Advanced Control Systems

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Module No. # 03
Time Domain Based Identification
Lecture No. # 12
Identification Procedures

Welcome to the lecture, titled identification procedures based on the limit cycle output wave form. We have derived earlier four important analytical expressions, and using those four analytical expressions, it is possible to identify a varieties of transfer function models. In this lecture, we are going to see how we can find or estimate parameters of the standard second order plus dead time transfer function model.

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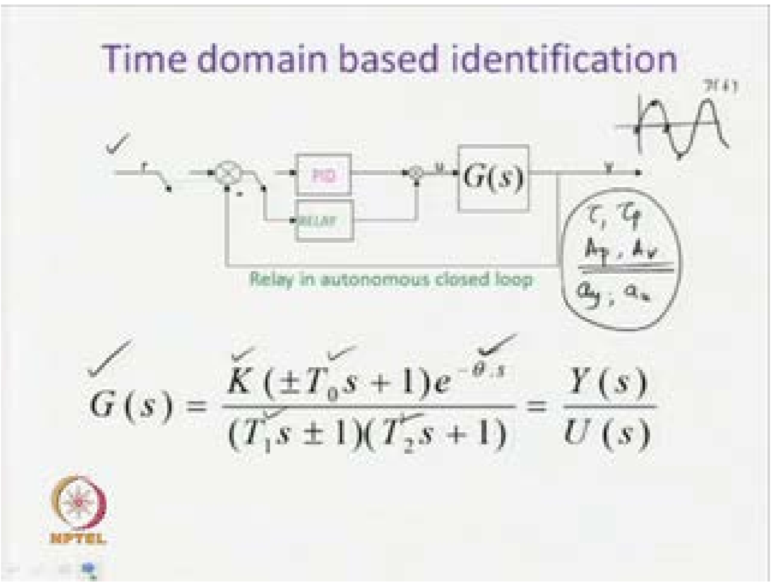

Earlier, we have seen also that the relay in autonomous closed loop yields are results in sustained oscillatory output. And let, the sustained oscillatory output be given by this wave form, and this is our y(t). Then, this is the arrangement that results in this type of sustained oscillatory output, what measurements are made from this sustained oscillatory output? The zero crossings, and the peaks with the help of those four measurements, it is possible to estimate the parameters of this standard second order plus dead time transfer function models. Those are the K, T 0, T 1, T 2, and theta. And we shall describe these in detail, how it is possible to estimate the parameters of this Transfer function model.

Now, so we make four measurements namely tau, tau p, A p, and A v, also it is possible to make two more measurements known as the area of the output signal, and area of the input signal. So, with the help of six measurements, it will not be difficult to estimate all the parameters associated with this transfer function model.

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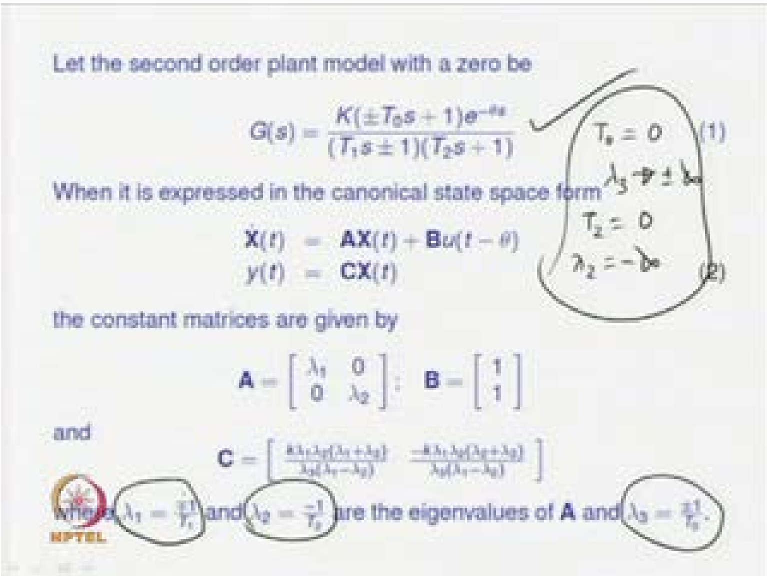

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Now, I will go to the next slide; where I had been showing, this slide since last 2 3 2 to 3 lectures. The reason for that is that the standard second order plant model with zero is given by this form. Each having, lambda 1 expressed h minus plus 1 upon T 1, lambda 2 expressed h minus 1 upon T 2, and lambda 3 expressed h plus minus 1 upon T 0. So, when we try to estimate parameters of lower order model? In that case, we need to approximate T 0 by some values like T 0 equal to 0. In that case, we get a second order planned model without zero. In that case, what happens lambda 3 will tend to your plus minus infinity; similarly, when T 0 equal to 0? In that case, lambda 2 will be equal to minus infinity

So, to emphasis on that point that when T 0 T 1 T 2 are... So, assume different values at that time the lambdas can be approximated by large or small numbers accordingly. So, the lambda values can assume some limiting values depending on the type of Transfer function model. We are going to estimate.

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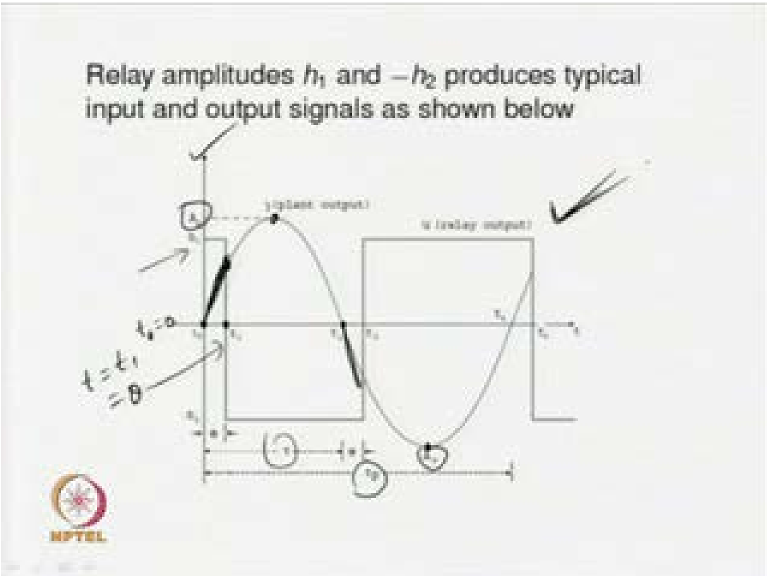

Now, we have this typical type of limit cycle output sustained oscillatory output from a from the dynamics of a system under relay control. Now, we will target the critical points like the zero crossings, and the peaks of the sustained oscillatory output signal. Thus we will measure thus tau, tau p, A p, and A v with the help of four measurements. It will be possible to estimate the parameters associated with a Transfer function model. Is it possible to obtain further or additional information from this output signal? That, we shall discuss in today's lecture, when have to try with the slope at the zero crossings? It is not difficult to obtain the slopes at the zero crossings or the expressions for the slopes at the zero crossings, and that will give you you additional expressions. Those can be exploited to estimate number of more number of parameters associated with a Transfer function model or we can choose suitable expressions for estimating the models model parameters. So, apart from the slopes associated with or the slopes at the zero crossings, Is it possible to get any additional information from the output signal? Yes, it is possible to obtain additional information if you look carefully at time T equal to theta at time T equal to theta what happens we have got the input changing from plus h 1 to minus h 2. So, when the input changes from plus h 1 to minus h 2? Definitely, there will be discontinuity at the of the output at time T equal to theta or in in in this figure. That is at time T equal to T 1 which is nothing, but T equal to theta for us if you take t 0 equal to 0.

So, what I mean by this is that at time equal to theta the output will have discontinuity, and this fact can be made use of to obtain additional information associated with the output signal.

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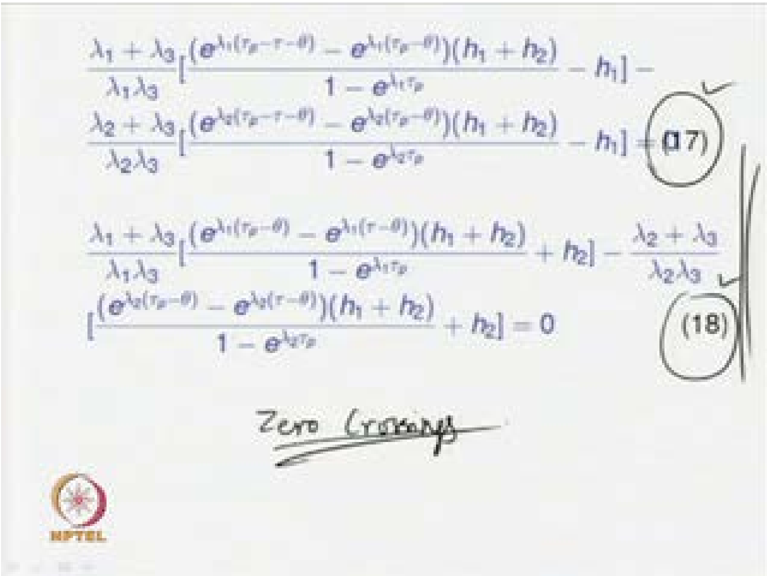

Now, these are the four standards equations; we have obtained earlier corresponding to the zero crossings, and the peak amplitudes A p,

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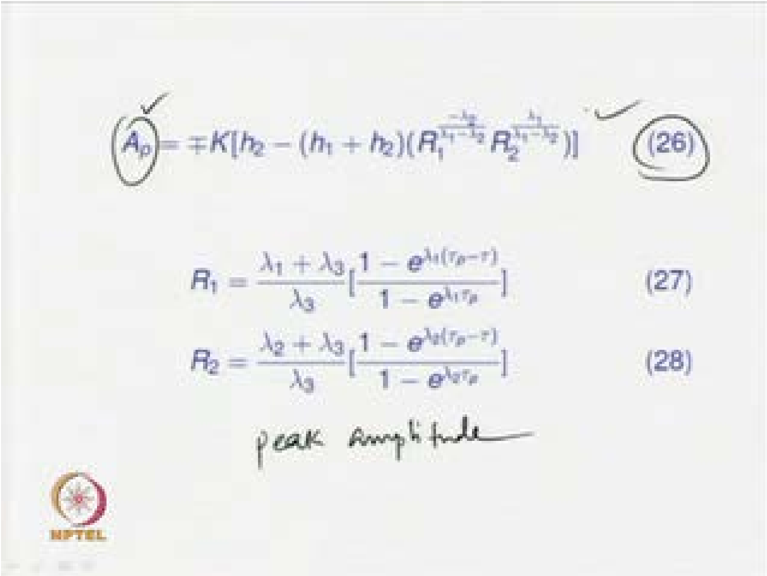

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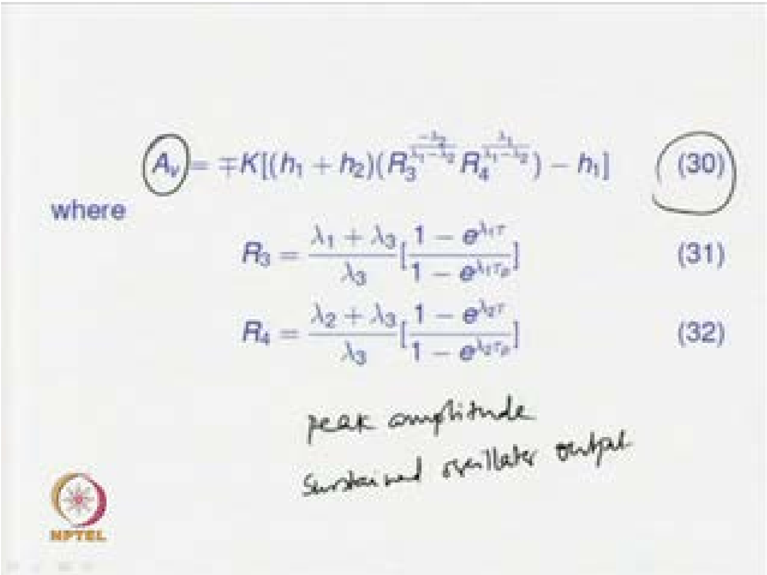

And another negative peak amplitude A V. So, the zero crossings yield equation number 17, and 18, and the peaks positive peak results in this equation. That, we have derived in our earlier lecture, and the negative peak results in this expression. Initially, the expressions appear to be more complex, but when it is solved using some routine or metlab routine or program the solution becomes easy also, and it is not difficult to get unique solutions for the variables associated with the expressions.

Now, I will go back to again the set of analytical expressions. So, thus we have got two expressions corresponding to the 0 crossings 0 crossings, and two expressions corresponding to the peak amplitude, and peak amplitude associated with the sustained oscillatory output sustained oscillatory output. So, those four equation number 17, 18, 30, and 26, are very important for us. Using, which it will be possible to or it it may be possible to estimate a number of parameters unknowns associated with the Transfer function model.

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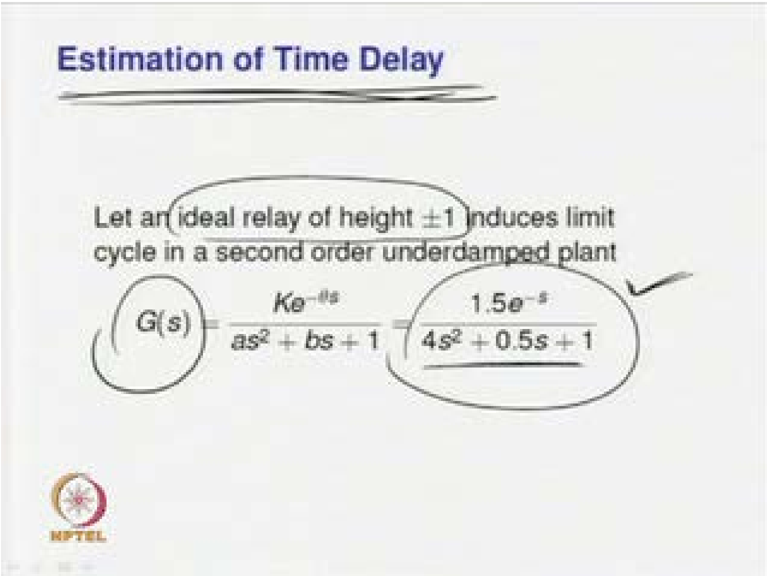

Now, can we as I have told earlier can we obtain additional information from the output signal? Yes, it is possible to obtain additional information from the output signal, and that we shall discuss now, how it is possible to obtain additional information. Before going to the analysis, what I will do? Let me, show the simulation result for a second order transfer function given by G s is equal to 1.5 e to the power minus s upon four s square plus 0.5 s plus 1. So, this process is an underdamped process. If you look at the denominator coefficients carefully, then obviously, it is evident that the process dynamics is a dynamics for an underdamped process.

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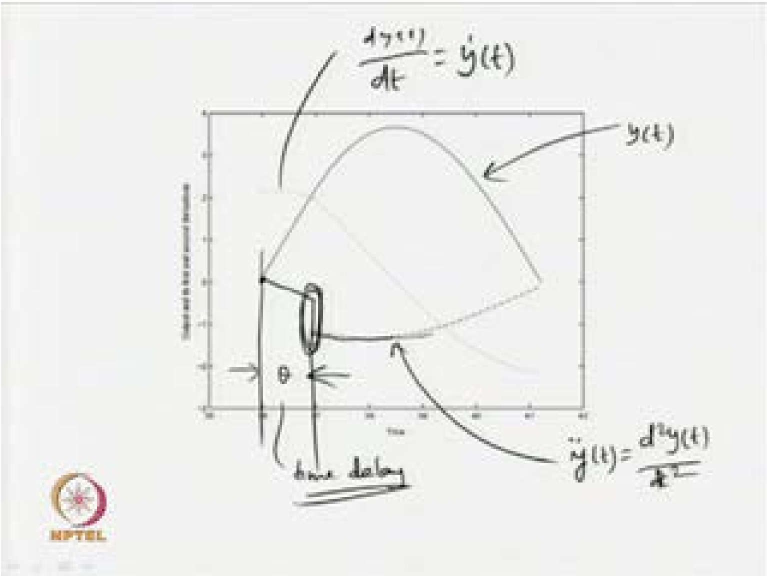

Now, for this when the relay settings are at plus minus 1 limit cycle signal is obtained in this form. So, the upper one gives us the limit cycle or sustained oscillatory output y t. Now, when the first derivate of the y t or output signal is taken? We obtain this plot. So, this plot is for d y t upon d t or I write y dot t. So, the first derivate of the output signal is obtain in this form, when I take further derivate of the output signal or that is the second derivate y dot dot dot t. That gives me, d square y t upon d y t square. So, in this case what I what I d t square sorry d t square? In that case, what from the second derivative of the output signal it is apparent that the output signal is having discontinuity at this time. So, please keep in mind the discontinuity is shown by this that the output signal without having monotonically increasing or decreasing in nature. Rather, there is some discontinuity appearing at this instant. So, this shows that using this phenomenon, it is possible to estimate one more parameter or one parameter of the Transfer function model out write and that parameter is nothing, but the time delay associated with the system dynamics. So, estimation of time delay can be accomplished using this technique the technique that go on taking the first, second, third. and So, on derivates of the output signal, and the wherever you see appreciable change in the output signal at certain instant of time. That means, this type of changes or abrupt changes is noticed at that time. The measurement can be made from the initial value or 0 first zero crossings of the output signal to that instant, and that will give you the time delay. So, that is how the time delay or information on the time delay associated with the dynamics of a system that is under relay control can be obtained.

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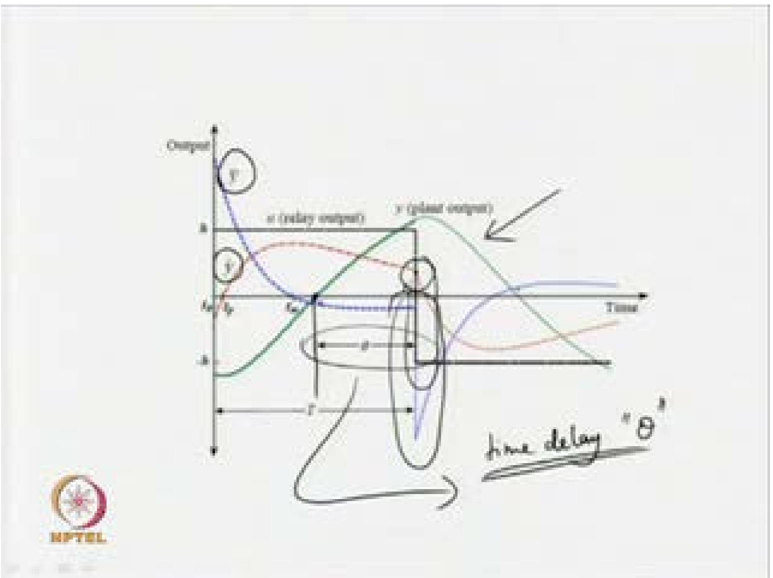

Now, this can be further illustrated by another figure. Let us, see this figure which has been obtained using simulation for another Transfer function model. Now, the output planned output signal is given by the green plot, and first order derivative of the output signal is given by the red one. So, the change is not a the the change is not so, appeared from this plot. Because although you see a change in non monotonous characteristic at this point, but the change or abrupt change is not evident from this figure; when second derivative is obtained which is given by blue plot. Then, we see that there is abrupt change in magnitude of the second derivative of the planned output. And now, measurement can be made from the first zero crossing as I have told the this is the first zero crossing or zero crossing. We have for the planned output, and when the measurement is made from they are till that the time at which is the abrupt change occurs that magnitude gives us the time delay associated with time delay, associated with a plant or process dynamics. So, this is how it is possible to obtain one more parameter or additional information from the output of a relay control system.

Now, we shall do little bit of analysis for this system, before going to identification procedures how can I make some analysis? Let me, first draw a plot this is a time x is output signal, and this is my output. As, I have told you that at h 1. Then, you have got the signal for this one is h 2, and so on. So, this is your minus h 2. So, h 1 changes from a time t equal to 0, h 1 changes at a time t h h 1 at time t equal to 0. The input to the system is h 1, which changes at time t equal to theta at time t equal to theta to minus h 2.

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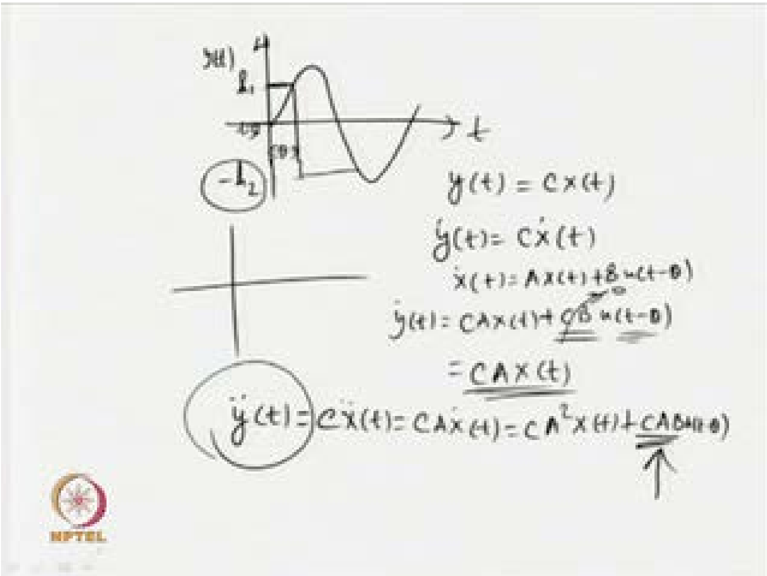

So, this fact this is already this has been already discussed earlier. This fact can be made use of to find the expression for the first derivative, and second derivative of the output signal. The output signal is given by y t equal to C x t. Thus the first derivative of the output can be expressed as C x dot t. Further, we know that x dot t is equal to A x t plus B u t minus theta. Then y dot t will be equal to C A x t plus C B u t minus theta. So, from here you can see that C B becomes 0 very (( )) for some time of processes. So, in that case y t will be simply C A x t now, when you take this plot here due to C, and A. You will get some changes definitely, because h the change input term is gone here, but when we take the second derivative y double dot t which is nothing, but now C x double dot t which can be ultimately written as C A x dot t is equal to C A square x t plus C A B u t minus theta. So, A B u t minus theta now C A B may not be zero, and this factor will contribute to the abrupt changes in the output signal derivative of the output signal. So, analysis also shows that, there will be abrupt changes in the second derivate of the output signal at certain instant of time, and if that time can be tracked properly or correctly then information about the time delay of the process can be obtained out right. And this is how one parameter of the Transfer function model can be obtained from the output signal or from derivative of the output signal conveniently.

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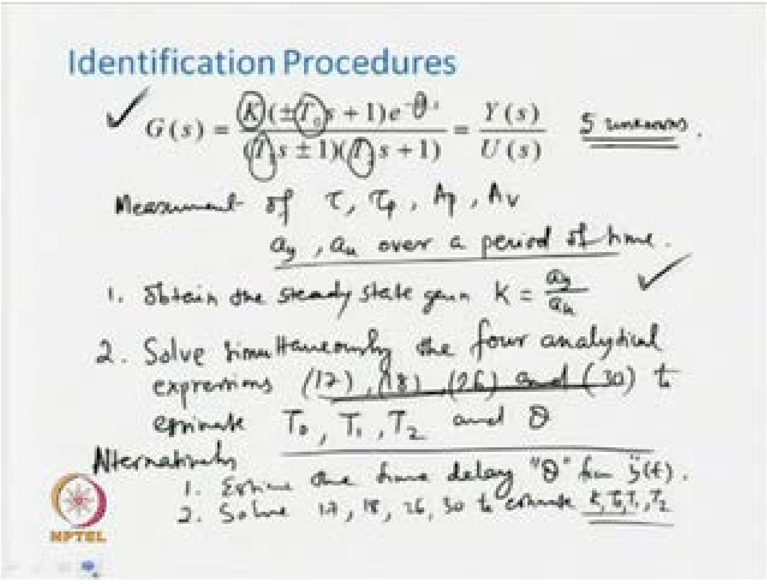

Now, we shall go to the identification procedures, now in this identification procedure what will be done? How many parameters are there in the process? Dynamics number of unknowns are k, T 0, theta T 1, and T 2. So, there are five unknowns: how can I estimate the five unknowns with the help of four analytical expressions? That, we have obtained we will see now, first make measurement of tau, tau p, A p, and A v. Next, we can also take measurement of the area of the output signal, and area of the input signal over a period of time. So, using the areas now it will be possible to obtain the steady state gain. Thus obtain the steady state gain, steady state gain k of the system from the ratio of the area of the output to area of the input signal. So, this is how one parameter is estimated.

Then, we are left with four unknowns, and those four unknowns can be found from the four analytical expressions. We have developed based on the zero crossings, and peaks, peak output of the output signal, sustained oscillatory output signal. So, second you solve simultaneously solve simultaneously the four analytical expressions, the analytical expressions given by 17, 18, 26, and 30. So, please make use of solve the four analytical expressions simultaneously, to estimate the unknowns to estimate the unknowns to estimate the unknowns those are now T 0, T 1, T 2, and theta.

So, this is how all the parameters associated with the Transfer function model can be estimated from a relay test or from the or or using the expressions that, we have obtained from the output signal based on the output signal of a relay test. Now, Secondly or alternatively alternatively what one can do? You can estimate the time delay estimate the time delay theta from the second derivative of the output signal or looking at the second derivative of the output signal estimate theta. Then, we are left with four unknowns, and those are k, T 0, T 1, T 2. Therefore, the remaining four unknowns can be estimated from the solution of simultaneous solution of equation 17, 18, 26, and 30. Then next, you solve 17, 18, 26, and 30, to estimate k, T 0, T 1, and T 2. So, this is how all the parameters associated with the Transfer function model can be estimated.

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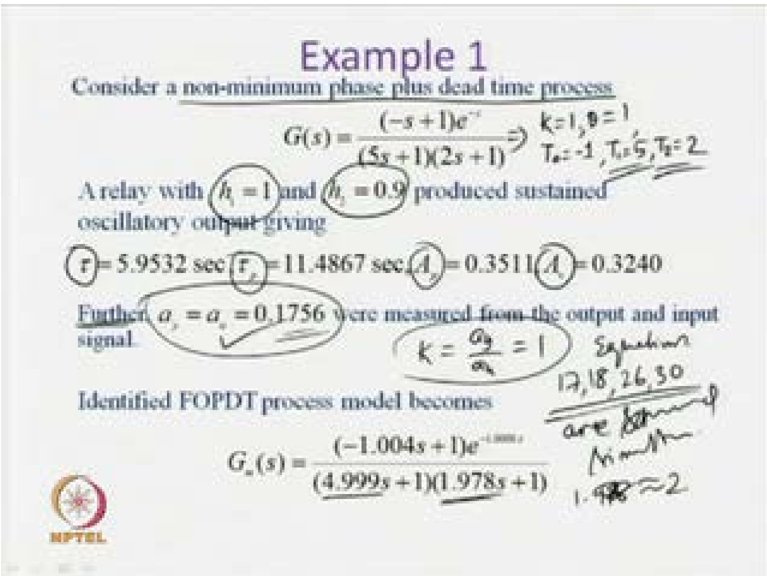

Now let us, go to some simulation example, in simulation example one: a non minimum phase plus dead time process dynamics is considered, where G s is equal to minus s plus 1 e to the power minus s upon 5 s plus 1 times 2 s plus 1. That means, what are those values k equal to 1, theta equal to 1, T 0 equal to minus 1, T 1 equal to 5, and T 2 equal to 2. So, these are the typical values. I have chosen to see the efficacy of the technique, we have developed for estimating the parameters of a Transfer function model. Now a relay with h 1 equal to 1, and h 2 equal to 0.5, is employed to generate or to induce limit cycle output, and from the limit cycle output measurement such as tau, tau p, A p, and A v, are made. Now, those values are shown over here; tau equal to 5.9532 second, tau p equal to 11.4867 seconds, and A p equal to 0.3511, A v equal to 0.3240. Further, the areas of the output signal, and input signal, are found to be a is equal to a u is equal to 0.1756, how can you measure? So, fine values you have to employ some routine to find accurately or to measure to make accurate measurements.

Now, using now the fact that the steady state gain is given by the ratio of a y upon a u which is nothing, but one because a y is equal to a u. Thus, the one parameter of the Transfer function model is estimated, whatever the remaining parameters of the Transfer function model when equation number 17, 18 equations, 17 18 26, and 30, are solved simultaneously. Then, we obtain theta h 1.0000. That means, theta has been estimated accurately, now T 0 h in place of one it has been found as 1.004 which is quite exact now, T 1 in place of 5; I have found 4.999, and similarly in place of T 2 equal to 2, the routine has given me 1.978. So, 1.978 which is fairly nearly equal to 2. So, that way using the identification technique, it is possible to estimate the parameters of a transfer parameters of a Transfer function model accurately. This shows the efficacy of the set of equations powerful equations. We have derived earlier based on the zero crossings, and peak amplitudes, of the output signal.

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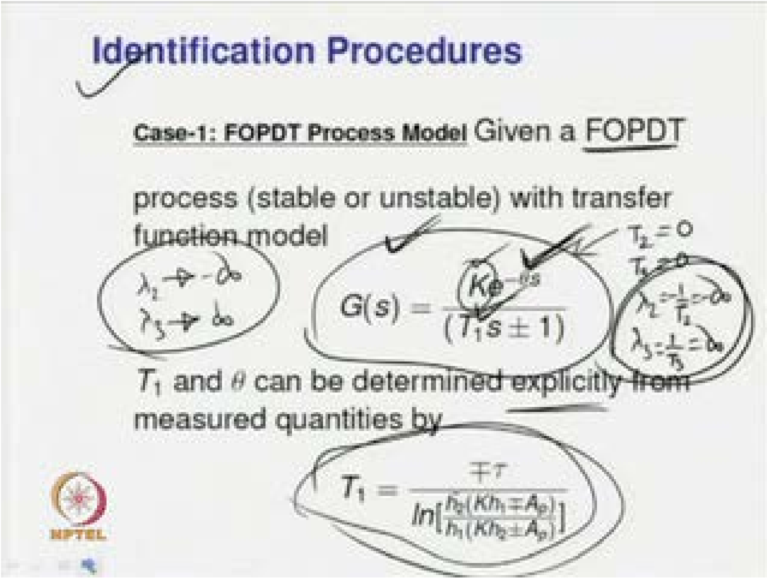

Now, identification procedures for simple Transfer function models, because here you need to solve a set of analytical expressions keep in mind. You have to solve four non-linear highly non-linear analytical expressions simultaneously, using some routine, and often the solutions may lead to false solutions be careful, but when we have got simple model like this. Let us, go for finding further simpler expressions. That can be used for finding the Transfer function model parameters of simple transfer functions. Now, consider a first order plus dead time Transfer function model given as k e to the power minus theta s upon T 1 s plus minus 1, that means what we have assumed T 2 is equal to 0, and T 1 is also equal to 0, that implies T 2 equal to 0, means what lambda 2 is equal to minus 1 upon T 2 is equal to minus infinity. So, lambda 2 equal to minus infinity, and T 1 is equal to 1 upon sorry lambda 3 is equal to not T 1 is 0. This is a T 3, T 2 is given their T 3 is equal to 0. So, lambda 3 is equal to 1 upon T 3 is equal to infinity. So, when these limiting values are put in the analytical expressions. It is not difficult to obtain expressions for explicit expressions for parameters associated with the Transfer function model.

Now, what are the parameters we have for the Transfer function model? The steady state gain k time delay theta, and one time constant, that is T 1. Now, it is possible to obtain explicit expression in terms of the measurements, what is that explicit expression? I have found an analytical expression or explicit expression T 1 which is equal to minus plus tau upon lawn of natural logarithmic of h 2 upon h 1 time k h 1 minus plus A p upon k h 2 plus minus A p. Now here, you look at the variables we have in the right half right side of this expression tau is measured from the output of the relay test or the output signal or sustained oscillatory from the sustained oscillatory output. We make measurements like tau, tau p, A p, and A v, as I have been repeating again, and again, therefore, what are the known's we have in the right half. So, tau is measured h 2, and h or h 1, are user defined. The user set those values; now, k can be estimated from different techniques. So, once k is known, then all other things are either measured or known. Once k is known h 1 is user defined h 2 is user defined A p is measured A p is measured. Thus, it will be possible to estimate the time constant associated with the Transfer function model explicitly. Using this expression is the beauty of this explicit expression no need of solving any analytical expression, no need of solving a set of non-linear equations. Simultaneously, to obtain the parameters of this Transfer function model. That is the beauty of this analysis; using the limiting values of course, that lambda 3 tends to minus infinity, and lambda sorry lambda 3 is equal to infinity, and lambda 2 is equal to minus infinity. So, what I am going to do? Now, whatever analytical four analytical expressions, we have derived earlier with the substitution of the limiting values. That lambda 2 tends to minus infinity, and lambda 3 tends to infinity. I am going to rewrite those expressions, and find explicit expressions. That can be obtained for the unknowns associated with the Transfer function model, and those unknowns are theta, and T 1, and k, can be found by some other technique. It will assume that k is known. So, we are left with two unknowns particularly. Those are theta, and T 1, and if it is possible to obtain explicit expressions for theta, and T 1. Then, our technique is quite powerful or I can say the analytical expressions, we have derived earlier are quite powerful, because those can be used to derive the explicit expressions that can be obtained for the Transfer function model given over here.

So, we will start our analysis now I believe that you have followed the objective of doing the analysis with the limiting values of lambda 2 tends to minus infinity, and lambda 3 tends to infinity.

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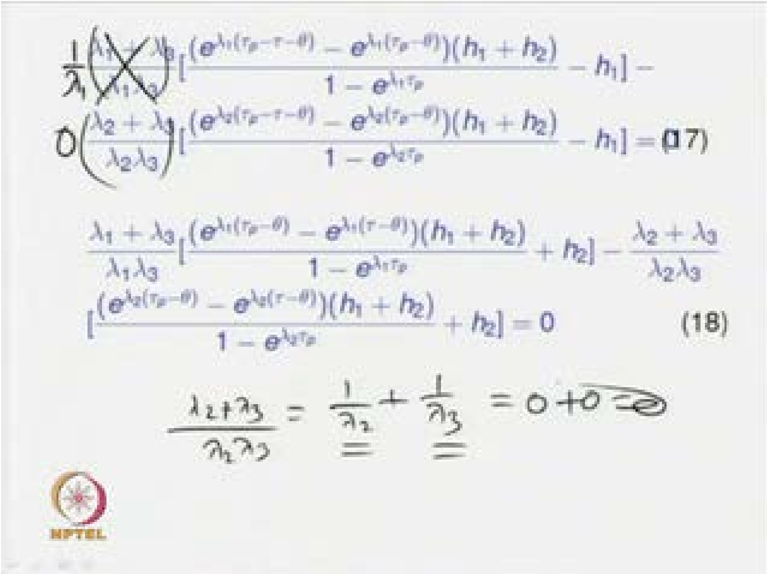

Now 17, and 18, I will rewrite those equations. Now, with the of course, the limiting values of lambda lambda lambda 2 tends to minus infinity, and lambda 3 tends to infinity. We will put those conditions now then, what 17 will look like now if you see this term all will be can be written by 1 upon lambda 1. Because we will get 1 upon lambda 1 plus 1 upon upon lambda 3, and lambda 3 is infinity therefore, I will be left with 1 upon lambda 1 for this factor. Now, coming to the second part lambda 2 plus lambda 3 divided by lambda 2 lambda 3 will give you, how much it will be 0? How that is So, because lambda 2 plus lambda 3 divided by lambda 2 lambda 3 is nothing, but 1 upon lambda 2 plus 1 upon lambda 3, and as. We know lambda 2 tends to infinity, and lambda 3 tends to infinity. Therefore, this as 0 plus 0 is 0, that is how what I will get zero multiplied by the whole factor we have here. So, that way this term can be removed whole term will be 0.

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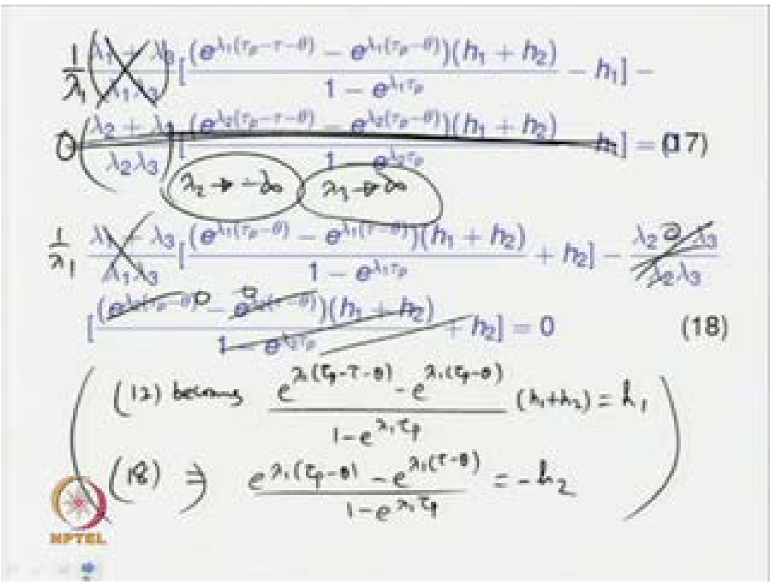

So, 17 can be rewritten as 17 becomes simply your e to the power 17 becomes e to the power lambda 1 tau p minus tau minus theta minus e to the power lambda 1 tau p minus theta divided by 1 minus e to the power lambda 1 tau p times h 1 plus h 2 is equal to h 1. Similarly, using the same thing that lambda 2 tends to minus infinity, and lambda 3 tends to infinity plus minus infinity, will make will make a sense because when lambda 2 tends to minus infinity. So, minus in e to the power minus infinity means this will also be 0 this will also be 0. Otherwise, also this is going to be 0, I need to not worry. So, that way these terms will be out from the equation 18, and thus 18 can further be written as the whole of this 1 will be again 1 upon lambda 1, because lambda 3 tends to infinity. Thus 18 can also be written as e to the power lambda 1 tau p minus theta minus e to the power lambda 1 tau minus theta divided by 1 minus e to the power lambda 1 tau p is equal to minus h 2. Please, keep in mind these two expressions are possible or 17, and 18 can be converted into these two expressions provided lambda 2 tends to minus infinity, and lambda 3 tends to infinity.

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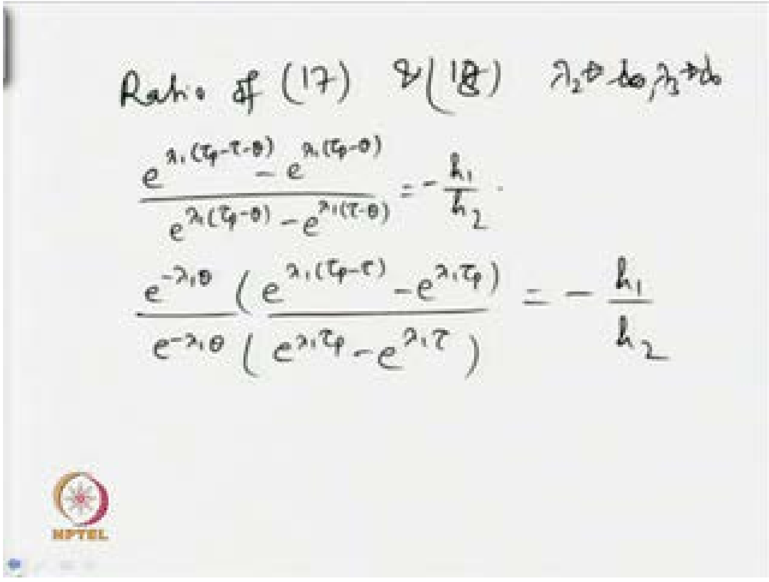

Next the ratio of 17, and 18, the ratio of 17 and 18 ratio of the 17 and 18 with the conditions that lambda 2 tends to minus infinity, and lambda 3 tends to infinity, gives us e to the power lambda 1 tau p minus tau minus theta minus e to the power lambda 1 tau p minus theta divided by e to the power lambda 1 tau p minus theta minus e to the power lambda 1 tau minus theta is equal to minus h 1 by h 2. Simply, the ratio of the two, what I mean by so, if you take the ratio of these left hand side by right left hand side this one this by this, and right hand side h 1 by h 2. Then you get sorry I have left a term here h 1 plus h 2 h 1 plus h 2. It will cancel out, when we take the ratio and finally, we will get the ratio in the form of minus h 1 by h 2. This will be used later on this equation will be used later on which can further be simplified as, I can simplify this expression left hand side further as. Because e I can take e to the power minus lambda 1 theta h common giving me e to the power lambda 1 tau p minus tau minus e to the power lambda 1 tau p divided by e to the power minus lambda 1 theta again in the denominator giving me e to the power lambda 1 tau p minus e to the power lambda 1 tau. So, this is equal to minus h 1 upon h 2.

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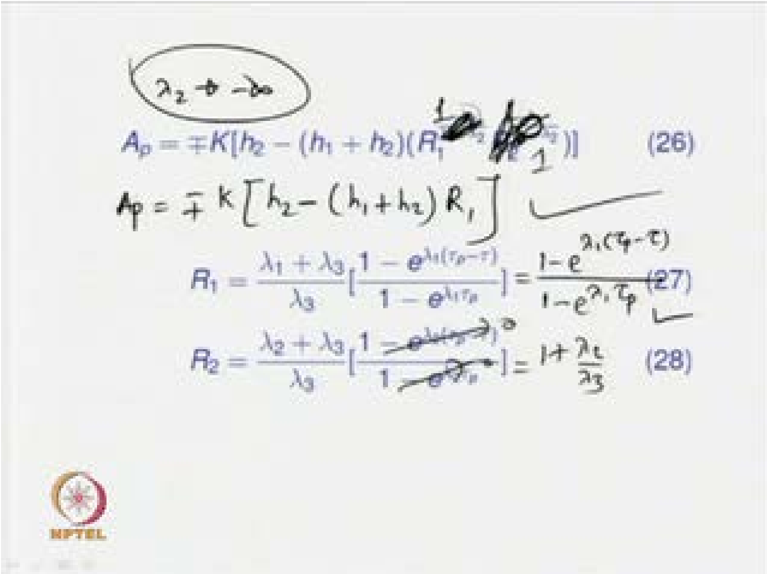

We will go to the analysis of this peak amplitude now when lambda 2 tends to minus infinity; what will happen to this term when lambda 2 tends to minus infinity? You see this will be 0 this will be 0. So, we were left with one by one, and this will be given by 1 plus lambda 2 by lambda 3. So, ultimately how much it will be sorry 1 plus yes lambda 2 by lambda 3. Now, R 2 will be this much now what about this one when lambda 2 tends to minus infinity, this lambda 1 will be neglected. That means, divide this or multiply this by 1. So, sorry this will go out this inner lambda 1 will go out this will be large value 1 upon large value will be 0. So, R 2 to the power 0 is 1. So, basically A p can be written as minus plus k h 2 minus h 1 plus h 2 times R 1 why that is? So, when lambda 2 tends to minus infinity this is a large value. So, lambda 1 can be neglected. So, we will have minus lambda 2 in the numerator, and minus lambda 2 in the denominator. Thus, this will be a power of 1. So, simply R 1 to the power 1, and R 2 to the power 0, that will give you one ultimately thus A p will be minus plus k times h 2 minus h 1 plus h 2 R 1.

So, this is the expression for this one, and similarly this will be simply lambda 1 by lambda 3 will be 0 plus 1. So, it will be equal to 1 minus e to the power lambda 1 tau p minus tau by 1 minus e to the power lambda 1 tau p. So, with this limiting value R 1 will be 1 minus e to the power lambda 1 tau p minus tau divided by 1 minus e to the power lambda 1 tau p, and A p will be minus plus k times h 2 minus h 1 plus h 2 together times R 1. Now, that will be used that concept will be used now.

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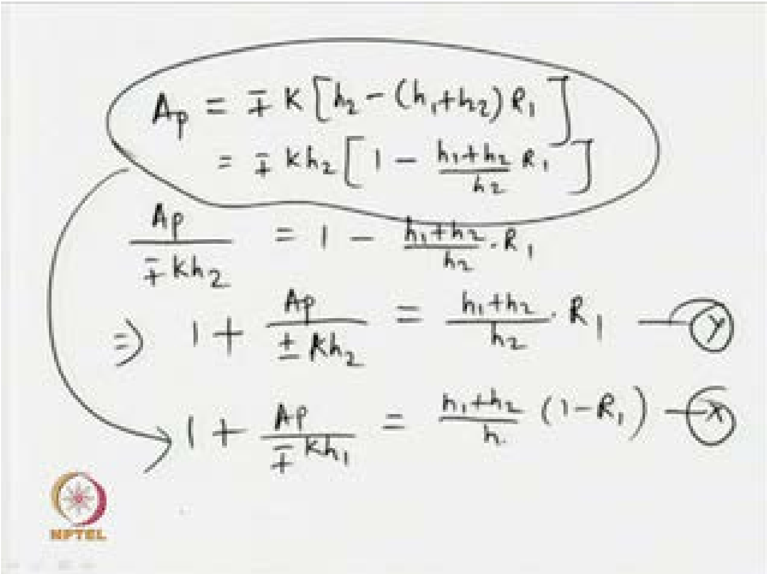

So, not be difficult to write expression like, your A p is equal to minus plus k h 2 minus h 1 plus h 2 times R 1. So, allow me to write 1 minus A p by or or it will be more direct you may confuse. So, let me take common here. So, minus plus k h 2 if I take h 2 as common, I will get a term like 1 minus h 1 plus h 2 by h 2 times R 1 is it. Then, I can write A p by minus plus k h 2 I take this term to the left hand side is equal to 1 minus h 1 plus h 2 by h 2 times R 1 or I can write 1 plus 1 plus A p by plus minus k h 2. Please, see the sign change; I have taken a minus sign over here. So, this will be equal to h 1 plus h 2 by h 2 times R 1 similarly it will be possible to obtain another expression like 1 plus A p by minus plus k h 1 is equal to h 1 plus h 2 by h 1 times 1 minus R 1. So, we will little bit of writing this in different form can yield this expression as well.

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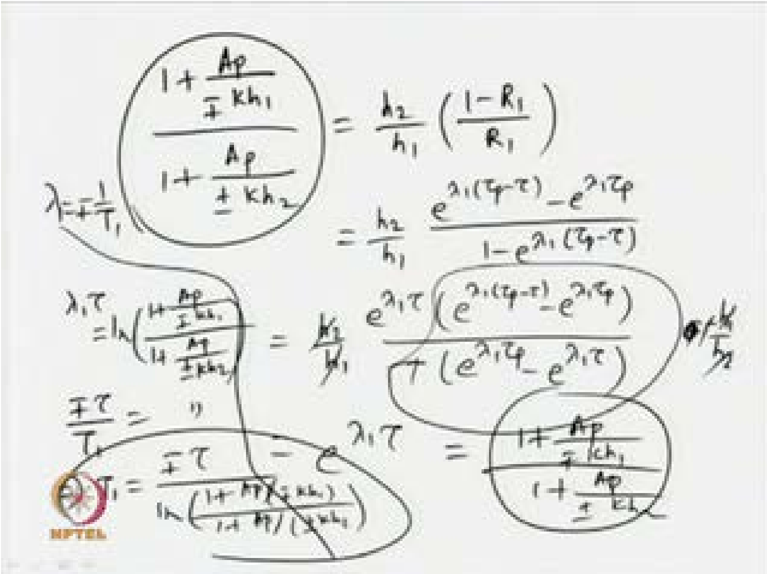

So, now take the ratio of these two when you take the ratio of these 2 upper 1 to or lower 1 to upper 1 upper 1 to lower 1. It does not matter, when I take the ratio of this x by y ratio of x by y? Will give me 1 plus a p by minus plus k h 1 divided by 1 plus A p by plus minus k h 2 is equal to h 2 by h 1 1 minus R 1 by R 1. All these things are very simple. So, ultimately you will see that, this is giving you a term when you substitute the R 1 R 1 what is R 1 what is R 1? R 1 is this much 1 minus e to the power lambda 1 tau p minus tau divided by 1 minus e to the power lambda 1 tau p. So, this is equal to R 1. So, when this R 1 is substituted over there, and this is also this concept is made use of you see this is this is cancelling out. So, this this expression is also used finally, what I will get this? I will get in the form of h 2 upon h 1 e to the power lambda 1 tau p minus tau minus e to the power lambda 1 tau p divided by 1 minus e to the power lambda 1 tau p minus tau.

So, this can be again written as h 2 upon h 1 multiply both numerator and denominator where the term e to the power lambda 1 tau that will give e to the power lambda 1 tau times e to the power lambda 1 tau p minus tau minus e to the power lambda 1 tau p in the denominator I will keep this as minus or allow me to write the term first e to the power lambda 1 tau minus e to the power lambda 1 tau p. So, I will make a sign change here here like minus of minus plus. So, ultimately I will get a term in the denominator h minus of e to the power lambda 1 tau p minus e to the power lambda 1 tau where I am doing? So, because you will see that this whole term. This term has been found out as h 1 by minus h 1 by h 2 earlier. Let me, show you e to the power lambda 1 tau p minus tau minus e to the power lambda 1 tau p by minus of this. We have obtained as minus h 1 by h 2. So, this term is obtained as minus h 1 by h 2. So, when I substitute this minus again minus minus minus will cancel out, and h 2 h 2 cancel out, h 1 h 1 cancel out, living us, because this this is giving us minus h 1 by h 2. So, finally we are getting e to the power lambda 1 tau. Thus, what you have got? That e to the power lambda 1 tau is equal to 1 plus a p by minus plus k h 1 by 1 plus A p by plus minus k h 2. So, now you can take the natural logarithm of both sides, and finally get it in the form of the expression, that I had shown over here. So, this is how it is possible to get the analytical expressions for the unknowns associated with the Transfer function model. So, let me since I have got space here let me write. So, if I take the natural logarithm I will get the lambda 1 tau is equal to lawn of 1 plus A p by minus plus k h 1 by 1 plus A p by plus minus k h 2.

Now, what is lambda 1? We know that lambda 1 is equal to minus 1 upon T 1 lambda 1 is equal to sorry minus plus, because we have got plus minus there. So, it will be minus plus 1 upon T 1. So, if I write here then minus plus tau upon T 1 is this much therefore, T 1 is equal to minus plus tau divided by lawn of 1 plus A p by minus plus k h 1 by 1 plus A p by plus minus k h 2. And this is what you have got this is how explicit expressions are obtained for the parameters associated with the Transfer function model.

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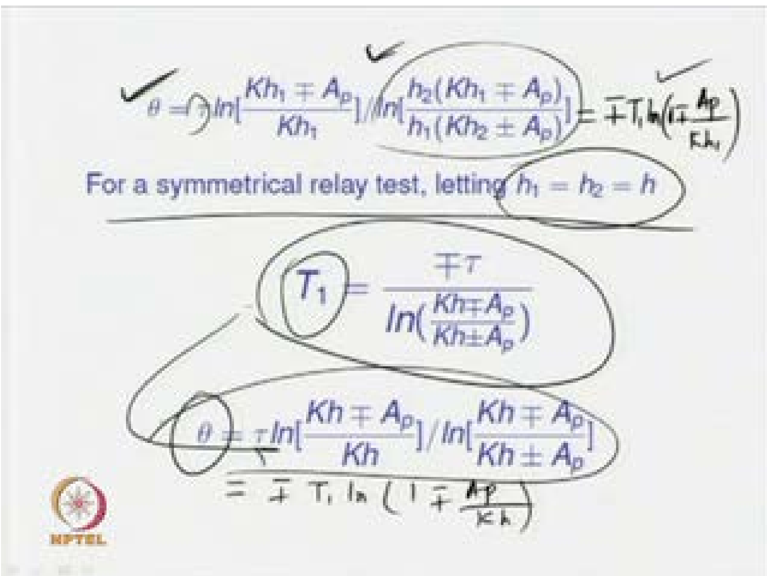

Now, how to obtain the analytical expression for an other parameter associated it with the first order plus dead times Transfer function model. This if you look this carefully, this this is nothing, but simply your this is expression for minus plus T 1 natural logarithm of 1 minus plus A p by k h 1. So, you when you substitute T 1, then T 1 is how much T 1 is tau divided by this denominator term tau divided by this denominator term. Therefore, the remaining term is this one. Thus I can get the explicit expression for the second unknown associated with the first order plus dead time Transfer function model as either minus plus T 1. So, minus plus T 1 lawn 1 minus plus A p by k h 1 or in this form.

So, once T 1 has been estimated then theta can subsequently be estimated using this explicit expression. Now, for a symmetrical relay what will happen h 1 is equal to h 2 is equal to h substitution of that will yield the explicit expressions in the form of T 1 is equal to minus plus tau upon lawn of k h minus plus A p by k h plus minus A p. Please, take care of the minus plus plus minus, because the first order plus dead time process could be stable or unstable. Now, the second expression again can be rewritten in the form of minus plus T 1 lawn of 1 minus plus A p by k h, and upon substitution of this T 1, you will get this full expression given by this. This is how we do obtain analytical expression or i can say explicit expressions for the unknown associated with the first order plus dead time Transfer function models.

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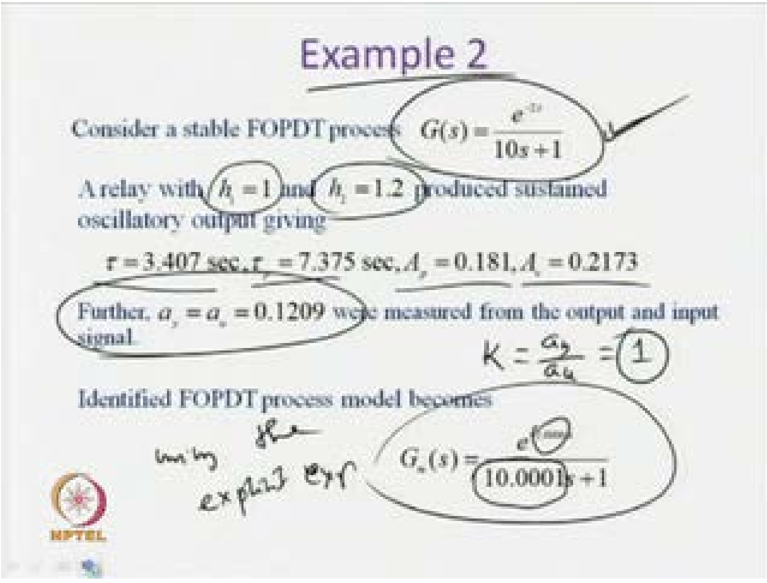

Let us, go to some simulation examples whether or analytical expressions or the explicit expressions. We have derived are truly yielding results or not or the efficacy of those can be tested from the simulation examples: consider a stable process of the form G s is equal to e to the power minus 2 s upon 10 s plus 1 a relay, and a symmetrical relay with parameter h 1 equal to 1, and h 2 equal to 1.2, produced sustained oscillatory output giving tau this much tau p, this much A p, this much, and A v this much. Now, I will not use the the four expressions or I will not use the set of expressions to find the unknowns associated with this Transfer function model for this process. Rather, I will use the explicit expression from the explicit expressions. The T 1 is found to be 10.0001, and the delay is found to be two. Therefore, the accuracy of the parameters are are obtained using the explicit expressions or I can say first of all the steady state gain is obtained from the ratio of a to a u h 1, and that we have got accurate value for the steady state gain. Similarly, accurate values for the unknowns associated with the Transfer function models are estimated using the explicit expressions using the explicit expressions. That we have described before this. So, these are the two explicit sorry these are the two explicit expression this is one, and the second one is this one, are used to estimate the model parameters.

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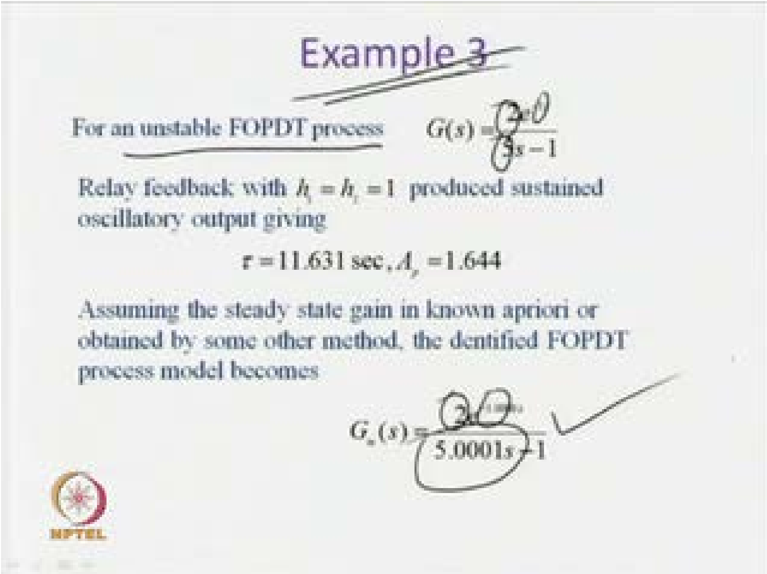

Now, coming to example three which considers an unstable first order plus dead time process for which k equal to 2, theta equal to 3, and T 1 equal to 5. We have used the explicit expressions, and found that k is found to be 2, theta is found to be 3, and T 1 is found to be 5.0001. Thus, the explicit expressions are found to be quite accurate.

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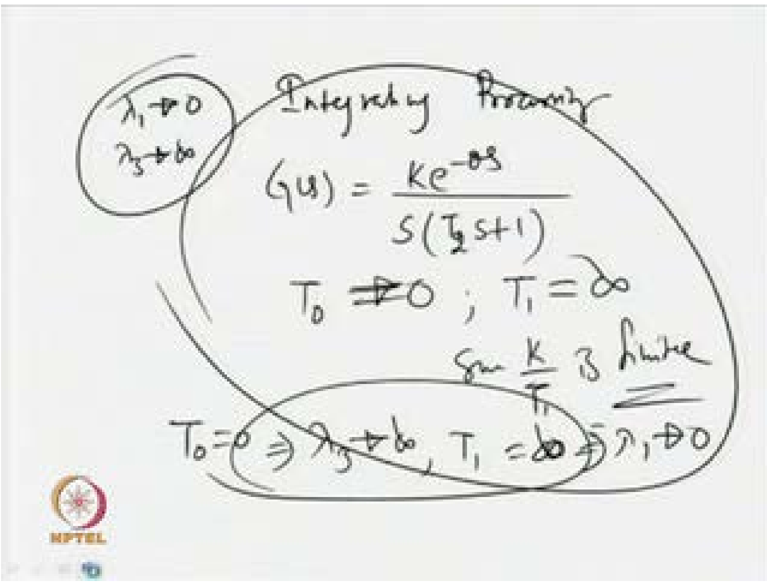

Similarly, the concept can be extended for integrating processes also integrating processes what are the integrating processes? We have got, I can say that this Transfer function model can be known as second order plus dead time Integrating process model.

Now, to find this process model what what is to be done? Your T 0 has to be approximated to 0, T 0 equal to 0, and T 1 is equal to a large number, such that k upon T 1 is finite. This is to be assumed, then we get this form. So, with the substitution of the conditions that T 0 equal to 0 or indirectly speaking your lambda 3 tends to infinity T 1 tends to 0. If this sorry lambda T 1 equal to infinity therefore, T 1 equal to infinity therefore, lambda 1 tends to 0. So, with the substitution of lambda 1 tends to 0, and lambda 3 tends to infinity, and using all the four analytical expressions. It is possible to estimate the parameters of integrating processes as well.

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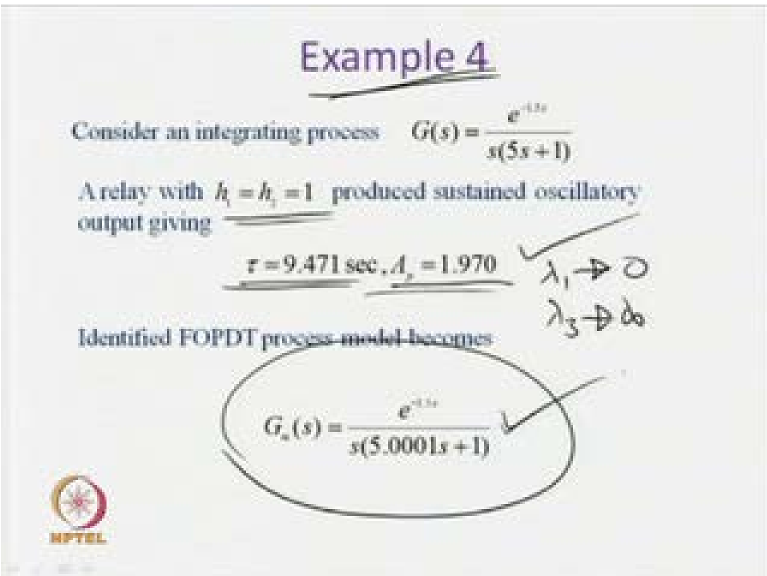

Now let us, consider this example: now, a symmetrical relay produced sustained oscillatory output giving half period as tau equal to 9.471 seconds, and A p as 1.970. Now with the substitution of again, I am the lambda 1 tends to 0, and lambda 3 tends to infinity, and using the analytical expressions. It is possible to find the Transfer function model for the integrating process as e to the power minus 1.5 s upon s times 5.0001 as plus 1. Again it shows that, we are, it is possible to obtain accurate Transfer function models using the four analytical expressions.

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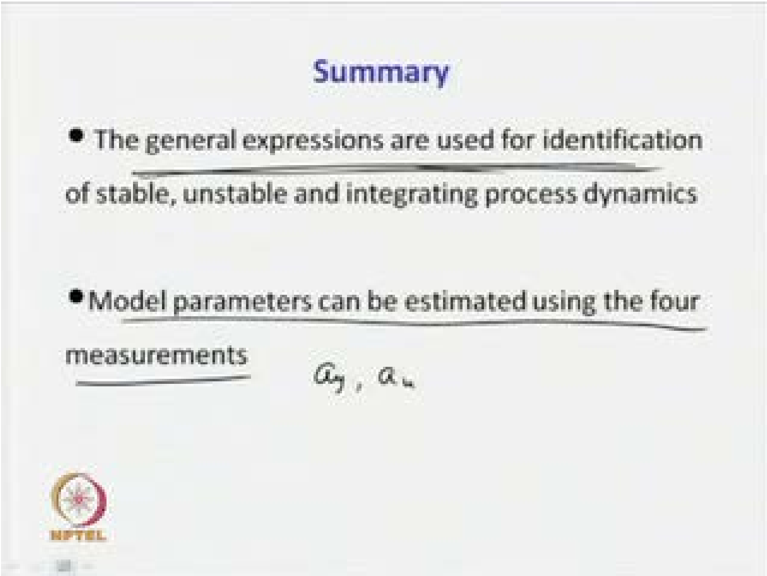

So, what we have discussed in this lecture? The general expressions are used for identification of stable, unstable, and integrating process, dynamics with proper substitution of the limiting parameters such as lambda 1 tends to infinity, or not or lambda 1 tends to 0 or not, and So on. So, with the proper substitution of limiting values for the variables of lambdas. It is possible to use the general expressions, and find the unknowns, associated with a Transfer function model. Now, model parameters can be estimated using only four measurements. If required further measurements, like the areas of the output, and input signal, like a y, and a u, can be used.

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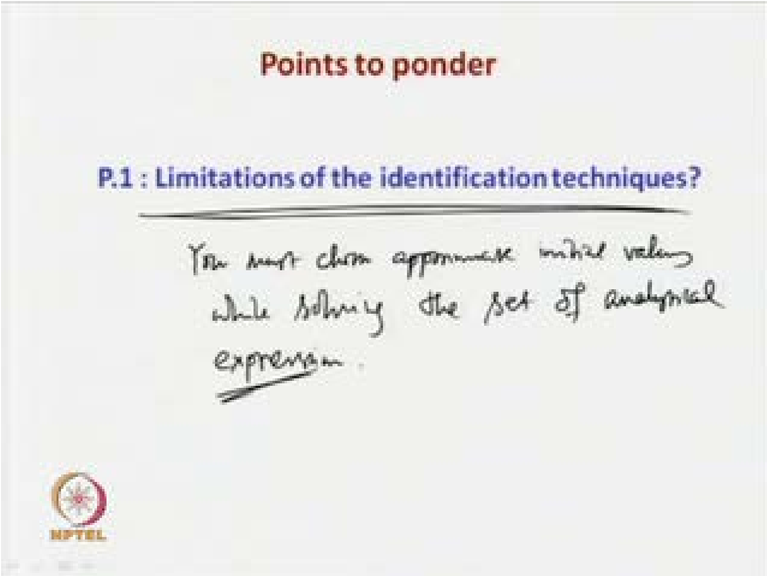

What are the limitations? We have for the identification techniques, one major limitation associated with the identification techniques. We have discussed, in this lecture is that we you must start with you must choose appropriate initial values initial values while solving solving the set of the set of analytical expressions. This is very important. So, when you solve a set of non-linear equations choice of initial values are of (( )) effect. You must choose judiciously, otherwise the solutions may lead to false solutions; in that case, the estimated parameters may not be accurate or may be quite absurd. That is all in this lecture.