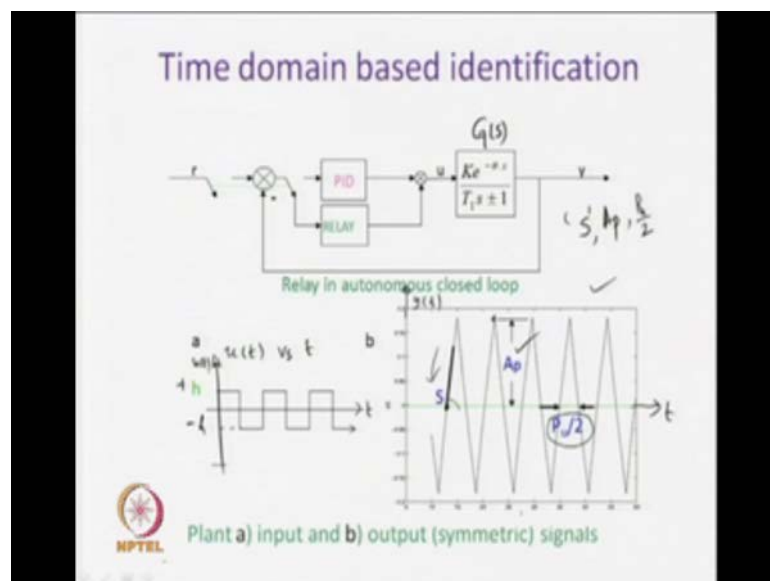


Advanced Control Systems
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Module No. # 03
Time Domain Based Identification
Lecture No. # 06
Identification of FOPDT Model

A new technique to identify dynamic model of a real time system will be discussed in today's lecture. The dynamic transform of function model can have a stable pole or an unstable pole.

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Let us see the relay autonomous closed loop system where the system, real time system is represented by the dynamics given as $K e^{-\theta s} / (T_1 s + 1)$. So, the denominator shows us that a pole in the right half or in the left half can be there for the dynamic model of the real time system. Then a relay is connected in this fashion disconnecting the controller, PID controller to induce limit cycle output. Then the limit cycle output or the sustained oscillatory output assumes this form, where this is $y(t)$ versus t , the output we get from the system corresponding to the input to the system

which is generated by the relay. So, the input to the system is now $u(t)$ versus t given as... this is our $u(t)$ versus t and we assume the relay amplitudes to have magnitudes of plus h and minus h .

So, we shall make now three measurements on the sustained oscillatory output unlike the previous lectures, where we used to take two measurements only and those are nothing but the peak amplitude given by A_p half period denoted by p_u by 2 where $p_u(s)$ stands for one period of output of the **output of the** system.

Now, we make additional measurement and that is nothing but the slope of the output signal at the zero crossing and slope will be designated by $\dot{y}(s)$. So, immediately after the zero crossing, we shall sense we shall measure the slope of the **sustained output** sustained oscillatory output of the relay control system. So, the relay induces this type of typical signal and we shall make three measurements namely: s the slope of the output signal at the zero crossing, A_p the peak amplitude of the output signal and half period of the output signal given by p_u by 2.

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• Consider the FOPDT plant transfer function $G(s) = \frac{Ke^{-\theta s}}{T_1 s \pm 1} = \frac{Y(s)}{U(s)}$
 $\Rightarrow T_1 s Y(s) \pm Y(s) = Ke^{-\theta s} U(s)$
 $\Rightarrow \dot{y}(t) = \mp \frac{1}{T_1} y(t) + \frac{K}{T_1} u(t - \theta)$
 Let $x(t) = y(t)$ (1) can be written as
 $\dot{x}(t) = \mp \frac{1}{T_1} x(t) + \frac{K}{T_1} u(t - \theta)$
 $\dot{x}(0) = \alpha x(0) + \beta u(t - \theta)$
 $x(0) = y(0) = 0$
 $\dot{x}(0) = \beta u(t - \theta) = \frac{Kh}{T_1}$
 • Its dynamics in state space becomes
 $\checkmark \dot{x} = \mp \frac{1}{T_1} x + \frac{K}{T_1} u(t - \theta) = \alpha x + \beta u(t - \theta) = \frac{Kh}{T_1} \dots (1)$
 where $\alpha = \mp \frac{1}{T_1}$ and $\beta = \frac{K}{T_1}$
 Initial slope of the output signal after a zero crossing is
 $\checkmark \dot{x}(0) = \dot{y}(0) = S = \frac{Kh}{T_1}$ (2)
 $u(t - \theta) = h \quad 0 \leq t < \theta$

Consider for analysis of the dynamic system now, closed loop system now relay control system now, consider the first order plus dead time plant transfer function $G(s)$ is equal to $K e^{-\theta s} / (T_1 s \pm 1)$ which is nothing, but equal to $Y(s) / U(s)$ where Y is the output and U is the input to the system. Then cross multiplication will result in $T_1 s Y(s) \pm Y(s) = K e^{-\theta s} U(s)$

theta s U(s). Again, when the inverse Laplace transform is taken, the same expression is given by $\dot{y}(t)$ is equal to $-\frac{1}{T} y(t) + \frac{K}{T} u(t - \theta)$. Thus, we have got delayed input to the system $u(t - \theta)$. When the state variable $x(t)$ is introduced, $x(t)$ is equal to $y(t)$, then the dynamic equation in state space can be written in the form of $\dot{x}(t)$ is equal to $-\frac{1}{T} x(t) + \frac{k}{T} u(t - \theta)$. That is what we have obtained.

Let us assume $\dot{x}(t)$ to be expressed **in the form** in the standard state equation form of $\dot{x}(t)$ is equal to $\alpha x + \beta u(t - \theta)$, where α is now $-\frac{1}{T}$ and β is equal to $\frac{K}{T}$. Why we are writing this in this standard state equation form? Because we know the solution of the standard state equation which can be used further for analysis.

Now, the initial slope of the output signal after a zero crossing which is nothing but this one (Refer Slide Time: 06:11) at the zero crossing. If we take the time T equal to 0 from this instant onwards, then the output at that instant will be $y(0)$, **$y(0)$** whereas the first derivative of the output at that instant will be $\dot{y}(0)$ and we know that $y(t)$ is equal to $x(t)$. Therefore, $\dot{y}(0)$ is equal to $\dot{x}(0)$ is equal to the slope of the signal as $K h$ by T . How do we get this $K h$ by T ? From this equation number 1.

Now, I can write the equation number 1. 1 can be written as **written as** $\dot{x}(0)$ is equal to $\alpha x(0) + \beta u(t - \theta)$. We know that $x(0)$ is equal to $y(0)$ is 0. $x(0)$ is equal to $y(0)$ is equal to 0 at the zero crossing. Therefore, $\dot{x}(0)$ will be equal to $\beta u(t - \theta)$ which is nothing but $\frac{K}{T} u(t - \theta)$. Now, $u(t - \theta)$ will have two values, either plus h or minus h depending on the time range. Now, $u(t - \theta)$ is equal to h for because we know that the input to the system $u(t - \theta)$ is equal to h for the time range $0 \leq t \leq \theta$. Therefore, $\dot{x}(0)$ will be equal to $\frac{K h}{T}$. That is what is given in equation number 2. So, the slope of the output signal at the zero crossing is given as s is equal to $\frac{K h}{T}$.

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What type of output and input signal we get? Let us consider half period of the output signal where the output signal will be of this form. This is the time axis. So, time pu by 2 because we are considering the half period. Then the input to the system will be of the form where this is h and this one is minus h (Refer Slide Time: 08:52). Now, what is the output - peak output? Peak output is nothing but y theta is equal to A p, the peak amplitude and also we know that x(0) is equal to y(0) is equal to 0. So, the output at a zero crossing is equal to 0.

This is the plot for u (t minus theta). Keep in mind this is not the plot for u (t). This is the plot for the rectangular pulse. We are getting the plot for u (t minus theta), delayed input to the system, whereas this is the plot for **output** sustained oscillatory output of the system. Now, we know that the general solution of the standard state equation given in 1 can be written as x (t) is equal to e to the power alpha (t minus t 0) x (t 0) plus integration from t (0) to t e to the power alpha (t minus tau) beta u (tau minus theta) d tau. So, this is the standard solution for a state equation given in the form of x dot (t) is equal to alpha x (t) plus beta u (t minus theta).

So, when the state equation is of this form, we get a solution of the state equation of this form. Now, we shall use this standard solution to find analytical expressions for the output, sustained oscillatory output of the system till time t equal to pu by 2. We know that x(0) is equal to y(0) is equal to 0. Therefore, the expression for the present case can

be written as $x(t)$ is equal to integration from $t=0$ to t of $e^{-\alpha(t-\tau)}$ $\beta u(\tau)$ $d\tau$, but for the time range **for the time range $0 \leq t \leq \theta$** is less than equal to t is less than equal to θ , we know that the input $u(t-\theta)$ is equal to $u(\tau-\theta)$ is equal to positive h .

Then the $x(t)$ can be written as $x(t)$ is equal to $e^{-\alpha t}$ then integral $t=0$ now $t=0$ equal to 0 we start the analysis from time t equal to 0 . Therefore, this will be 0 to t , then $e^{-\alpha(t-\tau)}$ βh $d\tau$ times βh .

This integral can further be simplified giving the expression $x(t)$ as $e^{-\alpha t}$ βh $\int_0^t e^{\alpha\tau} d\tau$. I will take minus α inverse. Then $e^{-\alpha(t-\tau)}$ with the limits 0 to t with βh , which becomes now $e^{-\alpha t}$ βh $\int_0^t e^{\alpha\tau} d\tau$ minus α inverse, or we can write this in the form of minus α whole inverse. Then it becomes $e^{-\alpha t}$ βh $\frac{1}{-\alpha} [e^{\alpha\tau}]_0^t$ minus α inverse $e^{-\alpha t}$ βh $\frac{1}{-\alpha} [e^{\alpha t} - 1]$ βh , which can ultimately be written in the form of α inverse $e^{-\alpha t}$ βh $(e^{\alpha t} - 1)$ βh .

So, what we have got from the analysis, that the output of the signal can be expressed. The analytical expression for the output of this part of the output signal can be given by $x(t)$ is equal to $y(t)$ is equal to α inverse times $e^{-\alpha t}$ βh $(e^{\alpha t} - 1)$ βh .

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• Solution of (1) is $x(t) = \alpha^{-1}(e^{\alpha t} - 1)\beta h$ for $0 \leq t \leq \theta$;

Then $x(\theta) = \alpha^{-1}(e^{\alpha\theta} - 1)\beta h$

For the time range $t > \theta$; $u(t-\theta) = u(\tau-\theta) = -h$

$$y(t) = x(t) = e^{-\alpha(t-\theta)} x(\theta) + \int_{\theta}^t e^{-\alpha(t-\tau)} \beta u(\tau-\theta) d\tau$$

$$= \alpha^{-1}(e^{\alpha t} - e^{\alpha(t-\theta)})\beta h + e^{-\alpha t}(-\alpha)^{-1} e^{-\alpha\tau} \Big|_{\theta}^t (-\beta h)$$

$$= \alpha^{-1}(e^{\alpha t} + 1 - 2e^{\alpha(t-\theta)})\beta h$$

• For limit cycle/self-oscillations to exist $x(t + P_s/2) = -x(t)$

• Using the condition one obtains

$$2e^{-\alpha\theta} - e^{-\alpha P_s/2} - 1 = 0 \quad \dots(3)$$

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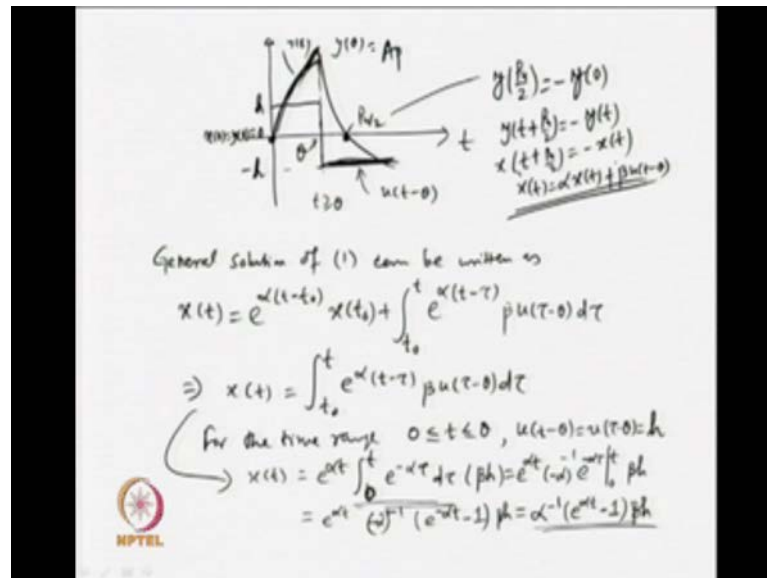
Now, the solution of (1) has been found in this form for the time range for $0 \leq t \leq \theta$ is less than equal to t is less than equal to θ . What about when t is greater than equal to θ ? Let

us find the solution for that dead time range first. So, since $x(t)$ is equal to $\alpha^{-1} e^{-\beta t}$, then $x(\theta)$ will definitely be equal to $\alpha^{-1} e^{-\beta \theta}$. But for the time range **for the time range** t is greater than equal to θ , we know that the input changes. $u(t - \theta)$ is equal to $u(\tau - \theta)$ is equal to $-\beta$; β we have seen that the input. This is time t equal to θ ; so, when t is greater than equal to θ , the input becomes $-\beta$. Then, the solution of the state equation $y(t)$ is equal to $x(t)$ will be $\alpha^{-1} e^{-\beta t} + \theta \int_{\theta}^t e^{-\beta(t-\tau)} (-\beta) d\tau$. While that is so, we need to substitute $t = \theta$ in the general solution for the state equation. Then, we get this expression.

Then, substitution of $x(\theta)$ in the above, results in $\alpha^{-1} e^{-\beta \theta} + \theta \int_{\theta}^t e^{-\beta(t-\tau)} (-\beta) d\tau$. This is what we get from the simplification of the first term and by substitution of $x(\theta)$ where $x(\theta)$ is given as $\alpha^{-1} e^{-\beta \theta}$. Then, the second part, the integral part can further be simplified giving us $\alpha^{-1} e^{-\beta t} + \theta \int_{\theta}^t e^{-\beta(t-\tau)} (-\beta) d\tau$ with the limits θ to t now and we are left with $-\beta$ because the input is $-\beta$ now; so, which further upon simplification, gives us $\alpha^{-1} e^{-\beta t} + \theta (1 - e^{-\beta(t-\theta)})$. So, this is what we get for the time range t is greater than equal to θ .

Let us see whether this expression is valid or correct or not. The correctness of this expression can be found by the substitution of t equal to θ . So, when t equal to θ , what do I get? t equal to θ will give me $x(\theta)$ is equal to $\alpha^{-1} e^{-\beta \theta}$. So, this is nothing but it is equal to $\alpha^{-1} e^{-\beta \theta}$; that we have obtained earlier. Therefore, the expression we have obtained for the time range t is greater than θ is correct.

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After ascertaining that, we will go to the analysis of the half period of the sustained oscillatory output, where we know that the output becomes 0 at time t equal to π by 2. So, that can be given in the form of y (π by 2) is equal to $y(0)$. Actually, since we get sustained oscillatory output, y (π by 2) has to be equal to minus $y(0)$. As we know the output **possess** possesses half wave symmetry, therefore, y (π by 2) has to be minus $y(0)$, or in general form I can write y (t plus π by 2) has to be minus $y(t)$ and since $y(t)$ is equal to $x(t)$ which can further be given in the form of x (t plus π by 2) is equal to minus $x(t)$.

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So, the limit cycle condition or the output will have sustained oscillatory form provided that condition is made. What is that condition? $x(t + \pi)$ has to be minus $x(t)$ or I can write this by substitution of t as $x(\pi)$ is equal to minus $x(0)$. What is $x(0)$? The $x(0)$ is equal to $y(0)$ which is nothing but 0 because we start our analysis from time t equal to 0, at which we have the output of the system 0. Therefore, this is equal to 0.

So, when I substitute t is equal to π , when t is substituted by π , then $x(\pi)$ will be equal to $e^{-\alpha\pi} e^{\alpha\pi} e^{-\beta\pi} - 1$ and this has to be equal to 0; that we know from this condition. Then, this expression now can be written in this form; $2e^{-\alpha\pi} - 1 = 0$. Why that is so? Because α is a constant and β is a constant, this expression is equated to 0.

Again, if I multiply both sides of this expression by $e^{\alpha\pi}$, So, multiplication of this factor to both sides of this equation will result in this (Refer Slide Time: 21:30). So, one can say this is the extended version of this one for each in analysis. So, this expression (3) meets the requirement on the limit cycle, condition for the limit cycle. So, this has to be satisfied; otherwise, one cannot guarantee sustained oscillatory output from the system.

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Handwritten mathematical derivation on a slide:

- Peak output at $t = \theta \Rightarrow y(\theta) = x(\theta) = A_p = \alpha^{-1}(e^{-\alpha\theta} - 1)\beta h \dots (4)$
- We know $\alpha = \frac{1}{T_1}$ and $\beta = \frac{k}{T_1}$
- $A_p = \frac{1}{T_1} (e^{-\theta/T_1} - 1) \frac{k}{T_1} h$
- $A_p = \frac{k}{T_1} h (e^{-\theta/T_1} - 1) \dots (4)$
- Substitution of α and β in (3) will result in
- $2e^{-\alpha\theta} - e^{-\alpha\pi} - 1 = 0$
- $2e^{-\theta/T_1} - e^{-\pi/T_1} - 1 = 0 \dots (3)$

The NPTEL logo is visible in the bottom left corner of the slide.

Now, peak output of the system occurs at time t equal to θ . Now, that peak output expression for the peak output can be given in the form of t equal to θ implies $y(\theta)$ is equal to $x(\theta)$ is equal to the peak amplitude given as $\alpha^{-1} e^{-\beta \theta}$. What has been done here in this expression? The t has been substituted by θ . Then, we get the expression for the peak amplitude, peak output of the sustained oscillatory signal.

Now, we know that we know that the constant α is equal to $-\frac{1}{T}$ and β is equal to k . So, substitution of α and β in (4) results in an expression of the form A_p is equal to $-\frac{1}{T} (e^{-k\theta})^{\frac{1}{T-1}}$ which is equal to $-\frac{1}{T}$; $T-1$ is cancelled out. So, that way we are left with $-\frac{k}{T} (e^{-k\theta})^{\frac{1}{T-1}}$. So, this is the expression for output of the first order plus time delay transfer function model.

So, when we take minus sign and when we take plus sign, so, for stable process models stable process models we shall use the upper symbol and the upper one and for unstable system for unstable system the systems dynamics will be or can be obtained using this expression where we shall choose the bottom sign.

So, for an unstable first order plus dead time model, we shall go for the lower symbols plus here and plus here (Refer Slide Time: 24:30 to 24:33), but for a stable first order plus dead time transfer function model, we have to use the upper symbol minus here and minus here. Now, after obtaining this one, let me give this as the same equation number equation number 4. Again, substitution of substitution of the same α and β in equation number 3 will result in what is equation number 3. We know that this is our equation number 3. $2 e^{-\alpha \theta} - e^{-\alpha \theta} = e^{-\alpha \theta}$ by 2 minus 1 is equal to 0.

So, substitution of α here will result in $2 e^{-\frac{\theta}{T}}$, this will be, plus minus θ by $T-1$. Because of this minus sign, the pair will get inverted and then we will get $2 e^{-\frac{\theta}{T-1}}$ plus minus θ by $T-1$ minus $e^{-\frac{\theta}{T-1}}$ plus minus θ by $T-1$ minus 1 is equal to 0. So, this is our equation number 3.

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Slope of the output at the zero crossing is

$$S = \frac{kh}{T_1} \quad (2)$$

$$2e^{\pm \theta/T_1} - e^{\pm \theta/(2T_1)} - 1 = 0 \quad (3)$$

$$A_p = \mp kh(e^{\pm \theta/T_1} - 1) \quad (4)$$


We know that the slope of the output signal, the slope of the output at the zero crossing is S is equal to kh upon T_1 which is given the same equation number 2. Thus, we have got three important equations. Let me write down those three important equations: S is equal to kh upon T_1 ; then $2e$ to the power plus minus θ upon T_1 minus e to the power plus minus θ upon $2T_1$ minus one is equal to 0, which is given the equation number 3 and the peak amplitude given as minus plus kh $(e^{\pm \theta/T_1} - 1)$. This is given as equation number 4. Then, we have obtained three important expressions and also we are making three measurements on the output signal. Thus, the three expressions can be solved simultaneously to estimate three unknowns: k , θ and T_1 .

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• From simultaneous solutions of (2-4), the unknown parameters of the FOPDT plant are estimated.

$$G(s) = \frac{k e^{-\theta s}}{T_1 s + 1} \quad (k, \theta, T_1)$$

• The estimated parameters are exact if there is no sensor measurement errors!

• In the face of noisy output signals, wavelet based measurement and noise reduction techniques can be used.

The slide includes two graphs. The top graph shows a noisy step response labeled y^* with a tangent line indicating the slope. The bottom graph shows a smooth step response labeled y with a tangent line indicating the slope. The NPTEL logo is visible in the bottom left corner.

From the simultaneous solution of those equations 2 to 4, the three unknowns of the transfer function model, we have got the transfer function model now - $k e^{-\theta s} / (T_1 s + 1)$. So, the three unknowns in the transfer function model k , θ and T_1 can be estimated using the three expressions. Now, one needs to solve the three equations simultaneously because they have the parameters in them; all the three parameters are found in all the three expressions so to say. That way, simultaneous solution of the three non-linear or linear equations are required to estimate the three unknowns.

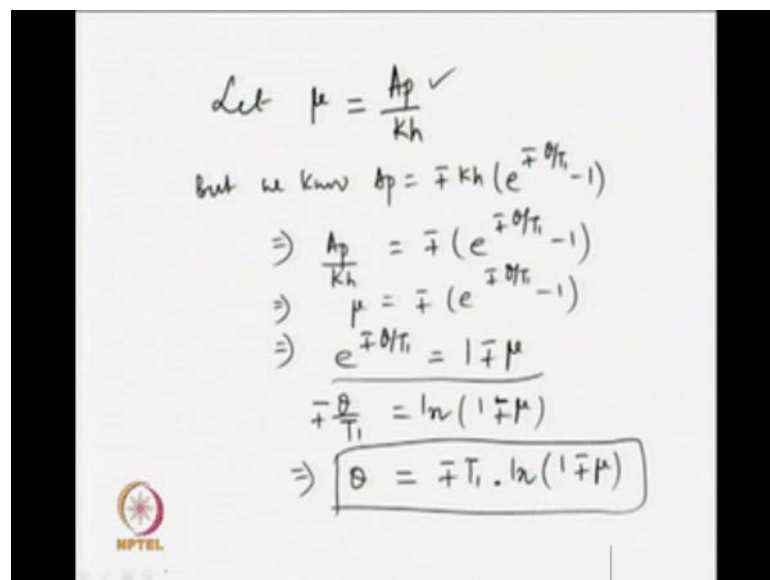
Now, the estimated parameters are exact; we will find exact estimation provided there are no sensor measurement errors; that means, **if the output is** let us take this case. If the output is not error free, rather we get some output of this form (Refer Slide time: 28:49). Then one may not be able to make measurement of the slope. So, the information, the slope information may be inaccurate. In that case, the estimation of k , θ and T_1 may be erroneous. So, one must take care of this fact.

In those situations what happens? One can make use of alternate techniques. Wavelet based measurement and noise reduction technique can be made use to obtain clean signal from the noisy output signal. So, the noisy output signal can be filtered out of all the noise components and then we can get some smooth output signal of this form. Now, this smooth output signal may enable us to make all the three important measurements

accurately. What are those measurements? The initial slope S , peak amplitude A_p and the half period p_u by 2.

So, when the measurements are error free and when the output is clean or output is not subjected to measurement errors or the output is not subjected to external errors, erroneous or not subjected to extraneous error signals, in that case it is possible to estimate the three unknowns using the three expressions we have found earlier. Those expressions are given by the equations 2, 3 and 4.

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$$\begin{aligned} \text{Let } \mu &= \frac{A_p}{k_h} \checkmark \\ \text{but we know } A_p &= -k_h(e^{-\theta/T_1} - 1) \\ \Rightarrow \frac{A_p}{k_h} &= -(e^{-\theta/T_1} - 1) \\ \Rightarrow \mu &= -(e^{-\theta/T_1} - 1) \\ \Rightarrow e^{-\theta/T_1} &= 1 - \mu \\ -\frac{\theta}{T_1} &= \ln(1 - \mu) \\ \Rightarrow \boxed{\theta} &= -T_1 \cdot \ln(1 - \mu) \end{aligned}$$

Now, let us little bit extend this analysis. Let us introduce one more parameter. Let μ is equal to A_p by k_h , but we know that A_p is equal to, the peak amplitude can be given in the form of minus plus k_h into e to the power minus plus theta upon T_1 minus 1. Then, the expression A_p can be written as A_p by k_h is equal to minus plus e to the power minus plus theta upon T_1 minus 1 or μ is equal to minus plus e to the power minus plus theta upon T_1 minus 1.

Likewise, then it is not difficult to write an expression of the form e to the power minus plus theta upon T_1 is equal to 1. Then 1 will come here; so, that way 1 minus plus μ . **why that is** How that is obtained? If I multiply minus plus on the both sides, if I multiply minus plus, then this will go out; this will be out (Refer Slide Time: 31:51).

So, that is how we are able to obtain an expression of this form, ultimately which taking the natural logarithm further gives us minus plus theta upon T 1 is equal to lan of 1 minus plus mu implies theta is equal to minus plus T 1 times natural logarithm of 1 minus plus mu. So, this is an important expression for us. The theta can be obtained using this expression later on or using mu, for mu is equal to A p by k h.

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$$\begin{aligned} \text{Eqn (3) can be written as} \\ 2e^{+\theta/T_1} - e^{+P_0/2T_1} - 1 &= 0 \quad \text{---(3)} \\ \frac{2}{1+\mu} - e^{+P_0/2T_1} - 1 &= 0 \\ \Rightarrow e^{+P_0/2T_1} &= \frac{2}{1+\mu} - 1 = \frac{1-\mu}{1+\mu} \\ \Rightarrow +\frac{P_0}{2T_1} &= \ln\left(\frac{1-\mu}{1+\mu}\right) \\ \Rightarrow T_1 &= \frac{+P_0/2}{\ln\left(\frac{1-\mu}{1+\mu}\right)} \end{aligned}$$

Similarly, we can start analysis for the third equation. So, again third can be written as 2 e to the power plus minus theta upon T 1 minus e to the power plus minus pu by 2 T 1 minus 1 is equal to 0. This is the equation number 3. Now, what I will do? We know that e to the power plus minus theta by T 1 can be obtained from here. Using this, e to the power plus minus theta by T 1, sorry not this one, the upper one, using this e to the power plus minus theta by T 1 will be 1 upon 1 minus plus mu. So, I can substitute that giving us 2 upon 1 minus plus mu minus e to the power plus minus pu by 2 T 1 minus 1 is equal to 0. That implies e to the power plus minus pu by 2 T 1 is equal to 2 upon 1 minus plus mu minus 1 is equal to 1 plus minus mu by 1 minus plus mu.

Further taking the natural logarithm of both sides, we can get plus minus pu by 2 T 1 is equal to lan of 1 plus minus mu by 1 minus plus mu giving us T 1 is equal to T 1 is equal to plus minus pu by 2 divided by plus minus pu by 2 divided by lan 1 plus minus mu by 1 minus plus mu.

So, this is another expression for the one of the unknown of the transfer function model. Earlier, we have got an expression for one unknown of the transfer function model theta.

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$$\mu = \frac{A_p}{k_h}$$

We know that $S = \frac{k_h}{T_1} \Rightarrow k_h = S T_1$

$$\mu = \frac{A_p}{S T_1} = \frac{A_p}{S \left(\frac{1+\mu}{2} \right)}$$

$$\mu = \frac{A_p \cdot \ln \left(\frac{1+\mu}{2} \right)}{S \cdot \left(\frac{1+\mu}{2} \right)}$$

Further analysis will be done and we know that mu is equal to A p by k h. We know that mu is equal to A p by k h; that we have taken the approximation, we started our analysis with the assumption that with the introduction of a new variable mu h A p upon k h. Then, we know that the slope of the output signal can be given as k upon k h upon T 1, which means k h is equal to S T 1.

So, substitution of this k h in this denominator gives us mu is equal to A p upon S T 1. Further, we know that T 1 is this much. Therefore, if I substitute the expression for T 1 over here (Refer Slide Time: 36:13), I get A p upon s times plus minus pu by 2 lan 1 plus minus mu by 1 minus plus mu. So, that will give us here plus minus pu by 2 divided by lan 1 plus minus mu by 1 minus plus mu. So, which will be ultimately mu is equal to A p times lan 1 plus minus mu by sorry lan 1 plus minus mu by 1 minus plus mu divided by s times plus minus pu by 2.

So, this is an important expression for us because if you look carefully then you can see A p is measured, S is measured, then pu is also measured. So, from the output measurement on the output, we get information about A p, s and pu. Therefore, in this expression, there is only one unknown - mu. Then this can be solved, this expression can easily be solved to find out find out the mu and once mu value has been found out, then

substitution of μ and making use the measurement p_u , we can estimate T_1 . Further, making use of T_1 and use of μ will give us θ . That is how all the unknowns, unknown of the transfer function model can be estimated.

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* Finally,

$$T_1 = \frac{\pm P_u / 2}{\ln \left(\frac{1 \pm \mu}{1 \mp \mu} \right)} \quad (5)$$

$$\theta = \mp T_1 \ln(1 \mp \mu) \quad (6)$$

$$K = \frac{S T_1}{h} \quad (7)$$

and

$$\mu = \frac{A_p \ln \left(\frac{1 \pm \mu}{1 \mp \mu} \right)}{S(\pm P_u / 2)} \quad (8)$$

* μ can be calculated from the solution of the equation (8).
 Then parameters of the model are obtained from (5-7).

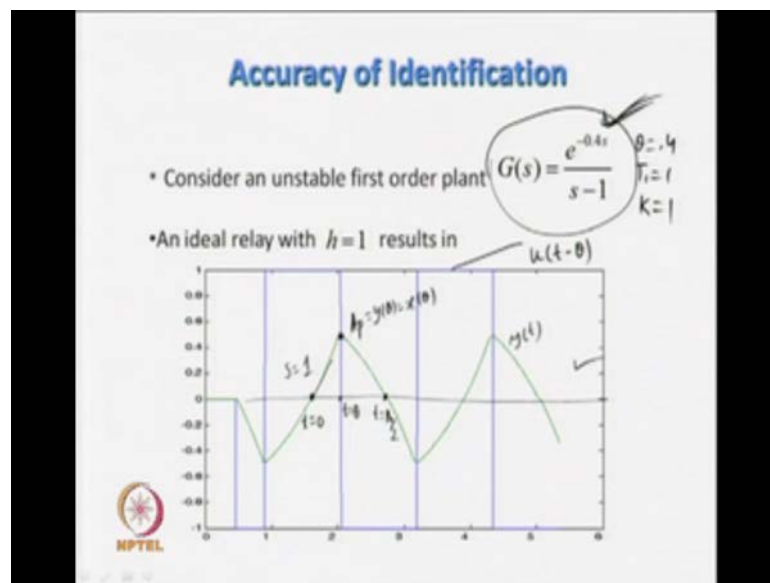
So, let me summarize. So, the expression for T_1 is given as plus minus p_u by 2 divided by $\ln(1 \pm \mu / 1 \mp \mu)$. θ is equal to minus plus T_1 time natural logarithm of $1 \mp \mu$. And the steady state gain k is equal to $S T_1$ by h because we know that the slope is given by s is equal to $k h$ by T_1 . Then your k is nothing but $S T_1$ by h ; that is how we get expression number 7. And also we have found in our last slide that μ is equal to A_p times natural logarithm of $1 \pm \mu$ by $1 \mp \mu$ divided by S times plus minus p_u by 2. So, μ can be calculated from the solution of the equation 4.

We have got one non-linear equation where there is only one unknown and since A_p , p_u and S are measured or obtained from the sustained output, sustained oscillatory output of the relay feedback test, in that case μ can be calculated estimated using expression number 8. So, using this expression 8 or equation 8, μ can be estimated or can be calculated. Once μ is known, then we can find T_1 because p_u is measured and μ has been estimated. Next, after getting T_1 , we can get k . Because h is known, T_1 has been calculated and S is measured, then also we can find θ using μ . μT_1 can be used to estimate θ . Thus, all the transfer function model parameters of the first order stable

or unstable transfer function model can be estimated using the three measurements **using the three measurements** made on the output of the system.

So, this gives us some freedom unlike the earlier case, where we had to obtain the steady state gain by some other technique or assume to be known a priori. We need not make any assumption in this case. The new technique enables us to estimate the three transfer function models conveniently using equations 5, 6, 7 and 8.

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Now, we shall go to some simulation example to verify accuracy of identification. Let us see whether our technique is accurate or not. So, in this simulation study, an unstable system, an unstable first order plus dead time system, transfer function has been considered where the time delay theta is equal to 0.4 second. Now, the time constant T 1 is equal to 1 second and the steady state gain k is also equal to 1. If I look at this transfer function model for unstable first order plus dead time model, the three parameters those have been given are k is equal to 1, theta is equal to 0.4 and T 1 is equal to 1.

Let us try to estimate back these values from the equations. Now, when the relay test is conducted on this plant, on this first order plus dead time transfer function model, the sustained output is obtained and the waveform of the sustained oscillatory output is shown over here, where this is our output $y(t)$. Now, let me draw the x axis and what is the blue line? This is nothing but the input to the system; delayed input to the system.

So, we have got the blue lines are nothing but the rectangular lines are nothing but plot for $u(t - \theta)$. Let me start my analysis from the first zero crossing over here. So, this time t is equal to 0 and we see that at time t equal to θ , **t equal to θ** a peak amplitude or peak output occurs. Now, what is this? This would be the half period t equal to p by 2 (Refer Slide Time: 43:36).

So, if I look at the plot, my peak amplitude is obtained here at time t equal to θ which is nothing, but, A_p equal to $y(\theta)$ is equal to $x(\theta)$. Now, how much is that A_p ? That accurate value we will try to estimate. If I look at this plot, how much I get this for the time half period of the signal? Half period of the signal is almost of one second or so and the time delay **the time delay** till t equal to θ will be also of magnitude 0.4 or so. **We will make** Measurement has been made using some software, or theoretically we have to make use of some software to accurately estimate the parameters A_p p by 2 and the slope at the zero crossing. So, if I look at the plot over here, the slope is found to be 1 **the slope is found to be 1**. So, let us see, what sort of measurements are obtained from this output signal.

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From simulation, the quantities $A_p = 0.4918$, $p/2 = 1.077$ and $S = 1.0$ were measured.

$$\mu = \frac{A_p \cdot \ln\left(\frac{1-p}{1+p}\right)}{\sqrt{S} \cdot (-R_{1/2})} = -0.4566 \cdot \ln\left(\frac{1-p}{1+p}\right)$$

$$\Rightarrow \beta = 0.4918$$

$$\Rightarrow T_i = \frac{-R_{1/2}}{\ln\left(\frac{1-p}{1+p}\right)} = 1$$

$$\Rightarrow \theta = T_i \cdot \ln(1+p) = 0.4$$

* Using (5-8), the identified model becomes

$$G(s) = \frac{1.0e^{-0.4s}}{1.0s - 1}$$

So, from the simulation with the relay testing with the relay setting of plus minus 1 a peak amplitude of magnitude 0.4918 is obtained; half period of the output signal is found to be of 1.077 and S is equal to 1.

So, these were estimated using some software. Now, I shall make use of the expressions we had for estimation of transfer function model for this system. Now, we know that μ is equal to $A_p \times \ln(1 - \mu)$ by $1 + \mu$ times minus $\frac{p}{2}$. While I am writing in this form, as I had mentioned earlier for unstable system, we have to choose pick up the bottom sign. So, μ will be equal to $A_p \times \ln(1 - \mu)$ divided by $1 + \mu$ upon S times minus $\frac{p}{2}$. So, that I have written over here (Refer Slide Time: 46:10).

So, as I know A_p and S are already obtained from the measurement. So, this is equal to minus 0.4566 times $\ln(1 - \mu)$ by $1 + \mu$. So, when this non-linear equation μ is equal to minus 0.4566 times $\ln(1 - \mu)$ by $1 + \mu$ is solved, the solution gives us μ as μ is equal to 0.4918. So, one may have to employ a non-linear equation all over or intuition can be applied. Many techniques are there to find or solve one unknown from one non-linear equation.

So, employing that technique, the μ can be estimated and the μ is found to be of the magnitude 0.4918. Then, the T_1 using the formula becomes T_1 is equal to minus $\frac{p}{2}$ divided by $\ln(1 - \mu)$ by $1 + \mu$ is equal to 1. Further, θ is equal to by formula it is $T_1 \times \ln(1 + \mu)$. So, substitution of μ and T_1 over here gives us θ is equal to 0.4. Thus we have found that if μ can be estimated accurately, in that case the T_1 and θ can be estimated accurately. What **what** was the transfer function model we had? we had $G(s)$ is equal to $e^{-0.4s}$ upon $s - 1$. As I have already mentioned, θ is equal to 0.4, T_1 equal to 1 and k is equal to 1.

So, our calculation from here, the k is equal to as we have seen the expression for k ; so, k is equal to **k is equal to** for our case, $S \times T_1$ by $h \times T_1$ by h and S is equal to $1 \times T_1$ has been estimated as $1 \times h$ is equal to 1. So, k is also equal to 1. Thus the estimated values θ , T_1 and k are found to be accurate and thus the dynamics of the real time system the unstable plus first order plus dead time system has been found as $G(s)$ is equal to $1 \times e^{-0.4s}$ upon $s - 1$.

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Summary

- All parameters of a FOPDT model can be estimated
- The general expressions can be extended for lower order dynamic models $G(s) = k e^{-\theta s}$ $G(s) = \bar{k} e^{-\theta s} / s$
- Output waveform can be observed to decide about the transfer function models
- Model parameters are estimated using the three measurements

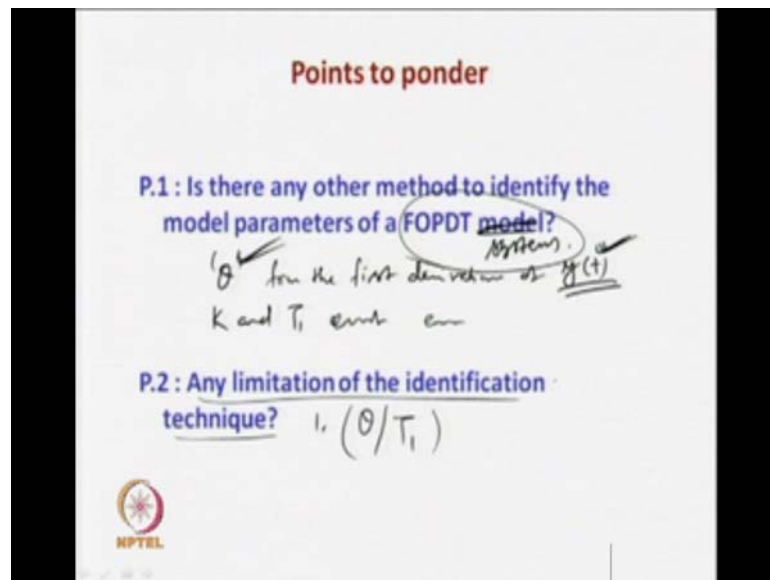


So, let me summarize. All the parameters of a first order plus dead time transfer function model whether the transfer function model is for stable or unstable, first order plus dead time systems does not matter. All the parameters of the transfer function model can be estimated using the expressions we have derived. So, three expressions can be made use of to estimate the three known unknowns of the first order plus dead time transfer function models.

The general expressions we have obtained can be extended for lower order dynamic models. What I mean by lower order a dynamic model? When $G(s)$ is equal to $k e^{-\theta s}$ or $G(s)$ is equal to $\bar{k} e^{-\theta s} / s$, those type of transfer function models or dynamics of real time systems can be obtained. Now, output wave form can be observed to decide about the transfer function model.

So, whether we have to identify simpler model or first order plus dead time model, those informations can be obtained from the observation of the output wave form. Now, model parameters are estimated using the three measurements and using the three expressions we have derived.

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Some points to ponder - is there any other method to identify the model parameters of a first order plus dead time model or first order plus dead time systems rather? Yes. There are many techniques. To name a few, one can estimate the time delay parameter from the first derivative **first derivative** of the sustained oscillatory output when there will be large change in the output or when there will be discontinuity in the output, then the time taken to that point from any zero crossing can be taken as the time delay associated with the dynamics of that system. Then the remaining two parameters K and T₁ can be estimated using any two expressions we have derived earlier.

So, there are other techniques also to identify the model parameters of first order plus dead time systems or third order systems in terms of first order plus dead time transfer function models.

The second point is any limitation of the identification technique. Certainly, the identification technique is subjected to mainly two limitations: one is the ratio between the dead time to unstable time constant of the first order plus dead time unstable systems. If this ratio is not large, then only a sustained oscillatory output can be obtained. That is one of the limitations of this identification technique. Secondly, sustained oscillatory output must be noise free; otherwise, it will be very difficult to make measurement of the slope which is an important parameter and which expression **which expression** has to be used to estimate the three unknowns associated with the transfer function model.

Thank you.