Transducers For Instrumentation
Prof. Ankur Gupta
Centre for Applied Research in Electronics (CARE)
Indian Institute of Technology, Delhi
Lecture - 3
Dynamic Characteristics of Transducer

Hello, welcome to Transducers for Instrumentation. Today we will discuss the Dynamic Characteristics of Sensor, what are the parameters which we use for characterization which are dynamic in nature unlike the static characteristics which are fixed with time. For example, the accuracy of a measurement which is a static characteristics whatever the error give sensor gives today if we measure tomorrow the error will be the same. So, it does not change with time. However, some characteristics are there of a sensor which is changing with time the the performance of a sensor for example, the output it the how how much is the output response with respect to time. So, that is the dynamic characteristic of a sensor. So, the dynamic response it involves the determination of transfer function of the complete system, the frequency response how the system is going to respond to a certain frequency the impulse response if we apply a sharp impulse at the input if we apply step response at the input and then evaluation of time dependent outputs. So, if we apply these functions which are time dependent functions if we apply this function at the input what will be the output response of a sensor. So, these are the dynamic characteristics. For determining these dynamic characteristics different inputs are given to the sensor and the response characteristics are studied how the sensor is responding to these inputs.

With step input the specification in terms of the time constant of the sensor are generally made and generally the sensor is a single time constant device it has a particular time constant and accordingly it produce the output. Though this topic is largely covered in control system we will just look at some basic characteristics which are dynamic in nature. So, we discuss a second order system which is a very common type of system which we consider in control system which we have in this example we have a mechanical system where we have this mechanical block with mass m and this is connected to this is spring with a spring constant k and also connected to a damper where the damping coefficient is b and this block is here on a friction less support where it is free to move this direction and the other side is connected to a fixed wall. Now if we write the force equation of on this on this block minus F b minus F k this is because of the damper and the spring respectively equal to minus b d x by d t minus a k x equal to the force on this block which is m into a m into d square x over d t square. So, this is the total force which is acting on this mechanical block and if we just rearrange this equation the equation comes out to be m d square x over d t square plus b d x by d t plus k x into equal to 0.

Now we can see this is a second order equation which is in with respect to time d square x by d t square this x is the displacement what we produce and this displacement is with respect to time. So, this is a second order differential equation. So, this is how we make a model or a equation of this mechanical block. Similarly, if we consider a electrical system the same kind of equations we can make. So, we have here a capacitor which is connected in parallel to a inductor which is in series with a resistor. So, this in this has R L and C and on the left we have the source connected to it. Now voltage across the resistance will be I into R I is the current flowing through this resistors into R. So, V R equal to I into R voltage across the inductor this is V L which is L di by d t where L is the inductance and I L is the current flowing through this inductor which is with respect to time. Current through a capacitor is given by C d V C by d t where C is the capacitance of this capacitor and V C is the voltage across the capacitor build up and divide by the d t. Again, we can write the equation across it and d d square I L by d t square plus R by L d L d I L by d t plus 1 upon L C I L equal to 1 upon L C I S this you can easily write using Kirchhoff current law. And again, if you see here this is a second order system. So, here if we see if we have a mechanical system that also we can model and that is a second order system and if we have electrical system in hand that also we can model and apart from the parameters where we have R and L in electrical system and in mechanical system, we have M and K etcetera. Apart from these constant parameters the equations are same. So, we can apply the same theory to both kind of systems. Before solving these equations actually, we use Laplace transform to solve this kind of equations.

Why do we use it? Because the Laplace transform converts a calculus problem for example, these linear differential equations to an algebra problem. So, instead of solving the differentiation and integration we apply Laplace transform and by doing that this becomes a algebra equation where we can just add multiply divide and subtract this these formulas we can apply. So, how to use this Laplace transform? What we do? We take the Laplace transform of this differential equation after taking this Laplace transform it becomes a algebraic problem. So, we solve this algebra problem and after solving we again take inverse Laplace transform convert it back to into the time domain. So, one example here is given we take Laplace transform of this time domain function and convert it into if into a F s which is in Laplace domain and again, we after solving we take inverse Laplace of this F s convert it back to the time domain. So, this is how we apply the Laplace transform to these equations and the standard form of this second order system when we apply the Laplace transform is this equation which is very general equation G s equal to omega n square divided by s square plus 2 zeta omega n s plus omega n square where zeta is the damping ratio and omega n is the undamped natural frequency of the system. This conversion into Laplace domain and other these calculations are basically covered in control system which we are not going to discuss completely here, but we start from this point where we convert it into Laplace domain and then we start solving the the the question. So, for example, we have this kind of system where we have gain in the system and we have a transfer function in Laplace domain. So, suppose loop transfer function we can write the output divide by the input which is the transfer function and we can write like this and this is this comes out to be second order system. This second order system have transfer function of the following form B upon s square plus As plus B we can use this or we can use the old one which is omega n square upon s square plus 2 zeta omega n s plus omega n square.

This also we can use and this transfer function may be multiplied by a constant which can be here as well this which affects the exact constant of time domain signal, but not its form. Now, we have this equation which is second order equation and we call it the characteristic equation. This equation is called characteristic equation and based upon the roots of this quadratic equation of this characteristic equation the system can be classified in one of the 4 forms which are shown here. So, if we have the poles or the roots of this characteristic equation which both are real it means there is no imaginary part to these roots then the system will be over damped. And the system responds if we see in the time domain it look like this over damped response. This is the final value which is steady state value or we can say and the system approaches this steady state value in a very sluggish way this is called over damped. And the characteristic of over damped system is the roots of this characteristic equation both are real. Next case may be when all these these 2 roots are complex they have real part as well as the imaginary part then the system will be under damped. Under damped system is shown this is a under damped system where the system shoots above the steady state value and then tries to settle it down. It means the damping in the system is not enough then there is a overshoot in the system. And the characteristic is when these both the roots are complex. If we have only the 2 imaginary roots of this characteristic equation means there is no real part to that then the system will be undamped means there is no damping in the system at all. So, if we apply a certain input to the system the system goes into the oscillation mode. This is the undamped case where there is no damper involved. For example, if we are talking about the mechanical system the first example the block is sitting on frictionless surface and damper is not there only spring is attached we disturb this equilibrium we pull the mass and leave it this system will be in oscillation state where there is no force available to stop the system to stop this vibration.

Friction is not there and the damper is also not there that is case of an undamped system and the root of this characteristic equation in that case will come out to be both imaginary. Another case is the critically damped where both the roots are repeated real and both roots are same and they are real. In that case the system achieves the steady state value fast and settles down there. So, this is the critically damped case where system achieves steady state value and the both of the roots are repeated real. So, here we have example for example, we have this as a system where characteristic equation is S square plus 9 S plus 9. This is the characteristic equation if we take out take the root of this characteristic equation those roots if we plot on real and imaginary axis those roots fall here and here. So, both roots are real with no imaginary part imaginary part is 0 j omega. So, the system response will be something like this which is a over damped case. Another case is when we have characteristic equation S square plus 2 S plus 9 this is the characteristic equation if we take out the root of this equation and plot it on real and imaginary axis both root comes like this both roots are real as well as imaginary part. So, in that case the system is under damped and system will have overshoots and undershoot.

So, this is called overshoot and this is called undershoot and this is in between we have steady state value. The third case is we have this type of equation where S square plus 9 where the zeta is 0 and in this case if we plot it plot the root of this equation on real and imaginary axis they comes out here where both are imaginary with without real part. In that case the system is undamped and it goes into the oscillatory mode if we apply a step input to this. So, this is the oscillation around this steady state value because there is no force existing there which is opposing this motion. The fourth case is the critically damped where we have equation for example, S square plus 6 S plus 9 this is the characteristic equation, and we have if we calculate the root and plot it on real and imaginary axis both roots are real and they are same if we plot it plot the output of this system that comes into the critically damped. The system achieves the output at very fast rate compared to the over damped case and reaches the steady state value. So, these are the four cases of output response which depends on the root of this characteristic equations. So, this is one way of characterizing the output. However, we can characterize the response of second order system using two parameters omega n and zeta instead of roots. So, instead of calculating the root of that characteristic equation we can define these two parameters which are omega n and zeta based on that we can classify the system.

Omega n is the natural frequency of the system. So, this is the frequency of oscillation without damping if we remove the damper from the sensor then the frequency of oscillation is this natural frequency. So, for example, in an RLC circuit if we remove the resistor from the system because resistance is the element which is opposing this which are kind of dissipating this energy and stopping this oscillation. If we remove the resistor then the frequency of oscillation will be the natural frequency. The damping ratio zeta this measures the amount of damping like how much damper or how much damping force is there in the system. So, damping ratio defines that and the typical range of zeta is between 0 and 1. Damping ratio zeta is defined as exponential decay frequency divided by the natural frequency and the exponential decay frequency zeta d is the real axis component of poles of a critically damped or undamped system. So, we discussed we can have a quadratic equation and we can solve for the roots or we can compare using standard characteristic equation which is S square plus 2 zeta omega n S plus omega n square. This is standard characteristic equation we can compare using this and find out the value of zeta and omega n. These zeta and omega n are more meaningful compared to finding the roots. When we start designing these are the more handy parameters. So, earlier we calculated the roots for example, for a undamped system we had roots like this, but if we calculate zeta, zeta is 0 for this undamped system. Similarly, for under damped system we calculated two roots which are two complex for these two complex system zeta comes out to be between 0 and 1. For repeated roots where we have both the roots are real and it repeated in case of critically damped zeta comes out to be equal to 1. And for under damped case where these both the roots are real zeta is greater than 1.

So, depending upon the value of zeta we can specify whether the system is undamped. If zeta is between 0 and 1 we can have under damped. If zeta is equal to 1 then we have critically damped and if the zeta is greater than 1 we have over damped. So, we can characterize the system using the value of zeta instead of finding out the root of this characteristic equation. So, here are some examples which we can consider. Let us say we have this first example where we have characteristic equation is S square plus 8 S plus 12. If we compare it using our standard equation where we have S square plus 2 zeta omega n S plus omega n square we can see omega n square is 12. So, we can calculate omega n is equal to under root 12 and 2 zeta omega n is equal to 8. So, we can calculate zeta is 8 upon 2 into omega n which is root 12. So, this is how you can calculate omega n and zeta. And based on the value of zeta you can specify which type of network it is. So, in first case zeta comes out to be 1.155 which is a over damped system. Similarly you can calculate for this this example S square S square plus 8 S plus 16 you can compare it with characteristic equation here and find out the value of zeta and zeta comes out to be 1 in this case which is critically damped. For for the case C we have S square plus 8 S plus 20 you can compare again with characteristic equation and zeta is 0.8 which is an under damped case. So, this is how using these characteristic equation and looking at the transfer function of the system we can we can calculate or we can know the system is over damped under damped and based on that we can calculate how the system is going to respond if we apply a step over input to the system. So, math expression for ah YT for ah under damped case let us say we consider the under damped case where damper is not very strong means the zeta falls between 0 and 1. This is the transfer function is Laplace domain this is Laplace domain and to convert it back into time domain we take inverse Laplace and the output in the time domain is given by this equation this is time domain equation is because there is a time t variable here present here. So, this is a time domain equation and we can now plot it using ah computer or we can plot it by varying this t. The output in time domain of the system looks like this.

So, we have this this is a time domain the time axis and this is the magnitude and the output is follows this this blue curve. So, this is the response of an under damped system this is the plot of this time domain equation. Few parameters we can see in this the output start from 0 and reaches a peak value which is Y max or the peak value in certain time which is let us say t p and then there is a decay in the magnitude because of the damping. Now we can define multiple parameters for this system based on the output characteristic. The first one is the peak time. So, the peak time is the time required to reach the first peak in the output when the output starts ramping it reaches a peak it comes down and then it oscillates and reaches the steady state value the first peak the time it takes to reach that that first peak is the peak time the t p we call it and in this graph if we see this is the peak value and this is the time which it takes to reach from 0 to this peak value which is t p. Percentage overshoot so, this overshoot is always represented in terms of percentage this overshoot is represented in percentage which is relative to the steady state value. So, percentage overshoot means how much percentage it goes above the steady state value. So, percentage overshoot or percentage os is the amount that the response exceeds the final value at t p final value is your steady state value steady state or SSV we call it in short. Settling time so, settling time is the time required for the oscillations to die down.

So, as we can see the oscillations take a lot of time in completely dying out. So, we define a range when the oscillations are within 2 percent of the final value 2 percent or 5 percent that depends on application. So, for example, we define when the output remains within 2 percent of the final value then that is the time taken to reach that that 2 percent within 2 percent value is your settling time. Rise time means when the output is rising how much time it takes reaching from 1 from 10 percent to the 90 percent of final output. Final output means the steady state value the how much time the system output takes reaching from 10 percent to 90 percent of steady state value that is the rise time. So, these are some parameters which we can calculate and the formula for these all these parameters are given here. If we have this let us say characteristic equation S square plus 2 zeta omega n S plus omega n square then the peak time is T p which is given by this equation peak value is given by this expression percentage overshoot is this and the settling time is 3 upon zeta omega n S or 4 upon z depending upon we specify 2 percent or we ah stay with the 5 percent value. So, depending upon this the time the settling time equation is that. We can see one thing all these parameters depends only the omega n and zeta. So, these 2 parameters are very much important in in describing the system. Some properties are the percentage overshoot depends on zeta, but not omega n. So, the how much system overshoots with respect to the steady state value that depends only on zeta the damping factor it does not depend on the natural frequency of the system. From second orders transfer function analytic expression of delay and rise times are hard to obtain if we have only the second order transfer function ah it was very difficult to obtain. Time constant is 1 upon zeta omega n which indicates the convergence speed. For zeta greater than 1 we cannot define peak time and peak value because for zeta greater than 1 this is over damped case where there is no overshoot of the output hence we cannot define the peak time, peak value and percentage overshoot.

So, let us consider some numerical example where we have the transfer function of the system is given by C s upon R s and which is 100 upon s square plus 20 s plus 100. So, find out the nature of the system means it is over damped, under damped or critically damped. So, this is the transfer function given to us and we need to compare it with the characteristic equation. So, by comparing this you can see omega n square is 100. So, the omega n comes out to be 10 and we compare 2 zeta omega n equal to 20 this is 2 zeta omega n equal to 20 we put value of omega n as 10. So, zeta comes out to be 1. So, when the zeta comes out to be 1 it means the system is critically damped. So, another example when a second order system is subjected to a unit step input the value of zeta is 0.5 and omega n is 6 radian per second determine the rise time, peak time, settling time and peak overshoot. So, for the peak time we have T p equal to pi upon omega n under root 1 minus zeta square we put the value of zeta and omega n which these 2 parameters are given we can apply omega n here and zeta here which is T p comes out to be 0.6 second. So, this is the peak time the system takes in reaching the its first peak. Settling time is 4 upon zeta omega n a we apply zeta and omega n here and we get settling time is 1.33 seconds. Maximum overshoot in the system is e to power minus pi into pi into zeta divide by e to the power under root 1 minus zeta square into 100.

So, M p comes out to be 16 percent. So, this 16 percent is the maximum overshoot above steady state value. So, we have steady state value at this value and the maximum overshoot is 16 percent this is 16 percent above the steady state value. Another example is let us say we have this second order system where the equation is S square plus 2 S plus 4. We again compare it with characteristic equation S square plus 2 zeta omega n S plus omega n square and we calculate omega n is omega n equal to 2 and zeta equal to 0.5. Once we know the value of zeta n and omega zeta n omega n then we can calculate omega d which is the damping frequency equal to omega n under root 1 minus zeta square this one. The peak time is T p equal to pi upon omega d which is 1.82 seconds. Settling time we can calculate 3 upon zeta omega n S which is 3 second and the maximum overshoot is 16.3 percent. So, these are the some examples how do we define a system and we know how the system is going to response to a unit step input and how the output will look like in time domain what will be the maximum overshoot what will be the peak time, settling time and other parameters. We can use this these parameters and design the system itself. We use these parameters let us say we need a system where the peak time should be x and the let us say the settling time should be y if these parameters are known we can even design the system. So, one example is given here we have this mechanical system which is a mass which is connected to again this this spring and a damper the transfer function we can write in Laplace domain like this 1 upon j divided by S square plus d by j into S plus k upon j. We compare it again with characteristic equation S square plus 2 zeta omega n S plus omega n square.

We know the value of zeta we know the value of omega n because we know the parameters of the values of peak time settling time overshoots from that we can calculate omega n and zeta and from this transfer function which we developed for this mechanical system. We compare it and we calculate omega n is equal to under root k upon j and 2 upon zeta is 2 zeta omega n equal to d upon j. We know the value of omega n and zeta. So, we can calculate the value of k and j which is the system parameters k is the spring constant j is this mass and d is the damping. So, these values we can decide based on how much peak time or settling time we want in the system. So, the specifications of let us say 20 percent overshoot allows us to calculate zeta is this and the specification of T S equal to 2 allows us to calculate zeta omega n and we can easily calculate the value of d and the value of j. So, these are some examples which shows that the system how the system performs when we apply a unit step input to the system. These are all the characteristics which are time dependent. So, these are dynamic characteristics of the system.

This is all for today.

Thank you.