Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 13

Identification of Underdamped Systems

Welcome to the lecture titled identification of underdamped systems. Lots of systems, subsystems, processes, components, and devices, in industries are found to posses underdamped characteristics. That is why there is a need for delivering a separate lecture on identification of Underdamped systems.

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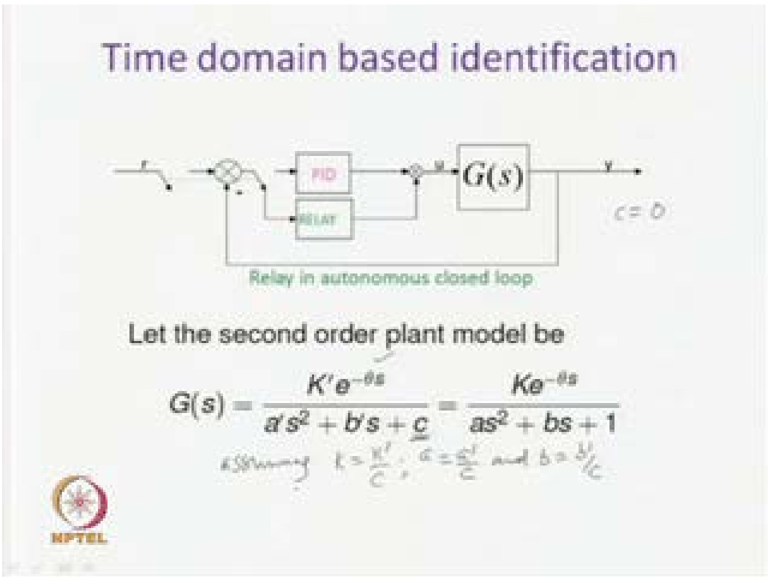

But how can we represent the dynamics of in Underdamped system? Often, it is given in the form of a transfer function model as G(s) is equal to K dash e to power minus theta s upon a dash s square plus b dash s plus c, which again can be converted to the form of K e to power minus theta s upon a s square plus b s plus 1. Assuming, assuming K is equal to K dash by c, a is equal to a dash by c, and b is equal to b dash by c, why we are we expressed the transfer function model in this form, because earlier we have found the transfer function models in this typical form. That is, why allow me to express the transfer function model in this from where the last term of the denominator is having one. Please, keep in mind the term in the denominator is having one in it. So, to obtain similar type of transfer function model allow me to assume K, a, and b in this form. That enables me to write the transfer function model as G(s) is equal to K e to the power minus theta s upon a s square plus b s plus 1.

Now, with various values of c, if I choose c equal to 0. Then, what happens? Then, the denominator this term will be 0 resulting in a transfer function model known as an integrating model or the transfer function for an integrating process. So, also when I choose c as negative value or positive value depending on the value of c. It is possible to express the second order plant model as the model for an unstable or for an stable for a stable process. So, those things are very easy, that you can easily make out now when the second order plant model is given in this form.

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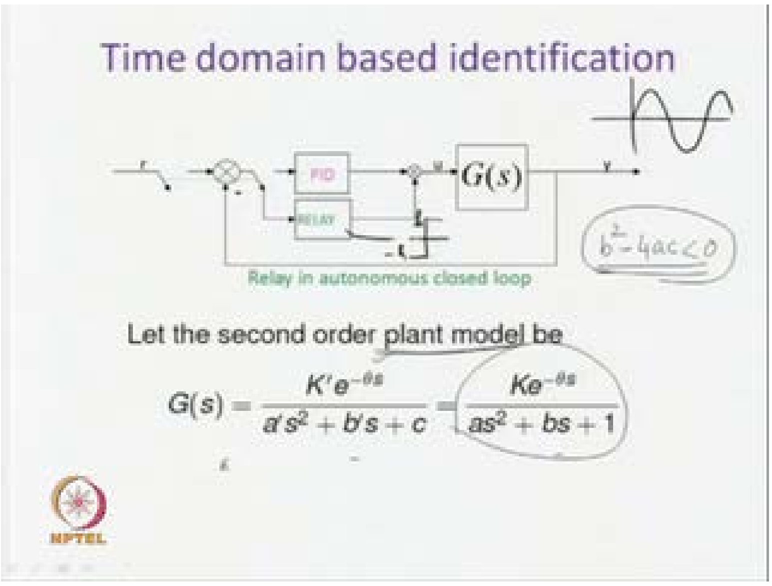

And when further assumption like b square minus four a c is less than 0 is made. Then only, this transfer function model or model plant model, transfer function is said to have underdamped characteristics, please keep in mind only when b square minus four a c is less than 0. Then only, the plant model becomes a plant model for an Underdamped system otherwise it is not.

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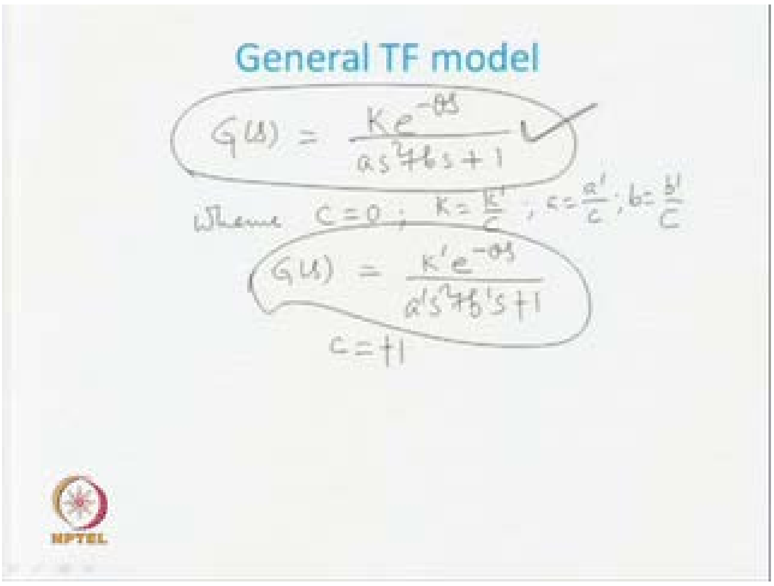

So, I will go ahead with the further analysis; the generality of the transfer function model can be explained by this fact that, when G(s) is given in the form of K e to the power minus theta s upon a s square plus b s plus 1, and various value of c are chosen, when c when c equal to 0 we do not say here, but as you know k is equal to k dash by c, a is equal to a dash by c, and b is equal to b dash by c. So, c is inherent in this in this coefficients now when c equal to 0. It is possible to get G(s) expressed as some form, which will give you the transfer function model for and integrating process given as k dash e to the power minus theta s a dash s square plus b dash s. Now, when c assumes negative value c equal to minus 1; similarly, I can get a minus 1 here, when c equal to plus 1, I get here plus 1. So, that is how we get a varieties of transfer function models; from this general transfer function model that is, why I say that this G(s) possesses some generality.

Now, what benefit you get from this that whatever expressions we will discuss can easily be extended for identifying a varieties of process dynamics. The process could can posses integrating characteristics can posses unstable dynamics still, the equations can be employed to find different type of transfer function. Now, we will assume that the relay in the relay control or closed loop system is an ideal relay. So, the relay characteristics will be given by h, and minus h. Then the output of the relay control system will be symmetrical output. So, for this analysis particularly, we will not assume or take any asymmetrical relay rather a symmetrical relay will be considered for each in analysis of this Underdamped systems.

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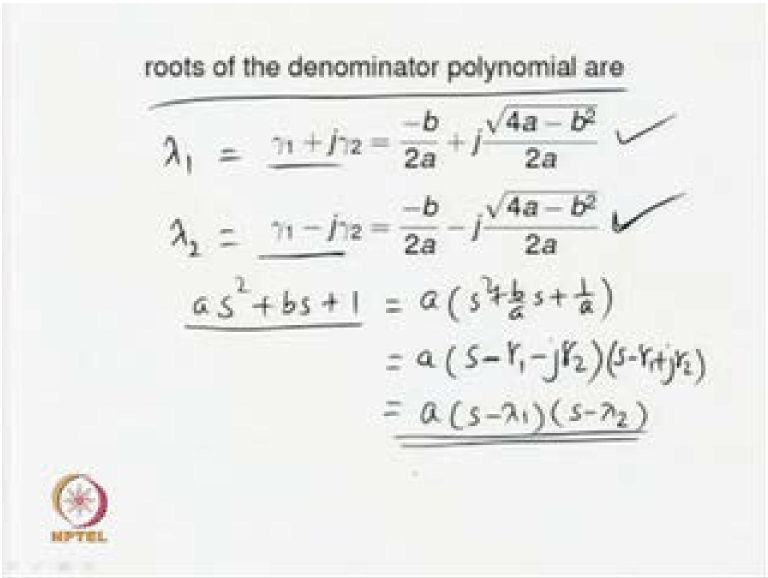

Now, when the denominator polynomial is considered, what is the denominator polynomial? That is a s square plus b s plus 1 this is the denominator polynomial. This can be expressed further, as a times s square plus by b by a s plus 1 upon a. Now, this can further be expressed as a times s minus gamma 1 minus j gamma 2 times s minus gamma 1 plus j gamma 2. I can always factorized this in this form, because using the roots of the denominator polynomial given by gamma 1 plus j gamma 2 is equal to minus b upon 2 a plus j times root of 4 a minus b square upon 2 a. This is the way, we find the root of a characteristic equation. So, the first root is like this; then it is conjugates can be given in the form of gamma 1 minus j gamma 2 is equal to minus b upon 2 a minus j root of four a minus b square upon 2 a.

So, when the two roots are assumed in this form, and subsequently, when you allow me to write the roots in the form of some variable lambda 1; lambda 1 is equal to gamma 1 plus j gamma 2, and lambda 2 is equal to gamma 1 minus j gamma 2. Then, the polynomial can further be written in the form a s minus gamma 1 s minus sorry s minus lambda 1 times s minus lambda 2. So, there are various ways the polynomial can be expressed. Now, why we are doing this? So, that the state space equation of the dynamics second order Underdamped system model can be obtained easily, when the polynomial is available in this form? It is very easy to find the state space representation of the dynamics.

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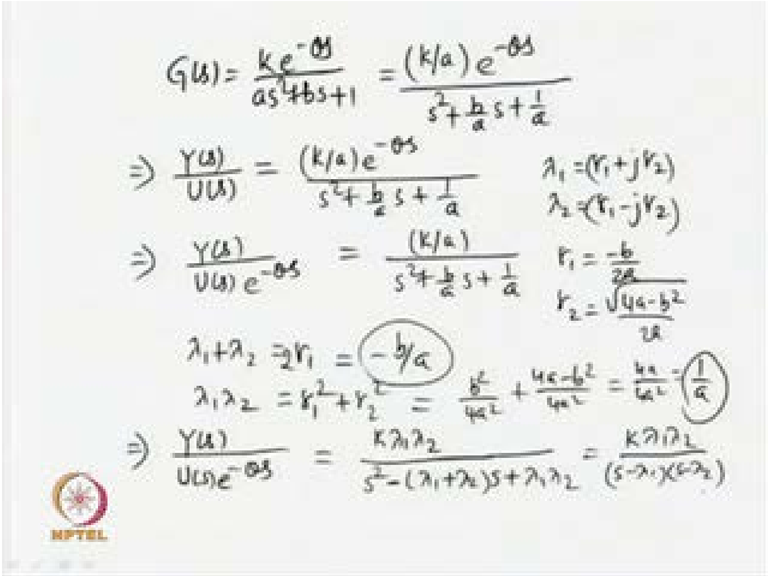

Now, the roots of the denominator polynomial, I have already given. Now, allow me to write the transfer function now G(s), which is nothing, but K e to power minus theta s upon a s square plus b s plus 1 can also be written in the form of k divided by a times e to power minus theta s. In the denominator we will have s square plus b by a s plus 1 upon a by writing so, what we get what is Y(s) sorry what is G(s) is nothing, but Y(s) upon U(s) the process the ratio of the Laplace transform of the process output to the process, input is given as now K by a times e to power minus theta s upon s square plus b by a s plus 1 by a why we are doing? So, to find the state space representation for the dynamics, why it is again important for us? Unless, we find the state space representation; then we may not be able to develop or describe the relevant expressions necessary to identify the model parameters of a of an underdamped second order plus dead time dynamics.

Now, again this can be written further in the form of Y(s) upon U(s) e to the power minus theta s is equal to K divided by a s square plus b by a s plus 1 by a. Now, already we have introduced lambda 1, and lambda 2, what is lambda 1? lambda 1 is equal to gamma 1 plus j gamma 2, and lambda 2, is equal to gamma 1 minus j gamma 2 where gamma 1 is equal to how much minus b by 2 a, and gamma 2 is equal to 4 a minus b square root by 2 a, these things already, I have explained. So, when these things are used here. I can write b by a, and these forms in the form of lambda 1 plus lambda 2 will be equal to now 2 gamma 1. So, 2 gamma 1 is nothing, but yes say gamma 1 is this 1. So, 2 gamma 1 will be minus b by a. Similarly, lambda 1 times lambda 2 will be equal to gamma 1 square then plus gamma 2 square multiply the 2 lambdas lambda 1, and lambda 2. Then, you will get gamma 1 square plus gamma 2 square, which will give us b square by 4 a square again, plus 4 a minus b square upon 4 a square giving ultimately 4 a upon 4 a square, which is nothing, but 1 upon a. So, I can write the denominator. Now, in the form of either b by a or 1 by a or I mean now, the same expression can be rewritten as y s upon u s times e to power minus theta s is equal to k times lambda 1 lambda 2 in the numerator. Please, keep in mind 1 upon a is equal to lambda 1 lambda 2 therefore, I can write k divided by a as k lambda 1 lambda 2. Similarly, in the denominator I get s square plus I have got this 1. So, I will get minus here. So, s square minus lambda 1 plus lambda 2 times s plus lambda 1 lambda 2. So, basically what you have got now k lambda 1 lambda 2 divided by s minus lambda 1 times s minus lambda 2. So, this is how we get the transfer function expressed in this form.

Now, this transfer function can further be written in the form of this, I will rewrite this derive the state equation using the basic principles. So, using the basic principle, how can we obtain the transfer function sorry a states space representation of the underdamped process dynamics.

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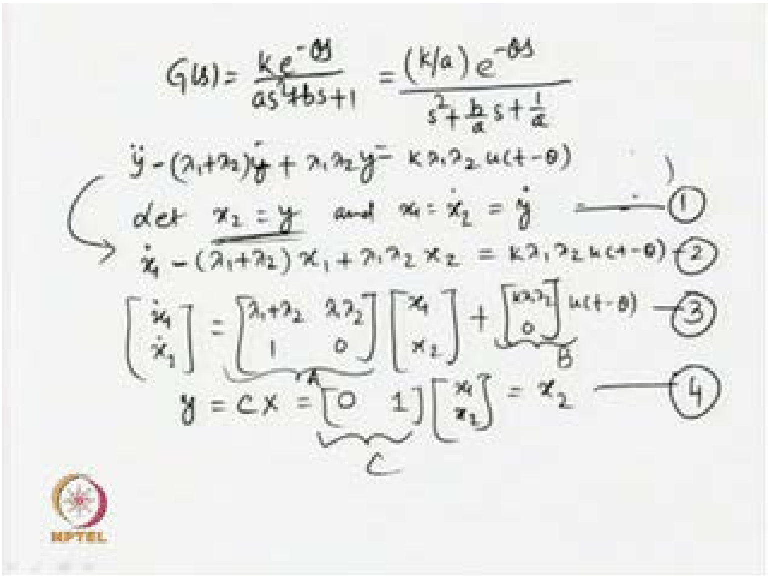

Now, then cross multiply, and take inverse Laplace transform that will give us e by cross multiply these terms like this. Then I will get, this as y double dot minus lambda 1 plus lambda 2 times y dot plus lambda 1 lambda 2 y is equal to k lambda 1 lambda 2 u t minus theta. So, this is the linear equation we have got the differential equation; we have got for the dynamics of the underdamped plant. Now, let me introduce the state variables let x 2 equal to y, and x 1 equal to x 2 dot first order derivative of x 2 is equal to y dot. That enables me to write this in the form. A state variable as x 1 dot minus lambda 1 plus lambda 2 x 1 plus lambda 1 lambda 2 x 2 is equal to a is sorry is equal to k times lambda 1 lambda 2 u t minus theta. Now, let this be equation number 1, and this be 2; 1 and 2, can be used to get the state equation now. So, allow me to write that further x 1 dot x 2 dot is equal to now x 1 x 2 plus further input that, we will have u t minus theta.

So, this the way one can write the state equation. Now x 1 dot has got lambda 1 plus lambda 2 here times x 1, then lambda 1 lambda 2 here, then I have got K lambda 1 lambda 2, and whatever the x 2 dot is equal to x 1 only. Therefore, one it will be 0, and it will be 0; this is, how we develop the state equation for the underdamped dynamics. And the output equation can be given out right in the form y equal to C x as 0 1 x 1 x 2. So, when you multiply you ultimately get y equal to x 2, and that is what we have assumed earlier. So, we have introduced the state variable x 2 as y. So, this is how we develop the state equations, and then the state equation constants, now are the A matrix is given by this the B vector or matrix is given by this and the C is given by this.

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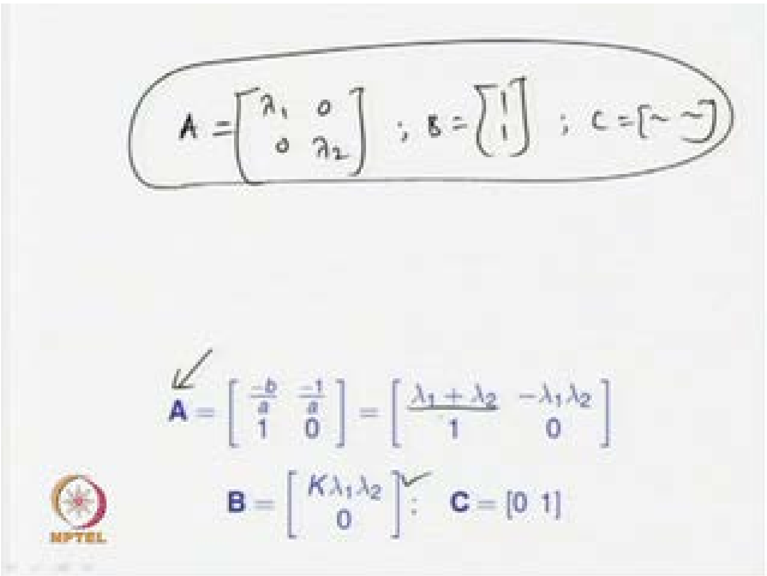

So, that is what we have got here. So, a can be the A matrix corresponding to the state equation can be given by lambda 1 plus lambda 2 minus lambda 1 lambda 2. sorry I have If I take this to this side it will be minus sorry; obviously, because x 1 dot is equal to lambda 1 plus lambda 2 x 2 minus lambda 1 lambda 2 x 2. So, this way we will get this manner. That is, how the a constant is found to be having components or elements lambda 1 plus lambda 2 minus lambda 1 lambda 2 1 0, and that is, what we have obtained lambda 1 plus lambda 2 minus lambda 1 lambda 2 1 0, and similarly, the B matrix is having elements K lambda 1 lambda 2, and 0. Thus it has got k lambda 1 lambda 2, and 0, and C is given by 0 1 C is given by 0 1. So, why I am doing all these analysis, when we get the state, and output equation expressed in some convenient from the powerful the set of powerful equations. We have derived earlier for identification of second order plus dead time transfer functions or transfer function models can easily be extended to the underdamped second order plus dead time dynamics. So, the earlier we had got for the earlier cases for the second order plus dead time system transfer function models, whatever our A was lambda 1 0 0 lambda 2, if you recall this. Then B was equal to 1 1, and C. We will having some different form there, where two elements these where the form now in place of the standard A B C forms we have been using during last few lectures; allow me to get the A B C in this form for identification of Underdamp systems.

Now, A is having the elements given in this form non-diagonal form, and there is no problem it is not absolutely not necessary to find A B C constants in diagonal or some other form, it could be available in any form. Now, when A B C are found in this fashion then subsequently; we can make use of these constants, and the state, and output equation, for deriving analytical expressions necessary for identification of the parameters of the second order plus dead time model.

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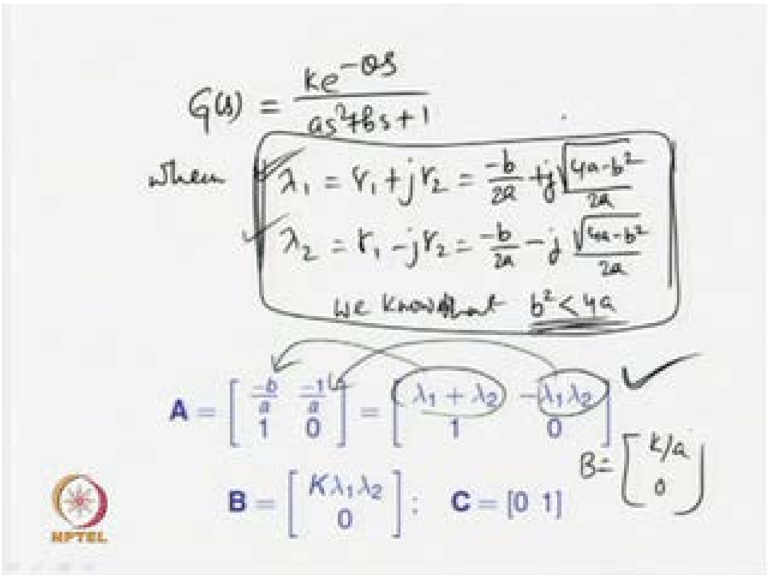

Let me, repeat how we have been able to find the state equations which has got constants A, B, and C, given G(s) in the form of k e to the power minus theta s a s square plus b s plus 1, when lambda 1 is equal to gamma 1 plus j gamma 2 is equal to minus b upon 2 a plus root of 4 a minus b square by 2 a, and lambda 2 is equal to gamma 1 minus j gamma 2 is equal to minus b upon 2 a minus j root of 4 a minus b square by 2 a. This is, you should not forget, because we know that we know that for underdamped dynamics we know that for underdamped dynamics b square is less than 4 a b square is less than 4 a therefore, 4 a minus b square will be positive, and therefore, j will appear over here or if you use when this is satisfied. Then the complex variable j will come into picture. So, please keep in mind, that for that Underdamped system dynamics b square is less than 4 a, and that result in the two type of variables lambda 1, and the lambda 2 in the form of complex numbers given by gamma 1 plus j gamma 2, and gamma 1 minus j gamma 2. If I have forgotten that; let me, again rewrite the gammas are yes j's are there must be their otherwise we get incorrect expressions.

So, only when lambdas, and gammas, are taking these form. Then, only A, B, and C, can be expressed in these forms. That is very important further, A can been written in the form of the A parameters of the dynamic model as lambda 1 plus lambda 2 can be expressed as minus b by a lambda 1 lambda 2 by 1 upon a. Therefore, here you will get as this B can be further written as K by a 0. So, these things you should not confused, because the B can be written either in terms of the constants of the transfer function model or in terms of the new variables we have introduced the lambdas or the gammas.

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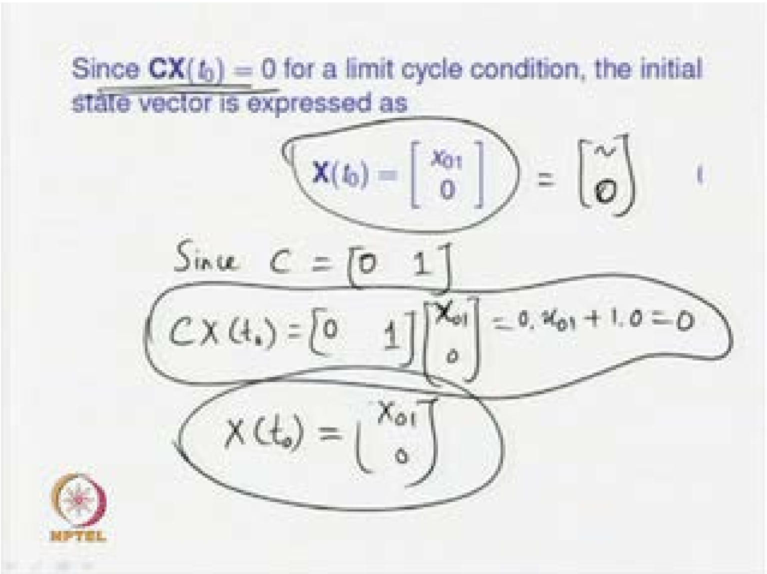

Now since c x t 0 has to be 0 for a limit cycle condition; that means, for to induce limit cycle for obtaining sustained oscillatory output this condition has to be made. So, when c X t 0 equal to 0, the initial state vector has to be given in some form as shown over here; then X t 0 has to be given as X 0 1 which is a non 0 value with zero element at the bottom, why that is so, since C is having the form 0 1; this vector having elements 0 and

1. Then C x t 0 will be equal to 0 1 times X 0 1, and 0 is equal to 0 times X 0 1 plus 0 times 0 is equal to 0. So, for obtaining C X t 0 equal to 0 X t 0 has to be made available in this form. this point is very important you cannot arbitrarily choose the form of X t 0; X t 0 must have some non 0 element in the offer, and some 0 element this must be 0 at the bottom, and the offer cannot be also 0; in that case, you do not get anything from the analytical expressions.

So, the form of X t 0 is chosen corresponding to the set of the constants A, B, and C. So, X t 0 will be given in the form of X 0 1, and 0. Please, keep in mind you must choose X t 0 in this form; otherwise, the analysis will result in faulty expressions or the erroneous expressions. Now, when a symmetrical or ideal relay is employed to conduct the relay test for obtaining sustained oscillatory output.

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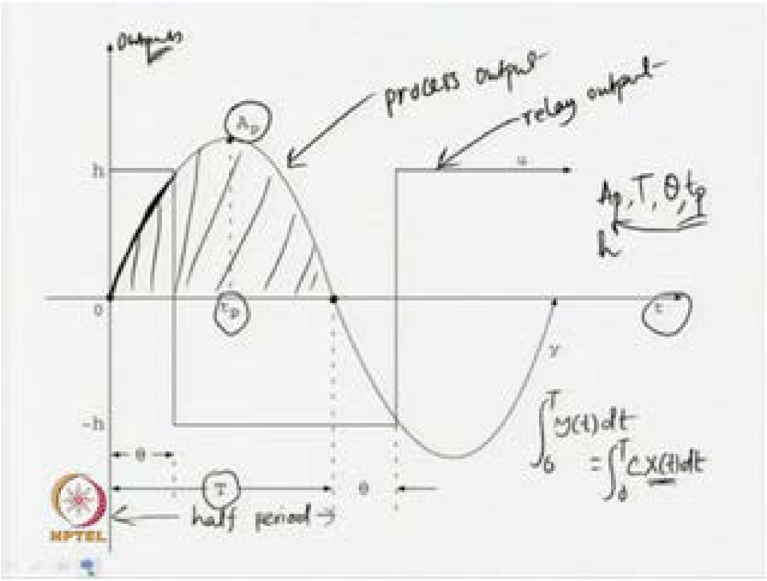

Then a typical output of this form can be obtained. Then, we have got the time access, and this is. You have got different type of outputs, what outputs we have shown here in this diagram here we have got the process output subjected to relay feedback. This is the process output, and what about this one? This is the relay output. So, corresponding to the symmetrical or ideal relay, what you have got? You have got some symmetrical process output, and symmetrical relay output. So, the process output is now having different parameters, the peak amplitude given by the variable A p which occurs at time T equal to t p, and the relay output has a (( )) relay as you know the process is subjected to (( )) input. So, that way the (( )) input or the relay output is given in this form when this is the this occurs the relay setting changes it is value after time T equal to theta, and the half period half period of the output half period of the output is shown by capital T. So, I will use this variable T for half period of the output signal.

So, the variables we have here or A p the peak amplitude half period of the output given by T, and all other parameters edge is the relay setting, and theta; theta is the delay associated with the plant dynamics, and t p; t p is the time at which the peak amplitude of the output occurs. Now, we will go to the further analysis the area of the process output over the half period. Please look at the half period. So, half period spends from time T equal to 0 to time T equal to capital T. So, this is our half period, then what will be the area of the output signal this area of the output signal i can write an expression of the form limit from 0 to T y t dt what is y t? Again y t equal to C X t. So, this can be further be written as integration from 0 to T C X t dt.

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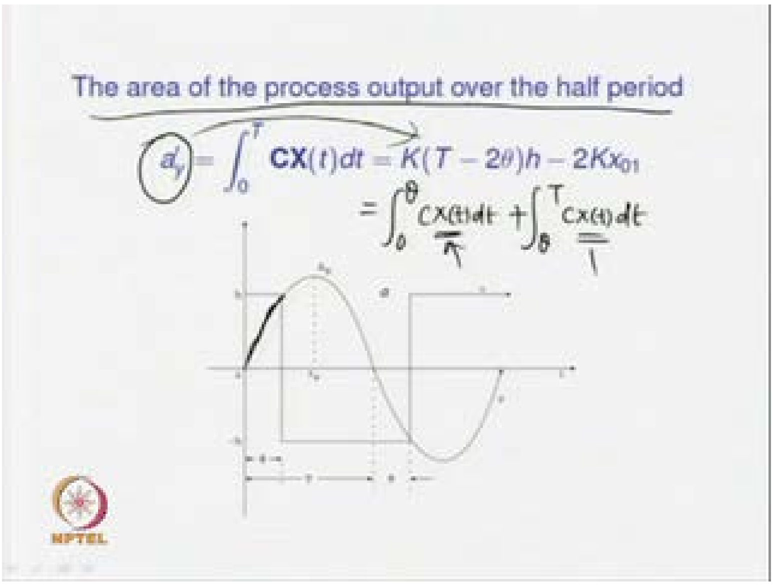

So, we can have expression for X t for different span of time. So, as you know I can find from wave form for this part of the wave form the corresponding output can be obtained using expression C X t, where X t can be given as you know, if the output is like this is the output half period of the output will consider, and relay switching takes place

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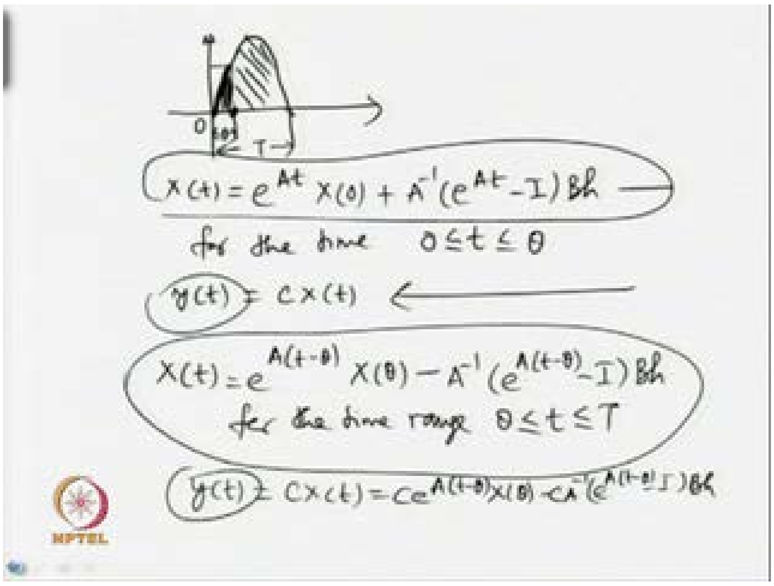

Here suppose, then in that case, we will have two segments one expression for this part of the output which has got the state equation of the form X(t) is equal to e to the power A t minus 0 X either 0 or t 0 x 0 plus A inverse e to the power A t minus theta minus I B h for the time 0 is less than equal to t is less than equal to theta. So, till time t equal to theta this is equal to theta. Please see, the way I have shown the theta. So, for this time we have got the state equation for that part of the output given by this then the output expression for the output will be simples y(t) equal to C X(t).

So, this is how the expression for output can be obtained or indirectly speaking, I can make use of X t in that expression to find the area, which area now this part of the area, what about the remaining part for this half period area for this part can be obtained provided we have got an expression for the state. And expression for the state can be given as X t is equal to e to the power A t minus theta X theta; please, keep in mind the change I am introducing here minus A inverse why this minus sign is coming the relay switching is taking place the input or the relay output or the relay output or the process input is going to be negative, then minus A inverse e to the power A t minus theta minus I B h. So, this will be the expression for the time range theta is less than equal to t is less than equal to T.

So, for this span from theta time till T, this is your till T. So, from time t equal to theta to time t equal to t for this part of the output the state equation is given by this. And once you get the expression for the state equation the output expression can easily be obtained as y(t) equal to C X t simply. So, multiply C over here, and if that the expression for output y t equal to c e to power A t minus theta x theta minus C A inverse e to the power A t minus theta minus I B h. So, use this y(t), and y(t) for different span for obtaining expression for the area.

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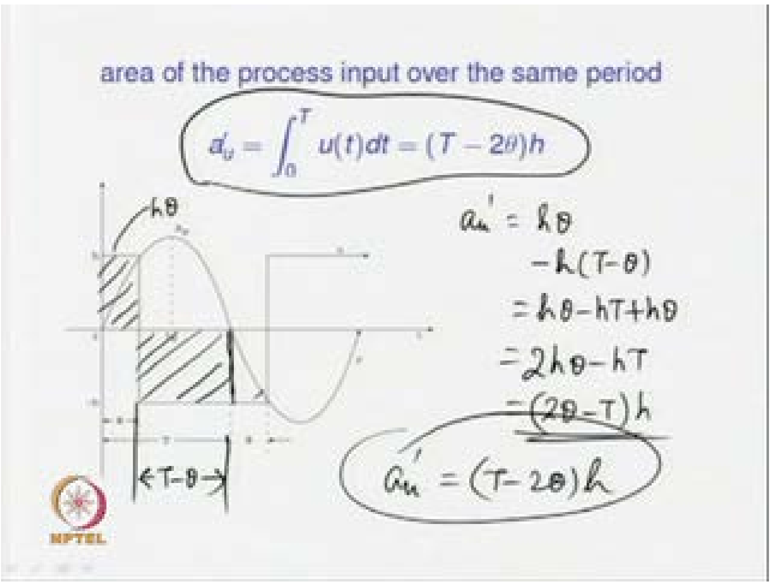

So, a u dash, a u area of the output signal or output over a half period of time denoted by a u dash is given by a u dash is equal to integration from 0 to T C X t dt is equal to integration from 0 to theta C X t dt plus integration from theta to T C X t dt, where use- please use correct expression for X t, and X t for different time ranges. Then upon simplification, it is not difficult to get an expression of the form a u dash is equal to K times t minus 2 theta times h minus 2 K X 0 1. So, this detail derivation I am not going to repeat here or give here. Obviously, if you substitute correct expression for x t, and x t for different time ranges you are going to get the expression for the output of the half period as a y dash is equal to this much.

Now, we shall see the area of the input signal now the area of the input signal can easily be obtained, how can you obtain area of the... what is the input signal the input signal is sorry this one. So, then this input signal area of this part will be how much simply h times this span height and length. So, that way this part will have area as theta what about the area of the bottom part of the output signal. This will be how much is this one this span is nothing, but if this is T minus theta.

So, sorry not up to this one this span is T minus theta therefore, this area please do not consider we are considering half period the area for the half period, we will have two pluses this upper one, and this bottom one. Therefore, this area can be found further h or the (( )) minus h, and the span is T minus theta. So, the upper area is area a u dash can be obtained as h theta minus h times T minus theta. So, please simplify this one h theta minus h T plus h theta. So, that gives you 2 h theta minus h T, and if you take h as common 2 theta minus T times h. Now, I have got an expression in different form, now the area of the output signal or process input signal or relay output signal is given by T minus 2 theta times h or h times 2 theta minus T.

Only sign change is their why sign change is their; when the output is positive when the system output or the sustained oscillatory output is positive? If you carefully, observe the block diagram the of relay control system, the input relay output or input to the system becomes negative. Therefore, we have to consider the negative of this one, I believe you are following; what I mean by that let me go back to that structure. So, when this becomes positive when y is positive, what will happen due to this negative feedback. So, your relay will be subjected to negative input, and the output of the relay, will be negative. So, when y is positive, you are getting some positive area; you will get some negative area given by the relay output, and that is where the correct expression for the output corresponding to a u dash will be T minus 2 theta times h; in place of 2 theta minus T times h. So, finally, allow me to write a u dash is equal to T minus 2 theta times h. So, this is how we get the areas of output and input signals over a half period of time

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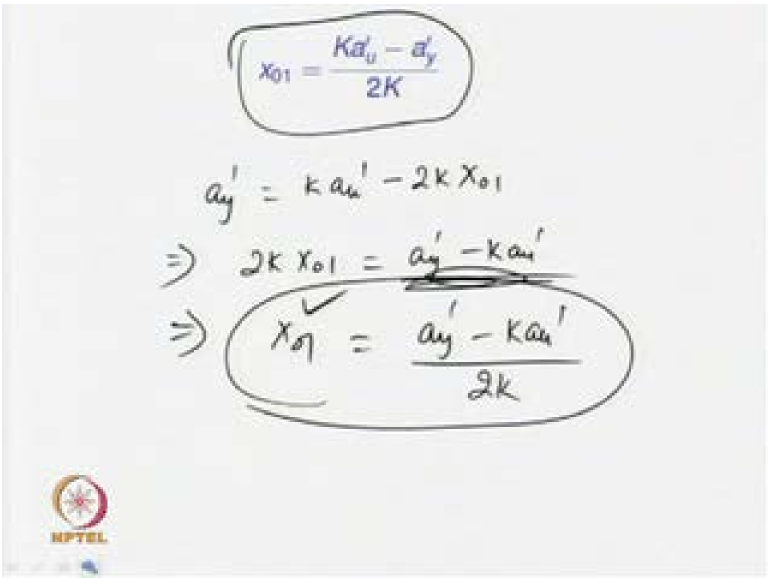

Now, what information we get from that can be used to get an expression of the form, X 0 1 is equal to K times a u dash minus a u dash upon 2 K. Since, you have got a u dash h t minus 2 theta h, and a u dash a u dash is K times t minus 2 theta h minus 2 K X 0 1. So, a y dash is equal to basically it is nothing, but k times a u dash minus 2 X 0 1 which upon simplification will give you 2 K X 0 1 is equal to a y dash minus K a u dash, and further, it will give X 0 1 is equal to a u dash minus K a u dash upon 2 K. So, when why we are doing all these analysis. So, when a y dash a u dash can be obtained from the output signals. Then, we can estimate X 0 1 conveniently. And this can be used to estimate this expression can be use to estimate one parameter associated with the transfer function model.

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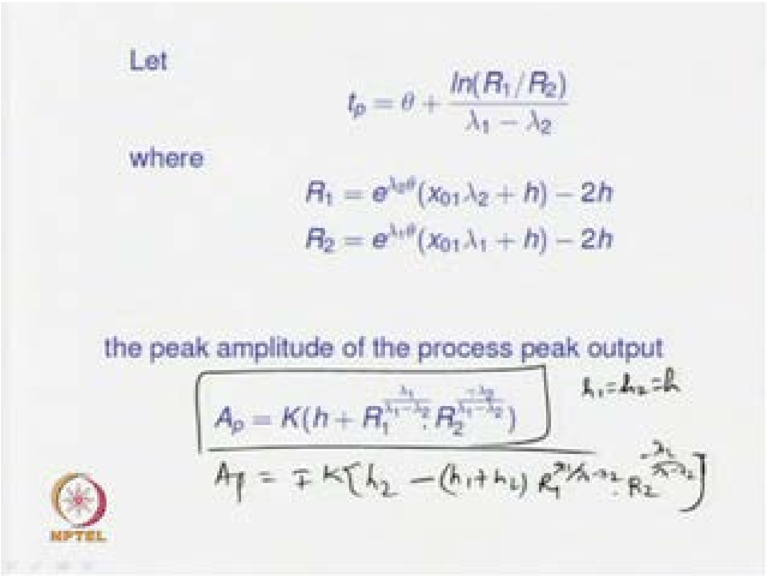

Now, I will go to other expressions, which are presented out right; I am not repeating the detailed explanation or the expressions required to find the final expressions or what I mean by this? Already, we have presented the derivation for these expressions in the form of A p is having A p was earlier having minus plus k times h 2 minus, then your h 1 plus h 2 times R 1 to the power lambda 1 lambda 1 minus lambda 2 times R 2 to the power minus lambda 2 lambda 1 minus lambda 2. If you do if you have not forgotten please keep in mind already, we have given some general expressions or we have derived the general expressions for peak amplitude of the output signal for general second order plus dead time transfer function model.

So, when h 1 equal to h 2 is equal to h. Then, it is not difficult to get the x same expression expressed in the form of A p is equal to k times h plus R 1 times lambda 1 R 1 to the power lambda 1 divided by lambda 1 minus lambda 2 times R 2 to the power minus lambda 2 divided by lambda 1 minus lambda 2.

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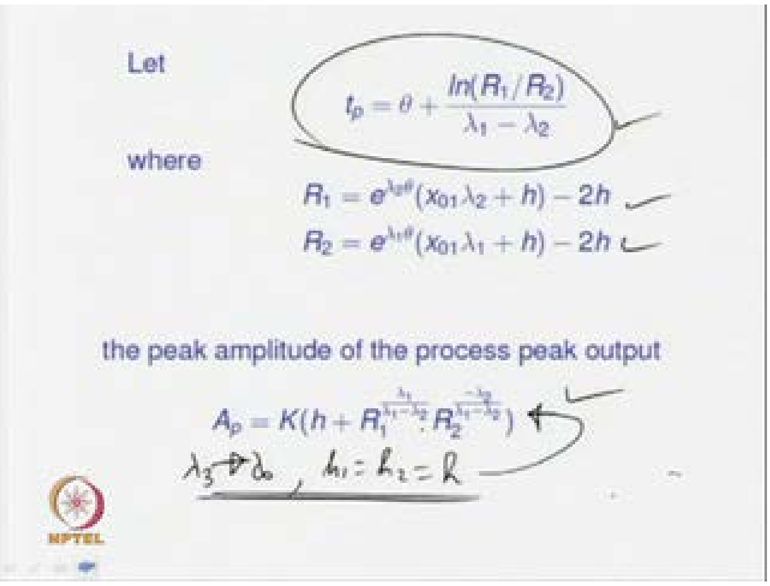

So, with the limiting values of with the limiting values of lambda 3 tends to infinity, and h 1 is equal to h 2 equal to h. It is possible to find the expression for the peak amplitude as given over here. And what are those R 1, and R 2? Now corresponding to this limited limiting values R 1 will be e to power lambda 2 theta times x 0 1 lambda 2 plus h minus 2 h, and R 2 is equal to e to the power lambda 1 theta x 0 1 lambda 1 plus h minus 2 h, and corresponding t p expression for the t p the time at which peak amplitude occurs can be given as theta plus (( )) of R 1 upon R 2 by lambda 1 minus lambda 2. Now, these expressions will be used out right to find analytical expressions for underdamped second order plus dead time dynamics.

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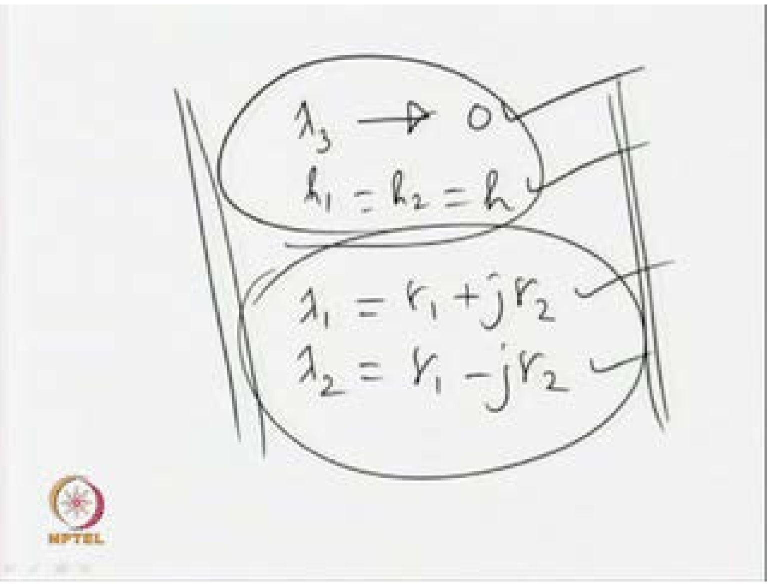

So, here what assumption you have to make lambda 3 tends to 0, and h 1 equal to h 2 equal to h, with those assumptions the peak time can be expressed as t p is equal to theta plus 1 upon gamma 2, where from gamma 2 came again. You need to substitute lambda 1 is equal to gamma 1 plus j gamma 2, and lambda 2 is equal to gamma 1 minus j gamma 2. So, with these substitutions the general expressions we have obtained.

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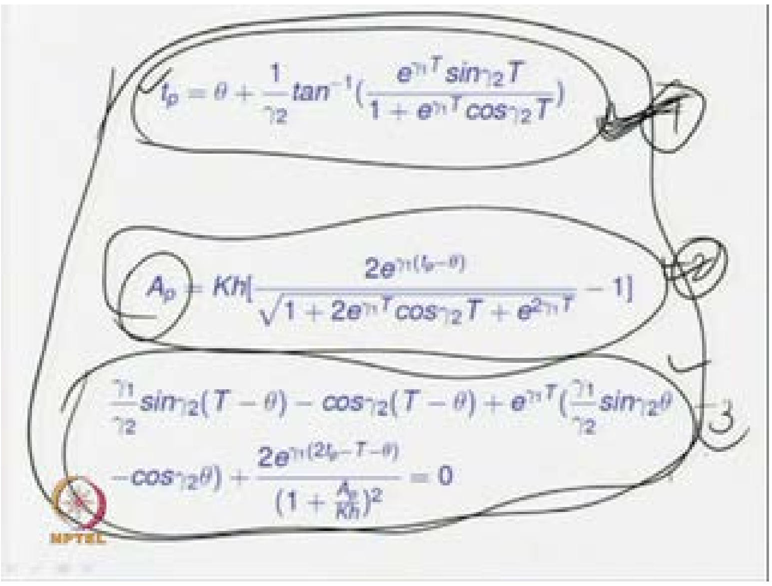

We have found earlier can be extended to find expressions for the t p time at which the peak occurs as t p is equal to theta plus 1 upon gamma 2 tan inverse of e to the power gamma 1 T sign of gamma 2 T divided by 1 plus e to power gamma 1 T cos gamma 2 T. Please, do not fear, because this expression can easily be obtained from the general expressions we have derived for t p, A p, and So on. Similarly A p can be found to be of this form, it appears to be very difficult, but the bottom one is the input and one for us. So, finally, the A p expression for A p can be like this, and the zero crossings will result in an expression for of this form. So, these are the three input, and expressions, these are the three input and expressions for us. Those can be used to identify the parameters of the transfer function model.

Now, how do we get 1, 2, and 3, again I am repeating when you substitute lambda 3 times to 0, h 1 equal to h 2 equal to h, gamma 1 equal to gamma sorry lambda 1 equal to gamma 1 plus j gamma 2, and gamma 2 is equal to gamma 1 minus j gamma 2. In those powerful expressions we have derived earlier for general second order plus dead time transfer function model. Then we can easily get t p expression for t p expression for A p, and for the zero crossings, as shown over here. So, please you can try and derive with these are not difficult only you need to substitute the limiting values as I have told only you need to substitute the limiting values in those expressions, and you do find this simpler 1, 2, and 3. Those can be used for identification of underdamped second order plus dead time transfer functions model.

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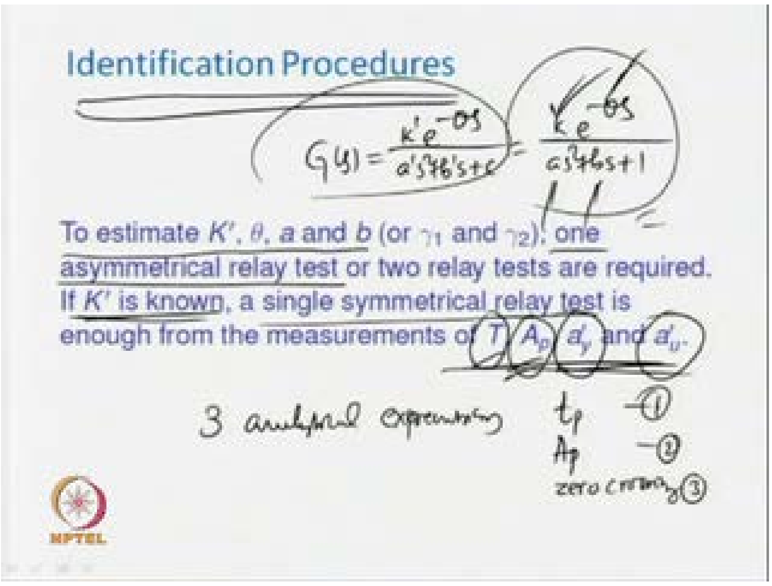

So, with this I will go to the simulations examples what could be the identification procedure how to identify the parameters of the transfer function model G(s) given by K dash e to the power minus theta s upon a dash s square b dash s plus c is equal to K e to power minus theta s upon a s square plus b s plus 1. So, the transfer function model has got how many unknowns? 4 unknowns, but we have derived three analytical expressions we have derived three analytical expressions of course, we are making two more measurements: the areas of the output signal, and the areas of the input signal. Now, to estimates K dash theta a, and b K dash theta a, and b or a dash, and a dash, and K dash theta a, and b, or indirectly speaking c is also inherent in K dash. So, indirectly speaking you are measuring either K dash, theta a, dash b, dash c or K, theta, a, b to estimate all those one asymmetrical relay test might be required.

Then, you will you can make more measurements, in place of the peak amplitude one peak amplitude, you will have two peak amplitudes, and you will have more number of expressions, which can be solved simultaneously to identify more number of unknowns associated with the transfer function or when a single symmetrical relay test is conducted. In that case, you have to make measurements like the half period the peak amplitude area of the output signal, and area of the input signal, over a half period. And then assuming k dash to be known, it is possible to estimate all the parameters associated with the transfer function model. We will go to one simulation example: now, because if you use this I mean, we are we are making, we have got three analytical expressions keep in mind we have got three analytical expression what are those three analytical expressions? One is for t p 1 is the peak value; A p, and another for the zero crossing another for the zero crossing.

So, we have got three analytical expressions 1, 2, and 3, and when we make additional measurements like the areas. In that case, it will be possible to estimate four unknowns, and we have got four unknowns in the transfer function model, what are those K, theta, a, and b. So, it is not difficult to estimate all the unknowns associated with the underdamped transfer function model with the measurements of a u dash, a u dash, or the areas of the half period, and T A p, and T A p.

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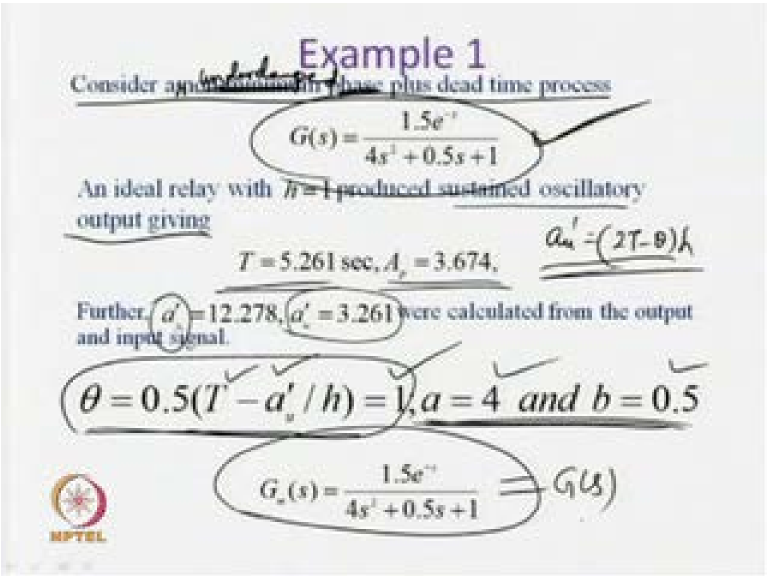

Now let us, consider a non minimum phase plus dead time process given by sorry this is non-minimum phase not consider, and underdamped plus dead time process model of this form. Then an ideal relay with h equal to 1 produces sustained oscillatory output giving the half period h 5.261 seconds, and A p as 3.674. This is the gain unit less for the time being, now further the half areas of the half periods output signal a y dash is equal to 12.278, and area of the input signal over a half period, over the same half period as a u dash is equal to 3.261. Then, we can develop an expression involving the dead time in the form of theta is equal to 0.5 of t minus a u dash upon h.

Because if you look at carefully, the expressions for a u dash; a u dash is given by 2 T minus theta times h. So, using that, I can easily write an expression for theta, and since T is measured a u dash is obtained or measured; then it is directly it is directly possible to obtain the estimation for the time delay theta. So, theta is found to be one or estimated to be one similarly using two analytical expressions now a and b can be obtained, a and b are estimated to be 4, and 0.5 therefore, the transfer function model we obtained for the Underdamped system dynamics is found to be quiet accurate, and the model is very much equal to the original system dynamics. This is how we see the efficacy of the expressions; we have derived for estimating the dynamics of Underdamped system.

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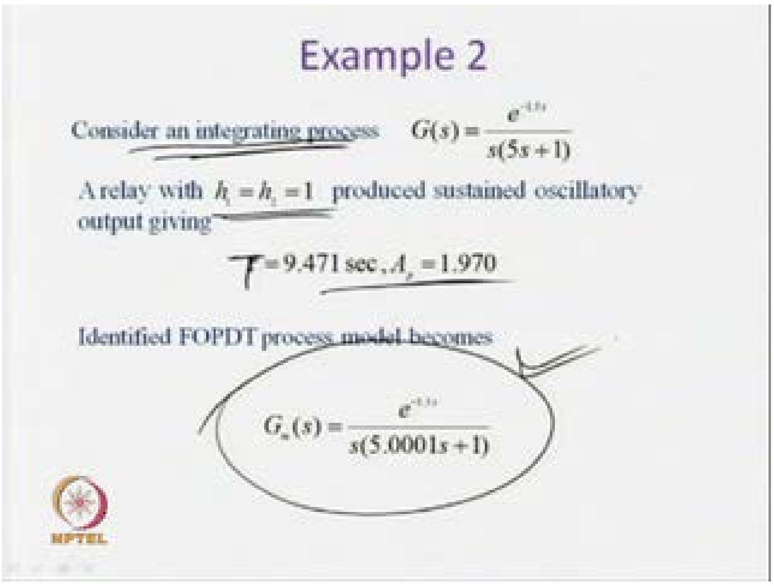

Now, let us go to second example: where we consider an integrating process also a relay with again h 1 equal to h 2 is equal to one produces the parameters T is equal to 9.471 seconds, and A p is this much, and using the set of analytical expressions. I have derived it is possible to identify the integrating process model, also identify the integrating process dynamics which model is given by now G m(s) is equal to e to power minus 1.5 s upon s times 5.001 s plus 1. So, the accuracy is very high to within 10 to the power minus 4. So, we see that the applicability of identification method is not restricted for only underdamped processes. It can be made applicable for other type of processes as well.

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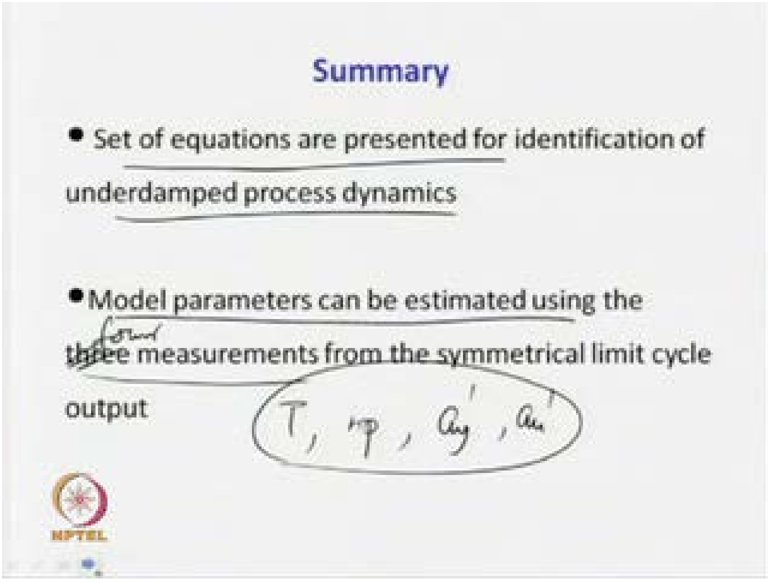

Let me, summarise the lecture; a set of the equations are presented for identification of underdamped process dynamics, and it is found that the identification technique can also be extended for other type of processes. Now, model parameters can be estimated using the three measurements from the symmetrical limit cycle output actually not three, we have to make four measurements, because those measurements could be T half period peak amplitude, and areas a u dash, and a u dash. So, using the four measurements actually, it is possible to estimate all the four unknowns or at least three unknowns associated with the transfer functions model.

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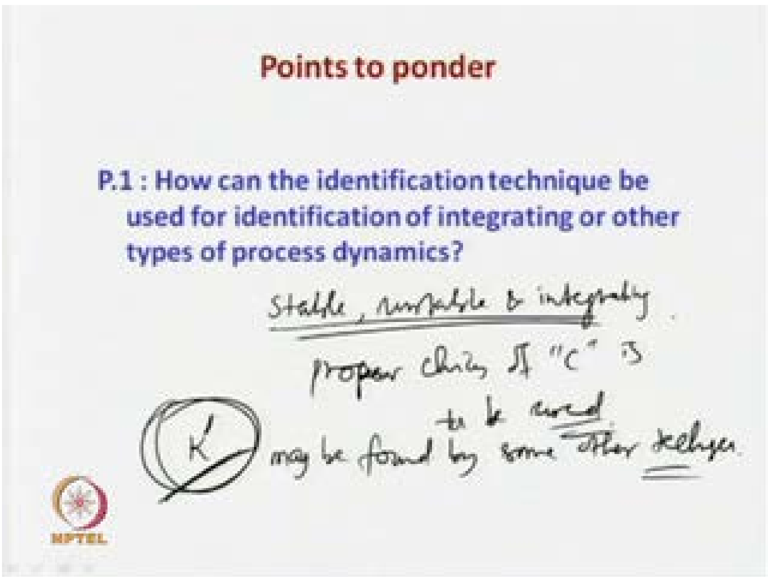

Now, how the identification technique can be used for identification of integrating or other type of process dynamics. They ask, I have already explained, now with proper choice C the parameter c associated with the underdamped second order plus dead time transfer function model with proper choice choices of C. It is possible to identify stable, unstable, and integrating process dynamics.

So, proper choice of C is required proper a choice of C is to be used; now, what is the limitation of this identification technique? This identification vary often used, you may need to find the k dash associated with the process model by some other method. So, k dash may be found by some other technique. So, accurate estimation or accurate information about the steady state gain associated with the Underdamped system model is necessary. So, care must be taken to find K dash accurately. Then only the set of analytical expressions can be solved easily, and correctly. Thank you.