Advanced Control Systems

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Module No. #03

Time Domain Based Identification

Lecture No. #07

Identification of SOPDT Model

So, we shall continue with the identification of second order plus dead time model a set of explicit expressions. So, will be derived to find transfer function model parameters. So, we shall have four unknowns for this second order plus dead time transfer function model. Actually there will be five unknowns, but four unknowns can be estimated using four non-linear equations. So, effort will be made to derive four analytic expressions, and four measurements will be made on the output sustained oscillatory output of the system.

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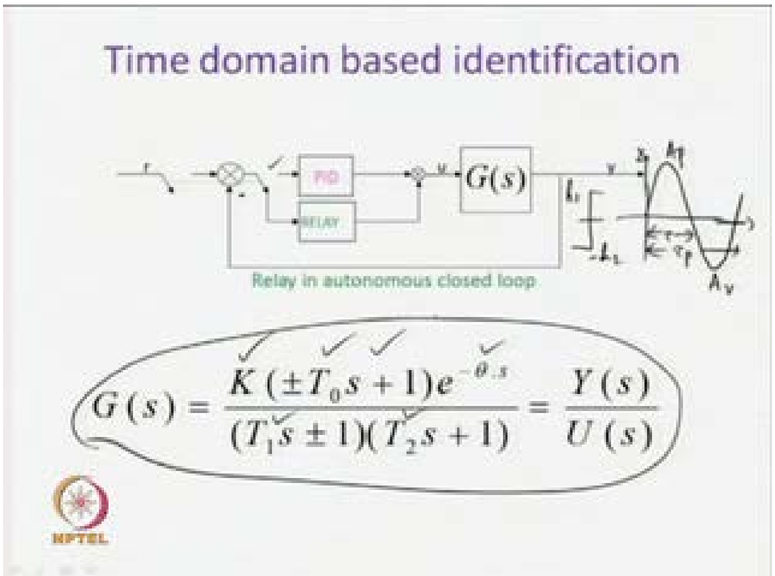

So, these things we have already seen in our last lecture, where we considered an autonomous closed loop relay control system, and the PID controller has been detached and the relay asymmetrical relay has been connected to the system to induce to induce sustained oscillatory output. So, the form of output depending on the type of input to the system will have some specified form, and the input to the system from the asymmetrical relay which has got heights h 1 and minus h 2 will be of the form of this one. So, this sustained oscillatory output can give us four measurements, and namely the peak amplitude A p, the negative peak amplitude A v, and will have two spans here. One from here tau, and tau p is the period for the sustained oscillatory output. So, making four measurements it is possible to estimate 4 unknowns associated with the transfer function model, whereas we have got 5 unknowns in the transfer function model, and those are the steady state can k the zero T 0 and 2 poles with time constants T 1 and T 2 and along with that, we have got the dead time associated with the system.

That is given by theta. So, we have to assume that either a new one of the parameter is found by some other method or estimated by some other technique. Then, it will be possible to make use of the 4 explicit expressions, we derived for obtaining the transfer function model. So, we have studied earlier in our earlier lecture, we have found the state space model of this second order plus dead time transfer function model.

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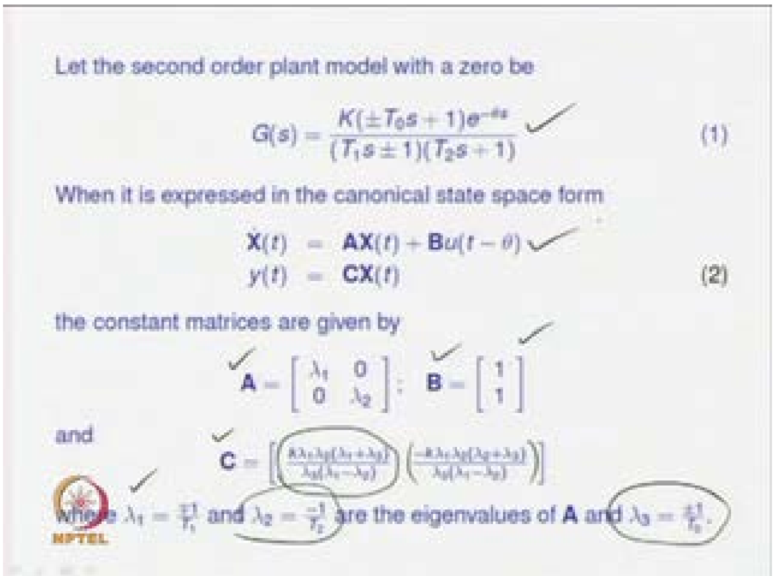

So, that I will not repeat, where we find that A B C D constants for the second order plus dead time transfer function model will have the variables lambda 1 0 0 lambda 2, for A and B will be made of a vector of elements 1 and 1, whereas C will have the variables lambda 1 lambda 2 lambda 3 k, and the constants of C can be found as k lambda 1 lambda 2 times lambda 1 plus lambda 3 upon lambda 3 times lambda 1 minus lambda 2, whereas the second element of C vector will be minus k lambda 1 lambda 2 lambda 2 plus lambda 3 upon lambda 3 lambda 1 minus lambda 2. So, these lambda 1 lambda 2 lambda 3 are nothing but the eigen values, we can write the expressions for that with respect to the transfer function model in the form of lambda 1 is equal to minus plus 1 upon T 1, and lambda 2 is equal to minus 1 upon T 2, whereas lambda 3 is given as plus minus 1 upon T 0. So, using these lambdas and the state equation.

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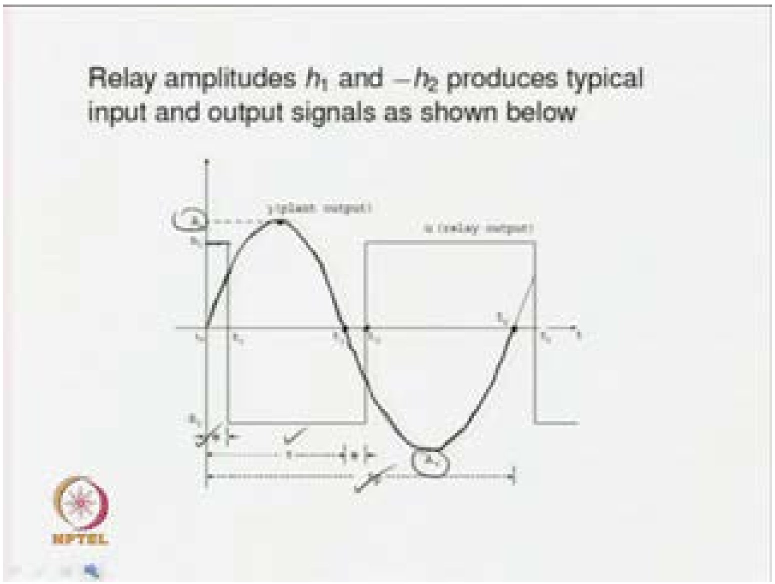

It is possible to find analytical expression for difference span of the sustained oscillatory output. So, the output will be considered for different time ranges for time t 0 to t 1. We have got input h 1, whereas for time t 1 to t 2, we have got input h minus h 2. So, also from time t 2 to t 3, we have got the input to the system h minus h 2, whereas for time t 3 onwards till t 4 another zero crossing, we get the input as h 1. So, since we have got different piecewise constant linear inputs to the system piecewise constant inputs to the system therefore, the expression for different segment of the output will be obtained. So, we will have one segment spanning from time t 0 to t 1, and another expression will be obtained for obtaining the output for this span. The third one for this time range and the final one for this output, we shall find another analytical expression.

So, in we have to find four analytical expressions to find one complete cycle of output of the system. Now, the peak amplitude is given by A p, whereas another peak or the value of the output is given as A v, and the time theta since the input appears after time theta. Therefore, the theta can be shown in this fashion, whereas tau and tau p are the half period and the period time period of the sustained oscillatory output.

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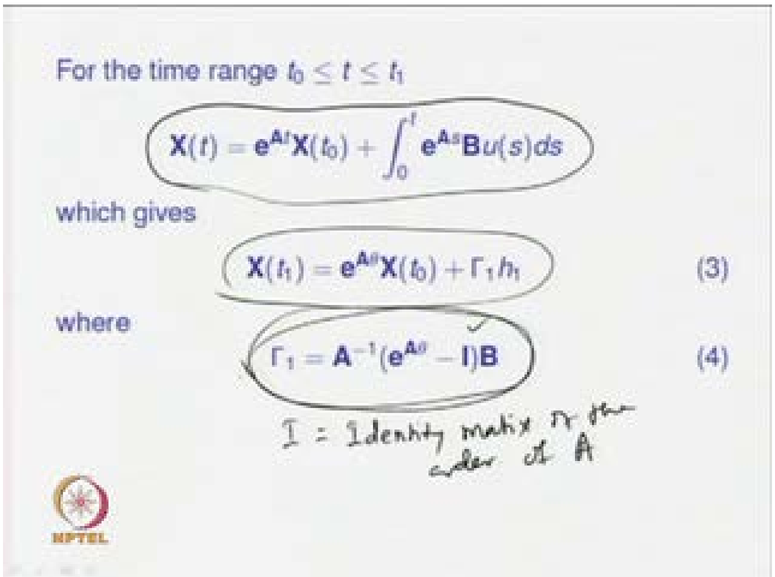

Now, using analysis and the state equation solution for which is given in this form. It is possible to obtain the solution as X(t 1) is equal to e to the power A theta X (t 0) plus gamma 1 h 1, where gamma 1 is equal to A inverse e to the power A theta minus I B. What is I, I is nothing but the identity matrix of the order of A. So, wherever we have go I in the expressions I will be of the order of A. Then, only we can have proper dimensionality for gammas.

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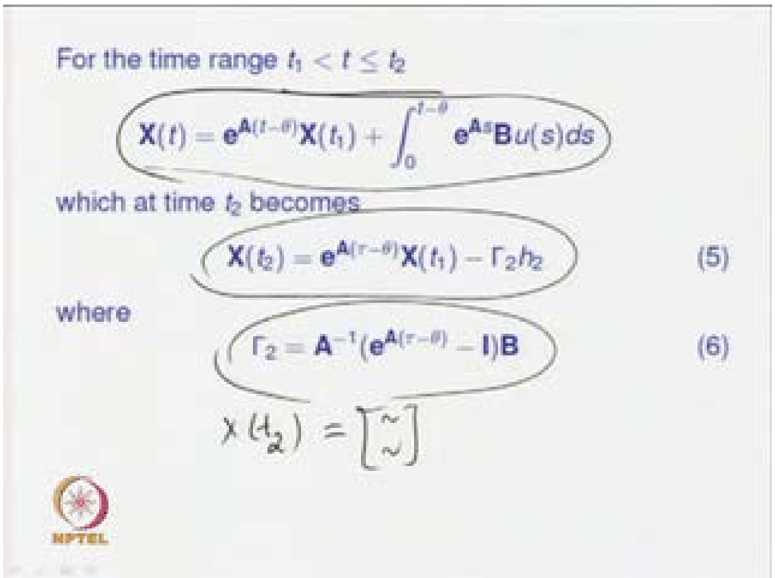

Then for the second time range, for the time range spanning from t 1 to t 2, the state equation gives us a solution of this form which upon again simplification gives us, X(t 2) is equal to e to the power A tau minus theta times X(t 1) minus gamma 2 h 2, where gamma 2 is obtained as A inverse time e to the power A tau minus theta minus I B. Now, X is the state variable, and we will get the values of the state variables. So, we shall have two values for the state variable at any instant of time therefore, X(t 2) will also give us two values. So, that one has to keep in mind that, the state we obtained are not scalar values rather for a second order system will obtain two elements for each state vector at any instant of time.

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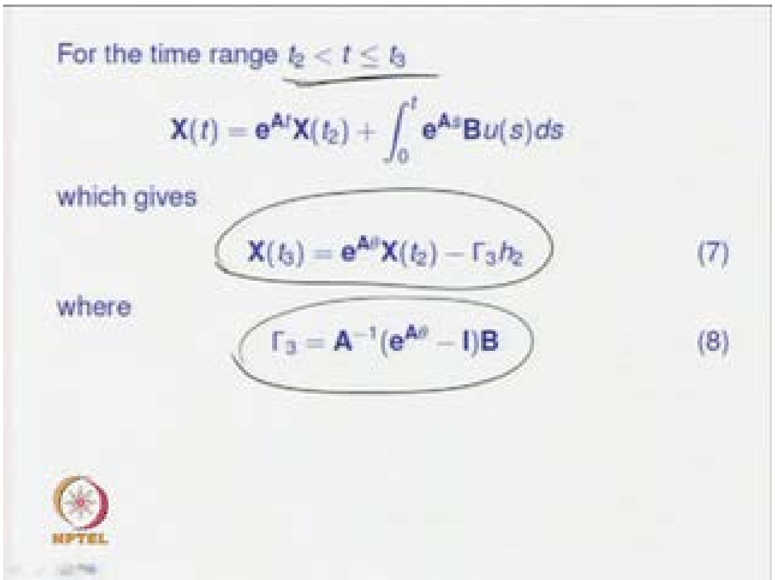

So, for the next time range, the solution of the state equation obtain in the form of the X(t 3) is equal to e to the power A theta X(t 2) minus gamma 3 h 2, where gamma 3 is given as A inverse time e to the power A theta minus I B.

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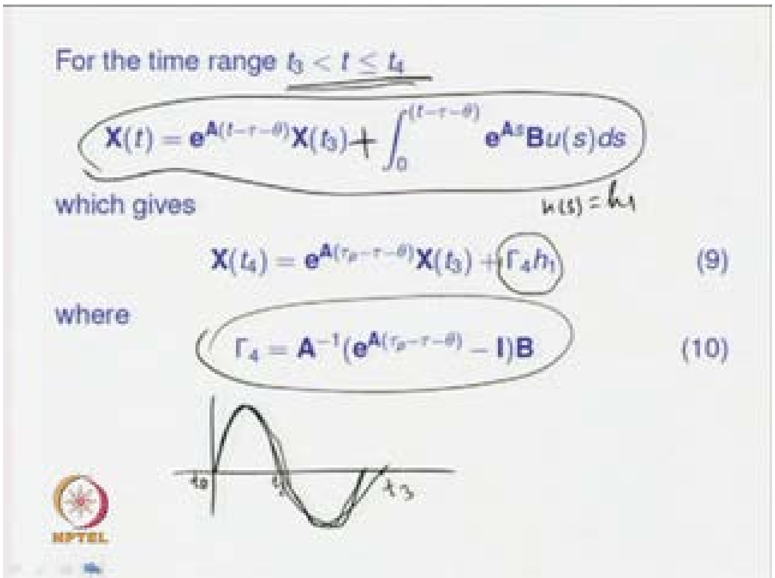

And for the final segment, which starts from time t equal to t 3 to time t equal to t 4, the solution of the state equation is given by this, where X(t) becomes, X (t)t is equal to e to the power A t minus tau minus theta X (t 3) plus integral from 0 to t minus tau minus theta e to the power A s B u(s) ds, but u (s) we find that u s is equal to h 1 for this case therefore, the integral can be found, which gives us the term gamma 4 h 1. So, gamma 4 becomes A inverse e times e to the power A tau p minus tau minus theta minus I B. So, for the 4 parts of one complete cycle, which is coming from t 0 to t 2 to t 4 for a different piecewise constant inputs, we have found analytical expressions. So, using the set of analytical expressions, it is possible to obtain the exact output sustained oscillatory output we get from the relay test. Now, using those set of analytical expressions. Now, we can find the unknowns associated with the same second order plus dead time transfer function model.

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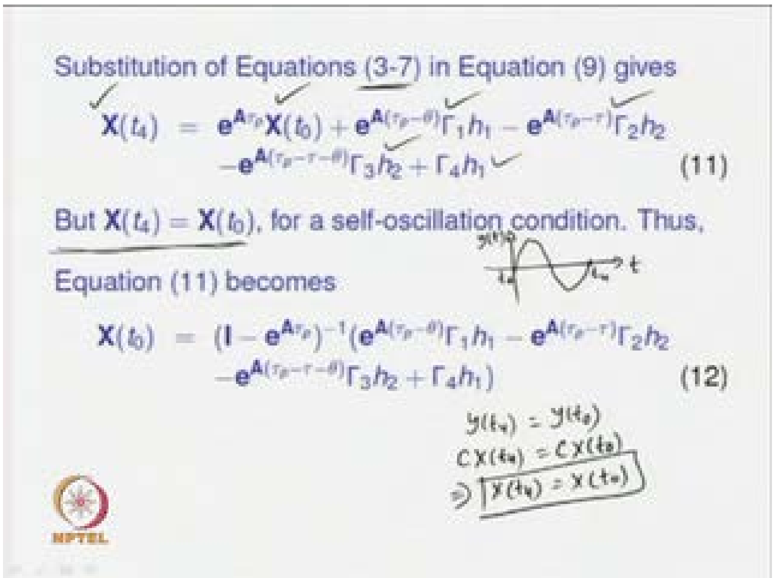

Now, that effort will be made to find that one, before that we need to find the value of state at time t equal to t 4. So, the value of states are state variable at time t equal to t 4 can be given in the form of e to the power A tau p X(t 0) plus e to the power A tau p minus theta times gamma 1 h 1 minus e to the power A tau p minus tau gamma 2 h 2 minus e to the power A tau p minus tau minus theta gamma 3 h 2 plus gamma 4 h 4. How we get this state solution of the state equation, if we substitute equation 3 to 7 in equation 9; equation 9 is nothing but the expression, we have obtained for X(t 4).

So, when the state variable X(t 3) solution of the state at time t equal to t 3 is substituted at here one obtains the expression for X(t 4). Now, X(t 4) has to be X(t 0) .Why that is so, to obtain sustain oscilladftory output please keep in mind that, this is our t 0, and this is our t 4, this is time axis, we have got sustained oscillatory output y(t).

Now, unless this is maintain we cannot guarantee sustained oscillatory output. So, what has to be maintain, that the output at time t 4 y (t 4) has to be equal to y(t 0). Then only one guaranties sustained oscillatory output from the second order system. Now, again we know that y(t) is equal to CX(t). So, using this one can write this expression in the form of CX(t 4) is equal to CX (t 0), which implies X(t 4) is equal to X (t 0). So, we find that X(t 4) has to be X(t 0) for obtaining a sustained oscillatory output from the relay test.

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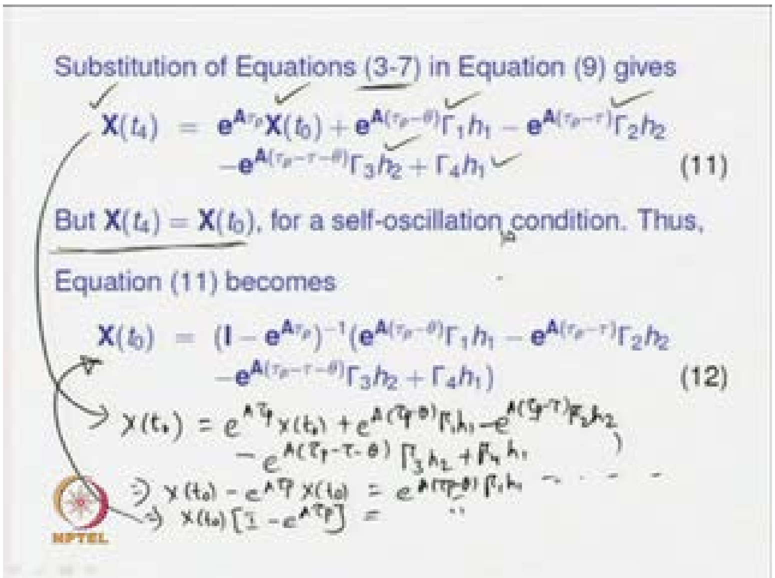

When that condition is substituted now, equation 11 that you have obtained earlier can be returned in the form of X(t 0) is nothing but X (t 4) or X (t 4) is equal to X(t 0) therefore, X (t 0) is equal to e to the power A tau p X (t 0) plus e to the power A tau p minus theta gamma 1 h 1 minus e to the power A tau p minus tau gamma 2 h 2 minus e to the power A tau p minus tau minus theta times gamma 3 h 2 plus gamma 4 h 1. So, when the terms associated with X (t 0) is collected, we obtain in the form of X (t 0) minus e to the power A tau p X (t 0) is equal to e to the power A tau p minus theta gamma 1 h 1 and so on. Then this first one can be simplified as X (t 0) times I minus e to the power A tau p is equal to these terms.

Thus one enables, this enables us to 11 expression can be finally obtained, in the form of X (t 0) is equal to I minus e to the power A tau p inverse times e to the power A tau p minus theta gamma 1 h 1 minus e to the power A tau p minus tau gamma 2 h 2 minus e to the power A tau p minus tau minus theta gamma 3 h 2 plus gamma 4 h 1. So, we have found expression for initial state of the system, and now using this initial state. Now, subsequent states of the system at any instant of time can be obtained.

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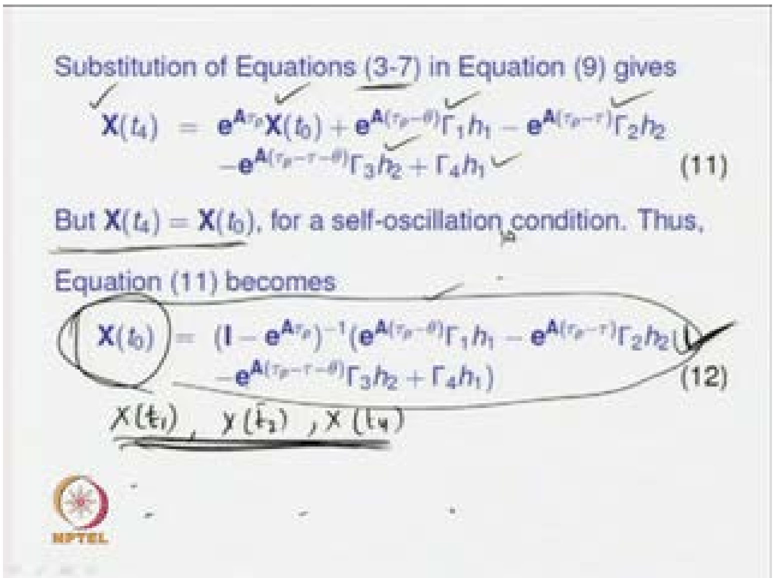

So, this X (t 0) expression is very vital, very important for us, because we can now find the state of the system at any instant of time using the state at time t equal to t 0. So, this is very important. Now, once we have found the expression for X (t 0), it is not difficult to find expression for X(t 1), X(t 2) and X(t 4) conveniently. So, all these are expression, the state expressions will make use of equation number 12 or will require this analytical expression to find correct expression for state variable at any instant of time. Now, one has to keep in mind that matrix exponentiation has to be found.

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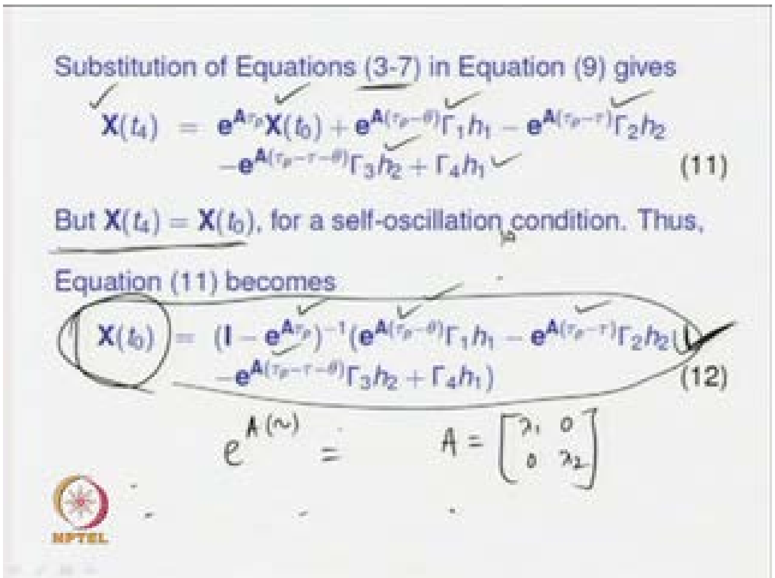

So, e to the power A with something is there, as you see we have got matrix exponentiation involved in almost all terms, and matrix exponentiation can be found conveniently provided the A matrix is available in diagonal form. So, care has been taken to find A in diagonal form, where we have found A as lambda 1 0 0 lambda 2. So, if it is not so, it is very difficult to obtain matrix exponentiation and subsequently, it will very difficult to simplify the analytical expressions we have obtained so, far.

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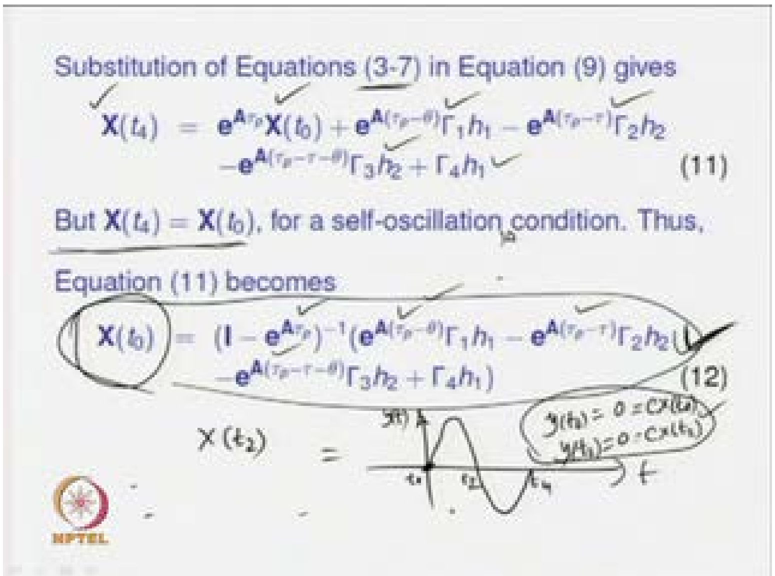

Now, we will try to find another expression for X (t 2) yX(t 0) and X(t 2) as you see the output waveform assumes this form and we have got time instance t 0 t 2 and t 4. Now, as you see the output at time t 0 is 0y, if this is the y then y (t 0) is equal to 0. We have zero crossing at time t equal to t 0 similarly at time t equal to t 2 y(t 2) the output is 0. So, when we substitute CX(t 0) is equal to y(t 0) is zero similarly, CX(t 2) is equal to y (t 2) is equal to 0. Then, we will get further simplified expressions for 12 and the other one. So, that way keep in mind this can conveniently, we used to find simpler analytical expressions are non-linear expressions which can subsequently, we solve to estimate the unknowns associated with second order plus dead time system.

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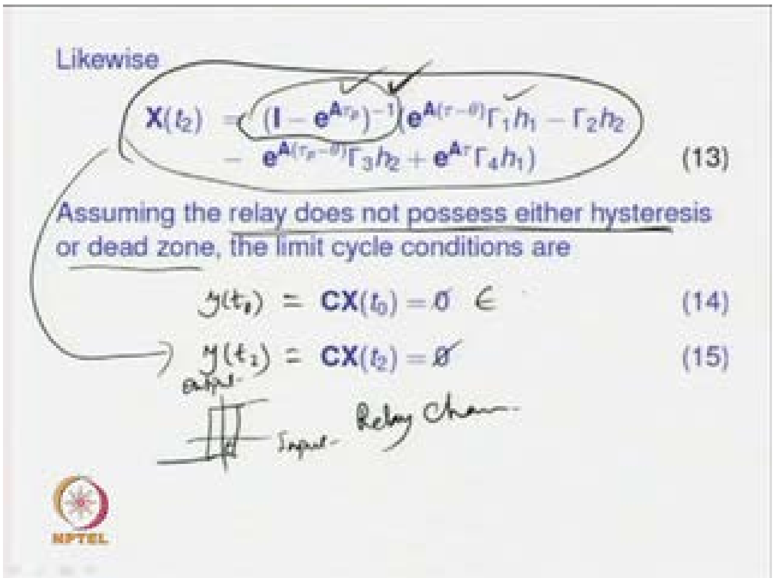

Now, we will try to find another expression for the state at time t equal to t 2. So, that comes out to be in the form of X(t 2) is equal to I minus e to the power A tau p inverse time e to the power A tau minus theta gamma 1 h 1 minus gamma 2 h 2 minus e to the power A tau p minus theta gamma 3 h 2 plus e to the power A tau gamma 4 h 1. You see the same term is present here, also in this same inverse term which present for the expression for X(t 0). Keep in mind why that is coming, because X(t 2) is making use of X(t 0) therefore, we are getting the same expression, same term present in the expression for X(t 2). Now, if the relay does not possess either hysteresis or dead zone, in that case the output y(t 0) is equal to CX(t 0) has to be 0 and we have got a zero crossing at time t equal to t 0. Similarly, the output at time t 2 y(t 2) has to be 0.

Now, y(t 2) is equal to CX(t 2) therefore, the X(t 2) expression can be substituted here and subsequently a simpler expression can be obtained. When this is not true what is not true, when the relay is not ideal and, if it possesses hysteresis or dead zone. In that case this zeros will not be there, in that case suppose the relay is having some hysteresis, if the relay characteristics is given by this. So, if the relay characteristics relay input and output this the relay characteristics. So, when the relay characteristics is not ideal, in that case y(t 0) will be some finite value depending on the hysteresis width. So, that way y(t 0) is equal to CX(t 0) will be equal to epsilon and y(t 2) will be also equal to minus epsilon.

So, we have to be very careful. So, only when we are employing an ideal relay for performing the relay test at that time at the zero crossings the outputs will be 0. So, finally, we get the two important expressions as far as the initial conditions of the state equations say time t equal to t 0 and t equal to t 2 are concern as y t 0 is equal to CX(t 0) equal to 0 y (t 2) is equal to CX(t 2) is equal to 0.

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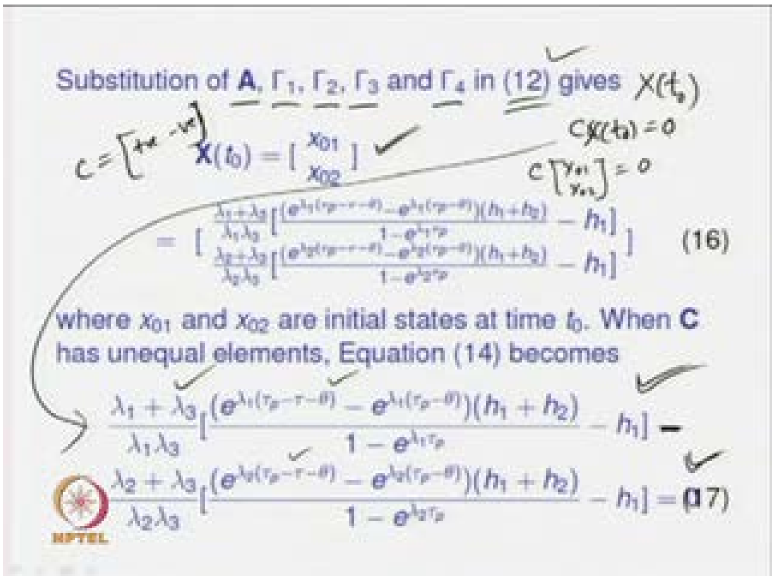

Now, when X(t 0) and X (t 2) expressions are put what form of expressions we get, the non-linear expressions we will get for that case will be of this form. How we are obtaining that one. So, when A gamma 1, gamma 2, gamma 3 and gamma 4 are substituted in the expression for X (t 0) that we have obtained earlier in equation number 12, that time X (t 0) gives us two values X 01 and X 02, and assuming that X 01 is not equal to X 02. It can be equal to also depending on the values for a gamma 1 gamma 2 gamma 3 gamma 4, but when it is not equal to 0. In that case equation number 17 is obtain, in which case X 01 is obtained as lambda 1 plus lambda 3 upon lambda 1 lambda 3 times e to the power lambda 1 times tau p minus tau minus theta minus e to the power lambda 1 tau p minus theta times h 1 plus h 2 upon 1 minus e to the power lambda 1 tau p minus h 1.

So, in the second case again, if you see the expression we get is X 02 will be lambda 2 plus lambda 3 upon lambda 2 lambda 3 times the expression minus h 1. So, keep in mind only one lambda 1 is equal to lambda 2 then, the two will be equal. So, X 01 will be equal to X 02, when lambda 1 is equal to lambda 2 in that case, we will have some problem. So, assuming that X 01 is not equal to X 02 in that case. Since CX(t 0) has to be zero for the sustained oscillatory output in that case, we obtain an expression of the form given as 17. So, finally, what we obtain when you substitutes CX(t 0) equal to 0, we obtain lambda 1 plus lambda 3 upon lambda 1 lambda 3 times e to the power lambda 1 tau p minus tau minus theta minus e to the power lambda 1 tau p minus theta times h 1 plus h 2 divided by 1 minus lambda 1 tau p minus h 1 minus lambda 2 plus lambda 3 upon lambda 2 lambda 3 times e to the power lambda 2 tau p minus tau minus theta minus e to the power lambda 2 tau p minus theta times h 1 plus h 2 upon 1 minus lambda 2 tau p minus h 1 is equal to zero. So, if you look at carefully, the upper and bottom terms have got only difference in lambda 1 and lambda 2.

So, the bottom term can easily obtained by the substitution of lambda 1 is equal to lambda 2. So, for the second order case always you will find two values for the states at any instant of time. So, when those two states are negated then, since it has to be zero CX(t 0) has to be 0. So, therefore, C times X 01 X 02 is equal to 0, and C is obtain in such a way C expression for C such that first term, we have got a positive and second term we have got a negative value. So, that way we have finally, obtained the negative symbol over here. So, this first term minus second term has to give you 0, that has to be maintained to obtain sustained oscillatory output.

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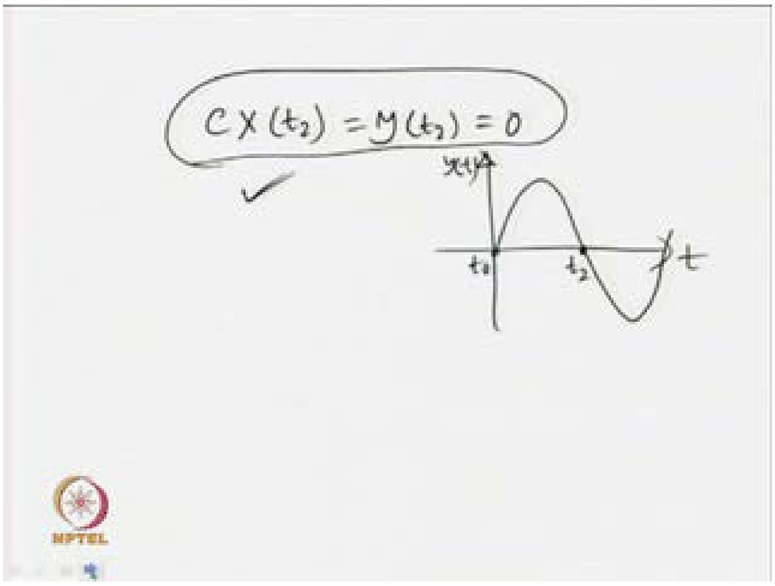

So, similarly from the second condition, the condition that CX(t 2) has to be equal to y(t 2) has to be 0, because we have got another zero crossing at time t equal to let me repeat this is the output waveform. So, we have got t 0 here, and t 2 here. So, another zero crossing appears at time t equal to t 2 resulting in the expression y (t 2) is equal to CX(t2) is equal to 0. So, when this is also done, when the X(t 2) expression is substituted here.

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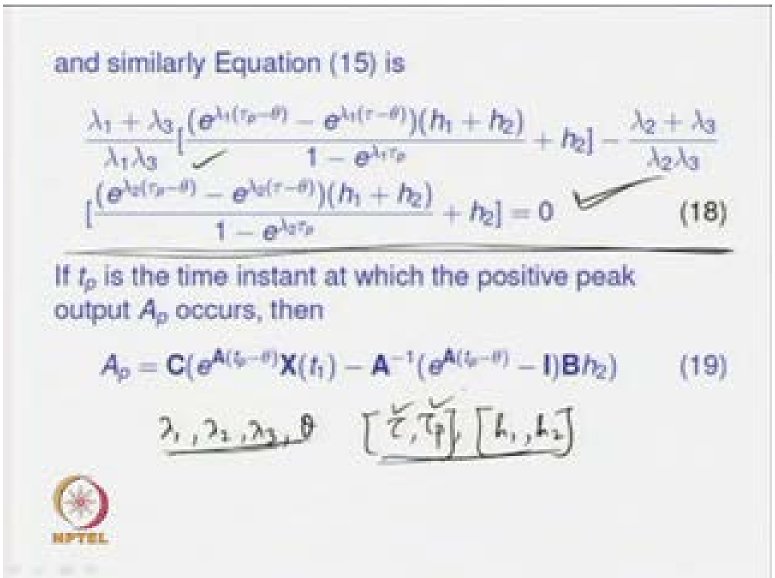

One obtains lambda 1 plus lambda 3 upon lambda 1 lambda 3 time C to the power lambda 1 tau p minus theta minus e to the power lambda 1 tau minus theta times h 1 plus h 2 upon 1 minus e to the power lambda 1 tau p plus h 2 minus lambda 2 plus lambda 3 upon lambda 2 lambda 3 times e to the power lambda 2 tau p minus theta minus e to the power lambda 2 tau minus theta times h 1 plus h 2 upon 1 minus e to the power lambda 2 tau p plus h 2 is equal to 0. So, we obtained another similar expression as you have obtained in equation number 17. So, these two expressions are highly non-linear in nature and those are to be solved using some non-linear equation solver. Then only it is possible to find the unknown associated with the equations 17 and 18, what are the unknowns we have in 17 and 18, if you look carefully lambda 1 lambda 2 lambda 3 theta, these are the unknowns we have, all other parameters are known or measured. What are the measured quantities we have in the expressions?

So, tau tau p are measured, and what are the known is we have for the expression h 1 and h 2, these are the user defined values. So, user sets the relay heights therefore, h 1 and h 2 are known. So, there are four unknowns, whereas two measurements are made. So, far so, that way we need to make further two measurements. So, that four unknowns associated with the transform of function model can be estimated. So, we have to find analytical expression for peak amplitude as well as the negative peak amplitude. So, those two values will not be same. Now, we have seen that, we have got the peak amplitude as A p, and another amplitude as A v. So, these expressions for these two amplitudes are to be found. So, that we will obtain 4 analytical expressions to estimate at least four unknowns associated with the transfer function model.

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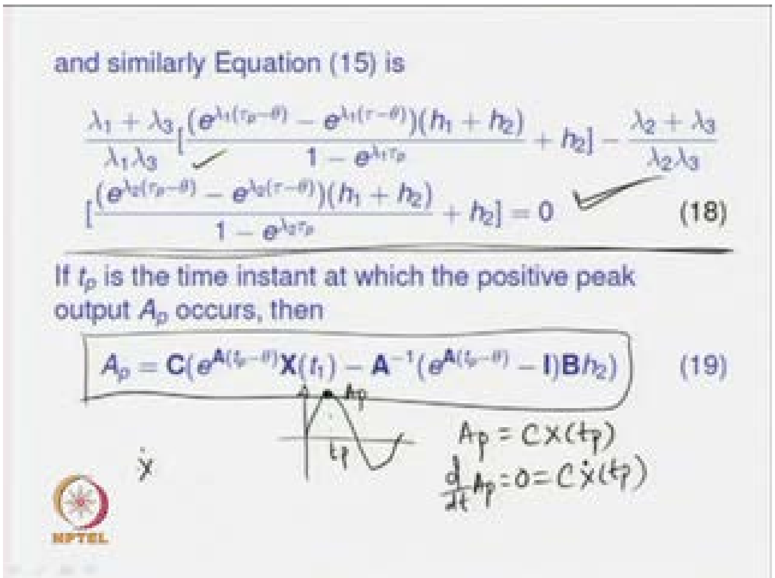

How we can develop those expressions, we know that the first order differentiation of any peak will result in 0. So, if this is the maximum value A p is the peak value maximum value. So, first order differentiation will give us 0. So, what is A p, A p is nothing but CX (t p). So, at time t p then, how to find the expression for A p, if t p can be obtained then, A p can easily be obtained, because we have got analytical expression for the state variables at any instant of time. Now, how can I find this one, I know that the first order differentiation d upon d t of A p will be equal to 0, which is nothing but CX. t p now, because first order differentiation of the state is taken.

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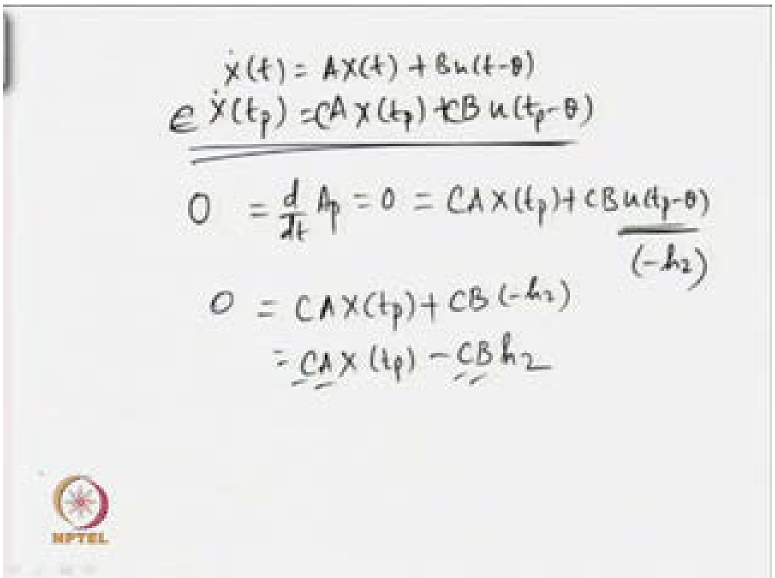

Then what is X.(t p), X.(t p) will be nothing but AX(t p)plus Bu t p minus theta. So, we will say at time, because the solution of any state equation or the state equation is given as X. t is equal to AX(t) plus Bu t minus theta. So, substitute t equal to t p, then you get this expression, but CX.(t p) will be now, CAX(t p) plus CBu t p minus theta, but the left hand side is equal to 0, because we have seen that first order differentiation of the peak amplitude has to be 0. So, this is equal to CAX(t p) plus CBu t p minus theta, but when the peak occurs, if we look at the output waveform, the peak occurs after time t equal to t 1. Therefore, we have to see that the input to the system at that time is minus h 2 therefore, I can write this expression as 0 is equal to CAX (t p) plus CB minus h 2, thus giving us CAX(t p) minus CBh 2. Now, substitute CA and X(t p) C and B.

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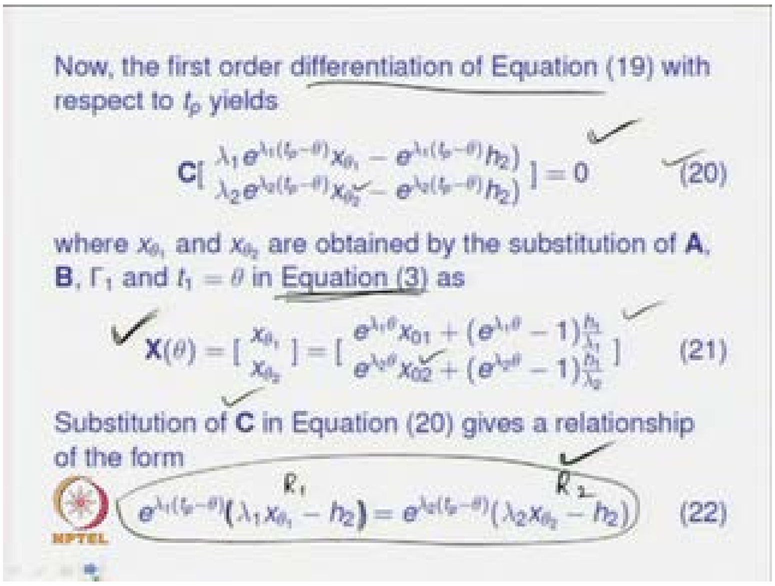

And we get the expression for the output difference, first order differentiation of the output in the form of C times lambda 1 e to the power lambda 1 t p minus theta X theta 1 minus e to the power lambda 1 t p minus theta h 2, and the second element as lambda 2 e to the power lambda 2 t p minus theta X theta 2 minus e to the power lambda 2 t p minus theta h 2 is equal to 0. So, when further X theta 1, and X theta 2 are obtained from equation three. It is not difficult to obtain the expression for X theta, which gives us X theta 1, and X theta 2. As I have said the state has got two values at any instant of times. So, those two values can be obtained with the expressions e to the power lambda 1 theta X 01 plus e to the power lambda 1 theta minus 1 times h 1 upon lambda 1, and e to the power lambda 2 theta X 0 2 plus e to the power lambda 2 theta minus 1 h 1 upon lambda 2.

So, when X theta is found in this form, and substitution of C in equation 20 gives us a simpler expression of the form e to the power lambda 1 t p minus theta times lambda 1 X theta 1 minus h 2 is equal to e to the power lambda 2 t p minus theta times lambda 2 X theta 2 minus h 2, this expressions can further be simplified to find the unknown (( )) to find the expression for t p.

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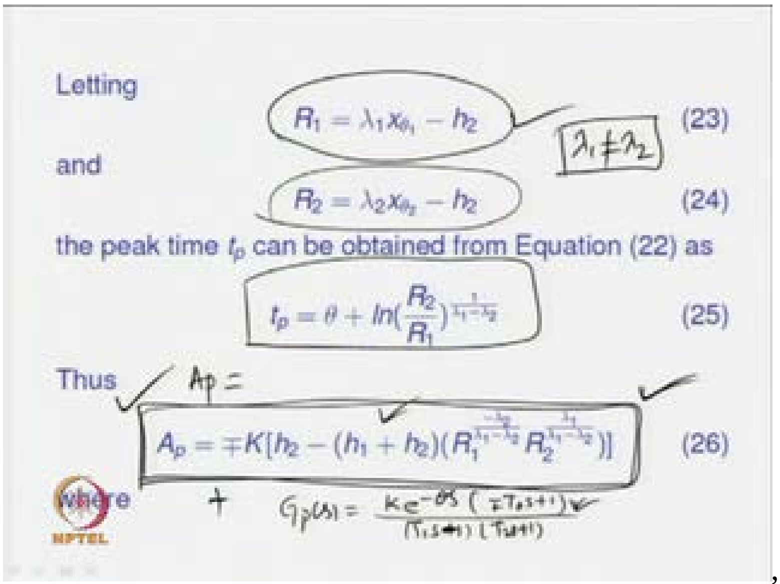

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So, I will try to find expression for t p with the assumption that. Let R 1, is equal to lambda 1 X theta 1 minus h 2 and R 2 is equal to lambda 2 X theta 2 minus X 2. In that case, how we get the expression. So, this is written as R 1, and this second part is written as R 2. So, the part we have within the parenthesis are written as R 1, and R 2 giving us an expression of the form e to the power lambda 1 t p minus theta.

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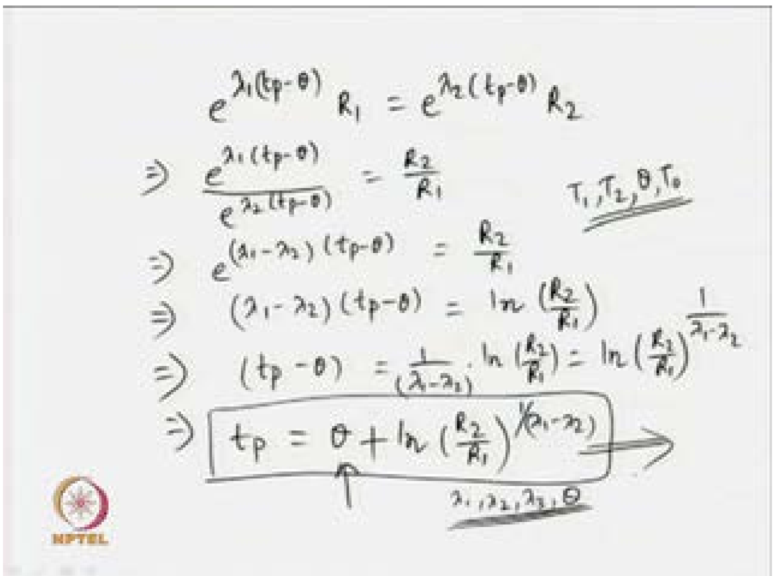

e to the power lambda 1 t p minus theta R 1 is equal to e to the power lambda 2 t p minus theta R 2, because we have assumed that R 1 is equal to lambda 1 X theta 1 minus h 2 and R 2 is equal to lambda 2 X theta 2 minus h 2. Thus further simplification will give us it can be written as e to the power lambda 1 t p minus theta upon e to the power lambda 2 t p minus theta is equal to R 2 upon R 1 or e to the power lambda 1 minus lambda 2 times t p minus theta will be equal to R 2 upon R 1. So, if I take natural logarithm of both sides in that case, we have left with lambda 1 minus lambda 2 times t p minus theta is equal to lon R 2 upon R 1, which further gives us t p minus theta is equal to 1 upon lambda 1 minus lambda 2 times lon R 2 upon R 1, which can further be written as lon R 2 upon R 1 to the power 1 upon lambda 1 minus lambda 2 or t p is equal to theta plus lon R 2 upon R 1 to the power 1 upon lambda 1 minus lambda 2. So, t p is obtained as t p is equal to theta plus natural logarithm of the ratio R 2 upon R 1 to the power 1 upon lambda 1 minus lambda 2. So, when this t p value is substituted in the expression for A p, A p the expression for A p we have already obtained.

So A p ,what is A p, A p is C times e to the power A t p minus theta times X(t 1) minus A inverse e to the power At p minus theta minus I to the I times B h 2. So, when the expression for the t p C and A are substituted. We get the expression for the peak amplitude in the form of A p is equal to minus plus K times h 2 minus h 1 plus h 2 together times R 1 to the power minus lambda 2 upon lambda 1 minus lambda 2 times R 2 to the power lambda 1 upon lambda 1 minus lambda 2. So, this is the correct exact expressions one obtains for the peak amplitude of the sustained oscillatory output. Now, what are the limitation of this expression, if you observe carefully lambda 1 should not be equal to lambda 2, when lambda 1 is equal to lambda 2, then it will be very difficult to find expression for A p. So, with this assumption that lambda 1 is not equal to lambda 2.

Let me once more read out the correct expression, exact expression the for the peak amplitude or peak output of the sustained oscillatory output as A p is equal to minus plus K times h 2 minus h 1 plus h 2 together times R 1 to the power minus lambda 2 upon lambda 1 minus lambda 2 times R 2 to the power lambda 1 upon lambda 1 minus lambda 2. Now, this negative value will be for what type of process, if you have taking a second order system with a right half plane pole. So, the negative sign the upper one will be chosen for a process of the form G p (s) is equal to K e to the power minus theta s minus plus t 0 s plus 1 times t 1 s plus 1 and t 2 s plus 1. So, the negative sign is for this process, whereas the bottom sign positive is chosen when this becomes positive, when A p equal to plus K times this term in that case, this t 1 S minus 1 will come here. So, one has to keep in mind this factor. So, using this general expressions, it is possible to estimate a second order stable or unstable dynamics. Now, similarly we have to develop one more expression for the negative peak for which we have to consider that A v occurs at time.

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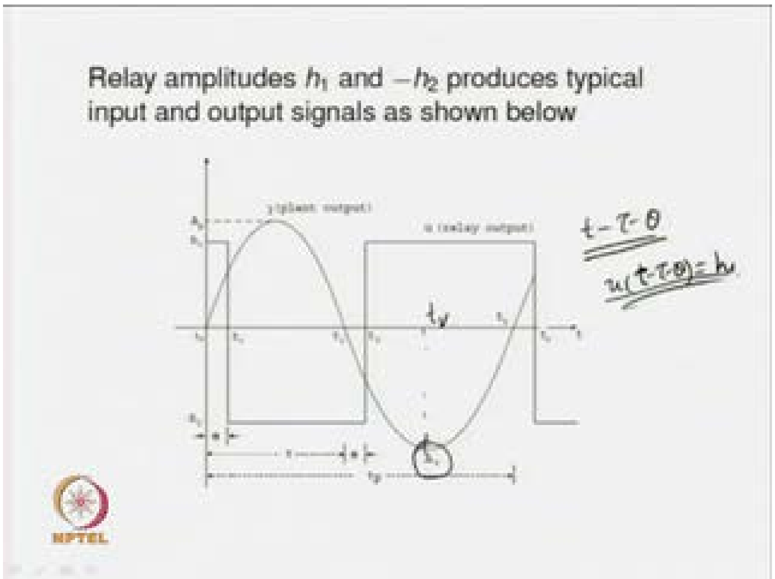

Let this time be t v, what is t v now, the time t v occurs after. So, the range for the time or where this t v is falling, it is falling beyond your time t minus tau minus theta. Keep in mind during, which we have got the input to the system u t minus tau minus theta is equal to plus h 1. So, proper input is to be applied to obtain the negative peak of the sustained oscillatory output.

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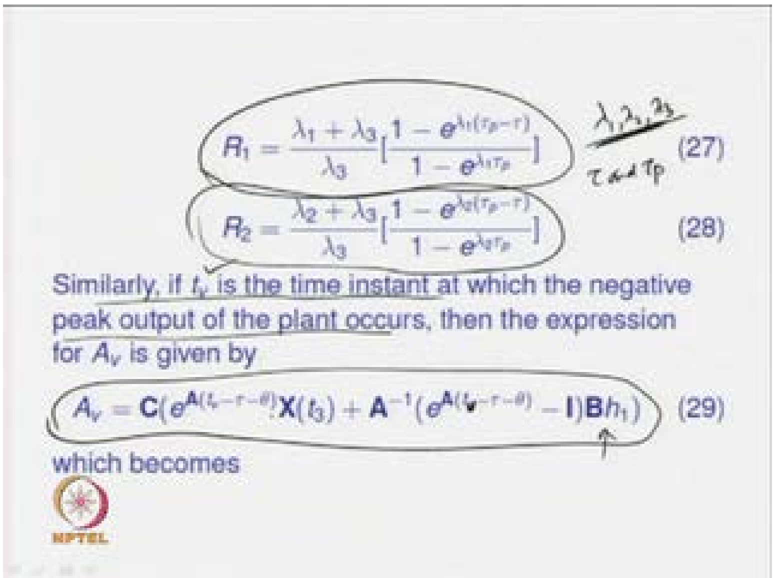

So, to find another analytical expression for A v, what we have to do, we have to consider that the time at which the negative peak output of the plant occurs is consider to be t v. Then the expression for A v correct expression for A v will be A v is equal to C times e to the power At v minus tau minus theta times X(t 3) plus A inverse e to the power A t v not t p, t v minus tau minus theta minus I times Bh 1. As I said earlier, the input to the system is plus h 1 now. So, we have got h 1 in the expression 29. The way, we have solved and found the analytical expression for A p similar procedure will be employed to find analytical expression for A v.

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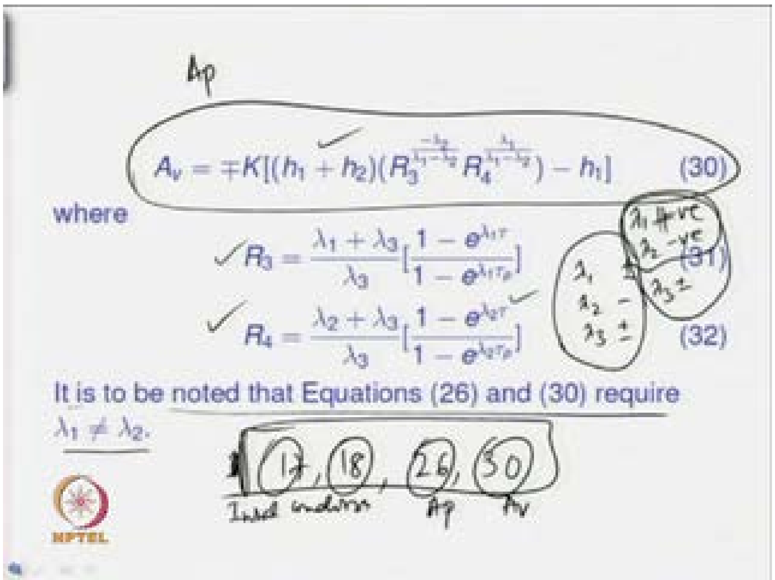

So, analytical expression for A v can be obtained taking the first order differentiation of A v, and found as A v is equal to minus plus K times h 1 plus h 2 times R 3 minus lambda 2 upon lambda 1 minus lambda 2 times R 4 lambda 1 upon lambda 1 minus lambda 2 minus h 1. What is R 3 and R 4 here. R 3 is equal to lambda 1 plus lambda 3 upon lambda 3 times 1 minus e to the lambda 1 tau upon 1 minus e to the power lambda 1 tau p, and R 4 is given as lambda 2 plus lambda 3 upon lambda 3 times 1 minus e to the power lambda 2 tau upon 1 minus e to the power lambda 2 tau p. So, this point is very important once more. Let me repeat that point, that lambda 1 must not be equal to lambda 1 otherwise, we get no solutions for either the earlier expression for the peak amplitude A p or A v.

So, R 1 and R 2 can how can we find, we have earlier we have taken that R 1 is equal to lambda 1 X theta 1 minus h 2, whereas R 2 is equal to lambda 2 X theta 2 minus h 2. So, when expression for X theta 1 is substituted and simplified that gives us R 1 h lambda 1 plus lambda 3 upon lambda 3 times 1 minus e to the power lambda 1 tau p minus tau upon 1 minus e to the power lambda 1 tau p and R 2 becomes, lambda 2 plus lambda 3 upon lambda 3 times 1 minus e to the power lambda 2 tau p minus tau upon 1 minus e to the power lambda 2 tau p. So, if we see what we have, what are the knowns and unknowns we have again in this expressions. So, the expressions let me consider the expression A p here. The unknowns are lambda 1 and lambda 2, whereas R 1 and R 2 involves the measurements tau and tau p and lambda 3. So, the unknowns associated with the all the R's are now, lambda 1 lambda 2 lambda 3, these are the unknowns associated with R 1 and R 2.

So, and what are the measurements we had for this or what are the parameters that is available from the measurement of the system output, we have go tau and tau p. So, h is there again for the final output expression for the output h 1, and h 2 are the user defined. So, if you look at carefully the unknowns we have in either A p or A v.

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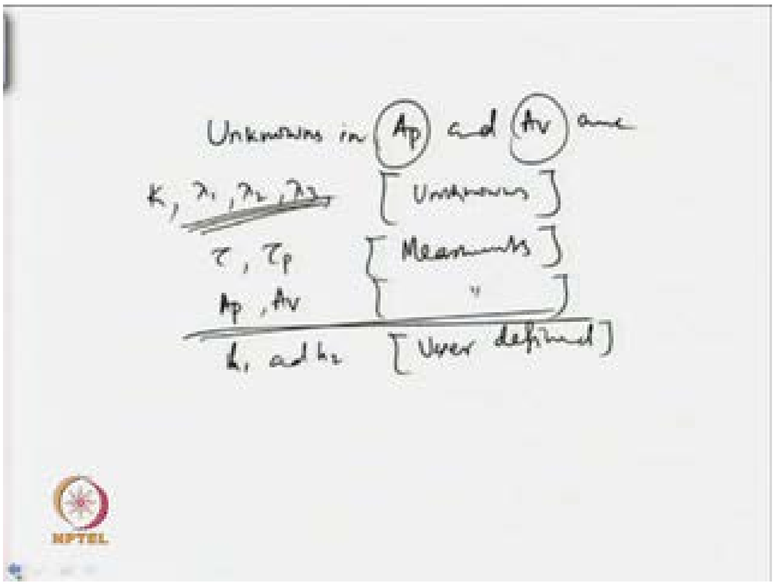

Expressions for A p and A v are the unknowns in A p and A v are unknowns are lambda 1 lambda 2 lambda 3, whereas we have got the measurements these are the unknowns. Now, we have got measurements tau and tau p. These are the measurements along with A p and A v are also the set of measurements and user defined values are h 1 and h 2. So, user defined. So, finally, if we see. So, we have got three unknowns associated with the expression A p and A v, whereas assuming that the steady state gain is known are found by some other technique.

So, if that is not assumed in that case, we will have four unknowns including K. So, we have finally, found 4 important expressions to estimate the unknown parameters what are those, those 4 expressions let me again show you those 4 expressions are equation number 17 and 18. So, 17 and 18, and similarly we have got expressions 26 and 20, 30. So, 17, 18, 26 and 30, these are the 4 expressions we have found for A p A v, this is for initial conditions.

So, finally, we have found four exact analytical expressions. Using this 4 exact analytical expressions it is possible to estimate four unknowns associated with the transfer function model. Then question comes how do solve these set of analytical expressions. Now, these analytical expressions are highly non-linear. So, you have to try with correct initial values, when you employ some non-linear solver to solve the set of equations. These equations 17, 18, 26 and 30 are highly non-linear in the sense, we have got. So, many exponential terms although, we have got few constants and unless those are solved carefully, the solutions may lead to false solutions. In that case the estimated parameters will be erroneous. These are very important point.

Now, how to begin with solving the set of non-linear equations. Now, you know that your lambda 1 can be either positive or negative, whereas lambda 2 has to be negative. So, and lambda 3 can be either positive and negative. So, when you solve the non-linear equations you choose the proper values you pass on correct values or correct initial values to the non-linear equations solver. So, what initial values you will send for identifying stable second order plus dead time transfer function model you pass on some negative lambda 1, lambda 1 has to be negative.

Similarly, lambda 2 has to be negative and lambda 3 has to be either positive or negative depending on what type of zero you have it. Does not matter, but it must take care must be taken to pass on negative lambda 1 and lambda 2 values, whereas when you are identifying an unstable second order plus dead time transfer function model. In that case lambda 1 must be equal to positive. So, when lambda 1 is positive lambda 2 is negative and lambda 3 can positive or negative in that case, the solutions will lead to correct solution. So, the set of non-linear equations, we have found are very powerful, because they are based on exact analysis.

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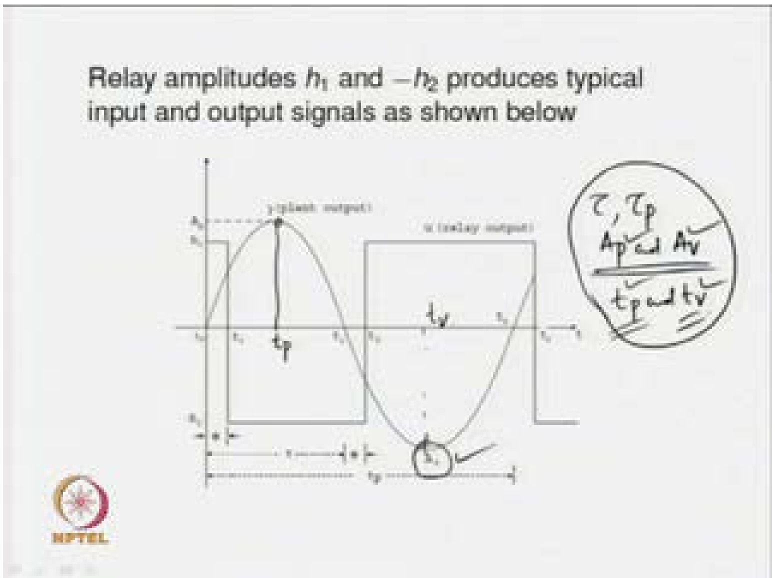

Now, how can I find analytical expressions for other parameters? Also we have found analytical expressions for the parameter t p, the time at which the peak amplitude occurs and t v the time at which the minimum or negative peak occurs. So, we actually speaking more measurements can be taken apart from A p, A v tau and tau p. So, what are the 4 measurements we have targeted? So, far we have targeted tau tau p A p and A v, whereas if one we says he can make use of two more expressions, we have found those are expressions for t p and t v. So, we have got two more expressions it is up to the user, if it is difficult to find solutions for the set of non-linear equations instead of using the non-linear equations for A p and A v.

One can make use of the expressions for the t p and t v by making measurements on the peak amplitudes and using the expressions for t p and t v. What are those expressions for t p and t v. Once as expression we have already shown, that the expression for t p each this one, which involves another unknown associated with the transfer function model the dead time. So, basically now the number of unknowns when you are using t p are lambda 1 lambda 2 lambda 3 and theta. So, theta comes into picture, if the aim is to identify the four unknowns T 1 T 2 theta and T 0. In that case the expressions for peak amplitude, the expressions for negative peak amplitudes can be made use of.

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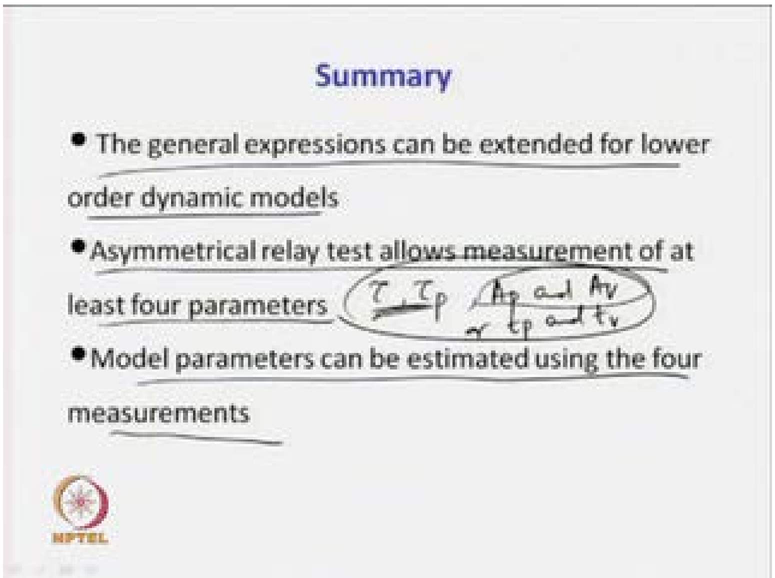

Now, I will summarize the lecture. Now, the general expressions can be extended for lower order dynamic models. What do you mean by this lower order dynamic models. Now, sometimes what will happen we can have simpler transfer function models or dynamics for the higher order dynamic real time dynamical system. So, in which case, you have to put some limiting values for lambda 2 or so. So, lambda 2 can be put either zero or can be put either infinity depending on the type of transfer function model we are targeting.

Then asymmetric relay test allows measurement of at least 4 parameters. What we mean by at least 4 parameters, as we have seen one can measure the span tau as well as the period tau p, whereas one can measure A p and A v or t p and t v. So, at least 4 measurements can be made, more measurements can be made, but only 4 expressions have to be solved simultaneously to find 4 unknowns associated with the transfer function model. So, the model parameters can be estimated using the 4 measurements either this two are must, whereas you have to choose expression for A p or A p and A v or t p and t v to find 4 unknowns associated with the transfer function model.

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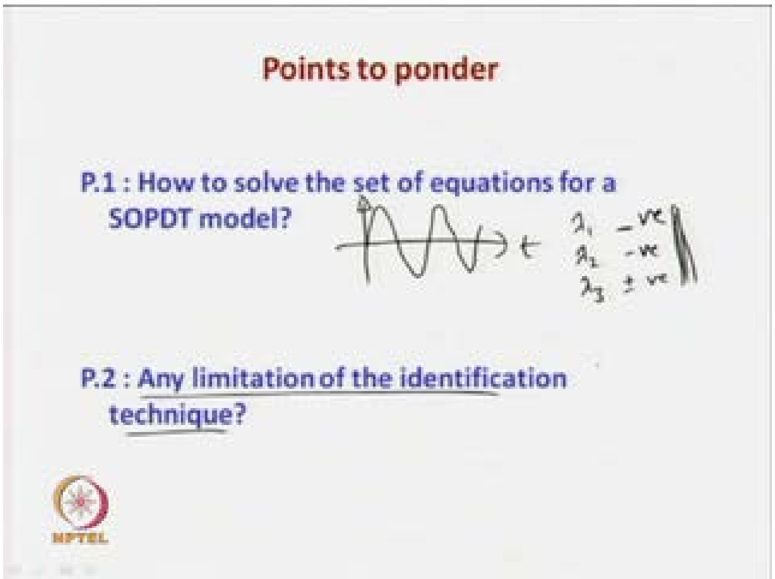

Now, points to ponder are how to solve the set of equation for a second order plus dead time model. I have already given the guidelines depending on the type of output from the sustained oscillatory output. It is very difficult to make out whether, the system dynamics has got some unstable poles or stable poles, but applying intuition now, you have to set the solve the set of non-linear equation. So, pass on lambda 1 as positive lambda 2 as negative and lambda 3 as either positive or negative and solve the equations. If you are not getting proper solutions, if the solutions are not realistic in that case. Next you choose lambda 1 to be negative.

So, this is one way one has to solve the set of non-linear equations to identify the unknowns associated with a second order dead time transfer function model. Any limitation of the identification technique, this the most important limitation is the solution, whether the set of non-linear equations lead to false solution or exact solution. So, we can get since we have got highly non-linear equations. So, the solutions can lead to false solution or can lead to local solution. So, one has to make proper equation, non-linear equation or use proper non-linear equation solver to overcome the limitation with the identification technique, that is all. Thank you.