

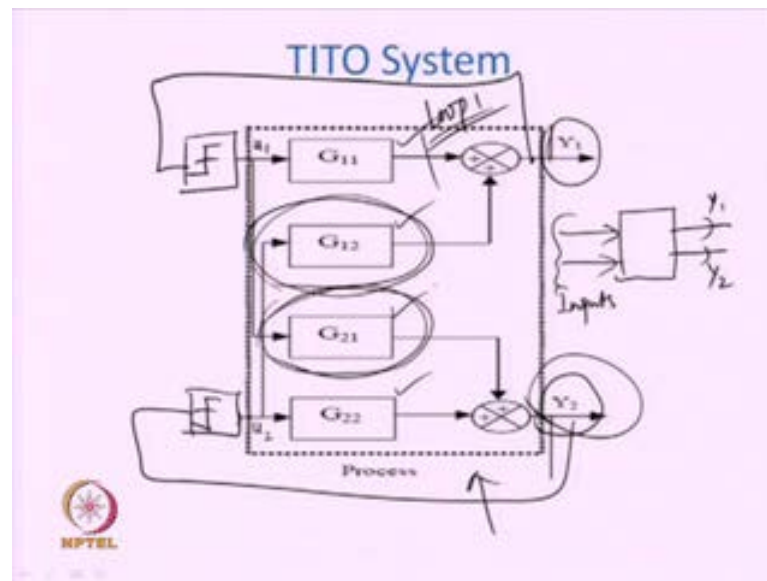
Advanced Control Systems
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Module No. # 03
Time Domain Based Identification
Lecture No. # 14
Off-Line Identification of TITO Systems

Welcome to the lecture titled off-line identification of TITO systems. Earlier, we have described a set of powerful equations, those can be used for the identification of stable, unstable and integrating process models. Now, those stable, unstable, and integrating processes are basically of single input, single output type. In today's lecture, we are going to discuss about the identification of two input, two output processes or systems. Now, this will be basically an off-line identification technique, and this technique is very much different from the earlier techniques, in the sense that we shall handle or deal with a TITO - two input, two output system.

How two input, two output systems are different from the single input, single output systems. In the single input, single output systems we do not have any loop interaction. Whereas, in the case of two input, two output system we have got loop interactions, and that makes the difference. And if the loop interaction can be handled suitably or properly, then the identification technique we have discussed earlier can easily be extended for the identification of TITO systems.

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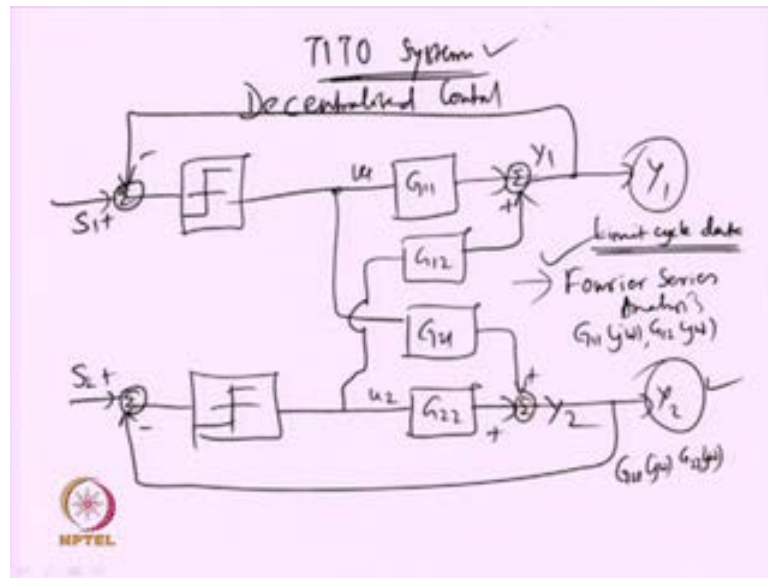
Now, how a TITO systems looks like, it has got at least four sub components given by G_{11} , G_{12} , G_{21} , and G_{22} . And the inputs to the systems are, there are two inputs. So, the input signal here can be given by u_1 , the input signal over here can be u_2 . Similarly, the system has got two outputs shown by y_1 , and y_2 . Now, the same TITO system can be represented in the block diagram form $\left(\begin{matrix} \square \\ \square \end{matrix} \right)$, a block having two inputs. These are the inputs, and two outputs. Now, it appears very simple, but when there is enough or sufficient loop interaction between the sub systems then, the two TITO system appears to be very much different from that of a single input, single output system.

Now, these loop interactions are shown by the transfer functions G_{12} , and G_{21} . Had there been no G_{12} , and G_{21} definitely one can employ two relays for the decentralized control of course, or for the decentralized relay control. Two relays can could have been easily employed here, in this fashion. So, let me add the two relays here, and I can take the feedbacks from the output. And get the simple relay control system for the TITO system, but what happens due to the interaction the outputs y_1 , and y_2 that we shall measure to estimate the parametric models or models parameters of the transfer function models, will not be straight forward.

In the sense that, because of the presence of G_{12} , and G_{21} . What will happen, the y_1 will get affected by the loop dynamics of second loop, and similarly y_2 will get effected or will be modulated by the dynamics of loop 1. So, that way the loop interactions plays

play a major role, and one must take care of enough effort to either get rid of the loop interactions or to divide suitable techniques, such that the loop interactions can be minimized, while conducting a closed loop relay control or relay test.

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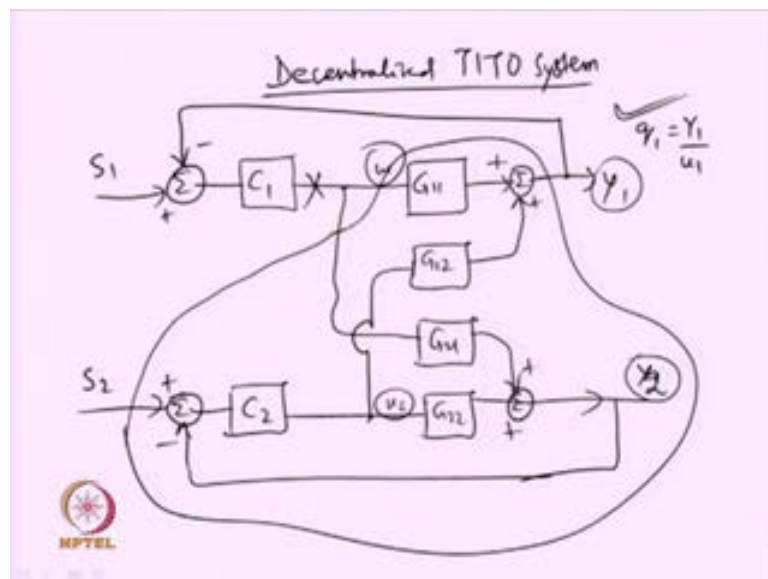
Now, let me describe the TITO system, a little bit more a TITO system. When it is presented in the decentralized control form **decentralized control form**, how it is shown; I will have G_{11} , G_{12} , G_{21} , and G_{22} . Outputs are y_1 , and y_2 over here, and inputs are u_1 , and u_2 . Now, what we have the loop interactions. So, we will get these added over here, and due to that we will have additional signals coming from the other loop. So, this is how we develop the 2 by 2 or two input, two output system. Now, how relay control is effected here, a relay is added here. Another relay is added here, and then the loop is closed. Of course, you will have negative feedback here. So, please allow me to put the reference inputs as well set point or set value one, and set value two.

So, this is how you get the decentralized relay control system. In this decentralized relay control system, how to find the dynamics of the TITO system in terms of either its frequency response characteristics or any other characteristics. So, initially what is done, basically the output signals are measured; the output signals will be sustained oscillatory output signal. So, they can assume different form, like this. Now, if the loop interaction is sufficient, then the frequency of the output signals y_1 , and y_2 could be different. If

the loop interaction is minimal or minimum, in that case I can have y_1 , and y_2 having almost same frequency.

And so, depending on the loop interaction it is expected to get the output signals with different frequencies or same frequencies, same frequency. Then how generally people identify the transfer function model, they collect the limit cycle data; so, collect limit cycle data. Then do the frequency domain analysis, and then use the Fourier series analysis, and find out that dynamics $G_{11}(j\omega)$, $G_{12}(j\omega)$, $G_{21}(j\omega)$, and $G_{22}(j\omega)$. So, this is how generally the decentralized relay control is used to identify the dynamics of a TITO system. Using of course, the Fourier series analysis, but we are not going to discuss this in this lecture, the main reason for that is that this procedure is quite cumbersome, and not straight forward. It is very difficult to unless you go through a series of analytical expressions, which are often very difficult why and quite non-linear in nature. It is not easy to make out, how the transfer function models can be obtained. So, that is to avoid that, I will present some simple technique that can be used to identify the dynamic models for TITO systems.

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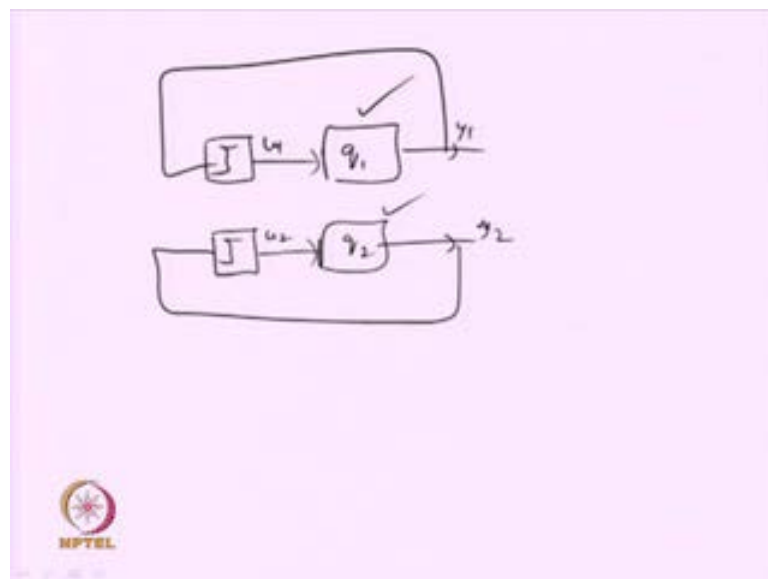


Now, allow me please to write down the T I decentralized control system once more, decentralized TITO system, where I shall describe the TITO system dynamics in the form of G_{11} , G_{12} , G_{21} , and G_{22} . Again we have got the output from the blocks

added in this form, and the input u_1 is passed on to **sorry**. So, that way I will have this in the second loop, and this will go to the first loop.

So, this is the TITO system now. Then, when it is connected with a controller - the controller could be a non-linear or a linear one. So, let me connect a controller of the form C_1 here, and C_2 for the second slope. So, with this controllers, we can have the closed loop system shown as, **sorry** this is y_2 , this is y_1 , negative feedback, and values S_1 and values S_2 . So, this is how I developed a decentralized TITO system with two controllers in the loop. So, these decentralized controller C_1 , and C_2 will result in different type of transfer function models. Now, in place of the transfer matrix which has got four elements G_{11} , G_{12} , G_{21} , G_{22} ; it is possible to find the a transfer dynamics, between y_1 to u_1 , and y_2 to u_2 or indirectly speaking this dynamics can be equivalently represented by 2 sub systems.

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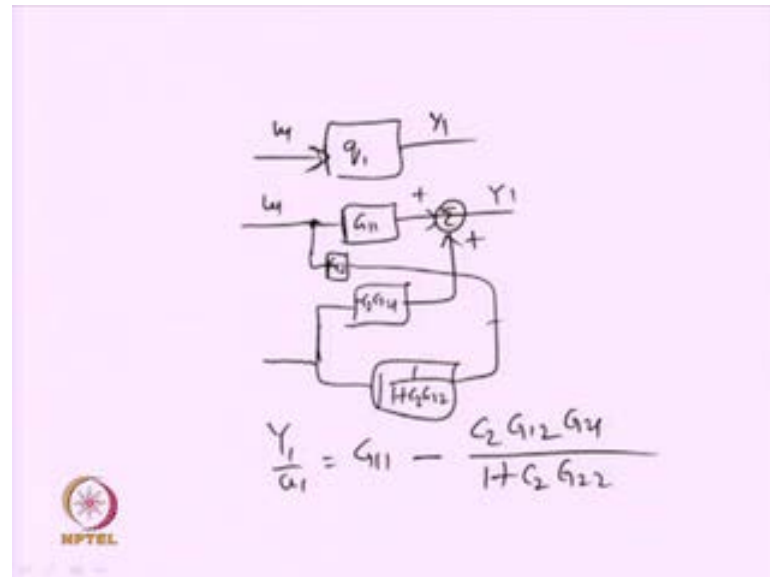


Given in the form of u_1 , q_1 , y_1 , and u_2 , q_2 , y_2 . So, whole dynamics - the dynamics the closed loop system, I have shown over here can equivalently be represented in this form. And if that is so, if that is possible in that case what happens. Now, one can easily employ a relay, here to identify the dynamics of the two sub systems.

So, I can put one relay over here, I can put one relay over here, and then I can have the relay test, and consequently the parameters of q_1 and q_2 can easily be estimated. So,

this is what, we are going to do in this lecture. Basically, how can we develop model, such that instead of having loop interactions or instead of representations of TITO by a transfer matrix, we can have two single input, single output dynamics for the whole TITO system. And that is the beauty of the technique. So, once we develop the technique for the upper case, then it will be extended for the lower case as well with very ease.

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So, what will be q_1 now, so before connecting this. So, what is q_1 ? q_1 is having the input u_1 , and output y_1 . So, please see the input u_1 , and output y_1 . So, how can I find this one, this is possible provided I consider this part of the **sorry**. So, as begin with this **this**. So, this will basically give you the ratio between y_1 upon u_1 or equivalent transfer function single input, single output system dynamics for this including that the effect of the interactions. So, I am going to find two equivalent sub systems. So, that equivalent sub system **system** q_1 . q_1 will be now the ratio between y_1 to u_1 . For that, I have to consider this.

So, I have to break the loop as well. So, if I break the over here, then I do not have this; then it will be possible to find the ratio y_1 upon u_1 . How can you find y_1 upon u_1 , you need to redraw this a little bit to find the ratio between y_1 , and u_1 , and subsequently q_1 . So, to find that one what we have in the reduced block diagram now or the modified

block diagram G_{11} , that will get connected the no signal is coming there, rather your G_{12} is coming over here, because this is basically this signal is the same as u_1 .

So, u_1 is passing on to this one, and that then we are getting the lower part. So, what I will have here basically signal u_1 , and then I have got the block G_{11} , and I have got the output y_1 . But the output is made up of two outputs, one given by the interaction. So, G_{21} will come into picture, and this is getting connected over here. So, basically I have the summer here, then I have got the controller here C_2 , and G_{22} , and we have got G_{12} coming from there. So, $G_{12} - G_{12}$ is adding over here, and I have got this. So, this will give us y_1 upon u_1 . So, how can you find reduce this block diagram now, this can be reduced further considering C_2 its, if I take minus C_2 minus C_2 over here. So, if I allow please allow me to write minus $C_2 G_{21}$, then this block will go out. Then this will be having minus C_2 times G_{22} . So, I will have here minus C_2 minus $C_2 G_{22}$. So, again this this full dynamics again can be given by 1 upon 1 plus $C_2 G_{22}$. So, this will get added. So, ultimately how much y_1 upon u_1 will be G_{11} , coming from here minus $C_2 G_{12}$, and G_{21} upon 1 plus $C_2 G_{22}$. So, this is how with the help of signal flow graph or reduced block diagram or block diagram reduction technique.

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$$q_1 = \frac{Y_1}{u_1} = G_{11} - \frac{C_2 G_{12} G_{21}}{1 + C_2 G_{22}}$$

$$q_2 = G_{22} - \frac{C_1 G_{12} G_{21}}{1 + C_1 G_{11}}$$

It is not difficult to obtain the expression for q_1 as, q_1 is equal to the ratio of y_1 upon u_1 is equal to G_{11} minus $C_2 G_{12} G_{21}$ upon 1 plus $C_2 G_{22}$. See this is very

important, why we are doing all these analysis, please look carefully the expression for q_1 or the sub system we have developed or we have converted the **the** two input, two output system into a system which has got two SISO transfer function models. Now, and all the loop interactions, all those things are taken care of all those are present here.

Similarly, I will get an expression for q_2 as G_{22} minus C_1, G_{12}, G_{21} upon $1 + C_1 G_{11}, G_{22}, G_{11}, C_1 G_{11}$. So, this is how we find two SISO transfer functions for the whole dynamics or the for the dynamics of the TITO system. And this is very important for us in the sense, that all the analysis we have discussed or derive all the equations we have derived, so far can easily be extended for identifying the dynamics of TITO systems provided, it is possible to convert the dynamics of TITO system into some single input, single output dynamics or single input single output transfer function forms.

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$$u_1 \rightarrow \boxed{q_1} \rightarrow y_1$$

$$q_1 = \frac{y_1}{u_1} = G_{11} - \frac{C_2 G_{12} G_{21}}{1 + C_2 G_{22}}$$

When $C_2 \rightarrow \infty$

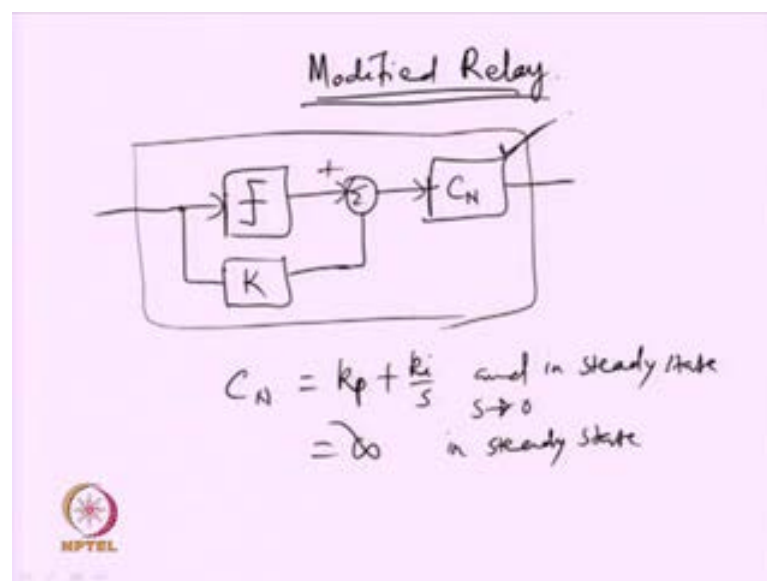
$$q_1 = \frac{y_1}{u_1} = G_{11} - \frac{G_{12} G_{21}}{G_{22}}$$

$$G(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \Leftrightarrow \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$

So, q_1 is this much now, then when C_2 is a large number that time what will happen q_1 will be equal to y_1 upon u_1 is equal to G_{11} minus G_{12}, G_{21} upon G_{22} . When **when** C_2 is a large number, then q_1 can be expressed in this form. And **C** when and **and** basically the loop interaction has gone out, I can say that when C_2 tends to infinity. Basically, we have been able to overcome the loop interactions between the two loops or the loop interactions had been reduced substantially. So, that relay tests can conveniently be conducted, and the parameters of the two models - the two transfer function models which can be given by the equivalent dynamics.

The equivalent dynamics now will be for $G(S)$, the dynamic was given by G_{11} , G_{12} , G_{21} , G_{22} . Now, its equivalence will be given by a transfer matrix of the form q_{11} , 0 , 0 , q_{22} . So, once it has been possible to convert this TITO dynamics into this equivalent diagonal form, then it will be very easy to extend the relay feedback technique, we have described for the single input, single output system. So, how can we make C_{22} tends to infinity during the relay test **yes**, that is possible provided we employ some modified relay, in place of the classical ideal relay or relay with hysteresis.

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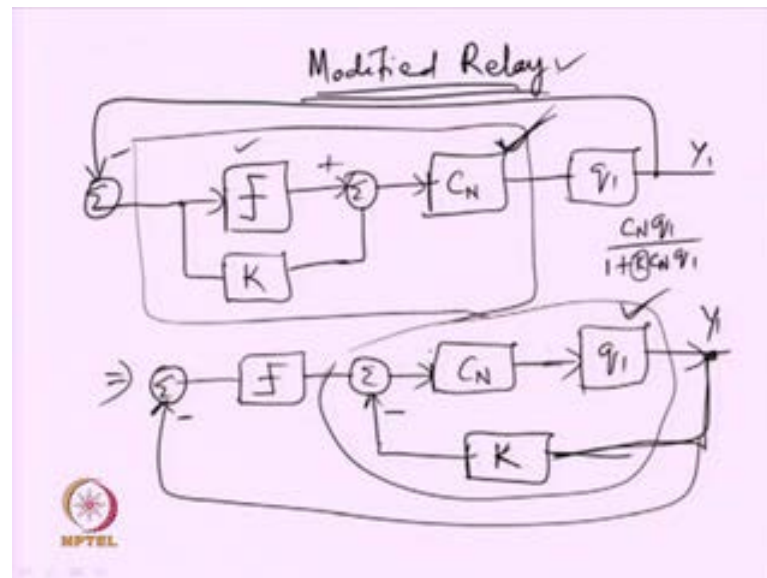


So, we I shall present you now a modified relay **modified relay** which will have a relay in parallel with a gain. Let the gain be given by K , and a controller C_N - **a controller C_N** . So, this gives a modified relay - the modified relay will be used now for identification of TITO systems. Why we are injecting, so many elements along with the conventional relay, the what **what** are the benefits we are going **going** to get from this arrangement, that will be apparent when I put the modified relay in the TITO decentralized control system. Now, this C_N can assume the form of API or PID controller, let us start with a simple PI controller. So, when C_N is given in the form of k_p plus k_i upon s . And in steady state **in steady state** s tends to 0 , that we know then C_N can assume a large value in steady state.

So, the may modified relay is going to help us in reducing the loop interactions with the help of C_N , that is present in the modified relay, it is possible to reduce or eliminate the

loop interactions, we have between the two loops of the TITO system. So, this modified relay will be used for conducting relay test.

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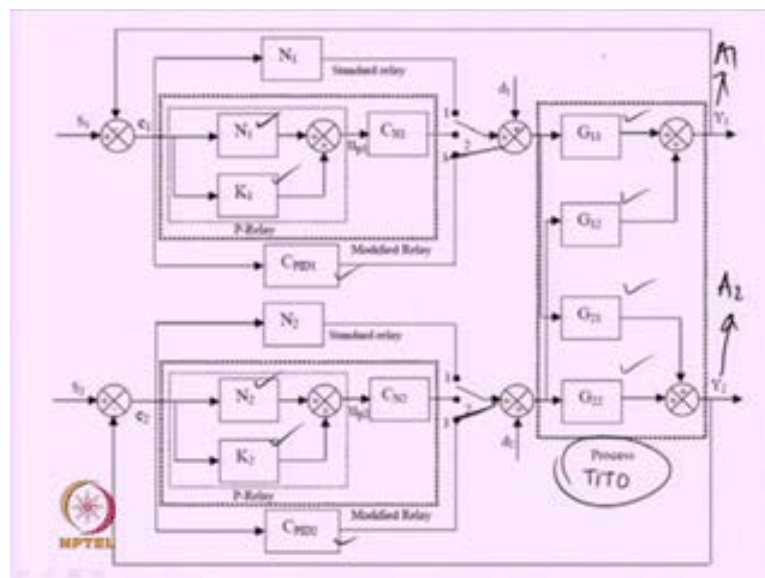
Now, how this modified relay can further be represented this is same as, a relay with the gain shown in the feedback path with C_N . Now, we will conduct the $(())$, we will connect the modified relay with the TITO system. So, please keep in mind. So, when I have got my q_1 here, because the loop interactions have gone with the proper choice of C_N , it is possible to the loop interactions. And the TITO system can equivalently be represented by a transfer matrix, which has got only diagonal elements q_1 , and q_2 . Then in that case, we will have our y_1 and this gives you a negative feedback. Because when q_1 is connected, output of the q_1 is fed back in this form, this is how the relay test is conducted.

Now, the equivalent form of that can be represented in this form, and subsequently you have got the negative feedback over here. So, the equivalent block diagram of the upper one is given below. And interestingly the gain I had connected in parallel with the relay is found to be available, in the inner feedback path. So, additional advantage for going for this modified relay is that, since we have got inner feedback controller now; if q_1 is unstable or having its pole located in odd positions can be relocated at some suitable positions, at the with the help of K_1 . So now, we can relocate the open loop unstable poles or odd poles or oddly located poles of q_1 at some convenient locations with the

help of K, because for this **this** dynamics which the relay sees can be equivalently be given by a transfer function of the form $C N q 1$ upon 1 plus $K C N q 1$.

So, please keep in mind the roots of open loop transfer function $q 1$, can be suitably replaced/relocated with the help of the gain controller K. So, we have got so many benefits from the use of the modified relay or using the modified relay, it is possible to get. So, many benefits or a number of advantages, apart from reducing the loop interactions with the help of C N; it is possible to roll/relocate the poles of $q 1$, at suitable locations with the help of the controller K. So, that is why I will go for this modified relay for conducting relay experiments.

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Now, this will be the block diagram that will be used finally, for identification of two input two, output systems. So, please observe carefully this is the two input, two output process or system dynamic dynamics given by $G 1 1$, $G 1 2$, $G 2 1$, and $G 2 2$. Now, what are these **these** are the load disturbances - static load disturbances. So, $d d 1$ and $d d 2$ stands for the load disturbances present during the relay experiment. So, in spite of having load disturbances I do not mind, because I have got a controller - PI controller while conducting the relay test. The effects of load disturbances can be nullified or can be minimized or can be eliminated with **with** the help of the PI controller.

Therefore, the **the** use of modified relay results in multiple benefits, it **it** can avoid the effects of ill effects of load disturbances also, while performing the relay test. So, if you connect the switches to one for both the loops, then a standard relay will be there in the decentralized relay control system. So, we will have two standard relays, when the when we connect to the second position; when I connect to two now, then the TITO system is subjected to modified relays. And when you connect the switches to the third position, then you get normal operation of the TITO system. So, to set the parameters of the PID controllers, that we have got decentralized PID controllers, that we have got for the TITO system. It is first necessary to identify the transfer function models of the dynamics of TITO process, and based on the transfer function model parameters - the parameters of PID controllers are set.

So, initially what will how it will be conducted, how to start with where we have shown a standard or some standard relays also in the block diagram. Initially connect two standard relays, and get the limit cycle outputs. And based on the limit cycle outputs use some simple formulae or technique, such as the technique given by Jiggler's and Nicole's. So, those technique can be used to find the parameters of C controllers - C N 1, and C N 2 which are nothing but the PI controller. So, connect the switches to one, let me repeat once more connect the switches to one, and find the dynamic model - and from that dynamic model set the parameters of C N 1, and C N 2.

Then you connect the switches to two. Now, when you connect the switches to two, you have to start with some values of K 1, and K 2. So, can be chosen K 1, and K 2 can be chosen from intuition or you **you** can start with very small values, initially. Then, perform the relay test, then identify the transfer function models - and based on the transfer function model parameters set the PID parameters. And then, switch on to 3 or connect the switches to number 3, and round the system your TITO system; then your TITO system will have a decentralized control system having suitable controllers in the loop.

So, this is how automatic tuning of the controllers are done for the decentralized control of TITO systems. Now, how can we identify the transfer function models when modified relays are connected with the TITO process? That we shall discuss in detail now.

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$$N = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} = \begin{bmatrix} \frac{4h_1}{\pi A_1} & 0 \\ 0 & \frac{4h_2}{\pi A_2} \end{bmatrix}$$

$N_1 = \frac{4h_1}{\pi A_1}$
 $N_2 = \frac{4h_2}{\pi A_2}$

$h_1 = \text{Relay parameter of } N_1$
 $h_2 = \text{Relay setting of } N_2$
 $A_1 = \text{peak amplitude of } Y_1$
 $A_2 = \text{peak amplitude of output } Y_2$

Please allow me to take the dynamics of the non-linear elements N_1 , and N_2 in some diagonal form N is equal to $N_1, 0, 0, N_2$; where N_1 is equal to $4 h_1$ upon πA_1 , and N_2 is equal to $4 h_2$ upon πA_2 . So, what are h_1 and A_1 in the expressions for N_1 , h_1 is the relay parameter of N_1 - relay setting of N_1 . Similarly, h_2 is the relay setting of N_2 , and when N_1 and N_2 are available, then we will have sustained oscillatory output for the TITO system, measure the peak amplitudes of the two output; and two outputs will give the two peaks A_1 and A_2 .

So, A_1 is the peak amplitude of the output signal peak amplitude of y_1 , the first output **output** of the upper loop. And A_2 is the peak amplitude of the output y_2 . So, please see y_1 will result in A_1 , and y_2 will result in A_2 . Measure the peak amplitudes and then you get N , this is how N is obtained

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$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$

$$C_N(s) = \begin{bmatrix} C_{N1}(s) & 0 \\ 0 & C_{N2}(s) \end{bmatrix} = \begin{bmatrix} K_{f1} \left(1 + \frac{1}{sT_{f1}}\right) & 0 \\ 0 & K_{f2} \left(1 + \frac{1}{sT_{f2}}\right) \end{bmatrix}$$

$C_{N1}(s) = K_{f1} \left(1 + \frac{1}{sT_{f1}}\right)$
 $C_{N2}(s) = K_{f2} \left(1 + \frac{1}{sT_{f2}}\right)$

Similarly, please allow me to write the gains in matrix form diagonal form given as K is equal to K1, 0, 0, K 2, and the controllers C N 1, and C N 2 are described by 2 PI controllers of the form. C N 1 is equal to K f 1 1 plus 1 upon sT f 1, and C N 2 as K f 2 1 plus 1 upon ST f 2. So, these are the PI controllers, we have in the modified relay.

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Estimation of Model Parameters

Two SISO transfer functions with delay are estimated based on limit cycle data $\leftarrow A_1, A_2 \Leftrightarrow h_1, h_2$

Let the transfer matrix be $G(s)_{2 \times 2}$

$$G_m(s) = \begin{bmatrix} G_{m1}(s) & 0 \\ 0 & G_{m2}(s) \end{bmatrix} \text{ with diagonal elements } 0$$

$$G_{mi}(s) = \frac{K_{pi}}{(1 + sT_i)^2} e^{-\theta_i s} \quad \forall i=1,2$$

So, with the introduction of K C N, and N in diagonal form particularly, it becomes very easy to get the dynamic equation for the TITO system under relay control. So now, we shall describe in detail the estimation technique, when the relay test is performed. So,

two SISO transfer functions with delay are estimated based on the limit cycle data. What do we mean by the limit cycle data, we measure A_1 and A_2 ; of course, with the help of the relay settings of h_1 , and h_2 .

Now, let the transfer matrix that is going to that **that** is to be estimated be given by $G_m(s)$, the subscript m stands for model - the transfer function model matrix in transfer matrix form be $G_m(s)$ is equal to $G_{m1}(s)$ 0, 0, $G_{m2}(s)$ with diagonal elements with **with** diagonal elements zero. Now, this the two transfer functions in general form can be represented by $G_{mi}(s)$ is equal to K_{pi} upon $1 + sT_i$ square $e^{-\theta_i s}$. So, for all i is equal to 1 to 2, I get two transfer functions. So, this is how the dynamics of the 2 by 2 or 2 input **sorry**, 2 by 2 system can be identified by the dynamics can be by 2 single input, single output sub systems which **which** have got the transfer functions G_1 , **G 1**, $G_{m1}(s)$, and $G_{m2}(s)$.

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Estimation of Model Parameters

Two SISO transfer functions with delay are estimated based on limit cycle data. $\leftarrow (A_1, A_2) \Leftrightarrow h_1 \& h_2$

Let the transfer matrix be $G(s)_{2 \times 2}$

$G_m(s) = \begin{bmatrix} G_{m1}(s) & 0 \\ 0 & G_{m2}(s) \end{bmatrix}$ with diagonal elements 0

$G_{mi}(s) = \frac{K_{pi}}{(1 + sT_i)^2} e^{-\theta_i s}$

$\leftarrow \begin{matrix} K_{p1} \& K_{p2} \\ T_1 \text{ and } T_2 \\ \theta_1 \text{ and } \theta_2 \end{matrix}$

Now, the transfer functions have got the parameters steady state gains given by K_{P1} , and K_{P2} for the two loops, and the time constants given by T_1 and T_2 for the two loops, and the delays given by θ_1 , and θ_2 for the two loops.

So, how many unknowns are there. So, we have got basically 6 unknowns for the dynamics of the, **for the dynamics of the** TITO system. How many measurements we are making, basically we are making the peak amplitudes. So, we are making, so far we have

made two measurements only, now further measurements will be made and that will be, then only it will be possible to estimate 6 unknowns associated with the dynamics of two input, two output system.

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Step1: The steady state gains can be calculated from

$$K_{pi} = \frac{Y_i(0)}{U_i(0)}$$

setting one of the set points at constant value and others at 'zero'

$S_1 = 0, S_2 \neq 0$ $S_1 \neq 0, S_2 = 0$

Step2: From the measurement parameters (A_1, A_2)

four unknowns parameters (T_1, θ_1) of the assumed plant model estimated for any $h_1 / h_2 = 0$

$\frac{h_1}{h_2} \neq 0$ K_{p1}, K_{p2}
 $T_1, T_2, \theta_1, \theta_2$

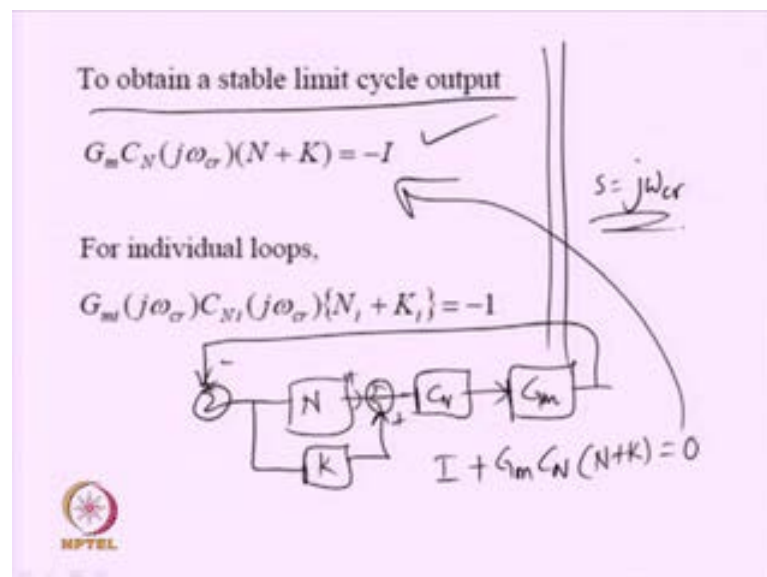
Now, the steps I shall follow for estimation of the parameters are given here. In step one - the steady state gains are calculated using the formula K_{pi} is equal to $Y_i(0)$ upon $U_i(0)$. So, the steady state values of the output signal $y_1(0)$, and $y_2(0)$ are measured, similarly the steady state values for the input signals $U_1(0)$, and $U_2(0)$ are measured. So, how many measurements have been taken so far - 1, 2, 3, 4, and A_1 and A_2 . So, please keep in mind, we are making six measurements now. So, with the help of six measurements outright, we estimate two parameters of the transfer function matrix those are the K_{P1} , and K_{P2} . So, with the measurements of four values now, it is possible to estimate the two steady state gains outright, and...

When you measure the steady the output signals in steady state conditions, please keep in mind set **set** the value for one set point at zero, and the other at non-zero. Then only, you should take these measurements. So, to obtain current correct measurement, what is to be done? Setting one of the set points at constant, setting one of the set values so, either S_1 has to be 0, and S_2 non-zero or S_1 non-zero, and S_2 equal to 0. Then you make the measurements of $Y_1(0)$, $Y_2(0)$, $U_1(0)$, and $U_2(0)$, and subsequently find the gains K_{P1} , and K_{P2} . Now, from the measurements of the peak amplitudes A_1 and A_2 ; and also

the loop frequency ω_{cr} . So, we are making one more measurement keep in mind. So, how many measurements we are making now, we are making seven measurements now. So, these are the four values - steady state values, and two peaks; so, 4, 2, 6 and another one. The frequency of the output signals, and we assume that both the loops have got same frequencies.

So, the both the loops have got same frequency. So, the loop frequencies are assumed to be of same. That means, what with the use of C N 1, and C N 2 we have been successful in eliminating the ill effects of interactions between the loops. Or the loop interactions had been nullified, and subsequently it is possible to get the two loops having outputs with same frequency. So, this is how we make seven measurements, and we are able to now estimate all the parameters associated with the transfer function models. So, with the measurements of A_i , and ω_{cr} four unknown parameters of the assumed plant models can be estimated, provided the ratio h_1 upon h_2 is not equal to 0. That means, both the relays should be present. So, both the relays settings must be non-zero; with non-zero relay settings. It is possible to measure A_1 , A_2 , ω_{cr} , $Y_1(0)$, $y_2(0)$, $U_1(0)$, and $U_2(0)$; and subsequently it is possible to estimate the unknowns $K_P 1$, $K_P 2$, and T_1 , T_2 , and θ_1 , θ_2 . This is how the identification of TITO system is accomplished.

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Now, let us go to little bit of analysis. Now, to obtain a stable limit cycle output definite the **the** loop characteristic equation is to be taken care of, from the loop characteristic equation I can get $G_m C_N(j\omega_{cr}) (N + K) = -I$. How we are getting that analytical expressions, I have got N here in parallel with the K , and I have got C_N here, and I have got G_m . So, this will give a characteristic equation of the form $I + G_m C_N(N + K) = 0$. And using that, I get the analytical expression $G_m C_N(j\omega_{cr}) (N + K) = -I$, when S is substituted by $j\omega_{cr}$. Now, for since we have got two loops, substitution of the diagonal matrices result in...

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To obtain a stable limit cycle output

$$G_m C_N(j\omega_{cr})(N + K) = -I \quad \checkmark$$

For individual loops,

$$G_{m_i}(j\omega_{cr}) C_{N_i}(j\omega_{cr})(N_i + K_i) = -1 \quad \checkmark$$

$s = j\omega_{cr}$

$$G_{m1}(j\omega_{cr}) C_{N_1}(j\omega_{cr})(N_1 + K_1) = -1$$

$$G_{m2}(j\omega_{cr}) C_{N_2}(j\omega_{cr})(N_2 + K_2) = -1$$

NPTTEL

Now $G_{m_i}(j\omega_{cr}) C_{N_i}(j\omega_{cr})(N_i + K_i) = -1$ or minus **sorry**, this will be minus for the individual loops it will be minus 1. That means, I can write this as $G_{m1}(j\omega_{cr}) C_{N1}$. Please keep in mind $j\omega_{cr}$ times $N_1 + K_1$ is equal to minus 1. Similarly, $G_{m2}(j\omega_{cr}) C_{N2}(j\omega_{cr}) N_2 + K_2$ will be equal to minus 1.

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Substituting the values of G_{mi} , N_i , K_i and C_{Ni}

$$\frac{K_{pi} K_{cri} e^{-j(\omega_c T_i + \phi_i)}}{(1 + j\omega_c T_i)^2} = -1$$

where, $K_{cri} = (N_i + K_i)K_{fi} / \cos \phi_i$ and $\phi_i = \tan^{-1} \frac{1}{\omega_c T_{fi}}$

C_N


Now, this can be substitution of G_{mi} , N_i , K_i , and C_{Ni} will result in this expression. Just simply substitute the well the expressions we have for G_{mi} , N_i , K_i , and C_{Ni} - that results in K_{pi} times K_{cri} $e^{-j(\omega_c T_i + \phi_i)}$ upon $1 + j\omega_c T_i$ whole square is equal to minus 1. Now, what is K_{cri} ? K_{cri} is found to be $N_i + K_i$ times k_{fi} upon $\cos \phi_i$, and ϕ_i is given by $\tan^{-1} 1$ upon $\omega_c T_{fi}$. Where from all these things the angles your are getting particularly, because I know that the expressions for C_N . It has got a PI controller, which is given by K_{fi} times $1 + 1$ upon $S T_{fi}$. So, that results in when you express this in the complex form or in **in in** magnitude, and angle form then ϕ_i is obtained.

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Equating magnitudes and phase of LHS and RHS
transfer function model are estimated as

$$T_i = \frac{\sqrt{K_p K_{cr} - 1}}{\omega_\sigma}$$
$$\theta_i = \frac{\pi - \phi_i - 2 \tan^{-1}(\omega_\sigma T_i)}{\omega_\sigma}$$

T_1, T_2
 θ_1, θ_2



Now, equating the magnitudes and phase of left hand side, and right hand side of the transfer function models. Now, if I take the magnitude of left hand side with magnitude of right hand side, and phase of left hand side with phase of right hand side, then that results in the expressions T_i is equal to root of $K_p K_{cr} - 1$ upon ω_σ , and θ_i is equal to $\pi - \phi_i - 2 \tan^{-1}(\omega_\sigma T_i)$ upon ω_σ . So, these are the two formulae associated with the estimation of parameters of the transfer function models, and from these formulae with the substitution of $K_p, K_{cr}, \omega_\sigma$; it is possible to estimate T_1 and T_2 , and from the bottom also it is possible to obtain the values for estimated values for θ_1 , and θ_2 .

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Example 1:

Consider the Wood-Berry binary distillation column process matrix

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{(16.7s+1)} & \frac{-18.9e^{-2s}}{(21s+1)} \\ \frac{6.6e^{-2s}}{(10.9s+1)} & \frac{-19.4e^{-s}}{(14.4s+1)} \end{bmatrix}$$

TITO System

It is a typical TITO process with strong interaction and significant time delays.




Now, I will go to one example. Consider this example, which has got a TITO transfer function matrix of this form. These two shows that, the TITO system has got enough loop interactions. So, the TITO system is subjected to significant or strong interaction between the loops.

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$$C_N = \begin{bmatrix} 0.201 \left(1 + \frac{1}{10.6639s} \right) & 0 \\ 0 & -0.1305 \left(1 + \frac{1}{10.6639s} \right) \end{bmatrix}$$

(10.6639s)


$$q_1(s) = \checkmark$$
$$q_2(s) = \checkmark$$


Now, using the initial controllers C_N of this form, the 2 PI controllers C_{N1} , and C_{N2} of this form, I get the q_1 and q_2 **q 2**. I get obtain the q_1 , and $q_2(s)$ or the models are estimated. So, we will estimate the models.

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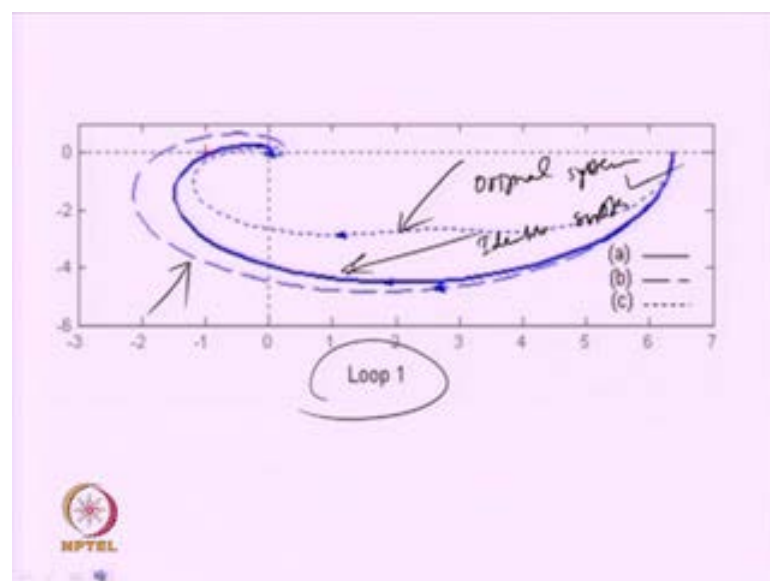
If the process is completely decoupled,
 diagonal elements are written as *TITO System*

$$q_1(s) = G_{11}(s) - \frac{G_{12}G_{21}(s)}{G_{22}(s)} = \frac{12.8e^{-s}}{(16.7s+1)} - \frac{6.43(14.4s+1)e^{-7s}}{(10.9s+1)(21s+1)}$$

$$q_2(s) = G_{22}(s) - \frac{G_{12}G_{21}(s)}{G_{11}(s)} = \frac{-19.4e^{-s}}{(14.4s+1)} + \frac{9.745(16.7s+1)e^{-2s}}{(10.9s+1)(21s+1)}$$


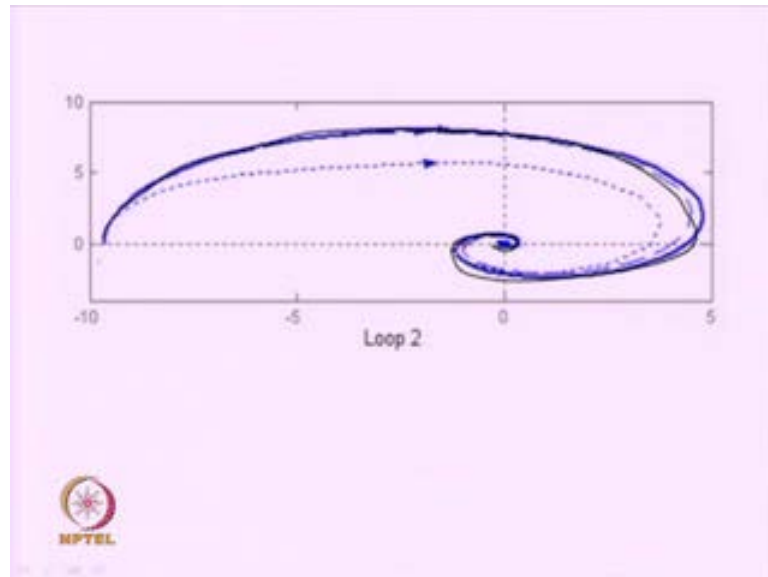
And now, for the original process the analysis gives us q_1 and q_2 of this form. So, simply substitute, we know that q_1 is equal to G_{11} minus $G_{12}G_{21}$ upon G_{22} . So, that will result in this form, and q_2 will give this expression. So, now to compare the efficacy of the identification technique, it has been the $q_1(s)$ and $q_2(s)$ are obtained. This is what we get from the original TITO system dynamics, and we have also identified the system parameter models $q_1(s)$, and $q_2(s)$ based on our technique.

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Now, what are those 2 queues. If you look at this diagram, the **the** dotted one gives you the original system or nyquist **nyquist** plot for the original system. And the solid one for the identified system, and as you see for the loop one, they are not farer from each other. The third one actually shows the nyquist plot given by some other technique. So, ours is definitely the better one, because it is approaching the original systems nyquist plot.

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Similarly, for the loop 2 rather, we have seen that the nyquist plot for the original system dynamics, and the identified system dynamics are very much in agreement. And it shows that, the identified technique or the identification technique is quite efficient.

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Summary

- A method for off-line identification of TITO process dynamics is explained
- Model parameters can be estimated using limit cycle data, steady state output and setpoint values
- Accuracy of identification depends on the efficacy of the modified relay

C₁₁ & C₁₂




Let me summarise the lecture. So, we have described **described** a method for off-line identification of TITO process dynamics. Now, model parameters can be estimated using limit cycle data, steady state outputs, and the set point values. And further accuracy of identification depends on the efficacy of the modified **modified** relay. So, modified efficacy of modified relay means, one must use correct C Ni's or C N 1, and C N 2 appropriate C N - C N 1, and C N- C N 2 to eliminate the effects, ill effects of loop interactions during relay experiment.

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Points to ponder

P.1 : How to reduce the loop interactions during the identification tests?

Modified Relay
Output filters ←



Now, how to reduce the loop interactions during the identification techniques, one way is that by employing a modified relay, as we have discussed here. So, you can use modified relay to eliminate the loop interactions between the two loops or you can use some output filters, **output filters**. That can also enable us to reduce or minimize the interactions between two loops. That is all in this lecture.