Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 11

Existence of Limit Cycles for Unstable System

Welcome to lecture titled, existence of limit cycles for unstable systems. Unlike stable systems, integrating systems, unstable systems may not yield stable limit cycle output. And unless we obtain stable limit cycle output, it will be difficult to identify the parameters of transfer function models. Some simple integrating processes, and all stable processes always guarantee stable limit cycle output under relay control. And that is not so, for the case of unstable systems, open loop unstable processes or systems. Today we shall see, what are the limitations we have for unstable systems? Under what conditions unstable systems can yield stable limit cycle output?

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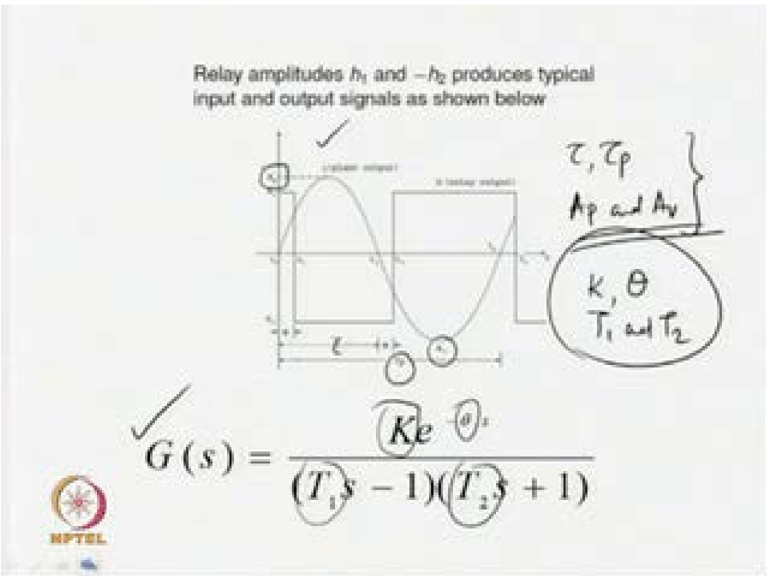

We have this type of typical limit cycle output from stable processes, when the process dynamics is given by G(s), which has got an unstable pole, which is located at 1 upon T

1 may not guarantee stable limit cycle output. Now for stable systems, the typical type of limit cycle output we obtain is of this form, we now assume that for unstable systems also we get such type of waveform. Then what are the measurements we make on the waveform? We take a measurement of the tau and tau p, the period of the limit cycle output, when the limit cycle output is stable. Similarly, the other two measurements we make are A p, and A v. Thus the list of measurements, we take from the limit cycle output can be given as tau, tau p, A p, and A v.

So, these are the four measurements we make on the limit cycle output. And based on the four measurements, what are the parameters? we can estimate the parameters associated with the dynamic model of the transfer function are the steady state gain k, the time delay theta, the time constant T 1 and the time constant T 2. Thus we with the help of four measurements, we are able to estimate the four unknowns k, theta, T 1 and T 2. Now we will subject the open loop unstable system, second order system to relay test and see what sort of output waveform we get from a typical second order plus unstable system.

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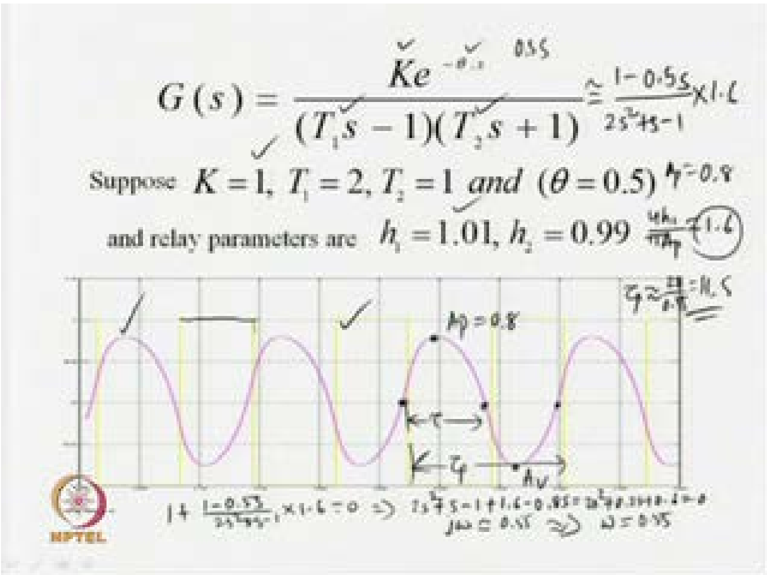

Now, this is the transfer function of a second order unstable system, where k is the steady state gain given as 1, and T 1 is assumed as 2, T 2 as 1 and theta as 0.5. Under that situation, a relay with parameters h 1 as 1.01 and h 2 as 0.99 yields a limit cycle of this form. So, the waveform shows us two signals, one is the output signal shown by this.

And the second one is the input signal shown by the yellow lines. Now, this is the input signal we have like this, is the input signal then from here. we see that the relay test is able to provide us stable limit cycle output signal from, which we can make measurements of the peak amplitudes A p and the negative peak amplitude A v and tau given by this and tau p given by this.

Assuming that, this is the zero crossings, we have zero crossings we have. Now, when I change this values k, T 1, T 2 and theta. Am I expected to get stable limit cycle output that may not be. So, that we shall see subsequent in our subsequent analysis. Now for the time being, let us analyze why this system is yielding us stable limit cycle output. Allow me, to approximate the transfer function dynamics as 1 minus 0.5 s upon 2 s square plus s minus 1. How I have obtained this? with the substitution of k equal to 1 theta equal to 0.5 t 1 equal to 2 and t 2 equal to 1. It is possible to obtain this transfer function of course; this will be the approximate one, because the time delay term e to the power minus 0.5 s has been approximated by the term 1 minus 0.5 s.

Now, what is the peak amplitude we are getting? The peak amplitude A p is approximately of the value 0.8. A p is 0.8, this A p is 0.8. Then the gain of the relay can be given as, 4 h 1 by pi A p which will be approximately 1.6, upon substitution of h 1 as 1.01 and A p as 0.8. Then the net gain loop gain we have in the system or the loop transfer function can be given by, 1 minus 0.5 s upon 2 s square plus s minus 1 times 1.6. And this will result in a characteristic equation of the form, 1 plus 1 minus 0.5 s upon 2 s square plus s minus 1 times 1.6 equated to 0. This will result in an expression of the form, 2 s square then plus s minus 1 plus 1.6 minus 0.8 s is equal to 2 s square plus 0.2 s plus 0.6 is equal to 0. Then, s can be obtained having the frequencies nearly of 0.55. Thus, s can be now substituted by j omega, thus I see that the frequency of oscillations is of roughly of the magnitude omega equal to 0.55. That will give us a time period, tau p of approximately 2 pi upon 0.55 is equal to 11.5, that is what we get. So, if I measure tau p it will be approximately of value 1 11.5. This ensures that we are able to obtain a stable limit cycle output in spite of having an unstable pole for the unstable system.

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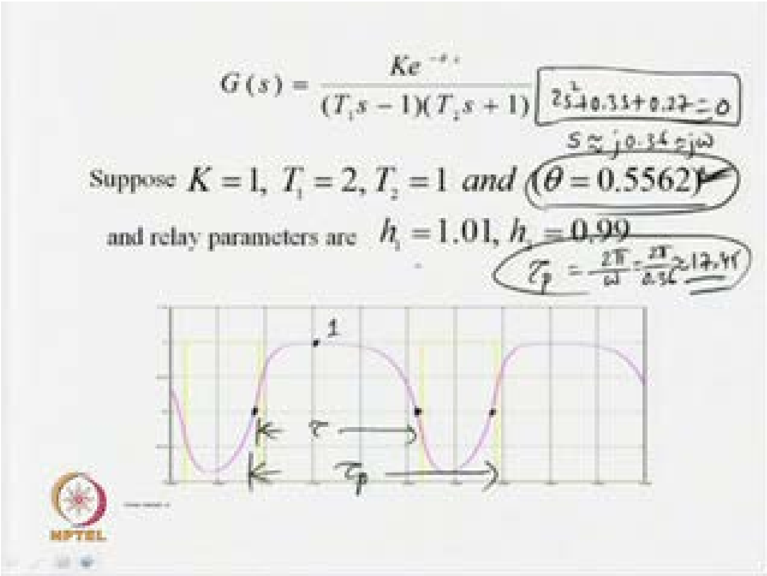

Now, let us go to the second simulation. Here, I have changed the value of the time delay, from theta equal to 0.5 to theta equal to 0.5562. Now, we get a typical output of this form. Now you see that, the tau p has increased significantly. if this is the first zero crossing then, I have got the second zero crossing at this point and the third one at this point. Now, what is tau? tau is from this to this and tau p is from this to this.

So, what we observe from here. Interestingly tau increases, when the value of theta or the time delay associated with the system unstable system increases. And similarly, tau p is also increasing. So, those are increasing gradually. If I further increase the theta value, definitely tau and tau p are going to increase substantially. So, this is one observation we have got. As far as analysis of system like the previous case will yield us a an equation, characteristic equation of the form 2 s square plus 0.3 s plus 0.27 is equal to 0. How do you get this characteristic equation? Again finding the peak amplitude, which is of the magnitude 1 now, that will give us the gain of the relay as relay gain as 4 into 1.01 upon pi into 1. So, approximately this is 4. So, approximately we will get this value to be of some magnitude 4 by pi 1.27 or so.

After getting this as 1.27, again you multiply this with the loop transfer function and get the characteristic equation finally. Now, what we get from this characteristic equation? This characteristic equation gives us, the location of the poles or the values of s. Now s will be roughly equal to a value of j 0.36, then this is also equal to j omega.

That it implies that, the period tau p will be now, 2 pi upon omega is equal to 2 pi upon 0.36, Which is roughly of the value 17.45. As expected the tau p has increased, earlier it was 11.5, roughly it was 11.5 that has gone up to 17.45. This is how the period, the time period of the output signal the stable limit cycle output signal increases with the increase of time delay term.

What is that value of theta, for which we will the system which is to provide us a stable limit cycle output. If I increase theta further, definitely it is going to come to a situation, where we shall not get any stable limit cycle output. Now, the value of theta is 0.5562

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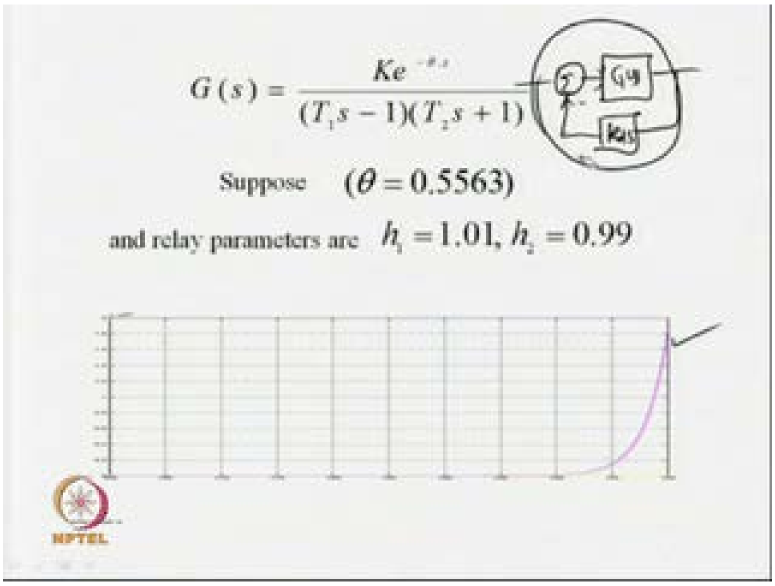

If that theta value is increased to 0.5563, I have increased a fourth decimal place value only. So, in place of theta equal to 0.5562, now the theta value is 0.5563. In spite of that, the same set of relay parameters, h 1 equal to 1.01, and h 2 equal to 0.99 is not going to yield stable limit cycle output. So, here you see it is blowing out, the output is blowing out, and we are getting very high output that means, the system or the relay control system is unable to provide us stable limit cycle output. So, in place of getting a pattern of this form, what I am going to get? I am getting nothing.

So, that way what we infer from this that. There is something constant associated with the unstable systems or processes particularly, that must be maintained otherwise one may not expect stable limit cycle output from unstable processes. How to overcome this limitation that, we shall discuss later on.

One way of overcoming this limitation is that, you have to stabilize the unstable open loop process. So, how can unstable open loop processes can be stabilized? Given an unstable process of the form G s e by provide some inner feedback controller K d s or so, or rate controller in the inner feedback path. Then this overall dynamics is such that, it provides stabilization to the open loop unstable process. With this it is possible to change or increase the value of theta, that we shall discuss later on or in some other lecture.

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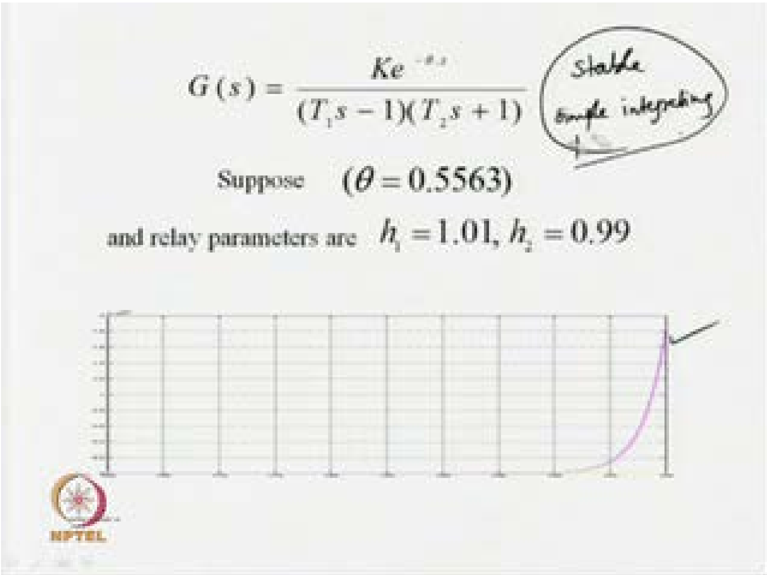

For the time being, we should concentrate on this fact that unlike stable and some simple unlike stable and some simple integrating processes integrating processes. One cannot guarantee stable limit cycle output from open loop unstable system. This fact is to be taken care of, when particularly dealing with unstable systems. And unless it is possible to obtain stable limit cycle output from unstable systems, it will not be possible to identify the parameter of transfer function model parameters of the dynamics of an open loop system. Now, we shall proceed to the analysis why that is happening? For analysis, we have to consider all the set of equations we have derived for the identification of systems.

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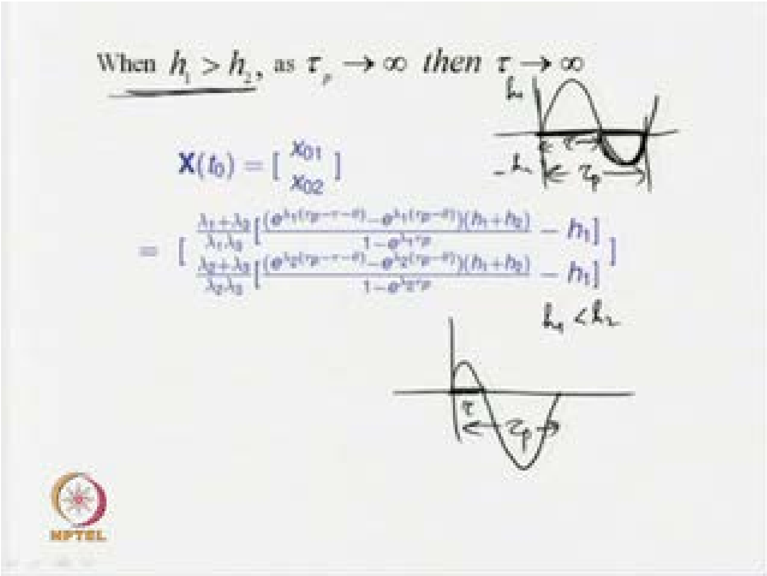

Now I will go to the analysis, when h 1 is greater than h 2, what type of waveform you get? Depending on h 1 and h 2, the span of the output signal will vary. Which span? Particularly, when h 1 is greater than h 2 h 1 and minus h 2 I have. Then I may get different type of output, where you see this is the span we denote by tau and this is the span we denote by tau p. When h 1 is greater than h 2, tau will be more then tau p minus tau or to avoid that one, indirectly speaking this span will be more then this span. The span you are getting from the negative pulse or the pulse the negative the pulse which has got negative output. And when h 1 is less than h 2, what will happen? h 1 is less than h 2, we may expect a typical output of the form this one. So, as tau will be tau will be substantially less than tau p.

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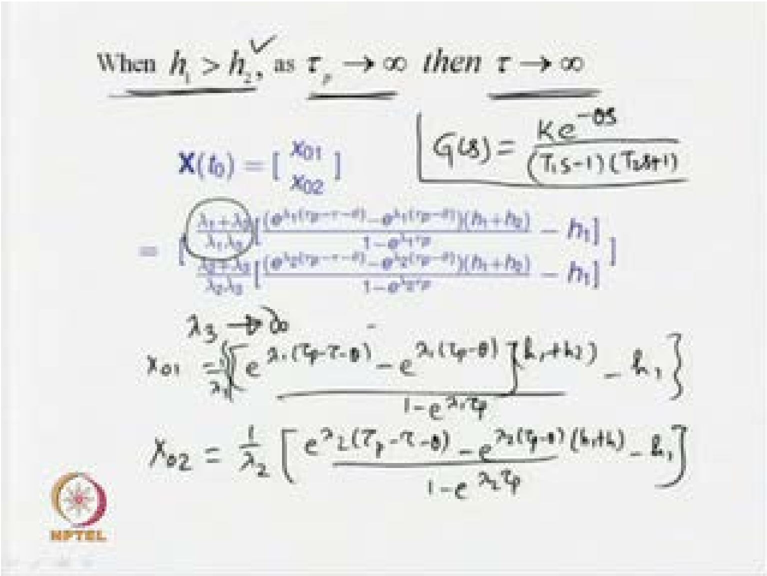

Basically when h 1 is greater than h 2, I mean by tau p and tau will be of significant values. And as we have seen, when h 1 is greater than h 2 for certain value of the time delay, what is happening? The tau p is assuming very large values, at that time certainly it is expected that, tau also will assume very large values. We shall apply now, the large value theorem to find analyzes the analytical expressions, and we have obtained earlier for the stable processes or the process dynamics general transfer function model. Now, what happens to the expression X t 0? When tau p tends to infinity and tau tends to infinity, as you have seen that the transfer function model we are dealing with is now, G s is equal to K e to the power minus theta s T 1 s minus 1 time T 2 s minus plus 1.

Here what happens, we do not have any 0 in the transfer function dynamic models. So, this is having no 0, I mean we have not added a term like T 0 s plus 1 or minus T 0 s plus 1 in the numerator. That means, what happens for this transfer function lambda 3 will be equal to infinity or lambda 3 tends to infinity, because the T 0 has been taken as 0. Please keep in mind, when t 0 tends to 0, t lambda 3 will tends to infinity. As lambda 3 is by definition given as lambda 3 is equal to plus minus 1 upon t 0. When lambda 3 is tending to a large number at that time the expression for X 0 1 will be equal to e to the power lambda 1 tau p minus tau minus theta minus e to the power lambda 1 tau p minus theta times h 1 plus h 2 divided by 1 minus e to the power lambda 1 tau p minus h 1.

So, this term will be giving us another term that is given as, 1 upon lambda 1. When lambda 3 tends to infinity, we will get 1 upon lambda 1 from here. We will get 1 upon lambda 1. Similarly, X 0 2 will be giving us 1 upon lambda 2 times e to the power lambda 2 it is whole of this one. So, one upon lambda 2 times lambda, this will be tau p minus tau minus theta minus e to the power lambda 2 tau p minus theta upon 1 minus e to the power lambda 2 tau p times h 1 plus h 2 minus h 1 the bottom one. So, basically what is what has been done here, that when lambda 3 tends to 0 X 0 2 is obtained in that form.

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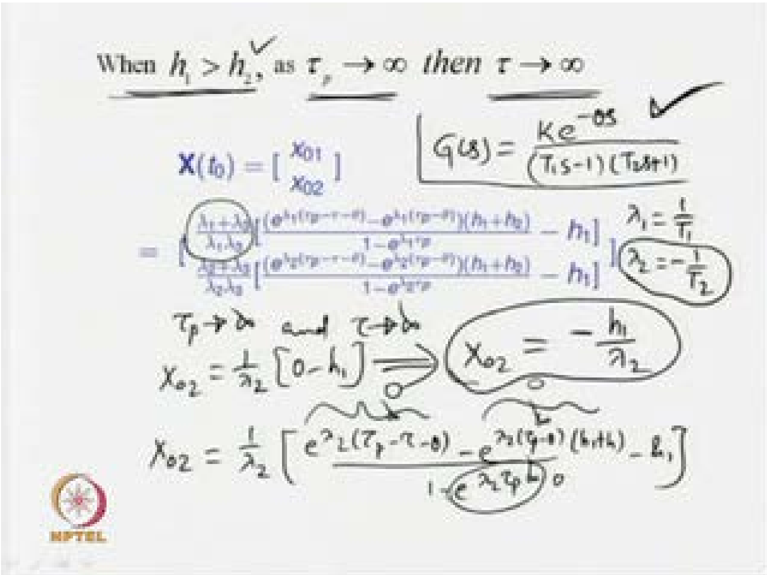

And with the condition that, tau p tends to infinity when the unstable open loop unstable process ceases to provide us stable limit cycle output. At that time tau p tends to infinity and subsequently tau tends to infinity, when that condition is put here. What we obtain? Basically, we obtain an expression of the form. Let us put the values over here, you see where we have tau p tau p minus tau also will be a large value. So, this is a large value again, this tau p minus theta will be a large value, here tau p is a large value. Now, what about lambda 2? As you see given this is the dynamic model of the open loop unstable system, lambda 1 is equal to 1 upon T 1 and lambda 2 is equal to minus 1 upon T 2. So, lambda 2 is a negative number. So, lambda 2 is a negative number therefore, the first term here will be something like, e to the power a negative number into a large number. So that way it will be equal to 0.

The first term will be approximately equal to 0. So, first term will be 0, second term will be 0 and in the denominator even this term will be 0. Leaving us, the terms x 0 2 is equal to 1 upon lambda 2 and whole of this bracketed part will give us 0 minus h 1. Thus, X 0 2 x o 2 will be equal to minus h 1 by lambda 2. This is how, when the large value theorem is applied expression for the X 0 2 can be obtained as, minus h 1 by lambda 2. Similarly, X 0 1 also can be obtained and now interestingly for this particular unstable open loop unstable second order dynamics. What will be the X 0 1 X 0 1 also will be equal to minus h 1 upon lambda 2.

Why that is so? if you look if you carefully remind, the form of the C matrix C matrix can be given as, k lambda 1 lambda 2 upon lambda 1 minus lambda 2 and the second term will be, minus k lambda 1 lambda 2 upon lambda 1 minus lambda 2. This is will be the form of c. And we know that, for limit cycle to happen or occur or to induce limit cycle for the relay control system C X t 0 has to be equal to 0. That implies that, both the X t 0 that is giving us X 0 1 X 0 1 and X 0 2 must be equal. Because if I look at carefully, this is if I take this as the common term, I will be left with C with a form of some expression with 1 and minus 1. Therefore, that compels us to get the X t 0 in the form of X 0 1 X 0 2 such that, X 0 1 is equal to X 0 2. What we are getting from this analysis? This analysis gives us that, x 0 1 is equal to x 0 2 is equal to minus h upon lambda 2, when tau p tends to infinity and tau tends to infinity.

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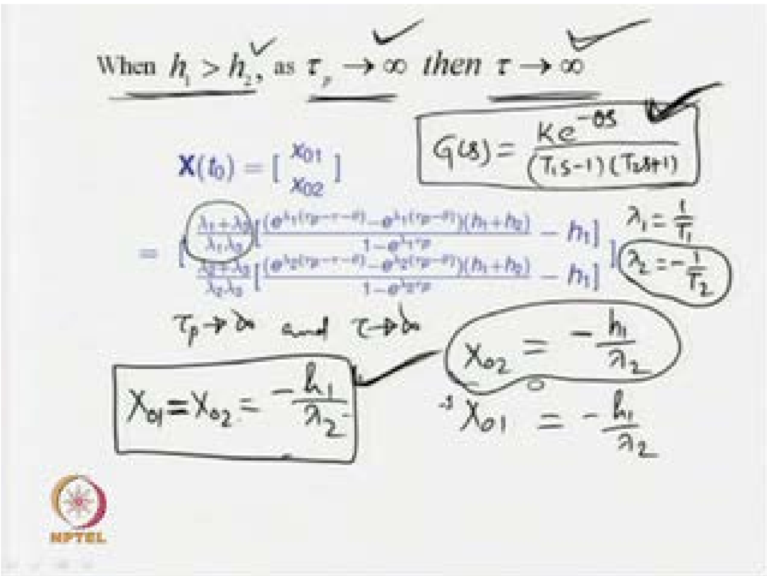

Let me repeat once again, X 0 1 is equal to X 0 2 is equal to minus h 1 upon lambda 2. When the theta value is such that the unstable process is unable to induce limit cycle output and at that time, what happens? Tau p tends to infinity and tau tends to infinity. Therefore, you do not get output or stable limit cycle output for the relay control system. This expression we shall use in our subsequent analysis. Now please keep in mind, X 0 1 is equal to X 0 2 is equal to minus h upon lambda 2.

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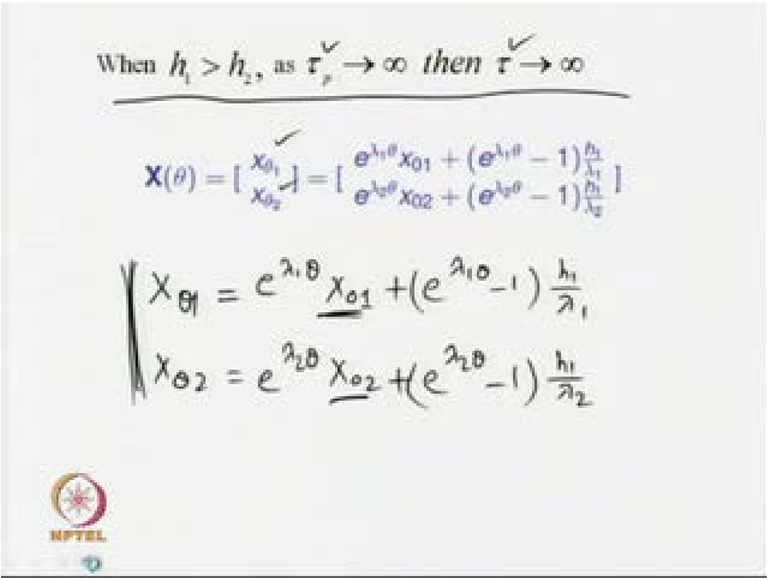

Now, when h 1 is greater than h 2, again I am writing when h 1 is greater than h 2, tau p tends to infinity and tau tends to infinity. What are those tau p and tau? These are the two parameters of the limit cycle output signal. Now, what will be the expression for X theta now? So, X theta will be as it is X theta 1. I can write down from this one given X theta, which has got two components or elements, X theta 1 can be written as e to the power lambda 1 theta X 0 1 X 0 1 plus e to the power lambda 1 theta minus 1 times h 1 by lambda 1. And X theta 2 is equal to e to the power lambda 2 theta X 0 2 plus e to the power lambda 2 theta minus 1 h 1 by lambda 2. Here please keep in mind, for our analysis has given X 0 1 is same as X 0 2 as minus h 1 upon lambda 2. So, upon substitution you can get X theta 1 and X theta 2 in correct form.

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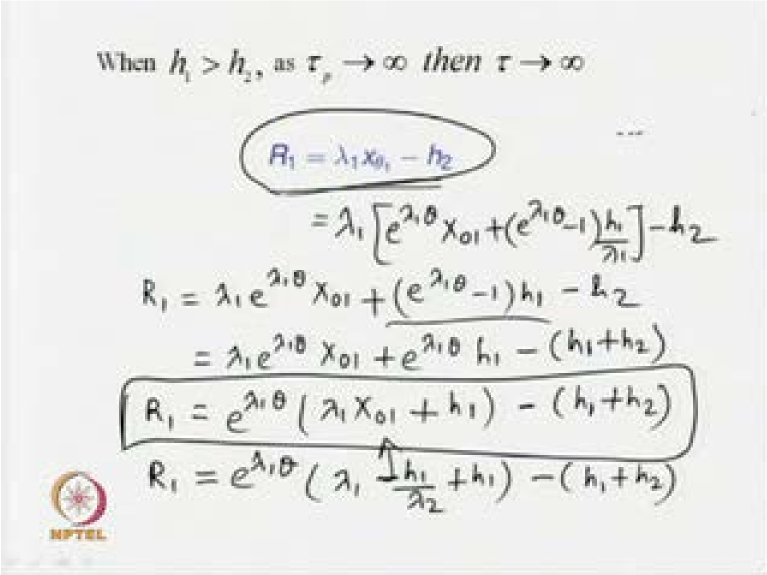

But we know that, R 1 that is used for finding the peak amplitude of the output signal can also be given by the expression lambda 1 X theta 1 minus h 2. I will expand this now, R 1 is equal to lambda 1, now X theta 1 X theta 1 is given as e to the power lambda 1 theta X 0 1 plus e to the power lambda 1 theta minus 1 h 1 upon lambda 1 minus h 2. I have substituted the expression for X theta 1 over here.

Let us simplify this expression further, R 1 can be simplified as lambda 1 e to the power lambda 1 theta X 0 1 plus e to the power lambda 1 theta minus 1 h 1 minus h 2. So, this lambda 1 and this lambda 1 in the denominator will cancel out, giving us this term. Then further simplification will yield, lambda 1 e to the power lambda 1 theta X 0 1 plus e to the power lambda 1 theta h 1 minus h 1 plus h 2. I have got an expression of the form, e to the power lambda 1 theta as common then lambda 1 X 0 1 plus h 1 minus h 1 plus h 2.

So, this is what we get for the R 1. What is this R 1? R 1 appears in the expression for the peak amplitude. Again let me write not the complete expression, A p is given as minus plus K h 1 or h 2 as you have seen minus plus K, then you will have certain terms like, then here your R 1 to the power minus lambda 2 upon lambda 1 minus lambda 2 then R 2 to the power.

We get an expression for the peak amplitude of the output signal in this form. Now, this peak amplitude is associated with R 1 and that R 1 is expressed or analytically is given by the expression, R 1 h lambda 1 X theta 1 minus h 2. So, which further can be simplified with the substitution of X theta 1 expression by the final expression given as R 1 as R 1 as R 1 is equal to e to the power lambda 1 theta times lambda 1 X 0 1 plus X h 1 minus h 1 plus h 2. Now I will substitute this X 0 1, what we have obtained earlier for this typical case, When the unstable process ceases to provide us a stable limit cycle output.

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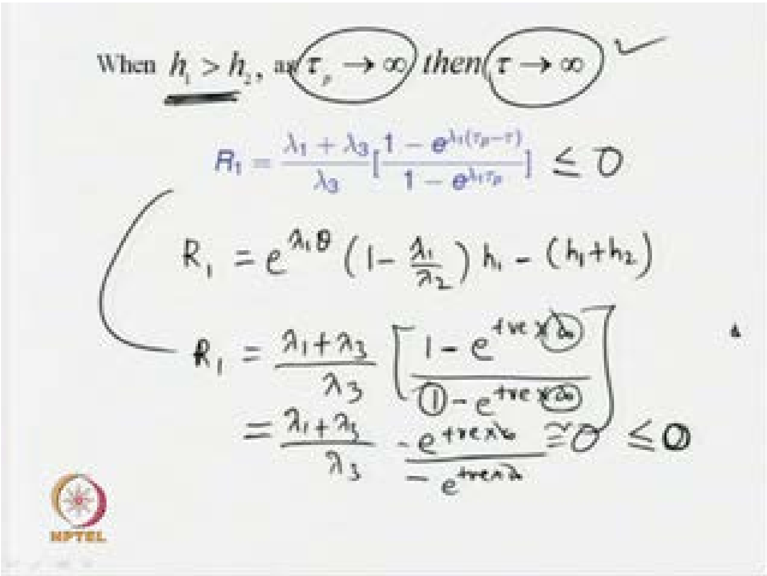

So in that case, the expression for R 1 will be finally given as, let me substitute over here. Because again to avoid writing in the next page, let me write over here. So, R 1 is e to the power R 1 is equal to e to the power lambda 1 theta lambda 1 X theta 1 is minus h 1 upon lambda 2 plus h 1 minus h 1 plus h 2. So, finally, I can write R 1 as R 1 is equal to e to the power lambda 1 theta times 1 minus lambda 1 upon lambda 2 times h 1 minus h 1 plus h 2.

Now, what about this expression? Further R 1 is given by this expression, where from we have earlier we have already derived the correct expression for R 1, which is available in two forms. when complete analysis is made, this R 1 can also be expressed in the form of lambdas given by R 1 is equal to lambda 1 plus lambda 3 upon lambda 3 times 1 minus e to the power lambda 1 tau p minus tau upon 1 minus e to the power lambda 1 tau p. And when h 1 is greater than h 2 and theta value is such that tau p tends to infinity and tau tends to infinity. At that time, what will be this R 1? This R 1 can be obtained as, R 1 is equal to lambda 1 plus lambda 3 upon lambda 3 times 1 minus, you please keep in mind lambda 1 is a positive number.

So, this is this will give you a value a large value. So, this will be approximately some a large value e to the power lambda 1. So, I will substitute this by a positive number positive number with infinity by one minus e to the power a positive number with again infinity, but this is this infinity or this denominator infinity is quite larger then this infinity. Therefore, yielding us approximately lambda 1 plus lambda 3 upon lambda 3 times, it will be approximately e to the power a positive number times infinity upon a very large number giving us a value of a you see 1 is there. So, I can approximated approximate the bottom one let me rewrite in the down here. So, this is same as lambda 1 plus lambda 3 upon lambda 3 and here, I have got e to the power a positive number with a large number times a positive number with a large number. So, this will give us basically finally, approximated to 0 or less than 0.

So, I can put a condition that R 1 can be equal to 0 or a number less than 0. Because finally, we can see that this is coming out to be a value near 0. Using the large value theorem, it is not difficult to get the R 1 expressed as R 1 is a value which is less than equal to 0.

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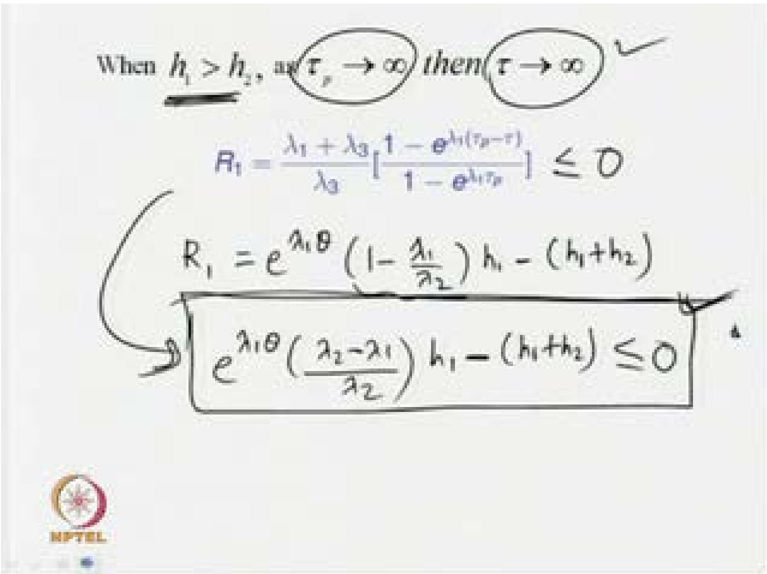

When this is so, then I will write this condition over here to finally get an inequality of the form. If R 1 is given by this expression then allow me to write, e to the power lambda

1 theta lambda 2 minus lambda 1 upon lambda 2. I am simplifying this term, times h 1 minus h 1 plus h 2 is less than equal to 0. So, when this expression is obtained, further simplification of this expression will give you a condition that has to be made to obtain stable limit cycle output for second order systems particularly and which analysis can be extended further to obtain conditions for obtaining stable limit cycle output for open loop unstable systems.

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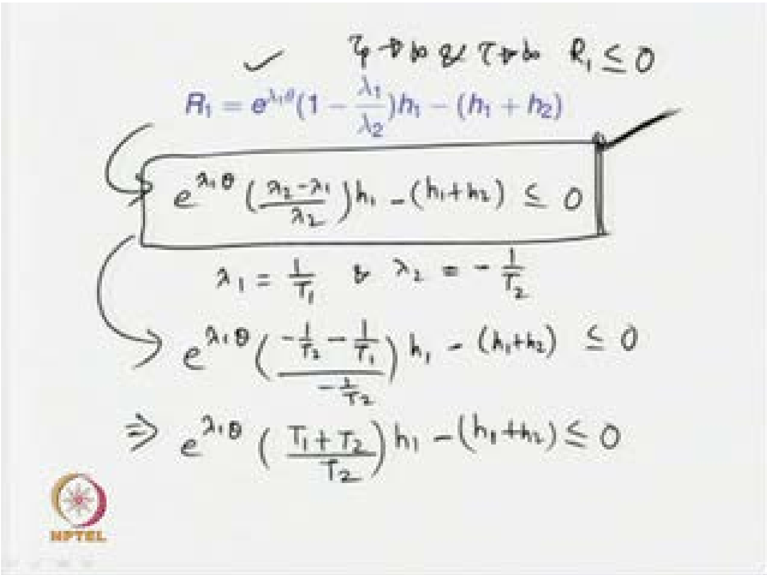

This is the condition we have obtained now, we shall write that condition finally in that form. The condition we have obtained is that R 1 given R 1 is obtained in this form. We have found that when tau p tends to infinity and tau tends to infinity R 1 is found to be less than equal to 0. So, that enables us to get e to the power lambda 1 theta lambda 2 minus lambda 1 upon lambda 2 times h 1 minus h 1 plus h 2 is less than equal to 0.

Finally, this is the condition that has to be made. Once more let me repeat this is the condition that must be satisfied to generate stable limit cycle output for open loop unstable systems. Further analysis of this expression can give us simpler expression. Let me substitute lambdas by lambda 1 is known is 1 upon T 1 and lambda 2 is equal to minus 1 upon T 2. As you have you have seen lambda 1 lambda 2s are defined and this lambda 1 is equal to 1 upon T 1 lambda 2 minus lambda 2 is equal to minus 1 upon T 2.

So, upon substitution the expression becomes e to the power lambda 1 theta times in the numerator. I will get this in the form of T 2. So, I will substitute this by minus 1 upon T

2 plus 1 upon T 1 by minus 1 upon T 2 h 1 minus h 1 plus h 2 is less than equal to 0. So, this will give further h e to the power lambda 1 theta and finally, we will get minus lambda 1 sorry. I will get here minus 1 upon; I will get in the numerator T 1 plus T 2 by T 2 finally. This time h 1 minus h 1 plus h 2 is less than equal to 0. So, I will carry on with this analysis further.

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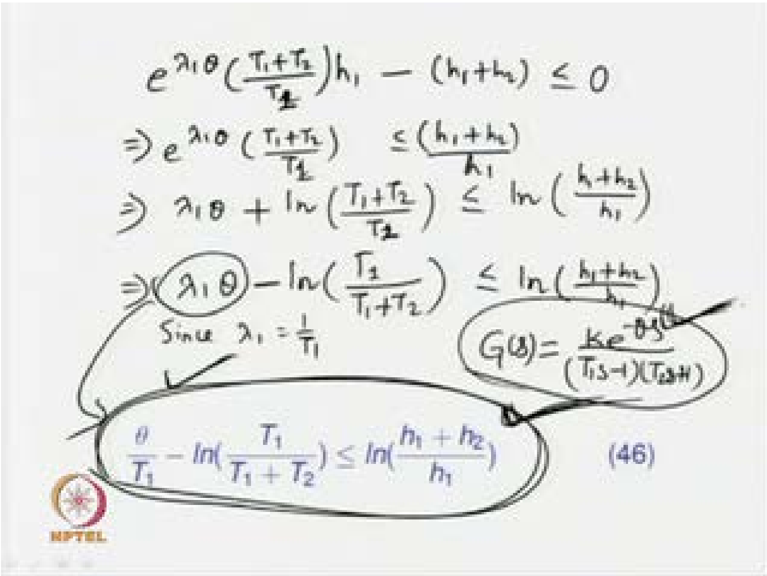

This will give me an expression of the form, e to the power lambda 1 theta T 1 plus T 2 by T 2 h 1, e to the power lambda 1 theta T 1 plus T 2 by T 2 h 1 then minus h 1 plus h 2 h 1 plus h 2 is less than equal to 0 or e to the power lambda 1 theta T 1 plus T 2 upon T 2 h 1 is less than equal to h 1 plus h 2. If you take natural logarithm of both sides, then you will get lambda 1 theta plus lon of, why not to take this h 1 to the other side for further simplification, we get all X will be in one side. Then this will give us lambda 1 theta plus lon of T 1 plus T 2 upon T 2 is less than equal to lon of h 1 plus h 2 by h 1.

Further, this can be written in the form of lambda 1 theta minus lon of T 2, where I have missed somewhere something, this is lambda 2 lambda 2 is minus T 2. It is alright, lambda 1 lambda 2 is minus 1 upon T 2 then minus lambda 1 minus 1 upon T 1 T 1 T 2 T 2. So, this will give us T 1, T 1, T 2 is the common T 2 will cancel out. We will be left with T 1 sorry that is why we are we have got some wrong expression here. So, simplification of this term will result in T 1 in the denominator then we will have T 1 in place of T 2 this will be T 1 T 1 and this is also T 1.

So, thus this will be minus lon T 1 upon T 1 plus T 2 in the denominator is less than equal to lon h 1 plus h 2 upon h 1. This is what we have got finally, if you see that since lambda 1 is equal to 1 upon T 1. The first term can be written as theta upon T 1. So, theta upon T 1 minus lon T 1 upon T 1 plus T 1 is less than equal to lon h 1 plus h 2 by h 1. This is the condition that has to be made, such that stable limit cycle output can be obtained from the open loop unstable system. If the system dynamics is of second order again let me repeat so, given a system of the dynamics of form k e to the power minus theta s T 1 s minus 1 T 2 s plus 1.

This is the dynamics of the original system then the conditions on theta T 1 T 2 and. So, on that is given over here has to be made to generate stable limit cycle output or indirectly speaking, the relay will be able to induce stable limit cycle output provided this condition on theta and T's are made. So, the relay parameters are also associated with the condition and the final condition is given as, theta upon T 1 minus lon of T 1 upon T 1 plus T 2 is less than equal to lon h 1 plus h 2 upon h 1. Then only the unstable system of this form, this type can generate or can induce or can expect a stable limit cycle output.

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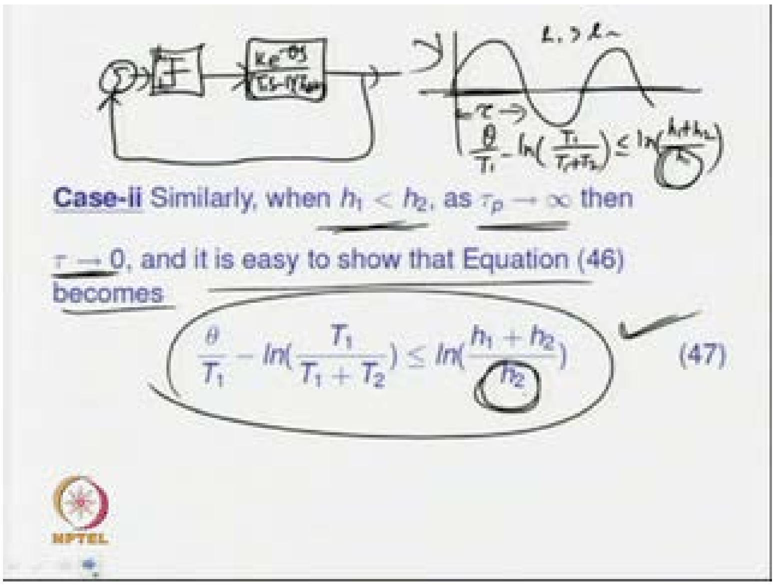

So, finally, let me rewrite these things. So, given an unstable system dynamics of the form k e to the power minus theta s T 1 s minus 1 T 2 s plus 1 and a relay in the loop with heights h 1 and minus h 2 can induce limit cycle outputs, stable limit cycle output stable limit cycle output provided the condition is met. What is that condition? the condition is theta upon T 1 minus lon T 1 upon T 1 plus T 2 is less than equal to lon h 1 plus h 2 by h 1. So, this is the condition that has to be made so that stable limit cycle output can be expected. Now when h 1 is greater than h 2, this span will be more than this span as repetitively I have told. So, this tau will be the larger span. Now when similarly, when h 1 is less than h 2 then also, we will have obtained tau p as a large number, when h 1 is less than h 2 tau p will be large, where as tau will be a small number. And using this analysis again, it is not difficult to obtain an inequality of the form theta upon T 1 minus lon T 1 upon T 1 plus T 2 is less than equal to lon of h 1 plus h 2 upon h 2. Please see the change; here in the denominator of this expression you have got h 2 in place of h 1.

When h 1 is greater than h 2, h is the user defined the user is choosing the or setting the relay parameters. That way you need not worry, you know what type of relay parameters you are using. Whether it is h 1 is greater than h 2 or h 2 is greater than h 1. Accordingly, please make use of this expression either 46 or 47 to accurately find the condition under which we will be able to generate stable limit cycle output for the open loop unstable system. Now can we use this expression further for other type of system?

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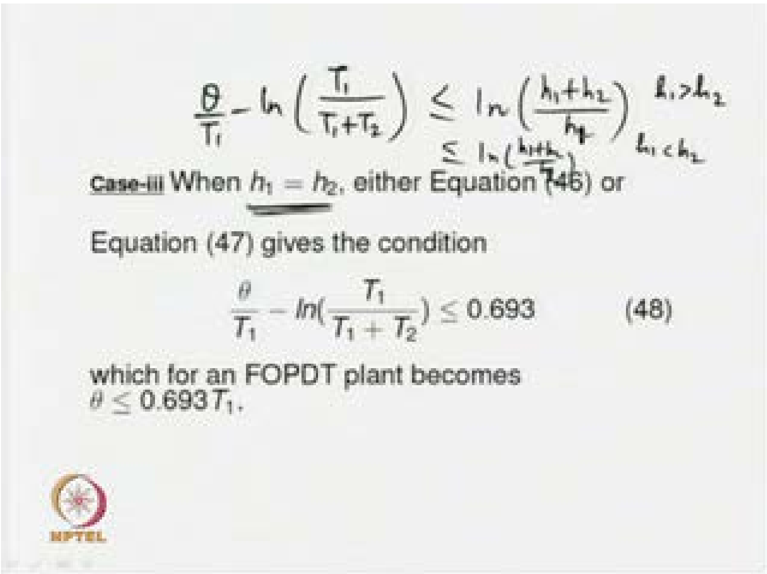

Yes we can use this for the systems where the relay parameters are same. When h 1 equal to h 2 the two expressions obtained earlier can be, what are those expressions?

Theta upon T 1 minus lon T 1 upon T 1 plus T 2 is less than equal to lon of h 1 plus h 2 upon h 2. This is what you got, when h 1 is greater than h 2. And you have got the second one lon h 1 plus h 2 upon h 1, when you got h 1 is less than h 2. So, these are the two expressions you have got already, when h 1 is less than h 2 you have got h 2 here h 1 here sorry.

You have got the expressions, now when h 1 equal to h 2 what happens? when h 1 equal to h 2, then you will have 2 h 1 in the numerator and h 1 in the denominator leaving us lon two term in the second half.

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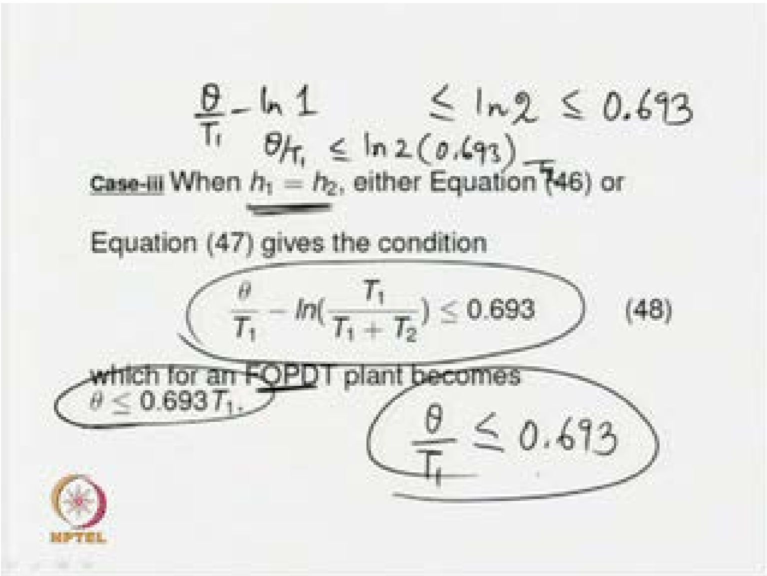

So in that case, basically theta upon t 1 minus lon T 1 T 1 upon T 1 plus T 2 has to be less than equal to lon 2 which is nothing but 0.693 and that is how we have got the expression given in equation number 48. When h 1 equal to h 2, when the symmetrical relay test is conducted at that time the condition for obtaining stable limit cycle output is given by theta upon T 1 minus lon tau T 1 upon T 1 plus T 2 is less than equal to 0.693. Can we extend this for simple unstable first order plus dead time unstable plant? We can extend, because for the first order plus unstable plant whose dynamics is given by G s is equal to k e to the power minus theta s T 1 s minus 1 only. Please keep in mind, the dynamics of a first order plus dead type plant is given by G s is equal to k e to the power minus theta s upon T 1 s minus 1 where T 2 is equal to 0. Substitute T 2 equal to 0 there. Simply if T 2 equal to 0, then I will be divide of this term and I will finally get, minus lon T 1 upon T 1 which is nothing, but again 1, that will be lon 1. Lon 1 is equal to 0 giving us theta upon T 1 is less than equal to lon 2, which is same as is less than equal to 0.693. So, that is the condition is given over here. For inducing limit cycle or sustain oscillatory output in the case of first order plus dead time unstable systems, the condition for obtaining stable limit cycle output is given by theta upon T 1 is less than equal to 0.693. And unless we obtain stable limit cycle output, we cannot make measurements and we will not be able to identify the parameters of the dynamic model or transfer function model of the dynamic system.

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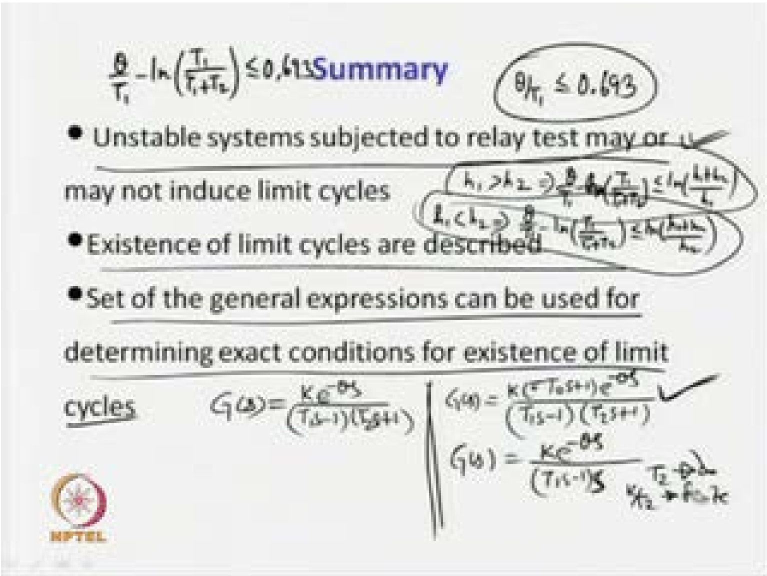

This is how the analysis goes. Now, I will summarize my lecture unstable systems subjected to relay test may or may not induce limit cycles. The meaning of this sentence is particularly, the relay may not be able to induce limit cycle output for unstable systems unless the conditions are made. What are those two conditions we have found? the conditions were h 1 is greater than h 2 is obtained as theta upon T 1 minus lon T 1 upon T 1 plus T 2 should be less than equal to lon of h 1 plus h 2 by h 1. And when h 1 is less than h 2 the condition that is to be made is theta upon T 1 minus lon T 1 upon T 1 plus T 2 should be less than equal to lon h 1 plus h 2 by h 2. And for first order plus dead time systems the condition is theta upon T 1 is less than equal to 0.693. And when symmetrical relay are used to induce limit cycle output, in that case the condition is given as theta upon T 1 minus lon T 1 upon T 1 plus T 2 is less than equal to 0.693.

So, these are the four equations one has to keep in mind or have has to apply to find, the condition under which it is possible to generate limit cycle output for an open loop unstable system. We have described in detail the analytical expression that gives us the inequalities that have to be made or the conditions that are to be made to induce limit cycle outputs. Now set of the general expressions can be used for determining exact conditions for existence of limit cycle. What I mean by this?

I have considered only few typical cases, where the G s is assumed to be of the form k e to the power minus theta s upon T 1 s minus 1 T 2 s plus 1. Now what about other type of processes unstable processes? when the unstable process G s is having 0 k plus minus plus t 0 as plus 1 e to the power minus theta s T 1 s minus 1 T 2 s plus 1. For this case please make use of the condition, that lambda 3 is not equal to infinity. Then you will get similar analytical expressions involving lambda 3 in all those expressions. Similarly, what other cases are there, you can use the same set of analysis for integrating processes also where G s can be of the form k e to the power minus theta s T 1 s minus 1 with s, where T 2 tends to infinity such that k upon T 2 is finite. The point of telling all those things are that yes the point of telling all those things are that it is possible to make use of the analysis or extend the analysis for many such cases and different type of unstable processes. So, this is what we have studied in this lecture.

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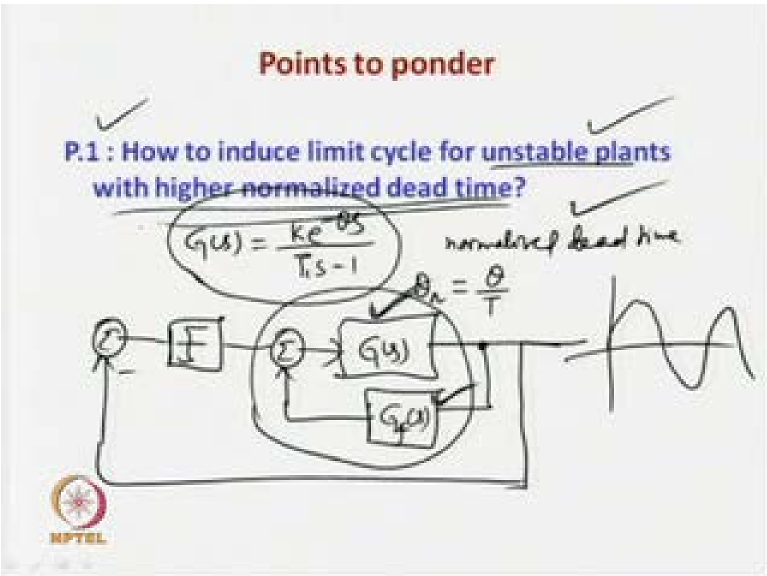

Now coming to the question that, how to induce limit cycle for unstable plants with higher normalized dead time? What you mean by dead, normalized dead time? Given G s of the form k e to the power minus theta s upon T 1 s minus 1, this is the dynamic model of a first order plus dead time transfer function. Now, the dead normalized dead time normalized dead time normalized dead time is expressed by the ratio theta upon T 1 that we show as theta n. So, the normalized dead time is theta upon T 1. How to induce limit cycle output for unstable plants with higher normalized dead time? We have found the condition, that theta upon T 1 minus lon T 1 upon T 1 plus T 2 is less than equal to lon of h 1 plus h 2 upon h 1.

Now, for higher theta upon T 1 ratio, what is to be done? You have to do something, you have to basically stabilize the open loop unstable plant, and that is how? That is the only way we can extend the condition on this normalized dead time, we can relax the condition on normalized dead time to induce limit cycle output.

As already I have described earlier, the simplest way to increase the normalized dead time or condition on normalized normalized dead time is that, you provide some inner feedback path, with a feedback control derivative feedback controller. So, and provide a relay over here, and conduct the limit cycle test or relay test. Then the output is expected to give sustained oscillatory output. Why that is so? Basically, the open loop unstable system or process will get stabilized or it the poles of the open loop unstable system will get relocated with the help of this inner feedback controller, and subsequently it will be possible to obtain or induce limit cycle output. So, this inner feedback controller is to be chosen properly, so with the proper choice of this inner feedback controller - feedback controller G f(s), it is possible to induce limit cycle output for unstable plants with higher normalized dead time. That is all. Thank you