Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 07

Identification of Second Order plus Dead Time Model

Welcome to the lecture, titled identification of second order plus dead time transfer function model. earlier In our earlier lectures, we have seen how to identify simple transfer function models, where the systems can have a delay and gain can have a fast order dynamics and delay and so on. Depending on the type of output sustained oscillatory output signal, we decide what type of transfer function model one has to identify. Thereafter, we employ the simple formulae we have derived in our earlier lectures, to decide and find the parameters of simple transfer function models. In this lecture, we shall consider a bit higher level transfer function model, where the transfer function will include a right half zero or left half zero apart from two poles. The poles can have also right half pole or right half left zero.

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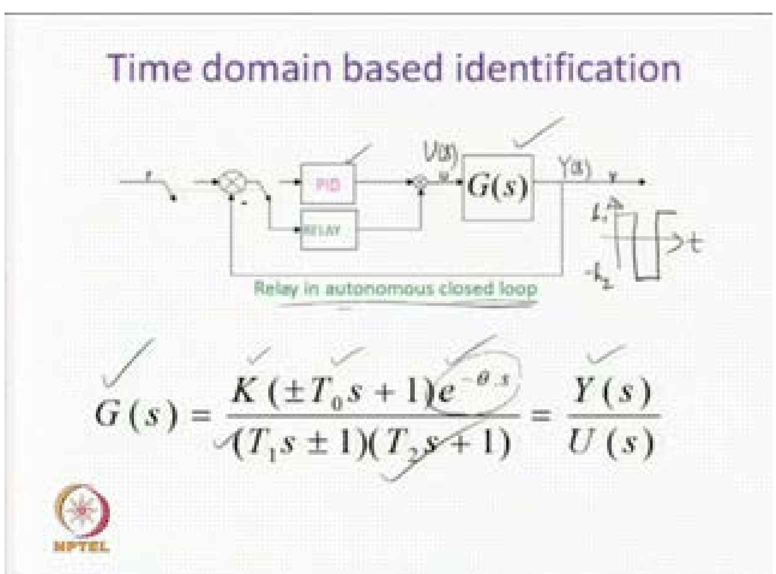

So, that way we will see, first the structure of the relay test. We shall employ for obtaining sustained oscillatory output. Now, the PID controller will be detest first; a relay will be connected to induce sustained oscillatory output. Now, the relay in autonomous closed loops shall induce sustained oscillatory output. Unlike the earlier cases, here in this case the relay will have different amplitudes. So, the relay characteristics can be shown by the diagram or plot given by t and h1 and minus h2. So, we will obtain piecewise linear output signal from the relay, but the output signal will have different amplitudes. Why we are they doing so? To obtain sustained oscillatory output with different amplitudes, peak amplitudes and different half periods; that will enable us to measure more than two parameters from the sustained oscillatory output.

So, when we measure more than two sustained parameters from the sustained oscillatory output, in that case the analysis will enable us to estimate more than two unknowns associated with a transfer function transfer function model for the dynamics of a real time system. So, the real time system is denoted by G(s). The dynamics of a real time system is given by G(s). Now, the G(s) can assume different form and one such form is mentioned over here; where the G(s) is of the transfer function model of the real time system is represented by this form; where we have got a steady state gain and we have got either one right half or left half s plane zero given by plus minus T 0 s plus 1.

Ofcourse, the system is expected or the system is assumed to be associated with some finite time delay, given by the term e to the power minus theta s. Further, the transfer function has got two poles; where one left hand side pole denoted by T 0 s plus 1 and either a left hand side or a right hand side pole given by the term T 1 s plus minus 1. Thus we see that, unlike the earlier cases the transfer function model has has got 0 2 poles and 1 0 and the zero can be located anywhere in the s plane. Now, the output and input to the system is assumed to have in the Laplace transfer form Y s and U(s). So, the system output in Laplace domain is denoted by Y(s) and the input is by U(s).

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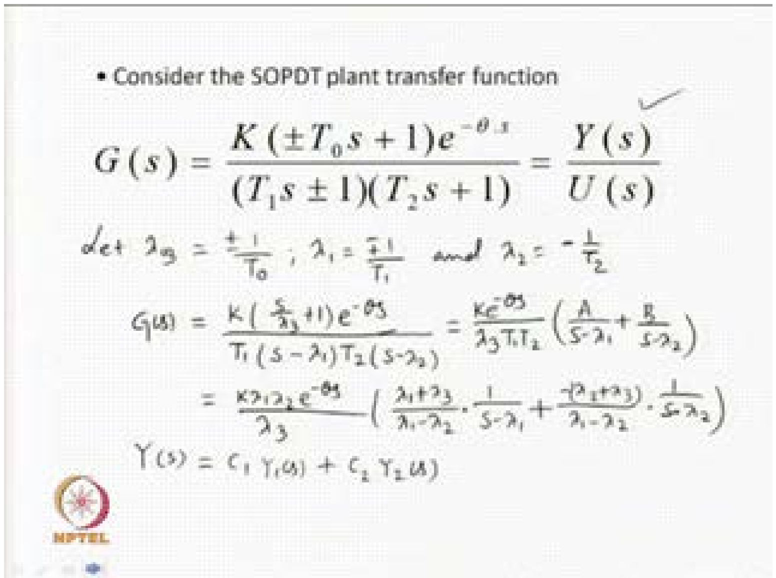

Now, when this transfer function is written in this form; further analysis of this transfer function can yield us, the representation of the transfer function in different time domain form. So, we can have atleast three type of representation of the transfer function model in time domain. Those are known as controllable canonical form, observable canonical form and diagonal form. Out of the three forms, it is often found that, the diagonal form of representation of dynamics of a system is convenient to handle. So therefore, we shall attempt to obtain the transfer function in diagonal state space equation form.

Now, let me introduce some variables; lambda 3 is equal to plus minus 1 upon T 0; let lambda 1 is equal to minus plus 1 upon T 1 and lambda 2 is equal to minus 1 upon T 2. Then G(s) can be written in simpler form in terms of lambda 1 lambda 2 and lambda 3; K times s upon lambda three plus 1 e to the power minus theta s divided by when I take T 1 common from here. I will be able to write this in the form of T 1 times s minus lambda 1 and similarly when I take T 2 out from the term, that will enable to write the second term as T 2 times s minus lambda 2. So, this expression can further be expanded and written in the form of K e to the power minus theta s lambda 3 T 1 T 2 times are A upon s minus lambda 1 plus B upon s minus lambda 2. Then, this can further be written as, K lambda 1 lambda 2 e to the power minus theta s upon lambda 3 times lambda 1 plus lambda 3 divided by lambda 1 minus lambda 2 times 1 upon s minus lambda 1 plus minus lambda 2 plus lambda 3 whole divided by lambda 1 minus lambda 2 times 1 upon s minus lambda 2.

So, why we are writing in this form? The benefit of writing in this specified form is that, we will be able to get the transfer function model expressed in some convenient form. Now, it will be able to get the state space form of this transfer function model in diagonal form. Now, if I let the G s expressed in terms of output and input that will enable me to write Y s is equal to your some c 1 constant times Y1(s) plus another constant times Y2(s). Then, if I take the Laplace transform inverse Laplace transform of the expression, ultimately we will be able to write expression like y 1 dot t is equal to lambda 1 y1(t) plus lambda 2 y2(t); where sorry this will be lambda 1 y1(t) plus u t minus theta.

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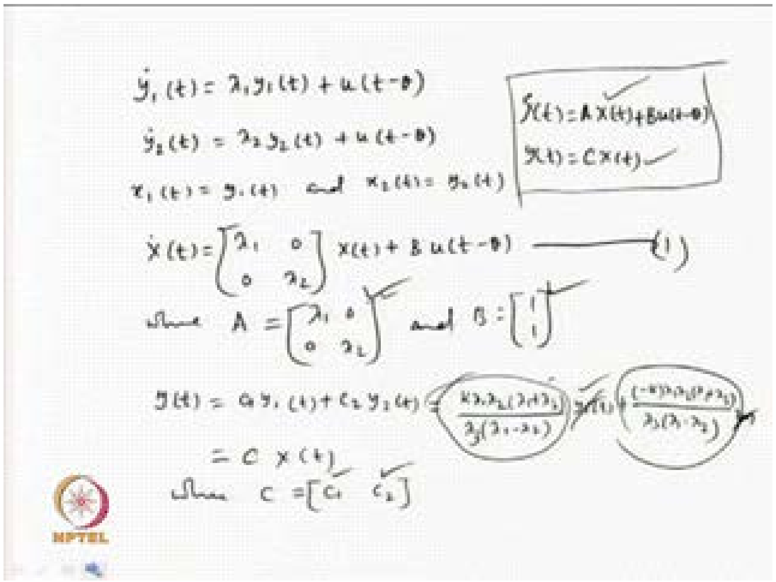

Similarly, one more expression will come from the second output y2(s) giving us, y 2 dot y 2 dot t is equal to lambda 2 y2(t) plus u t minus theta. Now, if I introduce the state variables x1(t) and x2(t); let x1(t) is equal to y1(t) and x2(t) is equal to y2(t), then that will enable me to write that in the state equation form giving us, X dot t is equal to lambda 1 0 0 lambda 2 with X(t) plus B u t minus theta. So this is how, we obtained a state equation form of representation of the dynamics of a real time system; where the A matrix now for us A is equal to lambda 1 0 0 lambda 2 and B is equal to 1 1 for the second order case. Now, A can be obtained in different form.

If it is expressed If the dynamics is expressed in the state other state variable form like controllable canonical form or observable canonical form, in that case A can have 0 1 with some constant say, c 3 and c 4 or A can again be given as 0 1 with some constant say, c 5 and c 6. Now, what is problem with representation of in these forms? It is very difficult to find the matrix exponentiation, if A's are written in these bottom two forms. If A is available in the diagonal form, out write I can find the matrix exponentiation; in which case, I will get e to the power A where this is the matrix exponentiation. e to the power A will simply will give us, e to the power lambda 1 0 0 e to the power lambda 2.

So, this is the benefit we obtained, by getting the A expressed in this particular diagonal form. Now, we shall see finally, what we have got for the expression for the output for the system. So, the output finally, now output has got two components; y(t) is equal to c 1 y1(t) plus c 2 y2(t). Therefore, we have got terms like K lambda 1 lambda 2 k lambda 1 lambda 2 times lambda 1 plus lambda 3 upon lambda 1 minus lambda 2 times lambda 3 in the bottom y1(t) plus minus K lambda 1 lambda 2 lambda 2 plus lambda 3 upon lambda 3 times lambda 1 minus lambda 2. So ultimately, putting the state variables x1(t) and sorry I will have y2(t) here; x2(t) over over here.

Then, we get the output expressed in the standard form of y t equal to CX(t); where our C vector will have two elements; those are c 1 and c 2. So finally, one can obtain the state space representation of the second order transfer function model in the form of standard form of X dot t is equal to AX(t) plus B u t minus theta and y(t) equal to CX(t); where A is this much; B is having elements 1 and 1 and c has got two element c 1 and c 2; where c 1 is this much and c 2 is this much. Thus it is possible to get the transfer function model expressed in the form of a canonical state space form given by the state and output equations of the form soon over here.

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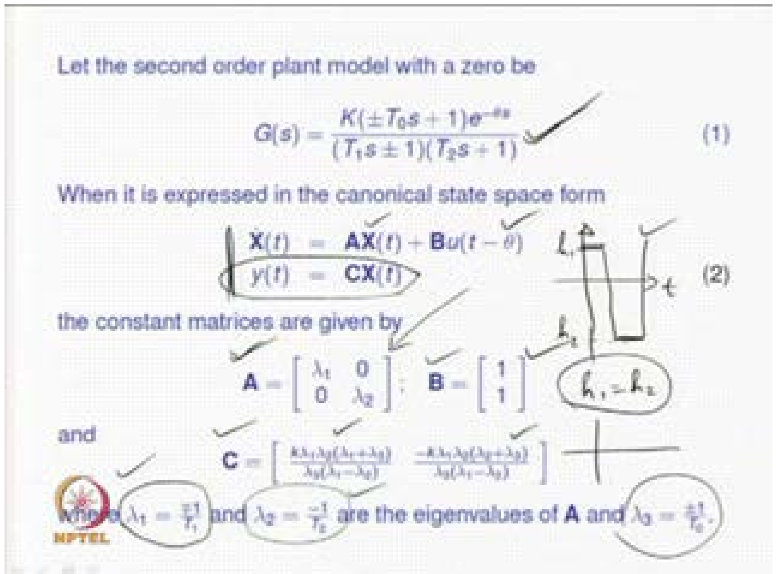

Let me summarize the representation. So, when the second order transfer function model with a zero is expressed in this form. Then, its canonical state space form in x dot t is equal to AX(t) plus B u t minus theta will give us A is equal to lambda 1 0 0 lambda 2; B is equal to 1 1. Similarly, when the output equation y t is equal to CX(t) is considered; in that case, C becomes C is equal to k times lambda 1 lambda 2 lambda 1 plus lambda 3 divided by lambda 3 times lambda 1 minus lambda 2 and minus k lambda 1 lambda 2 lambda 2 plus lambda 3 divided by lambda 3 times lambda 1 minus lambda 2. So, if you look carefully, the C has got the two elements which have different values.

So, the it is not like C is having c 1 and c 1; what I mean to say, I have got C expressed in the form of c 1 and c 2; where different values for lambda 1 lambda 2 or lambdas are chosen as lambda 1 is equal to minus plus 1 upon T 1 lambda 2 is equal to minus 1 upon T 2 and the Eigen values of A are nothing but, lambda 1 and lambda 2 and the 0 is expressed by the term lambda 3 is equal to plus minus 1 upon T 1. So, for convenience we have chosen the 3 variables, lambda 1 lambda 2 and lambda 3; such that, the dynamics of the second order transfer function model is conveniently expressed in state space form; where A B C are given as A is equal to lambda 1 0 0 lambda 2. Let me repeat. Why I am repeating this again and again? Because if you look at carefully, the form of A is such that it is possible to take the matrix exponentiation very conveniently and that is not the case, when A is not available in diagonal form.

What type of output signal one expects, when the asymmetrical relay relay with the parameters, relay with settings, h 1 and h 2 are employed to induce sustained oscillatory output from the higher order system. Now, when the relay dynamics or relay output is given by a signal of this form, h 1 and h 2; so, this is the characteristics given by the this is the characteristics for a relay, where we get piecewise constant output from the relay. But the piecewise constant outputs are of different magnitudes. When these two magnitudes become same, when h 1 is equal to h 2; in that case, the sustained oscillatory output will enable will will give us only two parameters.

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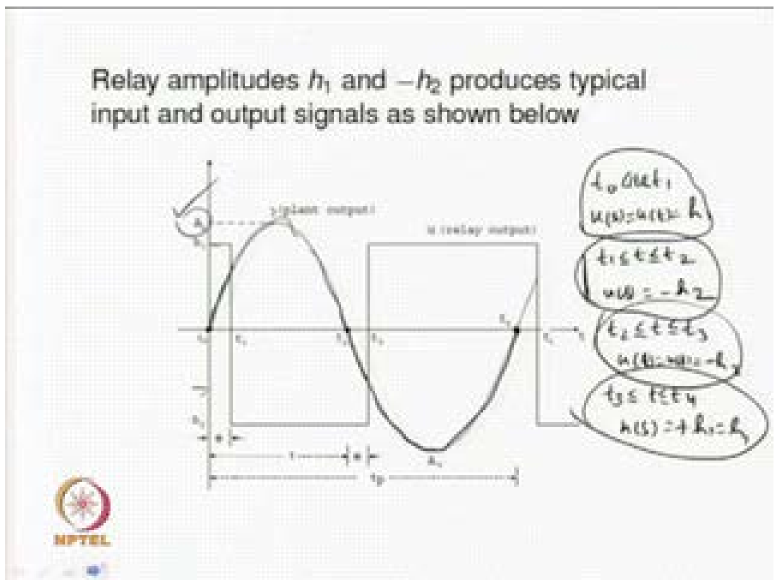

So, two measurements can easily with a gain when h 1 is equal to h 2, but when h 1 is not equal to h 2; in that case, the type of output signal one obtains is shown over here. Now, h 1 is not equal to h 2; in that case look carefully, that h 1 is resulting in depending on the type of input to the system is resulting in piecewise constant input to the system further of magnitude h 1 for some duration; the duration shown over here is from time t 0 to t 1 and for the time t 1 to t 3, we are getting negative input to the system with different relay height. Now, from time t 3 to t 4 and beyond, you see again the input provided by the relay is a piecewise constant with the same amplitude h 1 positive h 1, but for the duration t 3 to t 4.

So, when one considers one period of the output signal that is penning from time t 0 to time t 4, we have got 3 piecewise constant input to the system. So, this point must be taken care of while deriving analytical expressions for different segment, different parts of the output signal. So, when one tries to find analytical expression for the output signal from time t 0 to t 1, he moves from time t equal to 0 to time t equal to theta. As we know, because of the delay associated with the real time system, the output or the relay output gets delayed. So, when the relay output gets delayed, the switching does not takes place at time t equal to t 0; rather the switching take place at time t equal to theta. Similarly, the switching is getting shifted by the time theta seconds.

So that way, we have got the switching in place of at time t 0, at time t 2 and at time t 4. The switching are taking place after time theta. So, that is going from here to here. So, different time ranges, instants are given and few new variables are introduced. So that, we can consider whole span of one period of the output signal with the help of those parameters; so, let me introduce half period of the output signal by the term tau. So, tau now represents half period of the output signal. In our earlier analysis, we had used p u by 2 is the half period of the output signal; where one period was given by p u. Now, tau p in place of p u, I use tau p for one period of the output signal.

So, when one starts from time t 0 from this zero crossing goes through another zero crossing to the final zero crossing; thus giving us one period of the output signal, which is given by the time period tau p; whereas tau is the half period. If you look carefully, tau is not equal to tau p; tau is the half period; tau p is the full period. So, 2 tau is not becoming tau p or I can say that, tau is not equal to tau is not equal to tau p minus tau; that implies that, we are getting output signal which has got different half periods. The spans of half periods are not equal. So, one obtains it is possible to make measurements, more measurements from these output signal. The benefit we get from this is that, what measurements one will be able to take from this output signal?

We have the two peak outputs. Unlike the earlier cases for symmetrical relays, the peak amplitudes were same. The positive peak and negative peak values were found to be same. In this case, the positive peak denoted by A p is different from the negative peak denoted by A v. So, magnitude wise A p mod is not equal to A v and tau is not equal to tau p minus tau or I can say 2 tau does not give us tau p. These are happening, because we are employing asymmetrical relay during the relay test. So, the relay test is initiated such that, one obtains asymmetrical output; sustained oscillatory output from the system and enables one to make few more measurements, then two measurements.

So, what are the measurements one has to take from this output signal? Those are the peak amplitude, another peak amplitude given by A v, the half period tau and the full period tau p. So, these are four distinct and different parameters, measurements; one can obtain from the output signal. So, with what benefit we get from making more measurements from the output signal? It is possible to obtain more or it is possible to estimate more unknowns associated with the transfer function model. So, more measurements, more equations, more unknowns can be estimated. So, this is the added advantage, we have got from employing the asymmetrical relay in the autonomous system or during the relay test.

So, after introducing all those variables associated with the output signal, I will go ahead with the analysis of the output signal. So, the analysis will be carried out for different span of time. So, from time between t 0 to t 1, the input to the system U(s) is U(t) is equal to h 1. From time t 1 to t 2, the input to the system U(s) is equal to minus h 2; it goes beyond that, but I will restrict my analysis from time t 1 to t 2. Because after that, again we have got a span for theta seconds for which, we can start our analysis from t 2 assuming that, the output signal at that time y t 2 equal to 0. Then, I will consider another time span t 2 less than equal to t is less than equal to t 3, the input to the system U s U(t) is equal to U(s) is equal to minus h 2. Finally, time for the time range t 3 to t 4, the input to the system U(s) is equal to plus h 1 or h 1.

So, in this case, one has to consider the state equation for different time ranges and obtain four expressions for different time ranges. So, one cannot employ a single piecewise constant input signal to obtain the output; we have obtained for the sustained oscillatory output for the real time system. So, output starting from time t 0 to t 1, this exact output expression for this output can be employed with the consideration of this input. Similarly, when one wishes to obtain exact analytical expression for this part of the output ranging from time t equal to t 1 to t 0, then he has to take the input as minus h 2. Again for this segment of the output, the input is considered to be minus h 2 and finally, for this output, the input to the system is equal to h 1.

So, analysis will definitely give you expression exact expressions for this output. Why those expressions will be exact? Because we are not making any sort of approximation in our analysis;

rather, we are employing the state equations for finding expression for the output and which are exact expressions, without making any sort of approximations. Unlike the describing function case, where you approximate the relay by equivalent gain. So, no more or no nothing, no approximations are used in the analysis. Therefore, the expressions will get will be exact.

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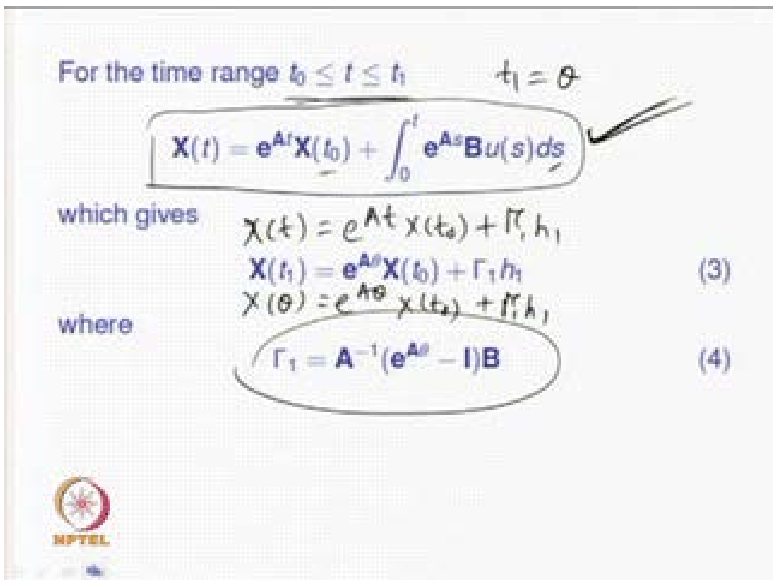

Now, I will start my analysis for different time ranges. So, for the time range t 0 to t 1, where t 1 is equal to theta; please keep in mind; at time t equal to t 1 that is having time t equal to theta. So, for that time range, the state equation solution of the state equation can be given by the simple expression; where X(t), the solution of the state equation is given as X(t) is equal to e to the power A t X(t0). We start from time t equal to t 0. Therefore, we have got X(t0) here plus integral from in limits 0 to t e to the power A s B U(s) d s. So, s is a variable; it is not the s domain; it is not the laplace domain for us; now s is a variable. So, that way what we have got, integral can be found and which will give us an expression of the form x t is equal to X(t) is equal to e to the power A t X(t0) plus gamma 1 h 1; offer from that h 1 is coming that is coming from the input.

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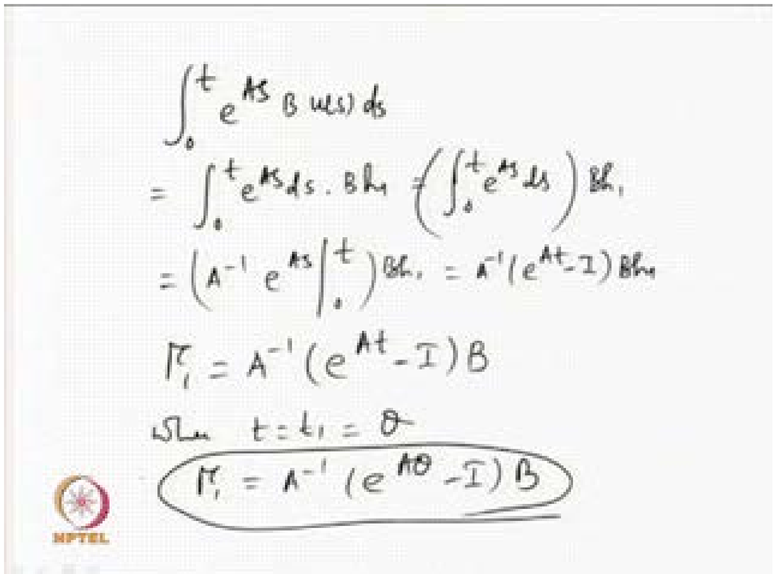

Let me self the integral, once more for the second part only; the integral can be written as integration from 0 to t e to the power A s B U(s) d s. So, this is equal to integral from 0 to t e to the power A s d s times B and u the input to the system is h 1. So, in place of U(s), let allow me to write h 1; thus giving us B h 1. So, I have got integral from 0 to t e to the power A s d s together times B h 1. Now, when the integral is found finally, we get in the form of A inverse e to the power A s with the limits 0 to t times B h 1. When the limits are port, we get A inverse e to the power A t minus the identity matrix times B h 1.

So, the gamma is introduced, where gamma 1 is assumed to have the part A inverse e to the power A t minus I times B. So, when t is equal to t 1 is equal to theta, at that time gamma 1 becomes A inverse e to the power A theta minus I times B. So, gamma 1 is obtained as A inverse e to the power A theta minus I times B. So, that is what we have obtained here, gamma 1 is given as A inverse e to the power A theta minus I B. So, when gamma 1 is used, then the three expression 3 sorry expression this solution can be expressed in equation number 3; which becomes X t(1) is nothing but, X theta is equal to e to the power A theta X t(0) plus gamma 1 h 1.

So, this is the analysis obtained for this part of the output this part of the output, which is obtained for the time range spending from time t equal to t 0 to t equal to t 1. Similarly, let us try to obtain analytical expression for the other part, where t goes from t 1 to t 2. So, for that part, again we can use the same solution. So, when the output this part of the output is considered now starting from here to here. When we go from t 1 to t 2 at that time, the input to the system U(s) is equal to minus h 2; keep in mind; one has to use this, in the integral to find correct expression for the output.

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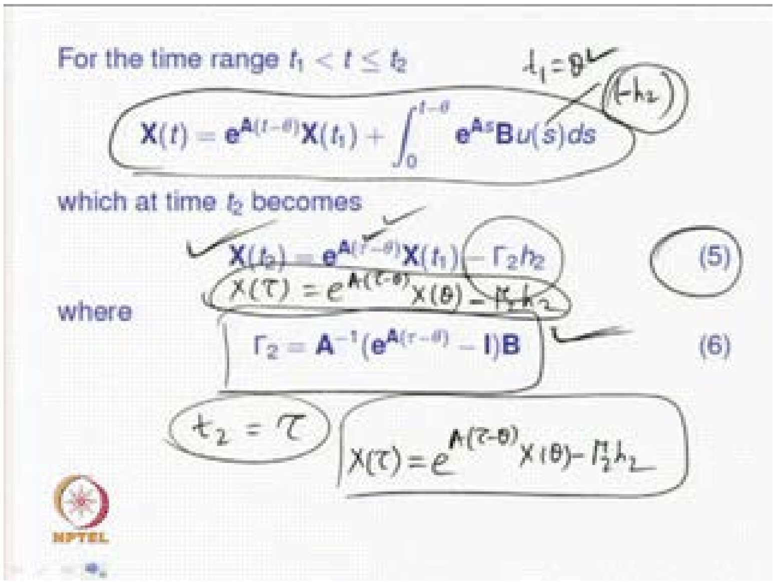

So, that gives us the solution of the state equation for that time range in the form of X(t) is equal to e to the power A t minus theta X (t1) plus integral from 0 to with limits 0 to t minus theta e to the power A s B U(s) d s. So, U(s) becomes minus h 2 now. Why I am writing t 1 here? This t 1 is nothing but, theta; keep in mind; that t 1 is equal to theta. So, which again upon solving gives us X (t2) is equal to e to the power A tau minus theta X (t1) minus gamma 2 h 2. Where from this minus sign is coming? It is due to the minus associated with the input to the system. So, input is negative; therefore, the second term is becoming negative. Then, the gamma 2 like the earlier case, is found to be A inverse e to the power A tau minus theta minus I B. Now, what is tau? At time t 2, that span is starting from t equal to 0 to tau.

So, let me show you the output waveform once more. This tau, tau is starting from time t equal to the first 0 crossing 0 or t 0 to t 2. So, this t 2 is nothing but tau; t 2 is equal to tau. So, when you substitute t equal to t 2, then we get the term tau or the variable tau in the expression. So, X (t2) is nothing but, I can write X tau is equal to e to the power A tau minus theta X theta; because t 1 is equal to theta, as I have already mentioned. So, this expression is nothing but, X tau is equal to e to the power A tau minus theta X theta minus gamma 2 h 2. So, the state at t 2, t equal to tau at time t equal to t 2 or the value of the state variable X tau is given as e to the power A tau minus theta X theta minus gamma 2 h 2. Now, this equation is given by equation number 5.

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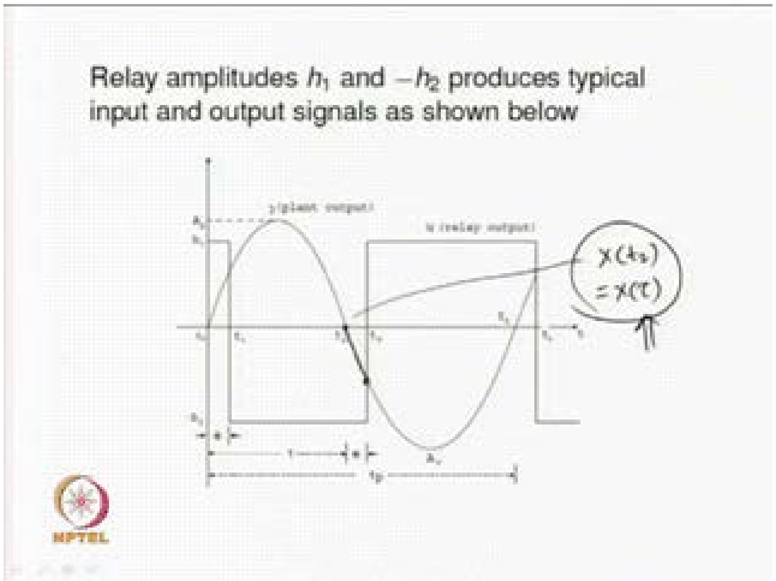

Similarly, we will target the other span now. The output starting from time t equal to t 2 and going up to time t equal to t 3. What is actually the time span here? It is nothing but, for theta seconds only. So, the output will be obtained for theta seconds only; but it is starting at time t equal to t 2. So, one need to one needs to the find the expression for X (t2), which is nothing but X tau; because this will enters the initial condition for the solution of the state equation. So, when you self the state equation, please keep in mind; you must consider the initial condition where from the solution starts. Depending on, you will get correct expression for the subsequent states and unless the starting time is correctly considered, it is very difficult to get correct expression for the output signal, unless that point has been taken care of. Now, for this segment, again I will start the analysis.

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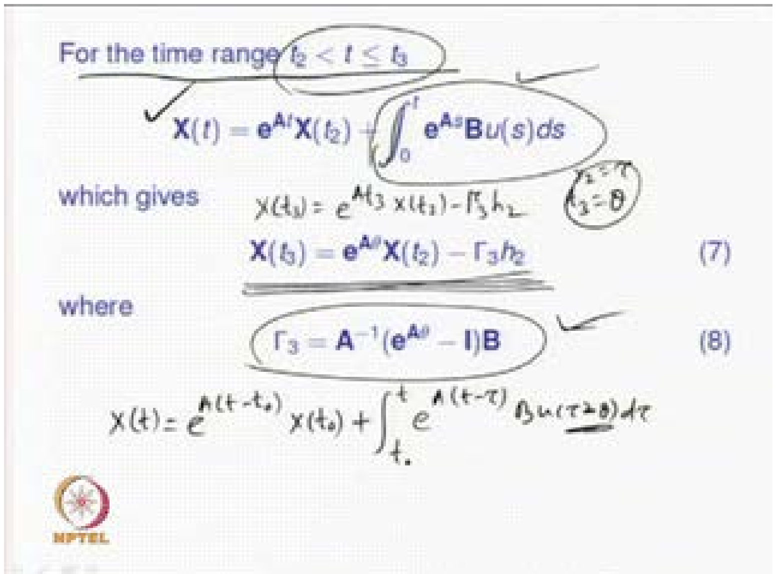

So, for the time range t starting from time t equal to t 2 to time t equal to t 3, the solution of the state equation gives us X(t) is equal to e to the power A t X(t2) plus integral from 0 to t e to the power A s B U(s) d s. Why I am not starting the limits of integral from time t 2? The reason is the integral has been expressed in such a way that, you need not keep in mind the starting time; rather, the integral will go from 0 to t means, it will be found for the time duration theta seconds. So, you start assume that, you are starting from time t equal to 0 and you are going up to theta theta seconds; because the integral can be found for that period.

If you have not forgotten the standard form for the solution of the state equation; in that case, the state equation solution is given as X(t) at any time is equal to e to the power A t minus t 0 X(t0) plus integral from t 0 to t e to the power A t minus tau B u tau minus theta d tau. So, this theta is taken care of over here and we get the integral expressed in the simplest form integral from 0 to t. You see the expressions in different solutions. It starts always starts from time t equal to 0 to some other time; because this u has been done in such that… So that, you start the integral from time t equal to theta. So, try to get the philosophy behind, finding integral in that convenient way. So, for this time range now, the solution of the state equation gives us X (t3) is equal to e to the power A t 3.

Actually, the solution is giving us X (t3) is equal to e to the power A t 3 X (t2), then I will get minus gamma 3 h 2. So, when t 3 is substituted now, what is t 3 for us? t 3 equal to theta as I have said, after the zero crossing x 2 is equal to tau. You go; you find the output for theta durations only; theta seconds only. So, that is why, the solution of the output can be or the solution of the state equation can be given in the form of X (t3) is equal to e A theta X (t2) minus gamma 3 h 2, where gamma 3 again is found from the integral as A inverse e to the power A theta minus I B. So, this integral is not difficult to found to find, because already I have shown you; how to find the gammas from this integrals.

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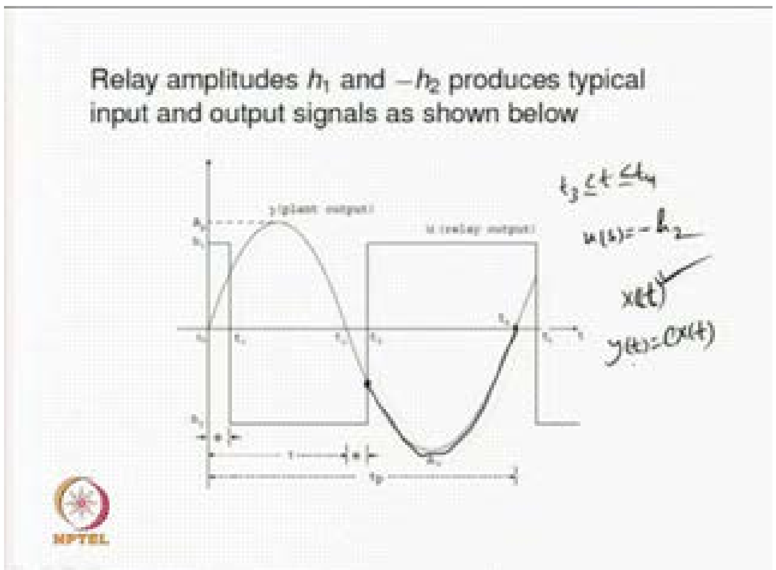

So, let me not repeat that once more; rather, I will go to the last segment of the of the output. So, the output expression for the output for this time range will be found. So, the input to the system for this time range, t 3 less than equal to t is less than equal to t 4 for that, the input is equal to minus h 2. So definitely, you will have minus h 2 in the expression for the state variable for this segment. Now, how can we find this output? And I am considering only the state variables, because it is very easy. Once you have got x expression for X (t) at any time, then y(t) the output is nothing but, CX(t); just you multiply the vector C and you get the output. So, it is all about finding correct expressions for the state and rest of the things will be automatically taken care of.

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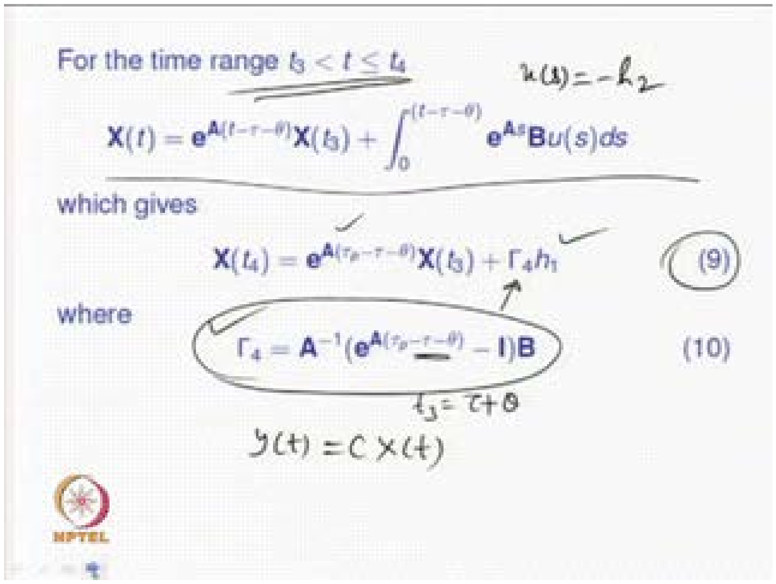

So, the output for that time range can be obtained with the assumption that, U(s) not assumptions; with the input U(s) is equal to minus h 2 h sorry for that time range sorry the input is; for this time range the input is plus h 1 sorry. So, this is not h 2; this is plus h 1. As you see, when the output is starting from time t 3, t equal to t 3; during that, the input to the system remains plus h 1. Therefore, U(s) is equal to plus h 1 and thus giving us h 1 factor in the right. So, when the state equation is solved, we get expression for that span and ultimately, the state at time t equal to t 4 is given as e to the power A tau p minus tau minus theta X (t3). t 3 what is t 3? x t 3 t 3 is nothing but, you are starting with time. So, this will be tau plus theta t 3, if you look at carefully.

So that, from t 0 to t 2, it is tau seconds and from t 2 to t 3, it is theta seconds. So, theta, tau plus theta seconds. So, from t 0 to t 3 it is nothing but, tau plus theta; that is why that is acting as the initial value and you are getting the term tau minus tau minus theta in the expression. So, again it is nothing but, if you recall the solution for the state equation, X (t) is equal to e to the power A t minus t 0. So, this t 0, the starting point for us is time t equal to tau plus theta; that is why, t 0 is becoming tau plus theta. And you are getting the expression X (t4) at particular time t equal to t 4 h. X (t4) is equal to e to the power A tau p minus tau minus theta times X (t3) plus gamma 4 h 1; where gamma 4 is given as A inverse e to the power A times tau p minus tau minus theta minus the identity matrix times B.

So, tau 4 appears over here. So, we have been able to find state equations for different time ranges associated with the output signal. Once the state added the state equations have been found, it is not difficult to get the output for whole one period of the sustained oscillatory output. Using the equation that y (t) is given as CX(t). So, all these keep in mind the equation numbers. What are the important equations for us for different segments? So, we have got, let me show you the equations one more. the We have got equation number 3, equation number 3 5, equation number 3 5 7 and 9. So, equation number 3 5 7 9 are important for us.

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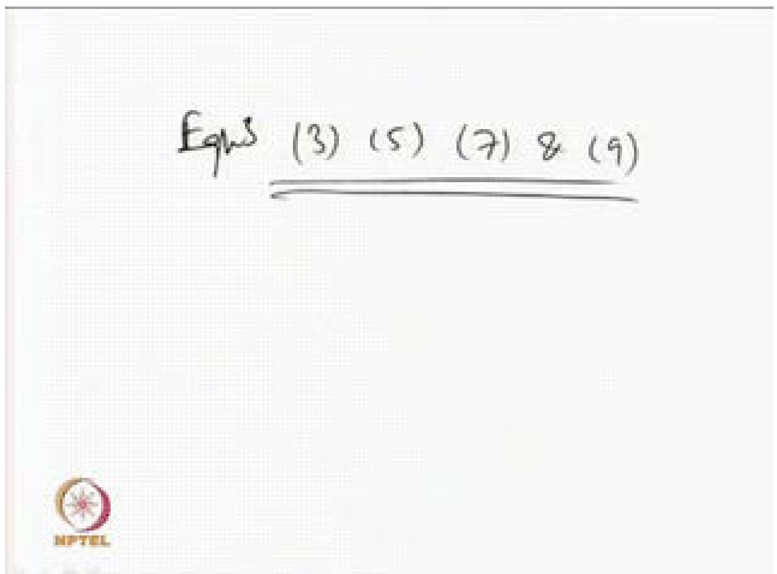

So, the four equations, equations 3 5 and 7 and 9 will be used further to find the final output expression.

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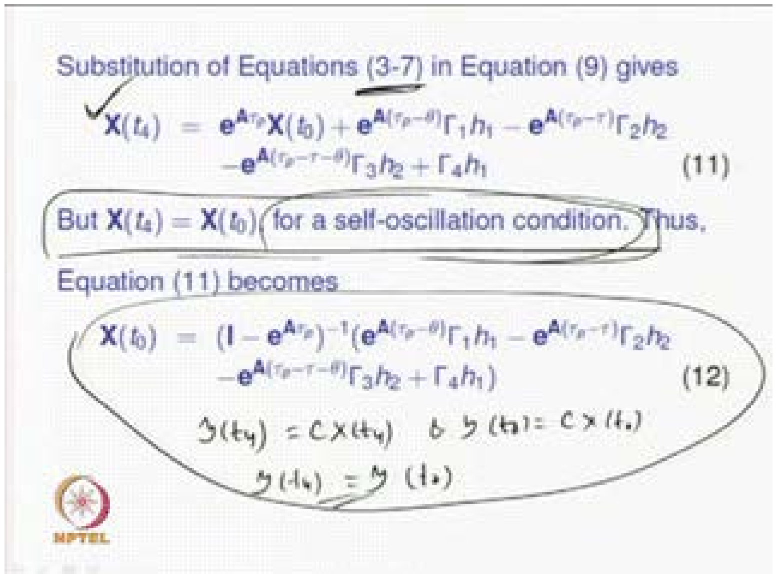

So, when the 3 to 7 are substituted in 9 further. What is 9? 9 is the expression for the output at time t equal to t 4. So, when the equations 3 to 7s are substituted in equation number 9; we get X (t4) is equal to e to the power A tau p X (t0) plus e to the power A tau p minus theta gamma 1 h 1 minus e to the power A tau p minus tau gamma 2 h 2 minus e to the power A tau p minus tau minus theta gamma 3 h 2 plus gamma 4 h 1. So, all the gamma values are present here as well as all the time spans have been considered to get this solution X (t4). So, the state the value of the state at time t equal to t 4 can be obtained using this.

Now, but we know that X (t4) has to be X (t0) for obtaining a sustained oscillatory output. This is a requirement; this must be satisfied. Why X (t0) has to be X (t4) has to be X (t0)? Because we know there is Y (t4) is equal to C X (t4) and Y (t0) is equal to C X (t0). So, that way Y (t4) has to be Y (t0) otherwise, you do not get limit cycle oscillations. So, where from you are getting you say that, we have got a positive going output at time t equal to theta. Similarly, you must have a positive going output after time t equal to t 4; then only, there will be sustained oscillatory output or the output will repeat itself after every period. So, for periodicity for periodicity of the output; it is a must that, at time t equal to t 0; whatever state you have got, you must get the same state at time t equal to t 4.

So, using that condition what you get? Using that condition, if you put X (t4) is equal to X (t0), we get finally this expression. So, this is very important for us. So, X (t0) what has been done? This X (t4) has simply been substituted by X (t0), because X (t4) has to be X (t0) for sustained oscillatory output. So, when I substitute X (t0) here and simplify now, because I have got another X (t0) in the right half, the first term; then X (t0) expression for the X (t0) will be like this. So, this expression equation number 12 can easily be obtained from equation number 11, with the substitution of X (t4) is equal to X (t0). So, this similar type of one condition will be find found in the next lecture, which are very essential for maintaining sustained oscillatory output.

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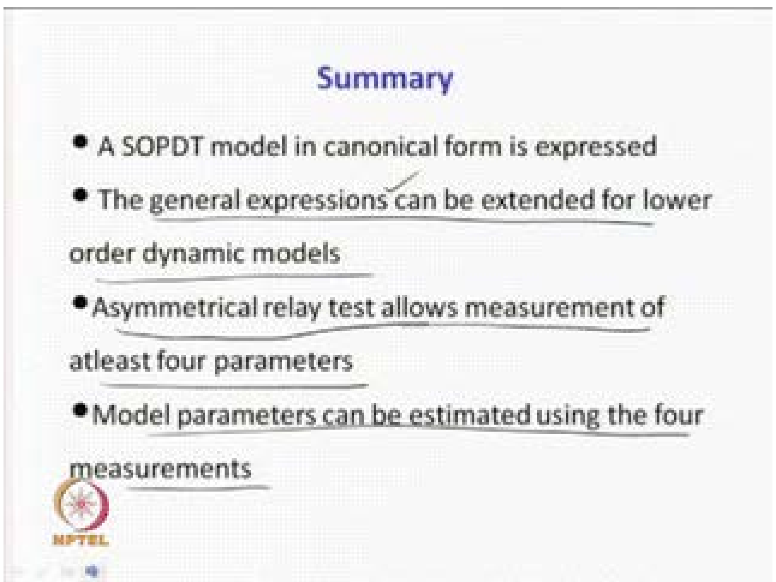

So, I will skip this thing as material for the next lecture. Let me summarize. So, a second order plus dead time transfer function model has been expressed in convenient diagonal for for obtaining expressions for output signal. Now, the general expressions we have been obtaining can be extended for lower order dynamic models. Keep in mind, the biggest benefit the the greatest advantage one obtains from this study is that, all the expressions are general in nature. Now, with suitable substitution of limiting values; now it is possible to derive analytical expressions for simpler system real time systems for which, you have got simple transfer function model.

Now, the asymmetrical relay test allows us measurement of atleast four parameters. Those are the two peak outputs and the the half periods are different. So, we have got different half period and different periods. So, the two segments or the time segments are different. Now, the model parameters can be estimated using the four measurements. So, making four measurements definitely, atleast four unknowns associated with the transfer function model can be found.

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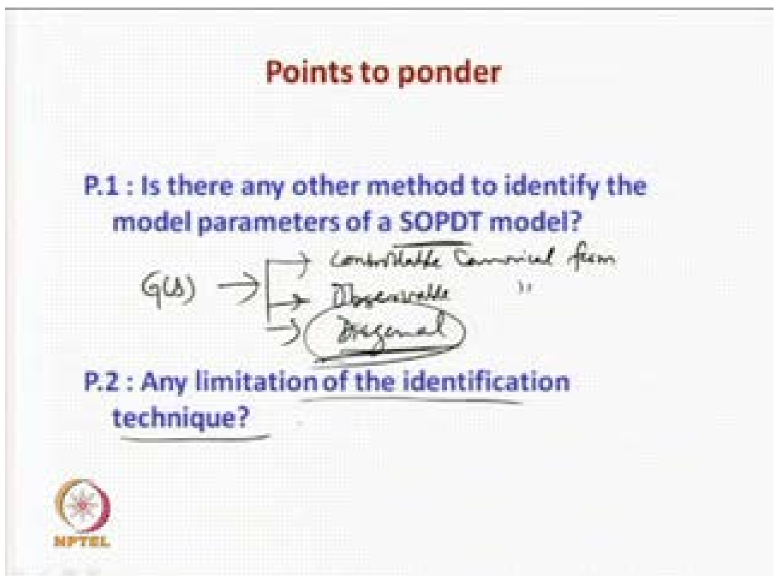

Any question we have? Yes. Is there any other method to identify the model parameters of second order plus dead time model? Yes. Now, one can go for other technique as well as I said the transfer function model can be expressed in different form; controllable canonical form, controllable canonical form observable canonical form and diagonal form. So, we have considered in our analysis, the diagonal form only. So, if you employ the controllable canonical form or or observable canonical form, then you will get different type; you will definitely get that the same expressions the the, but the technique analysis technique will be different. So, there are many methods to identity the model parameters of second order plus dead time transfer function model.

Any limitation of the identification technique? Yes. We have got many limitations associated with the identification technique, but those things will be discussed in detail towards the end of the identification of second order plus dead time model. One simple limitation is that, it will be very difficult to find the contribution, we get the asymmetrical output; we get at the sustained at the at the system is obtained from the different relay parameters or are they coming from any external disturbances? So, any external disturbances can also lead to asymmetrical output sustained oscillatory output. So, it is very difficult to make out from where we are getting the asymmetrical output. So, those things will be discussed in detail in the subsequent lectures. Thank you.