Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 10

Identification of SOPDT Model With Pole Multiplicity

Welcome to the lecture, titled identification of second order plus dead time model with pole multiplicity. Earlier, we have derived a set of analytical expressions for identifying parameters of the second order plus dead time transfer function models, but we have certain limitations as we have discussed earlier, when there will be pole multiplicity, one has to take care of the analytical expressions. In this lecture, we shall discuss how the same set of analytical expressions can be used to identify transfer function models with pole multiplicity.

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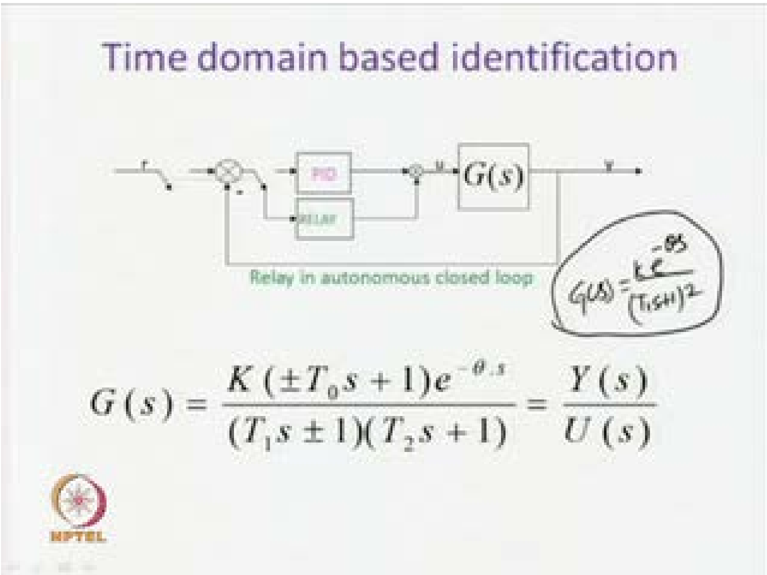

We shall say how we can identify a model transfer function of the form G(s) is equal to K e to the power minus theta s upon T 1 s plus 1 square. This type of model is known as models with pole multiplicity.

Now, before going to that, I would like to repeat certain things, we have already discussed in our last lectures. The set of analytical expressions, we have derived for identifying the second order plus dead time transfer function models. Now, the relay feedback system is arranged in this passion, when the relay test is conducted at the time the reference input is set to zero. And we have also assumed earlier the transfer function model that are identified to be of the form G(s) is equal to k times plus minus T 0 s plus 1 e to the power minus theta s upon T 1 s plus minus 1 times T 2 s plus 1. Now, the G(s) is the dynamics of the actual system which model transfer function model is to be identified.

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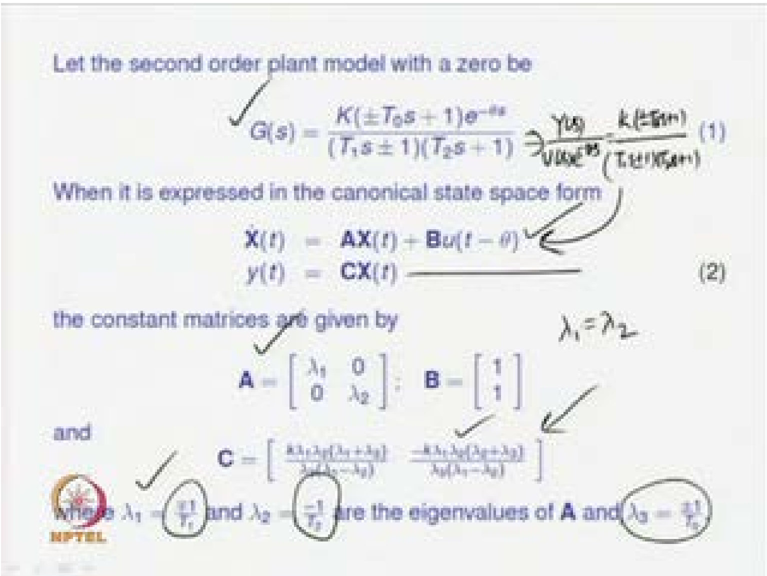

Now, what we have done earlier to identify to design a set of to derive a set of analytical expressions. We have to express the transfer function model in state space form, where we got the state equation of the form X dot t is equal to A X t plus B u t minus theta. So, we assume that the delay is there in the input. Therefore, this G s in equation number 1 can also be written in the form of Y s upon U s e to the power minus theta s is equal to K then plus minus T 0 s plus 1 upon T 1 s plus minus 1 times T 2 s plus 1. So, this is how we have formed or got the state equation given in equation number 2.

Similarly, the output equation is expressed as Y t is equal to C X t. Now the A B C constants of the state and output equations are given as, A in diagonal form as lambda 1 0 0 lambda 2 and B as 1 1 and c as k lambda 1 lambda 2 lambda 1 plus lambda 2 upon lambda 3 times lambda 1 minus lambda 2 and minus k lambda 1 lambda 2 times lambda 2 plus lambda 3 upon lambda 3 lambda 1 minus lambda 2. Where, lambda 1 is equal to minus plus 1 upon T 1 and lambda 2 is equal to minus 1 upon T 2 minus 1 upon t 2 and that of the lambda 3 is given as lambda 3 is equal to plus minus 1 upon T 0.

Now, it is evident from equation from the constant c that when lambda 1 is equal to lambda 2. We cannot get the state space presentation correctly or indirectly speaking; now this state space equation can be used when lambda 1 is not equal to lambda 2. With that condition, we started our analysis and we found a set of analytical expressions using the output wave form.

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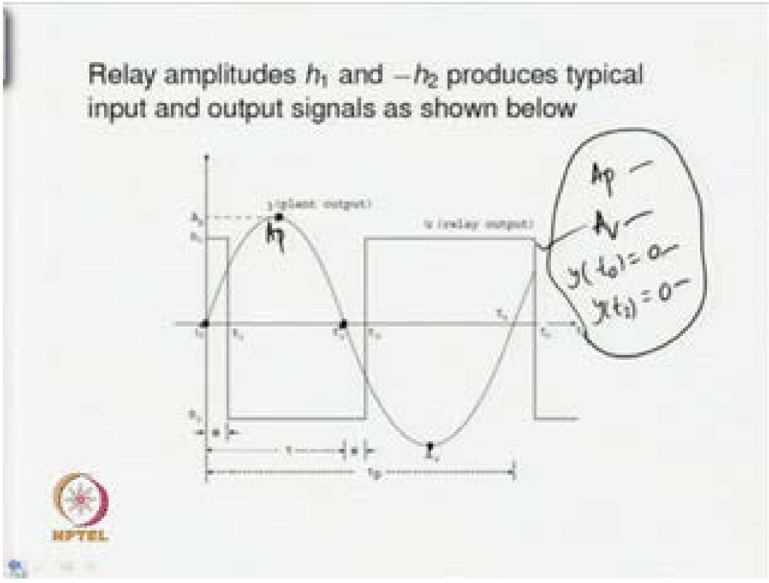

What is that output form we got? This is the type of limit cycle output we got, when the relay test was conducted. So, the relay test resulted in some typical output wave form of this form, where as the input to the system relay output is given in the rectangular signal form. Now, this output signal has got some critical points like the zero crossings zero crossings and peak amplitude and the negative peak amplitude or the (( )) amplitude. So, that way we identify these four points and derive analytical expressions for these four points resulting in four general expressions. Those are given by equation number 17, equation number 18 and equation number 31 and equation number 28.

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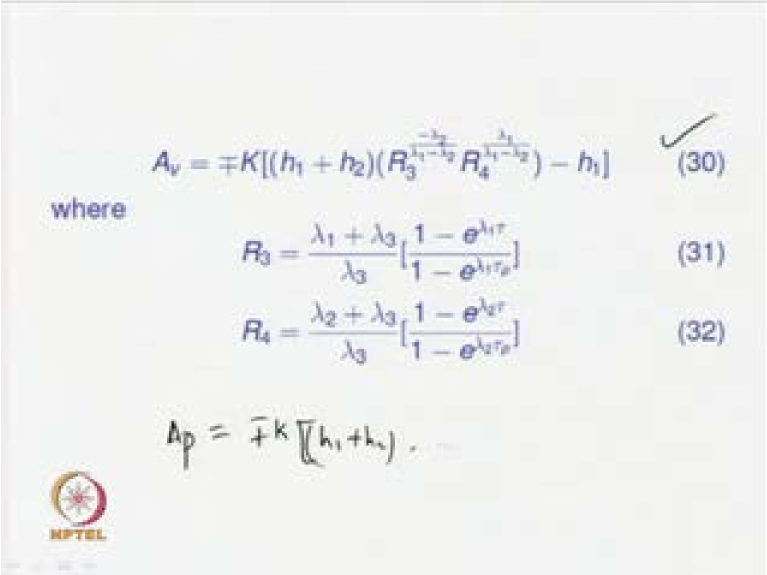

Perhaps, I have missed the expression for A p. So, A p also will get in the form of k a then h 1 plus h 2 and so on. Basically what we have done so far that, using the output wave form, some critical points of the output way form we have been we have been able to derive four analytical expressions corresponding to the peak amplitude A p is the peak amplitude. Similarly, the negative peak A v and the first zero crossing occurring at time T 0 t 0. Assume that y t 0 equal to 0 and y t 2 is equal to 0. So, these four outputs or four points of the output signal results in four analytical expressions.

Now, those analytical expressions can be used to estimate the parameters of the second order plus dead time transfer function models like, the steady state gain T 0, T 1, T 2 and theta provided lambda 1 is not equal to lambda 2. This point is to be taken care of. Now lambda 1 is equal to lambda 2, at that time the dynamic model will have pole multiplicity. I mean to say this expression analytical expressions are valid provided, lambda 1 is not equal to lambda 2. So, this is correct through provided lambda 1 is not equal to lambda 2.

Similarly, the expression A v is also correct or applicable for the case that lambda 1 is not equal to lambda 2. Now, when lambda 1 equal to lambda 2, what happens to the state space equation? Can we obtain the state space representation in the same form will little changes that we shall discuss in this lecture.

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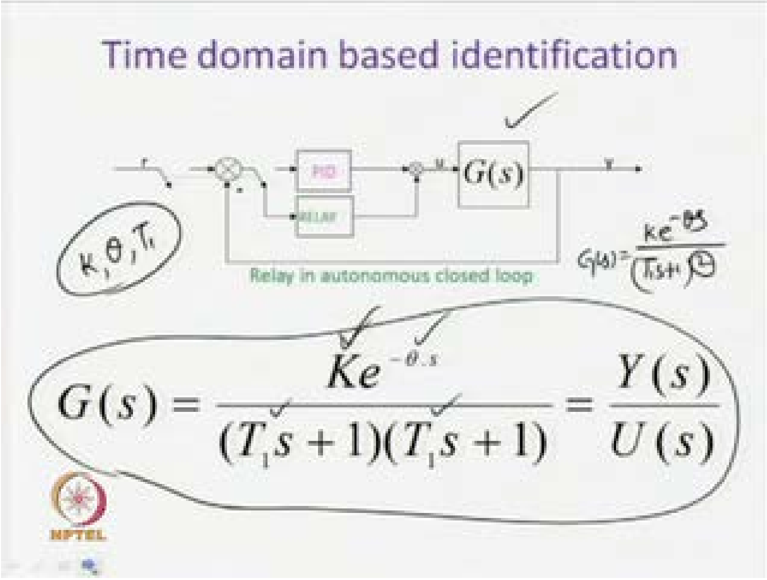

I will begin with the relay control system, where the relay system relay will be subjected or the relay will be experiencing a dynamic system with pole multiplicity. Let us assume that, G s is equal to K e to the power minus theta s upon T 1 s plus 1 times T 1 s plus 1. So, I can write G s as K e to the power minus theta s upon T 1 s plus 1 square. This is what we mean by pole multiplicity, we can have more number of poles also. Let us begin with the simplest case, where the system dynamics is assume to have got two identical poles located at the same place of the s plain.

Now in this case also, we can find the dynamic equations for identification of transfer function model parameters. In this identification this transfer function has got, how many unknowns now? We have got four unknowns, the steady state gain k, the time delay theta, the time constant t 1, and the time constant t 1. As evident from this one, in place of four unknowns actually how many unknowns? We have k, theta, and t 1. There are basically three unknowns associated with this second order plus dead time transfer function model with pole multiplicity. So, we need to derive three analytical expressions those that can be used to identify the model parameters. Now, why to repeat all those things and find three analytical expressions? If it is possible to use the previous earlier derived general expressions, then our life will be easy why to go for further analysis of this typical model.

Then effort will be made now to show that, the transfer the state space representation of this second order plus dead time model with pole multiplicity can be given by equation number 2. I mean, can we get the same sort of state space representation in a further model with pole multiplicity that, we will verify now.

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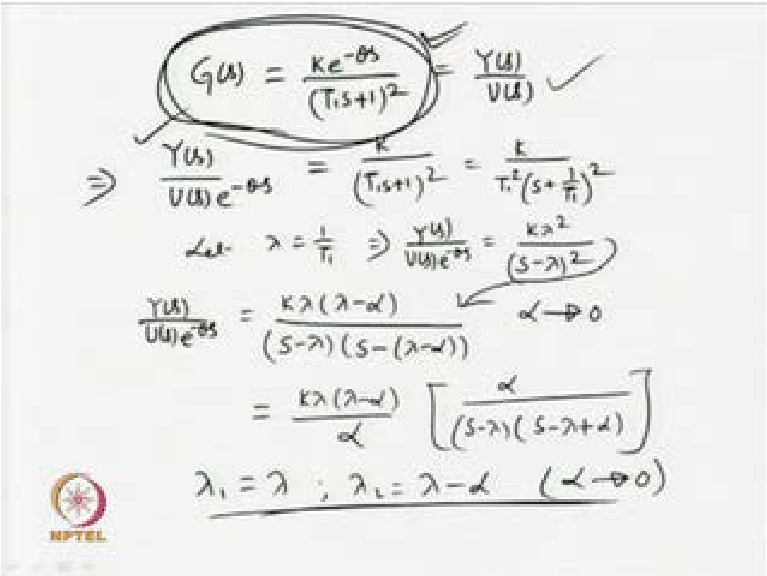

I will begin with the second order plus dead time transfer function model with pole multiplicity. Where there will be two poles located at the same locations. Now, let G s be given by the steady state gain k e to the power minus theta s upon T 1 s plus 1 square. Now, I can write this G s in the form of Y s upon since G s is equal to Y s upon U s. it is not difficulty to write, Y s upon U s times e to the power minus theta s as K T 1 s plus 1 square. Because our input to the plant is assume to be delete inputs, when the plant is subjected to relay test. Therefore, please allow me to write, Y s upon U s e to the power minus theta s as K upon T 1 s plus 1 square.

Now, this can be further be simplified in the form of, K T 1 square s plus 1 upon T 1 square. Which again, let lambda is equal to 1 upon T 1 implies Y s upon U s e to the power minus theta s is equal to K lambda square upon S minus lambda square. So, this again can be expressed as, Y s upon U s e to the power minus theta s is equal to K lambda lambda minus alpha upon s minus lambda s minus lambda minus alpha. So, what I have done here, I have introduced a small number, where the small number lambda tends to 0. When lambda tends to 0, I have basically got the same expression.

So, Y s upon U s times e to the power minus theta s becomes K lambda square upon s minus lambda square, when alpha tends to 0. Always it is possible to write this expression in this form provided alpha tends to 0. Why I have done so, we will see the effort is to bring this state space representation of this dynamics in the standard form. That we have obtained earlier for the second or general second order plus dead time model case. Then it is possible to further express this expression in the form of, k lambda lambda minus alpha by alpha times alpha by s minus lambda times s minus lambda plus lambda. Then next what I will do, I will write this in the partial fraction expansion form.

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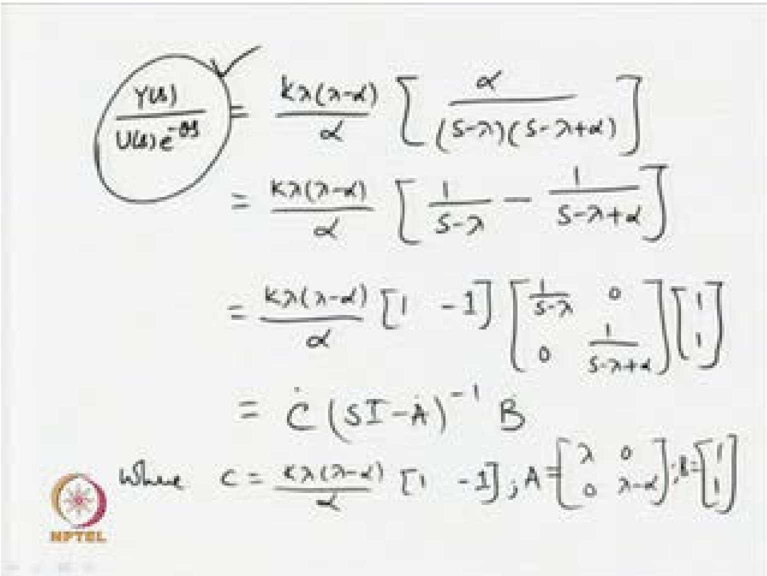

Now this same expression now, Y s upon U s e to the power minus theta s, which is given as k lambda lambda minus alpha upon alpha alpha upon s minus lambda s minus lambda plus alpha. When expressed in the partial fraction expansion form gives us, k lambda lambda minus alpha upon alpha 1 upon s minus lambda minus 1 upon s minus lambda plus alpha. Please keep in mind that, alpha tends to zero. Only then we are getting the same expressions that, we that are analysis begin with. Then this expression further can be written in the form of, k lambda lambda minus alpha by alpha times 1 minus 1 with 1 upon s minus lambda 0 0 1 upon s minus lambda plus alpha times 1 1. So, basically what I have been doing? I am trying to find the same dynamic model using the state space equation constants.

What is the state space equation constant? If you look minute carefully minutely observe this one, what I have tried to do? I am trying to get the same transfer function model using the equation C S I minus A inverse B. We know that for linear time in variant system, we have the transfer function model to state space conversion and state space model to transfer function model conversion with the help of the standard equation given by, C times s I minus A inverse B. Where, C A and B are the constants of the state equation. In that case, often comparison we get that the same transfer function model can be obtained using the constant C A and B. Where, C is equal to now K lambda lambda minus alpha upon alpha times 1 minus 1. It is of dimension one in to two.

Now similarly, A is now obtained has A is now obtained has lambda 0 and 0 lambda minus alpha. And B is obviously are 1 1. Thus, we have got a state space representation of the dynamic second order plus dead time model with pole multiplicity, but there are two poles given by a state equation.

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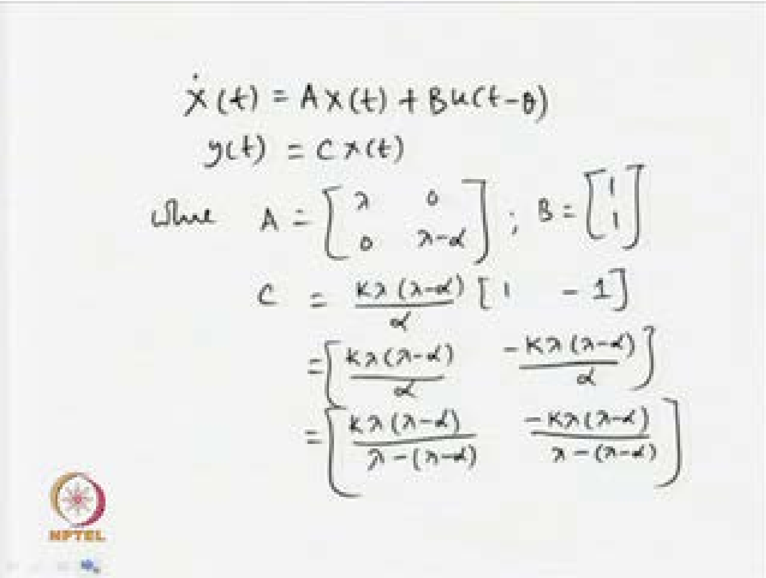

where the state equation is now, X dot t is equal to A X t plus B u t and y u sorry u t minus theta y t is equal to C X t. Where, A is found to be lambda 0 0 lambda minus alpha, B is 1 1, where at C is given as k lambda lambda minus alpha upon alpha times 1 minus 1. Allow me to write the c in some other form now. Other means, I will simplify the C now. C can also be extended to the form of, K lambda lambda minus alpha by alpha minus K lambda lambda minus alpha by alpha.

Again this can be written as, K lambda lambda minus alpha by lambda minus lambda minus alpha in the denominator minus K lambda lambda minus alpha by lambda minus lambda minus alpha. So, why we are doing? We are trying to get the state equation represented in the original form. What is that original form? We have got the system state equation represented by this form. Where, X dot t equal to A X t plus B U t minus theta Y t equal to C X t have got A, B, C given in this form. Now basically, I have been able to obtain this standard form using analysis.

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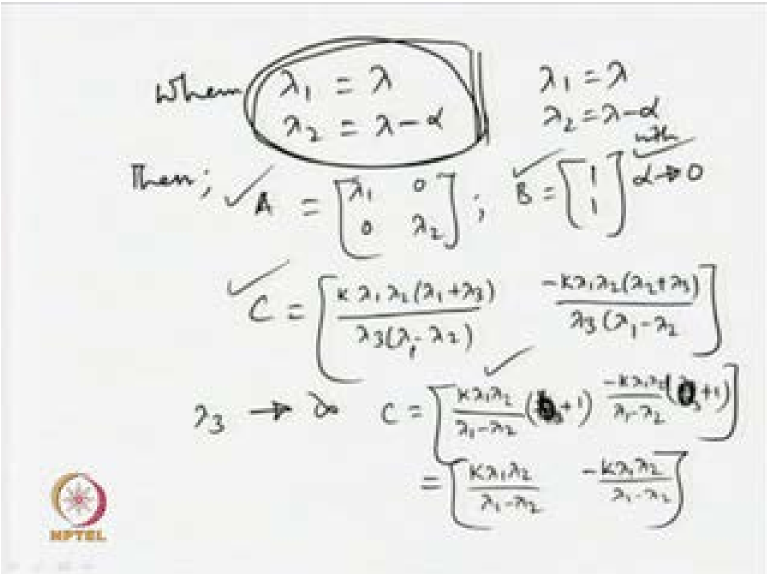

When that is possible that is possible provided I assume that, when lambda 1 equal to lambda and lambda 2 becomes lambda minus alpha. Then whatever A, B, C we have got, then A becomes lambda 1 0 0 lambda 2; B equal to 1 1 and obviously, C will be now in the standard form of k lambda 1 lambda 2 lambda 1 plus lambda 3 by lambda 3 lambda 1 minus lambda 2 minus k lambda 1 lambda 2 lambda 2 plus lambda 3 by lambda 3 lambda 1 minus lambda 2. This is the standard form.

So, we can obtain the same A, B, C or we can express this A in the standard form with the assumption of, lambda 1 equal to lambda and lambda 2 is equal to lambda minus alpha, and B no changes. Already we have got the B in the specified form. Now what about the C? C is available in this form and arranging it a little bit, where I am substituting lambda by lambda 1 and lambda minus alpha by minus lambda 2.

Therefore, I get a term as lambda 1 minus lambda 2 in the denominator, but we have got additional terms are also. In the general expression for the C therefore, with the assumption further assumption of when lambda 3 tends to a large number infinity. In that case, then what C becomes c becomes? When lambda 3 tends to infinity, then I will get this in the form of, k lambda 1 lambda 2 upon lambda 1 minus lambda 2 times 1 upon lambda 1 upon lambda 3 plus 1. And the second term will be similarly, K minus K lambda 1 lambda 2 and lambda 1 minus lambda 2 in the denominator with lambda 2 by lambda 3 plus 1. And as we have assumed that lambda 3 tends to 0.

Therefore, this will be 0. And similarly, this will be 0. We are multiplying this factor by one, only resulting in the expression for the C as K lambda 1 lambda 2 upon lambda 1 minus lambda 2 minus K lambda 1 lambda 2 upon lambda 1 minus lambda 2. So, finally the reason for doing all these analysis is that, with proper assumptions simple assumptions or introduction of the terms that, when lambda 1 equal to lambda, when lambda 2 equal to lambda minus alpha with the constraint that alpha tends to 0 alpha tends to 0. At that time it is possible to find the transfer function model, as a transfer function model with pole multiplicity. We are dealing with a second order plus dead time delayed transfer function model and where we have got a two dimensional A B and C.

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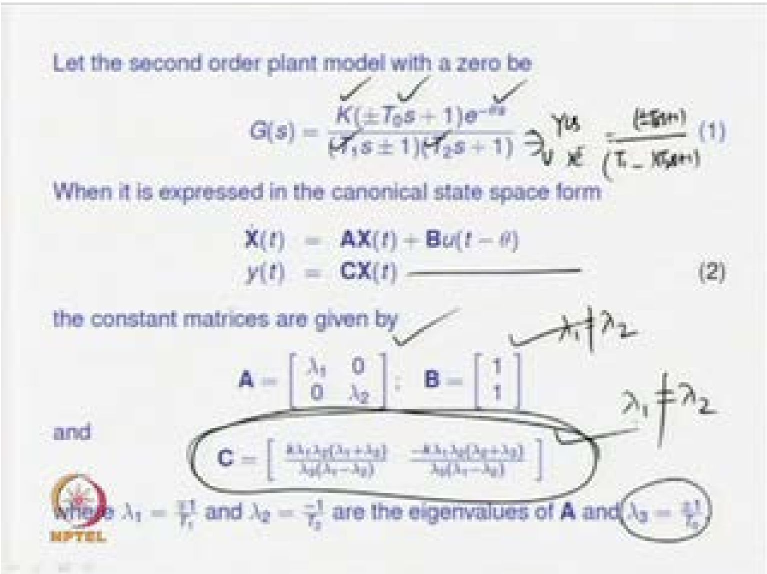

Ultimately this same set up, I mean this general expression can also be obtained in the form of this, G s can be expressed in the form of a transfer function model will pole multiplicity K e to the power minus theta s upon T 1 s plus 1 square. When lambda 1 is equal to... already we have got lambda 1 lambda 2 and lambda 3 keep in mind. When lambda 1 equal to lambda lambda 2 is equal to lambda minus alpha, where again alpha tends to 0. Please keep in mind alpha tends to 0 then only you will get the dynamic model in the form of model with pole multiplicity.

Again to obtain this second order plus dead time transfer function model, we have to assume model with no zero particularly, we have to assume that lambda 3 tends to a large number. If you allow me to make this assumption, then all the analytical expressions we have derived so far can be extended to identify the transfer function model parameters of these models with pole multiplicity. Because one has to properly substitute the lambda values the lambda 1, lambda 2, and lambda 3 then the same set of analytical expressions as I have said or shown earlier.

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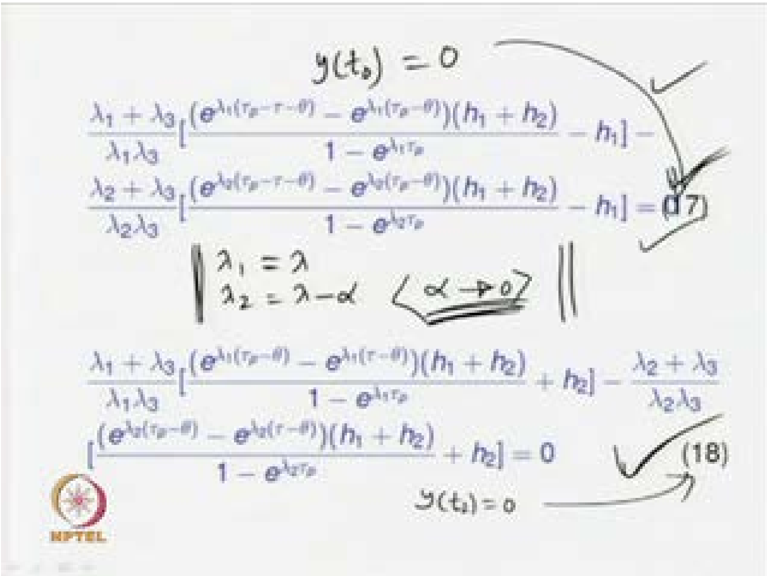

What are those analytical expressions? What we have obtained these is the analytical expression. Now, these analytical expressions are not valid or cannot be used when lambda 1 is equal to lambda 2. To avoid that, what has to be done in place of for the case of system or system dynamics with pole multiplicity? What the lambda values are to be taken? Now, lambda 1 can be assume that lambda lambda 2 will be lambda minus alpha. With alpha tends to 0 and lambda 3 can be a large number to identify the transfer function model with no zeros.

When you have a 0 in the transfer function model, then this constant is not required. lambda 3 can be as it is. There is no constant on lambda 3 as far as using the analytical expressions are concerned. Therefore, I can allow I can make use of equation number 17, equation number 18, which are obtained from the condition that the output at the first zero crossing is equal to 0, that results in equation number 17.

Similarly, the output at the second zero crossing is equal to 0 results, in equation number 18. These analytical expressions can be used for our case or the case with a plant with poled multiplicity provided, the lambdas are chosen in this form. Otherwise one cannot use, because you have seen the limitation the limitation is evident when you simply look at the C constant or the C. where it is you were not allowed to use lambda 1 is equal to lambda 2. It cannot be, when lambda 1 is equal to lambda 2. I cannot use C and subsequently, I cannot use the analytical expressions. So, to use the analytical expressions, please assume lambda 1 to be equal to lambda lambda 2 to be equal to lambda minus alpha where, alpha tends to 0.

With this assumption, it is possible to use the four analytical expressions we have derived for identifying the general transfer function model. Whatever I have thought so far, let me repeat. So, the set of analytical expressions, that we have derived so far can be or can be used for identifying a transfer function model of the form G s is equal to K e to the power minus theta s upon T 1 s plus 1 square provided... again let me repeat provided lambda 1 is equal to lambda lambda 2 is equal to lambda minus alpha with the condition alpha tends to 0 is used in the set of analytical expressions. How to find explicit expressions for this case? To identify this sort of transfer function model using the same general analytical expressions that we shall see subsequently.

Let us go to one case, how can we use the same analytical expressions we have used found earlier for identification of the second order plus dead time transfer function model with pole multiplicity? Now, we have found an expression for the negative peak output. When this plant is subjected to relay control, as A v is equal to minus plus k times h 1 plus h 2 times R 3 power minus lambda 2 upon lambda 1 minus lambda 2 R 4 times R 4 power lambda 1 upon lambda 1 minus lambda 2 minus h 1.

Where, lambda 3 and lambda 4 are given by equations 31 and 32. So, this is what already we have derived earlier in our earlier lectures.

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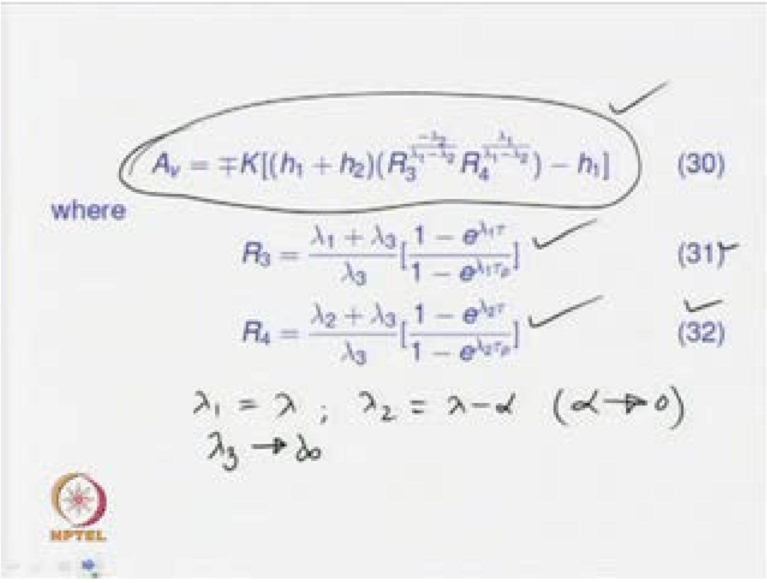

Now, how can we use those conditions to find simpler expressions for A v, when effort is made to identify second order plus dead time transfer function models with pole multiplicity. Now, I will go in that direction. Now, this R 3 and R 4 can be simplified further, when lambda 1 and lambda 2 are substituted by lambda 1 becomes lambda lambda 2 becomes lambda minus alpha with the condition alpha tends to 0. When this is used, then R 3 and R 4 can be obtained further in simpler form.

So, how our R 3 becomes? Now, R 3 is equal to... let me use the values here again we have seen that lambda 3 is there. I have to impose a condition on that also; I know that lambda 3 tends to infinity. when that is now, R 3 will be obtained in the form of R 3 will be equal to 1 minus e to the power lambda tau upon 1 minus e to the power lambda tau p. Similarly, R 4 becomes 1 minus e to the power lambda minus alpha tau upon 1 minus e to the power lambda minus alpha tau. Put please keep in mind, how I am getting this R 3 expression for R 3 and R 4? I am obtaining from here only, with the substitution of lambda 1 is equal to lambda lambda 2 is equal to lambda minus alpha and lambda 3 tends to infinity.

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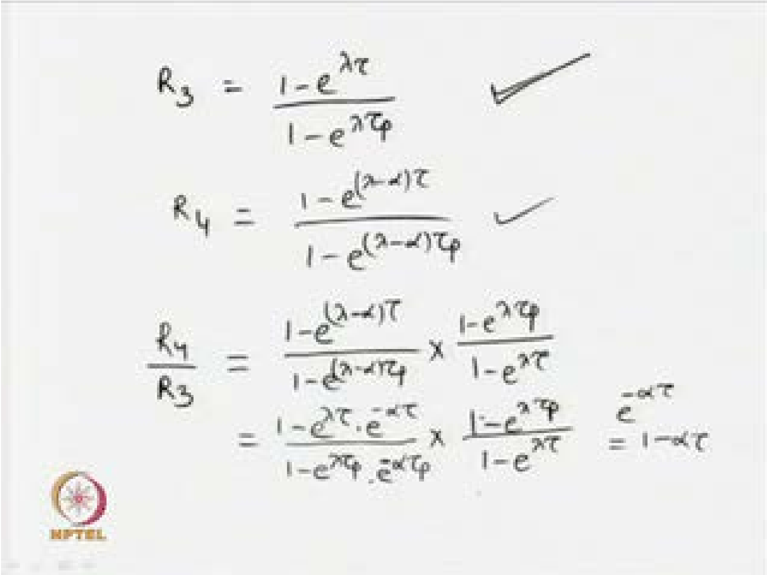

When lambda 3 tends to infinity, how do you get R 3 and R 4? obviously, simply in the form of as I have done earlier lambda 1 by lambda 3 plus 1 times, whatever you get now since lambda 3 tends to 0 this becomes zero and you are simply getting a term or an expression of the form 1 minus e to the power lambda 1 tau upon 1 minus e to the power lambda 1 tau p. Similarly, R 4 becomes 1 minus e to the power lambda 2 tau upon 1 minus e to the power lambda 2 tau p.

Now, when I substitute lambda 1 by lambda lambda 2 by lambda minus alpha, I get R 3 h 1 minus e to the power lambda tau upon 1 minus e to the power lambda tau p and, R 4 as 1 minus e to the power lambda minus alpha tau upon 1 minus e to the power lambda minus alpha tau p. I have to take the ratio of the two allow me to take the ratio of the two. When I found find a ratio of R 4 upon R 3. How it looks like, now R 4 upon R 3 will give us 1 minus e to the power lambda minus alpha tau upon 1 minus e to the power lambda minus alpha tau p into. So, R by R 3 therefore, 1 minus e to the power lambda tau p will go to the numerator and we will have 1 minus e to the power lambda tau in the denominator.

Now, I can write this in the form of 1 minus e to the power lambda tau times e to the power minus alpha tau. Similarly, the denominator term will be 1 minus e to the power lambda tau p times e to the power minus alpha tau p and rest of the things will remain as it is, 1 minus e to the power lambda tau p upon 1 minus e to the power lambda tau.

Again since alpha is a small number, the exponential term keep in mind e to the power minus alpha tau can be approximated as one minus alpha tau since alpha tends to 0. So, using that, I can write that numerator as 1 minus e to the power lambda tau plus or let me rewrite this in detail so, that we will not skip any in between expressions.

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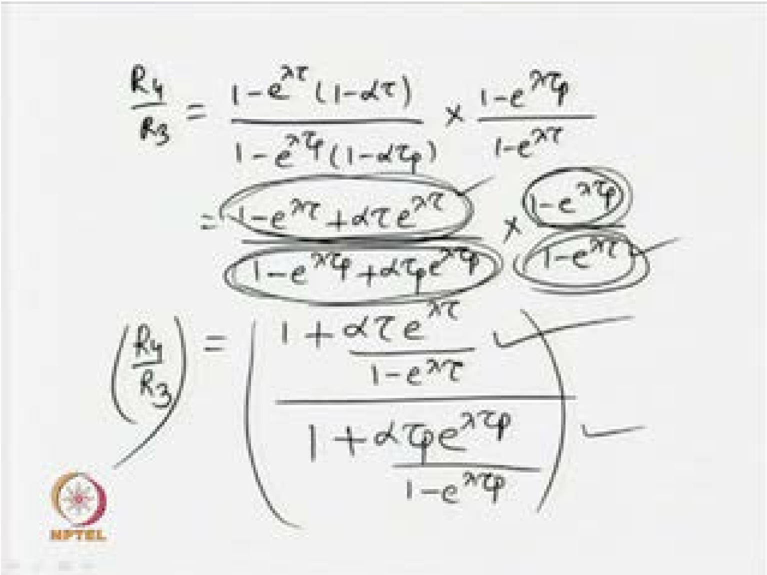

Finally, R 4 by R 3 can be written as 1 minus e to the power lambda tau 1 minus alpha tau divided by 1 minus e to the power lambda tau p 1 minus alpha tau p in the denominator. And further will have the terms, 1 minus e to the lambda tau p upon 1 minus e to the power lambda tau. So, upon expansion the numerator of this will give us 1 minus e to the power lambda tau plus alpha tau e to the power lambda tau by 1 minus e to the power lambda tau p plus alpha tau p e to the power lambda tau p times 1 minus e to the power lambda tau p upon 1 minus e to the power lambda tau.

Now, I will divide the numerator term this by this. That will give me an expression of the form 1 plus alpha tau e to the power lambda tau by 1 minus e to the power lambda tau. What I have done, I have divided this term by this similarly, dividing this term by this. I have to bring this to the down. So, that way that will enable me to write this as, one yes 1 plus alpha tau p e to the power lambda tau p upon 1 minus e to the power lambda tau p. I have believed that, how I have got this expression. You have followed now, why I have divided this by this and similarly, this by this.

That results in the expression R 4 by R 3 ratio in the form of, 1 plus alpha tau times e to the power lambda tau upon 1 minus e to the power lambda tau in the numerator and 1 plus alpha tau p e to the power lambda tau p upon 1 minus e to the power lambda tau p in the denominator. So, we are getting this in some convenient form. Now, why we are doing so? What is the purpose of getting this R 4 upon R 3 ratio? As you see this peak amplitude or negative peak amplitude has got R 3 and R 4 expressed in this form. And when I substitute lambda 1 by lambda and lambda 2 by lambda minus alpha your R 3 and R 4 will give will be available in the ratio form. That is why I am trying to find.

Let me write A v correct A v form with the substitution proper substitution then, what will be the expression for A v? Now, the same expression or equation number 30 can be written as A v is equal to minus plus K h 1 plus h 2 with R 3 minus lambda 2.

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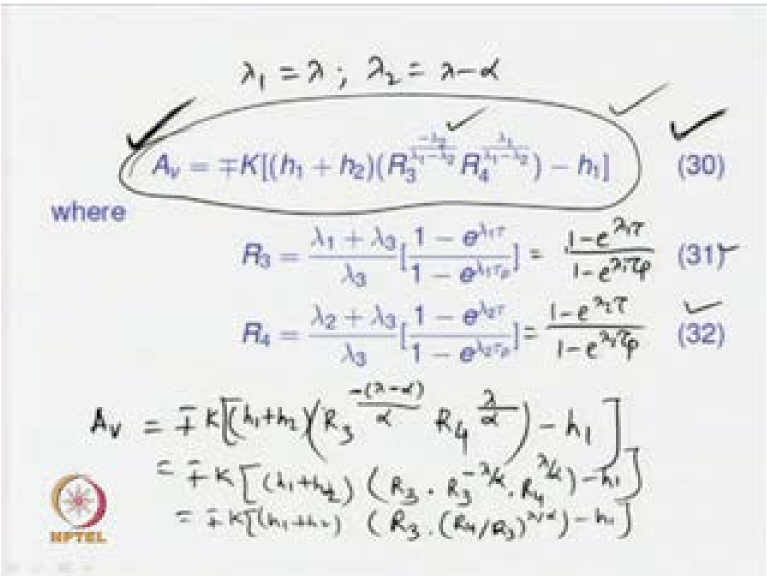

So, minus lambda minus alpha please keep in mind minus lambda minus alpha upon in the lambda 1 minus lambda 2 is nothing but alpha. What I am trying to do? I am substituting lambda 1 by lambda and lambda 2 by lambda minus alpha. that gives me R 3 to the power minus lambda 2 upon lambda 1 minus lambda 2 as R 3 times minus lambda minus alpha by alpha. Similarly, the next term becomes R 4 alpha by sorry this will be lambda 1 is lambda so, lambda by alpha. Then we have got minus we have got minus h 1.

if you see if you further expand this one, how I get minus plus k h 1 plus h 2 times this is become this becomes R 3 times R 3 to the power minus lambda by alpha times R 4 to the power lambda by alpha s minus h 1. Further it can be written in the form of, minus plus k h 1 plus h 2 then R 3 times R 4 by R 3 to the power lambda by alpha minus h 1. That is why I have taken the ratio R 4 upon R 3. Now we have got A v expressed in the form of A v has become So, A v A v A v has become available in the form of minus plus k h 1 plus h 2 times R 3 with R 4 by R 3 to the power lambda by alpha minus h 1.

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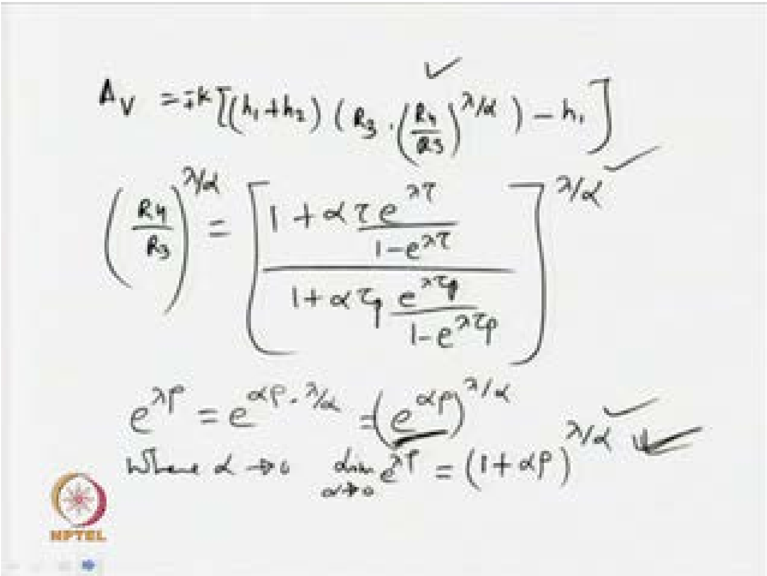

This is what we have got an expression for A v, but we have got R 4 by R 3 R 4 by R 3 is equal to 1 plus alpha tau e to the power lambda tau upon 1 minus e to the power lambda tau by 1 plus alpha tau p e to the power lambda tau p upon 1 minus e to the power lambda tau p. How to find? Now R 4 upon R 3 to the power lambda by tau, because this will be whole to the power lambda by tau yes it is possible to find simpler expression for this R 4 upon sorry this will be R 3 R 4 upon R 3 to the power lambda by pi.

It is not difficult to find, I will use some identity. How it can be used? We know that, e to the power some lambda rho can be return as e to the power alpha rho times yes e to the power alpha rho times lambda by alpha yes. So, I can write this as this one which is ultimately e to the power alpha rho to the power lambda by rho and when alpha tends to 0. Alpha is a small number the exponential term, e to the power. e to the power alpha rho can be expanded in the form of 1 plus alpha rho, this is known to us.

Using that, now I can write the upper expression as. Limit alpha tends to 0 e to the power lambda rho is equal to 1 plus alpha rho that is for this term 1 plus alpha rho to the power lambda by alpha. When alpha tends to 0 e to the power lambda rho is equal to 1 plus e to the power rho 1 plus alpha rho to the power lambda by alpha, keep in mind. So, this very much looks like this expression you minutely observe, we have already got the powers lambda by rho lambda sorry lambda by alpha here and here.

Now, I have got 1 plus certain thing 1 plus certain thing therefore, it will enable me to write this R 4 upon R 3 to the power lambda by alpha or this term in some simpler form. So, that will enable me simply to write the expression now in the form of, sorry in the form of A v as minus plus k times h 1 plus h 2.

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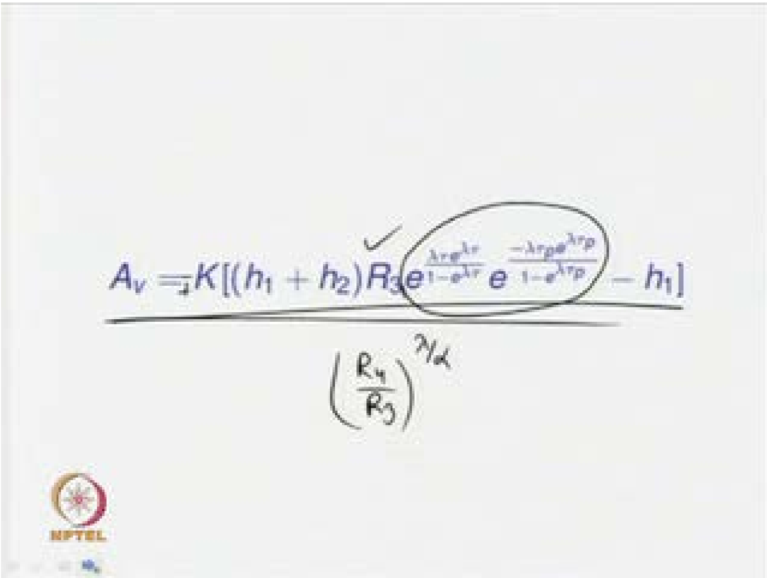

R 3 remains as it is, please see R 3 remains as it is. I am not changing R 3. And what is R 3? R 3 we have written already, R 3 is given by R 3 is given by this 1 R 3 is given by this one. What changes we have made, I have got R 4 upon R 4 to the power lambda by alpha is substituted by this two terms. Now, where from you get these two terms using this identity using this easily it is possible to get this term, R 4 upon R 3 to the power lambda by alpha as the as this one. I believe that we have followed how I am using. Please keep in mind, always it is possible to find e to the power lambda rho with the limiting value of alpha tends to 0 as 1 plus alpha rho to the power lambda by alpha, when this is used.

So, this factor simply can be obtained with the limiting value of limit alpha tends to 0 this much becomes. This gives us, when I put alpha tends to 0, you see alpha alphas are there alpha is here alpha is here. Therefore, this becomes this becomes your simply nothing, but or e to the power this.

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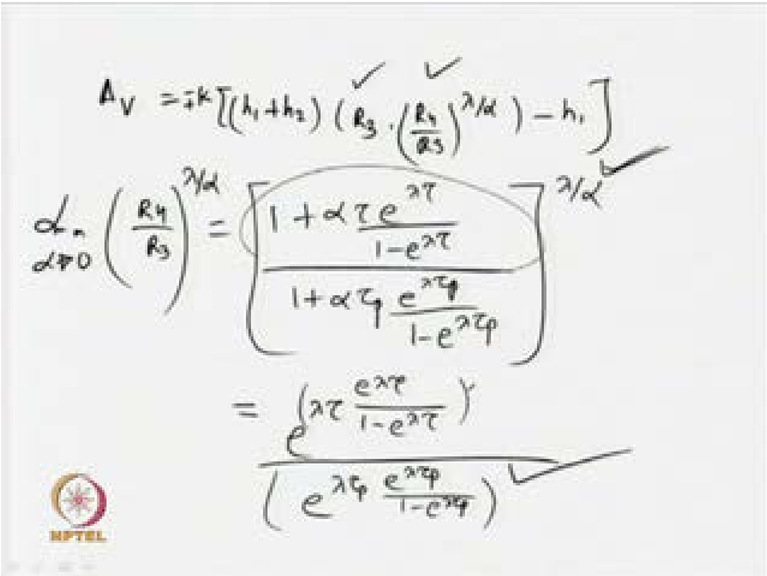

So, it will give us this ratio will be lambda tau e to the power e to the power lambda tau then again e to the power lambda tau upon 1 minus e to the power lambda tau. This is what you will get for the numerator part and for the denominator; it will be simply e to the power whole of this 1 e to the power lambda tau p e to the power lambda tau p upon 1 minus e to the power lambda tau p. So, limit alpha tends to 0 are R 4 upon R 4 R 3 to the power lambda by alpha becomes e to the power lambda tau e to the power lambda tau by 1 minus e to the power lambda tau upon e to the power lambda tau p e to the power lambda tau p upon 1 minus e to the power lambda tau p, and that is what we have got this expressions.

The final expression has been obtained from the analysis with the assumption that, lambda 1 is equal to lambda lambda 2 equal to lambda minus alpha with the condition alpha tends to 0 and lambda 3 is tends to infinity. When I use all these conditions, the general expressions result in simplified expressions which can further be used to identify the model parameters of a second order model with pole multiplicity. Which has got three unknowns given as k, theta, and T 1?

This is how second order plus dead time transfer function models with pole multiplicity are obtained. Using the same set of analytical expressions with the conditions that, alpha lambda 1 is equal to lambda lambda 2 equal to lambda minus alpha with alpha tends to 0 and lambda 3 tends to infinity.

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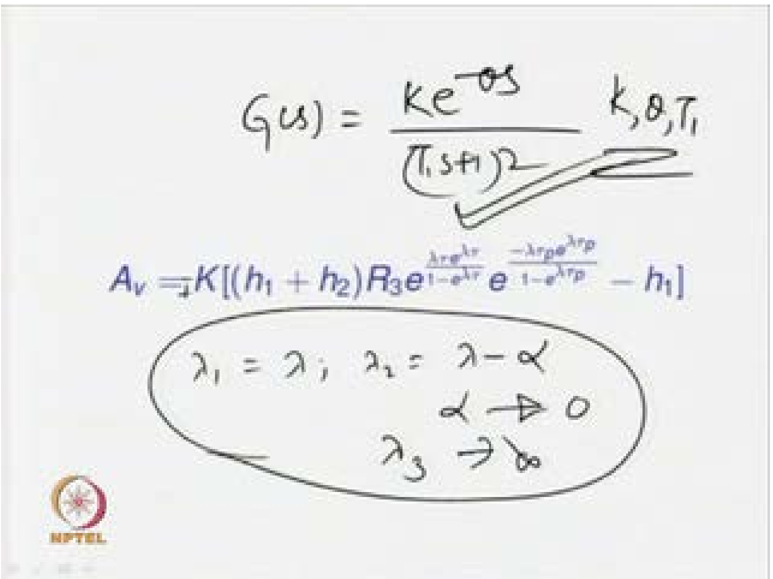

Let me summarize my lecture, the general expressions we have found earlier for identifying the transfer function model parameters of the general second order plus dead time transfer functions transfer functions can be extended for models with pole multiplicity. So, the model can have n number of poles also, it does not matter the same set of analytical expressions can be used with proper limiting values only. Where, I can have n number of poles located at the same point in the s plain.

Only thing I have to take care of the lambda. So, lambda 1 has to be lambda lambda 2 has to be lambda minus alpha and. So, on and lambda 3 can be constant or may or may not be constant depending on the type of the model. Now, model parameters also can be estimated using the four measurements four measurements can be used and the four analytical expressions can be simplified to identify transfer function models with pole multiplicity.

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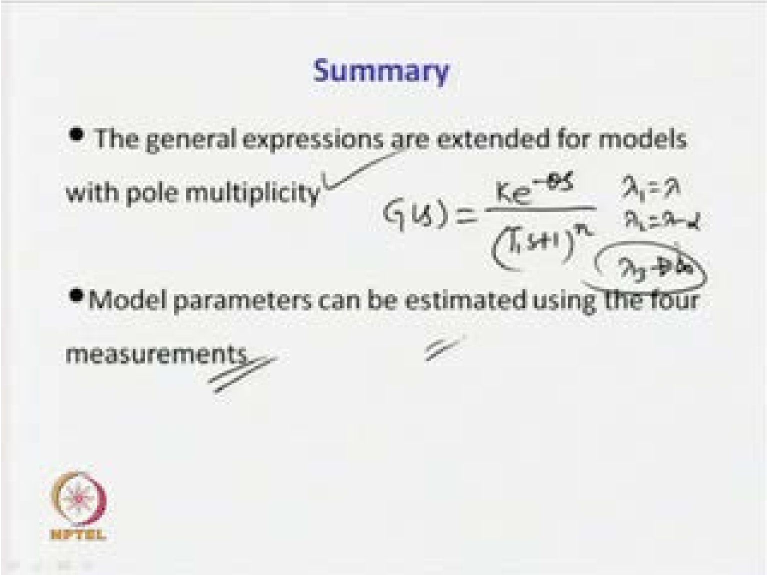

In which case, proper limiting values for different variables are to be used only. The beauty of this is that, the same set of powerful analytical expressions four analytical expansions and four measurements can be made use to identify a number of plant parameters or process model parameters where ,the processes can have pole multiplicity.

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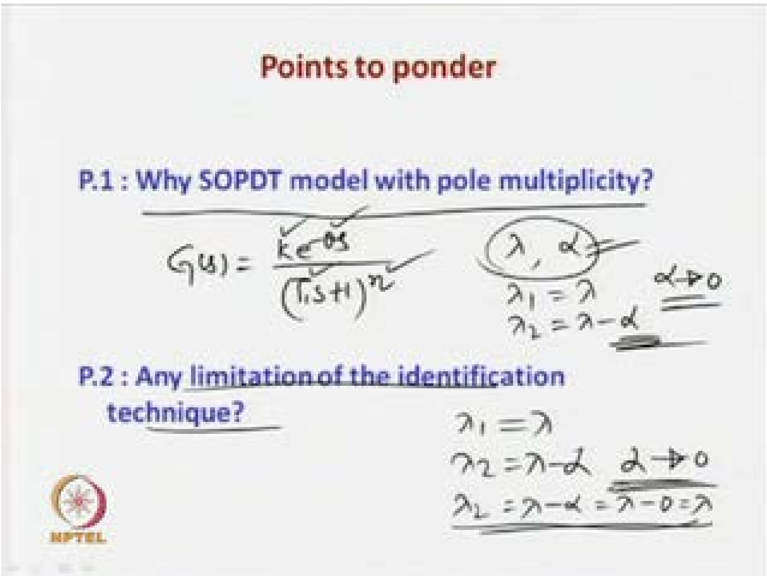

Now, we will go and to the questions, so the points to ponder. Why second order plus dead time model with pole multiplicity? Is it absolutely necessary to find transfer function models with pole multiplicity? yes sometimes we have got the system characteristics available in the form of k e to the power minus theta s upon T 1 s plus 1 to the power n. Particularly for poles with critically damped system, for systems with critically for systems with critically damped characteristics, we have got pole multiplicity characteristics.

We have here given by those systems and in those case, we need to divide or find analytical expressions to estimate the unknown parameters such as k, theta, T 1 and n. Therefore, the four analytical expressions can conveniently be used to identify the four unknown's: k, theta, T 1 and n with proper limiting values for the lambdas, and alphas. Where from you get alpha? This alpha you get with the assumption that, lambda one equal to lambda, lambda 2 equal to lambda minus alpha. So, some assumption on alpha has to be met as well.

Now, any limitation of the identification technique? Obviously, the technique is not free from limitations, one has to make proper use of small number theorem and large number theorems. So, large value theorems or small values theorems like one lambda 1 equal to lambda, lambda 2 equal to lambda minus alpha with alpha tends to 0. The please do not approximate this lambda 2 as lambda also. It is always lambda 2 equal to lambda minus alpha. Do not substitute alpha by 0, and get lambda minus 0 equal to lambda lambda 2 is equal to lambda. If you use this then you are not get going to get correct expression.

So, care must be taken to find or to apply the limiting theorems, small number and large number theorems properly, and accurately to rewrite the general expressions in appropriate form that is all in this lecture. Thank you.