Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 09

Steady State Gain from Asymmetrical Relay Test

Welcome to the lecture titled Steady State Gain from Asymmetrical Relay Test. In this lecture, we shall derive some explicit and exclusive expressions meant for getting the steady state gain from the relay test. The asymmetrical relay test will result in asymmetrical output, and we have attempted earlier to measure four parameters on the asymmetrical output. Now, we can make use of the asymmetrical output and input to the system to estimate one more parameter associated with the transfer function model.

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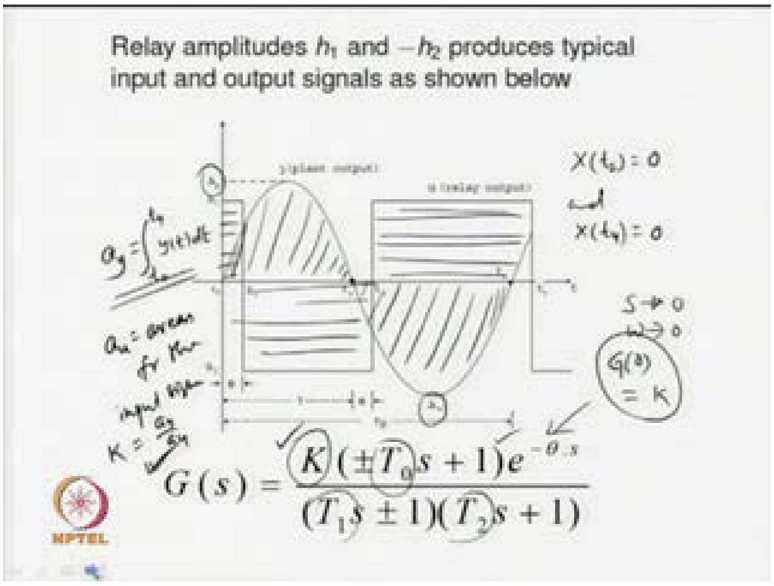

Now, the transfer function model is G s is equal to k plus minus T 0 s plus 1 times e to the power minus theta s upon T 1 s plus minus 1 T 2 s plus 1. So, the transfer function model has got five unknowns; and those are the steady state gain k, the 0 t 0, the time delay theta, and the two time constants T 1 and T 2. So, in all we have got five unknowns; but so far, we have been able to make four measurements on the asymmetrical output of the relay system.

Now, those measurements are A p, the peak amplitude of the output signal; A v, the negative peak amplitude of the output signal and two zero crossings those are namely t 2 and t 4; and we have also found that at time if 2, x t 2 is equal to 0 and at time t 4, x t 4 is equal to 0. Thus the four measurements have enabled us to estimate four unknowns associated with the transfer function model using four non-linear equations.

In this lecture, we shall try to develop analytical expressions, so that can be used to estimate the steady state gain associated with the transfer function model. What is steady state gain? When s tends to 0 or the frequency of a system output becomes 0 at that time G 0 becomes k and that is known as the steady state gain. How can we find the steady state gain? For that, we have to concentrate on the asymmetrical output and if I concentrate on one period of the asymmetrical output, the area of the asymmetrical output can be obtained conveniently using the analytical expressions we have derived earlier.

So, what will be the area of the asymmetrical output for one period, this will be the area for the asymmetrical output (Refer Slide Time: 03:12). Let us denote the asymmetrical output by the symbol a y, thus a y can be given in the form of an integral which is starting from time t 0 to t 4 with y t d t. So, that will give us the area of the asymmetrical output signal whereas, the area of the input signal can be also obtained, whereas the input signal area of the input signal can be shown as this one (Refer Slide Time: 03:52).

So, then we have to make use of this to find the two areas, where a u will be the areas for the input signal area for the input signal and next the ratio of the areas will give you the steady state gain. So, k will be equal to a y upon a u. So, all these things will be developed in sequence.

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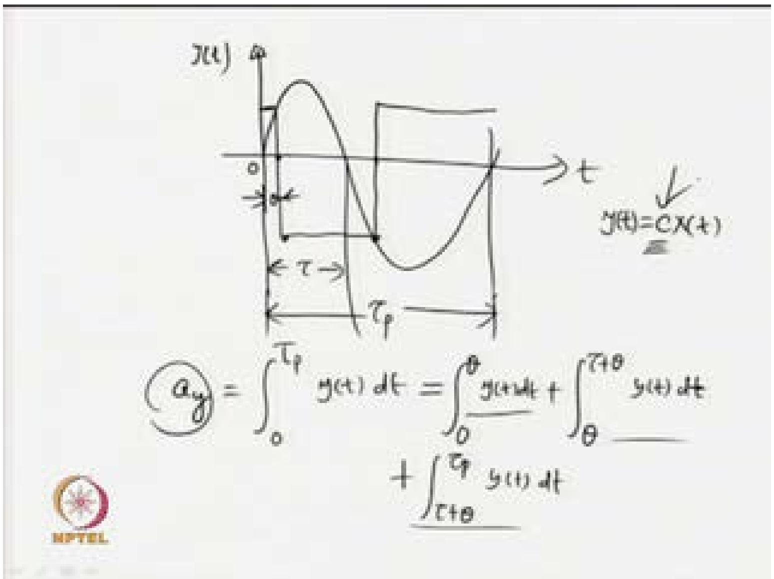

Now, I will start with the asymmetrical output first. So, the output will assume this form for one period of time. Now, y t is the output and I will denote different time instants to find the area of this signal. Now, I will start from time t equal to 0 till different span depending on the type of input signal we have for the system. So, this is our input signal, which has got a number of piecewise constant inputs during different instant of time.

Now, what is this, this is our theta, so this span is denoted by theta. And here, we get the period tau and for the full period we have got tau p starting from time t equal to 0; to find the area of this signal from time t equal to 0 to tau p, now I have to write the expression y t is equal to integral from 0 to tau p y t dt sorry not this is not the output, this is the area of the output. So, I write a y is equal to integral from 0 to tau p y t d t.

Now, how to find y t for different time segments, because we have got different type of inputs, a number of piecewise constant input to the system; therefore, the output during that particular piecewise constant input has to be found initially. So, this has to be expanded and written in the form of integral from 0 to theta y t dt plus theta to I will start from here, till we go to this point, so that will be our tau plus theta; so, we go from theta to tau plus theta y t dt plus tau plus theta to tau p y t d t; this will give the three terms combined together will give us the area of the output signal.

To find the area of the output signal, I need to find the state of the system at different instant of time, because we know that the output can be found using the state variables x t, where y t becomes c x t, c is the constant of the dynamics of the second order plus dead time system.

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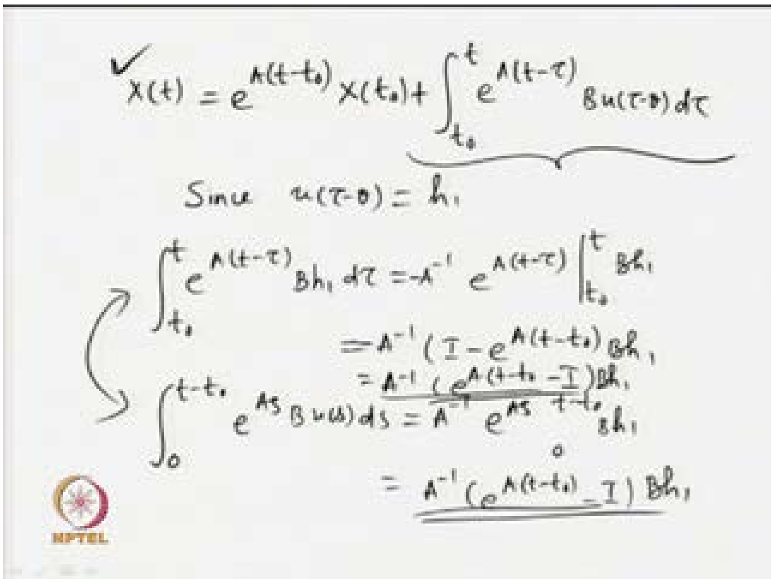

Now, for that I will start deriving the state equations for different segment of time. So, we know that the solution of a state equation is given by the general expression, x t is equal to e to the power A t minus t 0 x t 0 plus integral from t 0 to t e to the power A t minus tau B u tau minus theta d tau. This is what we get for the dynamics of a system at any instant of time, where x t stands for the state of the system as you know.

Now, since in our case luckily we have got u tau minus theta is a constant either h 1 or minus h 2 using that it is very convenient to find the second part of the x t, which becomes t 0 to t e to the power A t minus tau B h 1 d tau or minus h 2 d tau. So, let us take u tau minus theta is equal to h 1, which results in the integral part integral from t 0 to t e to the power A t minus tau B h 1 d tau.

So, let me find the integral which comes out in the form of A inverse then, we will have integral e to the power A t minus tau with the limits t 0 to t B h 1; when the limit is put then we get it in the form of A inverse I minus e A t minus t 0 B h 1. So, thus we get the integral in this form. Now, the integral can also be obtained using one simpler expression which can be given in the form of 0 to t minus t 0 e to the power A s B u s d s.

So, either one can use the upper one or the bottom one, because the bottom one gives you very simpler expression compared to the earlier one. Now, the bottom one also can be solved and found as, A inverse e to the power A s with the limits 0 to t minus t 0 and you have got u s equal to h 1, so B h 1; and that will give you A inverse e to the power A t minus t 0 minus I B h 1. So, there is difference in sign only. So, the you needs not worry about that, because here the integral will be found with respect to e to the power minus A tau so obviously, there will be a minus here and ultimately you get here minus and which gives you finally, in the form of A inverse e to the power A t minus t 0 minus I B h 1.

So, I get the same expression using the upper or the lower integral. So, I shall use henceforth, the lower integral in place of the upper one for finding the state of the system at any instant of time.

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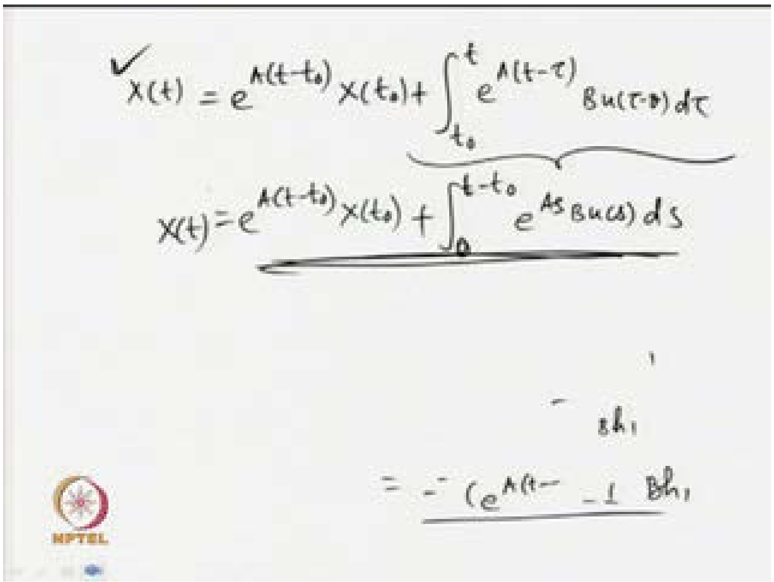

Thus I can write x t h e to the power A t minus t 0 x t 0 plus integral from 0 to t minus t 0 integral from 0 to t minus t 0; please keep in mind the limits of this integral, which always starts from 0 to the time span for that segment of the input, t minus t 0 e to the power A s B u s d s. Thus I shall make use of the lower one, the lower expression to find the output of the symmetrical sorry output of the relay control system for different time spans. Now, I I shall start finding the output for different time spans.

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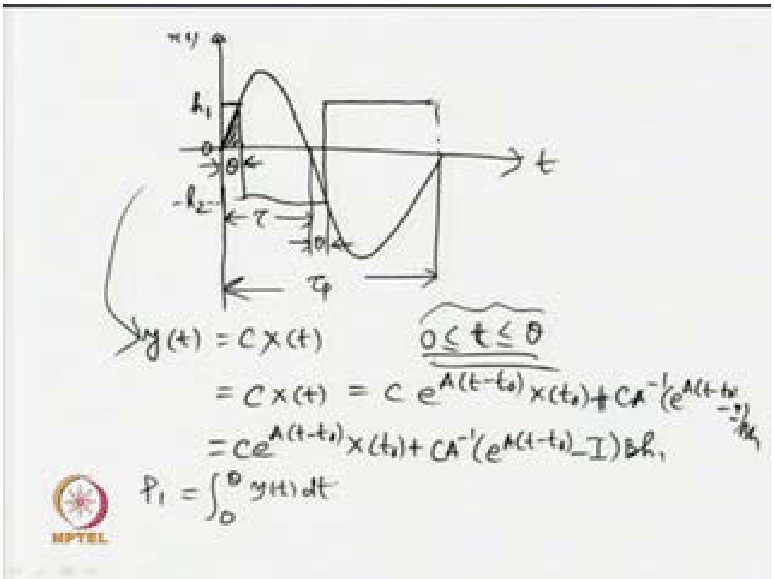

Let me redraw the output signal once more. So, that it will be possible for you to follow the different time spans conveniently. So, the output is given in this form, I will draw the input to the system y t is the output, t is the time axis. And the input to the system becomes like this for one period of the output signal, where the input has got heights h 1 and minus h 2. Now, as you know this span is the theta time delay associated with the dynamics of a system whereas, this span we take as tau from here; whereas, this span again becomes theta and finally, period of the output is given by tau p.

So, the output y t for different segments can be used using the expression y t equal to c x t. Now, when the y output is considered part of the output is considered for the time range 0 is less equal to t is less equal to theta; then, the output for this time span can be found using the expression c x t, where your time span is from t ranging from 0 to theta. So, x t again can be obtained using the general expression which is nothing but, e to the power A t minus t 0 x t 0; then I have the integral for the second part which is obtained in the form of A inverse, so I will have c again c plus c A inverse e to the power A t minus t 0 minus I times B h 1.

So, let me write once more. So, the output signal will be given by the expressions c e A t minus t 0 x t 0 plus c A inverse e to the power A t minus t 0 minus I B h 1; because the input to the system at that time is this piecewise constant input h 1 for which we have got the output starting from time t equal to 0 till time t equal to theta. So, this part of the output of this part of the signal is given by this expression y t. Now, to find the area under these let me designate that area by symbol p 1. So, p 1 will be from time 0 to theta y t dt then, the area can be found now using the expression yes I will I will find the area later later on. So, let me find the state where the output for different segments initially then, we will start finding the area later on. Then for this time range we have got the state expression as this one, which is multiplied with c to give the output y t.

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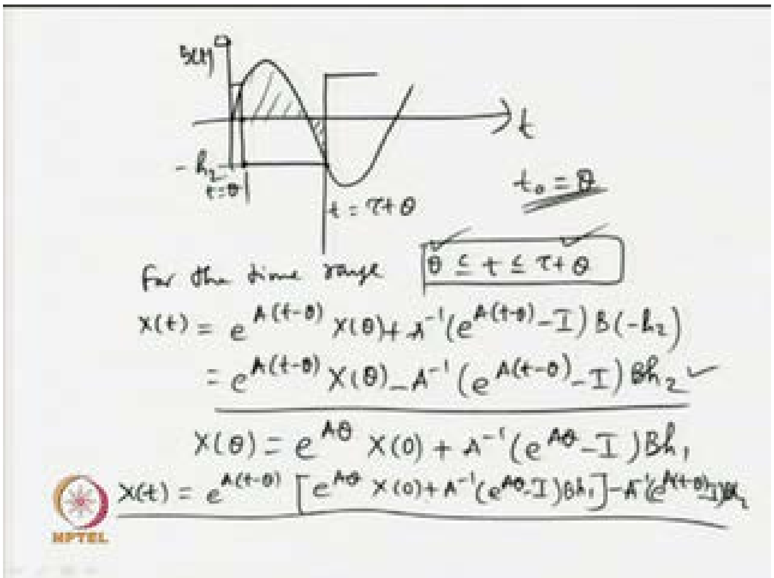

Similarly, for another time range which is starting from y t. So, for this time range where I start from time t equal to theta and I will go up to time t equal to tau plus theta now; in that case, the state variable for that time range for the time range, theta is less equal to t is less equal to tau plus theta. The state variable x t is given as e to the power A t minus theta x theta plus A inverse e to the power A t minus theta minus I now B; what will be the input at this time, the input at this time is minus h 2 therefore, we will have minus h 2 over here.

So, upon simplification which again gives us the expression e A to the power t minus theta x theta plus or I can write here minus A inverse e to the power A t minus theta minus I B h 2. So, this is what you get for the state variable for the second time range for the time range t between theta to tau plus theta. Then the area of that time span can be obtained using the output and in which case the area can be obtained as shown here in the see that part of the output signal.

Now, whereas why we have been able to get this expression, simply I have substituted the u s by its appropriate input in this case it is minus h 2, rest of the things will remain as it is. And what changes have been made to this state equation? We start the output signal from time t equal to theta till time t equal to tau plus theta therefore, the t 0 becomes theta.

Please keep in mind t 0 is theta therefore, you have got x theta over here, but x theta again we know that x theta is equal to e to the power A theta x 0 plus A inverse e to the power A theta minus I B h 1. How do you get this one, this can easily be obtained if you substitute t equal to theta here. So, when t equal to theta is substituted over here and t 0 equal to 0, one obtain x theta conveniently.

So, when this x theta is used over here finally, I get an expression for the x t as x t is equal to e to the power A t minus theta, x theta will be substituted here e to the power A theta x 0 plus A inverse e to the power A theta minus I B h 1 B h 1 then minus A inverse e to the power A t minus theta minus I B h 2. So, this is the expression for x t for the time spanning from time t equal to theta to time t equal to tau plus theta. So, let me repeat and rewrite this expression after simplification.

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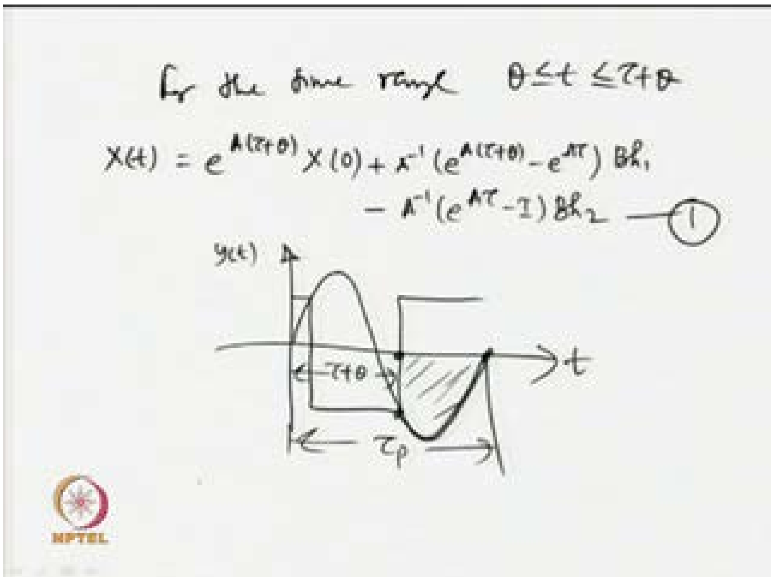

So, that x t for the time range for the time range theta is less than equal to t is less than equal to tau plus theta, x t will have the final form given by e to the power A tau plus theta x 0 now plus A inverse e to the power A tau plus theta minus e to the power A tau

B h 1 minus A inverse e to the power A tau minus 1 B h 2. Similarly, we are left with one more segment of the output signal; therefore, we have to make use of this concept to find the state variable for the last segment of the output signal.

So, the last segment of the output signal can be again shown as time t y t, where the asymmetrical output again is shown in this form and the input to the system is like this (Refer Slide Time: 20:06); therefore, the last segment starts from this instant time instant therefore, I have to concentrate the output from here till the zero crossing and for that, the area again all we obtained by this shaded part. Now, to find the state variable for the system for this time range time t, this is your tau plus theta. So, this span is now given as tau plus theta whereas, this is our tau p.

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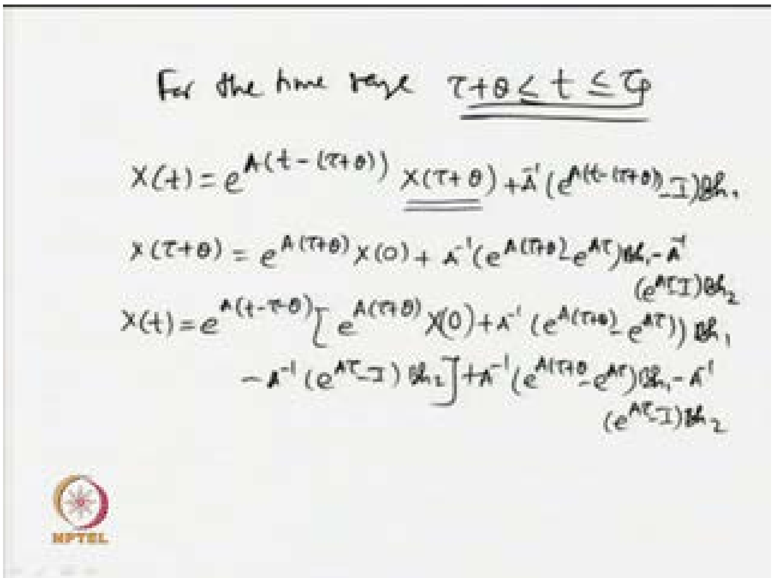

So, for the time range for the time range for the time range tau plus theta is less than equal to t is less than equal to tau p; for this time, the state variable will be given by x t is equal to e to the power A t minus tau plus theta x tau plus theta then plus A inverse e to the power A t minus tau plus theta minus I B h 1 now. Again what is x tau plus theta? x tau plus theta is nothing but, e to the power A tau plus theta x theta plus A inverse e to the power A tau plus theta minus e to the power A tau B h 1 minus A inverse e to the power A tau minus 1 minus I B h 2; what is I? I is the identity matrix of the order of A.

So, x t upon simplification can be found in the form of e to the power A t minus tau minus theta x tau plus theta will be substituted over here. So, finally giving us in the form of e to the power A tau plus theta x 0 sorry this is not theta (Refer Slide Time: 22:35). So, this is equal to x 0 then this is your x 0 plus A inverse e to the power A tau plus theta minus e to the power A tau B h 1 minus A inverse e to the power A tau minus identity matrix time B h 2 plus, the remaining part as it is e to the power A tau plus theta minus e to the power A tau B h 1 minus A inverse e to the power A tau minus I B h 2.

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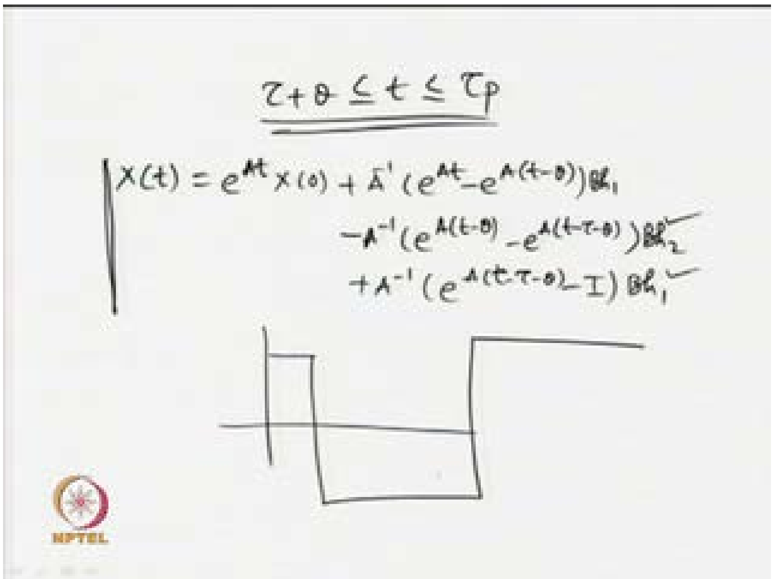

So, we have got a horrible expression which can further be simplified and written finally in the form of x t for the time range 0 for the time range tau plus theta to tau p. The state variable will assume the form e to the power A t x 0 plus A inverse e to the power A t minus e to the power A t minus theta B h 1 minus A inverse e to the power A t minus theta minus theta t minus theta minus e to the power A t minus tau minus theta B h 2 then plus A inverse e to the power A t minus tau minus theta minus the identity matrix time B h 1.

So, as expected the state for the final segment of the output is involving two inputs, two linear piecewise constant inputs h 1 and h 2. As you see here, the input to the system is assuming this form; whereas, during this input we have got the output which is going from negative to positive. Therefore, we have got the inputs h 1 and h 2 present in this expression. Now, after finding these expressions, what to do with these all these expressions? So, we have been able to find the expression for x t the state variable for different time ranges then it is possible to find the area of the output signal using the expression.

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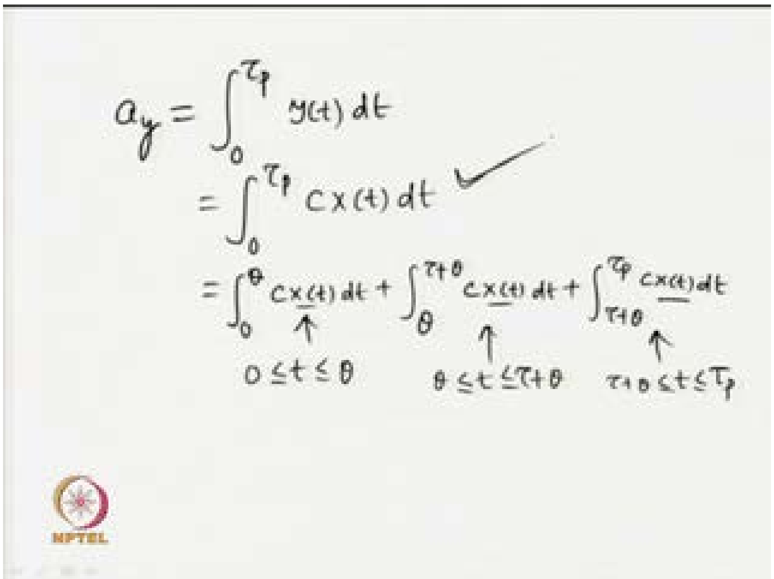

The area of the output signal asymmetrical output signal is equal to integral from 0 to tau p y t d t, which is nothing but for our case 0 to tau p c x t d t. Now, I will divide this into different time ranges as you have seen, because you have got different piecewise constant input. Therefore, we cannot have the expressions expressed by single expression. So, that way we have to make use of all the three time spans to find the final area of the symmetric asymmetrical output signal.

Now, this goes from the limits 0 to theta c x t dt plus theta to tau p tau plus theta c x t dt plus tau plus theta to tau p c x t d t. Now, I shall substitute the expressions x t, x t, x t we have found for different time ranges. Now, the integral is for the limits 0 to theta, so the t is between 0 to theta in this case, now for the second part it is your theta is less than equal to t is less than equal to tau plus theta and for the third part it is tau plus theta is less than equal to t is less than equal to tau p.

So, keep in mind one cannot make use of the single expression to find the area of the output signal, because we have got three different piecewise constant input to the systems which changes their magnitude at different instants of time. Therefore, the piecewise constants input are to be considered definitely to find correct expression for the output of the asymmetrical output of the system.

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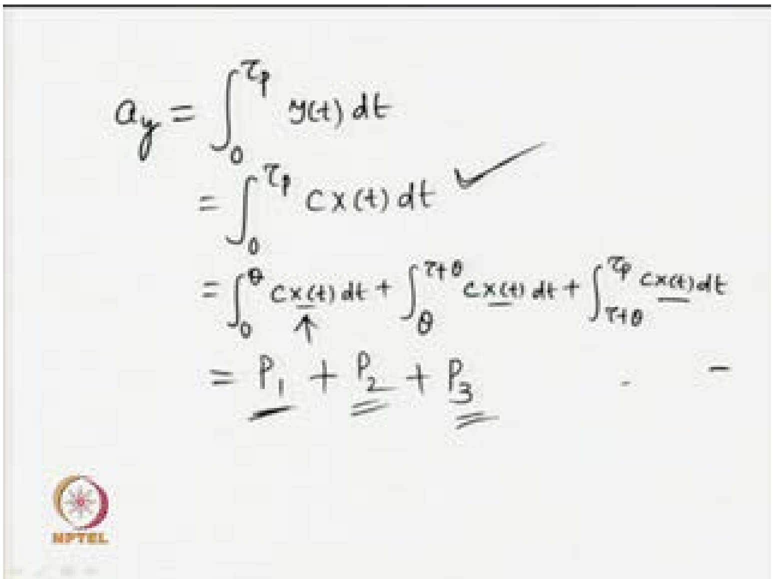

Now, I shall go on substituting the expressions for x t for different time range. So, let us write this a a y by three parts, p 1 plus p 2 plus p 3. And try to find the parts individually initially before combining and finding the final area of the output signal.

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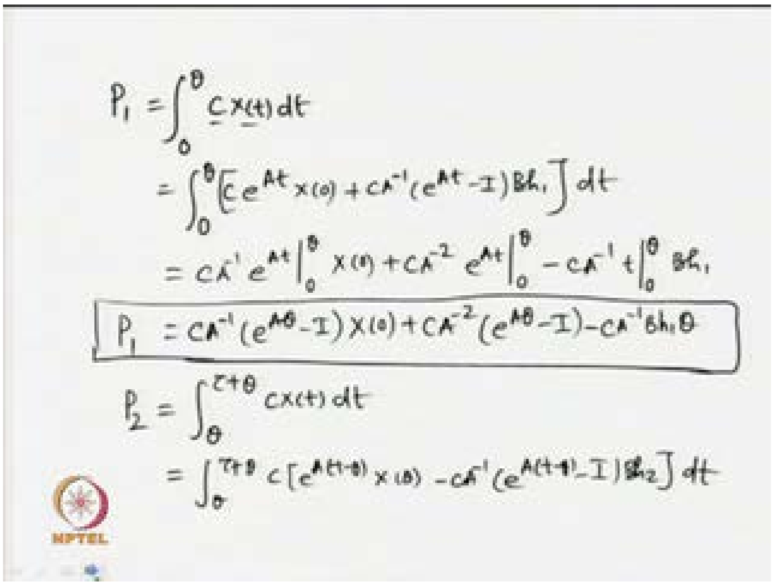

Then p 1 is given by the expression integral from 0 to theta c x t d t, which is again written as now integral from 0 to theta c e to the power A t x t 0; and in this case, t 0 equal to 0 therefore, directly I can write this as x 0 plus c A inverse e to the power A t minus I B h 1 times d t. So, what is this, the c has been multiplied with x t the state variables of the system and we have to integrate it for the time range t spanning from 0 to theta.

Now, when I start integrating this one I will find the expression C A inverse e A t with limits 0 to theta with the multiplier x 0 again for the second part it will be C A to the power minus 2 e A t with the limits 0 to theta; then, the remaining part will have the expression minus c A inverse integral from 0 to theta for t dt therefore, your left with t with limits 0 to theta times B h 1.

So, which upon simplification gives us, C A inverse e to the power A theta minus I times x 0 plus C A minus 2 e to the power A theta minus I minus this will be simply C A inverse B h 1 theta. So, the part p 1 has been found in this form p 1, let me repeat once more. The p 1, the area part of the area of the output signal given by p 1 is given by the expression p 1 is equal to C A inverse e to the power A theta minus I times x 0 plus C A to the power minus 2 to the power times e to the power A theta minus I minus C A inverse B h 1 theta.

Then, similarly I can get an expression for p 2, p 2 is from the integral from theta to tau plus theta c x t d t. Now, the correct expression for this time range when put in this expression gives us p 2 as, p 2 is equal to integral from theta to tau plus theta c e A t minus theta x theta minus C A inverse e to the power A t minus theta minus I B h 2 d t. So, this integral can also be found like the previous case.

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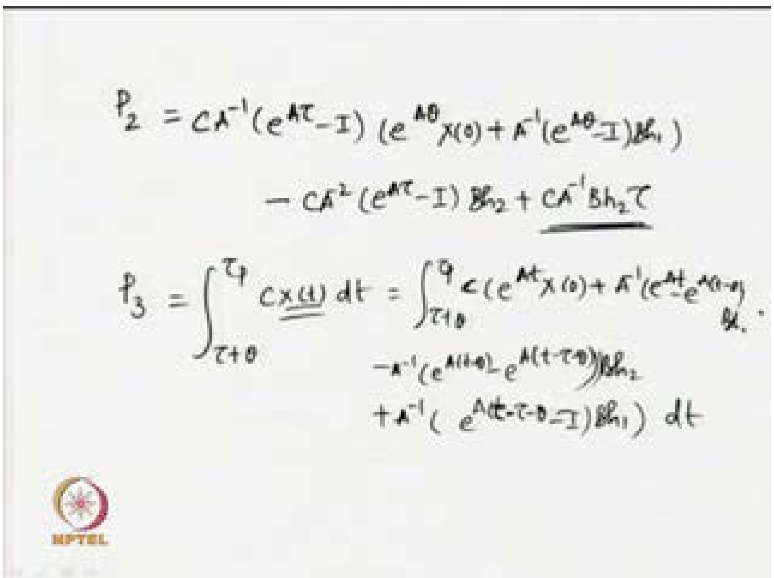

And simplified to give us finally p 2 as, C A inverse e to the power A tau minus I times e to the power A theta plus sorry e to the power A theta times x 0 plus A inverse e to the power A theta minus I B h 1 minus C A to the power minus 2 e to the power A tau minus I B h 2 plus C A inverse B h 2 tau. So, keep in mind the last term interestingly the last term is found in the form of C A inverse h 2 tau; if you go back, see the last term of p 1 again it is obtained in the form of minus C A inverse B h 1 theta. So, the last term of the integrals of different parts are found to have the expression, C A inverse B; so, C A inverse B that is one important observations we have made so far.

Now, I will try to find the third part, p 3 for which we have the integral starting from time tau plus theta to tau p c x t d t. So, when this x t for this one will be which one, the x t for this one is this one (Refer Slide Time: 33:01). So, the when this x t is substituted in p 3 then we obtain that expression in the form of integral from tau plus theta to tau p c e to the power A t x 0 plus A inverse e to the power A t minus e to the power A t minus theta B h 1, B h 1 will come here then minus A inverse e to the power A t minus theta minus e to the power A t minus tau minus theta B h 2 plus A inverse c e to the power A c e to the power A tau t minus tau minus theta minus I B h 1 d t.

So, C will not come here, because C is there already here. So, when again this is expanded and simplified will get an expression for this p 3 is equal to I will write the final expression of p 3, which will not be so simple.

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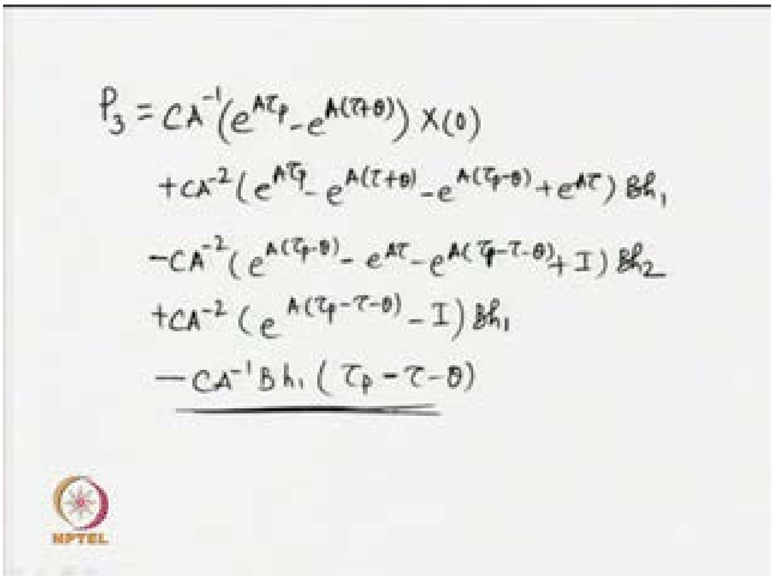

But, let me try to write the whole expression for p 3 which comes out to be in the form of c A inverse e to the power A tau p minus e to the power A tau plus theta times x 0 plus C A minus 2 e to the power A tau p minus e to the power A tau p no this will be tau (Refer Slide Time: 35:11), A tau plus theta minus e to the power A tau p minus theta plus e to the power A tau B h 1 plus sorry this is not plus minus C A to the power minus 2 with terms e to the power A tau p minus theta minus e to the power A tau minus e to the power A tau p minus tau minus theta plus I times B h 2, again plus C A to the power minus 2 e to the power A tau p minus tau minus theta minus I B h 1 with the last term minus C A inverse B h 1 times tau p minus tau minus theta. So, again see the last term, interestingly when all the parts are added together what do we get?

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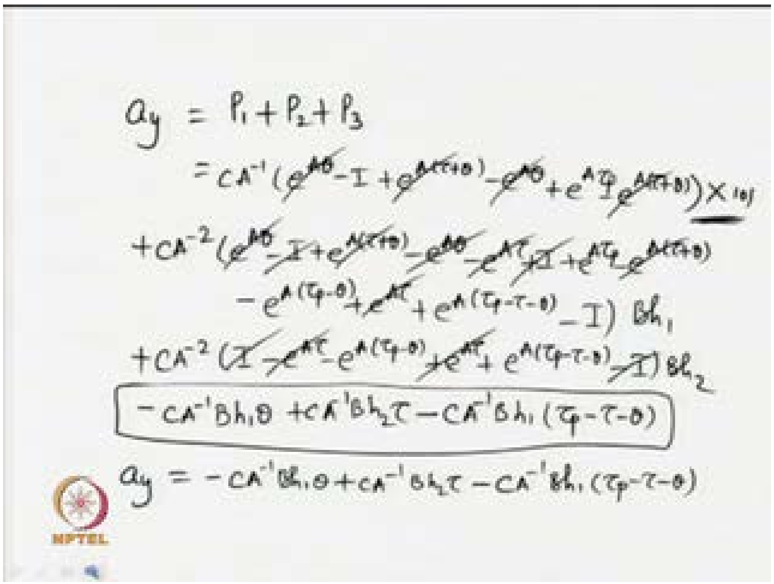

When p the area is found in the form of a y is equal to p 1 plus p 2 plus p 3. Then one obtains, C A inverse e A theta minus I plus e to the power A tau plus theta minus e to the power A theta plus e to the power A tau p minus e to the power A tau plus theta times x 0 plus C A to the power minus 2 e to the power A theta minus I plus e to the power A tau plus theta minus e to the power A theta minus e to the power A tau plus I plus e to the power A tau p minus e to the power A tau plus theta minus e to the power A tau p minus theta plus e to the power A tau plus e to the power A tau p minus tau minus theta minus I times B h 1 with another term as C A to the power minus 2 I minus e to the power A tau minus e to the power A tau p minus theta plus e to the power A tau plus e to the power A tau p minus tau minus theta minus I B h 2 minus C A inverse B h 1 theta plus C A inverse B h 2 tau minus C A inverse B h 1 tau p minus tau minus theta.

So, this is the expression for area of the asymmetrical output signal, where we will find that all other parts are terms then the last three terms becomes 0; when x 0 is substituted over here, we have an expression for x 0 which had been found in the previous lecture. So, when x 0 is substituted over here, then the first term first few terms get cancelled, leaving us a y as a y is equal to minus C A inverse B h 1 theta plus C A inverse B h 2 tau minus C A inverse B h 1 tau p minus tau minus theta.

So, interestingly we get only simpler three expressions remaining in the expression for the area of the output signal. This is an important observation. So, how the other parts are getting cancelled, if you look carefully so many terms are cancelling out like this one (Refer Slide Time: 39:42), this one cancelling out, e to the power A theta minus e to the power A theta, leaving us only two terms in the first part.

Similarly, e to the power A theta A theta cancelling out, cancelling out minus I plus I cancelling out then, minus e A tau plus e A tau cancelling out, and leaving three terms there; and here also e to the power plus minus A tau. And you have got plus minus I then, whatever remaining terms you have upon substitution of x 0 in that term.

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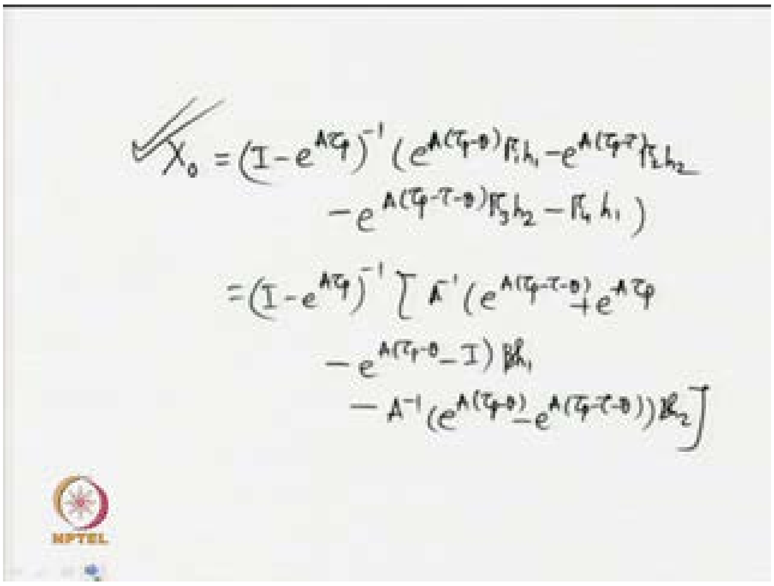

What is x 0? x 0 is found to be in our earlier lecture we have seen that, x 0 is given by the expression I e to the power minus A tau p inverse e to the power A tau p minus theta gamma 1 h 1 minus e to the power A tau p minus tau gamma 2 h 2 minus e to the power A tau p minus tau minus theta gamma 3 h 2 minus gamma 4 h 1; where again gammas are there, when the gammas are substituted finally one obtains x 0 in the form of I minus e to the power A tau p inverse with A inverse e to the power A tau p minus tau minus theta plus e to the power A tau p minus e to the power A tau p minus theta minus I B h 1 with another term minus A inverse e to the power A tau p minus theta minus e to the power A tau p minus tau minus theta times B h 2.

So, when this x 0 is substituted here, then the first few terms apart from the last three terms cancels out and we are left with only three terms.

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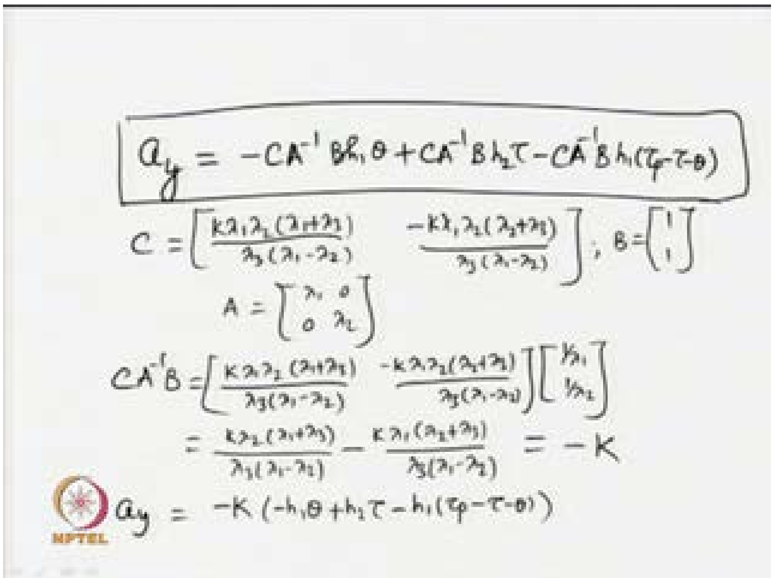

Therefore, the output of the relay test has the final form given as, a y is equal to minus C A inverse B h 1 theta plus C A inverse B h 2 tau minus C A inverse B h 1 tau p minus tau minus theta. Then, what is C A inverse B? We know that, from the state space model of the second order plus dead time transfer function model we have obtained C. C is equal to k lambda 1 lambda 2 lambda 1 plus lambda 3 upon lambda 3 lambda 1 minus lambda 2; and the other element is minus k lambda 1 lambda 2 lambda 1 lambda 2 lambda 2 plus lambda 3 upon lambda 3 lambda 1 minus lambda 2.

So, this is the C vector with B vector we know has got the elements 1 and 1; whereas, A is given by lambda 1 0 0 lambda 2 matrix. Now, C A inverse B becomes C A inverse B becomes k lambda 1 lambda 2 lambda 1 plus lambda 3 times lambda 3 lambda 1 minus lambda 2 minus k lambda 1 lambda 2 lambda 2 plus lambda 3 upon lambda 3 times lambda 1 minus lambda 2 multiplied by A inverse B; so, A inverse B will be 1 upon lambda 1 and 1 upon lambda 2.

So, when this is simplified we get, k lambda 2 lambda 1 plus lambda 3 by lambda 3 times lambda 1 minus lambda 2 minus k lambda 1 lambda 2 plus lambda 3 by lambda 3 lambda 1 minus lambda 2, which is nothing but equal to minus k. So, interestingly we have found C A inverse B to be minus k then, a y can be written as you see in all the three terms you have got C A inverse B. So, why do not you take common then giving us a y as C A inverse B times minus h 1 theta plus h 2 tau minus h 1 tau p minus tau minus theta; but, C A inverse B is equal to minus k.

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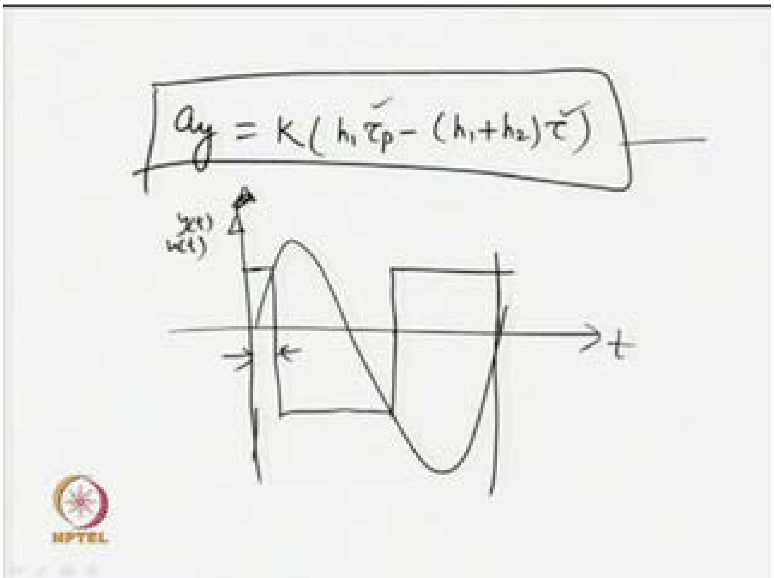

Therefore, allow me to write here minus k and writing like that I finally get the expression for a y is equal to k times h 1 tau p minus h 1 plus h 2 tau. So, this is the expression final expression for area of the output signal. So, whatever might be the area of the output signal, the real output area of the output signal can be easily obtained using this simpler expression; which involves the parameters relay heights and few measurements like first sampling first zero crossing instant, second zero crossing instant from the beginning which is considered to be 0 with the multiplication of k.

Now, let us try to find the area for the input signal now. Then what will be the area for the input signal? As you have seen, the span for different time instants for the input signal can be obtained looking at the output signal only. And for our case, the input signal appears to be of this form it does not overshoot. So, we will have a zero crossing at this instant of time. Now, I will write the difference spans now, what is this input signal? The input signal is going from sorry y t and u t versus t. Now, this is the span x axis is not correct now, because I have to start from 0. Let me redraw again, because it is getting confused.

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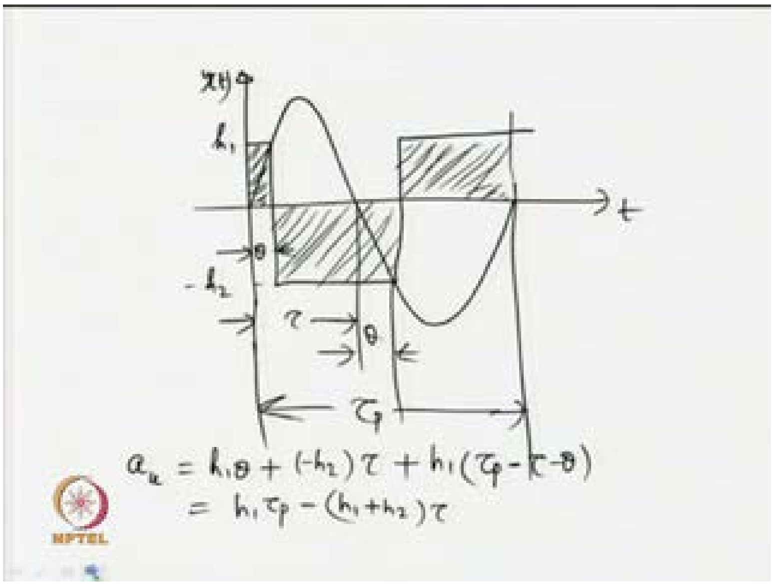

To find the input signal let me plot the output signal first t, y t. So, the y t is like this and we have got the input signal given by this zero crossing and this one. Thus, what are the spans here, this is theta; what is this, this span is equal to tau; what is this, this is equal to theta. And similarly, finally one period is given by tau p. Thus, if I try to find the area of the input signal I have to consider the shaded area shaded part only.

So, to find the area of the shaded part, what I have to do? I have to consider, the heights h 1 and minus h 2. Thus area of the input signal a u can be written as h 1 theta this rectangle; then coming to the second rectangle the bottom one, what its width and height? Height is equal to minus h 2 and width is equal to this will be tau plus theta minus theta.

So, this span will be equal to tau yes, and the area of the upper rectangle now can be found as h 1 time tau p minus tau minus theta; so, tau p minus tau minus theta. So, when this is simplified again what we get the expression for a u as a u as h 1 tau p minus h 1 plus h 2 tau. So, if you look at the expression for a y and a u, let me give you a y and a u once more.

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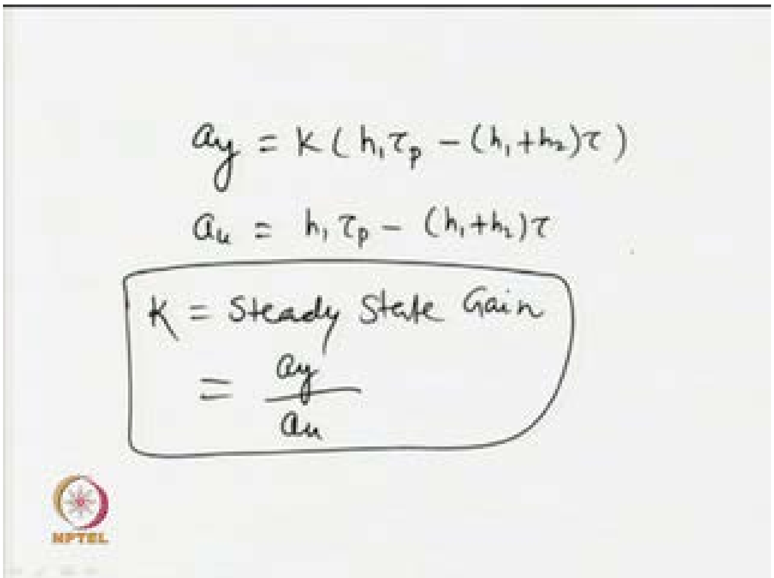

So, a y is found to be k times h 1 tau p minus h 1 plus h 2 tau; and a u is found to be h 1 tau p minus h 1 plus h 2 tau. Thus the steady state gain k, the steady state gain state gain can be obtained from the ratio of the areas of the output to the area of the input. So, this is how the steady state gain of the transfer function model is found. So, when you get the asymmetrical output signal measure the or find the area of the output signal and area of the input signal take the ratio of the two; that will give you the steady state gain of the transfer function model.

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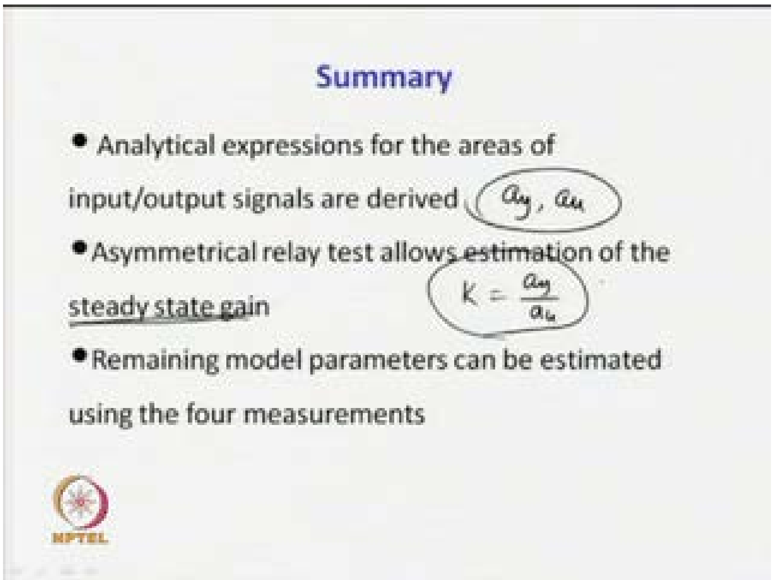

So, let me summarize my lecture now. Analytical expressions for the areas of input and output signals, a y and a u have been found. Now, taking the ratio of these two areas, it is possible to estimate the steady state gain of the system; thus k is equal to a y upon a u. Why we have been doing so because we know the transfer function model the in our transfer function model we have got five unknowns, earlier we had developed four non-linear equations to estimate four unknowns; now, the fifth unknown the steady state gain of the system can be estimated using the areas of the output and input signals.

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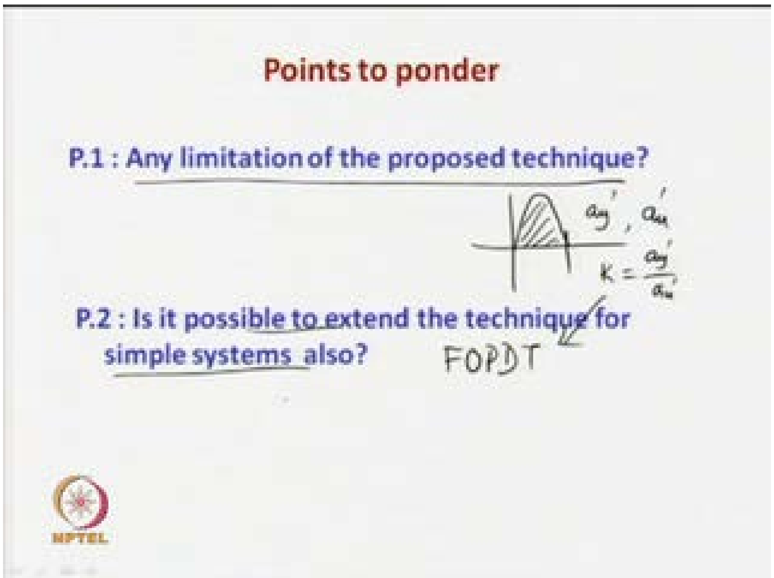

Any points to ponder, what are the limitations any limitation of the proposed technique? Yes, this technique is applicable only when the output the output is asymmetrical or I mean the average output a y does not becomes 0 or a u does not become 0 especially. So, this is one of the limitations of this method that, it cannot be used for sustained oscillatory signal of symmetrical measure. That means when the output is symmetrical in that case, we cannot make use of this technique to find the steady state gain of the system.

Is it possible to extend the technique for simple systems also? Yes, it is possible to extend this technique for simple systems in which in which case, luckily for first order plus dead time systems especially what happens, one can make the one can find the area of half period signal, unlike the full period signal one can find the area of half period signal.

So, a y dash and similarly area of half period of the input signal, a u dash and the ratio of the two will give you the steady state gain that is particularly for first order plus dead time systems; that is whereas, this technique may not be applicable for large order systems or systems with higher dynamics, thank you.