Transducers For Instrumentation
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Lecture - 4
Errors in Measurement and Instrumentation, Propagation of Errors.

Hello, welcome to the course Transducer for Instrumentation. In today's lecture, we will discuss about the errors in measurement and the propagation of error. Errors or I would like to call it uncertainty because with word error most of the students confuse it about the mistake or the blunder they have performed in some measurement which is not. The error is actually an inevitable part of a measurement you cannot actually remove the error completely, you can only suppress it to certain extent by using some very advanced instruments or taking very good care, but it is not possible to entirely eliminate the error. So, error is not something which a student or a scientist is performing during the measurement it is a part of the system it depends on the system characteristics as well. So, the error does not mean a mistake or blunder, error is in a scientific measurement it means an inevitable uncertainty it is a kind of randomness in a measurement that exists in all the measurements no matter what measurements we are performing it is a mechanical measurement or it is a electrical measurement or any other optical or any other measurement.

The error will be inevitable it cannot be removed. So, error cannot be eliminated by being very careful you can be 100% careful during all your measurements, but still the certain amount of error will be there in all the measurements it can be less or it can be big depending upon multiple factors which we are going to discuss, but it cannot be made 0. Some sources of error are intrinsic to the process for example, we use a multimeter or some voltmeter to measure certain parameter that voltmeter itself has some problem inside. So, some errors can be depending on some intrinsic parameters of the instrument and cannot be removed entirely. However, some errors are there which you can remove and all the errors can only be minimized so that they are very less or negligible compared to the measured quantities. So, for example, we measure for example, we have a 1 meter rod and we want to measure its length using a inch tape. So, depending upon the quality of our tape the measurement can be 1 meter plus few millimeter or 1 meter minus few millimeter. So, this few millimeter extra is it comparable to the original length of the rod so that actually dictates whether we can leave out this error or we need to improve on our system. So, error can be only be minimized so that they are very less or negligible compared to the base value. For example, these are the following general classifications of error for consumer purposes we tolerate around 5 to 10 percent of errors in measurements. For example, we have a fridge in our kitchen and the fridge temperature is let us say 30 degree inside. So, plus minus 5 or 10 percent error in the temperature reading is acceptable for consumer goods. So, we are not very much interested in knowing whether it is 30 degree or 30.1 degree whether it is 29 or 31 it is acceptable for consumer purposes.

However, for engineering purposes for example, we are building a dam now 1 percent is acceptable we cannot accept 5 or 10 percent error in measurements while we are making it a dam or a bridge. So, the amount of error acceptable is lesser in engineering purposes because the criticality of measurements is more. The third is for scientific purposes when we have a scientific experiment going on in the lab and we are interested in very much precise measurements. So, that time we cannot tolerate even 1 percent of the error in our measurement that time we go typically for 0.1 percent error in our scientific measurement.

Though these numbers are not very fixed that depends upon the application for example, some engineering applications can be so demanding that you need to have 0.1 percent error and some scientific measurements can be accepting like 1 percent of error. So, but in a nutshell these are the some numbers what we can accept in a measurement. So, the classification of error so we can classify those errors what generally occurs in all the measurements. We can have one is the gross error, the second is the systematic error and the third one is random or residual or accidental error.So, these are the three types of errors we can classify and further these systematic errors we can further classify into instrumental error and then environmental and observational. So, the systematic error we can further divide into three parts instrumental, environmental and observational. Let us discuss each of them in detail. The gross error is caused by human mistakes mainly when let us say you have some measurement to be performed and the human being who is measuring this experiment or the measurement he performs certain error which is not the fault of the instrument or any other equipment is purely the human way of measuring if that has an error that is called the gross error. Let us take the example of this mass which is hanging on this screen on this spring. So, we have a spring here and we have this mass which is hanging on this spring depending upon the mass there will be an extension in the spring and we want to measure how much this end goes down because of the mass. So, this end goes down and this is the stable state this is the state. We want to measure how much this spring is elongated using this tape measuring tape we have here a measuring tape. Now if the observer or the person who is measuring this is right perpendicular to this axis you see this there is axis y axis if the observer is exactly perpendicular to this to this axis then the measurement is accurate which is around 12. So, this is the exact measurement, but let us say the observer is at a angle to this axis let us say case 1 where the observer is a little bit higher what he will see typically this arrow which is connected to this spring this arrow is not at 12, but it is slightly below this 12.

So, this is 12.3 if the observer is at a certain height compared to this needle. In case 2 let us say the observer is below this level then observer will see this arrow is not right at 12, but it is actually above than 12. So, in this case the reading will be typically 11.8 or something. So, here the instrument is same in instrument is not moving anywhere there is no error in the instrument itself, but depending upon the location of observer the readings are different where 12 is the exact reading which is the best case here this is the best case where the reading is accurate and these are the two cases where observer is not at 90 degree then the reading is different that is purely depending upon how a person is measuring this experiment. However, there is a certain gap here if we see closely this needle which is connected to this spring and there is a gap between this needle and this inch tape this gap actually dictates how much the error will be if the observer is performing an error. So, if this distance is more than the error will be higher if the distance is less the error will be lesser. So, instruments are designed in such a way that these kinds of error be minimized, but they are not exactly 0 there will be some error in the measurement due to this gap here between the pointer and the scale there will be an error generated in the measurement. This gross error is a human error in reading, recording and calculating measurement results this is purely based on the human errors. The experimenter or the observer may grossly misread the scale. So, this also happens in the laboratory for example, the reading of the thermometer is 21.5 degree, but while noting down the temperature reading accidentally the observer writes 31.5 instead of 21.5 is just this digit is mistakenly taken which is not correct and because of that there is a huge error from 21 to 31 it is almost 50 percent of the error.

So, we call it the gross error the magnitude of this is not controlled by the instrument. Another kind of errors can be like they may transpose the reading while recording like reading 25.8 degree centigrade and record as 28.5. So, exact reading is 25.8, but when they are actually recording in computer or in the notebook these digits are misplaced. The record at the recording time the observer records 28.5 instead of 25.8. So, these are the errors made by human beings and these are called the gross errors. These gross errors these are caused by human mistakes in reading the instruments or using the instruments depending upon what kind of instrument it is this error can be higher or lower. If these are mechanical systems where because of the bad condition of instruments these errors can be more if the instrument is designed such a way that the mistake should be minimal then these errors are minimized. These errors may also occur due to incorrect adjustment of the instrument. For example, in the last slide we saw that the needle was connected to the spring, but that needle was very far from the tape. So, in that case the error will be higher if we bring this needle very close to the tape no matter then how where the observer is that error can be minimized. So, this can also occur due to incorrect adjustment of the instrument and of course the computational mistakes are always there 25.8 can be written at 28.5. So, that is always there well a human is recording the values. These errors cannot be treated mathematically because the origin of these errors are not mathematical in nature while doing a mistake a mistake can be of any form and we cannot actually treat these errors mathematically.

They are only can be treated by repeated measurements should repeat the experiments multiple times. So, probability of having an error of the of the same magnitude at the same point will be lesser. So, we perform multiple measurements and take the average values of that. So, these cannot be eliminated, but can be minimized by repetitive measurements. These happen because of the improper use of an instrument and this error can be minimized by taking proper care in reading and recording measurements. Though during experiments most of the people pay attention and do proper care in reading and recording, but these errors can can occur and which leads to the gross error. In general indicating instruments change ambient conditions to some extent when connected to a complete circuit. All several readings at least 3 we say, but the more is good we perform the same experiments multiple times and take the readings and we average them out. So, let us say in one experiment a human error is there that will show up there if all the readings are at one place and suddenly one reading only is misplaced or having higher deviation from the other readings. So, that will show up and that can be corrected using these multiple measurements typically 3, but more than 3 is good and these must be taken to minimize the effect of ambient condition changes. The other type of error is systematic error. Systematic error is due to the shortcomings of the instruments. So, instrument may have problems inside. So, because of that there is certain error generation in the measurement that is called systematic error. These shortcomings can be like defective or worn parts, aging effects or the effect of environment on the instrument.

For example, we are measuring a voltage using a voltmeter. This voltmeter it has multiple circuits inside those circuits age after a certain period 1 or 2 year the performance of that circuit actually degrades and because of that there can be an error generation by this voltmeter in measuring the voltage. So, this can be one example. Another is the voltmeter let us say the voltmeter is analog which actually the needle is moving on the scale. These moving parts have wear and tear over the time. So, if you use continuously this voltmeter of 1 year, 2 year, 5 year after certain time there will be wear and tear in the system and it will start showing us some drift with the actual voltage. So, these are some systematic errors which happen because of the instruments because the instruments has some defects inside and these systematic errors are of three types the instrumental error, the environmental error and the observational error. These are the systematic errors. So, the first is instrumental error. These errors are inherent to the instrument while measuring instrument because of their mechanical structure or electrical structure whatever that instrument is. These are inherent to that instrument. We took the example of voltmeter it has internal circuit which degrades with time and it shows a drift from the actual voltage. So, this is inherent to this error which is generated by this voltmeter is inherent to that instrument to this voltmeter. So, this is a instrumental type of error. Another error can be because of the friction in the bearings of various moving parts, irregular spring tensions stretching of springs etcetera.

There can be multiple sources of the systematic error or this instrumental error and this type of error can be avoided by selecting a suitable instrument for the particular measurement application. For example, we want to measure precisely we may either switch to the new voltmeter or we service this voltmeter find out what are all the errors and we correct this voltmeter for those errors and we then use this particular measurement system. We can apply correction factor that is also one way for example, the voltmeter is costly and not easy to repair. Then these systematic errors we know how much error is being produced. For example, because of the aging of this voltmeter 1 volt actual 1 volt of signal is measured at 1.1 volt by this voltmeter. So, we always know that actual voltage is 1 volt and voltmeter is giving 1.1 volt it means 0.1 is the error which is generated by this voltmeter and this 0.1 volt of error can be subtracted from all the subsequent measurements. So, this is one way of putting a correction factor by determining this instrumental error. The third option is to calibrate the instrument. Calibration of a instrument is when we have a known set of input and we generate the output by the instrument and we manually tweak this measurement system or this voltmeter to adjust the output to the desired output. So, we know the input output is known and we tweak the faulty output to the correct output by doing this external kind of adjustment this is called the calibration of the system. The second is environmental error due to external conditions affecting the measurement including surrounding area conditions such as change in temperature, humidity, barometer, pressure etcetera. So, temperature is a very very temperature is a surrounding parameter which impacts almost all the circuits.

If the temperature rises the performance of electronic circuits generally goes bad. So, every 10 degree change in the temperature every 10 degree rise in the temperature typically change that current of devices by a certain amount. This amount is practically known by our measurements. So, if the temperature is changing in the surrounding this causes a systematic error in the measurement of instrument. If there is a significant change in the temperature then our error will be more and this error need to be corrected using look up table or some other approach. To avoid this kind of error we use air conditioner with wherever it is possible. We want to maintain the external temperature at the same point no matter what outside temperature is we want to keep all the temperature in the laboratory typically to room temperature which is 27 degree. So, this can be used by using air conditioner. Sealing certain component in the instrument this is one way of protecting these internal circuitries from interference from temperature change from humidity and so many other environmental factors. We kind of put epoxy around these components and put inside the instrument so that these parameters does not impact the performance of these circuits

We sometime use magnetic shields magnetic shield is used to cut down the interference because of the RF signals. If we have a mobile phone or a some radiating antennas nearby these antennas generate EM waves which interfere with the electronic circuits. So, we put a magnetic shield to prevent this interference and to reduce the error. So, this is second types of error which is environmental error. The third is observational error. So, this is introduced by the observer the human being who is taking this measurement and this is most common is parallax error which we discussed earlier that is spring has a needle connected and which is moving on this tape. So, that is kind of parallax error and estimation error there is another kind of error while reading the scale. So, while reading a human being can make the mistake these are most common type of observational error. For example, an observer who tend to hold his head too far to the left while reading the position of the needle on the scale. So, this was a observational error. So, these are the third type of systematic error. The next type of error is random error. As the name says these are random in nature. So, due to these are due to unknown causes we do not know exactly what cause this kind of errors, but these errors exist in the measurements.

These occur when all systematic errors has accounted. So, for example, we put a temperature sensor outside the room and the outside the environmental temperature is let us say 30 degree C, but when we actually measure this the output of this sensor this never comes 30.0 it always comes with some error. There is a we see the signal generated by the sensor this is always fluctuating a little bit, because there are certain unknown factors which are affecting the performance of the sensor. Because of that there is a error generated in the measurement of temperature. So, these are called random errors and the source is unknown. There can be more than one sources for example, humidity for example, the radiation from the sun that also impacts the performance. The other thing can be like surroundings, the shielding there are so many factors which can contribute to cause this random error. And this is generally accumulation of small effects. There are multiple parameters which are affecting this error and this random error is generally the accumulation of all those effects required at high degree of accuracy.

This can be avoided by increasing number of readings. We increase the number of readings and then we average it out. So, that is one way of reducing this type of error or we use statistical means to obtain best approximation of true value. So, we perform certain standard deviation or the Gaussian distribution kind of analysis on the data to get the desired output which is estimating the error how much the error is possible, but it is not possible to completely remove that. All these errors we will discuss in detail. So, the difference between these random and systematic error is these random errors results from random effects in the measurement. The magnitude and sign of a random error changes from measurement to measurement. So, this random error can be positive for one measurement and can be negative for another measurement. The magnitude as well as sign can also change and measurements cannot be corrected for random errors. A measurement if it has random errors these measurements cannot be corrected for these type of errors. However, a systematic error results from systematic effects which we know what kind of parameters can impact my accuracy and the magnitude and sign of systematic error is constant from measurement to measurement.

For example, we measure a length and the error is plus 2 for example, 100 centimeter plus 2 centimeter this plus 2 centimeter will always be in the positive direction. For example, this will always be 100 plus 1 centimeter and 100 plus 3 centimeter it will never be 100 minus 1 centimeter the systematic error. The magnitude and the sign of this error will be constant and these kind of errors can be corrected because we know how much the error is generated by the systematic sources. So, we know these values and this whole of the measurement can be corrected for the systematic errors. So, as we discussed last time that we discussed accuracy and precision where we see this graph we have probability density function here and on the x axis we have the values. Our true value is here which is we are calling the reference value. Let us say this is the value of actual parameter we are measuring let us say the temperature the temperature is exactly at 30 degree. So, this is the true value or the reference value, but when we measure it using our sensor this 30 degree is not a 30 degree, but it is somewhere like 32 degree or 35 degree. So, there is a difference in the measurement between the true value and the actual value we get from the sensor. This difference we called accuracy which is 35 minus 30 this is the error and when we measure this temperature multiple times this follows some sort of a Gaussian kind of distribution. Sometime when we measure it the error comes in 35.1 degree the next measurement it may come 34.9 which is left side of this bar again 35.2 sometime on the right side or on the left 34.8 degree. So, if we perform multiple measurements this is going to follow a Gaussian kind of distribution and this is called the precision.

The precision means the difference in the multiple readings should be smaller. So, the narrower this graph is the more precise the instrument is this is what we discussed last time. So, now we define accuracy and precision in terms of error. So, this is the case that what we discussed where we have precise and accurate measurement this is precise and accurate where the measurement is precise right at the true value and accurate as well here. If we talk about in the random and systematic errors the random error is also small and the systematic error is also small in this case.If we talk about this case B where we have precise, but not accurate precise, but not accurate means the all the values what we measure they are very tightly bound no measurement is overshooting or undershooting compared to the rest of the values. So, they are all in a very tight space and that we call the random error is small. So, here we see this random errors which are generated in the instrument these are small it means these are precise the instrument is precise when the random error is small. However this is not accurate because the true value is right at the center and all the values are different than this it means there is a systematic error existing which is large here. So, this error is systematic error which is large and this is corresponding to not accurate. So, the accuracy you can say is caused by systematic error and precision is controlled by the random error. Other two cases are like random error is large and systematic error is small it means this is not precise, but accurate and this fourth case where this random error is also large and the systematic error is also large it means this is neither accurate and nor precise this is not accurate and not precise. So, these random and systematic errors actually define whether the instrument is accurate and precise or not. So, now let us have a look about the Gaussian distribution of these errors. When we perform these multiple measurements on a single instrument all the outputs generated by this instrument they form a sort of Gaussian distribution means none of the output will be very far from the true value.

Probability of the outputs will fall in and around to the true value and there will be some outputs which can be little bit far and that depending upon the distance from the true value their probability of finding at that place will be lesser. For example, we have this Gaussian distribution here which is the true value here which is let us say 1 and we are performing these experiments let us say 100 times then most of our measurements for example, here we take case of 68 percent these 68 percent cases will be falling within this range plus minus 1. So, if we are taking total 100 measurements 68 experiments will show a value which are closer to this between this and this only some 100 minus 68 will be this side or this side will be away from this region. So, the Gaussian distribution is when we have Gaussian type of distribution of these error or the uncertainty we start defining the error as x plus minus sigma where the x is the base value and the sigma is the variance where we call sigma as variance and between this plus minus 1 sigma let us say we have x plus minus 1 sigma in this range this is sigma this is x, x plus minus 1 sigma in this range 68 percent of the values will be falling. If we take sigma x plus 2 sigma the range increases from here to here and we cover more values for this range. So, we expect the true but unknown value of x to be an interval given by x minus sigma and x plus sigma. So, majority of the outputs will be between x minus sigma and x plus sigma between this range our 68 percent of the values will be falling if the error follows a Gaussian distribution. So, for example we have a line here which we have drawn let us say here and I ask you what is typically the length of this line let us say in centimeter. If I ask 100 students to tell me the length of this line most of them will say let us say 6 centimeter but most of the students say 6 centimeter less number of students will say it is like 5 centimeter or 5 to 4 centimeter or 6 to 7 centimeter. Very less students will say that this is a 1 meter long but there can be 1 or 2 outliers who will say it look like a 1 meter long. So, this is kind of a output we are generating where we have these errors which are in the measurement. So, the area value is let us say 6 centimeter but we have variance in the measurement. So, the area under the normal curve between this line and this line is this much 0.68 and depending upon the number of the number n we always write mu plus minus 1 sigma means this is the value mu and this is sigma between plus minus 1 sigma we have 67 percent or probability of exceeding mu plus minus n sigma is 0.5. If we have mu plus minus 1 sigma around 32 percent only will be the outliers 68 percent will be all within this curve. If I extend this n to 2 it means I am talking about mu plus minus 2 sigma. If I increase my range of acceptance then my error is then my range is from mu plus minus 2 sigma and I cover this whole of the graph and only 5 percent is the outliers 5 percent outliers. If I take n equal to 3 then only 3 percent outliers will be there in my measurement. So, if I keep on increasing my sigma values then I am covering more and more of outliers into my measurement.

So, if the variance or the sigma magnitude is less then mu plus 2 sigma will also be small and in 2 sigma I am covering almost all the measurements except 5 percent only 5 percent will be outlier to this and 95 percent will be covered in the measurement. So, it is very important to have a variance which is less. It is very unlikely that a measurement taken at random from Gaussian pdf will be more than plus minus 3 sigma because it is just the number is only 0.3 percent. So, most of the measurements will be covered if we consider mu plus minus 3 sigma range only 0.3 percent will be the outliers. Then we calculate the most probable value of these in the measurements. So, most probable value is it is also known as the arithmetic mean or the average value which is nothing but the summation of all the values divided by the number of values and the MPV is the sum of all the measurements divided by total number of measurements. Standard deviation sigma also known as standard error or the variance which is sigma square equal to sigma M minus MPW MPV 2 square divided by N minus 1 where M minus MPV is referred to as the residual M is the actual value minus MPV is the most probable value which we calculated last slide. The sigma is computed by taking the square root of this above equation.

So, this is sigma square we take sigma as under root of this. So, this is our variance. Let us take an example of this kind of measurement. So, we have a distance which is measured repeatedly in the field and we got these 4 values one is 31.459 meter, 31.458 meter, 31.460 meter and 31.457 meters. Now we want to compute the most probable value of the measurement and the standard error for this data. So, this is how we calculate these are all those 4 values 1, 2, 3, 4. So, first we calculate the MPV or the M bar which is the sum of all these value divided by the number of readings. So, we have this reading 1 plus 2 plus 3 plus 4 the sum is 125.834 divided by 4 is equal to this is the MPV value 31.459. Then we calculate M minus M bar which is the individual residual this reading 31.459 minus MPV which is 0 this minus MPV this third reading minus MPV and the fourth reading minus MPV and then we take the square here in this graph in this column. Then the standard error or the variance we can calculate using this M minus M bar whole square divided by N minus 1 which comes out to be 0.001 meter. So, the most probable value of the measurement is 31.459 and the value that is most likely to occur when you perform multiple measurements on the same field this is the value which most probably will come as a output. This value represents the peak value on the normal distribution curve. So, what distribution curve we saw there this is the peak value here of all the measurement let us say our 4 measurements over somewhere here and this is the peak value which is 31.459 this is the peak value. The standard error is 0.001 so it means 68% of the values would be expected to lie between this MPV which is 31.459 plus minus 0.001 this is the variance or let us say this is 31.459 plus 0.001 and this is this value is 31.459 minus 0.001 between these values 68% of our mean measurements would fall. If we measure the same field 100 times 68 measurements will fall in this range of length these values are computed using the MPV and the standard error. So, this is how we measure the errors. Now the next important point is the propagation of error when we do these measurements we definitely come up with some error in all the measurements.

Then these errors actually propagate to the final output for example, we have some intermediate steps of measuring for example, we have voltage measurement how do we do a voltage measurement we do a current measurement and a resistance measurement I and R and then we multiply this I and R to calculate V the Ohms law V equal to IR. So, measurement of I has its own error because this is a separate measurement and measurement of R is also a separate measurement so it also has its own error. So I has an error R has its own error these both errors will actually contribute to the error in voltage which is I into R. So, these errors generated by us at the first place they actually propagate throughout the system. So, the propagation of error is if we have X and Y which are independent random errors delta X and delta Y so X is given something like X plus minus delta X where X is the base value and delta X is error and Y is also given Y plus minus delta Y where delta Y is the error. Then the error in addition of these two parameters if we have third parameter Z which is addition of X and Y X plus Y then the overall error will be given by del Z equal to under root X del X square plus del Y square means the square root of under the square root variance in X square plus square of variance in Y that will be the total variance in Z. This is the one rule for means propagation of error. The second is the same if we have X is given as X plus delta X Y plus delta Y and now the output Z is like X into Y X is multiplied by Y this is exactly same as V equal to IR the Ohms law. We have error in I we have error in R then how much will be the error in V. So, the formula for calculation of this type of error is del Z upon Z where del Z is the error in final output this Z is the base value of final output equal to under root del X upon X to whole square plus del Y upon Y to whole square.

This is the formula if we have two variables which are in multiplication earlier they are here they were in addition here they are in multiplication and one more important thing is it is X plus Y and X minus Y also calls same error X plus Y and X minus Y both are same for error calculation. The third rule is if Z is some function of X then del Z or the variance of Z is derivative of FX which is F dash X into variation of variance of X which is del X. So, this is third rule and the fourth rule is if we have more than one parameter which can cause error for example, there is an error in current measurement and there is a error in the resistance measurement then how the error will be propagated. This is given by this formula sigma Q square with Q is the function here Q is a function of X and Y where this X and Y can be correlated or they can be uncorrelated. So, this is the generalized formula where we have variance of X square into partial derivative of Q with respect to X square plus sigma Y square into partial derivative of Q with respect to Y square plus 2 sigma XY dQ by dx into dQ by dy. Most of the time during these measurements these two parameters which are causing error they are most of the time uncorrelated. Uncorrelated means if one parameter is causing some error this generation of error does not impact the error generated by the second parameter. So, they are uncorrelated. So, in this uncorrelated case this sigma XY becomes 0 or this whole term goes away. So, we have the overall formula for uncorrelated cases this. We will see some examples here for this. For example, we have to find the volume of a certain cube you measure its side as this. So, the side is given which is let us say s of this cube side is s which is given 2 which is the base value of the side 0.02 which is the error in the measurement or the variance.

We need to find the error in the volume. So, volume is given by s cube s into s into s. So, that we can apply this rule number 3 the various variance in V will be equal to the derivative of s3. So, del V will be d by ds of s cube into delta s. So, this is what we calculate 3s square into delta s which comes out 0.24. So, this will be the variance of volume or the error caused in the volume if we get an error of 0.02 in the side measurement the total 0.24 will be the error in the volume measurement because volume is not directly measured volume is calculated from the side measurement. So, the actual volume will be 8 plus minus 0.2 we can neglect this 4 here as well. Another example is suppose we want to measure 3 numbers as follows x is 200 plus minus 2 this is the error in x or this is we can call it delta x this is delta y which is error in y and this is del z which is error in z.

Now, the expression for the output Q is x upon y minus z. So, this is the function x has its own error y has its own error z has its own error. Now, we want to calculate how much overall error will be there in Q. So, let us say d equal to y minus z this denominator we write separately y minus z and in the addition and subtraction the error is delta x square plus delta y square and the root of that. So, this comes out to be 10 plus minus 2 under root 2 this root 2 is because of that under root del x square plus del y square the rule number 1. So, the error in the denominator is 10 plus minus 3 which is del d I can call it then the overall error in Q will be x upon d and now we have this in x upon y. So, this is the formula for calculating the error and it comes out to be 20 plus minus 6. So, this is the overall error in Q which is 6 if we have individual errors as 2, 2 and 2. Another example is let us say we want to calculate a power in a electric circuit. So, power is given as P equal to I square R and I and R has its own individual errors I is 1 plus minus 0.1 which is let us say del I can call it which is the error in current measurement and resistance also has its own error 10 plus minus 1 ohm. So, this 1 ohm is the error in the resistance measurement. Now the base value of power will be 10 which is I square R the base value is I of I is 1, 1 square into R the base value of R is 10 so this is 10. So, the base value of power is 10 watts. Now we want to calculate the error produced in power if we have individual errors. So assuming I and R are uncorrelated means the error in I does not produce any extra error in R and while measuring R error in R does not produce any extra error in I.

In that case that sigma xy terms goes away which is 0. Now the formula is sigma P square the variance of power equal to sigma I square into del P by del I to whole square plus sigma R square del P by del R to whole square at R equal to 10. The partial derivative of P with respect to I which comes out to I R plus sigma R square into I square the whole square. We can now put the values in this formula comes out to be 5 watt square which is sigma square. So, the sigma or the variance of the error in power is under root 5 typically 2.238 will come so we neglect all these decimal values and the final power will be 10 plus minus 2 watt. So, the base value of power is 10 with a 2 watt of error or inaccuracy. So it means when we say power value we take 10 plus minus 2 watt all of our 100 measurements 68 percent will be in this range 10 plus minus 2 watt. If we perform 100 measurements 68 will be between 10 plus 2 which is 12 and 10 minus 2 which is 8. So, in this range if we perform 100 measurements on this instrument 68 times this instrument will show the number between 8 and 10 because the N we are talking here is 1. So, this is the some example of the propagation of error this is what we have for today.

Thank you.