Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 16

Review of Time Domain based Identification

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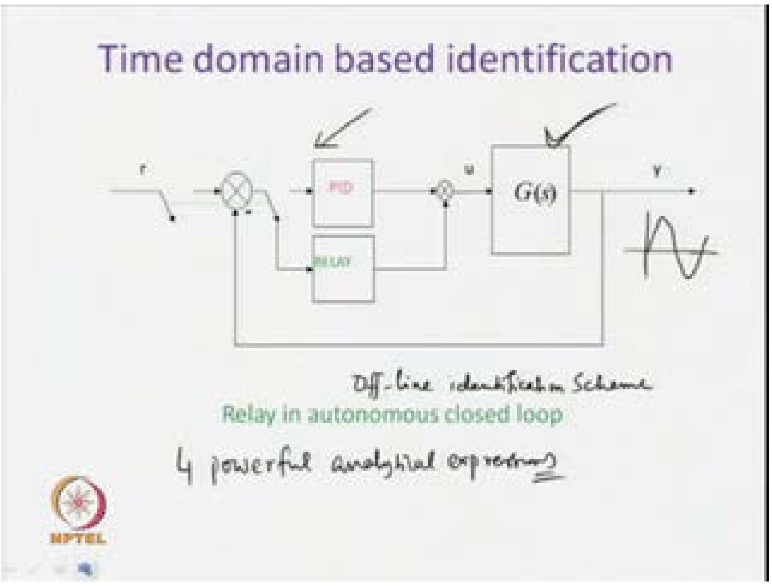

Welcome to the lecture titled review of time domain based identification. In this lecture, we shall discuss about some issues related to time domain based identification. Also using some statistical measures, measurement error effects on the model parameter estimations will also be described. In the offline identification scheme, we have already discussed earlier, offline identification scheme. Why it is called the offline identification scheme? When the relay is switched on that time the controller remains out of the loop. And when the controller is in action, relay remains out of the loop that is why, this scheme is known as the offline identification scheme.

Then how identification is accomplished, the limit cycle waveform is obtained, and analysis of the limit cycle waveform results in 4 powerful analytical expressions - analytical expressions. And now, solving the set of analytical expressions, it is possible to identify the dynamics of a real time process, in the form of transfer function models. Now, the transfer function models can assume various forms.

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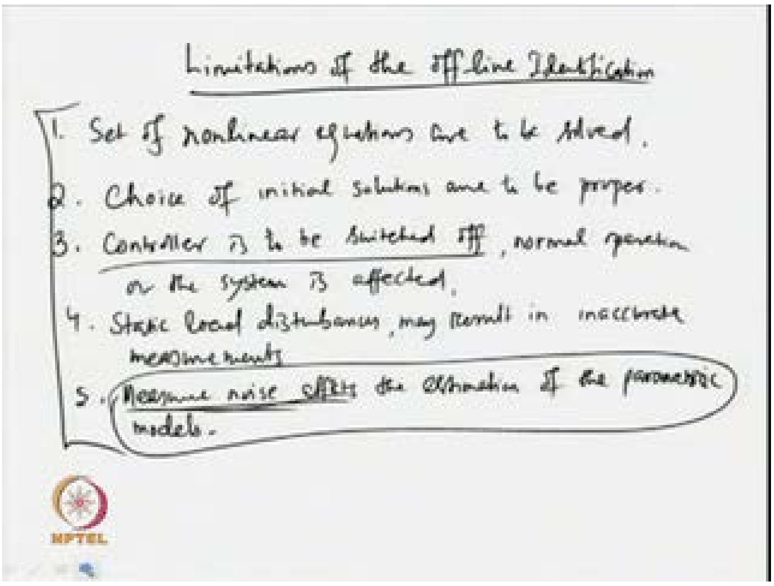

Now that will not discuss again rather, we shall see the limitations of this identification scheme. So, the measure limitations of the offline identification scheme are: First, you need to solve a set of non-linear equations using some non-linear equation solver. So, a set of non-linear equations equations are to be solved. And solution of the non-linear equation requires you to choose proper initial values or initial settings, while trying to solve the set of non-linear equations. So, choice of initial initial solutions, choice of initial solutions are paramount importance; means choice of initial solutions are to be proper.

Next controller is to be switched off, controller is to be switched off while conducting the identification test or relay experiment. That means, the process operation will get disturbed. So, when the controller is switched switched off, normal operation of normal operation of the system closed loop control system is affected. Now in the phase of presence of static load disturbances, we may get inaccurate pix or the measurement of the limit cycle output may yield in accurate measurements. So, static load disturbance may result in inaccurate measurements, and lastly the measurement noise measurement noise affects the estimation of the parametric models parametric models.

So, measurement noise results in inaccurate measurements, and which in terms may yield higher estimation errors or the parameters may be estimated in accurately. So, these are the five important limitations, we have with the offline identification scheme. In spite of that, for each in analysis analysis, and for each in identification - offline identifications are often chose or selected.

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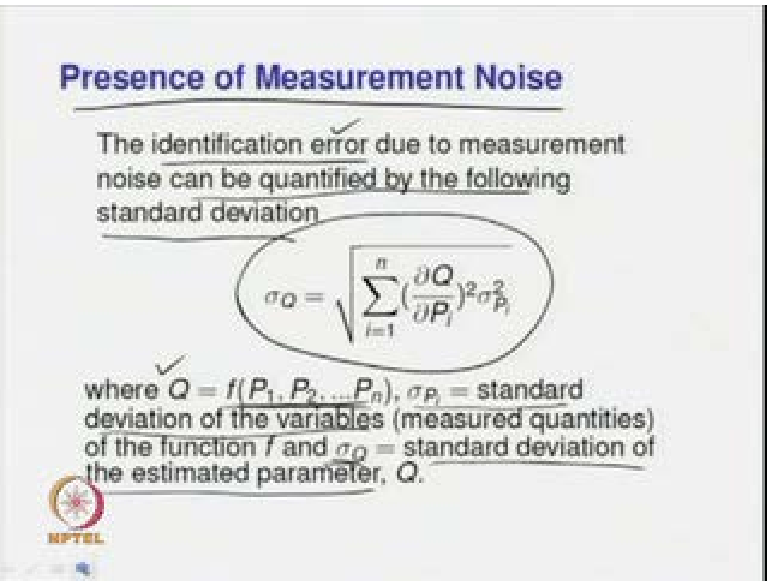

Now, since measurement noise plus an important role; it will be nice to see the estimation errors result in due to measurement noise. So, the presence of measurement noise relates to error in measurements, which indirectly gives identification error. And the identification errors, can be quantified by using some statistical measure, such as the standard deviation. So, the standard deviation for the estimation error can be given by this expression. What is that? Sigma Q is equal to square root of sum of i equal to one to n del Q upon del P i square sigma P i square. What are Q, P, and sigma - sigma is the standard deviation.

Now Q is a function of the parameters of the transfer function model, sigma P i is the standard deviation of the variables, those have there in the explicit or inexplicit expressions, those are to be used for identification of transfer function models. Now, sigma Q is the standard deviation of the estimated parameter that is Q.

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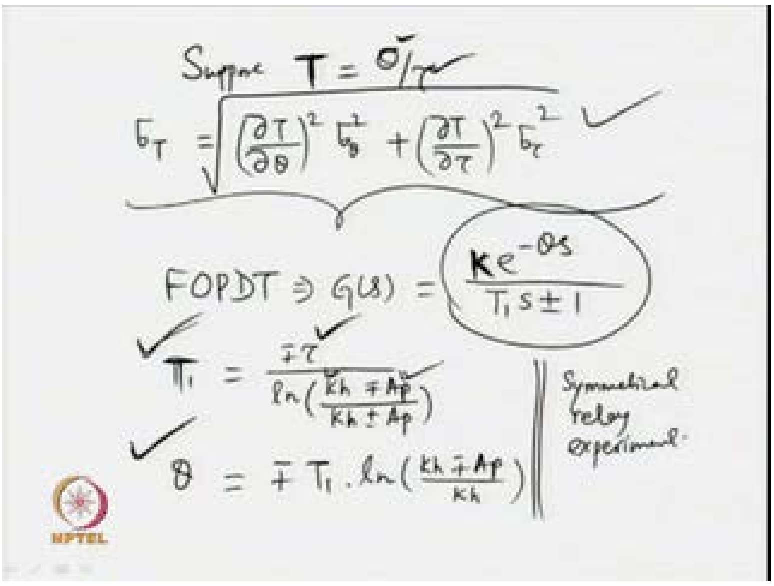

Let you give one simple example, suppose some T is a function of theta by tau, let us say. Then, how can I find the standard deviations; now sigma T will be equal to del T upon del theta square sigma theta square plus del T upon del tau square sigma tau square root. This is how we can find the effects of the parameters theta, and T on the estimation value of capital T. So, this is how the standard deviations are found, suppose theta is deviated by some value, tau is deviated by some value, then how much accuracy we have in the estimation of T can be obtained from sigma T or the standard deviation of T.

So, using this, now we shall try to find the estimation errors associated with a first order plus dead time model parameters. What is one first order plus dead time transfer function model parameter, FO OP FOPDT model can be given in the form of G(s) is equal to K e to the power minus theta s upon T 1 s plus minus 1. This is what we have already discussed earlier now, to estimate the parameters associated with this transfer function model. We have got some explicit expressions for T 1, and theta; and K can be found by some other technique or can be can be found from the measurement of the area of the output signal divided by the area of the input signal.

Now, we have got expression for T 1 as T 1 is given by plus sorry T 1 is given by minus plus tau divided by lon of K h minus plus A p divided by K h plus minus A p. So, this expression has already been derived in our earlier lecture, similarly the expression for theta is given as minus plus T 1 times lon of K h minus plus A p divided by K h. When you get, these type of expressions when we use symmetrical relay test - symmetrical relay experiment is conducted. Then the analysis of the limit cycle waveform results in explicit expressions of these forms T 1, and theta.

Now as I am telling due to the measurement noise, it is possible to get inaccurate values for tau K and A p; these are the three measurements, we are making. Now, assuming that K is also affected by the measurements of the ratio of output signal to the ratio of the input signal. So, let me find the standard deviation of T 1 with respect to that of the tau or variations in tau K and A p.

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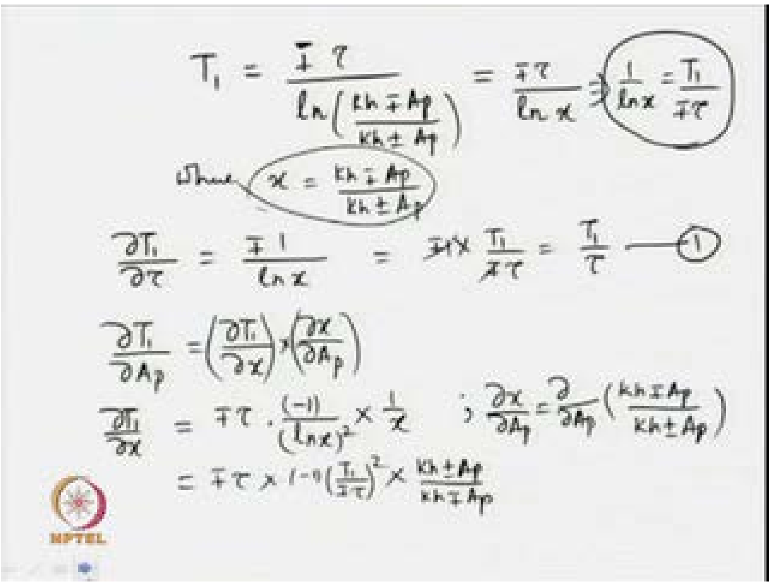

So, let me start with the basic expression T 1 is given as minus plus tau divided by lon of K h minus plus A p divided by K h plus minus A p. Let us write this in the form of minus plus tau divided by lon x, where x is equal to K h minus plus A p divided by K h plus minus A p; why I have written in this form. So, that I will be able to find the partial derivatives, conveniently. Now, how much will be the partial derivative of T 1 with respect to the tau, this will be equal to minus plus 1 upon lon x - and lon x is how much, that is lon x from here can be obtained as or 1 upon lon x 1 upon lon x will be equal to T 1 divided by minus plus tau.

So, substitution of that will give us is equal to minus plus 1 minus plus times, your 1 upon this is equal to T 1 by minus plus tau. So, minus plus will cancel out giving us T 1 upon tau. So, this is what I get one partial derivative of the variable T 1 with respect to the measurement tau. Similarly, we can find del T 1 with respect to del A p; now here, you need to find this in the form of del T 1 upon del x, because T 1 is a function of x now. T 1 is equal to minus plus tau upon lon x. So, I have to use this identity del T 1 upon del A p is equal to del T 1 upon del x times del x upon del A p. So, del x del x can cancel out giving us del del T 1 upon del A p.

So, I need to find two partial derivatives to find the partial derivative of T 1 with respect to A p. So, how much is del T 1 upon del x. So, del T 1 upon del x will be equal to minus plus tau times, the differentiation of 1 upon lon x formula is to be used which gives us minus one divided by lon x square times 1 upon x. And next will find del x upon del A p will be equal to del x upon del A p times sorry, this will be del upon del A p of x. So, what is x? x is K h minus plus A p divided by K h plus minus A p. So, this is not difficult, once you know the way partial differentiations are down, then it is very easy to find the partial differentiation.

Now, we know that one upon lon x, one upon lon x from here, one upon lon x is nothing but T 1 upon minus plus tau. So, I will make use of this. So, when you substitute this, then you get minus plus tau times minus 1, then you will have for this T 1 divided by minus plus tau square into 1 upon x. So, 1 upon x means, you will get K h plus minus A p in the numerator, and K h minus plus A p in the denominator, because x is this much. So, one upon x will be the inversion of x. So, we will get this term.

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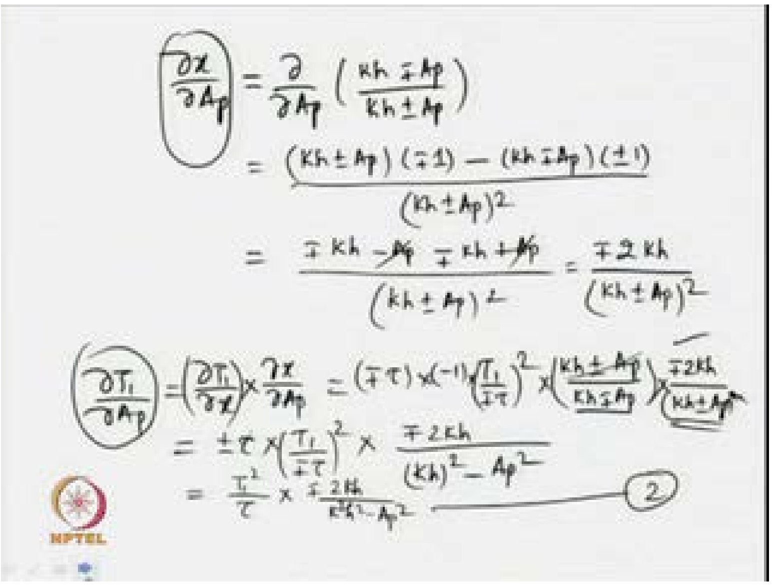

Now, let me find del x upon del A p which is given by del x upon del A p will be equal to del of del A p times. What is x now; x is K h minus plus A p. So, K h minus plus A p divided by K h plus minus A p. So, it will give now K h plus minus A p times differentiation of this. So, that will give your minus plus 1 minus you will keep K h minus plus A p here, times differentiation of this will give now plus minus 1 divided by K h plus minus A p square. So, this is how, you find the partial derivative of x with respect to A p. So, often simplification now, the numerator will give you minus plus K h, then, it will be minus A p and here. So, minus of minus minus times plus minus 1 will be minus plus K h, then minus minus plus. So, it will give plus A p. So, this is how we get the numerator, where A p A p will cancel out, and ultimately giving us an expression of the form minus plus 2 times K h divided by K h plus minus A p square. Now, finally how much will be the partial differentiation of T 1 with respect to A p. So, please substitute those expressions now del T 1 upon del x time del x upon del A p. So, the first part we have obtained as minus plus tau minus plus tau, and then let me see minus plus tau then minus 1 times T 1 by plus minus tau. So, minus 1 times T 1 by minus plus tau square into K h plus minus A p. So, into K h plus minus A p by K h minus plus A p.

So, all these terms are for the partial differentiation of T 1 with respect to x. Now, when it is multiplied with the partial differentiation of x with respect to A p, I get minus plus 2 K h divided by K h plus minus A p square at the end, A p square at the end. Now, K h plus minus A p in the numerator, and here the power will go out. So, leaving us an expression of the form minus will come. So, plus minus it does not matter, so plus minus tau times T 1 upon minus plus tau square into minus plus 2 K h due to this term divided by… If you multiplied now, this and this where the power has gone, K h minus plus A p times K h plus minus A p will result in K h square minus A p square. Please see the sign minus plus will give minus, plus minus multiplication will give minus. So, ultimately it will be like a plus b times a minus b giving us a square minus b square.

So, finally, what I get - the final expression for this one will be equal to… This is the final expression, no more simplification is required or further simplification is possible, because tau square will come in the numerator denominator cancelling this tau. So, I can get outright, your T 1 square. So, T 1 square divided by tau times minus plus 2 K h by K square h square minus A p square. So, this is what? We have got for the partial differentiation of T 1 with respect to A p.

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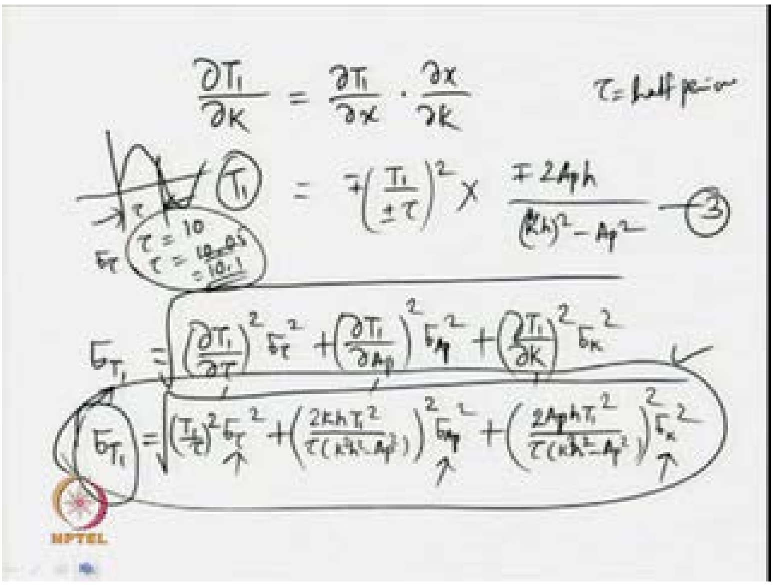

So, similarly the partial differentiation of T 1 with respect to theta sorry, we do not have theta here in the expression T 1. So, the T 1 expression has got three variables of three measurements tau K, and A p. Therefore, I need to find the partial differentiation of T 1 with respect to K, because K is now assume to be measured. So, del T 1 upon del K will similarly result can be written as del T 1 upon del x time del x upon del K, giving us the terms minus plus T 1 upon plus minus tau square times minus plus 2A p h now, divided by K square h square or you can outright write in the form of K h square minus A p square sorry K h square minus A p square. So, this this is our third expression.

Now the finally, this standard deviation of T 1 is given by the expression del T 1 upon del tau square sigma tau square plus del T 1 upon del A p square sigma A p square plus del T 1 upon del K square sigma K square root. So, this is how the standard deviation of the variable or the estimated parameter T 1 is given in terms of the measurement tau A p, and K. Now, you simply need to make use of the three expressions, we have found for partial differentiation of T 1 with respect to tau A p, and K over here to finally, get it in the form of T 1 upon tau square sigma tau square plus for this one will have 2 K h T 1 square divided by tau times K square h square minus A p square; square times sigma A p square plus for this one already when you take the square, how do you get this one.

So, you have already got terms like 2 A p h T 1 square divided by tau K square h square minus A p square square sigma K square root. So, this is the final expression for the standard deviation of T 1, in terms of the standard deviation of tau A p, and k. So, assuming that the measurements what deviation means what basically, when I am measuring the half period, tau means what? This is the half period, when we have got a limit cycle of the form this one, tau means this is the half period. So, when you make measurement due to the presence of measurement noise, there will be measurement error. We may not be able to accurately measure tau. So, suppose instead of measuring the correct value tau is equal to 10 by measure tau by 10.05 or 10.1; then we have got the measurement error.

This will lead to the standard deviation in tau, and that results in the standard deviation deviating from the actual value or the accurate value. So, how much inaccuracy there is in the estimation of the parametric model T 1, can be quantified using this expression. So, this is how the estimation errors are quantified using the statistical measure standard deviation.

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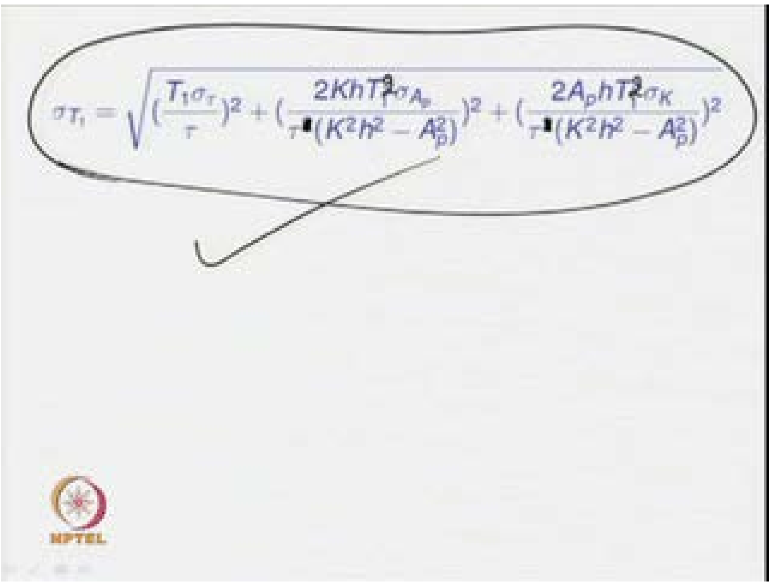

Similarly, let us go to the other parameters. So, this is the final expression that we have got, where this will be T 1 square, and this will be tau. I have make some mistake here. So, now it is correct, then this is what already we have derived.

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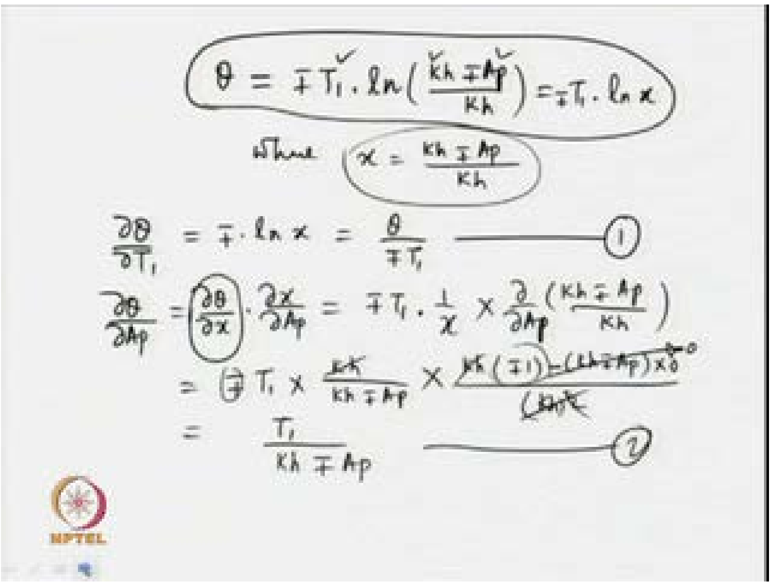

Similarly, allow me to derive the expressions for there of the other parameter associated with the first order plus dead time model. So, theta is given by minus plus T 1 times lon of K h minus plus A p divided by K h. Then, how can I get the partial differentiation now, for that take this as minus plus T 1 minus plus T 1 times lon x. Again, where x is given by now K h minus plus A p divided by K h. Then, to find the what are the parameters associated in this explicit expression for the time delay associated with the first order plus dead time model, we have got T 1, we have got K, and we have got A p. These are the three variables, there are going to affect the estimation error.

So, I need to find del theta upon del T 1. So, del theta upon del T 1 will be how much? This will be equal to minus plus lon x. So, this will be minus plus lon x, because you are differentiating with respect to T 1. So, lon x is how much theta by minus plus T 1. So, that way I can write this by theta by minus plus T 1. So, we got one partial differentiation of theta with respect to T 1, now another partial differentiation of theta with respect to the variable del A p can be obtained in the form of again del theta upon del x times del x upon del A p. So, please use this to find the partial differentiation.

Now del theta upon del x will be minus plus T 1 time; differentiation of lon x is x sorry, differentiation of lon x is 1 upon x only. So, giving us 1 upon x, this is for the first term. What about the second one - del x upon del A p will result in del x upon del A p will be del upon del A p times K h minus plus A p divided by K h. So, it will be minus plus T 1 times inverse of x will give you K h divided by K h minus plus A p. Now, the differentiation of this one will give you K h times differentiation of this with respect to A p will result result in minus plus 1 minus K h minus plus A p times differentiation of K h will be 0. So, the second term is ultimately zero divided by K h square.

So, when you simplify this K h square power will go out minus plus minus plus multiplied that will give you plus. So, I will be left with, and expression of the form T 1 K h in the numerator, and divided by T sorry, this this K h again will cancel out. We have got K h here. So, K h will go out. So, in the numerator I will be left with T 1 only. So finally, del theta upon del A p will be T 1 upon K h minus plus A p. Now, let us try to find the partial differentiation of theta with respect to K similarly.

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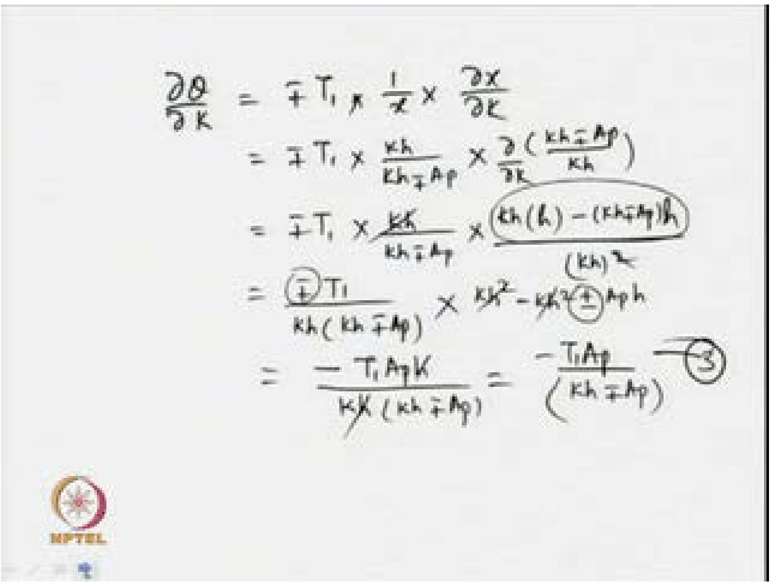

So, in this case del theta upon del K will be equal to again minus plus T 1 time 1 upon x, like the earlier case it will be 1 upon x into del x by del K is equal to minus plus T 1 into K h by K h minus plus A p minus plus A p into del of K h minus plus A p divided by K h upon del K. So, now, the differentiation is to be found with respect to K. So, simplification will give now, K h upon K h minus plus A p - this differentiation how to find, we can find this like the earlier case K h time differentiation with respect to K. So, it will leave as h minus then minus K h minus plus A p times, differentiation of K h with respect to K, it will leave as h divided by K h square.

So, I will cancel this giving us minus plus T 1 divided by K h times K h minus plus A p into the numerator of the last expression. So, how much it will be K h square minus K h square minus is there, therefore we will be left with plus minus A p h. So, you have got A p h finally, and due to this minus plus multiplication of minus plus with plus minus you will get minus here, leaving us in the numerator minus T 1 A p h divided by K h K h minus plus A p. So, further simplification can be obtained leaving us minus T 1 A p divided by K h minus plus A p. So this is how, the third partial differentiation is obtained for a finding the standard deviation of theta with respect to the measure quantities.

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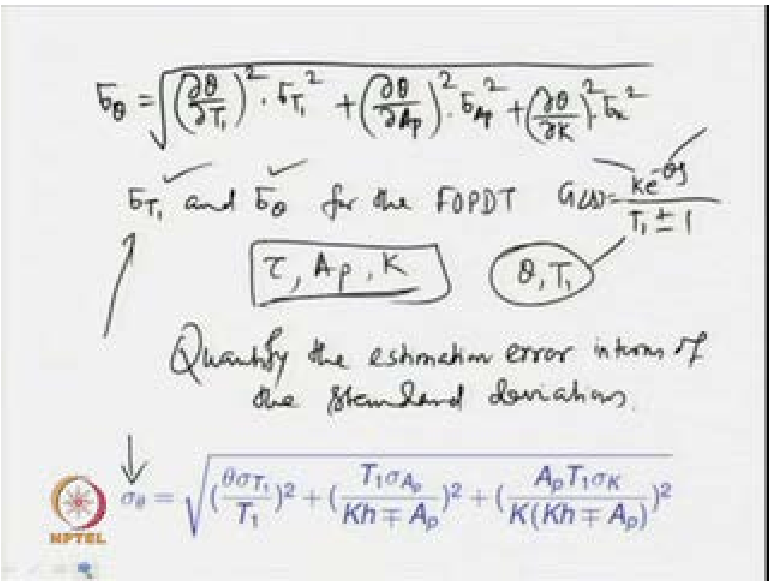

Now, I will write the final expression. So, sigma theta will be del theta upon del tau square plus sorry, no tau is there, T 1 is there; del T 1 square time sigma T 1 square plus del theta upon del A p square times sigma A p square plus del theta upon del K square sigma K square root. So, when you substitute the partial differentiations found earlier, we get the final expression as sigma theta is equal to square root of theta sigma T 1 upon T 1 square plus T 1 sigma A p divided by K h minus plus A p square plus A p T 1 sigma K upon K times K h minus plus A p square. Thus the standard deviations are found.

So, what benefit, we get from finding the standard deviations, basically we have been able to find sigma T 1, and sigma theta for the first order plus dead time model given by G(s) is equal to K e to the power minus theta s upon T 1 plus minus 1. So, the when there are measurement errors errors in the measurement of half period tau, the peak amplitude

A p, and the steady state gain K. Then the estimated parameters will be subjected to standard deviations of sigma T 1, and sigma theta. This is how, it is possible to quantify the, quantify the measurement errors or quantify the estimation error, I would like to say estimation error in terms of the standard deviations.

So this is how, one can quantify the estimation errors associated with the measurement errors particularly or with respect to the measurement errors - the errors in estimation of K theta, and T 1 and particularly theta, and T 1 can be evaluated.

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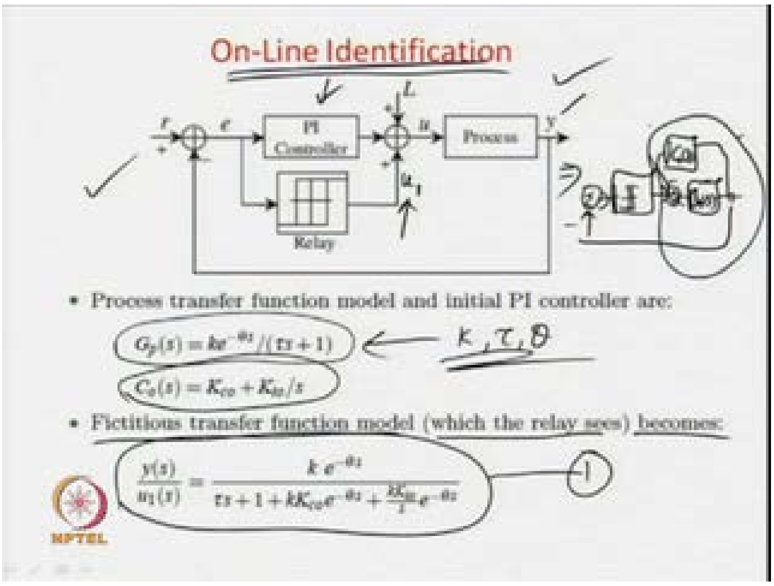

Next, will go to the on-line identification technique, because the lecture is on review of the time domain based identification techniques. So, in the on-line identification scheme, we have used frequency domain analysis earlier, but not necessarily one has to go for frequency domain analysis, although for ease in analysis of the relay control system. The describing describing function based frequency domain analysis has been used earlier. In today’s lecture, we will see how time domain based analysis can be extended for on-line identification of processes. Now, the on-line identification scheme with the help of a relay with hysteresis relay with hysteresis will be described very briefly. So, this is an on-line identification scheme, because the controller remains in operation throughout the operation of the process within during identification of the process. Therefore, this scheme is known as the on-line identification scheme, and we have have lot of advantages compare to the offline identification scheme, we have described earlier.

Now, let the process dynamics be described by a transfer function of the form G p(s) is equal to K e to the power minus theta s upon tau s plus 1. So, please do not confuse, now the tau here represents the time constant of the process, time constant of the process, not the measurement parameter. Similarly, the PI controller that is employed in this on-line scheme has the form c 0 s is equal to K c o plus K i o s. Why the subscript o is there, that is for initial value when you are conducting some relay test or experiment; and after relay test and experiment, then the c or the controller will be c(s), I believe you have follow. So, at the time of relay test, the controller dynamics is given by K c o, and K i o parameters with the help of K c o and K i o parameters.

Now, if you draw the equivalent circuit circuit diagram of this block diagram, then what relay sees basically. So, the relay will be subjected to… This is a relay with hysteresis, then you will have the process G(s) negative feedback, and you have got the controller c 0 s present here. So, the relay sees basically a a dynamics of this form. So, fictitious transfer function model, which the relay sees becomes y(s) upon u 1(s), where is u 1(s) - u 1(s) is here. Then the transfer function between y and u 1 can be expressed in the form of K e to the power minus theta s upon tau s plus 1 plus k K c o e to the power minus theta s plus k K i o upon s e to the power minus theta s.

So, what benefit you are getting from this fictitious transfer function model, this helps in the identification of the system in time domain. In place of using the frequency domain, I can use the state space analysis for identification of on-line identification of the system. How the state space model of this fictitious transfer function model can be found.

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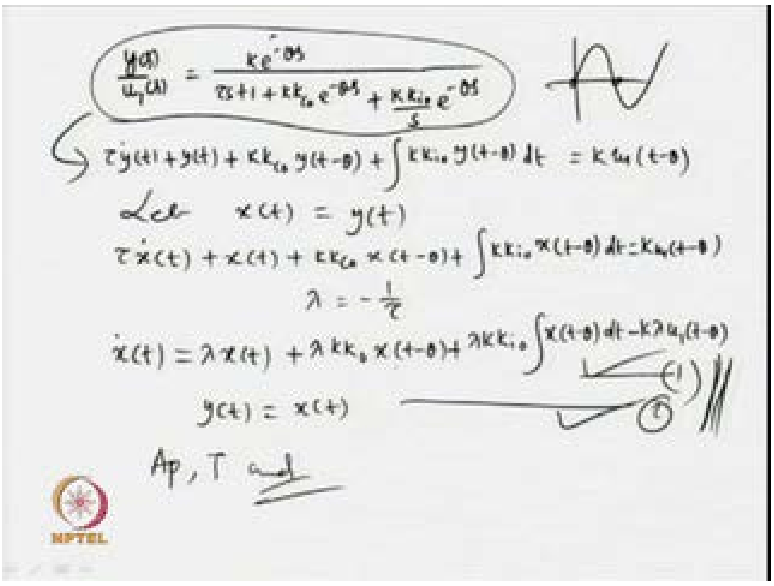

So, the state space model or state space equations can be obtained as given y(s) upon u 1 (s) h K e to the power minus theta s upon, K e to the power minus theta s upon tau s plus 1 plus k K c o e to the power minus theta s plus k K i o divided by s e to the power minus theta s. Now plus multiply, and write the expression in the form of the differential equation giving us tau y dot t plus y t plus k K c o y(t) minus theta plus integration of k K i o y(t) minus theta d t is equal to K times u 1 t minus theta. So, this is how, when you cross multiplied these terms, and take the inverse laplace transform.

We get the differential equation of this form. So, let x(t) the state variable be y(t) resulting in an expression of the form tau x dot(t) plus x(t) plus k K c o x(t) minus theta plus integral of k K i o x(t) minus theta dt is equal to K u 1(t) minus theta. Implies x dot t. So, let me take introduce one variable, suppose lambda is equal to minus 1 upon tau; in that case x dot(t) can outright be written in the form of lambda x(t), then these terms will go to the right half. So, that way you will have plus lambda k K c o x(t) minus theta plus lambda k K i o integral of x(t) minus theta dt plus, no there will be minus, because this lambda is minus 1 upon tau, it is getting divided. So, minus K lambda u 1(t) minus theta. So, this is the state equation, we are getting from that fictitious transfer function model.

And the output equation is given by y(t) equal to x(t). Now, using the state and output equations, and the limit cycle waveform, then now it is possible to find analytical expressions for the zero crossings, and for the peak amplitude, and how to find the expressions for zero crossings, and peak amplitude have been described in our earlier lectures. Only, you need to make use of this state equation, and this output equation in place of the earlier state, and output equations. This is how from the measurement of the peak amplitude, and half period, and so on, it will be possible to estimate the parameters of this transfer function model; and the parameters of this transfer function models are K tau and theta. So, this is how I can make use of time domain analysis for on-line identification of systems.

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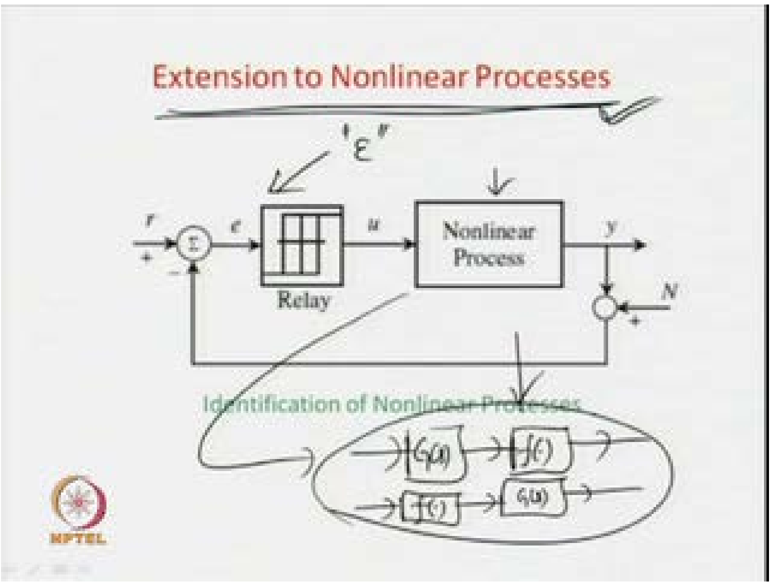

Next, we shall go to the extension of the time domain analysis to non-linear processes. So, how can we make use of the time domain analysis, state space based time domain analysis for a estimation of dynamics of non-linear processes. So far, we have discussed about or we have studied about the identification of linear processes. Now, the time domain analysis can also be used to identify non-linear dynamics. Now, the relay is having some hysteresis, you look at the relay. So, let the hysteresis of the relay be given by epsilon, then it is possible to conduct relay test like this, and the non-linear process dynamics can be represented by a linear dynamics given by G(s) along with some non-linear function. The non-linear function can have static nonlinearity, can have dynamic nonlinearity or now the non-linear process dynamics can be given in two form - the nonlinearity can appear first, and then the process dynamics or dynamic model can be given later. So, these are the two ways, one can get the block diagram representation for the non-linear process or the models for the non-linear processes.

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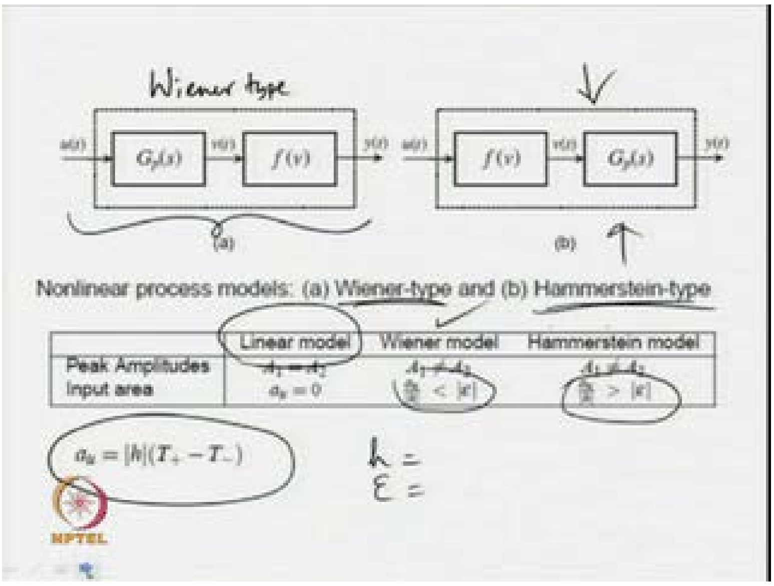

Now, how can you estimate, how can you find a certain that you will go for identification of linear system or non-linear system. Now, as I have already described when the non-linear system dynamics is given by this model, then we get some Wiener type model - this is called Wiener type non-linear process model, for non-linear dynamics associated with a system. And the second type of representation of non-linear dynamics in the form of block diagram is known as the Hammerstein type. Now, relay test is conducted with the relay having hysteresis, and one parameter a u is measured; what is a u?

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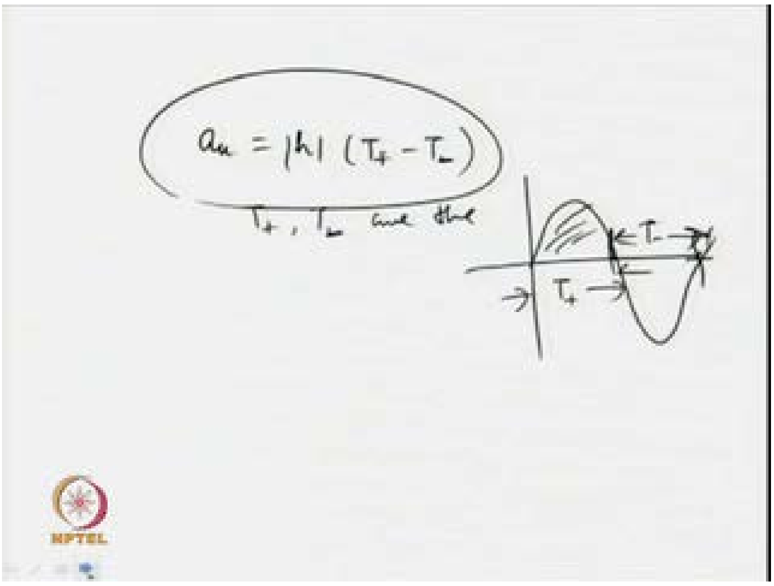

Now a u is equal to a u is equal to h times T plus minus T minus. So, what are T plus and T minus - T plus and T minus are the width of the limit cycle output signal for positive output you get this as T plus, and for the negative output the the time from here to here. The next zero crossing is shown by T minus. So, using this information when a u is equal to 0, then you can go for identification of a linear process model. When a u upon h what is h? h is the relay setting, and epsilon is the hysteresis hysteresis width with associated with the relay. So, when this is satisfied go for a identification of an wiener type model, and when this is satisfied go for identification of an of a Hammerstein type model. So, this is how non-linear process, are identified.

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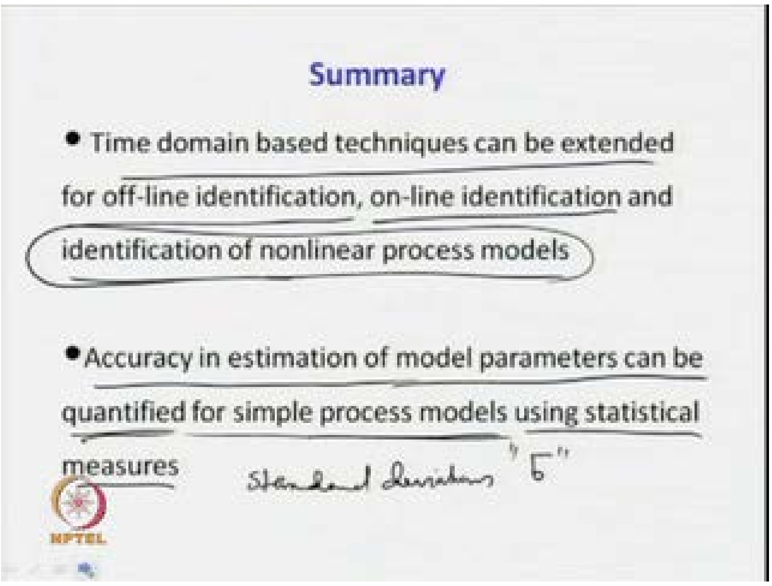

Now, let me summarize the lecture now, time domain based techniques can be extended for offline identification can be used for on-line identification, and also can be used for identification of non-linear process models. This is what to we have not studied in any lectures so far, earlier now further accuracy in estimation of model parameters can be quantified for simple process models. Like first order plus dead time transfer function model using some statistical measures, such as standard deviation. So, standard deviations or the sigma's.

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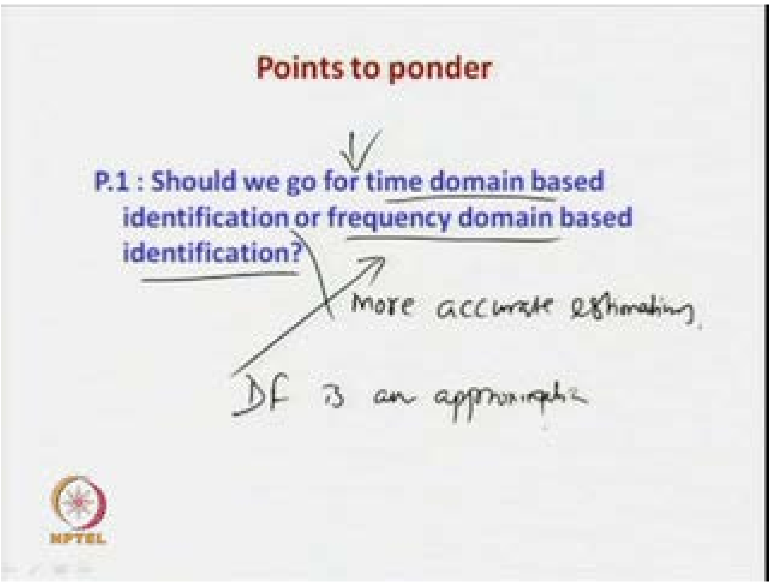

And lastly any point to ponder: should we go for time domain based identification or frequency domain based identification. So, we have studied two type of identifications during our last lectures, where we have seen that one can make use of time time domain based identifications, and frequency domain based identification. Out of the two although frequency domain based identification is quite easy or easier compared to the time domain based one, time domain based identification results in more accurate estimations.

More accurate estimations of the transfer function model parameters. Whereas, frequency domain based identification identification techniques are subjected to identification errors or estimation errors. And frequency domain based identifications using describing function are subjected to erroneous results, because the describing function itself is an approximation. That is all, in this lecture. Thanks.