Transducers For Instrumentation

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Lecture - 08

Thermal Sensors: Electrical vs Thermal Networks

Welcome to the course transducers for instrumentation. We are discussing thermal sensors and last lectures we are we discussed about electrical versus thermal parameters. So there is a correlation between the electrical parameters as well as the thermal parameters and we discussed that for example we have certain parameters in electrical systems like voltage, current, voltage difference. So the same type of parameters are exist in thermal parameters as well. For example the temperature difference, power dissipation, thermal resistance etc. So we are going to discuss today about the electrical versus thermal parameters. So we will discuss electrical versus thermal. So in electrical system we have like voltage difference. Similarly in the thermal system we have temperature difference. In electrical system we have current.

In thermal system we have the dissipated power. Similarly we have electrical resistance. And in thermal we have the thermal resistance. So there is a corollary between all these electrical parameters and thermal parameters and we have multiple laws in electrical system to solve electrical network. Similarly the same type of laws are applicable to thermal network as well and we can solve the thermal network using those laws.

We just need to replace voltage difference by temperature difference and electrical resistance by the thermal resistance. So similarly we can solve the thermal networks. For example we have Ohm's law in electrical system. We have a resistance. We apply a potential difference across this.

It is a positive and negative and there is a current flow in this resistor which is I. Similarly if we think about the thermal network we have a thermal resistance which is our TH the thermal resistance and we apply a temperature difference across this resistance which is delta T then the power dissipation will be there which is p dot. So this is very similar to electrical Ohm's law and we can write delta T is equal to pd into RTH the thermal resistance. For electrical network we can write V is equal to iR. So both of these examples are similar and we can apply the electrical laws to the thermal network as well.

So let's talk about electrical and thermal resistances. We have a material here. The thickness of this material is D and we apply a potential difference here across it which is V and the resistance of this is given by R equal to D upon A into I. Similarly we have a material in thermal networks and we apply a delta T across this and the power dissipation. So flowing through this material is pd and we can write RTH is equal to T upon A into lambda th where this psi is electrical conductivity and here the lambda is thermal conductivity.

Here R which is our normal electrical resistance and here RTH is thermal resistance. So this is the electrical versus thermal resistances. Both behave in a similar way except instead of voltage difference we apply the temperature difference and instead of the current flow which has heat flow flowing through the material. Similarly we have electrical and thermal capacitances which are analogous to each other. In electrical system we have electrical capacitance which is generally denoted by C and if we and if we apply a voltage across this capacitor then there is a charge Q which is stored in this capacitor.

Similarly in thermal networks we have C thermal which is the thermal capacitance and we apply a delta T across this element and there will be pdT stored within this thermal capacitance and we can write C is Q upon V or we can write I into T upon V. In thermal we can write the thermal is equal to E upon delta T or pdT upon delta T where C is the capacitance where the unit is farads. For the thermal capacitance the unit is joules per degree centigrade. So these are the difference in the units but both behave almost similar to each other. Using these elements for example thermal networks can comprise of thermal resistance and thermal capacitance very similar to electrical networks which can have electrical resistance and electrical capacitances.

We can make networks and solve using similar types of electrical laws. Now we discuss some electrical RC networks and see how we solve those electrical networks and we apply similar knowledge to solve the thermal RC networks. So let us move to electrical RC networks. So when we discuss electrical RC networks the very first topic of discussion is the RC delay because whenever we have an element which is resistance and it is connected to a capacitor there exists a delay which is RC delay or the time constant is RC and this delay is inherent to the network and let us discuss this in terms of some examples. We have a system and we are sending a pulse through this system.

So we are sending a train of pulses through this system which has R and C components joined together and at the near end or where we are transmitting these pulses we are sending this very perfect kind of pulses. You can see that rising and falling edge are very steep and it is almost a square kind of pulses. When we send this kind of signal at near end, so this is our near end pulses, we are sending it very perfect signals but when we receive these signals at the far end they don't exist like this when we receive at the far end they look something like having some exponents. For example, they will look like this. These are the waveforms at far end.

So we can see at near end we are sending very perfect signals but at the receiving end the signal is distorted. This happens because of the capacitive effect. Because charging is responsible for this kind of effect. So we can say that every node in the system, in an electrical system, every node has a certain capacitance with respect to ground and when we apply a transient signal this node need to be charged or discharged based on the signal applied. This charging of each and every node it takes time because we have finite amount of current which is provided by the input and this node need to be charged up to a certain value to respond to input signal.

So this charging and discharging takes time and when we apply a fast transient the system does not respond immediately to the input. So this is why we have this far end signal which is very much distorted compared to the input signal which we apply. Now let's say we discuss the RC delay of the system. We have a input node where we have a resistance connected to it which is R and there is a node which is output node. This is Vin and we have this capacitance which is connected to ground.

This is the simple RC delay model circuit and we need to analyze the behavior of this RC delay of this circuit. So when we apply a input at the Vin and we are assuming two cases when the output is zero and when the output is high when we have Vout we are plotting this Vout with respect to time and we are assuming that initially V at time t e qual to zero Vout is the voltage at the output node which is a low voltage. Now we apply a high input at Vin and my circuit respond to this. My output voltage rises like this and it saturates to certain value which is Vin. This is the case when Vin is higher than Vout at t equal to zero. The other case is when the output voltage is already high and we are bringing it down by applying a low input at the input voltage. This is our Vout at t equal to zero. This is the value of Vin. In both of these graphs we can see the output voltage Vout tries to achieve the value of Vin whatever it is. If it is higher than Vout it increases and if it is lower than Vout it actually decreases. And the charging is exponential. You see there is not a sudden kind of charging. There is an exponential behavior of Vout with respect to time which we can see. And theoretically Vout approaches Vin in infinite time because it is an exponential curve. So it never actually reaches the final value but it achieves a certain value within some reasonable time. So now we can write our KVL and KCL for this network. This is input which is applied. This is R. This is V. This node is Vout. And this is Vin. This is R. This is Vout. Now we can write KCL which is off current law at node out. The current into out from the left is Vin minus Vout divided by R. Which is this current. And we can write the similar expression for the capacitor. Which is C dV by dt. And because both of these currents are equal we can equate them. So we write Vin minus Vout divided by R equal to E dVout by dt. Or we can write dVout by dVout divided by dt is equal to one upon Rc Vin minus Vout.

So this is the equation we come up with and if we solve it the solution of this equation is Vout which is a function of time is equal to Vin plus Vout at t equal to zero. Minus Vin into e to the power minus t upon Rc. This is the solution of this equation and we can see now the output voltage here which is a function of time this is a exponentially rising equation where e is to the power of minus t upon Rc. So we can further rearrange it. Vout as a function of t is equal to Vin.

Vin minus e to the power minus t upon tau plus Vout at t equal to zero e to the power minus t upon tau. So in this equation we can see two things one is I have replaced Rc with a constant tau because R and C are circuit parameters which we are not going to change. So R and C combinedly we start writing with tau and which has a very important function in these equations and we call it the time constant. This Rc has a unit of time we can see t upon tau. So both t and tau have the same unit.

So this tau or Rc we call it time constant. The other thing we need to note is our Vout is a function of Vin plus Vout at t equal to zero. It means the initial charge which is already stored on the capacitor that is going to dictate my starting condition. So how much time it takes to reach to certain value that not only depends on the input but also depends on what is the previous state of the capacitor which is Vout at t equal to zero. Now we can again plot these charging and discharging graphs. This is for charging. So in charging case the capacitor is initially charged to Vout at t equal to zero and then we apply a Vin which is let's say here and the output voltage rises exponentially something like this and in one time constant which is tau. In tau time the output actually achieves the 63 percent of the total pull speed. We can write this value is 0.63 times Vin plus 0.37 times Vout at t equal to zero. In one time period if we assume that the capacitor was initially charged was not charged this does not have any charge then Vout is zero it means one time period the output will rise to 63 percent of the final value which is here. We can do this math on this equation and we can find out the output voltage will be 63 percent of the input voltage. This is for the charging case. For the discharging case when we have Vin is lesser than Vout. The initial charge of the capacitor is Vout at t equal to zero.

We apply a Vin which is of lower value and the capacitor discharges from this Vout to Vin exponentially. And it achieves the final value in infinite time because it's a exponential graph. Similarly just like the charging case here also we have a time constant which is tau and in one tau or one time period the discharging we can write is 0.63 times Vin plus 0.37 times Vout at t equal to zero. So both charging and discharging case we can say if the initial charge on the capacitor is zero then output reaches 63 percent of the input voltage in one time period. So 63 percent of the transition completes in one time period. So let's take an example for this. We have a RC network. And the input we apply a pulse input which is Vin plus minus. The value of R is one kilo ohm and the value of C is one kilo farad. We are assuming that Vin is zero for a very long time. It means the capacitor in this network is fully discharged. We can say the Vin is zero for a long time. Or we can say the capacitance is fully discharged.

Now what happens? This is the pulse we apply at input which is Vin and this is with respect to time. The output will respond to this input and will start rising. This is the output charging. And from the previous discussion we know that the output charges to 63 percent approximately in one time period. How much is the time period that we can calculate from here? The tau or R into C which is the time period. The time constant is equal to one kilo ohm multiplied by one picofarad which comes out one nanosecond. It means that the output charging which is this black curve in one nano-second it will reach 63 percent of the input value. And the input value we are applying here is 10 volt. This is a step of 10 volt. So that this value at one time period is 0.63 multiplied by 10 which is 6.3 volt. This value will be achieved in one time period. So this is how the RC charging time constant actually works. This charging and discharging of the internal nodes actually distorts the signal because as we saw in the very first slide we have a very perfect input applied at the input. But when the signal reaches at the far end the signal actually distorted that happening because of the charging and discharging of internal nodes. So this phenomena actually gives rise to phenomena something called pulse distortion. If we apply the pulses to this electrical network these pulses will be distorted by when they reaches at the far end. So let us discuss what is a pulse distortion.

So first I apply an input which is Vin here. This is the input I apply and the output will rise. This is the input and the output rises like this which is exponential curve. This is my output. However the output will reach the input in infinite time theoretically because this is exponential curve. If we wait long enough for practical purposes we can say the output reaches input in few time constants. But if my input frequency is high for example I have the same input but now instead of remaining high for a long time it comes back. This is my input then my output will be something like this and instead of continue reaching the steady state value immediately it will start following the input which is now discharging. So we can see there is a difference between the input and output. So there is a difference between these two values. Let us name it Vin and this is time. So there exists a voltage difference and that is because the output did not have enough time to reach a steady state value because the input is already being pulled down to zero. So the output will start following the input and there exists difference in the final value and output never achieves actually the final value. It remains lesser than that. Another thing happens if we increase the input again it is like a pulse then the output again will start increasing and again will start decreasing at this point.

So we can see the output is actually not rail to rail. The ground or the top supply the output is always in between the value is not reaching the final value or the steady state value because of the high frequency of the input pulses. If input is low frequency then the output will have enough time just in first case. If the input is low frequency it means the output will comfortably reach the steady state value and it will reach the highest value which is let us say VDD. But for high frequency signals the output will not reach higher or lower value completely and this gives rise to the difference in the voltage between input and output. Now let us discuss three cases where we can have pulse distortion and how this pulse distortion is affecting the shape of the pulses with respect to the frequency. So we have pulse distortion. This is input pulse positive and negative. This is R. This is C and this is ground. This is output node. And here we see three different cases. This is V out with respect to time. This is pulse width which is 0.1 times RC. The pulse width or the pulse duration is 10 percent of the RC time constant.

The second case we take for pulse width of equal to RC. Here we have pulse width equal to 1 time RC or equal to 1 time constant. And the third case we take where the pulse width is long enough. The pulse width here is let us say 10 times RC. So now we have three cases where the input frequency is very high. It means the pulse width is very small or equal to 0.1 RC. One frequency is moderate which is pulse width of equal to RC and one frequency is very low frequency where the pulse width of the signal is 10 times RC. So let us plot the inputs for a pulse width of 0.1 RC will be a very short pulse. This is the input pulse. For pulse width of 1 RC, the pulse width will be bit longer compared to the first one. And this is the input pulse. And for 10 RC, pulse width will be very long. So in case of pulse width equal to 10 RC, let us consider the low frequency case which is easy to understand. The output has enough time to charge and discharge. It can reach let us say high value and again to the low value.

So we can see the pulse is very good shape. This is the output. We can see the pulse is good. In case of pulse width equal to 1 RC, the frequency is moderate. It is not very low but not very high as well. Now we see the charging of this output, that output node, the characteristic remains same. It will take 63 percent, it will rise to 63 percent of this full value. This will be like 63 percent. And by reaching this, it will take time T which is equal to the pulse width. When it reaches 63 percent, the input will be again coming back to ground. That means this will again come down here and it will start discharging. Now we can see the output pulse here is not as good as in the case of pulse width equal to 10 RC.

The pulse here is very good but here the pulse has not even reached the top value, already reaches to 63 percent of its value. And then again goes down. Now the third case which is pulse width equal to 0.1 RC which is a very high frequency which we are applying to this RC network. The pulse width is very small and the network, the output node, it still take 0.1 time period to reach 63 percent and our time period, the pulse width is not even one time constant. So it just start increasing and by the time it reaches this point, the input again make a transition to ground and the output again follows this. So the output node, the output voltage will be like this and we can see for a input of let us say this top value is 5 volt across all the pulses, the output does not even rise to typically 1 volt or so. So for a pulse width of 10 RC, the output reaches comfortably to higher value. Pulse width equal to RC does not reach to maximum value, it only reaches to 63 percent.

But for a very high pulse, for a very high frequency pulse width equal to 0.1 RC, output does not even reaches to midpoint of the voltage which is not good because this small change in the output will not be detected comfortably with the next stage. So this is the effect of frequency. This is the effect of frequency on the circuit performance. The output node does not even rises to its higher value and that is happening not because of the circuit parameters but because of the input frequency which is high enough and circuit is not able to respond to that input signal. This is happening because of the inherent time delay which the time is taken by the internal nodes to charge and discharge.

We have a RC network where R is 2.5 kilo ohm and C is 1 nano Farad. If voltage pulse is applied. The voltage pulse has a width of 5 microsecond and a height of 4 volt. It is applied at the input and we need to sketch the output of the circuit. Now this is the input where we apply a voltage which is initial value is 0 and the height is 4 volt and the time period is the width of this pulse is 5 microsecond.

This time is 5 microseconds. This pulse is applied at the input. We are assuming that the capacitor initially is discharged. It does not have any charge initially with respect to ground and we need to plot the output. So this can be solved using our RC charging equation. So the first the output voltage will increase to approach the 4 volt because when the input is transitioning from 0 to 4 volt the output will try to reach this 4 volt and the charging is governed by that RC time constant and when after 5 microsecond when the input is transitioning back again to 0 the output will follow this the input and again goes back to ground.

We can plot the output versus input. It will look like this. We have this V out with respect to time and the input is 0 at 0 and the pulse is applied at T equal to 0 which is at 4 volt and it goes down to 0 at 5 microsecond. So this is 5 microsecond and this is 4 volt. Now the input will rise, the output will rise. It reaches to its maximum value whatever is there at 5 microsecond and again goes down following the input and the characteristic of this V out we can find out using the RC charging equation. For charging we can write V out is a function of time which is 4 minus 4 e to the power minus T upon tau which is 2.5 microsecond tau is R into C which comes out 2.5 microsecond minus T upon 2.5 microsecond. This is for the charging where the time is less than equal to 5 microsecond and for the discharging case the initial value on the capacitor will be this value. It's not the 0 because the capacitor is initially charged now when it is starting going down the initial value is not 0 but this value and we can write the output is 3.44 e to power minus T minus 5 microsecond upon 2.5 microsecond. This is for T greater than 5 microsecond. So this is the equation of charging and this is the equation of discharging when the output is following the input. So this is how we calculate the RC delays of electrical networks. Next we will see some examples for the thermal networks and we apply the similar kind of concept for the thermal networks where we have thermal resistances and thermal capacitances instead of the electrical resistance and electrical capacitance which we just had here. So next we will discuss the thermal networks.

That's all for today.

Thank you.