Advanced Control Systems

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Module No. # 03

Time Domain Based Identification

Lecture No. # 02

State Space Based Identification of Systems-1

Today's lecture is all about state space based identification of systems. In this lecture, we shall see how waveforms can be generated using state space and how wave forms can be correlated with explicit expressions that can be used for indentifying model parameters of a system.

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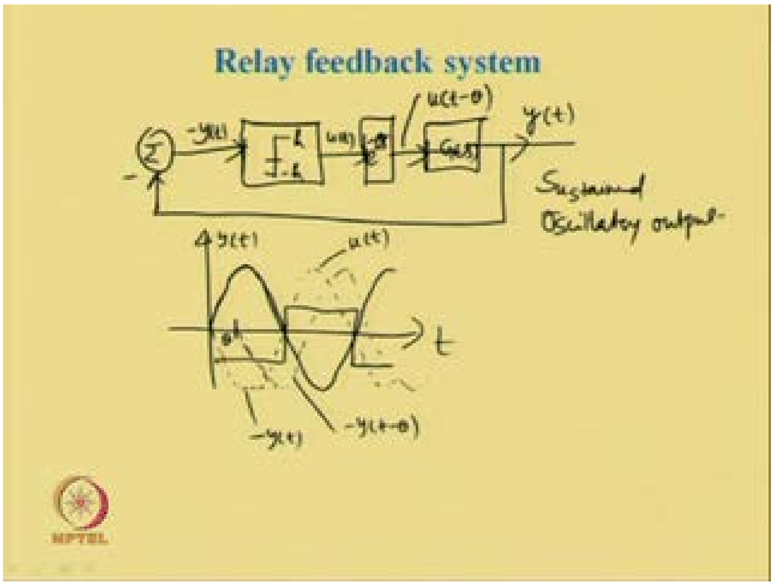

First, we shall discuss about the simple relay feedback system. We can draw a block diagram for that, where we have a summer, which is connected to a symmetrical relay of amplitudes h and minus h, which in turn is connected to a system; and, the system results in output y t. When this is simulated, the system generates sustained oscillatory output. The form of the output can be shown in this form, where we get periodic output of this form.

Now, when the system is subjected to time delay, then the system can be divided into two parts: one with the delay and the other part with the delay-free part. Then, the input to the system can be a delayed input, in which case, the time delay e to the power minus theta s gives a delayed input to the system u t minus theta. How can we show u t minus theta? When the output is positive, when the output is like this, during that, the input to the relay becomes minus y t. So, the input to the relay will be inverse of that. The dotted line shows the input to the system, which is nothing but u t.

When the input is delayed now… This is u t. When the input gets delayed by theta seconds, then there will be shifting by theta seconds. Let this be theta (Refer Slide Time: 03:01). Then, the same input can be derived from this inverted input signal. Now, the input to the relay is minus y t. So, this is minus y t and this is minus y t minus theta. Corresponding to that, the output from the relay will be rectangular pulses only. So, earlier, it was like this.

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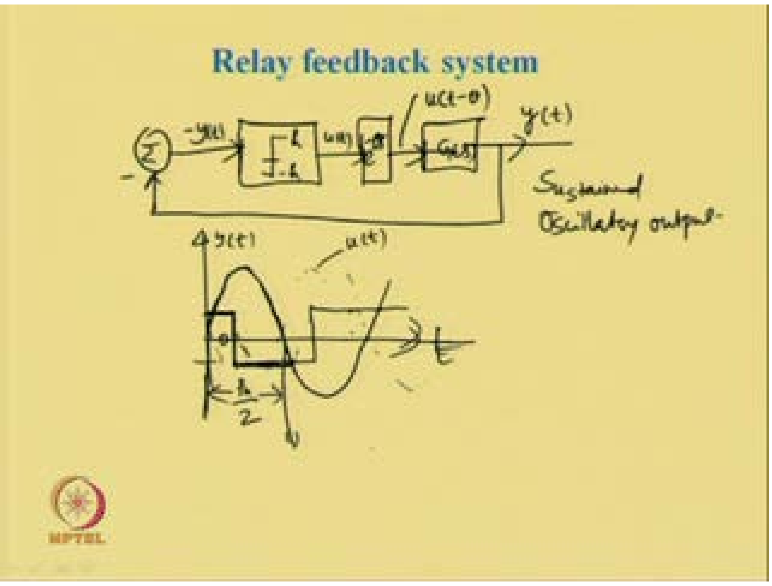

And now, due to the delay, the output of the relay will assume the form… So, this is the time axis. This is the output of the relay with delayed input to the relay. So, this output has got two piecewise constant inputs during half period of the output signal. This is the half period of the output signal, which can be given by p u upon 2. Now, during this half period, as we see, there will be two piecewise constant inputs to the system. Due to the two piecewise constant inputs, actually, the output goes monotonically from 0 to some peak value; then, it starts decreasing from some time theta onwards till it assume some negative value. This is how sustained oscillatory output is generated with the help of relay. Once this concept is thoroughly followed, then it will be very easy to analyze system dynamics using state space analysis. We shall make use of this concept. Once again, let me repeat these things, because this will be repetitively used in the state space based analysis of relay control system.

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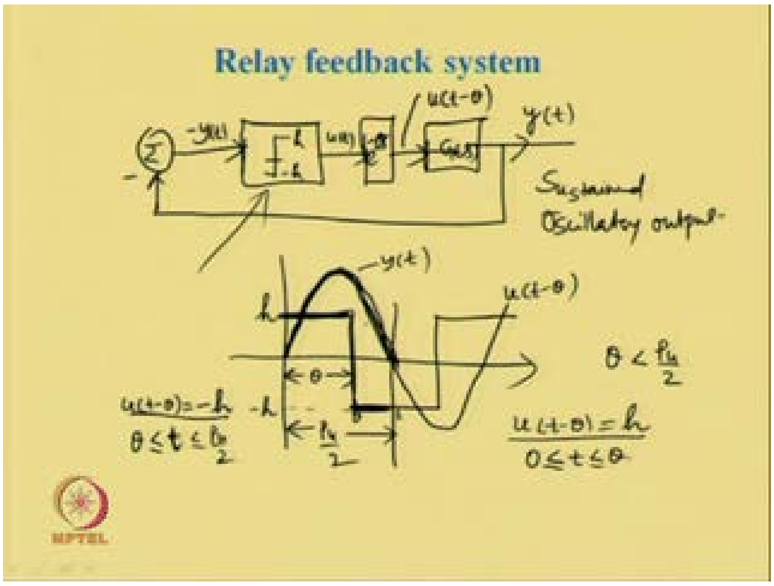

When the input to the system is some rectangular input of this form; whereas, this is a delayed input to the system u t minus theta; then, the output of the system will be of the form… So, this is y t. And, with the assumption that theta is less than p u by 2 – half period of the oscillatory or sustained oscillatory output signal; this is p u by 2.Then, when theta is less than p u by 2, always we shall have two piecewise constant inputs to the system during one half or positive half output of the system. So, when the output of the system is in the upper half, during that, the input has got two piecewise constant parts.

Now, the input of that form can be defined mathematically as u t minus theta is equal to positive h for the time range 0 to theta. So, when t is between 0 to theta, during that time, the input to the system is positive h. Similarly, u t minus theta will be equal to minus h for the time range theta is less than equal to t is less than equal to p u by 2. So, this part u t minus theta becomes minus h for the time t remaining between theta to p u by 2. So, this is point theta and this is (Refer Slide Time: 07:27) p u by 2. So, that way, this concept is very important. As I have said earlier, that we do get two piecewise constant inputs to the system due to the relay; relay device is such that it enforces two piecewise constant inputs to the system during half period sustained oscillatory output of the system. This will be used now for the generation of typical limit cycle output signal of different types of systems.

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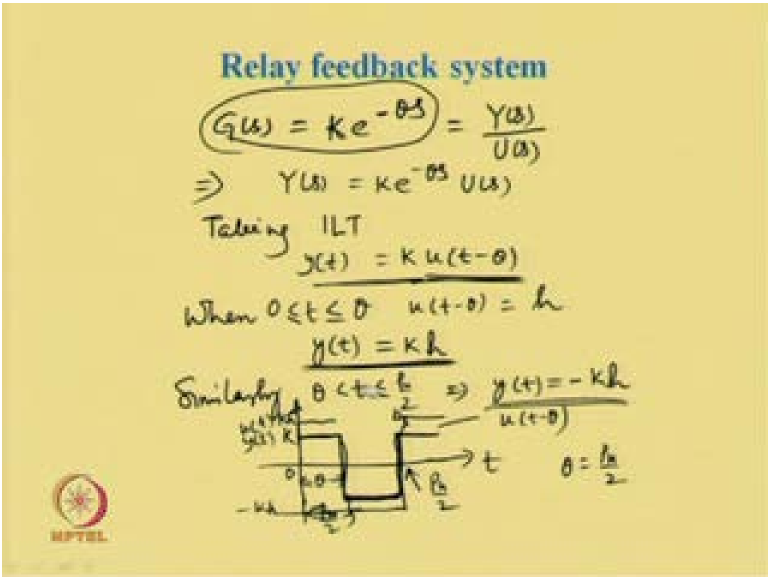

Let us for example, first, we shall consider a very simple system given by G s is equal to k e to the power minus theta s. We know that G s, the transfer function of the system, is the ratio of laplace transform of output to the laplace transform of the input, Y s upon U s, which can ultimately be written in the form of Y s is equal to k e to the power minus theta s U s. Taking inverse laplace transform, now, I get y t is equal to k u t minus theta. So, for such type of input, as you have seen earlier, the input is a delayed input; and, for different time range, we have got different magnitude for the input. So, when t is between 0 to theta, u t minus theta is equal to h. At that time, the output of the system will be equal to k h. Similarly, for the time range t is remaining between theta to p u by 2, the output of the system will be y t will be equal to minus k h, since u t minus theta at that time equal to minus h. Then, we can have the plot of the output signal for such type of input.

When the input is assuming this form – this is our time axis, this is y axis for two variables. It could be plot for either u t or y t. Now, we will have plot for u t minus theta. So, u t minus theta is of this form – this is the (Refer Slide Time: 10:16) plot for u t minus theta, which is going from positive to negative again for duration theta or so. That I will not discuss for the time being. When u t minus theta is equal to positive h, at that time, the output y t equal to k h. Therefore, this will be k. And, when time is between theta to p u by 2 – if this is p u by 2, at that time, we have got y t equal to minus k h spanning from time t equal to theta to t equal to p u by 2 – when this point becomes p u by 2. But, these cannot qualify as p u by 2, because we have got already one period of output, if I look at the input signal alone. So, the input signal has started from time t equal to 0 till this point and again it is going back. So, it already has made one period.

Half period can be denoted by this now; in which case, theta is equal to p u by 2. So, this is very important. For this typical case, when we have got a system, which has got a gain and delay only – a delayed system, but the system has got a gain and it is delaying the input signal. For such case, p u by 2 becomes theta. Then, what will be the output of the system? If I simply plot these two equations: y t equal to k h and y t equal to minus k h, definitely, the output of the system will be (Refer Slide Time: 12:20)… This is our k h and this is minus k h, minus k h, and again this will be k h.

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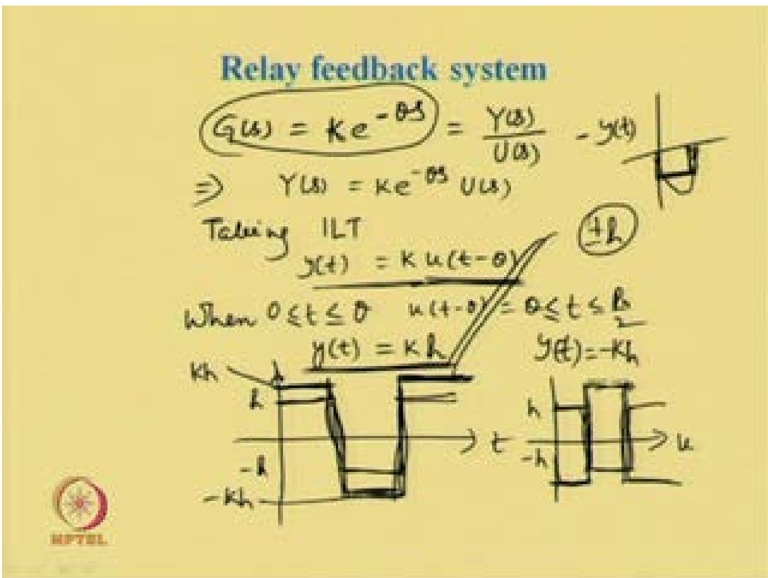

Let me redraw the plots for clarity. When the input to the system is obtained from a relay of amplitude plus minus h – in that case, the input of the system is given by h minus h and we will simply get rectangular pulses. And, the pulse width – I do not know now for the time being. It will be governed by the time delay associated with the system. Now, when we assume that the relay setting is of heights plus minus h – at that time, definitely, we will have input to the system of this form. And, corresponding output from the system will have magnitude k h when the input to the system is h. Then, the output will be k h. As we have already written over here, no need of writing once again. And, for the other time range – when time is between theta to p u by 2 – during that, y t will be equal to minus k h. So, the output will be like this. So, ultimately, the output is now given by this plot. Output plot is this one now.

Now, interestingly, in this case, what is happening? Whenever input signal is appearing, along with that, the output signal appears. This is happening due to the time delay associated with the system. Had there been no time delay, what sort of input and output we are expected to get? When the input is (Refer Slide Time: 14:44) h to minus h, at that time, the output should have come in this form. For positive input, we should have negative output. And for negative input, we should have positive output. Why that is so? As I have said, had there been no delay, then definitely, you will get this form, because the relay is such a device that it acts on the negative of the input. So, when the output y t is positive – as we have seen earlier, when the output is positive – at that time, the input to the relay is minus y t. So, minus y t will be like this. And, corresponding output from the relay will be negative rectangular pulse. So, going by that, I can tell that the output of the relay will be like this and the corresponding output of the system has to be of this form, if the system is not having any time delay. But, because of the delay, we are getting output and the input to the system appearing at same time. And interestingly, what happens? For the whole duration, time t between 0 to theta, when the input is positive, output is becoming positive. Let us do some simulation and see whether this is true or not.

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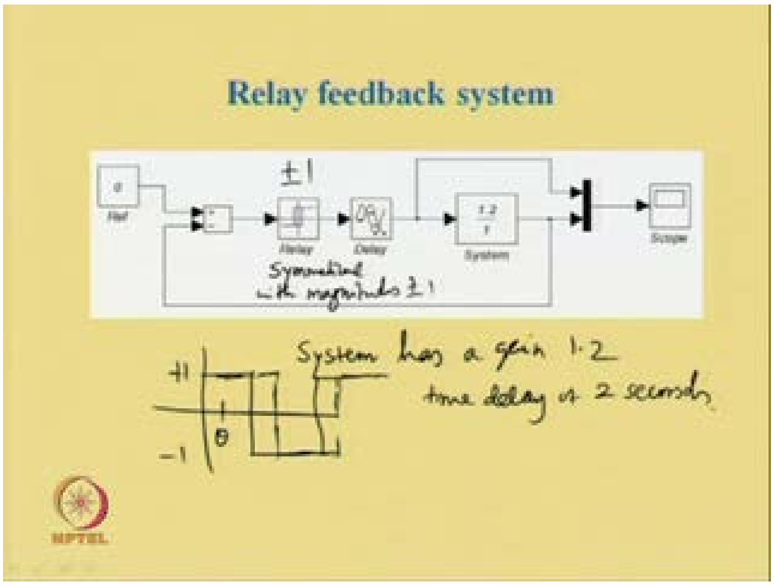

A typical system has been considered, where the system has a gain of magnitude 1.2 and time delay of magnitude 2 seconds. And, the relay setting is done at plus minus 1. So, we consider a symmetrical relay with magnitudes plus minus 1. That means, whenever the input is positive, at that time, output of the relay will be plus 1 irrespective of any value of input to the relay. And, when the input of the relay will be negative, for corresponding output from the relay will be negative 1 – minus 1. So, the type of relay output will have positive 1 and negative 1. So, this is what the type of relay output we will have. When this is delayed by theta seconds, again this will get shifted. So, if this is theta, then I will have signal shifted by theta seconds. So, the dotted one will be the shifted input, which is really passed onto the system which has a gain of 1.2.

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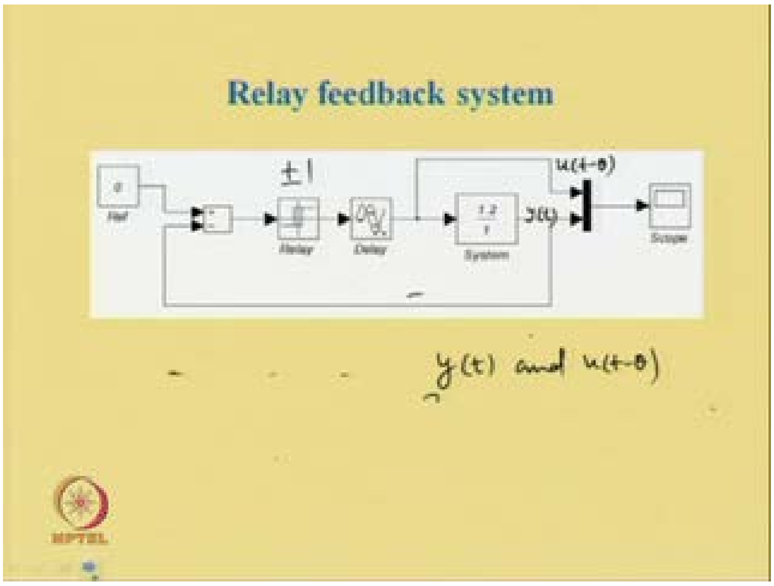

Then, when the relay test is conducted, what sort of output sustained oscillatory output we get from this system? That we shall see. Now, the scope has been connected to output of the system, which is denoted by y t and connected to delayed input. So, we get u t minus theta over here; that means the scope will be displaying the two signals: y t and u t minus theta.

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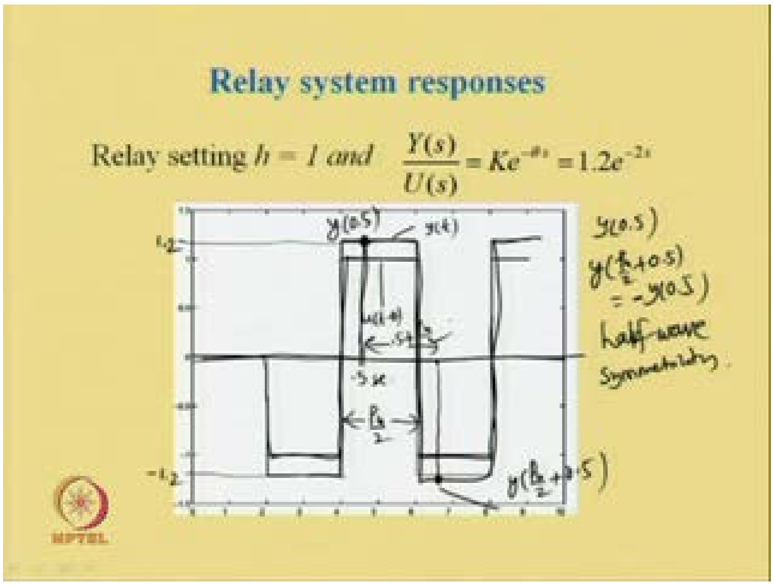

Let us see the type of wave form we will get from the simulation. So, the simulation is giving us this type of output signal. This is 1.2. So, this is minus 1.2; and, this one will be

1.2. So, whenever the input… And, let me show this plot; the inner one is for u t minus theta; the upper one is the plot for y t. So, y t is shown by this plot; and, u t is given by the inner one. Now, when you connect like this, then you get symmetrical input and output signal from the system. Now, it has symmetricity with respect to the zero axis or horizontal axis.

Now, it has got what type of symmetricity? It has got half-wave symmetricity. What do we mean by half-wave symmetricity? Whatever output you get at time t equal to suppose 0.5 second here; the output you get at that time is y 0.5 suppose – this one, then that output you will get – y p u by 2 plus 0.5 will be minus y 0.5. So, same time after half period, the output you will get will be negative of the value you will get over here. So, this value is denoted by y 0.5 and this is by y p u by 2 plus 0.5. Why I write like that? Because the half period is p u by 2. So, this 0.5 plus p u by 2 will be this point. This is (Refer Slide Time: 21:15) 0.5 plus p u by 2. This is how we get half-wave symmetrical output waveform from the system. So, if we carefully look all these typical type of output and input signals, from here, we can establish relationship among the parameters of the system along with that of the sustained oscillatory output signal.

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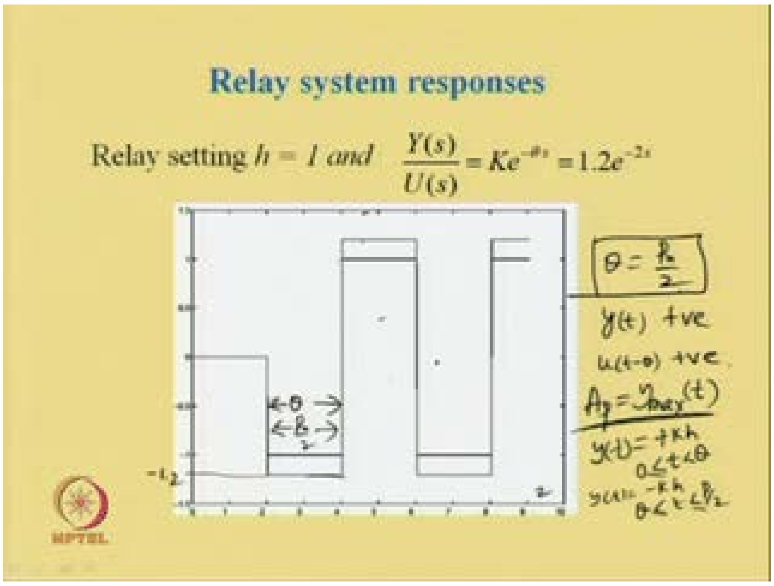

So, from the sustained oscillatory output signal, what we are getting? Interestingly, this theta is becoming p u by 2. So, theta is equal to p u by 2. This is a first one, why this is so? The main reason is that, whenever the output is going positive, y t is positive; during that, u t minus theta is also positive. So, in this case particularly, we do not have two piecewise linear inputs to the system during half period of the output of the system. So, we have got this relationship, theta equal to p u by 2. Another thing – the peak output of the system, A p is equal to y max t. How can we find this? Since we know the relationship, y t is equal to either plus k h for some time remaining between 0 to theta and y t is equal to minus k h for the time theta to p u by 2. So, the peak amplitude of the output signal is equal to k h. Thus, I can write A p is equal to k h.

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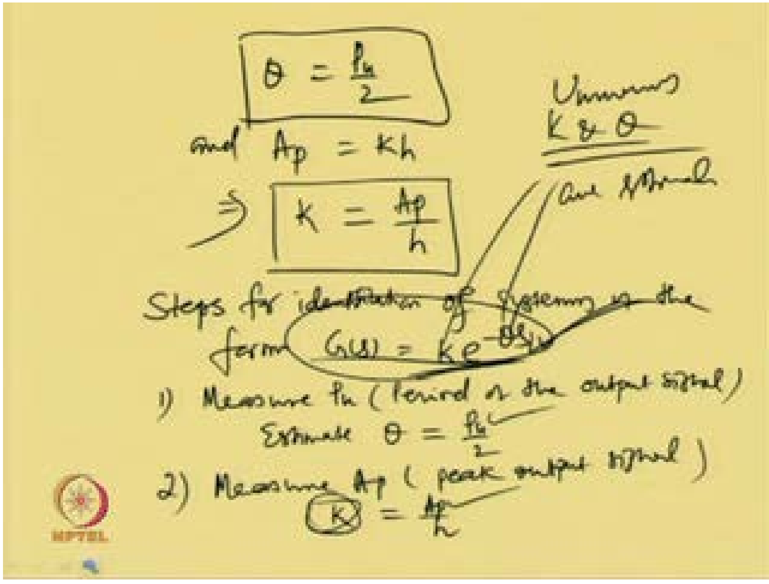

So, from the analysis of the input and output signal, what we have found? We have found that theta is equal to p u by 2 and A p is equal to k h; thus, giving us an expression of the form k is equal to A p by h. Now, the steps for identifying such systems; like a system having a delay and gain; a delayed system having gain only, the steps can be given like this. The steps for identification of systems of the form G s equal to k e to the power minus theta s is that, measure p u – period of the output signal; then, estimate one unknown of the model theta using this expression. Similarly, measure A p – the peak output signal; and, using the expression k equal to A p upon h, estimate the unknown k. Thus, the two unknown parameters of the model, k and theta are estimated; and thus, the unknown system is identified. So, looking at the type of output signal… Particularly, the output signal is a rectangular pulse. In that case, one has to choose the form of the transfer function model for a system is of the form G s is equal to k e to the power minus theta. Then, measure P u and A p; use the expressions and find the values for k and theta.

And ultimately, you get the mathematical transfer function model for the system. That is how the simple system, gain system with delay is identified.

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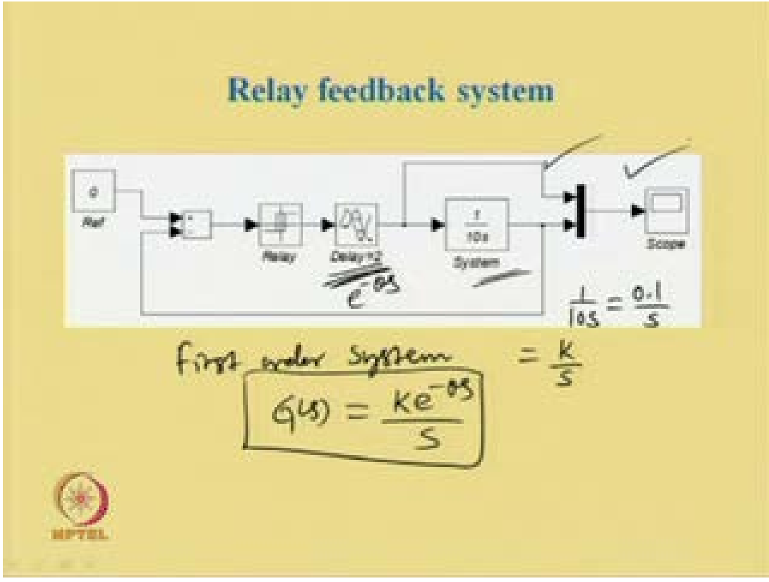

Let us discuss about the identification of some first order system. Here in this case, what has been shown? Relay test is conducted for a first order system. What type of a system we have? We have a time delay of 2 seconds, as shown over here. Therefore, this term is written by e to the power minus theta s. And, the system is given by a time constant with some integrator. Now, we have got an integrator basically. Therefore, the same system can be written in the form of 1 upon 10 s is equal to 0.1 upon s. Therefore, I can use some velocity gain k upon s. Then, the whole system can be represented in the form of k e to the power minus theta s upon s. So, this is a first order system. Then, how can we employ state space technique for identification of such system?

Now, we shall do little bit of mathematical analysis before getting the type of oscillatory output we expect, the type of rectangular input we expect during half period of output of the system.

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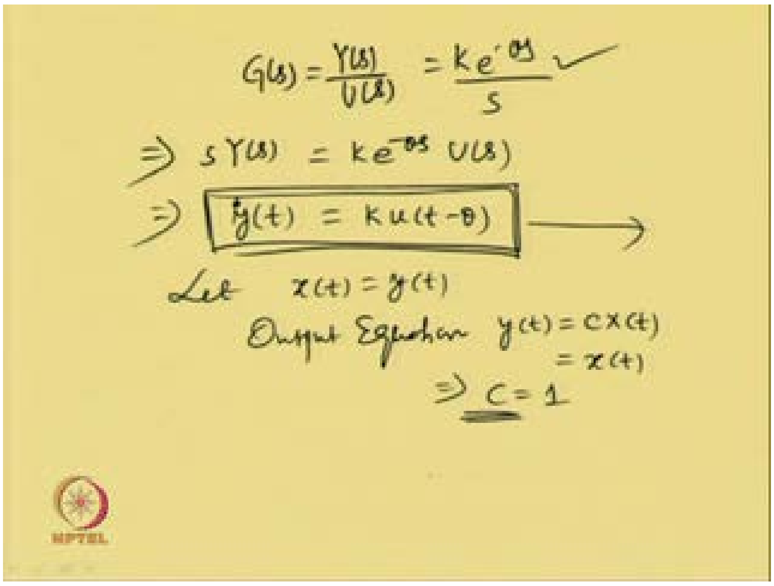

Now, the system itself is described by the transfer function G s is equal to Y s upon U s is equal to k e to the power minus theta s upon s, which again using cross multiplication, I get s Y s is equal k to k e to the power minus theta s U s; giving us y dot t is equal to k u t minus theta. So, this is the dynamic equation governing the dynamics of the real time system given by this form. Then, let us analyze this dynamic equation. As we have discussed in our last lecture, the state space analysis can be used to find the solution of state equation and subsequently the output expression for a real time system. Then, let me assume the state variable x t to be equal to y t. Then, the output equation, which assumes the general form, y t is equal to c x t is given by in this case x t; thereby, c is equal to 1. So, one parameter of the state equation – c is equal to 1. We are getting scalar values, because the order of the system is 1. When the order of the system is high, in that case, c will be a factor.

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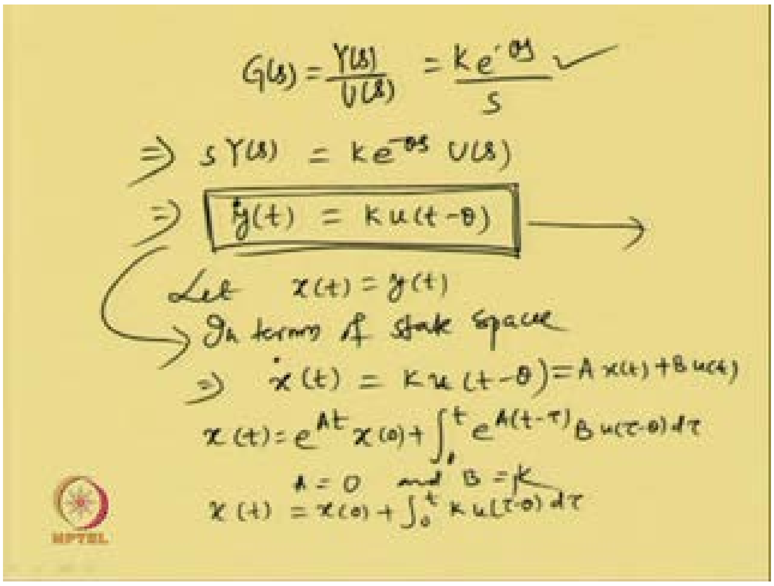

Now, the dynamic equation in terms of state space can be written as – in terms of state space, as x dot t is equal to k u t minus theta. The solution of the state equation using the standard formulae can be written as x t is equal to e to the power A t x 0 plus 0 to t e to the power A t minus tau B u tau d tau. What are these A and B? If I look at the state equation and compare with the standard form, A x t plus B u t, then A is equal to 0 and B is equal to k. So, in this case, the state equation becomes x t is equal to x 0 plus 0 to t. Then, k u… In this case, we have got delayed input (Refer Slide Time: 30:36). So, in this case, it will be tau minus theta d tau. Then, I will get u tau minus theta d tau. Thus, the solution of the state equation can be given in this form.

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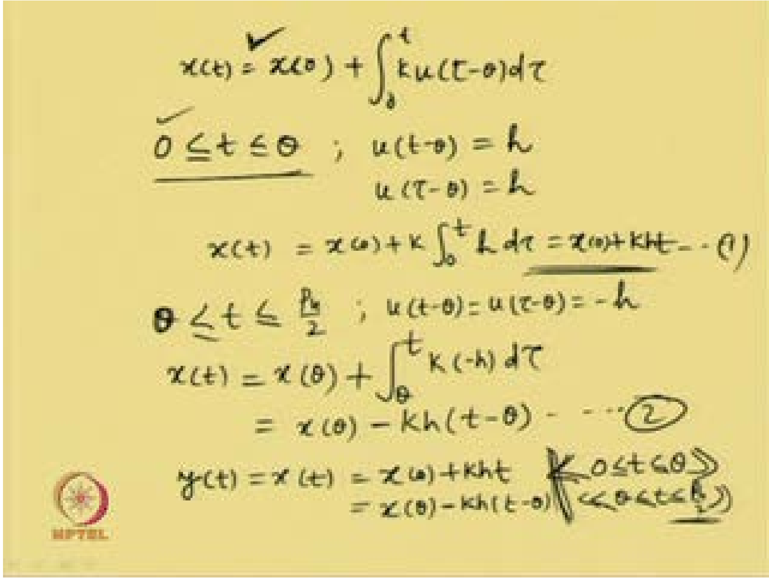

Let us simplify the expression further. x t is equal to x 0 plus integration from 0 to t k u tau minus theta d tau. Now, for the time range 0 to theta, we know that the input signal u t minus theta becomes plus h, when the system is under relay control and the relay setting has got amplitudes plus minus h. Then, in this case, u tau minus theta is also h. Changing in variable is not going to change the values; the expression can easily be written in that form. Then, the solution will be x t will be equal to x 0 plus – k we can take outside the integral; and, this becomes h d tau; thus, giving us x 0 plus k h t. So, this is what we get, the solution of the state equation for the time from 0 to theta.

Similarly, when the time is between theta to p u by 2, at that time, the input u t minus theta is equal to u tau minus theta is equal to minus h. But, when I will write the solution, the solution x t, where we have to consider t, a value greater than theta, this x 0 will become x theta. Keep in mind the way I am writing, because the initial value becomes x theta because we start the simulation from time t equal to theta; but, in the earlier case, we started the simulation from time t equal to 0. So, that way, x t will become x theta plus integration from theta to t k minus h d tau. Then, when this is integrated, then we will get this in the form of x theta minus k h t minus theta. So, let us designate this by the second equation and the first solution by the first equation. Then, in that case, the output waveform we get from this system can be given by y t is equal to x t is equal to x 0 plus k h t for the time range 0 is less than equal to t is less than equal to theta. And, y t will be equal to x t is equal to x theta minus k h t minus theta for the time range theta is less than equal to t is less than equal to p u by 2. When these two equations are plotted, then we will get the actual output waveform we get during relay test. But, the waveform will be available for half period only.

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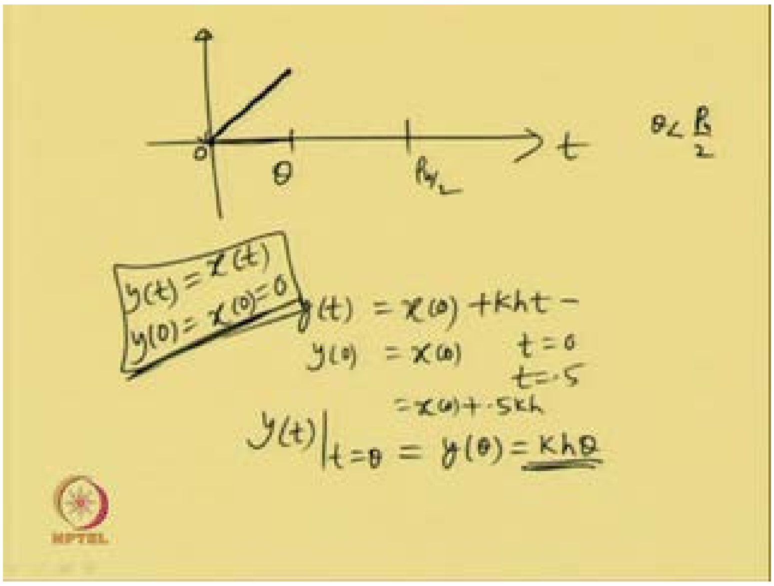

Let me do the plot; then, we shall have the time axis. Let this be theta and let this be p u by 2. Then, we assume that theta is less than p u by 2. Now, from time t equal to 0 to t equal to theta, the plot is governed by y t is equal to x 0 plus k h t. So, when time t equal to 0, it will be equal to x 0 for time t equal to 0. Some non zero value time t is 0.5, then this will be equal to x 0 plus 0.5 k h and so on. So, we will have a linear plot of this form, where the peak value will be given by x 0 plus k h t. Now, what is x 0 in this case? When time t equal to 0, y 0 is equal to x 0; so, the output at time t equal to 0 is y 0 over here, which is nothing but 0. So, the type of plot I will get for the time range from 0 to t is the straight line of this form. And, this peak amplitude is now will be y t for time t equal to theta, will be equal to y theta is equal to k h theta, because x 0 is equal to 0 or y 0 is equal to 0, because earlier we have assumed y t is equal to x t. Therefore, y 0 will be equal to x 0 will be equal to 0. So, using that, I find that at time t is equal to theta, the output will be equal to k h theta.

Now, what happens to the plot when time is greater than theta, when t is greater than theta? So, for that, we have already develop an expression, y t (Refer Slide Time: 37:39) is equal to x theta minus k h t minus theta, when t is greater than theta.

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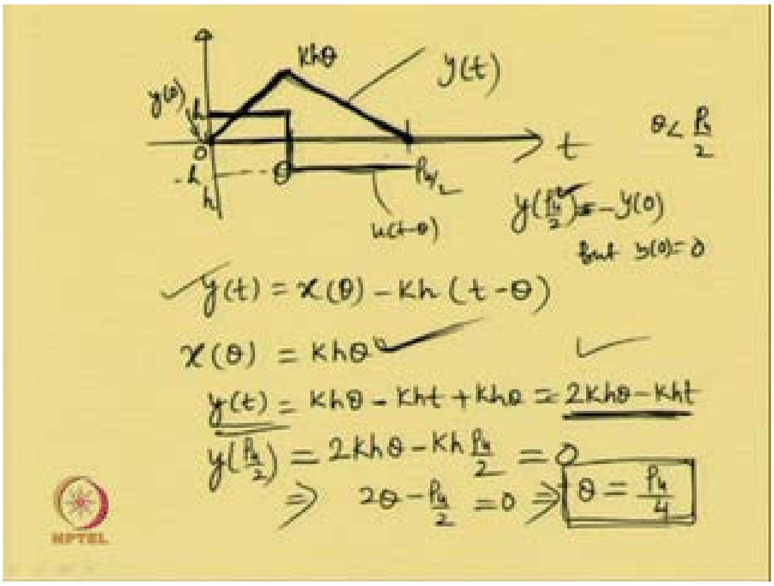

So, writing that again, y t is equal to x theta minus k h t minus theta. But, what is x theta? x theta will be equal to k h theta, as we have seen. Since, we have got till time t, time t is between 0 to theta, the output y t or the state space value x t is given by x t is equal to k h t. Therefore, x theta will be equal to k h theta. Then, in that case, substituting this value in the earlier expression, we get y t is equal to k h theta minus k h t plus k h theta; thus, giving us two k h theta minus k h t. So, this is the output we will get for time t is greater than equal to theta.

When time t equal to theta, what do we get? Are we getting the same value or not? Let us cross check. When time t equal to theta, at that time the output becomes y theta is equal to – upon substitution here, I get 2 k h theta minus k h theta is equal to k h theta. And since y t is equal to x t, this is same as x theta. So, this is verified that this expression is absolutely correct as far as t is greater than equal to theta is concerned, but t is less than equal to p u by 2. So, what sort of plot we will get? Interestingly, if you will have a plot for different values of k and theta, definitely, this plot will be like this.

With increasing time, it will be another straight line starting from the peak value of k h theta and gradually coming down to 0 at time t equal to p u by 2. Then, I can write the expression for the output at time t equal to p u by 2 h; y p u by 2 is equal to 2 k h theta minus k h p u by 2. And, at time p u by 2, what is the output? y p u by 2 must be equal to minus y 0. As you have seen, due to half wave symmetry of the output signal we get from relay testing, this is a property of half wave symmetry that at the half wave or at the half time period p u by 2, the output must be negative of the output you have at time t equal to 0. So, when this satisfies the high half wave symmetry; that means, y p u by 2 has to be equal to minus y 0. But, y 0 is equal to 0 (Refer Slide Time: 41:29) as we have already described how we get y 0 is equal to 0. So, this can be equated to 0. And upon solving, I get the relationship – k h is taken out; so, 2 theta minus p u by 2 is equal to 0; implies theta is equal to p u by 4. So, this is one important relationship we get from the analysis.

Now, the type of output signal you get is like this. This is the y t (Refer Slide Time: 42:05). What about the input signal? As we have seen, the input signal must be like this. When the input is positive h; that is there when time t… because by definition, we have taken u t minus theta is equal to h for the time range 0 to theta. So, with this assumption, we have derived the expressions for output. And, upon plotting the output expressions, we get two linear segments; and, connecting the two, ultimately, we get half period of output of the system. Then, the input to the system beyond time t is theta, is given by minus h. And, this type of output and input u t minus theta signal you must get. Since the analysis cannot fail, the analyses based on state space are exact ones. And, analysis shows that the output has to take this shape for the time duration from time t equal to 0 to t equal to p u by 2; during which, there will be two piecewise constant inputs of this form. Let us see really we get such type of output signal and input signal when the system is simulated or when the relay test is performed in real life.

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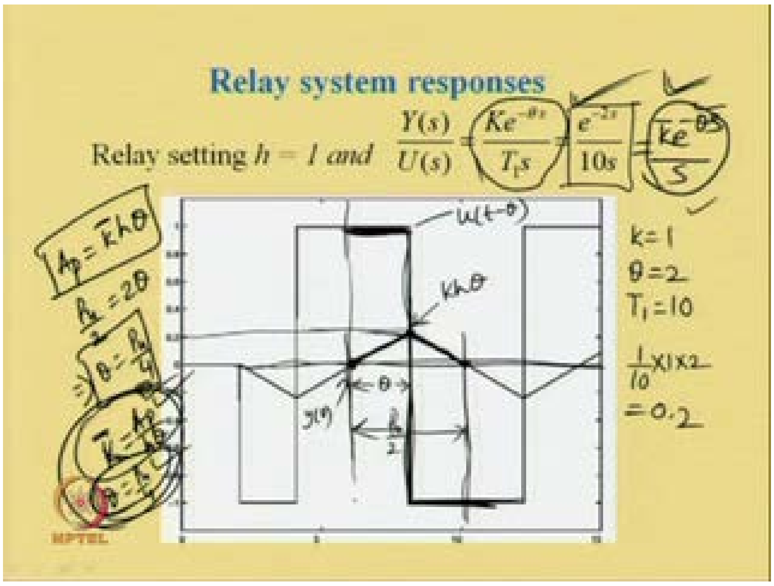

Then, this is what we get from the simulation of the system. Now, if I locate the horizontal axis, then in that case… and, start the analysis from any zero crossing; so, this is y equal to 0; output is equal to 0. It is going up like this. And then, after half period, it must come back to the zero crossing point. So, this span is now p u by 2. And, this is the peak value given by k h theta. So, the analysis gave us a half period signal of this form. And in practice also, we get half period signal of this form. And, we get the exact form, because the analysis using state space is exact. There are no approximations; even the initial conditions have been also taken care of. And therefore, the exact wave shape is obtained.

Now, what is this theta? Theta spans from here to here (Refer Slide Time: 45:41). And, during that, I see, what is the input? This is the input to the system. The rectangular pulses show the input u t minus theta. Keep in mind, it is not u t. Had it been u t, then at that time, the relay input would have been like this. When the output is positive, during that, input has to be positive for a whole duration of the positive output; whereas, we have got a positive input of this span; if I draw like this, a positive input of this span. And, subsequently, input is going negative; for how long, I do not mind. But, definitely, from time t is equal to theta to time t equal to p u by 2 the input is negative. So, the input signal for the positive output of the system is described by this.

Now, from the analysis, let us try to establish relationship between the parameters of the output signal, the input signal and that with the transfer function model of a system. Now, if you minutely observe, the peak amplitude is given by the analysis, is giving us the peak amplitude has to be k h theta. Is it so or not? For this system, k is equal to 1. If you look at the transfer function for which the simulation has been carried out, k is equal to 1, theta is equal to 2 and T 1is equal to 10. Then, in that case, for our case, this is 1 by 10; h is 1 and theta is 2. So, this is equal to 0.2 (Refer Slide Time: 47:45). So, the peak amplitude definitely is a magnitude 0.2. So, this is confirmed from here.

Next, what we have to see? The expression for the peak amplitude is (Refer Slide Time: 48:20) A p is equal to k h theta. In this case, we have got T 1. Then, k and T 1 are clubbed together. So, in this case, it appears as k upon T 1 h theta. Then, no need of writing in this form; this can ultimately be written in the form of k bar e to the power minus theta s upon s. Then, in that case, the expression for the peak amplitude will be k bar h theta. And also, we have found that p u by 2 is equal to 2 theta; thereby, theta is equal to p u by 4. So, these two expressions can be made use of to find the parameters of the transfer function model.

We have got two parameters now: k bar and theta. So, k bar can be estimated as k bar is equal to (Refer Slide Time: 49:17) A p by h theta and theta is equal to p u by 4. Like the earlier case, measure the peak amplitude; no. First, measure the period; divide it by 4; that will give the time delay of the transfer function model. And again, use the other expression to find k bar. k bar is equal to a peak. Measure the peak amplitude; divide it by h theta; and, you do get the other unknown k bar. That is how the two unknowns associated with this system are identified.

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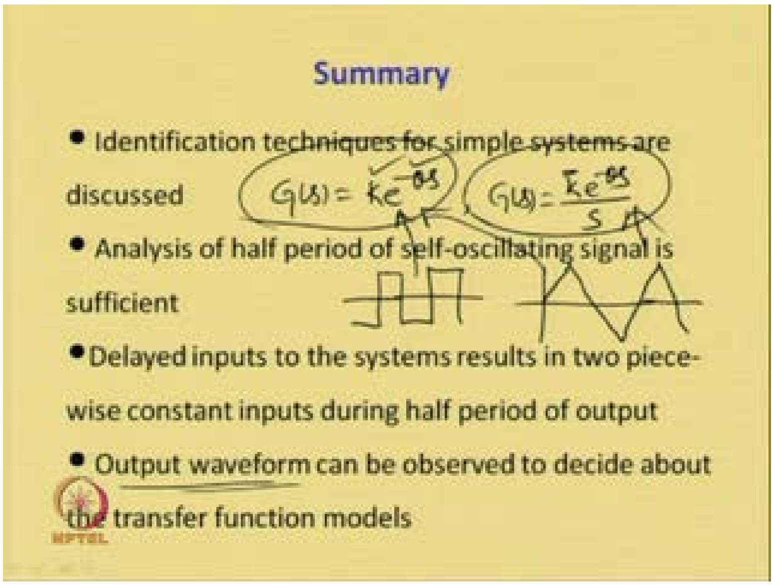

Now, what we have discussed in this lecture? Identification techniques for simple systems are discussed. For the system, dynamics are given by a gain with a delay or a velocity constant with delay and integrator. For these two simple systems, we have developed analytical expressions using state space; and, based on that, we have been able to establish relationship between the measurements of the output signal with that of the parameters of the transfer function model.

Now, also we have observed that during one half period of output of the system, we do get two piecewise constant inputs to the system; that is due to typical symmetrical characteristics of the relay. And, output waveform can be observed to decide about the transfer function model. When the output of a relay test is of rectangular type, then go for identification of such type of models, k e to the power minus theta s; whereas, if the output of a relay test is yielding triangular pulses, then go for identification of the system of this form (Refer Slide Time: 51:26). We should not confuse with each other. Suppose the output of this form, then this transfer function model should not be assumed for identification of dynamics of that system.

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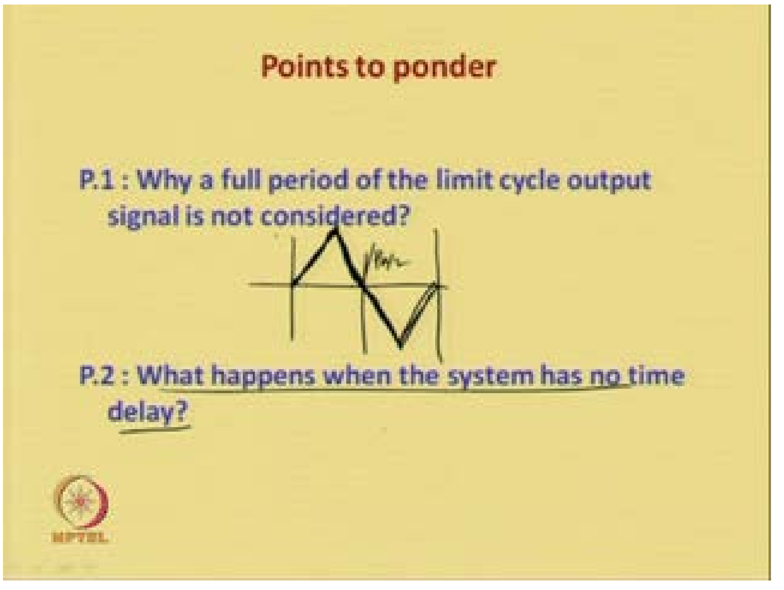

Some points to ponder. Why a full period of the limit cycle output signal is not considered? Since we do get half wave symmetricity of the output signal, there is no need of considering full period of limit cycle output signal, because the information that we get from half period is enough, in the sense, the negative of the half period is available during the time instant beyond half period. So, whatever you do get, replica of that with negative value is there. So, there is no need for considering whole period of output signal, rather the information we do get from half period is sufficient to identify the simple transfer function models of systems.

Secondly, if the question arises like this – what happens when the system has no time delay? In that case, we may or may not get limit cycle output or sustained oscillatory output. If the system is of higher order, an order more than 3 or so without time delay, in that case, symmetrical sustained oscillatory output can be obtained. But, when there is a time delay, sustained oscillatory output are guaranteed. That is all.