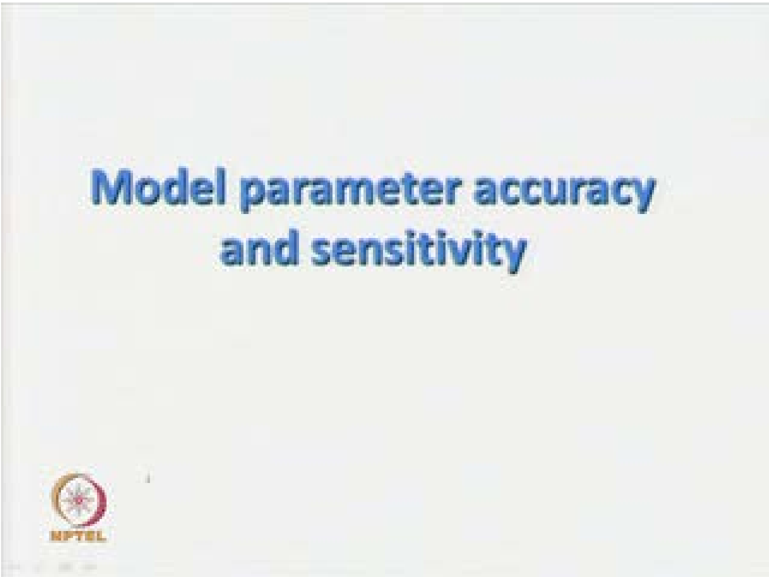
Advanced Control Systems

Prof. Somanath Majhi
Department of Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati

Module No. # 03
Time Domain Based Identification
Lecture No. # 18
Model Parameter Accuracy and Sensitivity

Welcome to the lecture 13 th Model Parameter Accuracy and Sensitivity.

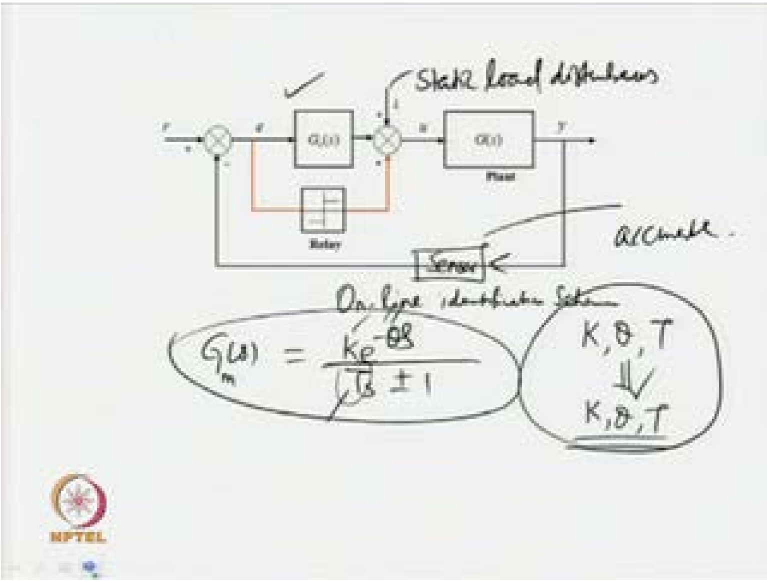
(Refer Slide Time: 00:24)



In this lecture, we are going to see, how the model parameters are sensitive to not only the measurements or measured quantities of limit cycle output rather, how the parameters are also getting affected by inaccuracy in the identification techniques. So, we have gone from offline identification technique to online identification technique, just to improve upon the transfer function models. Then, in spite of going from one to other technique, the model parameters may not be free from measurement errors, may not be free from identification errors or estimation errors.

Now, errors can be quantified by various statistical measures, such as average values, mean values, standard deviations, variance and so on, using also root mean square values. But, in spite of using all those techniques it it is felt that, you will must have some sensitivity analysis of the parameters of a transfer function model to accurately judge, what is happening with the transfer function model parameters, when there are measurement errors, when there are errors associated with the identification techniques and when you are going for more number of unknowns associated with a transfer function model.

(Refer Slide Time: 02:02)

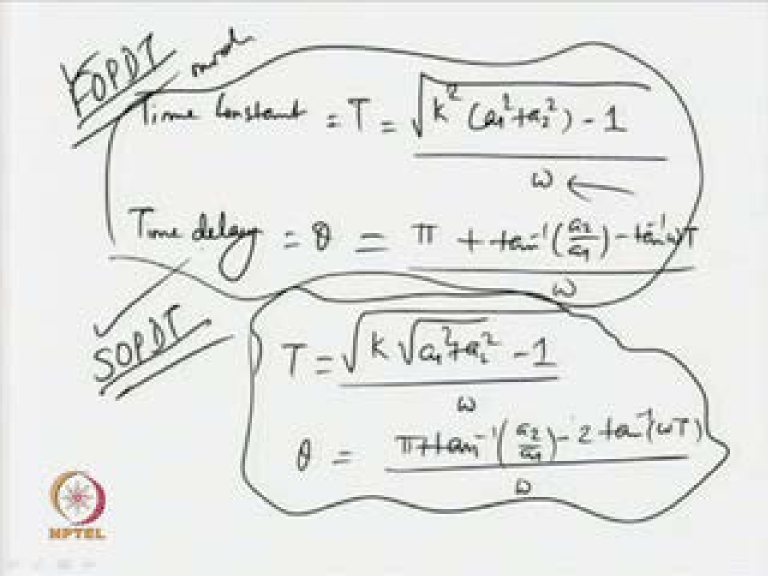


Now, in the online identification technique; online identification scheme, we use a relay in parallel with a controller, just to improve upon, basically to improve upon the estimated parameters or to minimize the estimation error associated with transfer function model of a plant. So, the plant transfer function model, G m s, I have to write now is given by k e to the power minus theta s T s plus minus 1. So, this transfer function model has got three unknowns; k, theta and T.

Now, if I go for offline identification without using a controller in the loop during the relay test, then I will get certain values of k, theta and T. Whereas, when I go for online identification; I will get improved values of k, theta and T, not only improved values for k, theta and T rather, during online identification; we can overcome the ill effects of static load disturbances static load disturbances in particular.

And if there is sensor inaccuracy, we have the sensors in the feedback path, so if we have sensor inaccuracy, then we will get erroneous measurements of the output signal and to resulting in erroneous values for the parameters of the sustained oscillatory output signal. So, sensors are assumed assumed to be accurate. So, for this identification technique in our last lecture, we have found the model parameters explicit expressions for the model parameters theta and T and let me repeat those expressions once more.

(Refer Slide Time: 04:16)

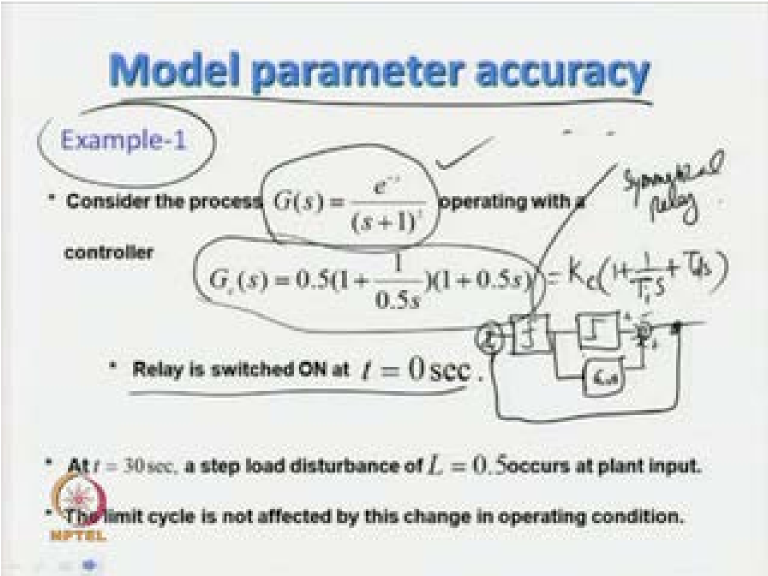


So, theta or the time constant of the transfer function model, T can be obtained using k square a 1 square plus a 2 square minus 1 root divided by omega u, so where omega u is also equal to omega, the frequency of the output signal, why I am using u not necessarily let us use omega in place of omega u. And similarly, for the time delay associated with the transfer function model, we have an expression given as, theta is equal to pi plus tan inverse a 2 by a 1 minus tan inverse omega T divided by omega.

Now, again a 1 is given by 4 h by pi A plus the parameters of the controller will come into picture, so depending on different type of controller, we will have different type of expression, so this is for the first order plus dead time model, so this will have plus k c and b 1 sorry a a 2 will have again an expression given as, k c times your function of k c times function of T i and T d, so T i and T d, so this is how you get a 1, a 2 and so on. Using this T and theta, now we can estimate the transfer function model parameter, when omega, a 1, a 2 are available.

Similarly, this is for the first order plus dead time transfer function model. Similar expressions for the time constant and time delay for the second order plus dead time transfer function model can be obtained as, T is equal to k time's root of a 1 square plus a 2 square minus 1 square root divided by omega. And theta is equal to pi plus pi plus tan inverse a 2 divided by a 1 minus 2 tan inverse omega T divided by omega. So, this is how we have obtained explicit analytical expressions for the parameters of the first order plus dead time transfer function model and second order plus dead time model based on online identification scheme, based on the online identification technique.

(Refer Slide Time: 07:40)

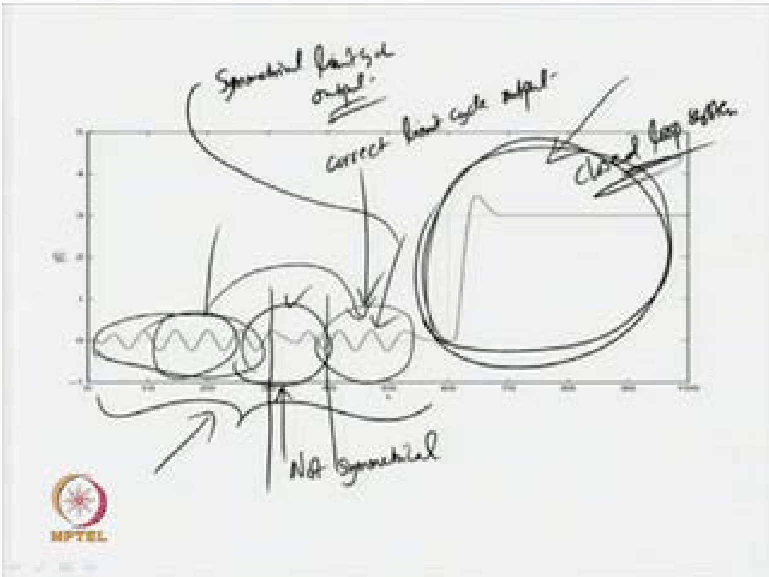


Now, the model parameter accuracy will be described with the help of two example; in the first example, we consider a plant with dynamics given as, G s is equal to e to the power minus s upon s plus 1 square, which is operating with a controller G c s is equal to 0.5 times 1 plus 1 upon 0.5 s times 1 plus 0.5 s that means, now we have got k is equal to 1 or the steady state gain of the process or plant is 1.

And I am going to estimate the transfer function model for this dynamics for this dynamics. Now since, we are going for an online identification scheme, the p i d series form of p i d controller is used, where it has got the general expression; k c times 1 plus 1 upon T i s plus T d s, so the proportional gain of the p i d controller is of magnitude 0.5, the integral time constant of the p i d controller is of value 0.5 and similarly, the derivative time constant of the p i d controller is also having a magnitude of 0.5.

So, we have got a p i d controller of this form, now when the controller and relay are connected in parallel, controller G c s and relay is connected; the relay is a symmetrical relay, then limit cycle is induced or sustained oscillatory output is obtained. Now, when the relay is switched on at time, t equal to 0 second, we will definitely be able to since I am using a stable process, definitely limit cycle will be limit cycle output will be obtained. Then, what sort of limit cycle output is expected?

(Refer Slide Time: 10:23)



You see the type of output we get when relay experiment is conducted. So, this is the output we get initially, when the relay is switched at time t equal to 0. Then, at time t equal to 30 seconds, a step load disturbance of magnitude 0.5 is injected, so where do we inject that step load disturbance static load disturbance here. So, when it is injected sorry I have forgot to put the process here, so process G s is here and you have the negative feedback, this is the correct block diagram for the online identification scheme that already we have given here.

So, simply I am redrawing the same over here at time t equal to 30 seconds, a static load disturbance of magnitude 0.5 occurs due to that, there will be changes to the limit cycle output for certain time you see during this, we do not get symmetrical limit cycle output for the second part for the duration from sorry 30 seconds onward for few seconds; the type of limit cycle output you get is not symmetrical not symmetrical.

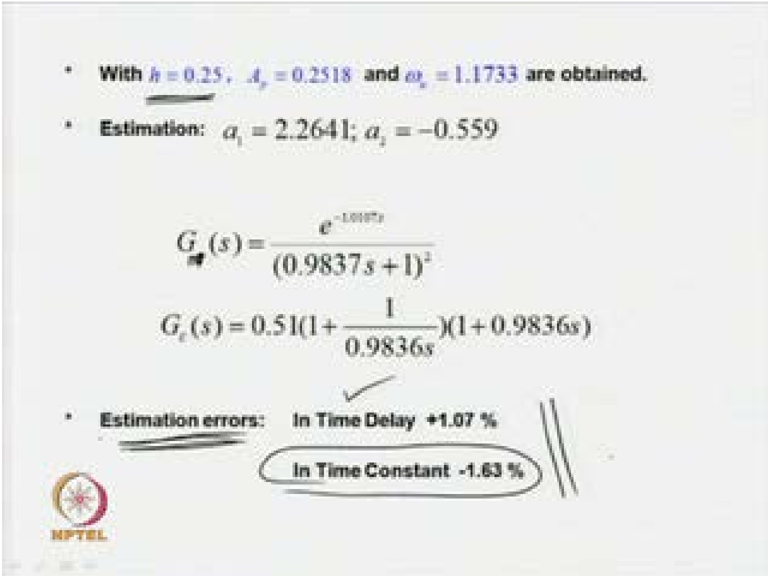
So, we cannot make any measurement, it is very difficult to obtain correct information from that output signal or if at all you make measurements during this period, when you have a static load disturbance or immediately after occurrence of the static load disturbance, then you will measure erroneous values for the peak amplitude and frequency, then what is observed after sometime again the original limit cycle output is restored.

So, the correct limit cycle output occurs after few seconds, why that is happening as you know the controller is there in the loop and the controller has got integral action the controller has got integral action and due to the integral action of the controller, the effects of static load disturbances get nullified or rejected after certain time; that means, in steady state condition, what happens the effects of static load disturbances are rejected are eliminated. So, when the effects are eliminated, we get back the correct limit cycle output as it was there prior to the occurrence of the static load disturbance. So, you look at the waveform before 30 seconds, whatever you have you get similar waveform in the steady state condition after occurrence of the static load disturbance.

Now, the measurement should be made either from here or from here, so basically it matter you have to target symmetrical limit cycle output, so target symmetrical limit cycle output, consider few cycles and make measurements from the stable limit cycle output symmetrical output signal. Then, when the tuning phase or auto tuning test phase is over, then the relay is switched off then, what happens, then you get the normal operation of the system and the controller provides you, if you have tuned the controller properly provides you proper dynamics of the closed loop system. This is the dynamics or output you get from the closed loop system (Refer Slide Time: 14:53).

The limit cycle is not affected by this change, which change? By the static load disturbance or the step load disturbance in normal operating condition that means, because of the presence of integral integral action of the p i d controller, the ill effects of static load disturbances are eliminated successfully in steady state condition.

(Refer Slide Time: 15:32)



So when a relay with a setting of 0.25 is connected in parallel with the controller, the limit cycle output results in peak amplitude of value A p is is equal to 0.2518 and a frequency of 1.1733 using A p and omega u. Now, for using this value in the formula I assume that, omega u is the ultimate frequency ultimate frequency which is nothing but, the frequency omega. So, for all practical purposes, we will use omega equal to omega u omega equal to omega u, so when A p, omega u, h are put in the formulae that we have found for the parameters of this second order plus dead time transfer function model, then T and theta are estimated.

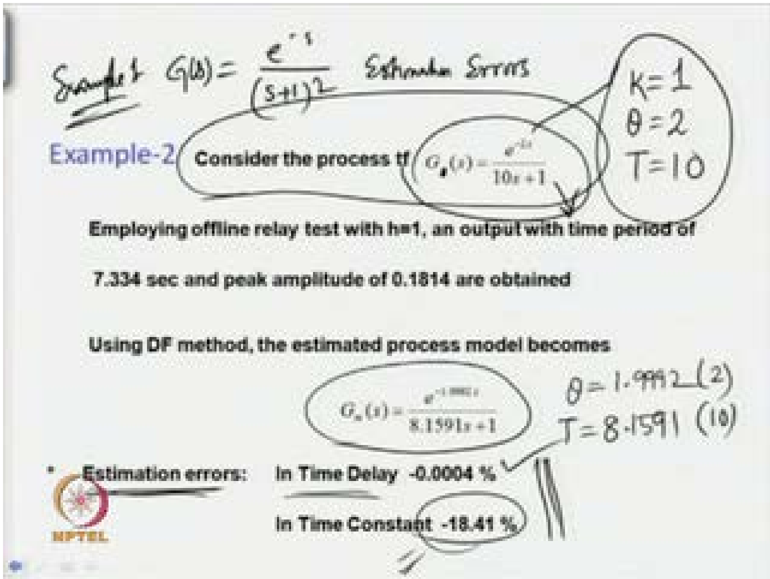
Now, k is found by some other technique or assumed to be known and then, a 1, a 2 are estimated. And when you substitute all those values, you get the estimated values for T and theta for the second order plus dead time transfer function model.

So thus, the values are estimated, so what is the T we obtain, the T we obtained is 0.9837 and the theta we obtained is 1.0107. So, in the transfer function model for the estimated parameters are shown over here and the the model has got a second order plus dead time transfer function of the form, G m s is equal to e to the power minus 1.0107 s divided by 0.9837 s plus 1 square. Now, how the controller is designed that will not be discussed now, rather we have interest in the identification in the accuracy of the parameters T and theta or the estimated values of T and theta.

Now, if you see ideally the T should have been 1 you see here the T is equal to 1 and theta is equal to 1. So, ideally (( )) analytical expressions should have given as, T should have given as T equal to 1 and theta equal to 1 in place of that, we have estimated T as 0.9837 therefore, the estimation is having errors of minus 1.63 percent in the time constant. And similarly, the time delay should have been 1 in place of 1.0107 therefore, the errors or error estimated with the estimation is now plus 1.07 percent, so the estimation errors for time delay is plus 1.07 percent and for the time constant is minus 1.63 percent.

So of course, these estimation errors are acceptable not so bad, but in spite of that, it all depends on how much minimum estimation errors can be, if you go for proper identification technique or if you go for some exact analytical expressions. So, the estimation errors or the inaccuracy in estimations are found to be of these values, this is how the model parameter accuracy is ascertained. Let us go to the second example.

(Refer Slide Time: 20:27)



In this example, we consider a process of dynamics G s is equal to e to the power minus 2 s upon 10 s plus 1, so the original process has got a time a steady state gain of 1, a time delay of 2 and a time constant of 10, the original process. Now, when the identification technique is employed or the set of formulae we have therefore, the first order plus dead time transfer function model that is this and this are used in that case, we obtain the transfer function model as, G m s is equal to e to the power minus 1.9992 s divided by

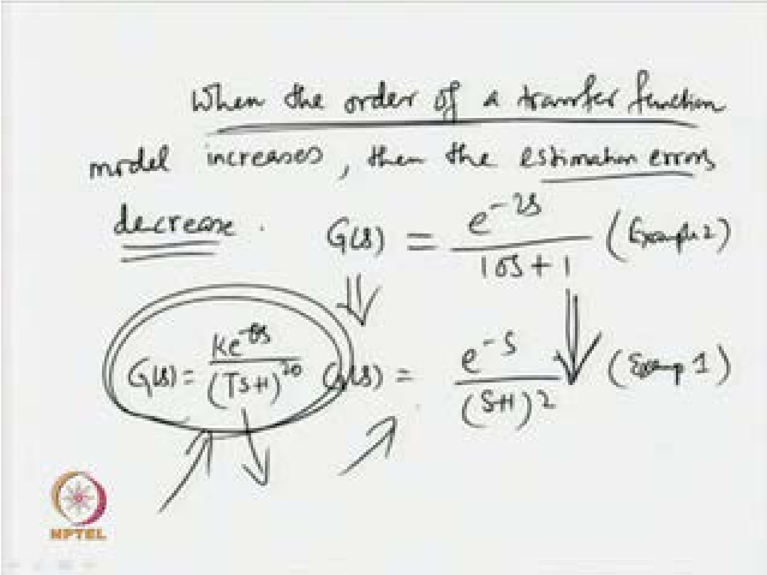
8.1591 s plus 1 that means, the estimated values for the time delay is equal to 1.9992 and that of the time constant is equal to 8.1591 in place of 10 of course, and in place of 2 (Refer Slide Time: 21:05).

So, now the estimation errors in absolute value term are obtained for the time delay as, minus 0.0004 percent and for the time constant as, minus 18.41 percent. Now, these values not acceptable that means the identification technique has not yielded proper identification or estimation of the transfer function model parameters.

So, the identification technique is subjected to model parameter inaccuracy. So, also one more observation we have that, when the transfer function model order decreases earlier in the example, one we had considered, a second order process a second order process, but in the second example; we have considered a first order process. So, in the example 1 we had G s is equal to e to the power minus s upon s plus 1 square, so we had a second order process and the estimation errors are found to be, estimation errors for the time constants and time delays are found to be negligible.

Whereas, for the second example the estimations errors are not so negligible, because in the case of time constant, the estimation error is something more than 18 percent, which was not the case in example 1, so this point is to be taken care of what I mean by that, when the order of a transfer function model increases, then then the estimation error decrease when the order of a transfer function model increases, then the estimations estimation errors decrease what I mean by that, let me again give some example; when I consider a higher order from the first order to higher order transfer function, so initially G s is equal to e to the power minus 2 s upon 10 s plus 1 and later on when G s becomes e to the power minus s upon s plus 1 square, this is what the process dynamics we have in example 2 and example 1. So, we have got less estimation error in example 1 than that of in example 2.

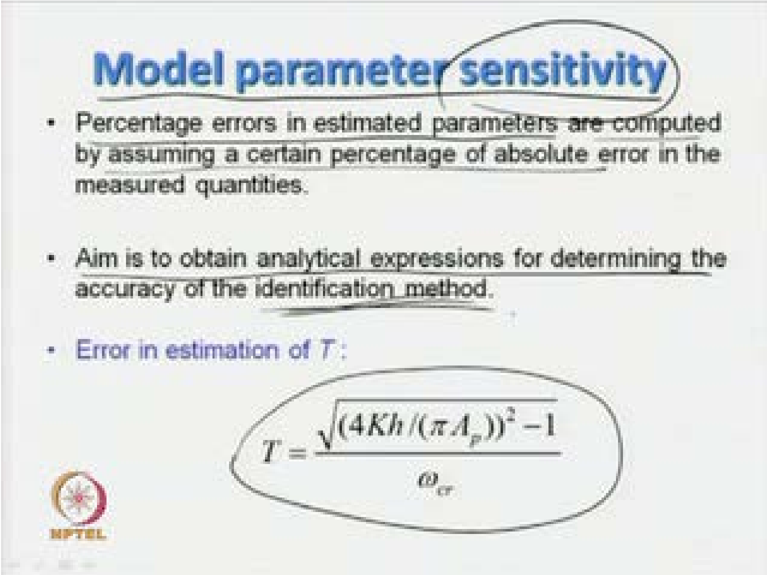
(Refer Slide Time: 25:15)



So, that can be stated in the form of or that can be generalized in the form of, when the order of a transfer function model increases, so when the order of the transfer function model sorry going from here to here increases when the order increases, then the estimation errors decreases.

So, if you use further higher order transfer function models like G s is equal to some k e to the power minus theta s T s plus 1 to the power 20, then some of the poles of this transfer function can behave as filters and smoothen the measurement noise associated with the limit cycle output and give you a transfer function model with parameters having less estimation errors or the estimation errors of the parameters of this transfer function model or or the transfer function model for this dynamics will be this dynamics will be having less values. So, this is how the parameter accuracy associated with the model parameters can be described. Now, we shall go to discuss about the model parameter sensitivities.

(Refer Slide Time: 26:48)

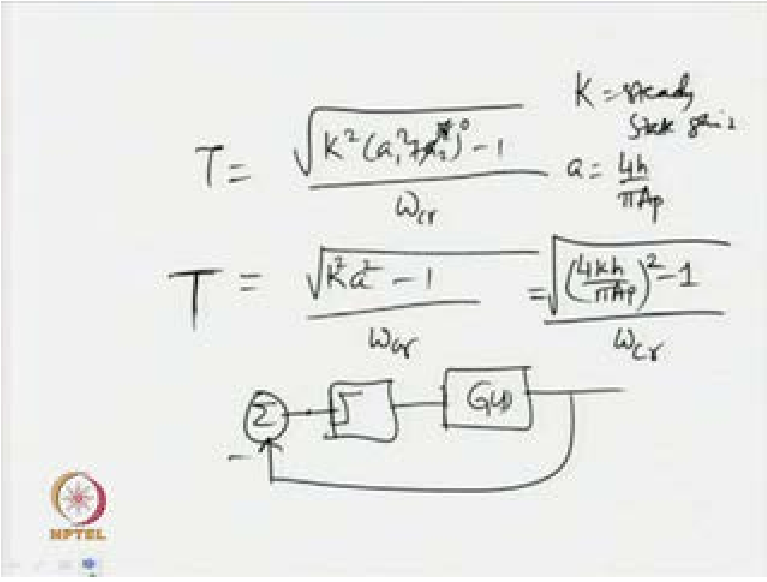


So, what we mean by model parameter sensitivity; the dependency of the process model parameters on that of the measured measured quantities or measurements or that of the other parameters of the transfer function model is known as the model parameter sensitivity.

Now, percentage errors in estimated parameters can be computed by assuming a certain percentage of absolute error in the measured quantities, what we mean by this, suppose I have got a transfer function model parameter T, how can I compute the errors associated with this absolute error associated with this for that, I have to consider the change in the parameter delta T, so delta T upon T will give you further the relative absolute error and when you multiply this by 100 and find the percentage, then you get the absolute relative error in percentage, this is how you can find the errors particularly the accuracies of the transfer function model parameters.

Now, same can be extended to find the sensitivity of the transfer function model parameters, now our aim is to obtain certain analytical expressions for determining the accuracy of the identification method. Now, I will use the simplest expression we can have for the time constant, which is given by T is equal to square root of 4 k h upon pi A p whole square minus 1 divided by omega c r, where from you are getting this T, expression for T basically as you have seen, you can find the expression for T from the earlier expressions, I have already given at the beginning.

(Refer Slide Time: 29:20)

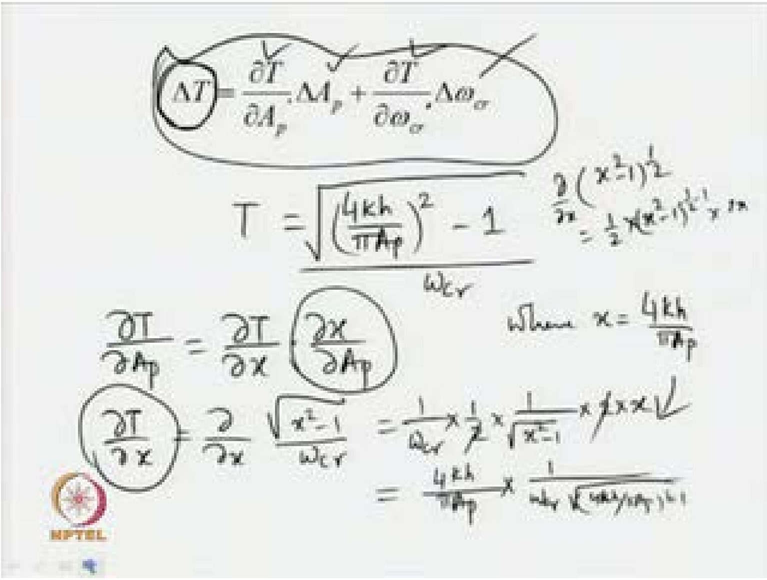


So, T is equal to T is equal to k let me consider the second one than better of the first one. So, T is equal to k square a 1 square plus a 2 square minus 1 root upon omega, so I will I will use omega c r; the critical frequency, omega c r is now same as omega u is now same as omega. So, the the limit cycle output frequency is denoted by various variables sometimes, it is by omega, sometimes by the ultimate frequency; omega u, where the subscript stands for ultimate and sometimes by the critical frequency; where the subscript c r stands for critical.

So, I will use the term omega c r here, now this can be expressed as k a minus 1 divided by omega c r, when a 2 is equal to 0, so when a 2 is equal to 0 sorry when this is a 2 is equal to 0, I have got k square a square minus 1 root divided by omega c r, when you have got a 2 equal to zero, when i have no controller in the loop or I mean to say, what I use offline identification scheme at that time a 2 becomes 0. And we have an identification scheme, which can be given in the block diagram for (( )) a relay and a process put together in closed loop with negative feedback.

So, for this case we do not have any controller either in series or in parallel with the relay in that case a 2 becomes 0, because if you look carefully a 2 can be expressed as in terms of k c times certain things and k c will be equal to 0, when there is no controller in the relay control system or during the relay experiment. So, when a 2 equal to 0, then the expression for T is equal to square root of k square a square minus 1 divided by omega c r, what is k? k k k is the steady state gain and what is a, is given by 4 h by pi A p, so finally, how much I will get upon substitution of k and a in this expression; we get 4 k h divided by pi A p square minus 1 root upon omega c r, so this is how you get the expression for the time constant T. So, I have got the simplest expression for the time constant given by T is equal to root of 4 k h upon pi A p whole square minus 1 upon omega c r. Now, how to find the sensitivity of T, what I have to do?

(Refer Slide Time: 32:58)



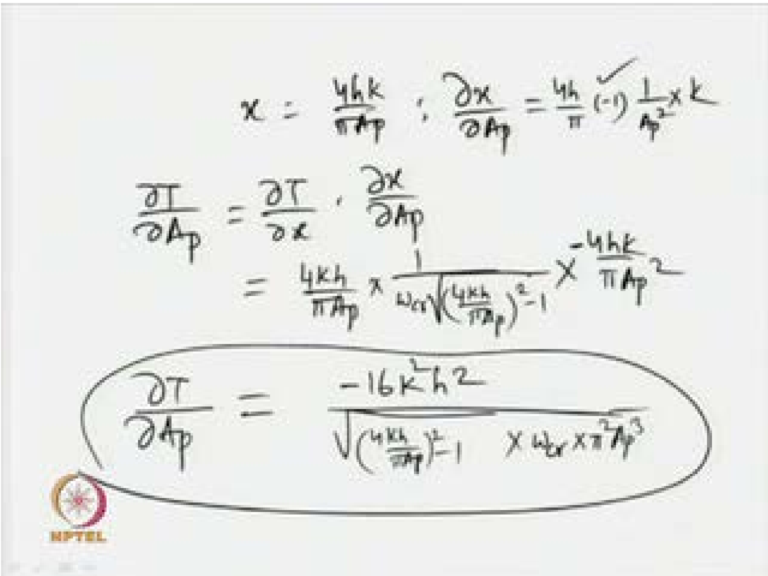
I can make use of this analytical expression or this expression, what is this, the change in time constant; delta T can be given in terms of the partial differentiation of T with the respect to the variables the expression T h that means; delta T is equal to partial differentiation of T with the respect to the peak amplitude times delta A p plus partial differentiation of T with the respect to the critical frequency omega c r times changes in omega c r, so this is one very simple expression, which is often found in many textbooks.

Now using that, I will be able to find the change in T due to the changes in the measured quantities peak amplitude and the critical frequency. So, let us find analytical expressions for delta T now, now we know that, T is equal to 4 k h divided by pi A p square minus 1 root upon omega c r, how to find to find the changes in or delta T I need to find, two partial derivatives; so, let me first find the partial derivative delta, del T upon del A p, this will be to find this one again I will make use of delta T upon del del T upon del x time del x upon del A p, where x is equal to 4 k h upon pi A p, then delta del T upon del x will be equal to del upon del x of root of x square minus 1 upon omega c r.

So, this will give as 1 upon omega c r, then differentiation of this will give you half times 1 upon x square minus 1 root minus 1 of course, into minus 1 sorry, so it is half if I take x square minus 1 half, so differentiation of this with the respect to x will be half times x square minus 1 half minus 1, so it will be minus half definitely it comes to the denominator times 2 x. So, this is how you will get, no minus here rather, you will have 2 into x, so 2 2 will cancel out giving us finally, an expression as 4 sorry root will come 4 k h upon pi A p that is for x into 1 upon omega c r root of 4 k h upon pi A p square minus 1.

Simply, substitute the value for our expression sorry expression for x over here to find delta del T upon del x similarly, we need to find del x upon del A p.

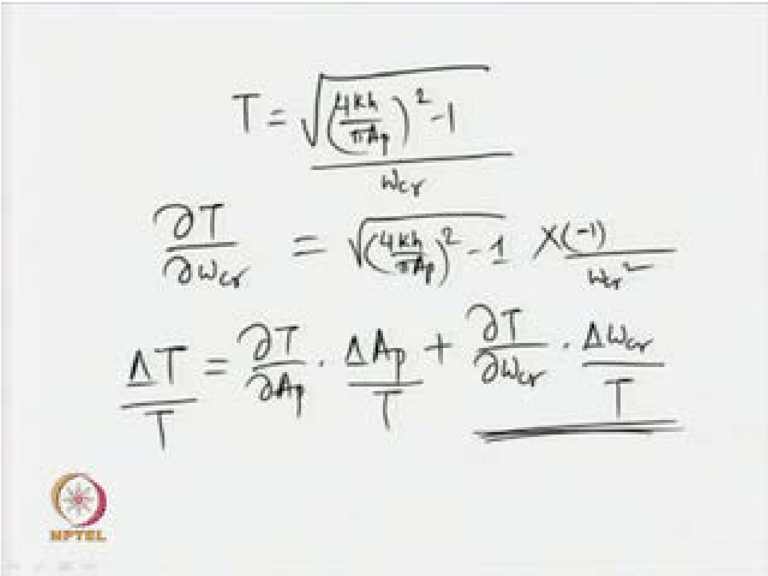
(Refer Slide Time: 37:23)



Since, x equal to 4 h by pi A p, so del x upon del A p will be, simply 4 h by pi minus 1 then 1 upon A p square, so that will give us finally, your del T upon del A p as del T upon del x time del x upon del A p as, minus 16 k square h square, so when you substitute let me substitute back certainly you will get those values, if time permits I will substitute all those values. So, I will get 4 k h upon pi A p 4 k h upon pi A p times 1 upon omega c r root of 4 k h upon pi A p square minus 1, that is what we have got for this one. And for del x upon del A p is given by 4 h minus 4 h due to this minus 1 minus

4 h upon pi A p square, which is giving us minus 16 k sorry 4 yeah minus 16 k where is 4 h by A p, so minus 16 k h square divided by root of 4 k h upon pi A p square minus 1 with terms like, omega c r into pi square A p cubed, so this is how I get x is 4 h 4 k h sorry x is equal to 4 k h by pi A p 4 k I am missing a k over here, so 1 k will come in the expression, so it will be k square finally, it will be k square, 1 k is missing. So finally, we get the partial differentiation of T with the respect to the peak amplitude A p as this one.

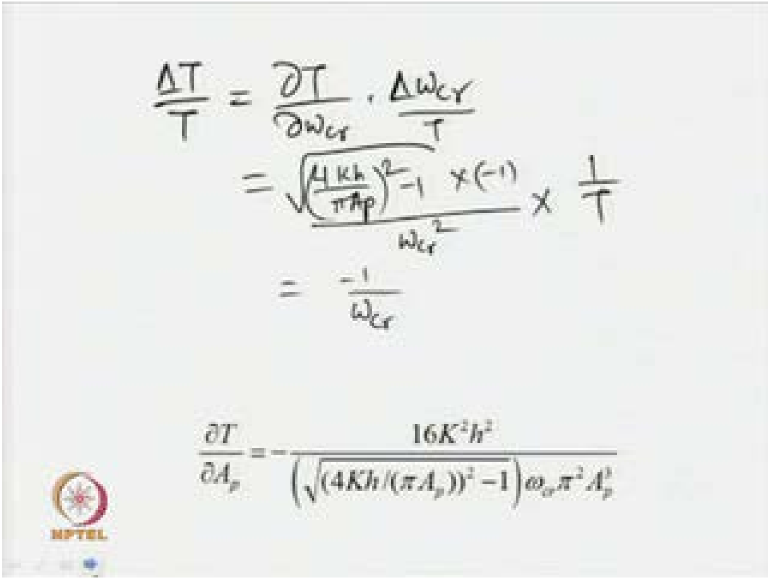
(Refer Slide Time: 40:17)



Now, since again T is equal to 4 k h by pi A p square minus 1 root upon omega c r, then the second partial differentiation with the respect to omega c r, so with respect to omega c r, del T of del omega c r will be simply your root of 4 k h upon pi A p square minus 1 into minus 1 divided by omega c r square. So, I will get the final expression for del, expression for delta T now. So, delta T given by del T upon del A p times delta A p plus del T upon del omega c r times delta omega c r.

So, go and substituting when you substitute these values; you get appropriate expression, but we have interest in finding, what the relative error, so due to that all please allow me to divide this expression by the time constant, T that means; it will be divided by T and this will be divided by T, so when the expression for T is substituted over here, whatever you have found suppose for this case, the last term, how we will find the last term for this one?

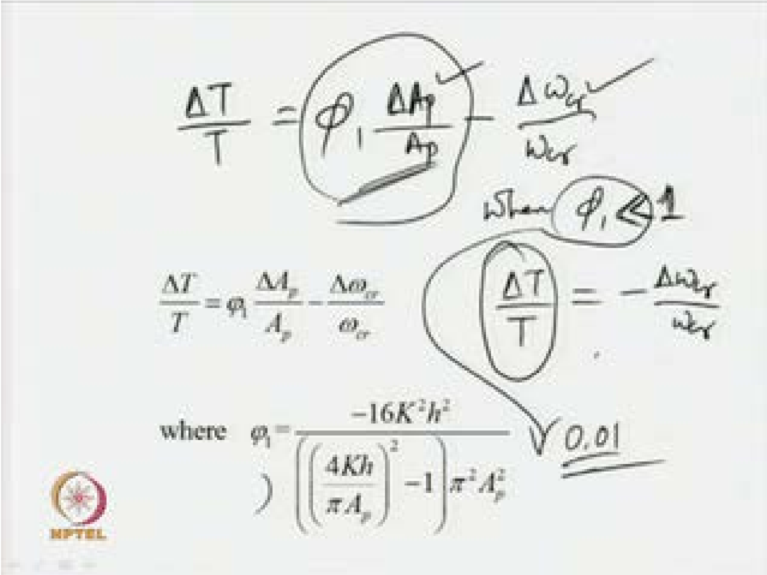
(Refer Slide Time: 42:20)



So, delta T upon delta delta T upon T we will have the last term now given by delta del sorry delta T upon T will have the last term given by this, so del T upon del T upon del omega c r times delta omega c r, so I have found del T upon del omega c r as this, so I will have 4 k h upon pi A p square minus 1 root times minus 1 upon minus 1 upon omega c r square.

Now, when you divide this by T sorry when you divide this by T we have seen that, this is getting divided by T therefore, you will have further division by the T giving us this expression, but we know that, T is given by this, so ultimately I will get a term like minus 1 upon omega c r there, so that is where I get the last term appearing as minus delta omega c r upon omega c r. So, finally what happens when you substitute the partial derivative terms in the expression?

(Refer Slide Time: 43:59)

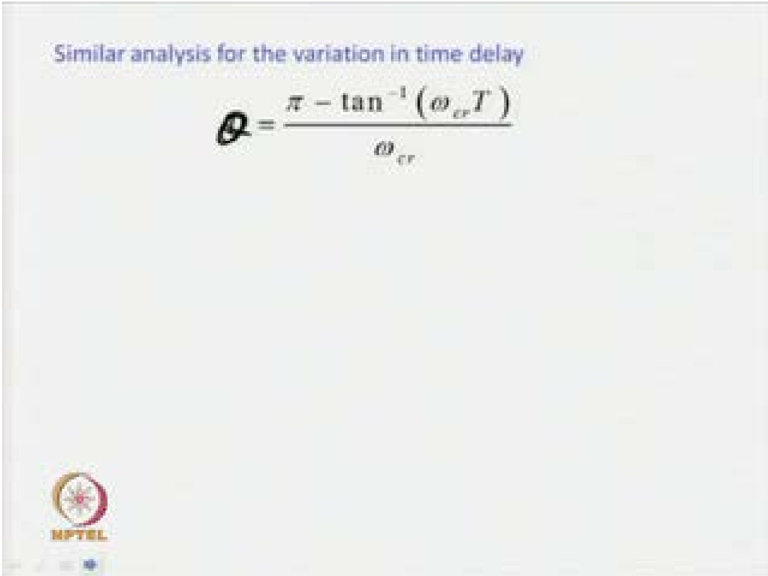


The relative error of the time constant can be found, delta T upon T as, phi 1 delta A p upon A p minus delta omega c r upon omega c r, where phi 1 is given by this constant (Refer Slide Time: 44:20). Now, what is this relative error, is going to give us what information we will get from here you see, if there will be relative error in the measured quantities omega c r and the peak amplitude, which one is going to contribute more to the relative error associated with the time constant.

If, I look at this expression certainly, the first term that means; if there is little error in the measurement of peak amplitude that is going to affect more than that of the error associated with in the measurement of omega c r, because phi 1 could be greater than 1, but when phi 1 is less than 1 when phi 1 is less than 1 when phi 1 is less than 1, then the significance of the first term is reduced. And consequently what happens, the del delta T upon T can be approximated by minus delta omega c r upon omega c r.

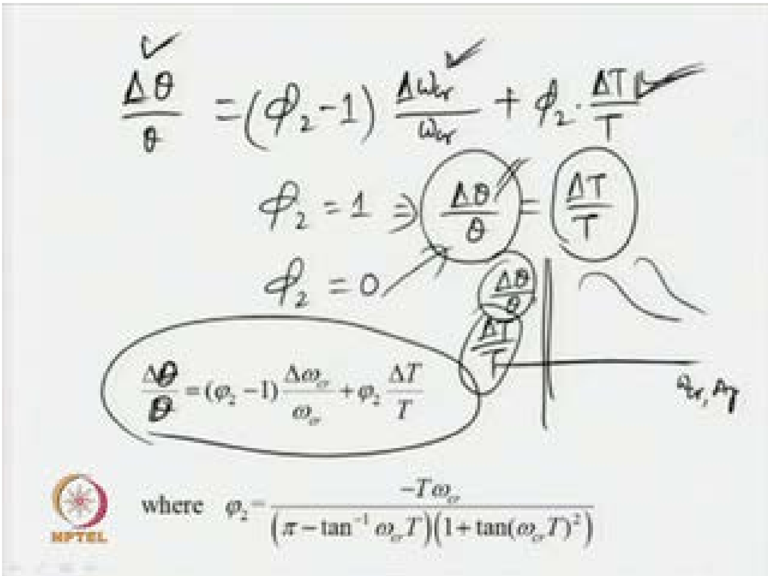
So, if this is less less than 1 actually when phi 1 can be made such that, it is a very small value of the order of 0.01 or so, then the relative error in the estimation of the time constant will (( )) depend on the relative error of that of the measured value of the critical frequency, so you need to measure accurately the critical frequency.

(Refer Slide Time: 46:08)



Similarly, the analysis can be extended for the time constant theta, which is given by theta, is equal to pi minus tan inverse of omega c r T upon omega c r.

(Refer Slide Time: 46:27)



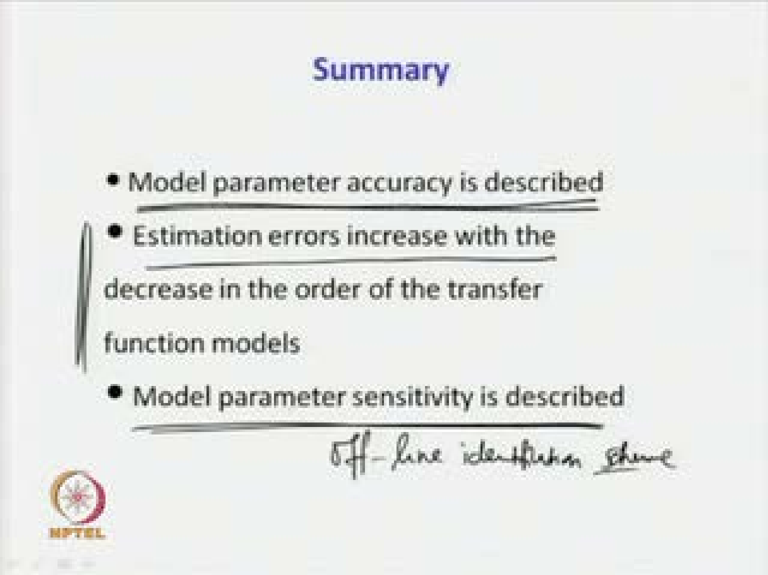
Where for this case finally, the relative error associated with the time delay is given by, phi 2 minus 1 times delta omega c r divided by omega c r plus phi 2 times delta T upon T. So, the relative error in the estimation of the time delay depends on the relative error in the measurements of critical frequency in the estimated value of the time constant. Now, where phi 2 is given by this expression, now can you have very small very large phi 2 values, when phi 2 equal to 1, what happens? When phi 2 is equal to 1, then delta theta upon theta will be equal to phi 2 is 1, so this will be equal to delta T upon T.

So, the relative error associated in the estimation of the time delay will be same as that of the time constant; when phi 2 equal to 1, when phi 2 equal to 0; what will happen? The relative error in the estimation of the time delay will depend on that of the measurement of the critical frequency accurate measurement of critical frequency, it will not depend on the accuracy in estimation of the time constant.

So, this is how one can explain the effects of either measurements or the estimated parameters on the estimation of various parameters associated with this transfer function model. So, these simple expressions are quite powerful of course, you need to find phi's; phi 1, phi 2 accurately, now how to we know that, the exact for for any simulation study you know T, you know omega c r, so all these values phi's can be estimated and it is not difficult to see or when you make a plot, the effects of different parameters like delta, theta upon theta and delta T upon T or effects of this with the respect to that of measurement of omega c r and A p can be plotted and then, from here you can (( )), which measurement or measurement is going to influence much as far as, the estimation errors are concerned with that is that that is how, the two expressions are quite powerful.

And similar expressions can be obtained similar expressions can be obtained for the analytical explicit expressions we have obtained for the identification of first order plus dead time and second order plus dead time transfer function models.

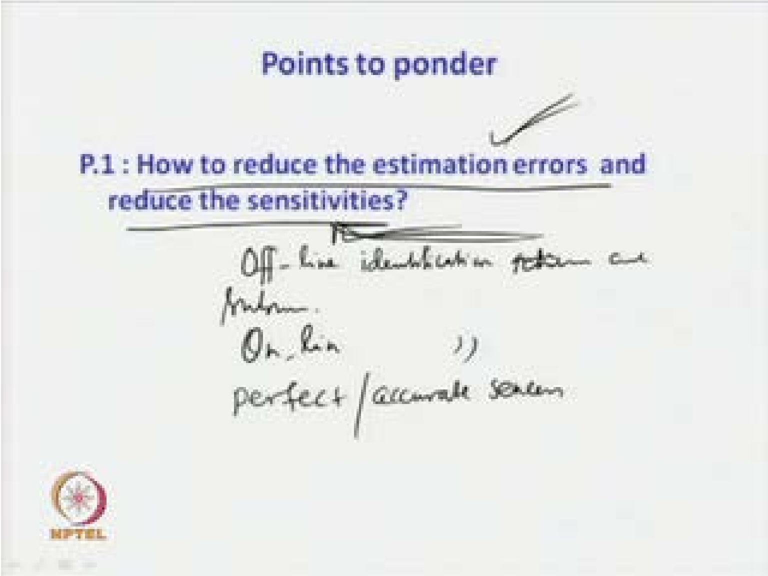
(Refer Slide Time: 49:44)



So, the model parameter accuracy is described, where we have seen the absolute error contributed by errors in the measurements particularly, also estimation errors increase with the decrease in the order of the transfer function models, this has been explain in detail, when the order of the transfer function models increases then, the estimation error decreases. But, when you go for higher order of transfer function model, what happens? More parameters could be there or the complexity involved will be more, when you find the analytical expressions, it may not be so easy, as for that for the transfer function model with less number of parameters or with lower order.

Now, model parameters sensitivity is also described for a simple case, where we have considered the off-line identification scheme only offline identification scheme and this can be extended to the online identification scheme as well, to find analytical expressions for the sensitivity of parameters associated with various transfer function models.

(Refer Slide Time: 51:18)



Now, one point to ponder: How to reduce the estimation errors and reduce the sensitivities? There are many ways as I have told you off-line identification identification are are identification schemes are, subjected to high value of estimation errors and sensitivities, whereas on-line identification techniques can be used to reduce the estimation errors and reduce the sensitivities. Further, if you use perfect sensors or accurate sensors, you can also reduce the estimation errors and the sensitivities associated with estimation of the parameters of a transfer function model that is all, thank you.