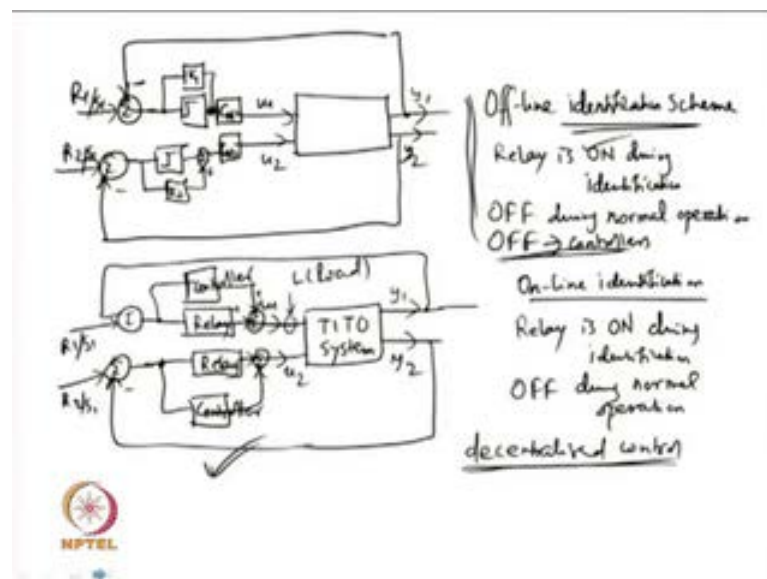


Advanced Control Systems
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Module No. # 03
Time Domain Based Identification
Lecture No. # 15
On-Line Identification of TITO Systems

Welcome to the lecture titled on-line identification of TITO systems. In our last lecture, we have studied the off-line identification of TITO systems. In this lecture, we shall see how simple on-line identification scheme can be applied for TITO or two input, two output systems.

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What we have described in our last lecture is that given a TITO system, which has got two inputs U_1 and U_2 , and two outputs y_1 and y_2 . We can add a modified relay of the form - a relay with one controller C_{N1} , and again in parallel K_1 . And similarly, a controller C_{N2} - a relay, and again in parallel with the relay K_2 can be added; these modified relays can be put in closed loops with negative feedback. With of course, the set values R_1 , and R_2 . So, this how we get an off-line **off-line** identification scheme,

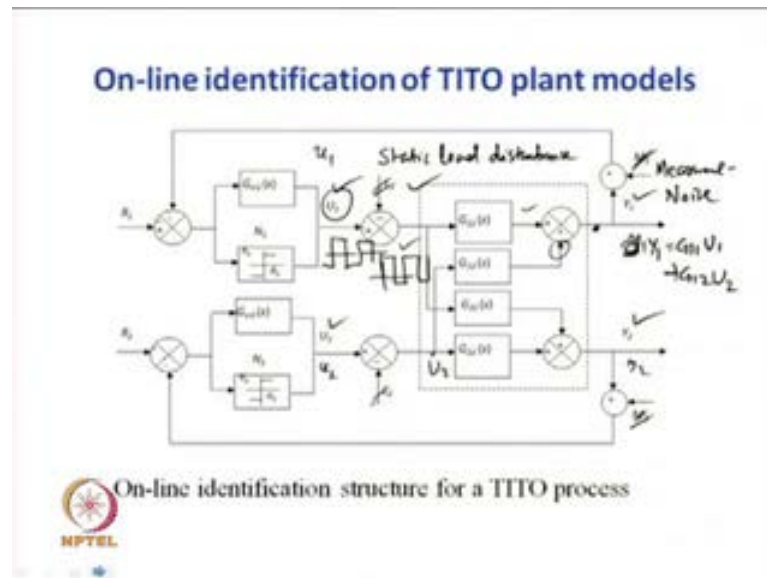
where the relay is switched on or modified relay is on during identification, whereas it is switched off. The modified relay is off, during normal operation of the decentralized control system, normal operation. So, how it is different from an on-line identification scheme that, we shall see. Given a TITO system again this is your TITO system which has got - two outputs y_1 and y_2 , and corresponding inputs U_1 and U_2 .

I can add relay **relay** of course, in parallel with a controller. So, we will have controllers in parallel with the relay. And then, put the relay in closed loop in this form. So, similarly for the second loop, we will have the negative feedback in this form. And of course now, we can have the set values R_1 and R_2 or s_1 and s_2 , as we have used in our last lecture S_1 and S_2 - these are the reference inputs or set point inputs. So, how this is accomplished, how this is **on** on-line identification now is done on-line identification. In the on-line identification, relay will be on during identification, and off during normal operation.

Now, when relay is off during normal operation, what happens? The TITO system will be subjected to decentralized control **decentralized control**. Whereas, in the earlier case, when the relay is on, the controllers are to be switched off. So, basically not only the relay, the controllers are also to be switched off **off**. So, controllers here unlike the on-line identification case, in off-line identification scheme; the controllers remain off, during the identification which is not so in on-line identification scheme. I mean to say, the controllers are always there in the loop. That means, when decentralized control is effected that time relay is switched off, otherwise relay is switched on, for identification of parameters of parametric models of the TITO system.

So, these are the basic difference between the two, and not only we are getting **getting** a simple scheme, simple on-line scheme. The other advantage of this one that, if the TITO system is subjected to some load inputs; some static load disturbance has like load inputs, then the output is not getting affected by the presence of controllers, in the loop during identification. This is the major advantage, we get with the on-line identification scheme. And unlike off-line identification scheme, the controller need not be switched off during identification. So, this is the fundamental difference, we have between the off-line identification, and on-line identification schemes.

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Now, I shall go to the on-line identification scheme which can be given in this form. Now, in this structure as you see the L_1 and L_2 represents the static load disturbances, and M_1 and M_2 also represents the measurement noise, **measurement noise**. So, we have got a controller connected in parallel with the relays throughout the operation of the system, and during the relay experiment as well. So, that is the basic difference, we have between the off-line identification scheme, and the on-line identification scheme.

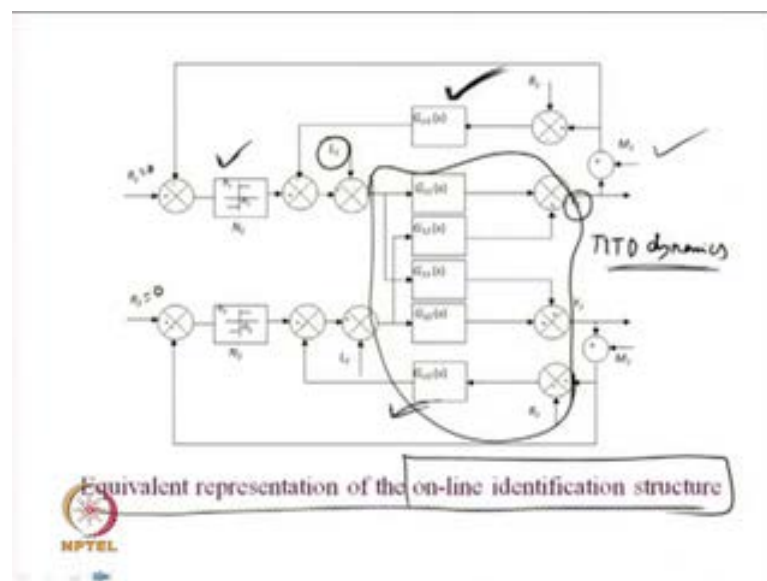
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1. Relay is connected in parallel with a controller
 2. Controller is assumed to have integral action
 3. The effects of static load disturbances are cancelled.
 4. We obtain symmetrical limit cycle output for asymmetrical relays.
 5. Equivalent block diagram representation for the on-line scheme.
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So, what are the major benefits one get from this arrangement, this on-line identification scheme. First is how you have to do, the relay is permanently connected in parallel with **with** a controller; and the controller is assumed to have, **controller is assumed to have** integral action, **integral action**. So, what type of controller we have to used either PI or PID or I controller. Now, what is the benefit you get by that, then the effects of **the effects of** static load disturbances - **static load disturbances** are cancelled. And we guarantee, we obtain symmetrical limit cycle output **output** for symmetrical relays. If the controller is disconnected as you can see from here, suppose I disconnect the controller during the relay experiment, what will happen? The static load disturbance will influence the input to the TITO process.

That means, whatever signal you are getting whatever rectangular pulses you are getting, that will get added by the static load disturbance, and ultimately you will **you will** get a symmetrical input to the system. Now, a symmetrical or shifted version of the input signal to the system. So, you will get a an input of this form, and corresponding to this asymmetrical input, the output will become asymmetrical. So that, **that** will not happen provided, we have a controller during the relay experiment. So, that is the major benefit we get from the scheme. Now, the further we can get a simpler equivalent representation, equivalent block diagram representation for the on-line scheme compared to that of the off-line scheme, a simpler one can be obtained.

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How it will look like, then the equivalent representation of the block diagram for on-line identification scheme looks like this. Where the relays is basically a TITO system **sorry**, this will cover like this. So, the relay will experience, and inner feedback controller subject the TITO system is subjected to inner feedback controllers. So, that is the beauty of the scheme, when the on-line identification scheme **scheme** is given or shown in equivalent form, then it is apparent from this figure that the relays is basically the TITO dynamics, subjected to **to** inner feedback controllers.

And what is the benefit of this inner putting inner feedback controllers, that the TITO dynamics can get modified or the open loop TITO dynamics can be modified with the help of the two inner feedback controllers $G_{c1}(s)$, and $G_{c2}(s)$. And thus, we can relocate, the poles of the open loop TITO dynamics to suitable positions. Thus the relay feedback will result in not only symmetrical output, rather it can result in symmetrical output for difficult TITO processes as well. So, that is the major benefit one gets from the on-line identification structure, we have provided.

(Refer Slide Time: 12:24)

Handwritten mathematical derivations for TITO system transfer functions and steady-state response:

$$\frac{Y_1}{L_1} = \frac{G_1}{1 + G_1 G_{c1}} = \frac{G_1}{1 + N G_1 + G_1 G_{c1}}$$

$$\frac{Y_2}{L_2} = \frac{G_2}{1 + N G_2 + G_2 G_{c2}}$$

where G_{c1}, G_{c2} are the equivalent representation of the dynamics of the TITO system

$$\lim_{s \rightarrow 0} \frac{Y_1}{L_1}(s) = \frac{s \times G_1}{1 + N G_1 + G_1 G_{c1}} = 0$$

where G_{c1}, G_{c2} have integrators

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Now, also I can get the relation between the output to the static load disturbances in the form of Y_1 upon L_1 is equal to G_1 upon $1 + G_1 G_{c1} + N G_1 + G_1 G_{c1}$. What I mean by that, let us find the transfer function between Y_1 to L_1 . Then assuming R_1 to be 0 that time, R_2 to be 0 that time for the upper case, when the TITO process or dynamics is assumed to have decoupled dynamics, that **that** time the

transfer function between the output to that of the static load disturbance can be given in this form. Upon simplification is which gives us in the form of $G_1 \text{ upon } 1 \text{ plus } N G_1 \text{ plus } G_1 G_{cl1}$.

This is for one loop, similarly I can find the transfer function of $Y_2 \text{ upon } L_2$ to be of the form $G_2 \text{ upon } 1 \text{ plus } N G_2$, in this case this will be N_1 , this will be $N_2 - N_2 G_2 \text{ plus } G_2 G_{cl2}$. So, what are G_1 , and G_2 here; G_1 and G_2 are the equivalent representation **equivalent representation** of the dynamics of TITO system of the dynamics of the TITO system. When it is assumed that the TITO system is decoupled. So, when the TITO system is decoupled; obviously, we get the equivalent representation of the whole TITO system dynamics in the form of G_1 , and G_2 . That means, as if we have got two single input, single output systems of the form $G_1(s)$, and $G_2(s)$. Corresponding to that the transfer function between the output, and load disturbances can be expressed in this form. So, why I am expressing this the reason is that, if you find the steady state values of the output corresponding to the load inputs or static load disturbances, limit S tends to 0, $Y_1 \text{ upon } L_1(s)$ will give you zero.

When the G_{cl1} or G_{cl2} or the controllers have integral action or integrators, when the controllers have integrators basically one S will appear in the numerator. So, ultimately it will be it will be S times your $G_2 \text{ upon } 1 \text{ plus } N_2 G_2 \text{ plus } G_2 G_{cl2}$ dash. It will be available in this form, when S tends to 0 whole term becomes zero. And that is how the effects of the load disturbances L_1 , and L_2 are nullified in the output of the system. So, the limit cycle experiment or the relay feedback experiment will not be disturbed by the presence of static load disturbances. So, this the major benefit you get from the on-line identification scheme.

(Refer Slide Time: 16:18)

• Aim is to identify two SISO second order plus time delay transfer function models of the TITO process from the relay experiments

TITO system \Leftrightarrow two SISO models

Let the model transfer matrix be

$$G_m(s) = \begin{bmatrix} G_{m1}(s) & 0 \\ 0 & G_{m2}(s) \end{bmatrix}$$


which possesses the SISO transfer function models as:

$$G_{mi}(s) = \frac{K_i e^{-D_i s}}{(sT_i + 1)^2} \quad \forall i=1,2$$

where K_i , T_i and D_i are the process model parameters to be estimated

K_i are steady state gains
 T_i, T_2 are the time constants
 D_1, D_2 are time delays

Handwritten equations for $G_{m1}(s) = \frac{K_1 e^{-D_1 s}}{(sT_1 + 1)^2}$ and $G_{m2}(s) = \frac{K_2 e^{-D_2 s}}{(sT_2 + 1)^2}$ are also present.



Now, I will describe the **the** dynamics of the TITO system by the model transfer matrix. Now, the TITO system will be identified by or will be represented by 2 single input, single output second order plus time delay transfer function models. So, the TITO system will be represented by two single input, single output models - transfer function models. Then the transfer model transfer matrix will be $G_m(s)$ is equal to $G_{m1}(s), 0, 0, G_{m2}(s)$. So, we can write the model transfer matrix in the diagonal form. Now what **what** form of G_{m1} , and $G_{m2}(s)$ will help, we one can choose the single input, single output transfer function models to be of the form $G_{mi}(s)$ is equal to $K_i e^{-D_i s}$ upon $(sT_i + 1)^2$ for all i equal to 1 to 2. That means, $G_{m1}(s)$ will be equal to $K_1 e^{-D_1 s}$ upon $(sT_1 + 1)^2$, and $G_{m2}(s)$ will now become $K_2 e^{-D_2 s}$ upon $(sT_2 + 1)^2$.

So, the TITO system dynamics has got how many unknowns means as far as the two single input, single output transfer function models are concerned number of unknowns will be 1, 2, 3, 4, 5, 6. So, we will have 6 unknowns - those are to be identified or estimated. What are those K's, D's and T's; K's are the steady state gains. So, we will have two steady state gains K_1 , and K_2 . So, K's are the steady state gains, what are the T's? T's are the time constants. So, T_1, T_2 are the time constants, and what are the D's? D's are the D_1 , and D_2 are the time delays.

Please see the changes, earlier I have used the theta variable for the time delays, but in this study, we are introducing D's for the time delays. So D 1, D 2 are the time delays of the transfer function models. Now, we have assumed a second order transfer function, model for the dynamics of for two SISO models of the TITO dynamics.

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$$G_{m1}(s) = \frac{k_1 e^{-D_1 s}}{(T_1 s + 1)^2} \leftarrow \text{SOPDT model}$$

3 unknowns

$$G_{m1}(s) = \frac{k_1 e^{-D_1 s}}{T_1 s + 1} \quad \text{FOPDT model}$$

3 unknowns

$$G_{m1}(s) = \frac{k_1 e^{-D_1 s}}{T_1 T_2 s^2 + (T_1 + T_2) s + 1} \leftarrow$$

4 unknowns

$$G_{m1}(s) = \frac{k_1 e^{-D_1 s}}{4 \tau_1 s^2 + \tau_2 s + 1} \leftarrow$$

4 unknowns

Why we have assumed the second order dynamics I mean to say, $G_{m1}(s)$ I have written by $K_1 e^{-D_1 s}$ upon $T_1 s + 1$ square. Often one can use any sort of transfer function model here. So, $G_1(s)$ can be of the form $K_1 e^{-D_1 s}$ divided by $T_1 s + 1$ also. So, we can represent the SISO transfer function model by some first order plus dead time model. Unlike the second order plus, dead time model that, we have assumed for our study. Now, can we take any other set of models $G_{m1}(s)$ is equal to say your $K_1 e^{-D_1 s}$ times $T_1 T_2 S$ square plus $T_1 + T_2 S$ plus 1. Can we take any other form like G_{m1} is equal to $K_1 e^{-D_1 s}$ upon your $\tau_1 S$ square plus $4 \tau_1 S$ square plus $\tau_2 S$ plus 1. Can I take in this form?.

That means, this represents the dynamics for an under damped system, this represents the dynamics for a second order transfer function model with two distinct poles. So, we can no doubt the analysis cannot be or may not be restricted to only the second order plus dead time model, we have chosen rather we can use any sort of model, but there is one limitation which shall make use of describing function in our analysis. So, when you

make use of describing function in your analysis, the describing function does not result in more analytical expressions. Only the phase, and magnitudes of describing function or the analytical expressions involving describing function can be considered, in that case number of unknowns - **the number of unknowns** that can be identified will be limited.

So, since we are using describing function, and the describing function does not give more analytical expressions, we are forced to choose simple transfer function model, that has got less number of unknowns. So, if I look at the transfer function models, number of unknowns who have in the second order plus dead time model; this second order plus dead time model are 3. So, we have got 3 unknowns here. Similarly, the second one we have got 3 unknowns, you have got 1, 2, 3, 4 - 4 unknowns, and 1, 2, 3, 4. So, you have got 4 unknown.

Now, since the last two transfer function models are having more number of unknowns, I am not using those in our analysis. Coming to the first two, now the first order plus dead time model has got 3 unknowns; whereas, the second order plus dead time model of this form has got 3 unknowns. So, why not to use the higher order transfer function model, that can replicate the dynamics of many practical systems. So, that is the benefit, we get from this second order plus dead time model. And that is why I am going to use this transfer function model in our analysis.

(Refer Slide Time: 23:49)

PID controller matrix

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 \\ 0 & G_{c2}(s) \end{bmatrix} \quad \checkmark$$

where


$$G_{ci}(s) = K_{ci} \left(1 + \frac{1}{T_{i1}s} \right) \left(1 + \frac{T_{d1}s}{1 + \beta_1 T_{d1}s} \right) \quad \text{Series PID controller} \quad \forall i=1,2$$

Let describing function matrix be

$$N = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$$

where N_1 and N_2 are the gains of the ideal relays of loop 1 and 2, respectively

As per the describing filters
 $\beta_1 = \beta_2 = 0$



Now, the PID controllers will be given by the series form of PID controller. So, this give the dynamics of a series PID controller, where you have got beta in the PID controller, as beta 1 and beta 2, when i equal to 1 and 2, beta 1 and beta 2 are the derivative filters. Which are of very small value. So, for practical purposes for each in analysis of analytical expressions, we are going to handle later on what will be done beta 1, and beta 2 will be assumed to be 0. There is no harm, because normally these values are very small percentage of that of the time constants T d i dash.

Now so, the controller in the matrix form, and that to in diagonal form are given by G cl (s) is equal to G cl1(s), 0, 0, G cl2(s) square, G cli(s) will be equal to K ci dash times 1 plus 1 upon T Ii dash S times 1 plus T d i dash S upon 1 plus beta i T d i dash S. So, we are using series form of PID controllers. Now, let the describing function matrix for the two relays be given by the non-linear matrix, and is equal to N 1, 0, 0, N 2 - where N 1, and N 2 are the gains of the ideal relays for loop 1 and 2.

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$$\begin{cases} N_1 = \frac{4h_1}{\pi A_1} & \text{for ideal relays} \\ N_2 = \frac{4h_2}{\pi A_2} \end{cases}$$

h_1, h_2 are the relay settings and
 A_1, A_2 are the peak amplitudes of the
sustained oscillatory outputs

$$N_i = \frac{4h_i}{\pi A_i} \quad \forall i=1,2$$

So, how the gains are described now N 1, as you know N 1 is equal to 4 h 1 upon phi A 1, and N 2 will be equal to 4 h 2 upon phi A 2 for ideal relays. So, when the relay possesses, when the relay possesses some hysteresis in that case these expressions are to be modified. Now, what are h 1, h 2, A 1, A 2? h 1, h 2 are the relay settings, and A 1, A 2 are the peak amplitudes of the sustained oscillatory output . So, thus we get the

dynamics of the relay in general form; N_i is equal to $4 h_i$ upon ϕA_i , for all i going from 1 to 2. So, this is how, we get the dynamics for N also.

(Refer Slide Time: 27:24)

Estimation of Steady State Gains (K_i) $\forall i=1,2$

- Measure average values of the process inputs u_{1avg} and u_{2avg} and the process outputs y_{1avg} and y_{2avg}
- Use temporal disturbance at the setpoints or reference inputs R_1 or R_2 to measure the above average values

$$K_1 = \frac{\chi}{(u_{1avg}^1 y_{2avg}^2 - u_{1avg}^2 y_{2avg}^1)(u_{1avg}^1 u_{2avg}^2 - u_{1avg}^2 u_{2avg}^1)}$$

$$K_2 = \frac{\chi}{(u_{2avg}^2 y_{1avg}^1 - u_{2avg}^1 y_{1avg}^2)(u_{1avg}^1 u_{2avg}^2 - u_{1avg}^2 u_{2avg}^1)}$$

$$\chi = (u_{2avg}^2 y_{1avg}^1 - u_{2avg}^1 y_{1avg}^2)(u_{1avg}^1 y_{2avg}^2 - u_{1avg}^2 y_{2avg}^1) - (u_{1avg}^1 y_{2avg}^1 - u_{1avg}^2 y_{2avg}^2)(u_{2avg}^2 y_{1avg}^2 - u_{2avg}^1 y_{1avg}^1)$$

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Now, we will start with the ((C)) process or stages or the procedure for estimation of the parameters associated with transfer two SISO transfer function models. First, we will try to identify the steady state gains given by K_i for all i equal to 1 to 2. That means, how to find K_1 , and K_2 will be described now. Now, measure average values of the process inputs, what are the process inputs U_1 and U_2 ? These are the two process inputs, but in first instant or at any instant of time when R_1 , and R_2 the two set values are not equal to 0, that time you measure the process inputs, and denote that by U_{11} , and U_{21} . When you measure the average values of the process inputs - the average of process inputs that time denote the average values of the process inputs by U_{11} average, and U_{21} average. Where are those U_{11} , U_{21} ; you see what are the process inputs, and output, if you go back to the model.

28 52

We have got the input U_1 , and output y_1 . Similarly, input U_2 and y_2 - the second output. Now, in **in in** variable form you have to right the lower case letters, U_1 in laplace transfer form is the upper case, and in variable form are lower case in the signal form it gives you U_1 . Similarly, the output is y_1 , and U_2 , and y_2 . So, measure those values average values when R_1 , and R_2 are having some values, and that time denote the

process inputs by U_{11} average, and U_{21} average. Similarly, the process outputs at that instance are denoted by y_{11} average, and y_{21} average.

Now, use some temporal disturbance at the set point or reference inputs to measure the above average values in the second instance, where the process inputs will be when you give some disturbance like. Change the value of R_1 or R_2 by some small magnitude, then you will have different inputs, and outputs variables or measurements. So, at that time let the process inputs, when the temporal disturbance is applied be process inputs be U_{12} average, and U_{22} average at the second instance. Similarly, the process outputs y_{12} average, and y_{22} average.

So, this is how I obtain how many 1, 2, 3, 4, 5, 6, 7, 8 - 8 measurements are obtained. So, with those measurements, it is possible to estimate the steady state gains using the expressions, K_1 is equal to y_{12} average times U_{11} average minus y_{11} average times U_{12} average whole of times U_{11} average U_{22} average minus U_{12} average U_{21} average. Similarly, K_2 can be having an expression of this form. So, use the measurements, and estimate the steady state gains K_1 , and K_2 , where x_i is given by this horrible expression. Now, these are very simple to obtain the procedure for obtaining **obtaining** the expression for steady state gains will be explained now.


(Refer Slide Time: 32:12)

$$h_1 = h_2 \neq 0 \quad R_1, R_2 \neq 0$$

$$\begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_1^2 \\ Y_2^2 \end{bmatrix} = \begin{bmatrix} U_1^1 & U_2^1 & 0 & 0 \\ 0 & 0 & U_1^1 & U_2^1 \\ U_1^2 & U_2^2 & 0 & 0 \\ 0 & 0 & U_1^2 & U_2^2 \end{bmatrix} \begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix}$$

$$Y_1^1 = G_{11} U_1^1 + G_{12} U_2^1$$

$$Y_1^2 = G_{11} U_1^2 + G_{12} U_2^2$$



How do I get those expressions now, if limits cycle occurs for some non-zero h_1 , and h_2 , and non-zero R_1 , R_2 . So R_1 , R_2 are not equal to 0 **not equal to 0** at that time make

the measurements - and take the measurements in laplace variable form first. Let then the outputs Y_1 , and Y_2 at first instance will be denoted by Y_{11} , Y_{21} . And similarly, the outputs at the second instant, like when the temporal input is given is given by Y_{12} , and Y_{22} .

When you put that in the matrix form, what you get basically these are nothing but your measurements U_{11} U_{21} upper case, U_{21} , 0, 0 with G_{11} , G_{12} , G_{21} , G_{22} . And let me complete this 0, 0, and you will have U_{11} , U_{21} . Similarly, you will have U_{12} , U_{22} , 0, 0, 0, 0, U_{12} , U_{22} or if you expand this, let me write one simple expression, how do I get Y_{11} . Y_{11} is nothing but $G_{11} U_{11}$ plus $G_{12} U_{21}$. So, in the first instant; forget about this first instant. Then what you are getting Y_1 is equal to $G_{11} U_1$ plus $G_{12} U_2$. Is it so or not **yes**, definitely if you see the block diagram now, what is your $Y_1 - Y_1$ will definitely is made up of two components $G_{11} U_1$ plus $G_{12} U_2$.


So, Y_1 is given by, this Y_1 the signal at this point is given by G_{11} times U_1 . So, $G_{11} U_1$ plus this signal is getting added here. So, G_{12} , and the input is U_2 now. So, at that time assume this to be U_{11} , and U_{21} to be 0, and signal measurements M_1 and M_2 errors are to be 0. Then you get the expression $G_{12} U_2$. So, it is not difficult to get those simpler expressions first, which can ultimately be converted to some matrix form, and to some ultimate form given by this. So, after obtaining this, what I will do then if this way we can find the expression for first instant - put U_{11} at the first instant. Similarly at different instant of times, second instant the output Y_1 will be Y_{12} will be nothing but U_{12} or $G_{11} U_{12}$, U_{12} plus $G_{12} U_{22}$.

(Refer Slide Time: 35:48)

$$\begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix} = \frac{1}{(U_1' U_2^2 - U_1^2 U_2')} \begin{bmatrix} U_2^2 & 0 & -U_2' & 0 \\ -U_1^2 & 0 & U_1' & 0 \\ 0 & U_2^2 & 0 & -U_2' \\ 0 & -U_1^2 & 0 & U_1' \end{bmatrix} \begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_1^2 \\ Y_2^2 \end{bmatrix}$$

TITO system dynamics 3

$$G_1 = \frac{G_{11} G_{22} - G_{12} G_{21}}{G_{22}}$$


$$G_2 = \frac{G_{11} G_{22} - G_{12} G_{21}}{G_{11}}$$


So, we obtained the signal in transfer in **in** laplace domain in this form, which ultimately can be given written as, now G_{11} , G_{12} , G_{21} , G_{22} will be equal to the matrix inversion will get involve. So, which can be solved, and found as U_{11} , U_{22} minus U_{12} , U_{21} times, you will have again your U_{22} , 0, minus U_{21} , 0 then minus U_{12} , 0, then U_{11} , 0, then 0, U_{22} , 0 minus U_{21} , 0 minus U_{12} , 0, U_{11} . So, when you find the matrix inversion - **inversion** of this one, because ultimately G_{11} **sorry** stop.

(Refer Slide Time: 36:42)

$h_1 = h_2 \neq 0$ $R_1, R_2 \neq 0$

$$\begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_1^2 \\ Y_2^2 \end{bmatrix} = \begin{bmatrix} U_1^1 & U_2^1 & 0 & 0 \\ 0 & 0 & U_1^1 & U_2^1 \\ U_1^2 & U_2^2 & 0 & 0 \\ 0 & 0 & U_1^2 & U_2^2 \end{bmatrix} \begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix}$$

$$\begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix} = \begin{bmatrix} U_1^1 & U_2^1 & 0 & 0 \\ 0 & 0 & U_1^1 & U_2^1 \\ U_1^2 & U_2^2 & 0 & 0 \\ 0 & 0 & U_1^2 & U_2^2 \end{bmatrix}^{-1} \begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_1^2 \\ Y_2^2 \end{bmatrix}$$


So, when I write this in the form of G_{11} , G_{12} , G_{21} , G_{22} is equal to, take this to the left hand side. So, you will have the inversion now, matrix inversion U_{11} , U_{21} , 0 , 0 , 0 , 0 , U_{11} , U_{21} , then U_{12} , U_{22} , 0 , 0 , 0 , 0 , U_{12} , U_{22} inverse times Y_{11} , Y_{21} , Y_{12} , Y_{22} . So, this is how get? You get. So, I find the matrix inversion which comes out to be of this form; times it will be your y_{11} , y_{21} , and y_{12} , y_{22} . So, this is how you get the expression for G now.

Now when the TITO dynamics, **when the TITO dynamics** - **dynamics** is represented by 2 single inputs, single output dynamic model form; assuming that there is decoupling between the two loops. Then we know that G_1 can be given in the form of G_{11} , G_{22} minus G_{12} , G_{21} divided by G_{22} . Similarly an expression for G_2 can be written, for in this case what will happen G , it will be G_{11} to G_{22} minus G_{12} to G_{21} divided by G_{11} . So, these are the two decoupled transfer function representation of the TITO system dynamics. So, when I consider the upper one, then upon substitution of G_{22} - upon substitution of G_{11} , G_{12} , G_{21} , G_{22} here in this expression, then I get finally, these K expression for K_1 , and K_2 .

(Refer Slide Time: 39:10)

$$\begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix} = \frac{1}{(U_1'U_2'' - U_1''U_2')} \begin{bmatrix} U_2'' & 0 & -U_2' & 0 \\ -U_2'' & 0 & U_1' & 0 \\ 0 & U_2'' & 0 & -U_2' \\ 0 & -U_1' & 0 & U_1' \end{bmatrix} \begin{bmatrix} Y_1' \\ Y_1'' \\ Y_2' \\ Y_2'' \end{bmatrix}$$

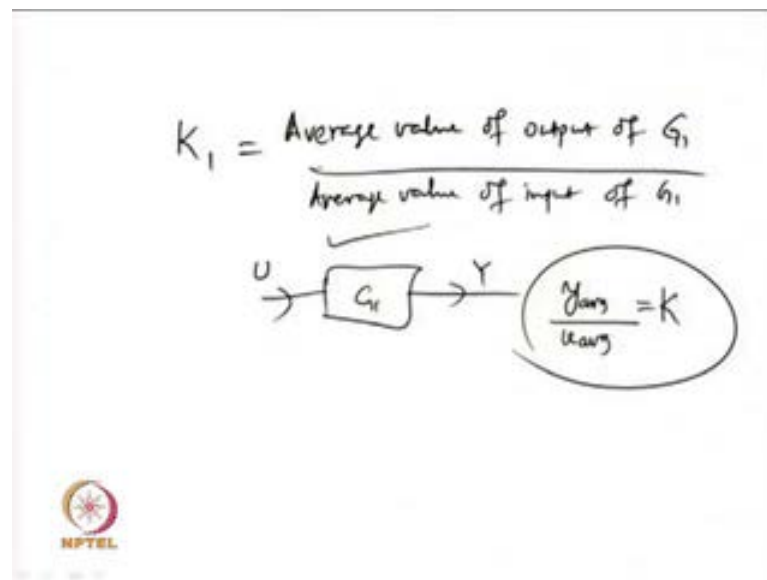
TITO system dynamics 3

$$G_1 = \frac{G_{11}G_{22} - G_{12}G_{21}}{G_{22}} = \frac{(U_2''Y_1' - U_2'Y_1'') (U_1'Y_2'' - U_1''Y_2') - (U_1'Y_2' - U_1''Y_2'')}{(U_1'Y_2'' - U_1''Y_2') (U_1U_2'' - U_1''U_2')}$$

How do you get, after obtaining the substitution of this one here, how much you will get basically G_1 will result in $U_{22} Y_{11}$ minus $U_{21} Y_{21}$ times $U_{11} Y_{22}$ minus $U_{12} Y_{12}$ minus of $U_{11} Y_{21}$ minus $U_{12} Y_{11}$ times $U_{22} Y_{11}$ minus $U_{21} Y_{22}$. What is, when I substitute this G 's values G_{11} , G_{12} , G_{21} , G_{22} in this expression, I

get in this form divided by $U_1 Y_2$ minus $U_2 Y_1$ times $U_1 Y_2 U_2$ minus $U_2 Y_1$. Now, when you take the... How we find the steady state gains, now when I take the average value of this one, **when I take the average value average value of this one**, then I get the K.

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What is that K 1? K 1 is now average value of **average value of** output of G 1 upon average value of input of G 1. So, basically what has happened, I have got a G 1 dynamics suppose, and the outputs, inputs is now U, and output is Y. So, when I take the average; that means, y a v g upon U **sorry**, this will be the small letter now, because I am measuring the signal, and finding the signals average value then U a v g will give you the steady state value or d c value. Which gives you, the steady state gain K. This is how, the steady state or static gains associated with the transfer function models are estimated.

(Refer Slide Time: 41:39)

Estimation of Process Model Parameters

Auto-tuning test results in stable limit cycle outputs for any $h_1 / h_2 \neq 0$ when $\overline{NG}_n(j\omega_{cr}) = -I$

where $\overline{G}_n(j\omega_{cr}) = [I + \overline{G}_m \overline{G}_{cl}(s)]^{-1} \overline{G}_m(j\omega_{cr})$

After substituting controller and process transfer function models


$$\frac{K_i e^{-j\omega_{cr} D_i}}{(1 + j\omega_{cr} T_{di})^2} (a_{1i} + a_{2i}) = -1 \quad i=1, 2$$

where

$$a_{1i} = \frac{4h_i}{\pi A_i} + K_{ci}' \left(1 + \frac{T_{di}'}{T_{di}} \right)$$

$$a_{2i} = K_{ci}' \left(\omega_{cr} T_{di}' - \frac{1}{\omega_{cr} T_{di}} \right)$$

(Handwritten notes: G_{m_i}, G_{cl_i} with arrows pointing to the equations)



Next, we will go to the estimation of the remaining parameters of the TITO transfer function matrix models. So, the auto-tuning test results in stable limit cycle outputs for any $h_1 / h_2 \neq 0$. That means h_1, h_2 are not equal to 0. Then from the loop equation, that loop dynamics gives out the characteristic equation of the form identity matrix I plus $\overline{NG}_n(s)$ is equal to 0. And that in frequency domain can ultimately be expressed in the form of $N \overline{G}_m(j\omega_{cr}) = -I$, where ω_{cr} is the loop frequencies. So, ω_{cr} we assume the ω_{cr} to be the loop frequency of both loops.

So, loop one, and loop 2 are assumed to have same signal assumed to have signal with same frequency - critical frequency. That is given by ω_{cr} . Now, $\overline{G}_m(j\omega_{cr})$ is further, when you look at the block diagram from there, you can **you can** easily find out, what is $\overline{G}_m(j\omega_{cr})$. This in transfer function form will be $\overline{G}_m(s) = [I + \overline{G}_m \overline{G}_{cl}(s)]^{-1} \overline{G}_m(s)$. So which in, frequency domain is given in this form. So, this is how we get from the loop equation or the characteristic equation of the loop this analytical expression.

Now substitution of \overline{G}_m , and \overline{G}_{cl} in the expression results in this expression. So, simply substitute for $\overline{G}_{m1}, \overline{G}_{m2}, \overline{G}_{cl1}, \overline{G}_{cl2}$, and get that in the general form of $K_i e^{-j\omega_{cr} D_i} (a_{1i} + a_{2i}) = -1$, for all $i = 1, 2$. So, we will get two analytical

expressions basically, but in general form I put the this expression upon substitution of various G's **G's**, G m and G c l, either I get an expression of this form. Where $1 + a_1 i$ is equal to $4 h i$ upon $\phi A i$ plus $K c i$ dash times 1 plus $T d i$ dash upon $T i$ dash. And $A_2 i$ is equal to $K c i$ dash times $\omega c r T d i$ dash minus 1 upon $\omega c r T i$ dash. These are not difficult to get these expressions, it is very easy simply substitute the expressions for G m and G c l or G m i and G c l i in this expression - general expression which will result in another general expression of this form.

Where the constants A - A's are given by these. So, after obtaining this what you do, since we have got an equation in complex variables. So, that way often simplification means equating the magnitudes of both sides of that equation. So, when you equate the magnitudes of both sides; take the magnitudes of both sides. So, what you will get basically, when you take the magnitudes of both sides of this expression.

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$$\frac{K_i}{\sqrt{1 + (\omega_c r T_i)^2} \sqrt{a_1^2 + a_2^2}} = 1$$

$$-\omega_c r D_i - 2 \tan^{-1} \omega_c r T_i = -\pi$$

So, I will get expression in the form of $K i$ upon $1 + \omega c i T i$ square root is equal to or again we have got complex number say $a_1 i$ square plus $a_2 i$ square root is equal to 1. So, this is what you get from the magnitude, and from the phase angles you get minus $\omega c r D i$ minus 2 times tan inverse of $\omega c r T i$ is **equal to**, then again given by angles **minus** plus tan inverse of no these are constants. So, no need of tan inverse. So, angle wise they do not contribute is equal to minus phi. So, using these two expressions what I will get, when those are solved again, when the first two are solved.

(Refer Slide Time: 47:04)

After equating magnitudes and phase angles of both side of the expressions


$$T_i = \frac{\sqrt{K_i \sqrt{a_{1i}^2 + a_{2i}^2} - 1}}{\omega_{cr}}$$

$$D_i = \frac{\pi + \tan^{-1} \left(\frac{a_{2i}}{a_{1i}} \right) - 2 \tan^{-1} (\omega_{cr} T_i)}{\omega_{cr}}$$

Load Disturbance Rejection:
Output of a TITO process for a step load disturbance is

$$Y(s) = [I + NG(s) + G_{cl}G(s)]^{-1} G(s)L(s)$$

where $G_{cl}(s) \approx \frac{1}{s}$ and $\lim_{s \rightarrow 0} Y(s) = 0$



Then obviously, one gets T_i is equal to root of K_i times root of a_{1i} square plus a_{2i} square minus 1 upon ω_{cr} , and D_i is equal to π plus $\tan^{-1} \left(\frac{a_{2i}}{a_{1i}} \right)$ minus $2 \tan^{-1} (\omega_{cr} T_i)$ upon ω_{cr} . So, this is how you get the expression final or explicit expressions for unknowns associated with the transfer function models. Now, load disturbance rejection already I have described. So, this is repeated in the matrix form as $Y(s)$ is equal to I plus $N G(s)$ plus $G_{cl} G(s)$ inverse $G(s)L(s)$, and G when $G_{cl} L(s)$, $G_{cl} L(s)$ has got some integrator then this in final limit s tends to 0, $Y(s)$ becomes 0. So, the contribution or the effects of load disturbances are nullified.

(Refer Slide Time: 48:10)

Simulation study


Consider a TITO process with the transfer matrix

$$G(s) = \frac{1}{den(s)} \begin{bmatrix} (1.5s + 1) & 0.2(0.75s + 1) \\ 0.6(0.75s + 1) & 0.8(1.2s + 1) \end{bmatrix}$$

where $den(s) = (1+s)(1+2s)^2(1+0.5s)$ *Two SISO*

Controller settings and Identification results after two tests

	Loop 1	Loop 2
Stage 1	$K_{c1}^* = 0.1, T_{i1}^* = 1, T_{d1}^* = 0$	$K_{c2}^* = 0.1, T_{i2}^* = 1, T_{d2}^* = 0$
	$K_1 = 0.849, T_1 = 1.6698, D_1 = 0.4370$	$K_2 = 0.678, T_2 = 1.3343, D_2 = 0.4272$
Stage 2	$K_{c1}^* = 3.5298, T_{i1}^* = 1.6711, T_{d1}^* = 1.6698$	$K_{c2}^* = 3.7481, T_{i2}^* = 1.3343, T_{d2}^* = 1.5343$
	$K_1 = 0.849, T_1 = 1.8790, D_1 = 0.3422$	$K_2 = 0.678, T_2 = 1.7573, D_2 = 0.3271$




Let us go to a stimulation study, consider a TITO process with transfer matrix of this form, where the denominator $d e n(s)$ is given by $1 + s + 2s^2 + 0.5s^3$. For this what initially the PID controller values are chosen as K_{c1} is equal to 0.1, $T_{i1} - T_{i1}$ is equal to 1, T_{D1} is equal to 0. And for loop 2 the PID controllers have the values K_{c2} is equal to 0.1, T_{i2} is equal to 1 and T_{D2} is equal to 0, with these two controllers settings relay experiment is conducted. And the dynamic model parameters are found to be K_1 is equal to 0.849, T_1 is equal to 1.6698, and D_1 is equal to this much.

Similarly, for the second loop - the second SISO process model parameters are obtained as this. Now, another stage of relay experiment is conducted with these settings of the PID controllers, with the then you get the final parameters for the 2 SISO transfer function models, as K_1 is equal to 0.849, T_1 is equal to 1.879, D_1 is equal to 0.3422, and K_2 equal to 0.678, T_2 equal to 1.7573, and D_2 equal to 0.3271. Thus we obtained 2 SISO transfer function models with these parameters, after two relay experimentations or two relay tests. And these are the final values, and found to be representing the dynamics of the TITO process.

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Summary

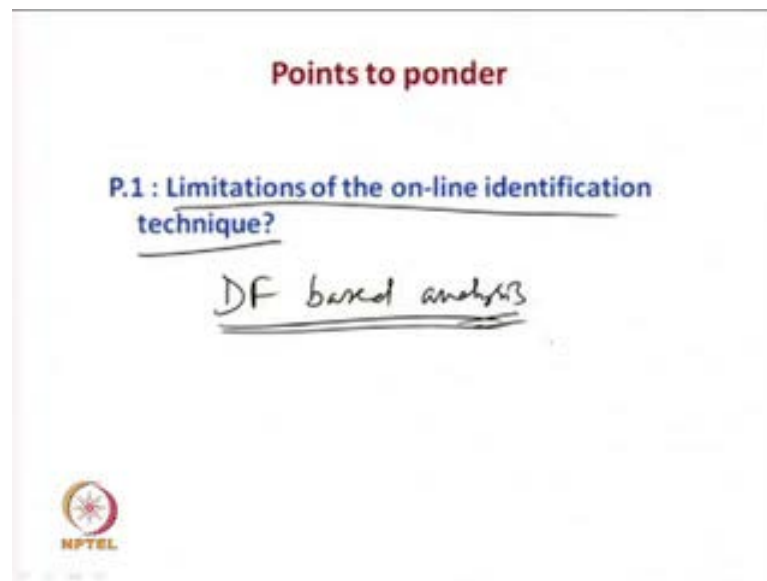
- Set of equations are presented for on-line identification of TITO process dynamics *Two SISO Transfer matrix*
- Presence of the controllers in the loops during identification results in zero steady state value of the outputs to the step load disturbance inputs
- Symmetrical limit cycle outputs obtained automatically during symmetrical relay experiments
- No effect of static load disturbances on identification result



So, what we have studied here, set of analytical expressions are presented discussed for on-line identification of the TITO process dynamics. So, two SISO transfer function models, transfer matrix are obtained. Now, presence of the controllers in the loops

ensures that the starting load disturbances, do not affect the system **system** outputs or sustained limit cycle or sustained oscillatory outputs. Now, symmetrical limits cycle outputs are obtained automatically; during the symmetrical relay experiments, because of the presence of the controllers during the relay experiments. Now, no effect of static load disturbances on identification result, and the analytical expression has been given, and proven.

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Now, what are the limitations of the on-line identification technique - one major limitation associated with this technique is that, we are using the describing function based analysis. Now, the describing function does not yield more number of inequalities, as far as analysis based on describing function is concerned. Therefore, one has to assume the transfer function models. The SISO transfer function models having less number of unknowns that is all, thank you.