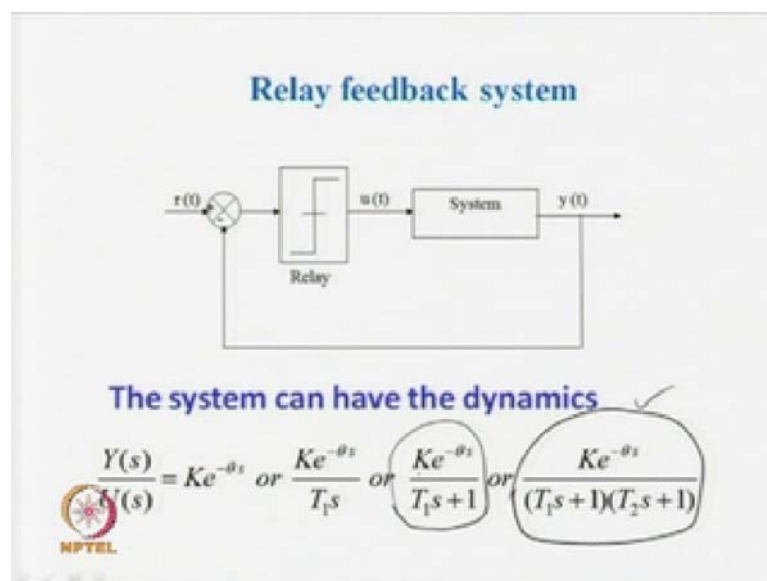


**Advanced Control Systems**  
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**Indian Institute of Technology, Guwahati**

**Module No. # 03**  
**Time Domain Based Identification**  
**Lecture No. # 01**  
**State Space Based Identification**

Today's lecture is about relay control based identification of systems. In our earlier lectures, what we have seen, how to identify a system dynamics using frequency domain technique, the describing function technique. But, there are some drawbacks associated with those techniques. That is why we may go for some alternate technique. Today's lecture will be mostly on the relay based identification using the time domain based techniques. The benefit of using time domain based technique is that effects of initial conditions can be taken care of using time domain based analysis. That is not so when we use frequency domain based analysis. So, using describing function or any other frequency response based technique, it is very difficult to handle the effects of initial conditions.

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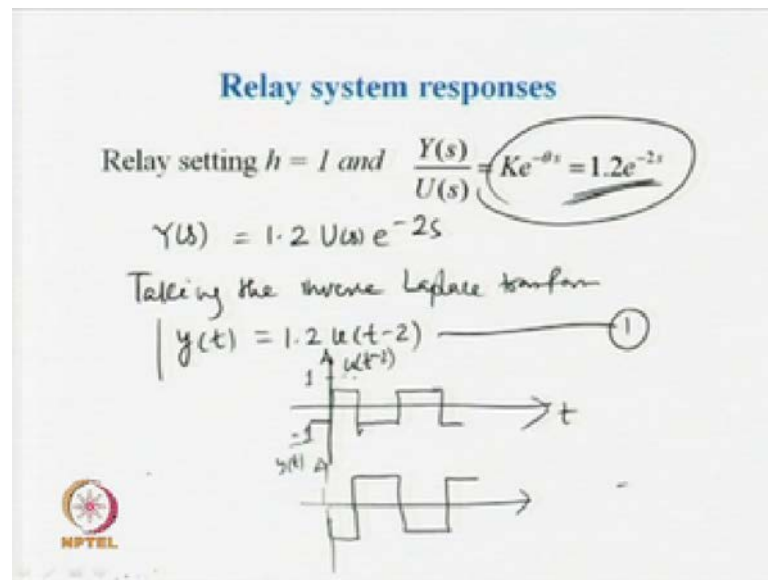
In today's lecture, basically, we shall see the output of a relay feedback system and the way one can obtain the output. Now, this simple relay-feedback system has got a relay, a symmetrical relay and system represented by the output and input ratio as  $Y(s)$  upon  $U(s)$ . So, the system input is  $u(t)$  and output is  $y(t)$  in time domain; whereas, in frequency domain, those are  $U(s)$  and  $Y(s)$  respectively.

Now, what sort of output is expected from this relay feedback system? When the input to the relay is rectangular, pulses only, the output of the system will be limit cycle output. Based on the rectangular input, the outputs of the systems will be limit cycle. But, the form of the waveform, output signal can vary depending on the system parameters. Now, for the simplest case, let us assume that the reference input  $r(t)$  to be 0. With this assumption, we shall have output signal of this periodic output signal of this form suppose. And, as I have already said, depending on the system parameters or the constituents of a system, the output will vary accordingly.

Now, the system dynamics can be represented in different form. And earlier, we have seen such a general transfer function form for the system representation. When the system dynamics is given in this form, it encompasses includes so many characteristics of a system. Now, with different limiting values, it is possible to obtain the system dynamics. In the first order plus dead time form, which is given in the form of  $K e^{-\theta s} / (T_1 s + 1)$ ; or, with further simplification, it is also possible to get the system dynamics in this (Refer Slide Time: 03:54) form; where,  $k$  is the steady-state gain of a system;  $\theta$  is the time delay associated with the system; and,  $T_1$  is the time constant of the system.

Now, when a system dynamics is given in different forms, different type of output signal is expected from this relay feedback system. We shall discuss one by one. Let us consider the simplest case of the system, which is given by the ratio  $Y(s)$  upon  $U(s)$ , is equal to  $K e^{-\theta s}$ .

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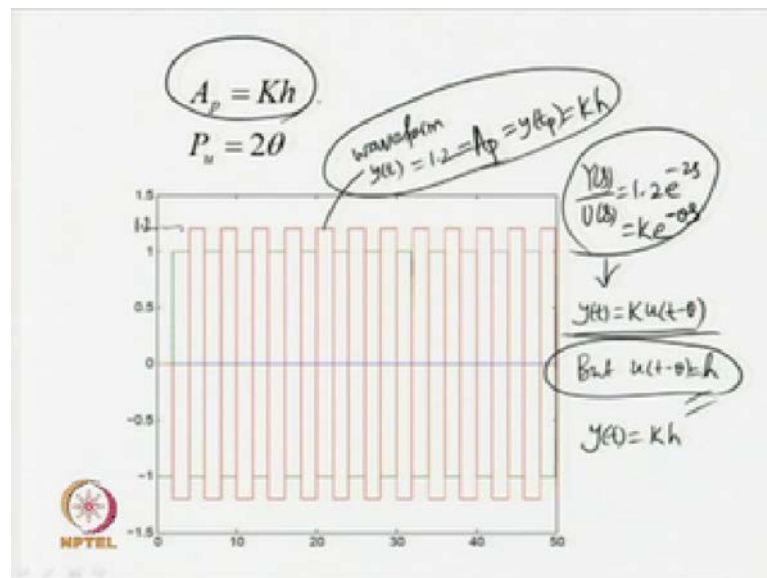
In that case, what happens, our system is having a pure gain with some delay. Then, that type of system with the relay setting of  $h$  equal to 1, can be again written in the form of  $Y(s)$  upon  $U(s)$  equal to  $K e^{-\theta s}$  equal to  $1.2 e^{-2s}$  when  $K$  equal to 1.2 and  $\theta$  equal to 2 seconds. Now, I can write the expression using cross multiplication as  $Y(s)$  is equal to 1.2 times  $U(s)$   $e^{-2s}$ . Why I am targeting to specific values, so that we shall be able to follow the type of waveform we expect from the system when the input has some typical form. Taking the inverse Laplace transform, I get  $y(t)$  is equal to  $1.2 u(t-2)$ . So, what sort of equation we have got? A linear equation, but it is a delayed linear equation. This time delay gives us typical type of responses of a system. The system input is  $u$  and the output is  $y$ , but we have got delayed input to the system.

Now, had there been no time delay, when  $y(t)$  becomes  $1.2 u(t)$ , in that case, suppose the input is given by  $u(t)$  as some rectangular pulses like this (Refer Slide Time: 06:42). And, in that case, what sort of output we will have? Simply it will be 1.2 times of the corresponding input signal. So, it will be magnified a bit only. But, we will get same type of output signal. But, because of the presence of the time delay, I cannot draw the output or input signal in a straightforward manner. So, what sort of signals we can expect from here? If the input sequence is now given by rectangular pulses of the form, suppose I start with this and this and this (Refer Slide Time: 07:31) like this, then correspondingly, the output sequence that I will find  $y(t)$  will be something different. Now, this magnitude

is 1; this is minus 1. Why I am writing here 1 and minus 1? Because the relay setting, height of the relay or amplitude of the relay,  $h$  equal to 1. Therefore, the type of input to the system will be rectangular pulses or square pulses only.

Now, when we have got such type of input to the system, a system, which has got a gain and a delay only, the type of output you will get is corresponding to this; this is a delayed. So, when you plot  $y(t)$  minus 2, I have to shift this by... So, the signal appears after 2 seconds or so. So, that way, this signal has to be shifted. Now, corresponding to this input, we will have some output of specific forms. So, for the negative input, the type of output I will have like this (Refer Slide Time: 08:36) and positive input like this, like this, like this. This is not correct; output is correct, but input has to be shifted. When  $u(t)$  is like this, when I have got  $u(t)$  only in place of  $u(t) - 2$ , then the output will be like this;  $u(t)$  only in place of  $u(t) - 2$ . When  $u(t) - 2$  is there, in that case, what will happen? The input has to be shifted. So, when I shift this input signal, I will get a signal of the form and so on. So, when I shift this (Refer Slide Time: 09:23) one, I have got different type of input and output pattern. So, in all, what sort of real life input output signals one can obtain? From this simple linear equation, we can see.

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Using simulation, it has been found that the output will be of this form and the input will be of this form. So, the red one is the output signal,  $y(t)$ ; and, the green one is the input signal  $u(t) - 2$ . So, the input output signal will appear like this. Because of the time

delay associated with the system, the input has been shifted; otherwise, when the input is going negative, output should have gone negative. And, when the output is going negative, that time input should be negative; or, when the input is going positive, at that time, input is going positive, but we find the output negative. That is happening because of the time delay.

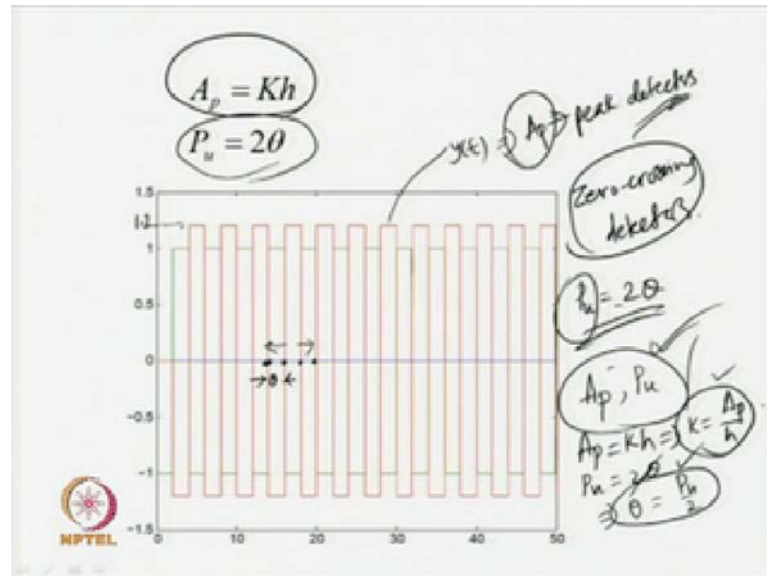
Now, can we find some relationship between the system parameters and the waveforms we get from the system? So, this is the output of the system; red one is the output  $y(t)$ . Can I establish some relationship between the parameters of the system with the output waveform of the system? When you see carefully the waveform of  $y(t)$ , the maximum that one gets is nothing but 1.2. So,  $y(t)$  is equal to 1.2 for some duration or minus 1.2 for some duration. So, the peak value of the output is 1.2 now. This can be translated in the form of the peak value  $A_p$  as  $y(t)_p$ , the time at which the peak appears in the form of  $K$  and  $h$ . For this typical case, when the simulation is done for this system, what is the system? The system  $Y(s)$  upon  $U(s)$  is equal to  $1.2 e^{-2s}$ . For this gain with delay system, this is the  $K e^{-\theta s}$ . That is the form. So,  $A_p$  invariably is to be found to be of the form  $A_p$  equal to  $K h$ .

And, if one writes the time domain equation, we have found that  $y(t)$  is equal to... If I take inverse Laplace transform, obviously,  $y(t)$  can be obtained in the form of  $K u(t - \theta)$ , because  $Y(s)$  upon  $U(s)$  is  $K e^{-\theta s}$ . So, if I cross multiply and take the inverse Laplace transform, it is not difficult to get the expression for the output in the form of  $y(t)$  equal to  $K u(t - \theta)$ . But, what we know, that  $u(t - \theta)$  is equal to  $h$ , the relay parameter. So, the output of the relay varies between or less to some constant value  $h$ ; either it is plus  $h$  or minus  $h$ , depending upon the input to the relay. So, when the input to the relay is positive, at that time,  $u(t - \theta)$  equal to  $h$ . When the input to the relay is negative, at that time, the input to the system is minus  $h$ . So, let us consider positive input to the relay; in which case,  $u(t - \theta)$  is equal to  $h$ ; thus, giving us  $y(t)$  equal to  $k h$ . And, that is what we have found also. The expression for peak amplitude, the output of the system, can be given as  $A_p$  equal to  $K h$ .

Similarly, it is not difficult to find out the periodicity of the output waveform. And interestingly, it is found to be  $P_u$  equal to 2 times the delay of the system  $\theta$  (Refer Slide Time: 13:45). So,  $P_u$  is equal to  $2\theta$ . So, these two important equations give us

some relationship between the system parameters and the output waveform; waveform of the output signal.

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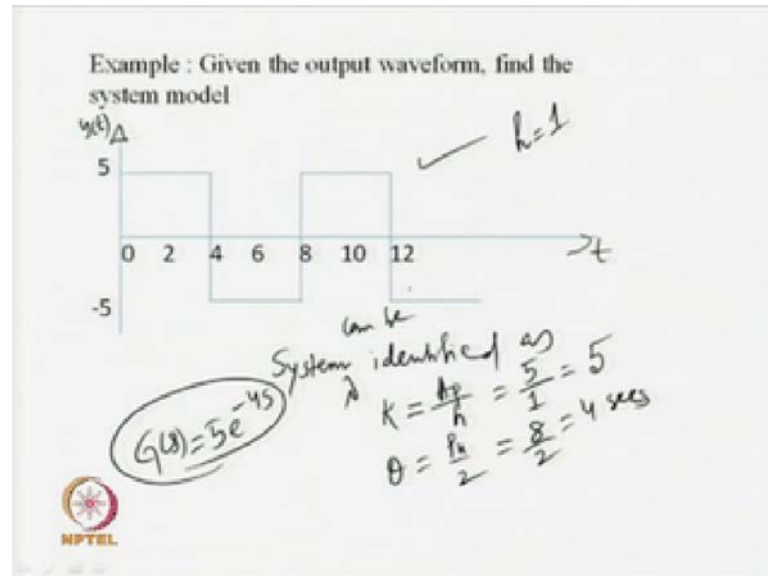


So, if someone takes measurement of the output of the system  $y(t)$ , the peak value of the output designated as  $A_p$  can easily be obtained using peak detectors. So, using peak detectors, measure the peak of the output signal. And now, using zero-crossing detectors, it is possible to locate the zero crossings. And then, you measure the span between these zero crossings. So, interestingly, this span happens to be  $\theta$  by  $\theta$ . So, this span happens to be  $\theta$ , so that from this to this, which makes one period complete period, gives you the ultimate period given as  $P_u = 2\theta$ . So, using the zero crossing detectors, I can estimate  $P_u$  or the period ultimate period of an output signal and using the peak detector, I can measure the peak amplitude of that signal. So, what we get from the measurements? Measurements give you  $A_p$  and  $P_u$ . Since we get these from the measurements of the output waveform, this is very readily available so to say.

Now, relationship with  $A_p$  and  $K_u$ , the system parameters can be made use of to find the unknowns:  $\theta$  and  $K$ . This gives  $K$  equal to  $A_p$  by  $h$ . And, this expression gives  $\theta$  is equal to  $P_u$  by 2. So, the unknowns of the transfer function model of the system,  $K$  and  $\theta$  can be estimated using  $A_p$  upon  $h$  and  $P_u$  by 2;  $A_p$  and  $P_u$ , we have got from the measurements of the output signal. Now, from those values, it is very easy to get the dynamic model of a simple system such as a system, which has got gain and a

delay. So, these concepts can be made use of to decide the type of dynamic model we can have for one system and the parameters of that dynamic model.

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Suppose a system gives us an output waveform of this one. This is our  $y$  and this is  $t$ . Suppose when relay test is conducted or performed on a system, the output of the system assumes this form. This is the waveform of the output. So, that way, from here, we can find out the transfer function model of the system. The system can be identified as... As we have seen, our first parameter  $K$  equal to  $A_p$  by  $h$  and  $\theta$  equal to  $P_u$  by 2. So, measure  $A_p$ ;  $A_p$  is found to be 5. So, this gives 5. Suppose the relay height,  $h$  is equal to 1; in that case, I get  $K$  as 5. And, when I measure the period, it is of 8 seconds now. So, that way, 8 by 2 is equal to 4 seconds. Then, the transfer function model of the system can be given as  $5e^{-4s}$ . How easy it is now to find the transfer function model or to identify the dynamics of a system? It is very easy if relationship can be established between the output signal and the parameters of transfer function model of a system; then, it becomes very easy to find the dynamic parameters of the system model.

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**Relay system responses**

Relay setting  $h = 1$  and  $\frac{Y(s)}{U(s)} = \frac{Ke^{-\theta s}}{T_1 s}$

$Y(s) T_1 s = K e^{-\theta s} U(s)$


$\Rightarrow \boxed{T_1 \dot{y}(t) = K u(t - \theta)}$  Inverse Laplace transform

$\Rightarrow \dot{y}(t) = \frac{K}{T_1} u(t - \theta)$

$\Rightarrow y(t) = \frac{K}{T_1} u(t - \theta) t + C = \int \frac{Kh}{T_1} dt + C$

$\Rightarrow y(t) = \frac{Kh}{T_1} t + C = \frac{Kh}{T_1} t + C$

$\Rightarrow \boxed{y(t) = \frac{Kh}{T_1} t}$



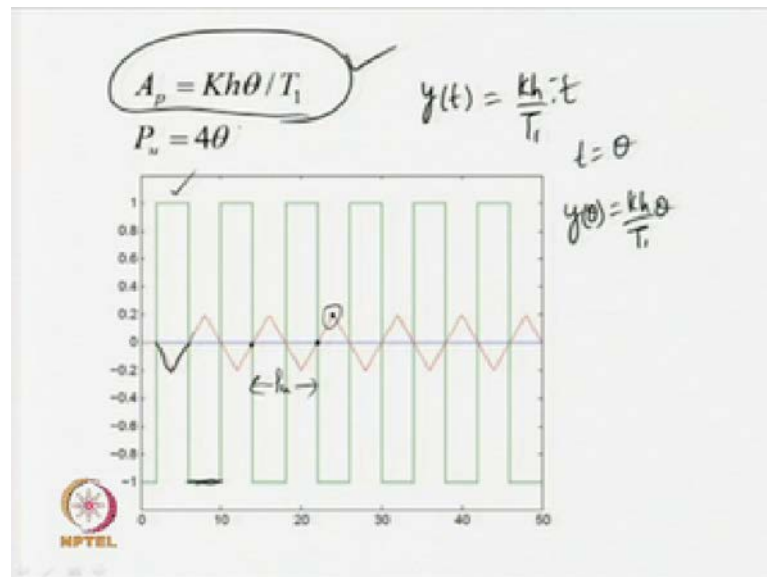
Let us consider some other case. Suppose the system is possessing now a gain time delay along with some time constant  $T_1$ . Now, when we look at this system, a pole is located at the origin and this type of system is known as integrating systems. It has got typical characteristics. What we mean by integrating systems? When the output goes on fluctuating in spite of any sort of inputs, that type of systems are often known as integrating systems. Now, when we have got such an integrating system, we can find the governing equation between governing equation relating the output and the input of the system.

Now, again cross multiplying, I can find  $Y s T_1 s$  is equal to  $K e^{-\theta s} U s$ . This can ultimately be written in the form of  $T_1 \dot{y}(t) = K u(t - \theta)$  using inverse Laplace transform. So, **which** inverse Laplace transform to find the dynamic equation in time domain form? Now, further simplification of the equation can be made;  $\dot{y}(t) = \frac{K}{T_1} u(t - \theta)$ . Then,  $y(t)$  will be equal to integration of  $\frac{K}{T_1} u(t - \theta)$  plus some constant. Now, as you know, what values you get for  **$t, u(t - \theta)$** ? We get, this is equal to  $h$ ; that means, we get **...** This is either in the form of plus minus  $h$ . So, let us concentrate on the positive output given by the relay. In that case, that enables me to write the expression in the form of  $\frac{Kh}{T_1} dt + C$ . So, this equation  $y(t)$  can be further written in the form of  $y(t) = \frac{Kh}{T_1} t + C$ . All these are constants; I will take out integration of  $dt + C$ , giving us  $\frac{Kh}{T_1} t + C$ . Then, when output equal to 0 at time  $t$  equal to 0



plus C, this gives us C equal to 0. So, ultimately, I get a simpler equation of the form  $y(t)$  is equal to  $K h t$  upon  $T_1$ . So, this dynamic equation in time domain can be obtained for this first order system (Refer Slide Time: 22:26). Now, we have got some specific values to see what sort of output waveform we will get from this system.

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So, let me write down this; once again,  $y(t)$  is equal to  $K h t$  upon  $T_1$ . This is the equation. So, when we have got the positive input,  $h$  is equal to positive, that time what we should get? We should have got positive output; **whereas**, the simulation shows that we are getting some negative output during that duration. Similarly, when the input is having some negative value minus 1, during that, the output is positive. Why that is happening? That is happening because of the time delay associated with the system. Had there been no time delay, you get very simpler equation.

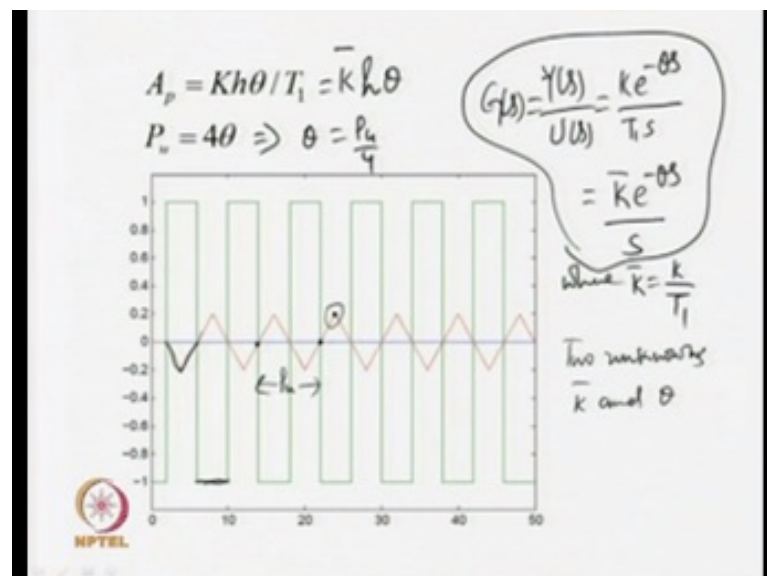
Now, the simple first order dynamic equation, which is known as (Refer Slide Time: 23:31) delayed differential equation has to be solved properly. We cannot solve it the way we solve it in the dynamic equations without having time delay terms in it. So, the delayed differential equation or the time delay term in the signal is to be taken care of very carefully. When the input is becoming  $h$ , when the input is not becoming  $h$  or it is becoming minus  $h$ , that has to be carefully decided. Can you decide from time 0 to  $\theta$ ? When the input  $u(t - \theta)$  is concerned, when this becomes  $h$  (Refer Slide Time: 24:21) for the time range beyond some  $P_u$  by 2, you cannot take  $u(t - \theta)$  equal to

h; it changes. These factors must be taken care of. So, these things come from the solution of delayed differential equation.

We will not go to much mathematics about this delayed differential equation, rather we shall see what sort of output we get. So, as you have seen, we get some simple expression like  $y(t) = K h \frac{1}{T_1} e^{-\theta/T_1}$ . So, the peak amplitude of this can be easily obtained. But, for integrating systems, it has been found that the peak amplitude comes after time  $t$  equal to  $\theta$  or after the time delay. So, when time  $t$  equal to  $\theta$ , we get the peak amplitude. So, that way, the maximum value of the output  $y(\theta)$  can be obtained as  $y(\theta) = K h \frac{1}{T_1} e^{-\theta/T_1}$ . That is what has been mentioned here.

The peak amplitude of the output signal is now related to the parameters of the dynamic model of the system by this equation. Similarly, one can measure the period and find out, is there any relationship between the parameters of the dynamic model, which that of the ultimate period of a system that is given by  $P_u$  over here? Yes, it is found that for integrating system on the relay control, gives us a typical relationship of the form  $P_u = 4\theta$ . Thus, I can make use of these two equations now to find the unknowns associated with the transfer function model of the system.

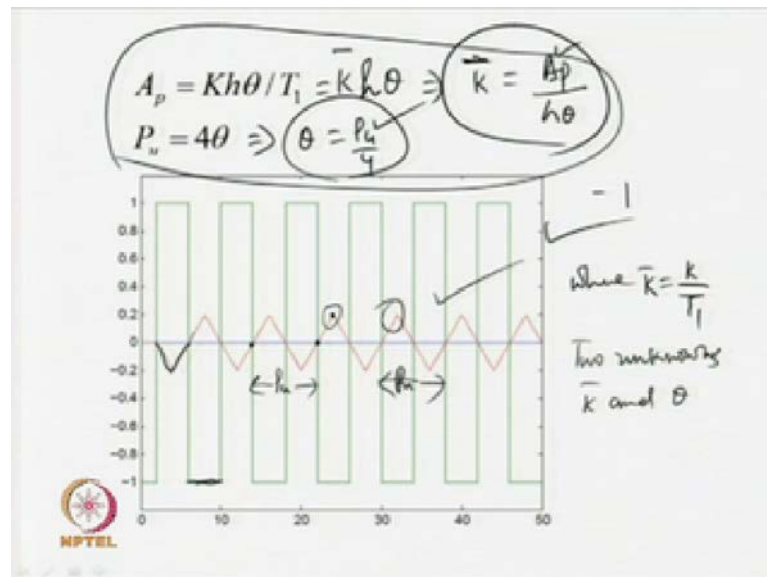
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What unknowns we have now?  $G(s)$ ; the transfer function model of the system is given by  $Y(s) \text{ upon } U(s) = K e^{-\theta s} / (T_1 s)$ . This can further be written in the form of  $\bar{K} e^{-\theta s} / s$ ; where,  $\bar{K}$  is equal to

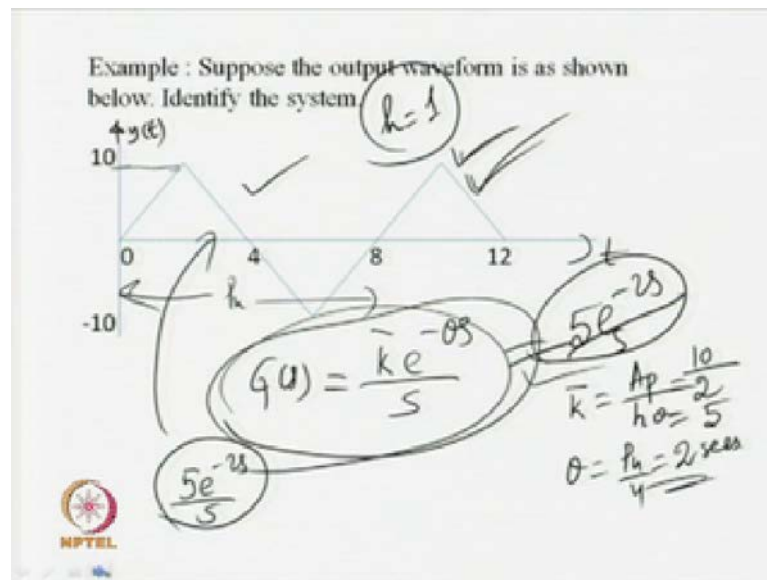
K upon T 1. So, basically, the integrating system model has got two unknowns. And, the two unknowns are K bar and theta. The two unknowns: K bar and theta can be estimated using these relationships. How can I find those? So, this is equal to K bar h theta. And, this can be written as theta equal to P u by 4.

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And, the upper one can be written as K bar is equal to A p by h theta. So, using these two, I can easily find the two unknowns associated with the integrating process model. The two unknowns can be obtained provided it is possible to accurately measure the peak amplitudes of the output signal and the period of the output signal. It is evident from the simulation results obtained for some integrating system that it is not difficult to measure the peak amplitude or the ultimate period associated with the waveform using peak detectors and zero crossing detectors. What basically the purpose of showing all those things? That the signal waveform, output signal waveform can certainly have relationship with that of the transfer function model parameters of a system. So, if some accurate relationship among them can be established, then it is always possible to identify the parameters of a transfer function model accurately.

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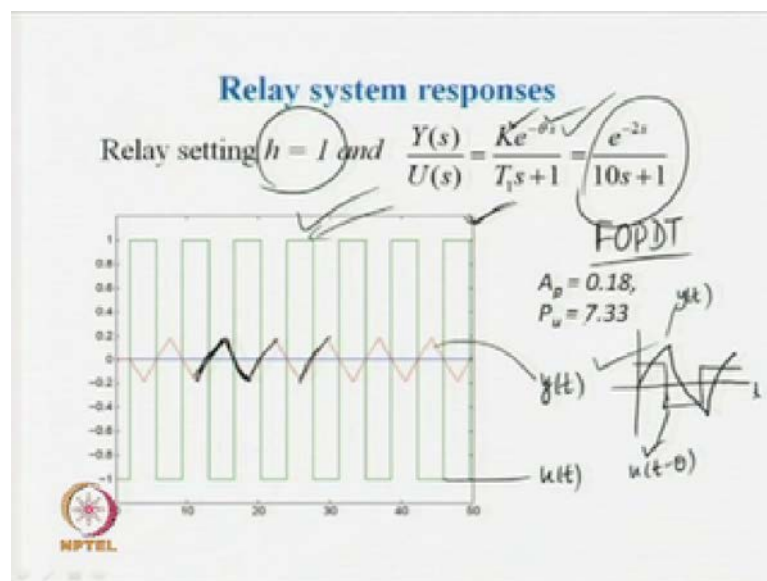


Now, consider this example. Suppose the output waveform as shown over here with the time axis and the output axis  $y$   $t$ , is obtained from some relay test of a system. In that case, how to find the system model? Now, as you have seen, looking at the output waveform, first of all, we decide the form of the transfer function model. What sort of transfer function model the system will have? So, I will go for the integrating one  $\bar{K} e^{-\theta s} / s$ , because we know that such type of integrating systems result in **triangular** output of systems under relay control. So, this type of system output you get under relay control, because relay control systems ensure limit cycle output or output with periodic oscillations. So, that way, a periodic output signals are induced using relay control. And therefore, we can now find from here the parameters of the model. As you we have seen,  $\bar{K}$  is found to be  $A_p / h\theta$  (Refer Slide Time: 30:28). So,  $\bar{K}$  is equal to  $A_p / h\theta$ . And similarly, the other relationship,  $\theta$  is equal to  $P_u / 4$ . The period is now of 8 time units. This is the ultimate period. Therefore, the time delay will be up to two time units or two seconds **suppose**.

Similarly, the peak amplitude is found to be of magnitude **10** suppose. Then, in that case, 10 upon  $h$  is 1; by default, let us assume  $h$  is equal to 1 for all the subsequent simulation studies in all the lectures. So, in that case,  $h$  is equal to 1 will us one times. Here we have got  $\theta$ ; already, we have found 2. So,  $\bar{K}$  is found to be 5. So, the transfer function model of the system becomes  $5 e^{-2s} / s$ . So, the point of giving this one

(Refer Slide Time: 31:37) is  $2s$  upon  $s$ . That, if there is a system, which has got the dynamics  $5e^{-2s}$  upon  $s$ , it is subjected to a relay test. In that case, such typical output waveform is expected from the **closed-loop** system. And, so, this model parameter has got relationship with the typical output waveform we are getting. So, if the waveform can be translated into some dynamic equations, if the waveforms can be expressed in some equations in time domain form, then definitely those equations can be made use of to find the transfer function model parameters effectively. That is the objective of time domain based identification technique.

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Now, let us see a first order plus dead time system that is subjected to relay test. The relay amplitude is set at 1; then, as it is evident from the simulation results, the output waveform is shown by the red; whereas, the input is given by this one, input signal. Now, this is actually  $u(t - \theta)$ . When I plot the  $u(t - \theta)$  versus  $y(t)$ , then it will appear in some different form. Now, you will have a form like this; where,  $u(t)$  will have like shifted one. So, there is no shifting here when you show the shifted version. So, in that case, this becomes the  $y(t)$  and this becomes the  $u(t - \theta)$ . So, now, using **the waveform of this waveform**, it is not difficult to find relationship between the system parameters and the form of this output waveform. So, this typical output waveform **form** if we look at minutely, it has got exponential over here and again you have got exponential over here. So, these responses are not triangular pulses. So, it does not take that form. Actually, we have got exponential responses. That we shall see in our next

lecture, how accurately exponential responses can be obtained using the time domain analysis of such type of transfer function model.

Now, this transfer function model – why we have considered this transfer function model? The objective is to see that can we get some relationship between the system parameters and the model. Some typical values are shown over here (Refer Slide Time: 35:09). Those typical values like the peak amplitude of the output signal is found to be 0.18. So, when it is measured accurately, the peak amplitude is equal to 0.18. And, the period of the output signal – if I start from here till here, that gives me the ultimate period, period of the output signal to be of 7.33. Now, if it is possible to establish relationship among  $A_p$ ,  $P_u$ ,  $K$ ,  $\theta$  and  $T_1$ , then  $k$ ,  $\theta$  and  $T_1$ , the unknowns in the transfer function model can be estimated accurately. That will be the objective of time domain based identification of system. How can we develop such expressions for this first order plus dead time system model? One can go for analysis.

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$$G(s) = \frac{Ke^{-\theta s}}{T_1 s + 1} = \frac{Y(s)}{U(s)}$$

$$\Rightarrow Ke^{-\theta s} U(s) = Y(s)(T_1 s + 1) = T_1 s Y(s) + Y(s)$$

Taking Inverse Laplace transform

$$\Rightarrow Ku(t-\theta) = T_1 \dot{y}(t) + y(t)$$

$$\Rightarrow \dot{y}(t) = -\frac{1}{T_1} y(t) + \frac{k}{T_1} u(t-\theta)$$

$$u(t-\theta) = \begin{cases} +h & 0 < t < \theta \\ -h & \theta < t < \theta + h \end{cases}$$

Now, when the system model is expressed in the form of  $K e^{-\theta s} / (T_1 s + 1)$ , which is nothing but the ratio between the Laplace transform of output to the Laplace transform of input. Then, cross multiplication of the terms **give us**  $K e^{-\theta s} U(s)$  is equal to  $Y(s) T_1 s + Y(s)$ , which can further be expanded and written in the form of  $T_1 s Y(s) + Y(s)$ . Now, again taking inverse Laplace transform, I can get the expression  $K u(t - \theta)$  is equal to  $T_1 \dot{y}(t) + y(t)$

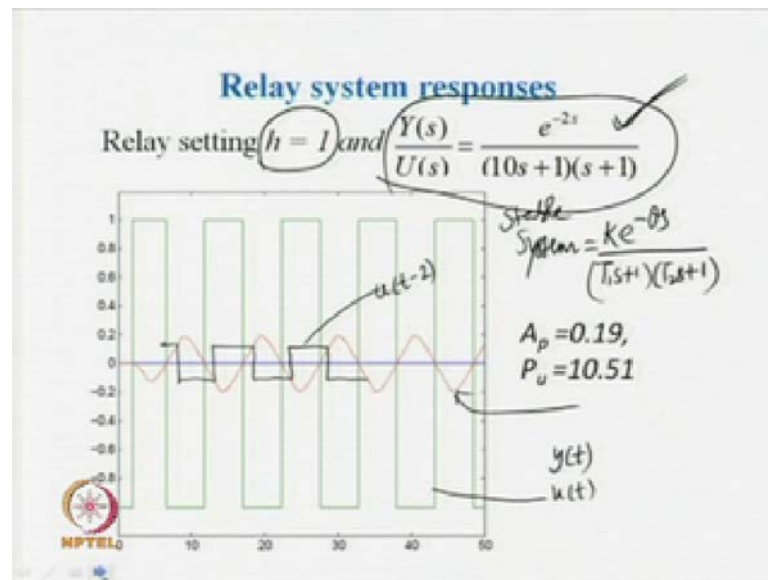
$y(t)$ . So, rearranging gives us  $\dot{y}(t)$  is equal to  $-\frac{1}{T+1} y(t) + \frac{K}{T+1} u(t - \theta)$ .

Now, I get a first order delayed differential equation. Why I say this delayed differential equation, because of the time delay term  $\theta$  in the differential equation. So, this gives us a first order differential equation. But, fortunately, how we overcome complex solutions of this first order delayed differential equation? Because we know that the  $u(t - \theta)$  gives us two values: plus  $h$  or minus  $h$ ; depending on whether time is between 0 to  $\theta$  or beyond that  $\theta < t < P$ ; all these things you will learn in our subsequent lectures, but this is the thing. So, what ultimately we find? Interestingly, we find that the input signal assumes either positive  $h$  or negative  $h$  values. So, the  $h$  has been stated. Again and again, the input signal is given by this (Refer Slide Time: 38:57). So, rectangular pulses – either positive  $h$  or negative  $h$ . So, that way, we get two piecewise constant values for the input. So, the input signal can be substituted by two piecewise constant inputs.

So, when you have got two piecewise constant inputs, in that case, the differential equation assumes the form of  $\dot{y}(t)$  is equal to  $-\frac{1}{T+1} y(t) + \frac{K}{T+1} u(t - \theta)$  or  $-\frac{1}{T+1} y(t) + \frac{K}{T+1} h$  or  $-\frac{1}{T+1} y(t) - \frac{K}{T+1} h$ . Then, this differential equation can easily be solved. There is no delay term in this. And fortunately, we can solve this to correlate the pattern or output waveform with that of the transfer function model of a system. So, interestingly, what one can see from here? If I take some limiting values, we know that  $Y(s) = \frac{K}{T+1} \frac{e^{-\theta s}}{s} + \frac{1}{s}$ . When  $T+1$  assumes large value, then in that case, this can be approximated to the form of  $\frac{K}{T+1} \frac{e^{-\theta s}}{s} + \frac{1}{s}$  since  $T+1$  is a large value. So, with these types of approximations or limiting, what basically we are getting? That when you have developed expressions for some second order model or higher order models, those same expressions can easily be used for identification of lower order models with the limiting values only. Simply you go by the limiting values; substitute those limiting values in the complicated expressions you have obtained from the waveform of higher order systems; and, those can enable us to find dynamic model of simpler order systems.



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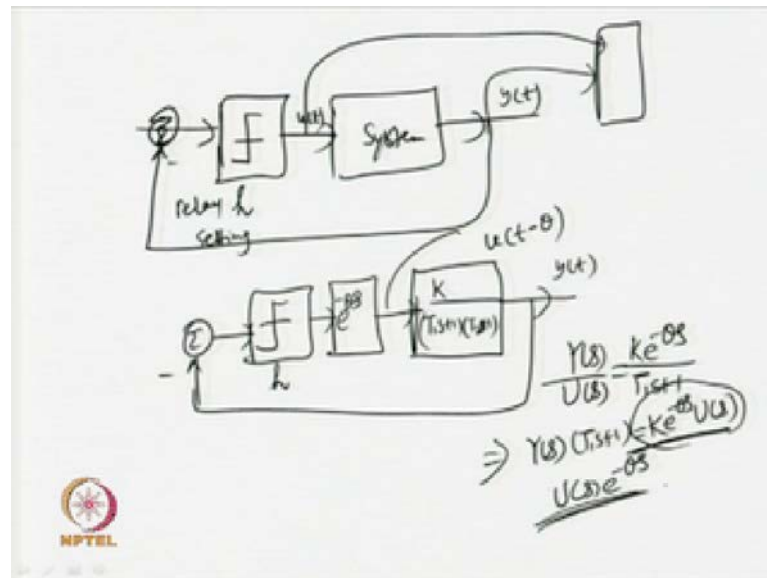
Next, we shall see what sort of output waveform is expected from higher order system dynamics. When a system is of higher order, it can always be given in the higher order transfer function form, can be represented in the higher order transfer function form of  $K e^{-\theta s} / (T_1 s + 1)(T_2 s + 1)$ . For the time being, we are considering only stable systems. Gradually, we will go to all sorts of systems: integrating systems, resonating systems and constable systems. To avoid complexity, for each analysis, we are concentrating on the stable systems now. So, when the stable system is represented by this form and when I have different values for  $K$ ,  $\theta$ , and  $T_1$ , then I get this transfer function model. So, this transfer function model when subjected to a relay control with the relay magnitude of  $h$  equal to 1, produces output signal of this form. So, this is the output signal you get from simulation. And, this is (Refer Slide Time: 42:56) the input signal we get.

Again, how can I draw the delayed input signal to the system? Only thing I have to do is that I have to delay this by the delay of 2 seconds. So, that will enable me to draw the waveform in the form of  $u(t-2)$ . This is delayed (Refer Slide Time: 43:20). So, that way, I can get waveform of the form like this. So, this can be  $u(t-2)$ . So,  $u(t)$  is shown in this form; whereas,  $u(t-2)$  has to be of that form. So, one has to make use of correct input signal in the time domain analysis to get expressions for the output signal accurately. This point has to be taken care of. To bring out that point, actually, this output signal has been drawn in this form (Refer Slide Time: 44:16). And, when you



have got delayed input signal, then the input has to be delayed by that time only. So, what I have done here, I have delayed the input by 2 seconds and obtained this signal. So, it is very important to get the correct input signal when getting the output and input waveforms together. So, let me redraw the relay control system a little bit for each in analysis.

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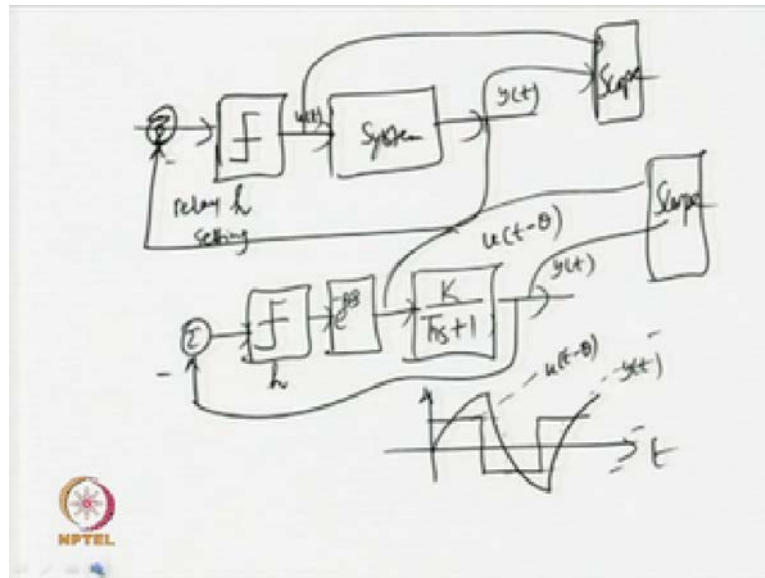


Now, we have got the relay over here – symmetrical relay of magnitudes plus minus  $h$ , but it is shown like this. So, this is the relay setting. Then, we have got the system output  $y(t)$ , input  $u(t)$ . So, when you are going to simulate this using mat lab or simulink, then in that case, you have to be very careful. How can we obtain the delayed input signal practically? If you simply connect  $y(t)$  or  $u(t)$  to scope, the type of signal you will get for a second order system is this one; the red (Refer Slide Time: 45:50) and the green one. The type of signals you will get will be of this form.

Now, when I re-adjust this one, this is the negative feedback (Refer Slide Time: 46:08). I have the relay parameter,  $h$  and I put the delay here, delay  $e^{-\theta s}$ . Then, rest of the things will remain here;  $K$  upon  $T_1 s^2 + T_2 s + 1$ . Then, what is the input to the system? The system input is now  $u(t - \theta)$ . The delay has been taken out from the system, so that the system is subjected to delayed input. Why we are writing in that typical form? Because we know that when you have got a transfer function of the form  $Y(s)$  upon  $U(s)$  is equal to  $K e^{-\theta s}$  upon  $T_1 s^2 + T_2 s + 1$ .

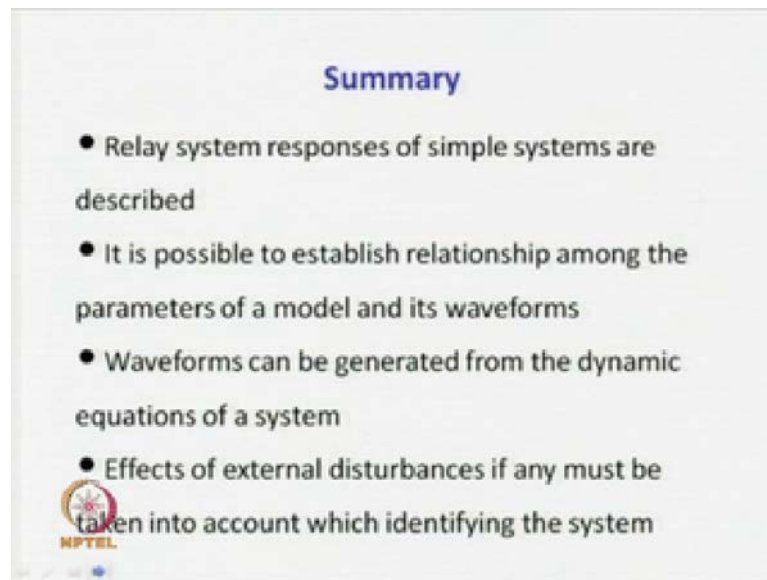
plus 1 or so, in that case, when you cross multiply, always you get  $K e^{-\theta s}$  together. So, the delay is going to **affect** the input. So, input will be delayed. Basically, the input signal to the system is delayed by sometime  $\theta$  seconds. So, this point is very important.

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Now, you need to obtain the plots of  $y(t)$  and  $u(t - \theta)$ . So, this is a scope. Then, for a first order system, what sort of input and output signals you will expect? For a first order system, this will have  $K T 1 s + 1$ . So, if the output is like this, exponential one, exponential like this, the input is going to be of the form like this. **So, this is the time existing.** Let me redraw again. So, the type of signals you will get: output signal exponential for a first order plus the time delay system – this is what you get for  $y(t)$ , and the delayed input signal will be of the form  $u(t - \theta)$ . So, the output-input **relay (( ))** will be  $u(t - \theta)$ ; whereas, if you draw  $u(t)$ , how it will appear?  $u(t)$  will simply be the expanded version. So, one has to be very careful the way you get the input and output waveform for the system. Because of the delay, you get typical output and input waveforms. So, these waveforms are to be minutely observed and proper measurements be made to obtain the system model parameters accurately.

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In summary, what we have discussed in this lecture, we have seen the relay responses of some simple systems. The system can have a gain and a delay; or, can have gain, delay and integrator; or, can have two poles or one pole, thus giving us second order plus dead time or first order plus dead time system model. So, for identification of varieties of system transfer function models, often it is possible to make measurements on the output of the system. And, using the relationship among the output waveform parameters and the system model parameters, it is possible to estimate the model parameters accurately.

Now, waveforms can be generated from the dynamic equations of a system. So, there are many ways one can go for identification of systems; either you make use of analysis of dynamic equations or you make some approximations to the relay and go for simpler identification. But, in that case, you may have to forego accuracy, because time domain based analysis of waveforms results in correct relationships among the model parameters and the waveform parameters.

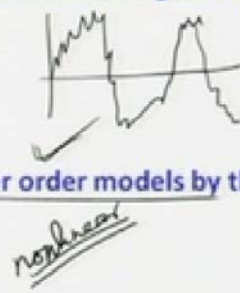
Now, in all our discussions so far, we have not taken care of external disturbances in the system. When we have got external disturbances, the output waveform varies significantly. Then, care must be taken to employ some technique to get rid of external disturbances or to nullify the effects of external disturbances, so that the output waveforms from where we take measurements can remain error free.


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**Points to ponder**

**P.1 : Are there any guidelines for setting the relay amplitudes?**

**P.2 : Can we identify higher order models by the state space analysis?**



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Now, some points to ponder. Are there any guidelines for setting the relay amplitudes? This issue already we have discussed earlier; the relay amplitudes can be set judiciously keeping in mind the type of waveforms we are obtaining from the relay test of many practical systems. If the output is found to be quite noisy, in that case, then to overcome the effects of noise, sometimes, relay amplitudes are set in such a manner that the actual output waveform can be distinguishable from noises.

Now, can we identify higher order models by the state space analysis? Yes, always it is possible to go for analysis of higher order system models using time domain and state space analysis. And, it is always possible to get relationships among all those system parameters and parameters of the waveform of output signal of a system. But, what one must take care of that although for higher order models we may get accurate expressions, often it is found, **expressions are to be** of nonlinear in nature. Then, care must be taken to solve the set of nonlinear equations such that there will not be multiple solutions or the solutions should not lead to erroneous results in the phase of different initial conditions.