

**Advanced Control Systems**  
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**Module No. # 02**  
**Frequency Domain Based Identification**  
**Lecture No. # 01**  
**Identification of Dynamic Models of Plants**

Welcome to the lecture titled Identification of Dynamic Models of Plants. Model identification is very important we must have studied identification of models or modelling of systems in the basic level control systems course. **If** for effective designing of **control** controllers and for analysis of closed loop system identification of models is necessary; when they process has been identified properly, definitely a controller an effective controller can be designed which can keep satisfactory time and frequency performances.

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The slide is titled "Dynamic Models" in a blue box. It contains a bulleted list of definitions and types of models, with handwritten annotations. The list includes:

- Model: description of relationship among related variables of a plant
- Theoretical Models: from basic laws of components in a plant  
Difficult and time consuming and involves lots of approx
- Empirical models:

Below the list, there are three lines of text with handwritten notes to their right:

- Observations of plant variables → *time domain*
- Relationship among variables → *frequency domain* (circled)
- Models linking the variables → *differential equations, partial diff. equations, difference equation*

The NPTEL logo is visible in the bottom left corner.

Now, we shall see what a model is a first. So, model of a plant, in broad sense is nothing but, the description of relationship among related variables of a plant. Now, models can be available in many forms, we know that the models may be of theoretical form or in

empirical form. Also, models of many real time systems are available in time domain and in frequency domain forms **in frequency domain form in**. When the model is expressed in time domain generally it is available in the form of differential equations **differential equations** or partial differential equations depending on the complexity of a process.

Again we can have in digital form, the model available in difference equation form **difference equation form**. So, all these forms of representation of model of a system are in time domain; in frequency domain, generally the models are expressed in the form of transfer function, some for assumed form of transfer functions are taken to represent the dynamics of a process v i b r.

Now, we shall go back to the basics of a model first, how the dynamics of a process can be identified. So, it can be identified in the form of theoretical models, where theoretical models are obtained from basic laws of components in a plant. However, the procedure is found to be difficult and time consuming and involves lots of approximations; whereas, empirical models of process dynamics are developed relatively, easily; it is done with the observations of plant variables or from the relationship among variables or from the models linking the variables.

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Time domain based Identification

- Measure plant i/p and o/p and establish a model structure  
Very sensitive to measurement errors

$y(1) = ay(0) + bu(1)$   
 $y(2) = ay(1) + bu(2)$

Suppose  $y(n) = ay(n-1) + bu(n-1)$

$\begin{bmatrix} y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} y(0) & u(1) \\ y(1) & u(2) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

Identification of the plant => estimating accurate  $a$  and  $b$  values

Suppose  $u(0) = 0; u(1) = 1; u(2) = 1.2$

$y(0) = 0; y(1) = 0; y(2) = 2; y(3) = 4.3$

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y(1) & u(1) \\ y(2) & u(2) \end{bmatrix}^{-1} \begin{bmatrix} y(2) \\ y(3) \end{bmatrix}$

$= \begin{bmatrix} 0.95 \\ 2 \end{bmatrix}$

$a = 0.95$   
 $b = 2$

Now, we shall go to some identification technique, we shall study some time domain based identification of dynamics of a real time process. First one has to, let us consider

we have some unknown process denoted by  $1/d_n(s)$ , then we can apply some known input and we can measure the output from the process. So, from the measurement of input and output, a model structure can be established, but the model is found to be very sensitive to measurement errors.

Let us see how we can identify a process model. Suppose, the process dynamics is given by the difference equation  $y_n$  is equal to  $a y_{(n-1)}$  plus  $b u_{(n-1)}$ . Identification of the process is nothing but estimating accurate  $a$  and  $b$  values for this case, because once  $a$  and  $b$  are estimated accurately then, the process dynamics has been obtained accurately.

Now, suppose  $u_0$ , the input to the process is 0,  $u_1$  is 1, and  $u_2$  is equal to 1.2. So, again we have got the process here, the process inputs are denoted by  $u$  and the outputs are denoted by  $y$ . So, corresponding output for the set of inputs be  $y_0$  is equal to 0,  $y_1$  is equal to 0,  $y_2$  is equal to 2, and  $y_3$  is equal to 4.3. So, with the set of inputs and outputs measurements, it is possible to estimate the model parameters  $a$  and  $b$ ; how can we estimate, since we know that  $y_n$  is equal to  $a y_{(n-1)}$  plus  $b u_{(n-1)}$  I can write a set of equations here for various values of  $n$ .

So,  $y_1$  will be equal to  $y_0$  plus  $b u_0$ ; and  $y_2$  will be equal to  $a y_1$  plus  $b u_1$ ; and  $y_3$  will be equal to  $a y_2$  plus  $b u_2$ . So, collecting the terms in matrix form, we get  $y_2$   $y_3$  vector is equal to  $y_1$   $u_1$   $y_2$   $u_2$  times the vector  $a$   $b$ . So, using this I can find an expression for  $a$  and  $b$  as  $y_1$   $u_1$   $y_2$   $u_2$  inverse times  $y_2$   $y_3$ , which gives us the values for  $a$  and  $b$  as 0.95 and 2. So,  $a$  is estimated as 0.95 and  $b$  is estimated as 2; this is how in time domain, process dynamics are obtained or dynamic model of a process can be obtained.

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**Time domain based Identification**

- Consider  $y(n) = ay(n-1) + bu(n-1) + \varepsilon(n)$

Suppose  $u(0) = 0; u(1) = 1; u(2) = 1.2$   
 $y(0) = 0; y(1) = 0.1813; y(2) = 2.0517; y(3) = 4.6809$   
 because of the measurement errors of  
 $\varepsilon(1) = 0.1813; \varepsilon(2) = -0.1205; \varepsilon(3) = 0.3318$

Now

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y(1) & u(1) \\ y(2) & u(2) \end{bmatrix}^{-1} \begin{bmatrix} y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1.2097 \\ 1.8324 \end{bmatrix}$$

$a = 1.2097$   
 $b = 1.8324$

The model parameter  $a$  is overestimated by 27.34% and  $b$  is underestimated by 8.38%.

Now, let us consider another situation, where the set of inputs be  $y_0$  is equal to 0,  $y_1$  is equal to 1,  $y_2$  is equal to 1.2; whereas due to some measurement errors  $\varepsilon_n$  the outputs are now  $y_0$  is equal to 0,  $y_1$  is equal to 0.1813,  $y_2$  is equal to 2.0517, and  $y_3$  is equal to 4.6809, because of the measurement errors of  $\varepsilon_1$  is equal to 0.1813,  $\varepsilon_2$  is equal to minus 0.1205 and  $\varepsilon_3$  is equal to 0.3318; due to the measurement errors, the output is different from the set of outputs we had obtained earlier.

Now, **the**  $a$  and  $b$  can be estimated in a manner we had done for the earlier case; and  $a$  and  $b$  are found to be  $a$  is equal to 1.2097 and  $b$  is equal to 1.8324; whereas, earlier we found  $a$  to be 0.95 and  $b$  to be 2. Therefore, in place of 0.95, we have got 1.2097 and in place of 2, we have obtained  $b$  as 1.8324. Therefore, the model parameter  $a$  is overestimated by 27.34 percentage and  $b$  is underestimated by 8.38 percentage. So, **there are** due to the measurement errors certainly there will be estimation errors, errors in the estimation of the model parameters.

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The slide contains handwritten mathematical derivations and notes. At the top, the equation  $y(n) = ay(n-1) + bu(n-1) + \varepsilon(n)$  is written with a checkmark to its left. Below it, the error term is derived as  $\varepsilon(n) = y(n) - ay(n-1) - bu(n-1)$ , which is circled. A horizontal line separates this from the next section, which is headed "Using the minimum square error criterion". Below this, the performance index is given as  $J(a,b) = \sum_{n=2}^N \varepsilon^2(n)$ , with checkmarks above the summation and the expression. A large oval contains the text "a and b can be estimated with certain accuracy.", and below it, the parameters "a, b" are circled. In the bottom left corner, there is a small NPTEL logo.


$$y(n) = ay(n-1) + bu(n-1) + \varepsilon(n)$$
$$\Rightarrow \varepsilon(n) = y(n) - ay(n-1) - bu(n-1)$$

Using the minimum square error criterion

$$J(a,b) = \sum_{n=2}^N \varepsilon^2(n)$$

a and b can be estimated with certain accuracy.

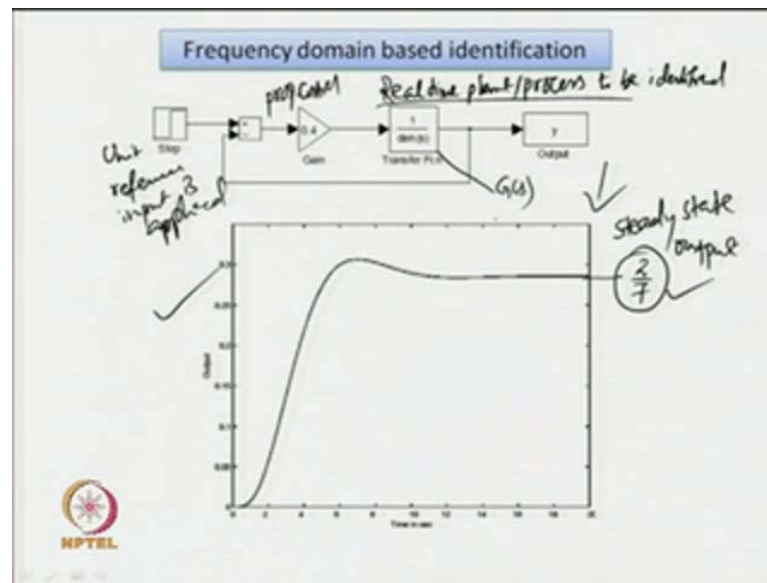
a, b



Now, how can we minimize that, from little bit of analysis we can write the expression for  $y_n$ , the difference equation can be re-written in the form of  $\varepsilon_n$  is equal to  $y_n$  minus  $a y_{(n-1)}$  minus  $b u_{(n-1)}$ . Then using the minimum square error criterion, it is possible to optimize the performance index  $J$ , which is defined as  $J$  is equal to sum of  $n$  from 2 to  $N$   $\varepsilon_n^2$ . Now, these optimization can  $a$  and  $b$  with certain accuracy; this is how one can get better values of  $a$  and  $b$  with less estimation errors.

Now, this is how one can obtain dynamic model for a real time process using time domain based technique. A real time system may be made of if not thousands hundreds of components; so, it might not be possible to apply basic laws of nature and derive its model in terms of differential equation or difference equation; so, an alternate approach discussed next that is known as frequency domain based identification.

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In the frequency domain based identification, we have the real time plant to be identified real time plant or process to be identified is put in a closed loop in this fashion. So, in the closed loop we have a fit forward proportional controller **proportional controller** with certain gain 0.4. And when some unit step reference input is applied at that time, suppose the output becomes like this (Refer Slide Time: 12:06), then from the output we can see in steady state, the output has assumed some value with exact value is 2 to the power 7; now, this is known as the steady state output **steady state output**.

So, when some unit reference input is applied, we get some steady state output of magnitude 2 upon 7. Now, what we get from this output, response of the closed loop system, when a proportional controller with gain 0.4 is put, one can extend some simple analysis; we know that, the closed loop transfer function can be written as  $T(s)$  is equal to  $0.4 G(s)$  upon  $1 + 0.4 G(s)$  (Refer Slide Time: 12:55); where the dynamics of the real time process is assumed to be  $G(s)$  in frequency domain.

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$$CLTF = T(s) = \frac{0.4 G(s)}{1 + 0.4 G(s)}$$
$$\lim_{s \rightarrow 0} T(s) = T(0) = \frac{0.4 G(0)}{1 + 0.4 G(0)} = \frac{2}{7}$$
$$\Rightarrow G(0) = \frac{T(0)}{0.4(1 - T(0))} = \frac{\frac{2}{7}}{0.4(1 - \frac{2}{7})}$$
$$= \frac{\frac{2}{7}}{\frac{2}{7}} = 1$$

$G(0)$  is the steady state gain of the unknown process

Then in that case, the steady state value limiting  $s$  tends to 0  $T(s)$  is equal to  $T(0)$  is nothing but,  $0.4 G(0)$  upon  $1 + 0.4 G(0)$ , this is found to be  $2$  upon  $7$ . So, after simplification, it is not difficult to obtain  $G(0)$  is equal to  $T(0)$  upon  $0.4(1 - T(0))$ ; so, which upon substitution gives  $2$  upon  $7$  by  $0.4(1 - 2/7)$ ; so, which is again equal to  $2$  upon  $7$  by  $2$  upon  $7$  is equal to  $1$ .

So, what is  $G(0)$ ,  $G(0)$  is nothing but the steady state gain of the real time process. So,  $G(0)$  is the steady state gain, state gain of the unknown process or of the process which is to be identified; that is this is how one is able to estimate unknown parameters of specific models, transfer function models of process dynamics.

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The image shows a handwritten transfer function  $G(s) = \frac{K}{(T_1 s + 1)^n}$  enclosed in a box with a checkmark to its right. Below the box, the parameters  $K$ ,  $T_1$ , and  $n$  are listed. A block diagram shows a circle containing  $K$  with an input arrow from the left and an output arrow pointing down to a circle containing  $1$ . The NPTEL logo is visible in the bottom left corner of the slide.

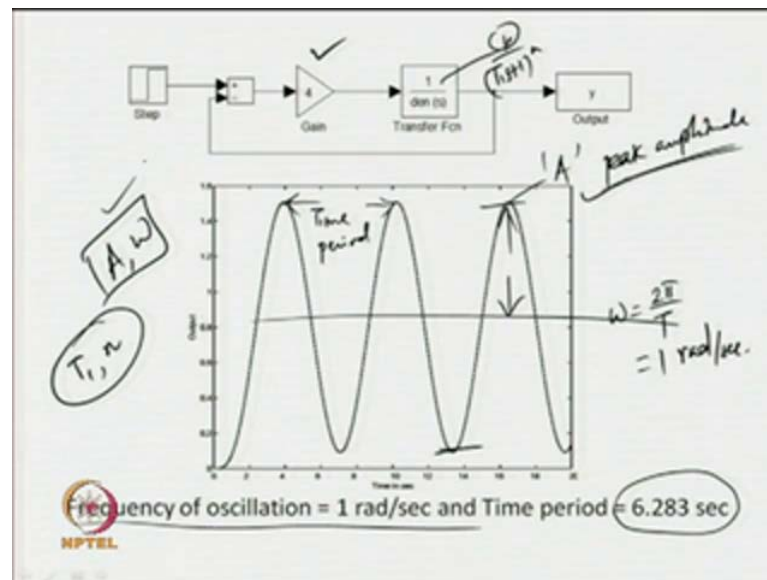
Now, one unknown has been estimated in this fashion, similarly this can be extended or we can go for some other technique to find the unknown parameters of model of transfer function model of a real time process. Let the real time process  $G(s)$  has the transfer function form  $K$  upon  $T_1 s + 1$  to the power  $n$ , before identification generally it is **assumed that** assumed that the transfer the process dynamics assume some specific transfer function form.

So, in this case we assume that the process has got some all pole transfer function form of the form defined as  $K$  upon  $T_1 s + 1$  to the power  $n$ . So, identification of the process dynamics means finding the values for  $K$ ,  $T_1$  and  $n$ .  $K$  has been found we have found  $K$  to be 1, from the steady state output of the closed loop system in some unit state reference is given; we found the closed loop, the steady state output to be 2 upon 7 and from analysis the steady state gain of the process model to be 1.

Then how to find the remaining two parameters  $T_1$  and  $n$ ; once,  $T_1$  and  $n$  are found then we can tell that the process dynamics has been obtained in the form of some transfer function, for that what we will do? We will go on increasing the gain of the proportional controller.



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So, when the gain of the proportional controller becomes 4, we get some typical output from the closed loop system, the output becomes some sustained oscillatory output, the output does not die down. So, we have pushed the system to the verge of instability, the closed loop system has gone to the verge of instability. Now, one can measure make simple measurement, one can measure the period of the output, this is known as or time period; also from the time period, it is not difficult to obtain the frequency of the sustained oscillatory output of the closed loop system.

So, the period of the output is found to be 6.283 seconds and the therefore, the frequency of oscillation becomes 1 radian per second. So, the frequency of oscillation  $\omega$  is nothing but  $2\pi$  upon  $T$ . So, from simple calculation it is not difficult to obtain of  $\omega$  as 1 radian per second.

Now, what benefit we get from oscillating or getting the output in this sustained oscillatory form; now, we can make some measurements like, one measurement is the time period, another is the peak amplitude we get for this signal. So, one can measure peak amplitude for this signal and from there, suppose this is the base line then we will get the peak amplitude in the form of some  $A$ , let  $A$  be the peak amplitude of the output, peak also can be obtained from the measurement of peak to peak value.

So, measure the peak to peak values and divide by 2 that will give us the peak amplitude  $A$  of the output signal. So, making two measurements  $A$  and  $\omega$  of the output signal,

it is possible to estimate the two unknowns of the model parameters, what are the two unknowns of the model parameters? We assume the dynamics of the process to be of the form of  $K$  upon  $T_1 s + 1$  to the power  $n$ ,  $K$  has already been found earlier; now, the remaining two unknown parameters  $T_1$  and  $n$  can be estimated using the information of peak amplitude and frequency of sustained oscillatory output signal.

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Let's assume a transfer function model  $G(s) = \frac{K}{(T_1 s + 1)^n}$  ✓

The steady state gain  $K$  can be obtained injecting a dc signal at the plant input or by  $G(0) = K = \frac{T(0)}{G_c(0)(1 - T(0))}$

$G(0) = \frac{K}{(T_1 \cdot 0 + 1)^n} = K$   $T(0)$  steady state response

The controller gain is  $G_c(s) = 4$

Plant output oscillates at critical frequency

$\omega = \omega_p = \omega_g = 1 \text{ rad/s}$

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Now, let us assume the transfer function model of the process dynamics to be of the form of  $G(s)$  is equal to  $K$  upon  $T_1 s + 1$  to be the power  $n$ . Now, the steady state gain we have already obtained, we need not repeat the discussion over here. We know that, the controller gain  $G_c(s)$  is equal to 4 and the plant output oscillates at critical frequency a frequency  $\omega$  which is nothing but,  $\omega_p$  is equal to  $\omega_g$  is equal to 1 radian per second; what is this  $\omega_p$  and  $G$ ?  $\omega_p$  is the phase cross over frequency and  $G$  is the gain crossover frequency.

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Plant output oscillates at critical frequency

Therefore,  $\angle G_c(j\omega)G(j\omega) = -n \tan^{-1}(\omega T_1) = -\pi \Rightarrow T_1 = \tan\left(\frac{\pi}{n}\right)$

Further,  $|G_c(j\omega)G(j\omega)| = 1 \Rightarrow (\omega T_1)^2 + 1 = \sqrt{4} \Rightarrow T_1 = 1$

Next,  $T_1 = \tan\left(\frac{\pi}{n}\right) \Rightarrow 1 = \tan\left(\frac{\pi}{n}\right) \Rightarrow n = 4$

Thus, the transfer function model for the plant dynamics becomes

$$G(s) = \frac{1}{(s+1)^4}$$

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Now, when we get sustained oscillatory output from the system? When the loop, closed loop gain is equal to 1 and loop phase is equal to minus pi at that time we get sustained oscillatory output from the closed loop system. So, using that, the loop gain is equal to 1 and loop phase is equal to minus pi using these two information, it is not difficult to write the expressions for loop phase given by angle  $G_c(j\omega)G(j\omega)$  is equal to minus  $n \tan^{-1} \omega T_1$ ; how we get this? Because, we know that  $G_c(s)G(s)$  is now given as  $4K$  upon  $T_1 s + 1$  to the power  $n$  again  $K$  is equal to 1.

Therefore,  $G_c(s)G(s)$  is equal to  $4$  upon  $T_1 s + 1$  to the power  $n$ ; therefore,  $G_c(j\omega)G(j\omega)$  will be equal to  $j\omega T_1 + 1$  to the power  $n$ . So, using this the angle, phase angle is found to be minus  $n \tan^{-1} \omega T_1$ ; and the loop phase is equal to minus pi, which gives us an expression  $T_1$  is equal to  $\tan \pi$  by  $n$ ; because,  $\omega$  is equal to 1, here  $T_1$  is equal to  $\tan \pi$  by  $n$  divided by  $\omega$ , but  $\omega$  is equal to 1. Therefore, we have written the expression for  $T_1$  as  $\tan \pi$  upon  $n$ .

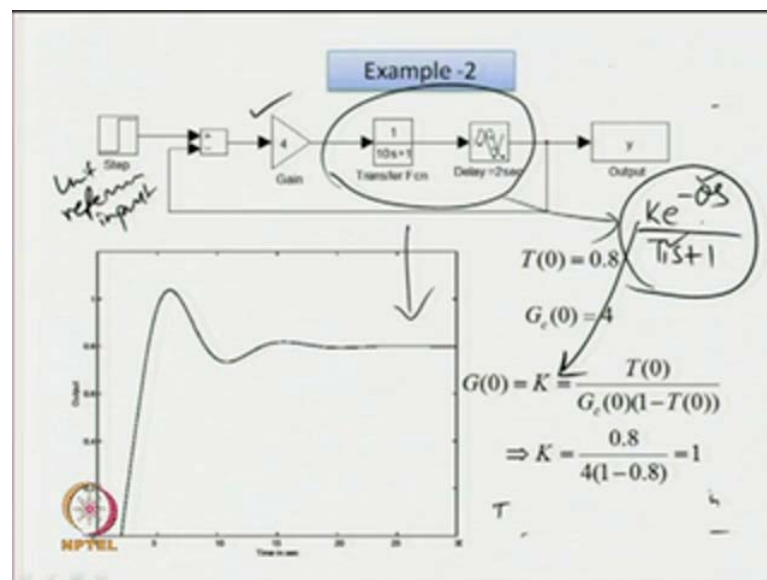
Further, the **gain** loop gain is equal to 1 using that condition, we get  $\omega T_1^2 + 1$  is equal to root 4, which gives us  $T_1$  as 1; how we find the loop gain, loop gain can easily be obtained, because your loop gain  $G_c(j\omega)G(j\omega)$  magnitude is equal to  $4$  upon  $\omega T_1^2 + 1$  to the power  $n$  by 2. So, using that one can easily find the expression and using this, again we get this magnitude to be is equal to 1, loop gain is

equal to 1. Therefore, simplifying this expression one gets  $\omega T \sqrt{1 + 1}$  equal to root of 4, what has been given over here.

So, from the analysis what we have obtained?  $T$  is found to be 1 and then making using this value in this expression gives us  $T$  is equal to 1, implies  $n$  is equal to 4. So, because  $\tan \pi/n$  is equal to 1 implies  $n$  is equal to 4. So, thus the unknowns of the process models,  $T$  and  $n$  are estimated. Then the transfer function model of the process dynamics, the real time process dynamics is given as  $G(s)$  is equal to  $1/(s+1)^4$  to the power 4; the general form of  $G(s)$  is  $K/(Ts+1)^n$ , and  $n$  is 4,  $T$  is 1,  $K$  is 1; therefore, the process model is found to be  $G(s)$  is equal to  $1/(s+1)^4$  to the power 4.

So, using the sustained oscillatory output from the closed loop system, it has been possible to estimate two unknowns of the transfer function model; and the two after finding the two unknowns and finding the steady state gain, the process dynamics can be expressed in the form of transfer function model of this form (Refer Slide Time: 24:12).

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Now, let us consider one more example. In the second example, we have considered a process,  $G(s)$  given as  $e^{-2s}/(10s+1)$  to the power minus 2 s upon 10 s plus 1. So, we have considered a first order plus dead time process. So, for this process the procedures for identification will be carried out.

Now, again we will go on **we will** we will not increase the proportional gain, let the proportional gain of the proportional controller be 4 with this gain, when the unit step reference input is applied, then the process output is found to be of this form (Refer Slide Time: 25:20). So, from the process output, the steady state value of the output is found to be 0.8; therefore,  $T(0)$  is equal to 0.8, but we know that  $G_c(0)$  is equal to 4,  $G_c(s)$  is equal to 4 this is the proportional controller. So, in spite in steady state value of this one is also  $G_c(0)$  is equal to 4; so, limiting  $s$  tends to 0, we get  $G_c(0)$  is equal to 4.

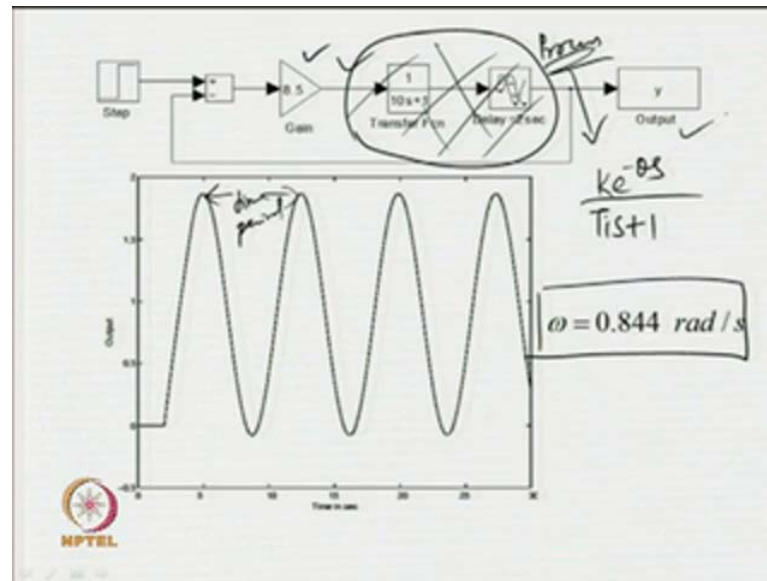
Again, using the relationship that the closed loop transfer function  $T(s)$  is given as  $G_c(s)G(s)$  upon  $1 + G_c(s)G(s)$ ; and when we take the limiting value  $T(0)$  becomes  $G_c(0)G(0)$  upon  $1 + G_c(0)G(0)$ . So, solving this we get the expression for  $G(0)$ , which gives us  $G(0)$  is equal to the transfer function model of the plant is having a steady state gain of  $K$ , then the steady state gain of the transfer function this  $G(0)$  is equal to  $K$ .

So, when  $s$  equal to 0, this term will not be there, then when  $s$  is equal to 0, this is also not there ultimately  $G(0)$  becomes  $K$ . Then the  $G(0)$  can be estimated from the equation  $T(0)K$  is equal to  $T(0)$  upon  $G(0)$  times  $1 - T(0)$  as 1. So, the steady state gain again the steady state gain **steady state gain** of the process dynamics or transfer function model of the process dynamics is equal to 1.

So, once the steady state gain has been found, again we will try to find the remaining parameters of the transfer function model. How we are going to model the dynamics of this process, let us assume some specified transfer function form. Let the model of this process dynamics be represented by  $K e^{-\theta s}$  upon  $T_1 s + 1$ . So,  $K$  has been already estimated, now the remaining two parameters,  $\theta$  and  $T_1$  can be estimated from the information from sustained oscillatory output of the closed loop system.

To obtain **to obtain** sustain oscillatory output, what we have to do? We have to go on increasing the proportional controller gain. So,  $K$  the gain proportional controller is gain  $K$  be 4 will be increased; so, we will try with various values.

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So, by read then trial, when we have the proportional gain of a value of 8.5, then we obtain a sustained oscillatory output for the closed loop system. When the proportional gain or proportional controller gain **gain** is equal to 8.5, then the closed loop system output becomes oscillatory a sustain oscillatory output. So, that is what we want, because these enable us to estimate two unknown parameters of a process model.

Now, again making or making measurement of the time period, from the time period, we find the frequency of the sustained oscillatory output signal to be of value 0.844 radian per second. Therefore, the signal has got frequency of omega is equal to 0.844 radian per second.

For other things we have now, we assume that we do not know the process, the process dynamics is not known to us; we are going to estimate a working transfer function model for the dynamics of the process. So, we assume this process to be of the form  $K e^{-\theta s} / (T s + 1)$ ; please keep in mind that, we do not know the dynamics, we do not know the model parameters of this process.

So, this is a black box for the sake of estimation accurate to demonstrate, how the identification is done we have taken some specified process dynamics. But, we assume that the process dynamics is not known to us, rather we are going on measuring the output; and from the output of the closed loop system, we are trying to estimate the transfer function model of a unknown process dynamics.

Now, we have found the frequency to be of 0.844 radian per second; next, we can measure the peak amplitude of this signal from measuring the peak to peak values and that values can be used later on.

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Let's assume a transfer function model  $G(s) = \frac{Ke^{-\theta s}}{(T_1 s + 1)}$  ✓

The steady state gain  $K = 1$  ✓

and the controller gain is  $G_c(s) = 4$  ✓

Plant output oscillates at critical frequency  $\omega = 0.844 \text{ rad/s}$

Further,  $|G_c(j\omega)G(j\omega)| = 1$  ✓

$\Rightarrow (\omega T_1)^2 + 1 = 8.5^2 \Rightarrow T_1 = 10$

Next,  $\angle G_c(j\omega)G(j\omega) = -\omega\theta - \tan^{-1}(\omega T_1) = -\pi$

$\Rightarrow \theta = (\pi - \tan^{-1} 8.44) / 0.844 = 2$

The transfer function model for the plant dynamics becomes

$G(s) = \frac{e^{-2s}}{(10s + 1)}$

Handwritten notes in circles:  $\theta = 2 \text{ sec}$ ,  $T_1 = 10 \text{ sec}$

NPTel logo

Now, again like the earlier case, let us assume a transfer function model for the dynamics of the process, time delayed process  $G(s)$  is equal to  $K e^{-\theta s} / (T_1 s + 1)$ , we have already estimated the steady state gain of the process. Therefore,  $G(s)$  can be given as now  $e^{-\theta s} / (T_1 s + 1)$ . And we know,  $G_c(s)$  is equal to 4, now again making use of the concept of loop gain and loop phase; loop gain of 1 and loop phase of minus pi, we get magnitude of  $G_c(j\omega)G(j\omega)$  is equal to 1; after simplification, which gives us  $\omega T_1^2 + 1 = 8.5^2$  square which gives us  $T_1 = 10$ ; and using the loop phase condition angle of  $G_c(j\omega)G(j\omega)$  is equal to minus  $\omega\theta - \tan^{-1} \omega T_1 = -\pi$ .

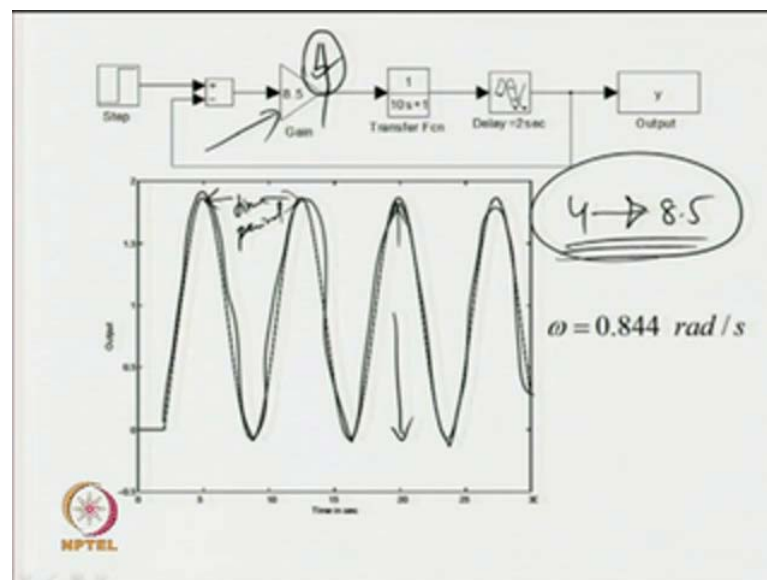
So, using this using  $G_c(j\omega)G(j\omega)$ , which is given as in our case 8.5 then  $e^{-\theta s} / (T_1 s + 1)$ ; so, this will give a phase angle of minus  $\omega\theta - \tan^{-1} \omega T_1$ , which has to be of the value minus pi. So, solving this again we obtain the theta as 2. So, what we have tried to do in this case, we have estimated the two unknowns of the process models, theta and  $T_1$ ; now, theta is equal to 2 seconds, and  $T_1$  is equal to 10 seconds.



So, we have accurately estimated the dynamics of a process. So, the dynamics of the process as you see over here is nothing but,  $e^{-2s} / (10s + 1)$ . So, this is the process we had employed for identification; and what we have identified? We have also identified the transfer function model of the process dynamics to be of the form  $G(s)$  is equal to  $e^{-2s} / (10s + 1)$ .

So, the frequency domain based identification can be used to estimate transfer function models of process dynamics in a convenient way. And one can estimate more accurate values for the parameters of the transfer function models by using this frequency domain identification technique.

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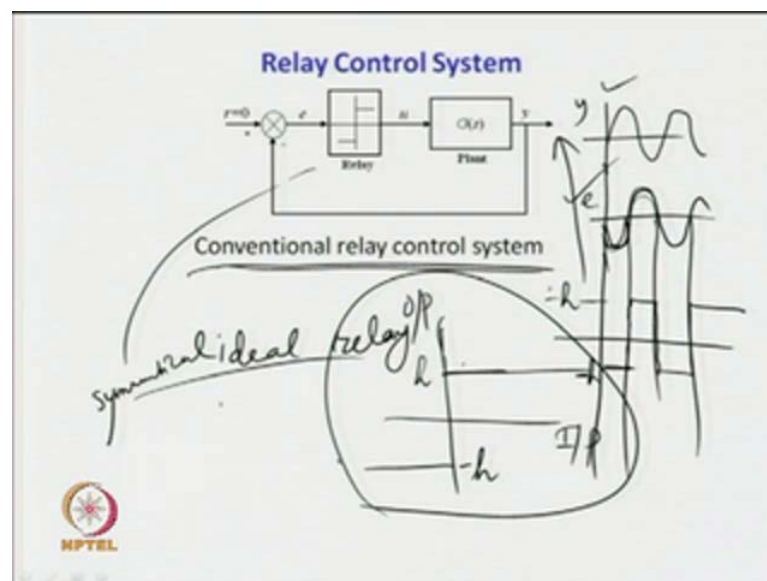
Now, why we are getting sustained oscillatory output for the closed loop system? Let us go back and analyze the closed loop system a little bit, now when the loop gain is equal to 1 **when the loop gain is 1**, and loop phase is equal to minus pi; then whatever output I get over here, this output will come here and due to this 180 degree phase shift due to the negative feedback, that output will get inverted, then same output is going over here, **and due to** due to this we get another 180 degree phase shift by the loop.

So, loop phase shift, this controller and plant is giving us a phase shift of minus pi. So, due to that, this again will get inverted; that is why we are getting some sustained oscillatory output from the system, when the loop gain is equal to 1, and loop phase is equal to minus pi.



Now, there is one risk with this technique, this frequency domain based identification method, **when the gain** proportional gain is become more than 8.5, what happens to the closed loop system? We may get unstable output from the closed loop system. So, there is some risk. So, one has to go on trying various values for the proportional controller. So, we started with a value of 4, from 4 we came to 8.5; so, 4 to 8.5 one has to go on gradually increasing the proportional gain. This is the drawback of this method of frequency domain based identification technique; to overcome this, there is one more powerful method that is known as a relay control system.

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What a relay control system? The gain or proportional gain has been substituted by a relay, earlier we had some proportional controller here. Now, the gain of the proportional controller where changes gradually till some sustained oscillatory output is obtained from the closed loop system, but there some risk as we have discussed earlier; if the gain of the proportional controller is more than some then some specified value in that case, the output may blow out.

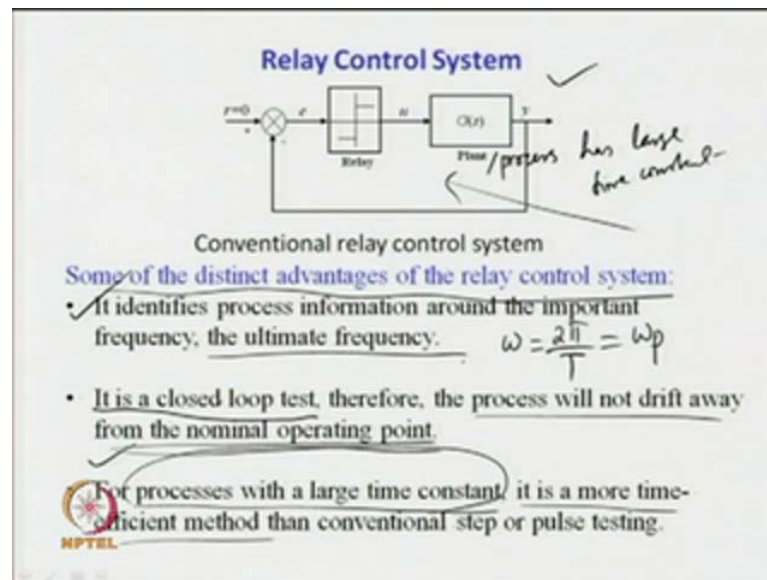
So, when proportional controller is replaced by a relay a symmetrical relay, then it is found that the output of the plant becomes oscillatory we get some limit cycle output from this arrangement. So, we get some limit cycle output also which is known as the sustained oscillatory output. So, this is the beauty of the scheme, a relay control system ensures are guarantees are induces limit cycle output for a closed loop system.

So, when a relay is put in the forward path, then the output of the closed loop system becomes oscillatory. So, in a conventional relay system an ideal relay, rather than some symmetrical ideal relay is put; and the symmetrical ideal relay is shown by the diagram of this form (Refer Slide Time: 39:27), where this is the input to the relay and this is the output from the relay. So, the relay characteristic can be shown by this plot.

Now, for any input, the output will be less to some fixed value; suppose, the relay setting is made at some positive value of  $h$  and negative value of minus  $h$ , then what type of output will expect from the relay? The input is sustained oscillatory, because output of this scheme is sustained oscillatory corresponding to that, the input  $e$  will be negative of that, **sorry it is not to the scale**; so, this will be the input and corresponding to this what type of relay output will have? Will get for negative inputs, the output of the relay will be less to some negative value which is minus  $h$  and for positive input to the relay, when  $e$  is positive, this is  $e$ , this is  $y$ .

So, this is when the output is sustained oscillatory, the form of the  $e$ ,  $e$  can be given in this form, then the output of the relay **the output of the relay** will assume some rectangular form, where the peak amplitude of the rectangular pulses will be minus  $h$ , and the negative peak amplitude of the rectangular pulses will be minus  $h$  and the positive peak amplitude of the rectangular pulses will be plus  $h$ . So, this is how we will get signals at different points of a conventional relay control system. So, we shall study the input-output Characteristics and input-output signal of a relay control system in detail later on.

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Let us see, what we have now some advantages of the relay control system are: it identifies process information around the input and frequency that is known as the ultimate frequency. The frequency at which we get the sustained oscillatory output signal is known as the ultimate frequency. The frequency of the sustained oscillatory output signal is known as the ultimate frequency.

So, omega is the frequency which can be obtained from the measurement of time period of sustained oscillatory output and which can be given as omega is equal to  $2\pi$  by  $T$ . Now, we says that, we identifies the process information around the important frequency means **this is the** not only this is the ultimate frequency, this is also the phase crossover frequency of the relay, closed loop control system.

Now, a beauty of this is says that, there is no need for worrying the level or amplitudes of the relay or the relay settings. So, the relay heights can be of any value, relay heights means what we mean by relay heights, a relay is represented by this and the positive amplitude is shown as  $h$  negative as minus  $h$ ; these are the relay settings  $h$  minus  $h$  are known as the relay settings.

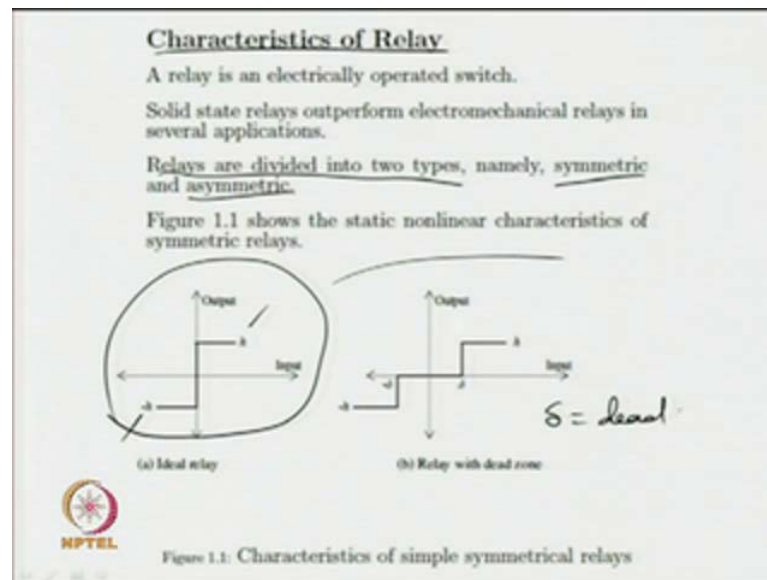
Now, this setting  $h$  can be of any value, we can vary go on changing  $h$  to any value till we get some appreciable sustained oscillatory output in the output of the closed loop system, which can be measured easily. So, the setting of  $h$  there is no condition for setting of  $h$  values. So, one can try from 0.01 to some higher values like 1 and so on.

Unlike, the earlier cases where one has to be very careful, when one increases the proportional gains here, the relay heights can be increased gradually without fearing for anything, because the output will definitely remain sustained oscillatory in spite of any setting of relay heights. So, this is what we get from the first point. What other advantages we have from the relay control system? It is a closed loop test; as you see, relay is put in closed loop, when the loop is broken, when the feedback is broken at that time we cannot ensure sustained oscillatory output. Therefore, there must be negative feedback for the relay control systems.

So, we have a closed loop test system which is known as relay control system for us. Therefore, the process will not drift away from the nominal operating point. Since, the process or plant always operates in closed loop and we are inducing limit cycle or oscillatory output in closed loop; therefore, the process will not drift away from the nominal operating or normal operating conditions.

Hard point is that, for processes with large time constant, it is a more time efficient method; when the process unknown process has got large time constant in that case, setting of proportional gains in place of relays will be very difficult; whereas, when we have a relay in place of proportional controllers in the closed loop, when we have got relay control systems we will have no difficulties in obtaining sustained oscillatory output in spite of large time constant in the process. So, it is an efficient method compared to the earlier techniques.

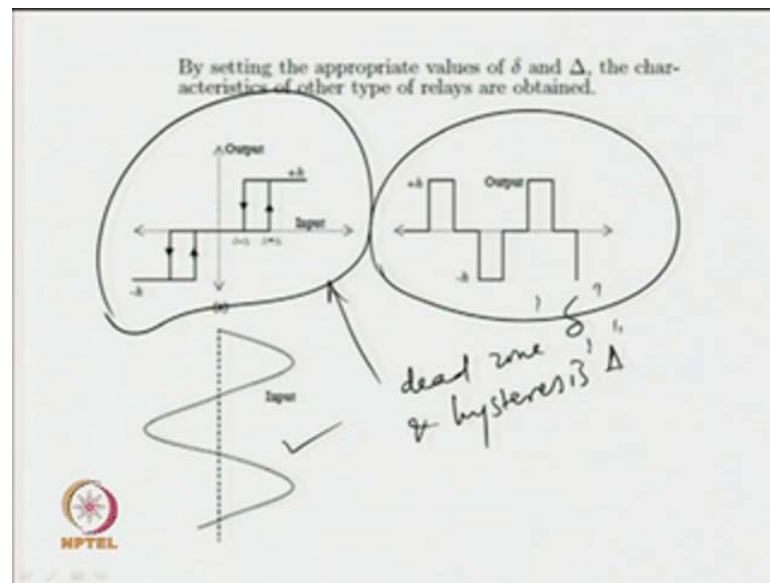
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Now, let us see the characteristics of a relay; what is a relay? A relay is an electrically operated switch. So, relay can be made up of solids state devices and can be of electro mechanical type as well. Relays are basically divided into two types, symmetric and asymmetric. What we mean by a symmetrical relay? The input-output Characteristics of a symmetric relay is shown over here (Refer Slide Time: 46:39); when the relay settings are equal, when we have got a positive plus  $h$  and minus  $h$  setting, then the relay is symmetric and ideal.

Now, what do you mean by asymmetric relay? When the relay amplitude will be relay heights will be different; so, we have got like this  $h_1$  and minus  $h_2$ , then we get an asymmetric relay.

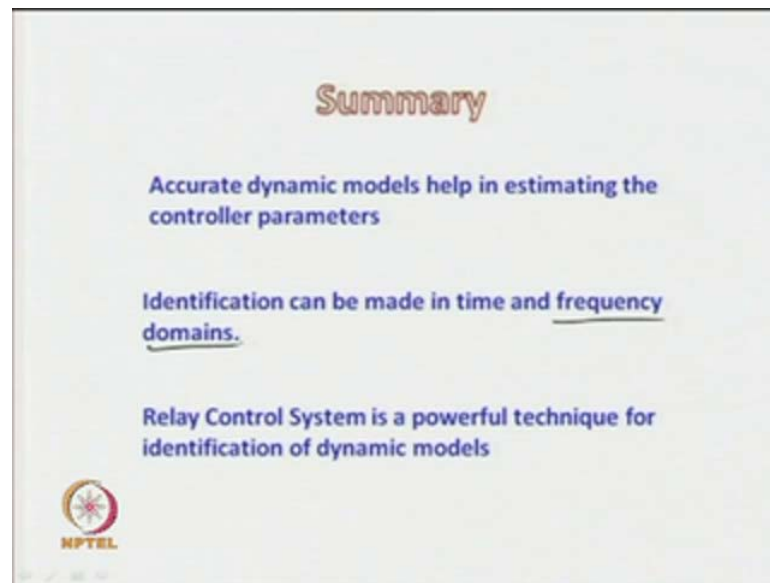
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Now, a relay with dead zone can be shown like this, where  $\delta$  is equal to the dead zone. Again we have got relay with dead zone and hysteresis, the input-output characteristics of a relay with dead zone and hysteresis can be shown like this. So, the relay has got dead zone and hysteresis. When the input to the relay is like this (Refer Slide Time: 47:50), the output will be of this form, when both dead zone and hysteresis in the relay are present.

Now, one can have asymmetrical relay of this form also. So, these are the characteristics of input-output characteristics of asymmetrical relays. And now, we have got to the outputs of asymmetric relays as shown over here (Refer Slide Time: 48:17).

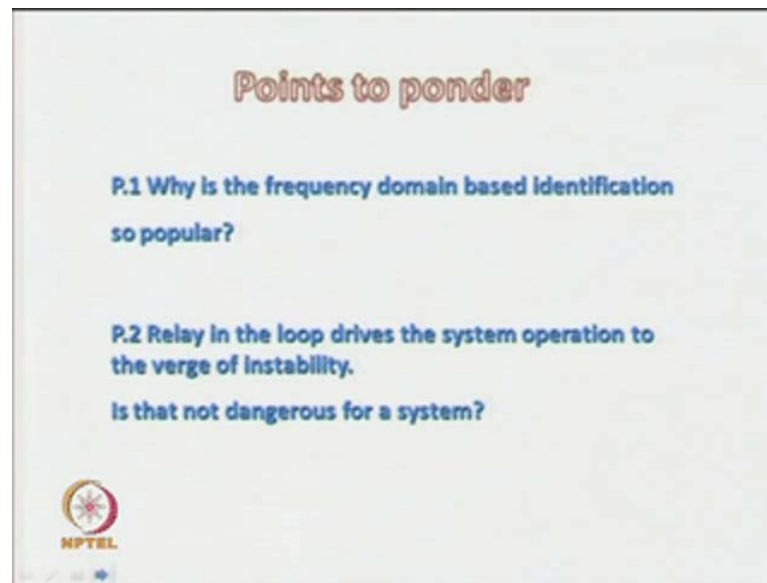
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So in summary, accurate dynamic models help in estimating the controller parameters; therefore, effort should be made to estimate the parameters of transfer function models or time domain models as accurately as possible. Now, identification can be done in time and frequency domain; whereas, the frequency domain based identification techniques are found to be more convenient and useful.

Relay control system is a powerful technique for identification of dynamic models of real time processes, because there is no worry for changing or getting suitable values for the proportional gains, proportional controller gains.

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Coming to the points to ponder one may ask, why the frequency domain based identification is so popular? **Frequency** we have got time and frequency domain based identification techniques, it is possible to estimate parameters of simple process model transfer functions using frequency domain based identification; also, it is a closed loop test that is the advantages we get from frequency domain based identification techniques.

The second point is, relay in the loop drives the system operation to the verge of instability, is it not dangerous for a system? No, relay ensures limit cycle output sustained oscillatory output, unlike the earlier case, where when proportional gain is more than certain value, then in that case the output may blow out; this is the not case with relay control system, relay induces sustained oscillatory output, thank you that is all in this lecture.