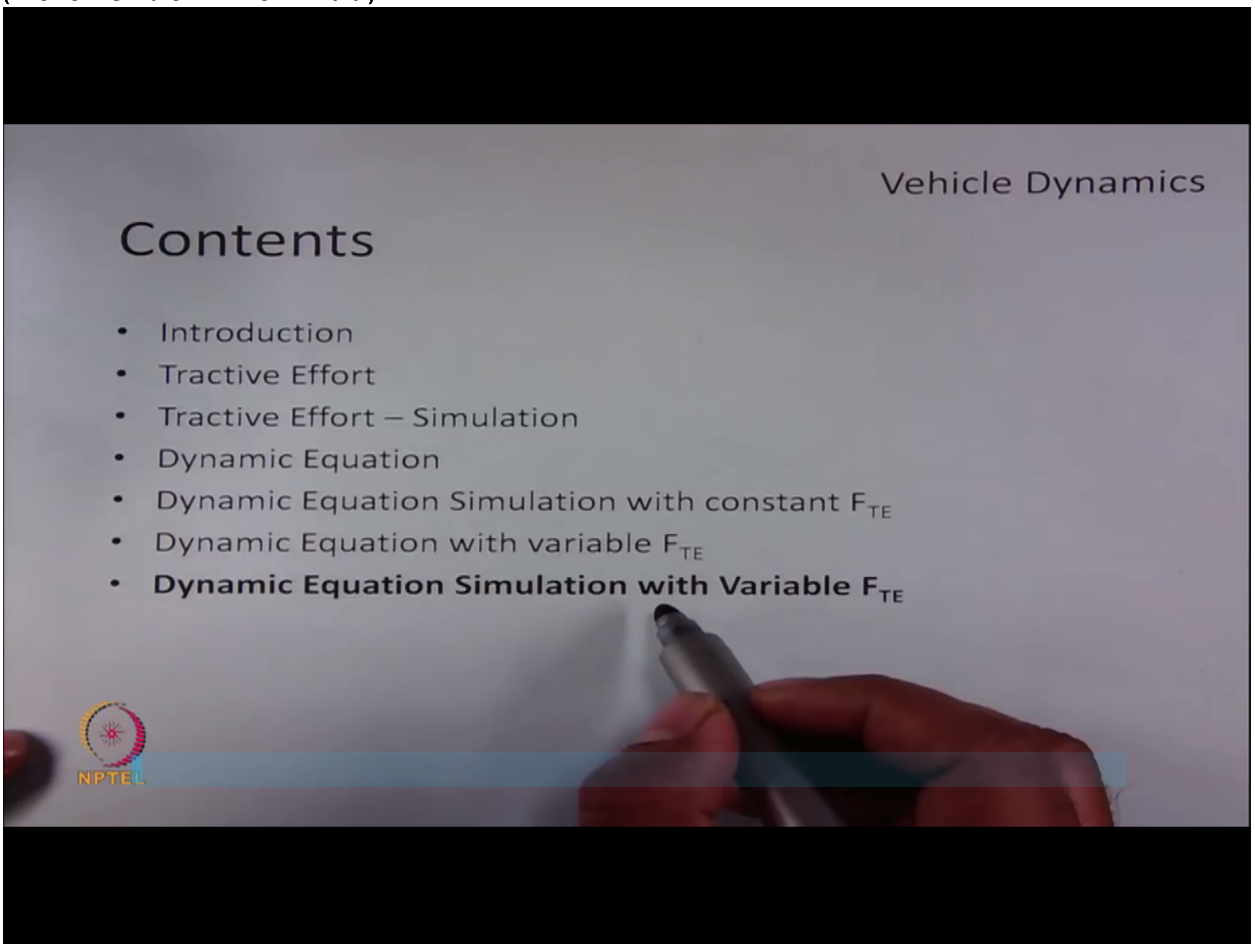
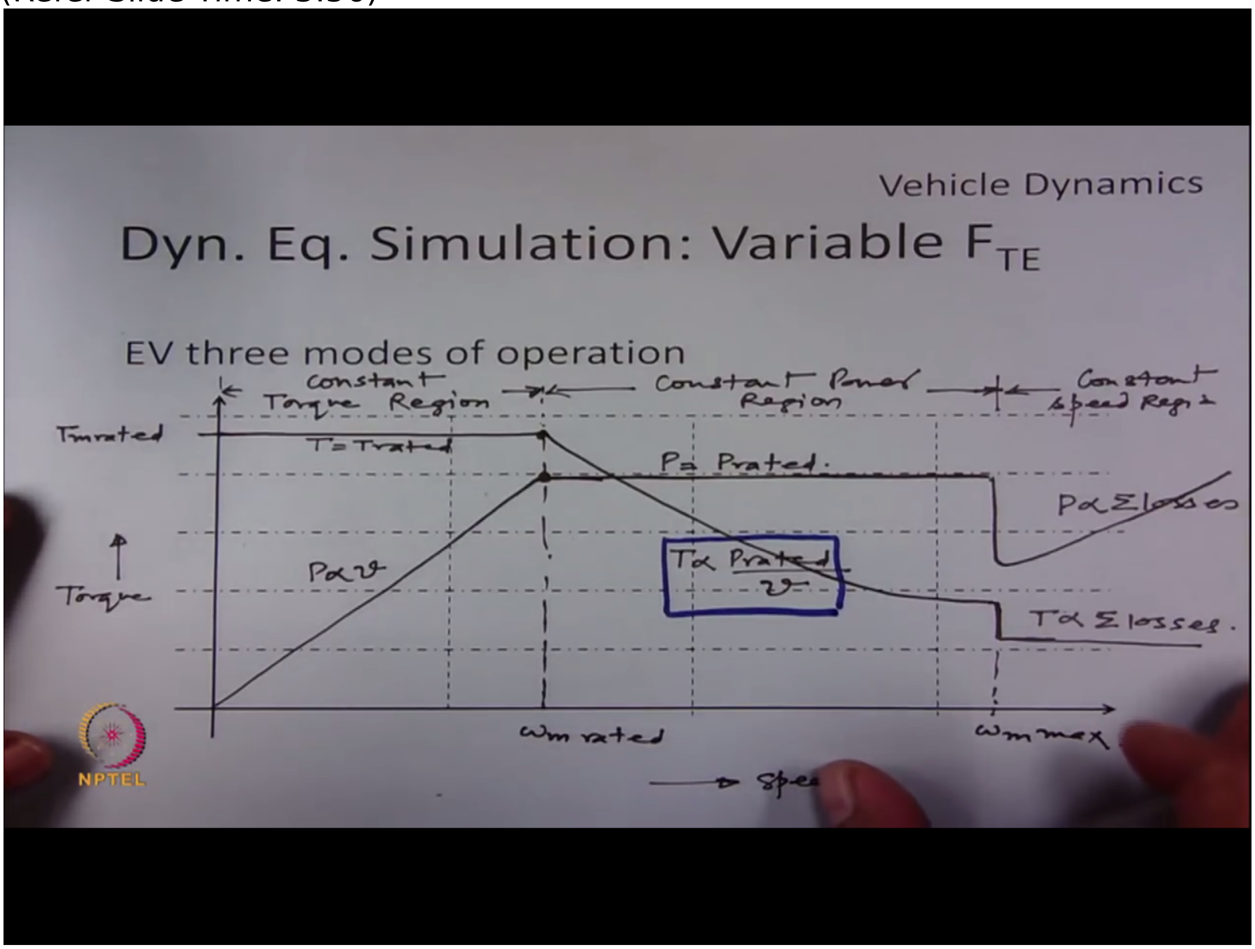
Hello everyone. Welcome to NPTEL online course on electric vehicles. So in today's interaction we will discuss the next topic under vehicle dynamics, which is simulation of dynamic equations with variable tractive effort. So in our previous interaction, we have derived different dynamic equations pertaining to variable tractive effort input to the vehicle. So let us try to see simulation of those dynamic equations in today's interaction.

(Refer Slide Time: 1:00)



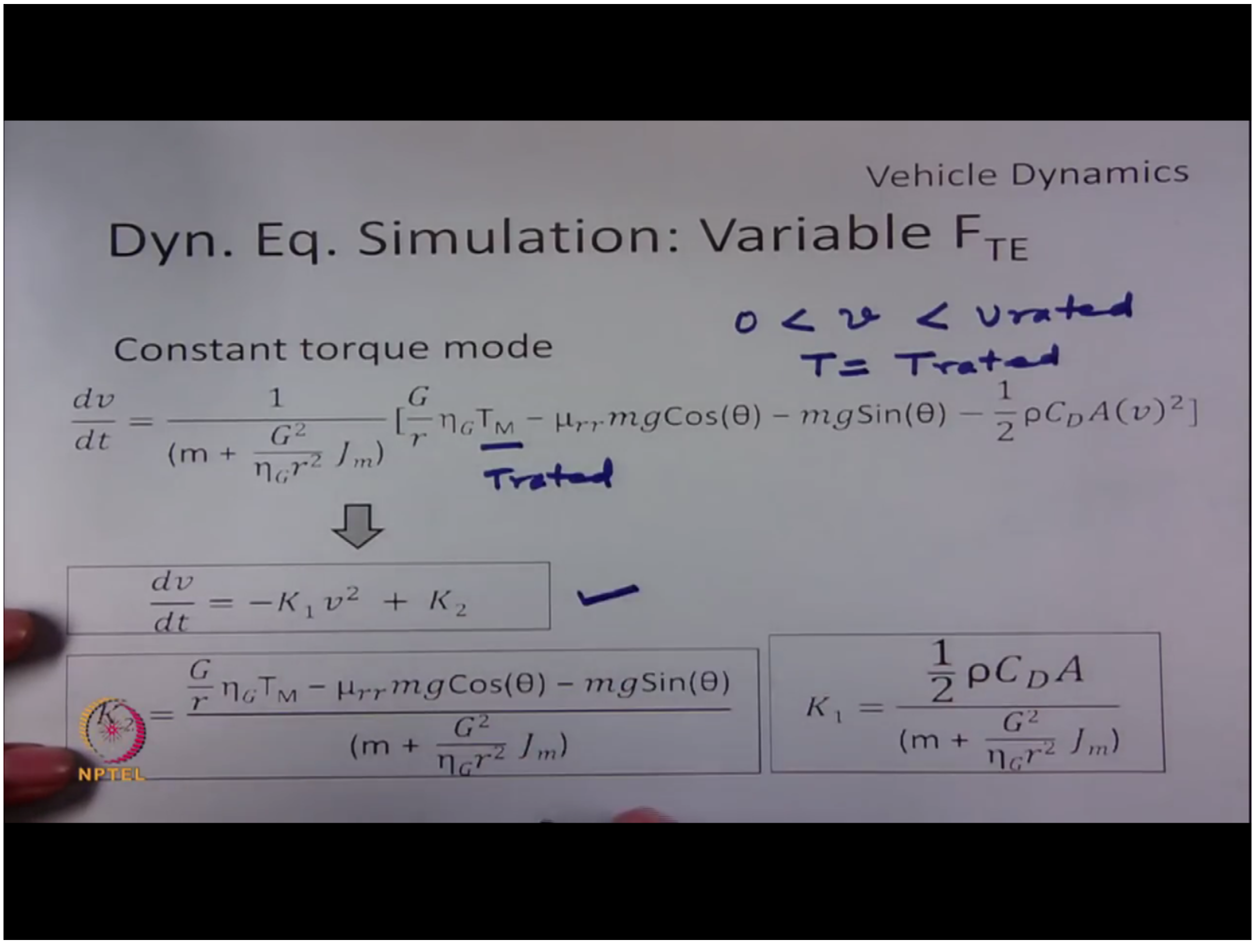
So as we have discussed an EV works on three modes of operations. So these modes of operations are divided based and the speed of the vehicle. So when the vehicle velocity is between 0 and rated conditions, it is known as constant torque region and when the vehicle velocity is between the rated velocity and the maximum velocity, the region is known as constant power region. And the speed operation greater than omega max is known as constant speed region. So if we see the torque graph. So torque graph is this one. So torque will be equal to T rated in constant torque region and we will look for maximum acceleration in this region. When the speed of the vehicle reaches the rated value, the machine will reach its rated condition and therefore the further acceleration is possible as a inverse function of vehicle velocity. So torque will be function of P rated by vehicle velocity. So this is the torque equation in constant power region. In constant speed region, there is no accelerating force required and therefore the torque requirement is only to support the resistive forces and other losses due to transmission system. Now let us see the graph of power. So power will vary in direct proportion to velocity in constant torque region. Since the torque is constant, power is directly proportional to velocity and it will increase linearly and it will reach its rated value. So P is proportional to V in constant torque region. In constant power region, the P will be = to P rated and in constant speed region, the power requirement is only to support losses. So this is a quick recap of our discussion we have done yesterday.

(Refer Slide Time: 3:56)



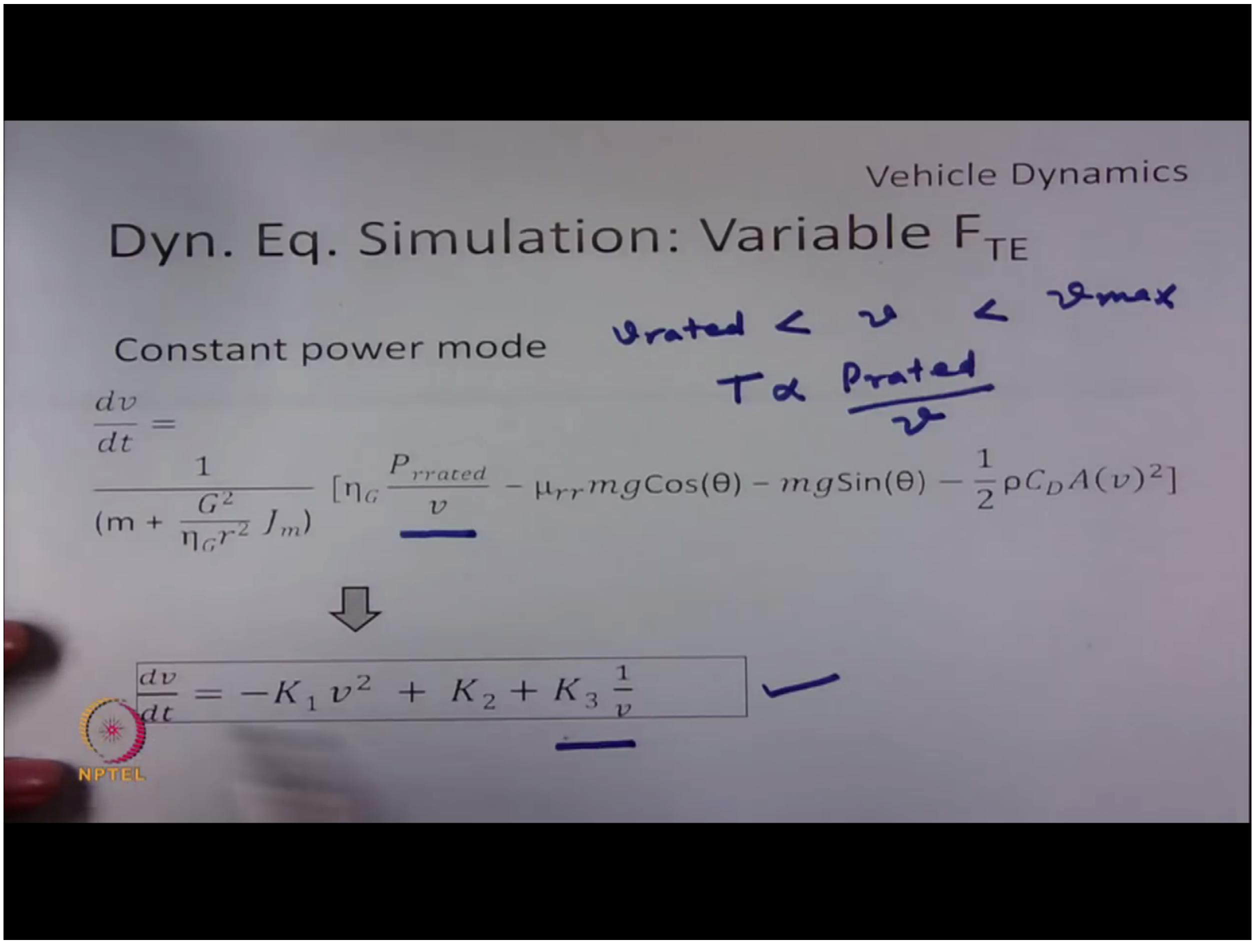
So we know that constant torque region operates when the vehicle velocity is between 0 and V rated and the torque is proportional to or equal to T rated. So in this DV/DT equation we generally substitute TM = T rated and derive the DV/DT equation, which is this.

(Refer Slide Time: 4:32)



In constant power mode, the vehicle velocity there is between V rated and V max and torque will vary as inverse proportion to V rated by vehicle velocity. So therefore the torque proportion to P rated by V will be substituted here and we will get a DV/DT equation, which is this and we can now see that we have a additional term here which is K3/V in comparison to the equation of DV/DT in constant torque mode.

(Refer Slide Time: 5:24)



In constant speed mode as the name suggest, DV/DT = 0. So there is no acceleration and the torque and the power is only required support the losses due to resistive forces and transmission system. So the equation of torque will be just addition of all the resistive forces.

(Refer Slide Time: 5:53)

Dyn. Eq. Simulation: Variable F_{TE}

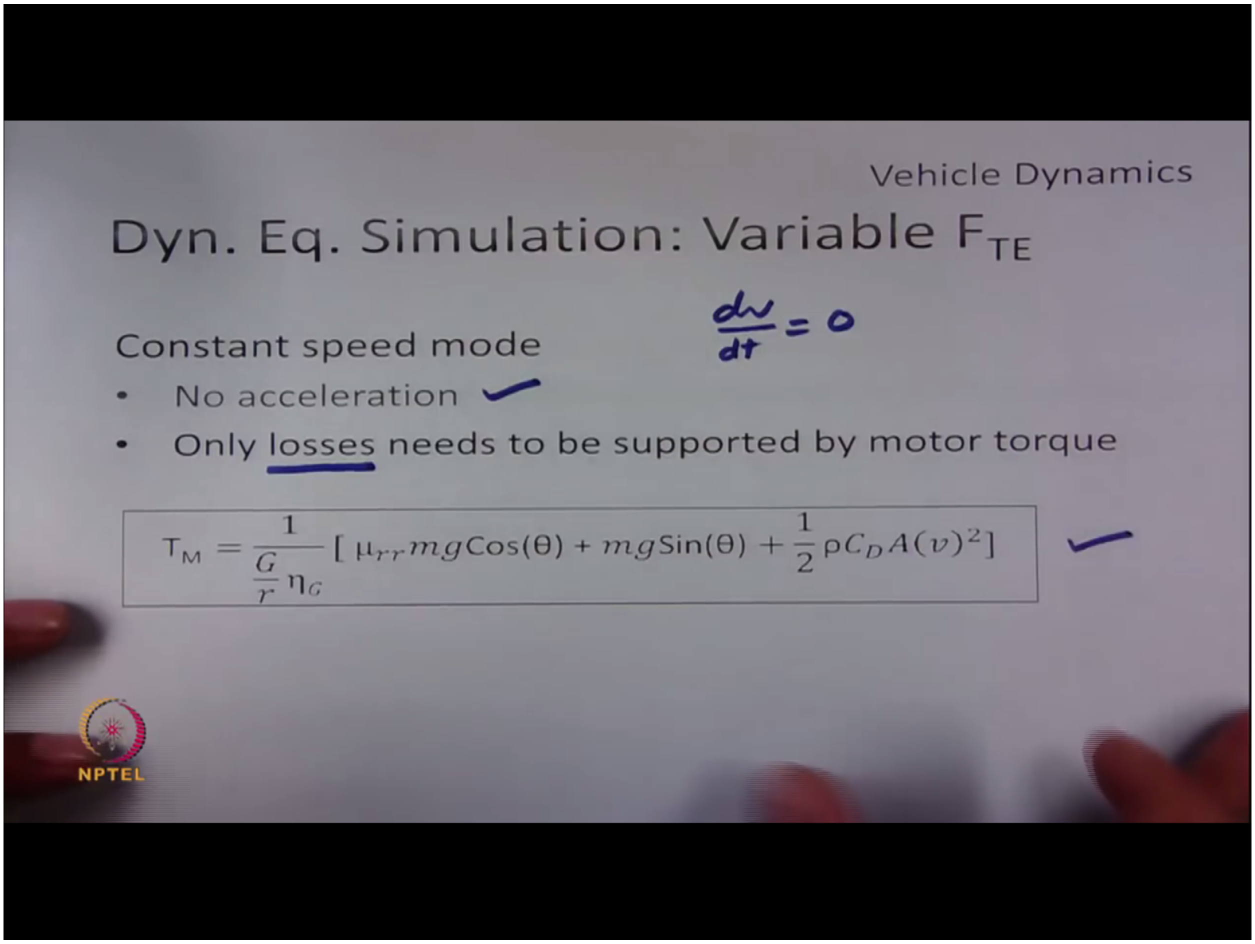
$\frac{dw}{dt} = 0$

Constant speed mode

- No acceleration
- Only losses needs to be supported by motor torque

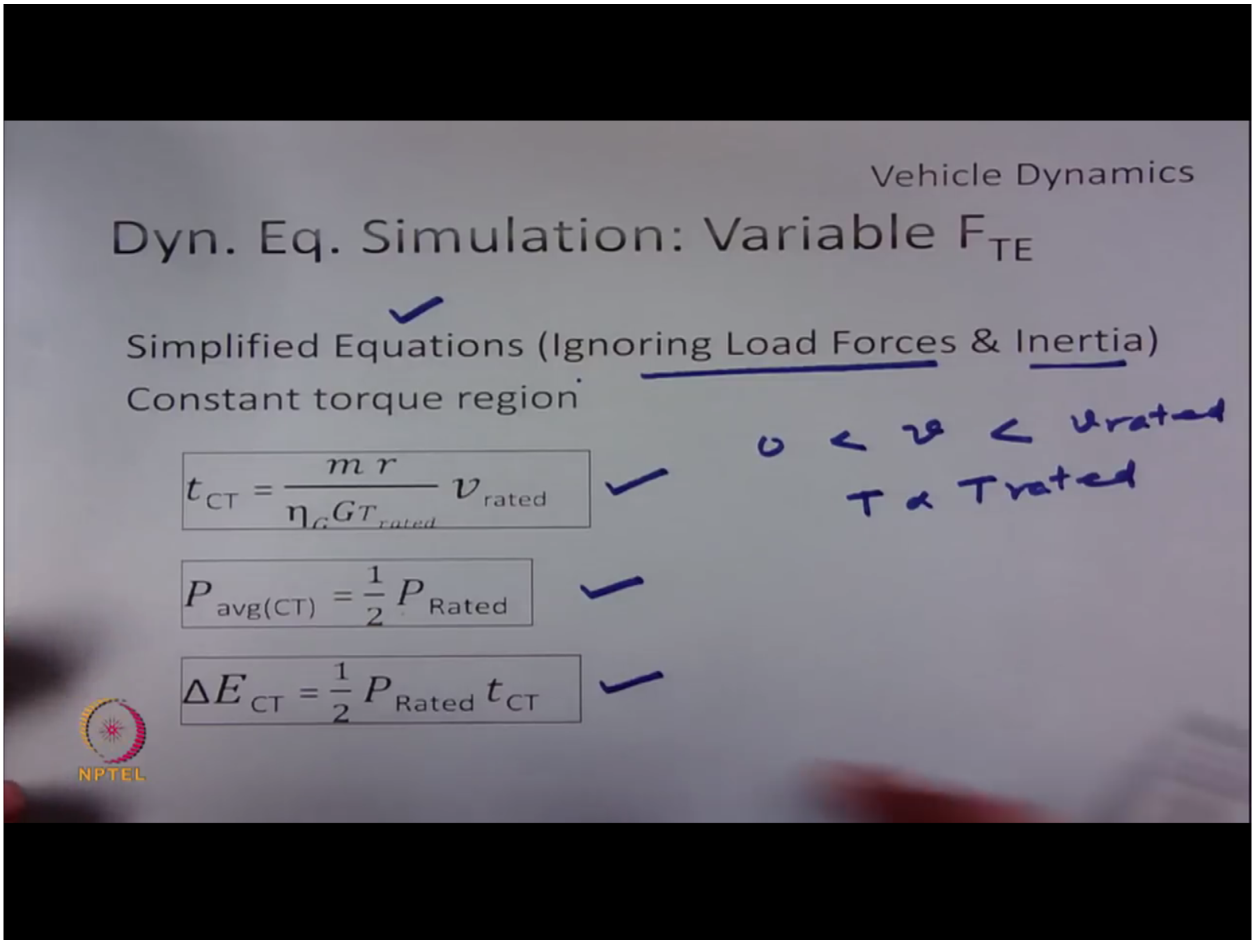
$$\boxed{T_M = \frac{1}{\frac{G}{r}\eta_G}\left[\mu_{rr} mg\mathrm{Cos}(\theta) + mg\mathrm{Sin}(\theta) + \frac{1}{2}\rho C_D A(v)^2\right]}$$

NPTEL



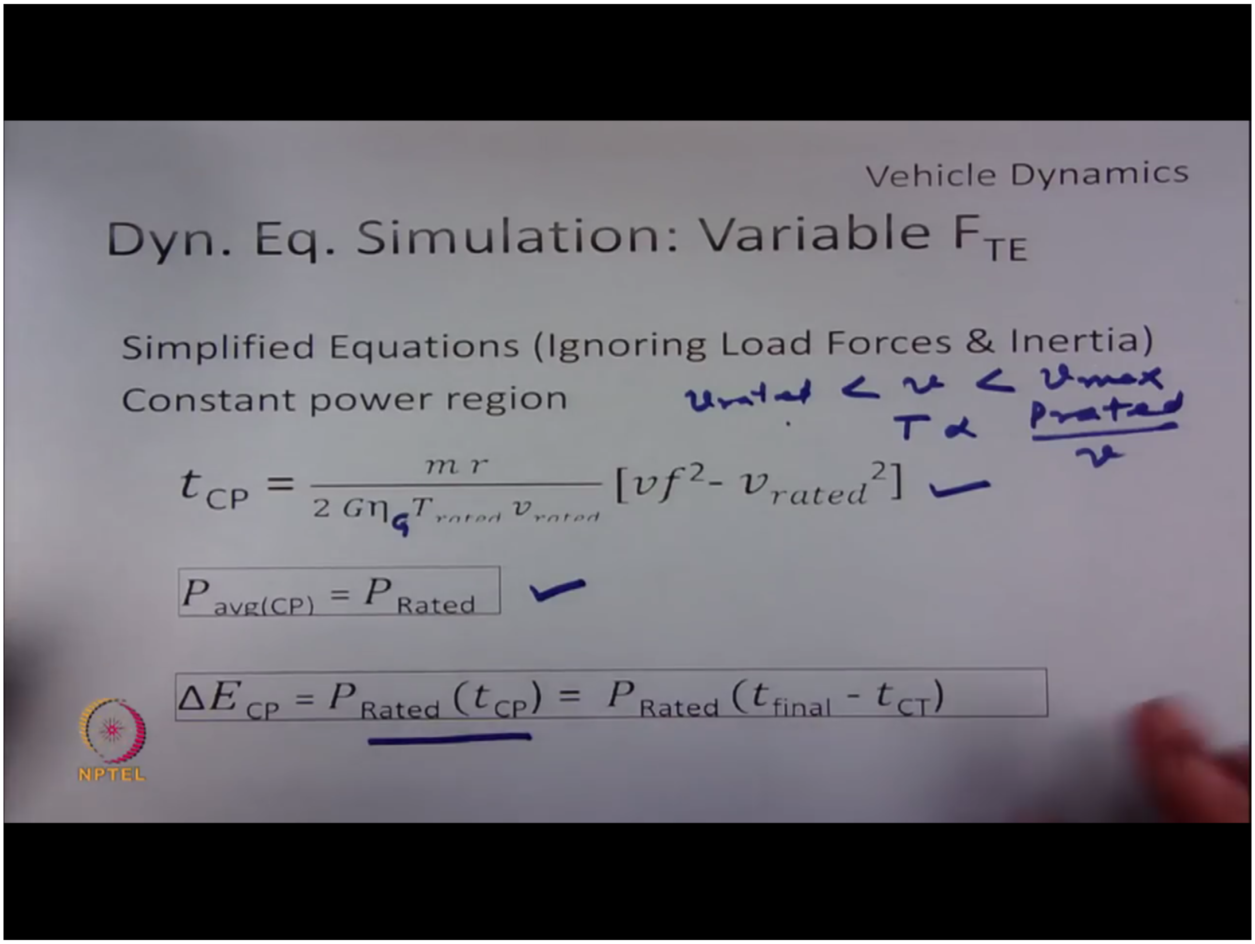
So we have also seen that it is possible get very simplified equation for the equation of various performance parameters, if we just ignore the load forces and inertial forces. And we were able to see that in constant torque region, so the velocities between 0 and rated torque is T rated. So the time required to reach the vehicle to V rated speed is this equation. So this is a simplified equation, when we ignore the load forces. Similarly since we know that the power variation is proportional to V, the average of power can be calculated as P rated/2 in constant torque mode. We have also seen that the energy required, which is simply a product of P average into the time elapsed, so it will be this equation.

(Refer Slide Time: 7:07)



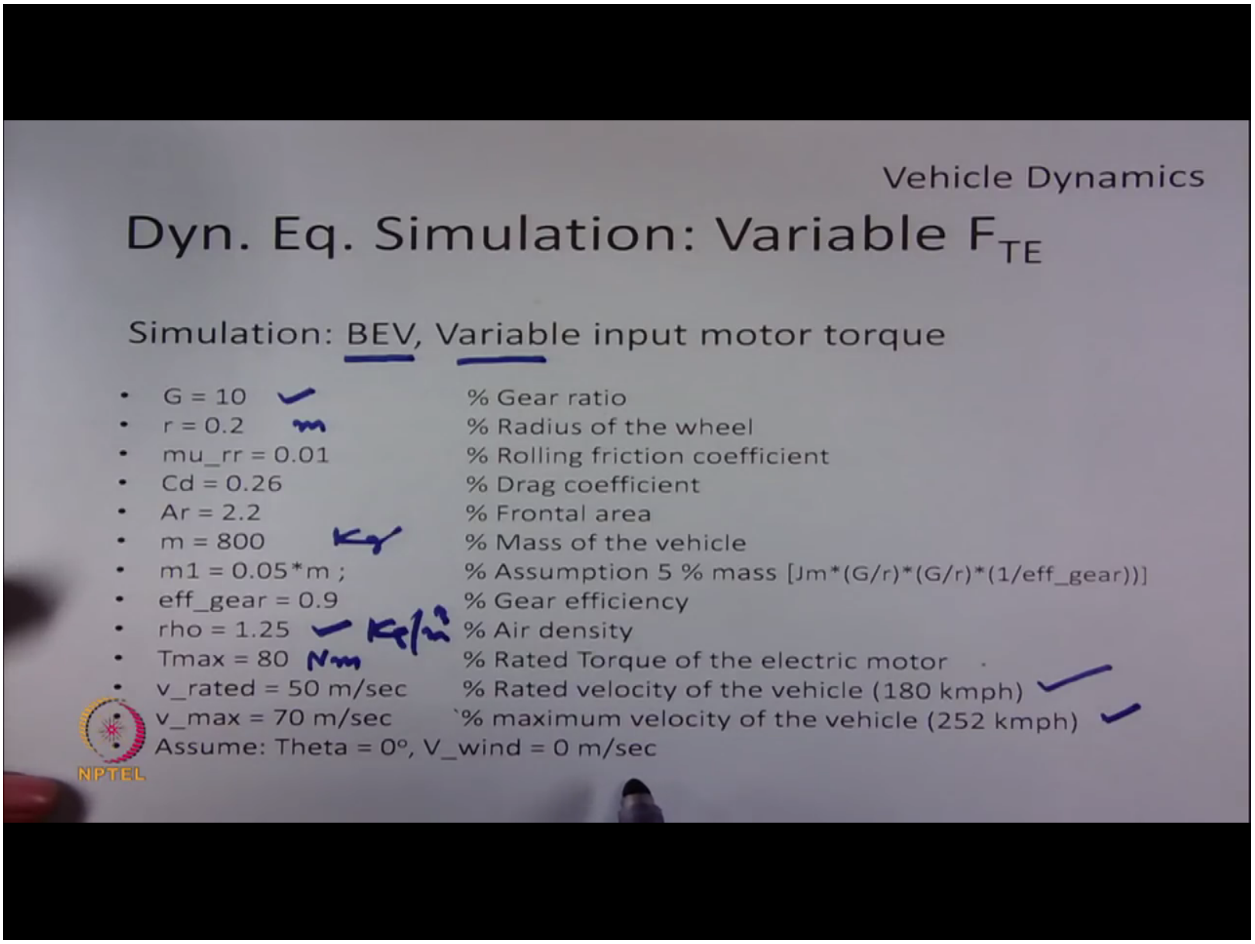
In constant power region the vehicle velocity is between V max and V rated and torque is proportional to V rated by velocity, so if we do that, we can calculate the time elapsed, the time the vehicle takes to operate between V rated and V max is equal to TCP = this equation. So… and we also know that the power is constant in this region, so therefore the average power is same as rated power. So if you want to calculate the energy that is required to operate the vehicle between V rated and V max velocities, the energy will be just V rated by TCP. So these equations will be used to get approximate values of performance parameters.

(Refer Slide Time: 8:18)



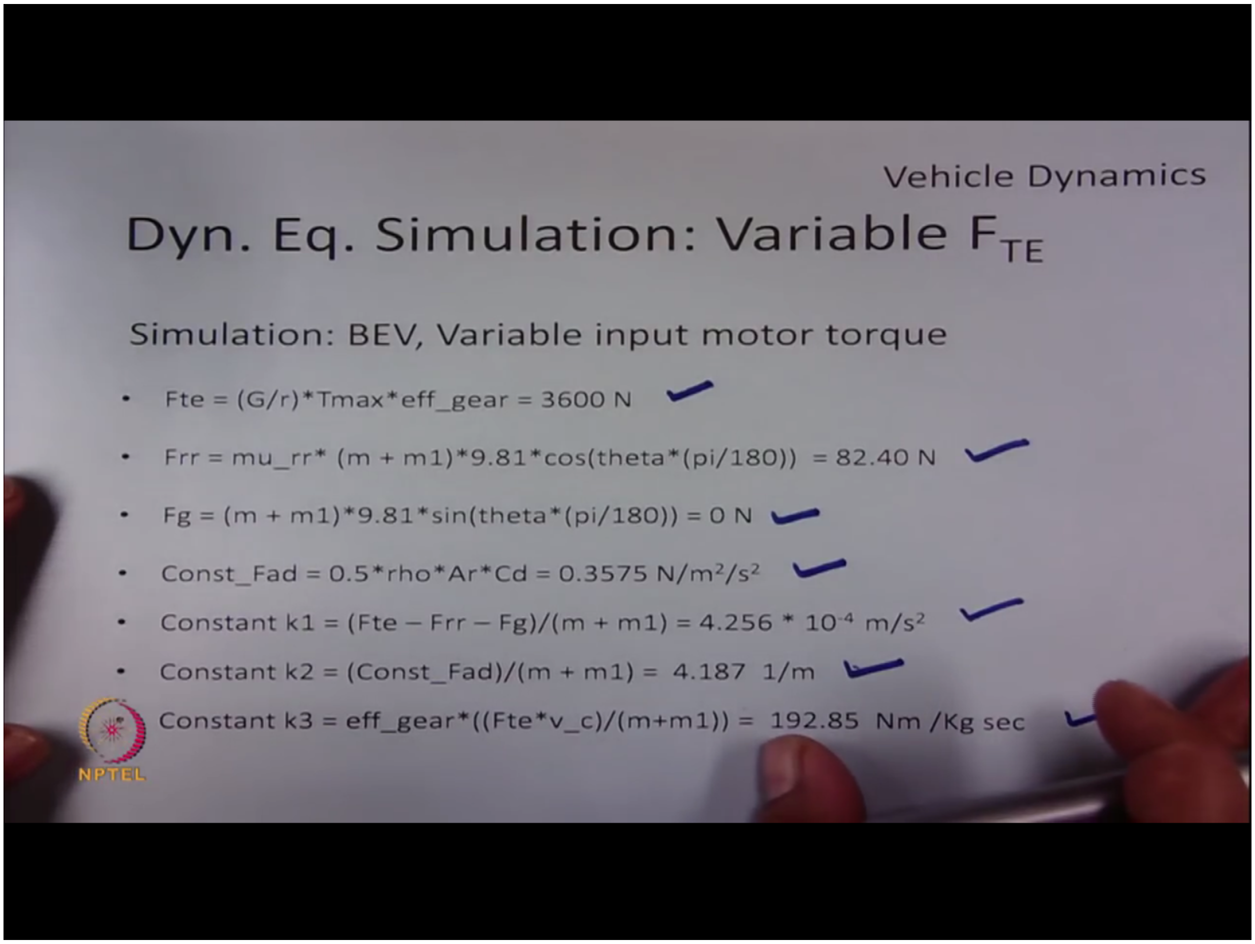
So let us quickly see some analytical calculations that what we have just seen and see whether they can be used to get reasonable values with respect to real time simulation. So again let us take a vehicle, let's say take a battery electric vehicle with a variable input motor torque, so gear ratio, let us take as 10, radius 0.2 meters, Mu RR 0.01, drag coefficient 0.26, frontal area of 2.2 meter square, mass of 800 kg, and similar to last approximation, let us take the mass equivalent to the inertial force is equal to 5% of this mass, so it is 0.05 M, efficiency of the gear is 0.9, Rho is 1.25 kg meter cube, T max let us take as 80 Newton meter. So in comparison to the last example we have taken, the mass is now reduced from 1200 kg to 800 kg and T max has increased from 40 Newton meter to 80 Newton meter. So since we are simulating a variable input motor torque conditions, we have to define the rated vehicle velocity. So let us take 50 meter per second as rated vehicle velocity, which turn out to be 180 km/hr speed. Also let us take the maximum velocity of the vehicle equal to 70 m/sec, which turns out to be 250 km/hr. Also let us assume that the slope is 0 and wind velocity is also 0 to start with.

(Refer Slide Time: 10:43)



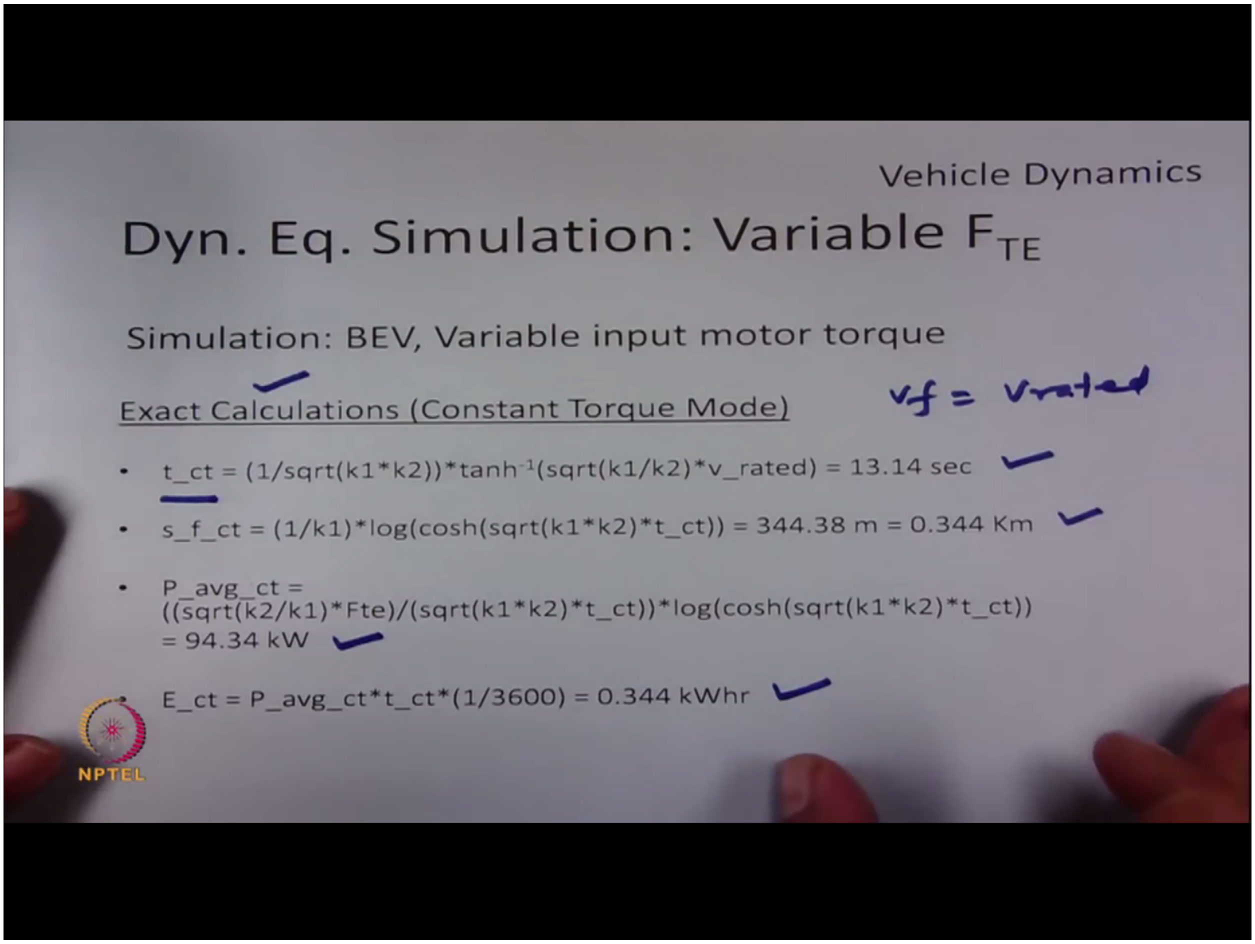
So once we define the parameters and their values, we can now start calculating the different forces. So tractive effort this equation G/R Tmax in efficiency care between 3600 Newton, rolling resistance force will be equal to 82.40 Newton, gradient is 0, because there is no slope involved. The constant term of the aerodynamic force, which is 0.5 Rho AR x CD, it is 0.3575. Now we have to calculate three constants, so K1, K2, and K3. So K1 is FTE - FFRY – FG/total mass, so it is 4.256 x 10 raise to -4 meter per second square. Constant K2 comes out to be 4.187, and the unit is 1 by meters. Constant K3, which is new constant in this mode, so K3 comes out to be 192.85 Newton meter per kg second.

(Refer Slide Time: 12:20)



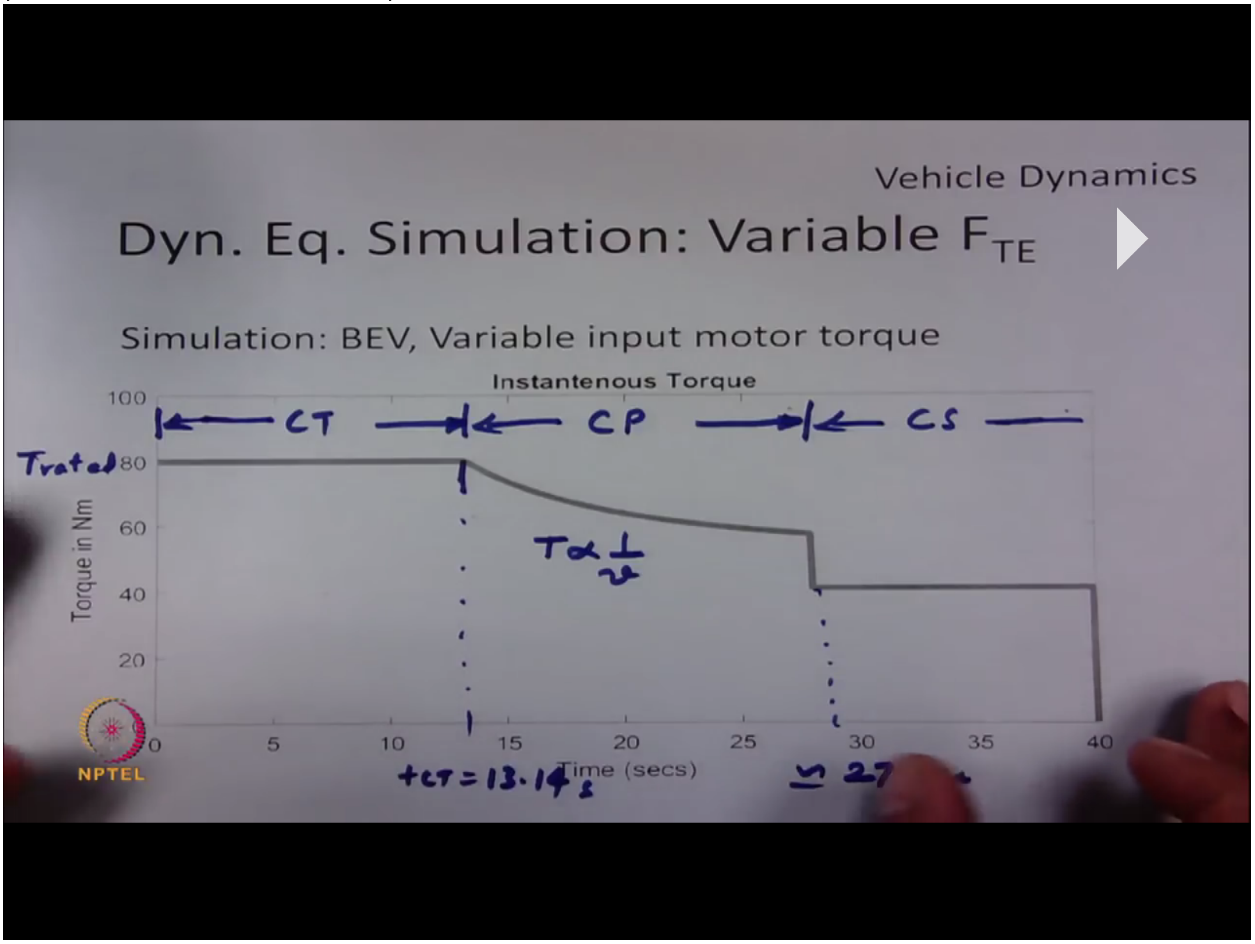
So once we have all this calculations with us we can calculate the different performance parameters. So we know that we do exact calculations based on the equations we have derived for constant tractive input and we can do so. So the time... so we know that in constant torque mode, the VF = V rated. So if we know this, we can substitute it in the time equation and we can get the time that is required to bring the vehicle from 0 speed to V rated speed. So we know this equation and it turns out to be 13.14 seconds. Similarly if we substitute this time in the distance equation, we can again get the distance that the vehicle travels when the vehicle moves from 0 to V rated speed is equal to 0.344 km. So again let us try to calculate the average power that is required in the constant torque region, so we substitute the required values of force, constants, and time elapsed. We can see that the average power in constant torque region is 94.34 and the energy required by the battery will be product of P average into time and constant torque region, which is... comes to 0.344 KW.

(Refer Slide Time: 14:21)



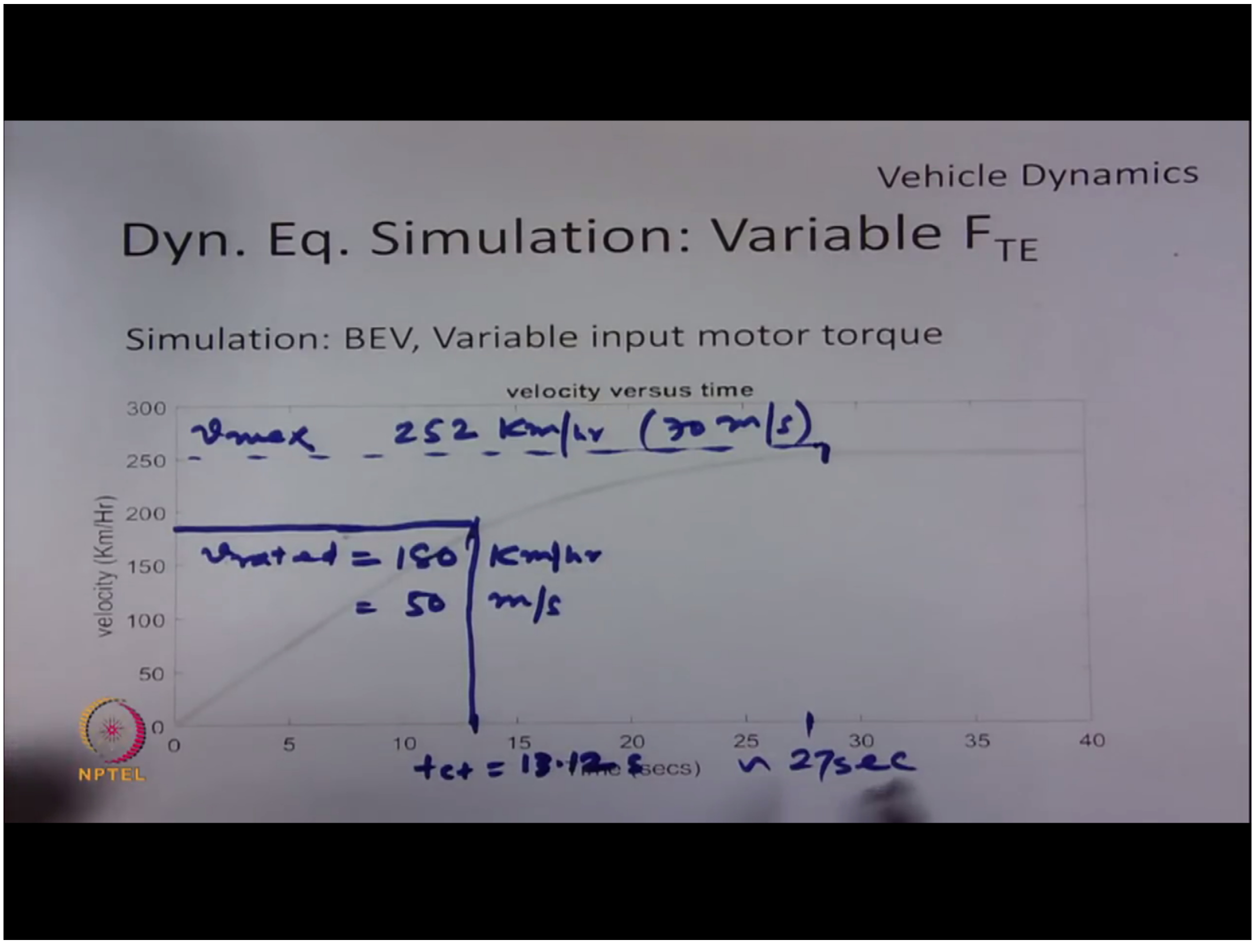
So now let us try to plot the different parameters such as velocity, torque, and distance. Since we know the equation of velocity with respect to time, distance with respect to time, and also the torque variation with respect to different vehicle speeds. So if we do the MATLAB simulation, we will be able to plot these graphs. So let us see the variation of the torque input, we are suppose to give, when the vehicle speed moves from 0 to V rated, V rated to V max, and speed beyond V max. So we know that in constant torque region we generally keep the value of tractive effort or the motoring torque as constant. So in our example, which we have taken, we have kept T rated = 18 Newton meter. So you can seen that, so the vehicle is operating in constant torque region here. So of 80 Newton meter. So this is basically T rated and we can see that, so this region is basically constant torque region. So typically the time required for the vehicle speed to vary from 0 to rated condition, turns out to be somewhere here. So it will somewhere be around 13. So our calculations shows TCT = 13.14 seconds. So it is coming as per calculation. So once the vehicle reaches its rated velocity, we go for constant power mode, where the torque variation will be inversely proportional to vehicle velocity. So we can see that that is happening, so here torque is proportional to 1 by vehicle velocity. So this region of operation is known as constant power region. So typically the time required is probably around the 27 seconds. So when we reach... so in 27 seconds the vehicle will reach its maximum velocity of 70 m/sec or 252 km/sec. So once we reach the maximum velocity, we will go for constant speed region. So the torque required will be equal to the resistive forces.

(Refer Slide Time: 18:20)



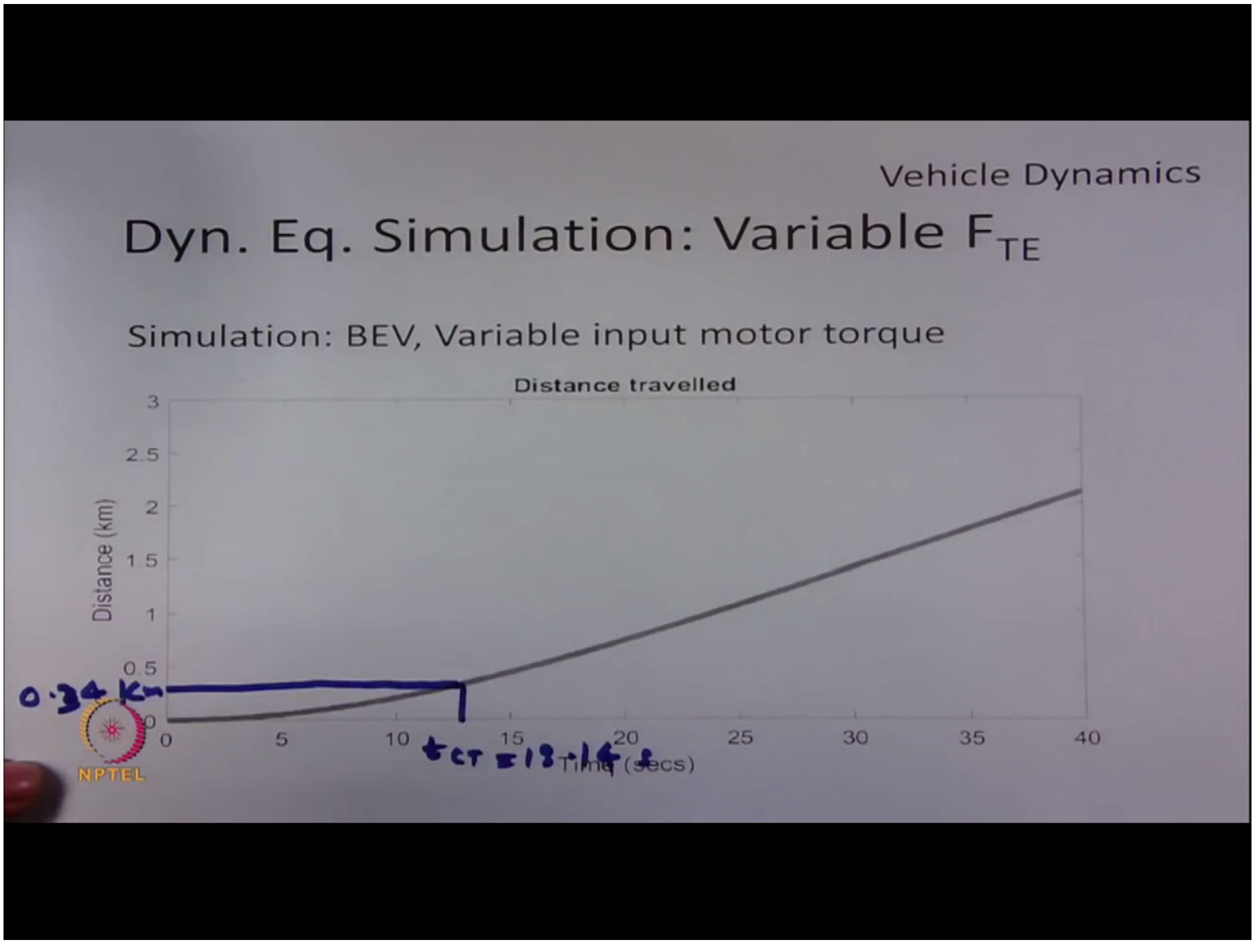
So if we see the variation of velocity with respect to time for this variation of vehicle velocity, we can see that, so we know that the constant torque mode is somewhere up to TCT=13.12 seconds. So you can see that roughly up to this speed, so we will be in... so we know that... so this is, we know that... so V rated = 180 km/hr or 50 m/sec. And when we reach, we will start this constant speed mode, when the time is around 27 seconds. So we can see that it has reached its value of 252 km/hr or 70 m/sec. So this is Vmax. So this is V rated and this is V max and we have a constant velocity of 252 post 27 seconds.

(Refer Slide Time: 20:05)



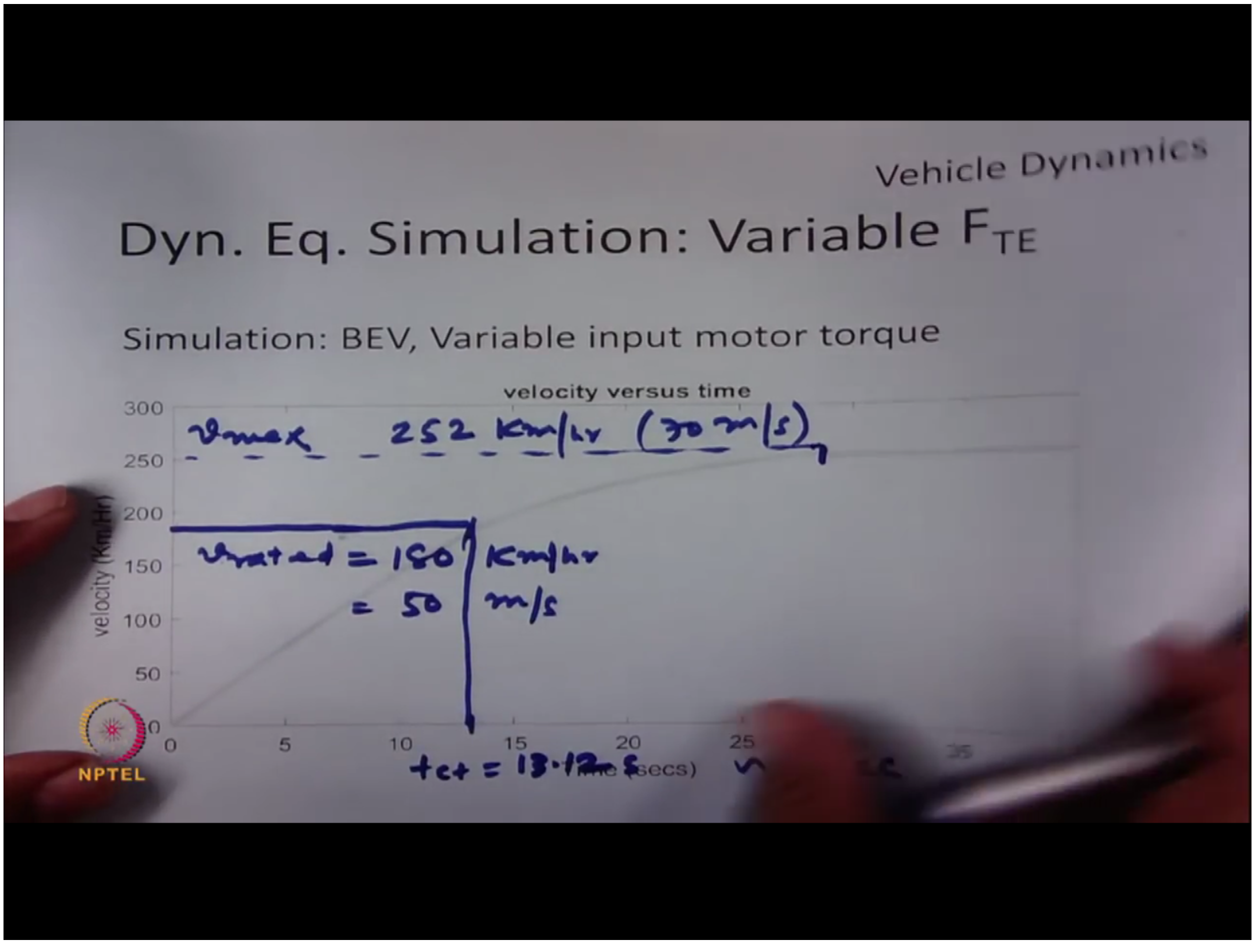
So if we see the distance curve with respect to time, we can calculate the distance that is travelled. So as per our calculations, we have calculated a distance travelled of 0.35 km in TCT mode or constant torque mode, so constant torque mode of, so it is around this 13.14 seconds. So we have distance equal to 0.34 km, so this is distance in km.

(Refer Slide Time: 21:00)



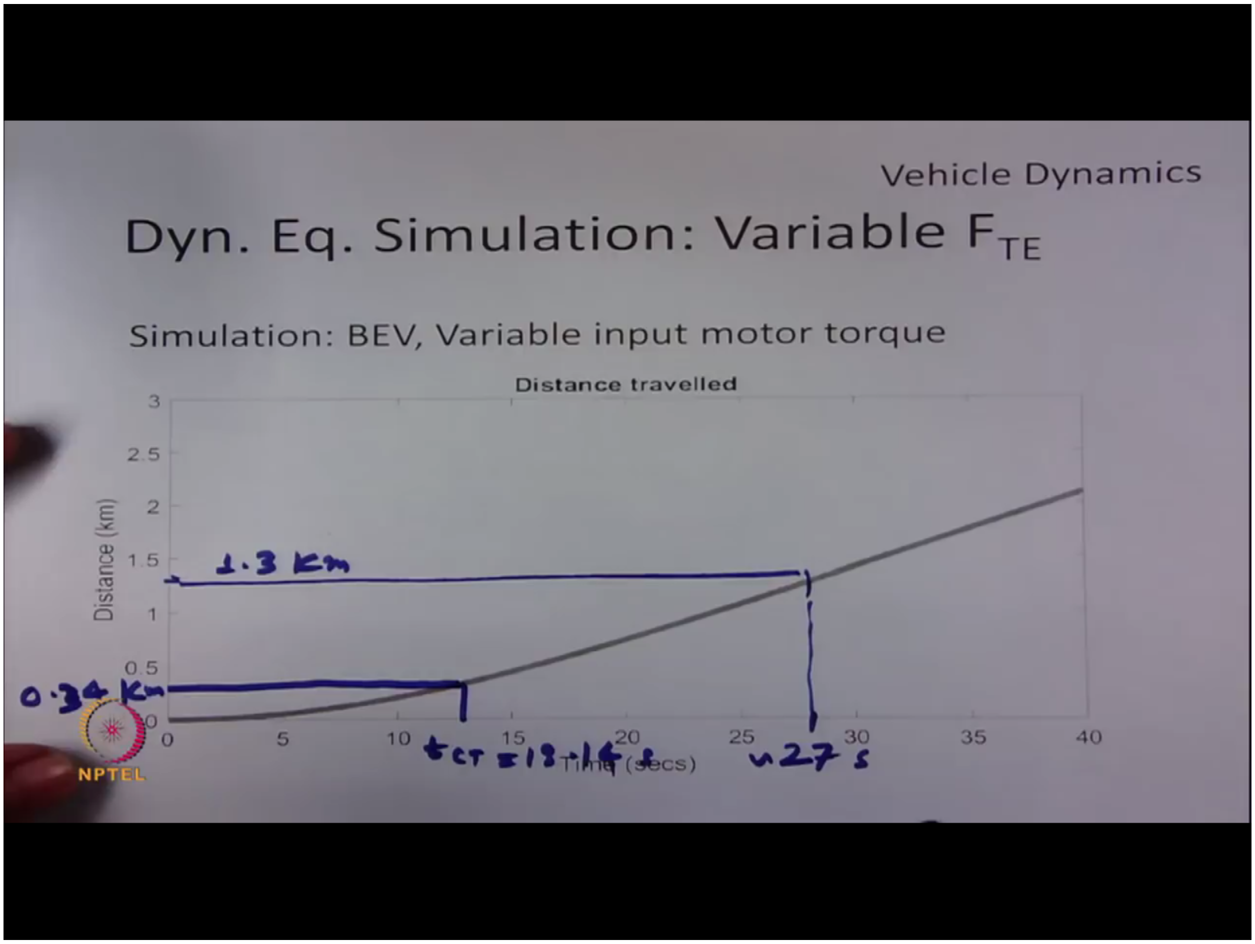
So also in the velocity graph, we can see that the acceleration between 0 to V rated is faster compared to the acceleration what we have from V rated to V max. So this is understandable. So acceleration is quite high and its almost linear and it reduces in constant power region as a function of speed. So it becomes lesser and lesser, it becomes and then becomes 0. So this can be observed from the velocity graph.

(Refer Slide Time: 21:42)



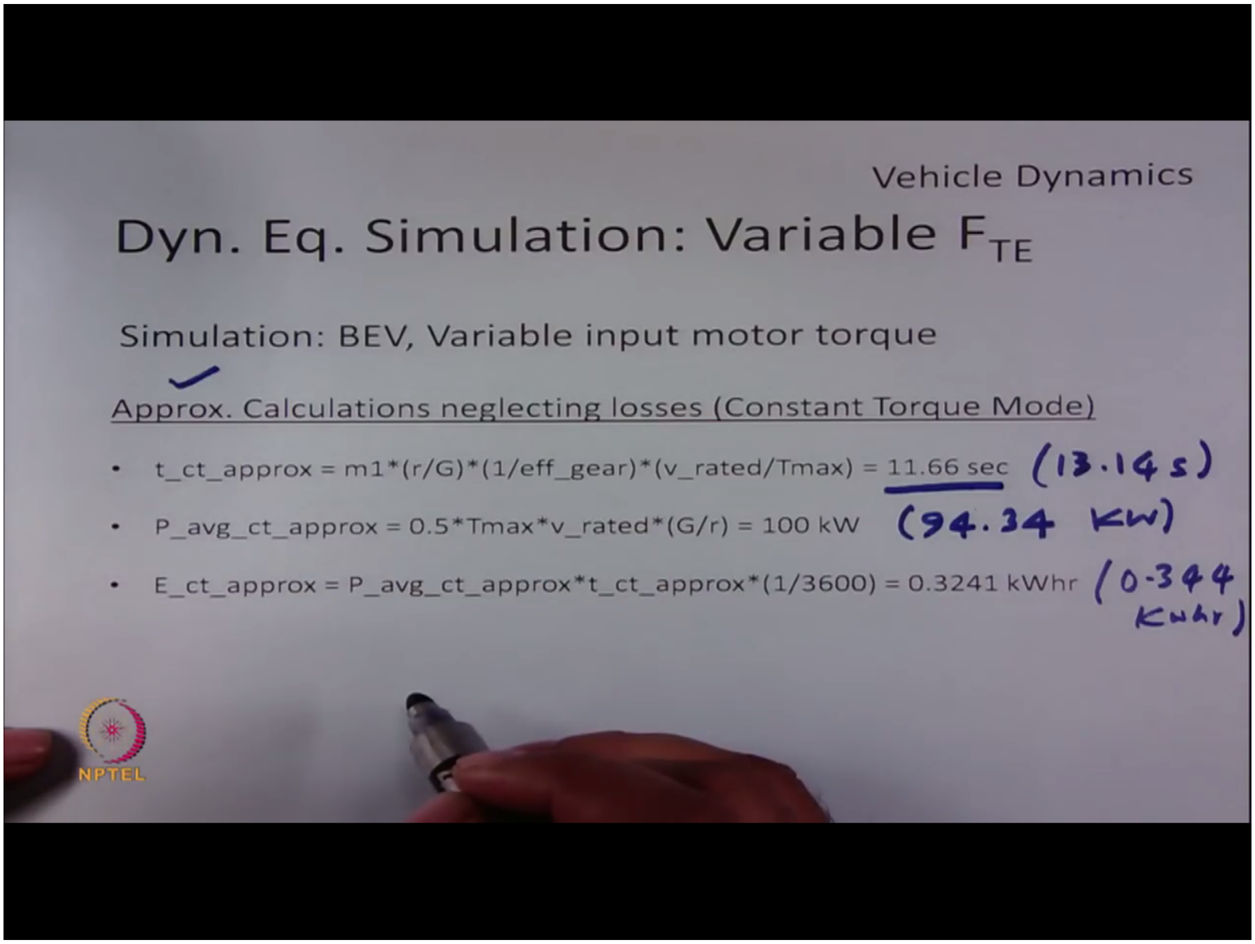
Also we can see that the distance which the vehicle travels is on… time proportional to 27 seconds will be somewhere around 1.3 km in the constant power mode.

(Refer Slide Time: 22:15)



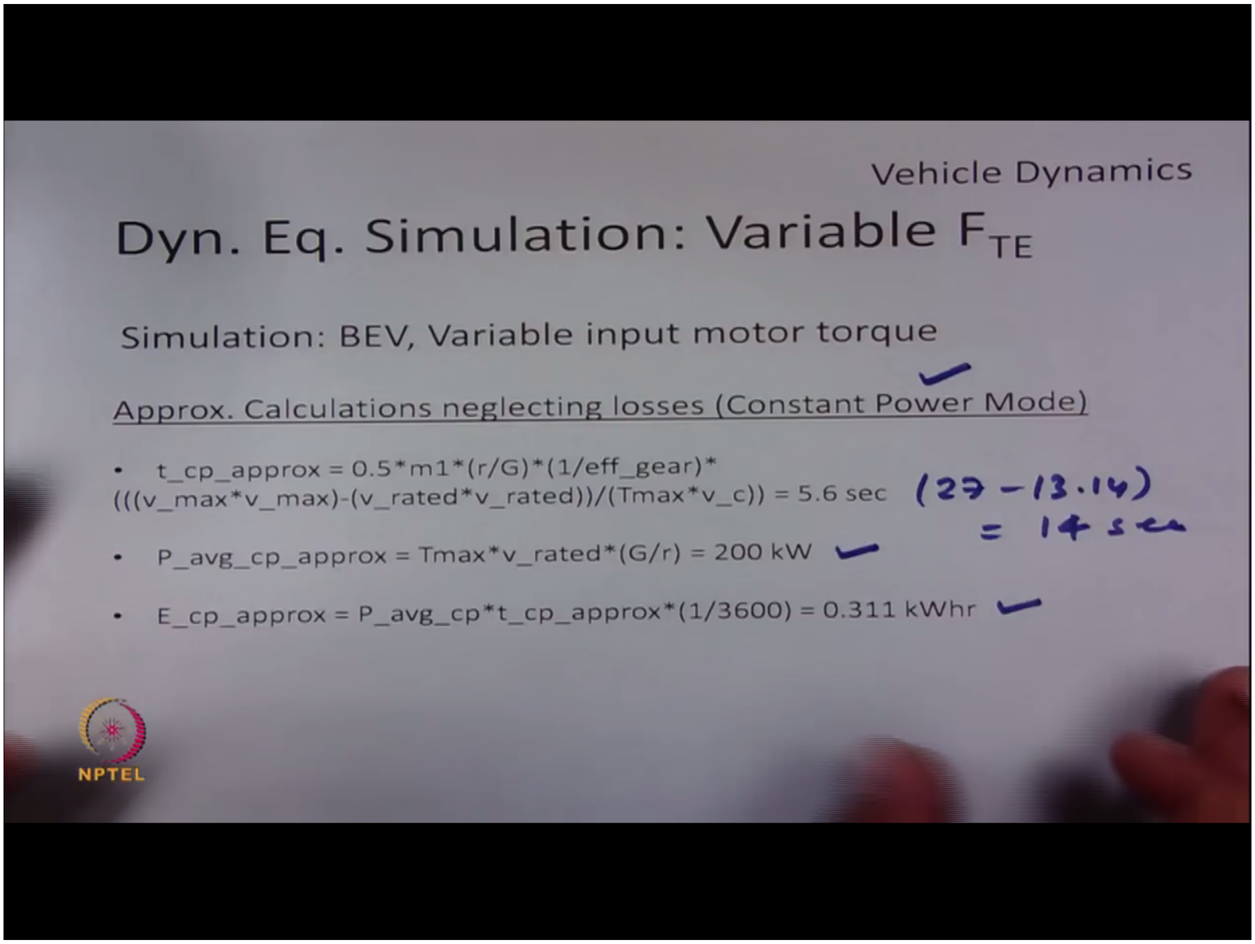
So we have also seen the method of calculating this performance parameter by some approximation such as neglecting resistive forces and other losses. So in constant torque region or constant torque mode we can calculate the approximate value of time elapsed as this equation, which gives the time = 11.66, but the exact timing is 13.14 seconds. And the power calculation if we see, the average power calculation, come for to be 100 KW, but the exact calculation shows that it is 94.34 KW. So it is basically under 10% accuracy, you can say. The energy calculation with the approximations comes out to be 0.324 KW hour, but on the result calculation it was 0.344 KW hours. So to check this approximations with respect to the simulation, we can do another simulation, where while plotting the velocity, distance, and the torque plot, we will neglect the losses. So if we do that, this approximate values should match with the simulations values.

(Refer Slide Time: 24:14)



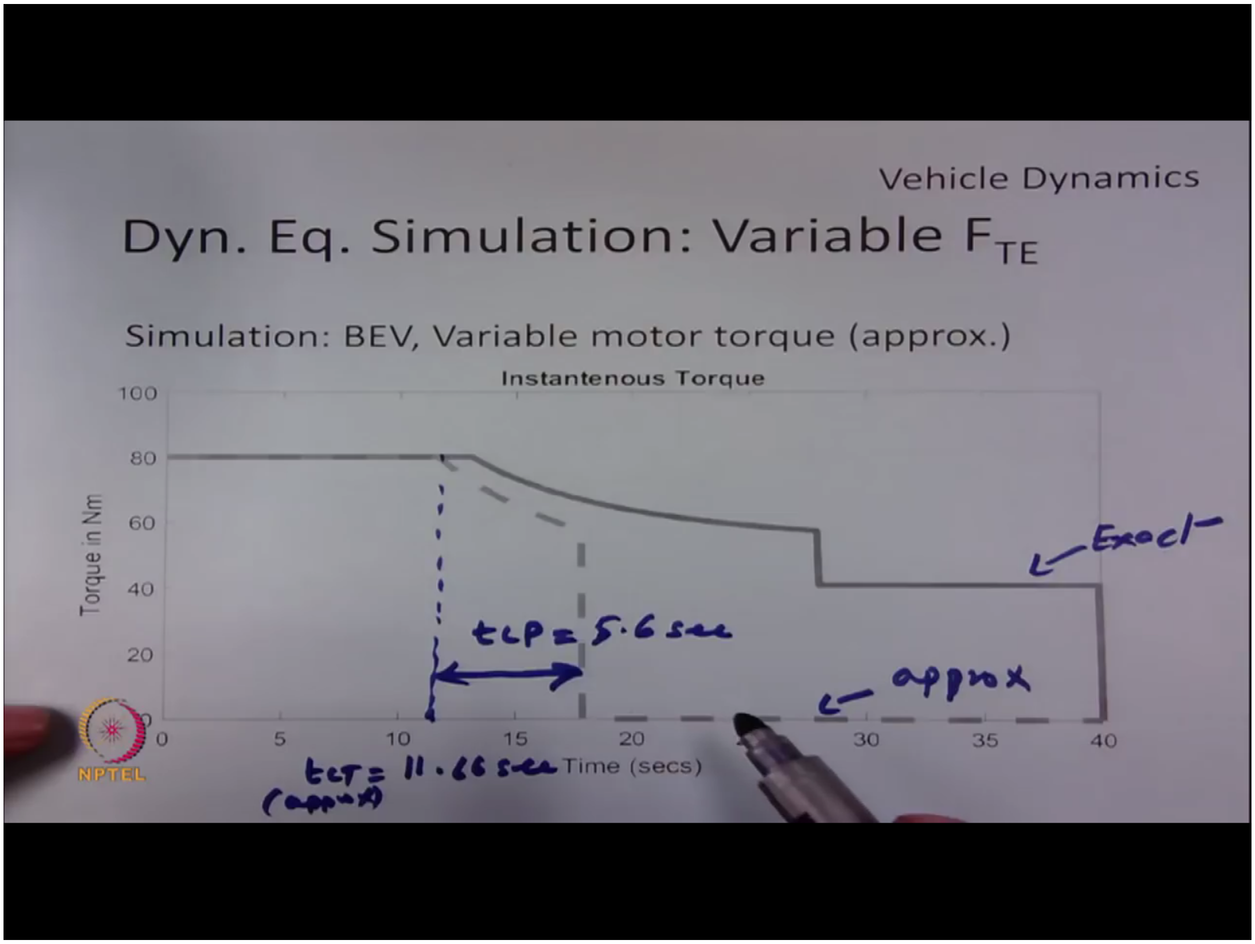
So the approximate values of time, average power, and energy required in constant power region, with this approximation comes out to be TCP of 5.6 seconds, but in actual exact graph it is 27-13.14, so which comes out to be around 14 seconds, so this approximation is not good and it has lot of errors, you know, quite reasonable error. The average power required in constant power mode is 200 KW and the energy required will be 0.311 KW hours. So let us try to do the simulation with this approximation, where we actually neglect the losses due to resistive forces like rolling resistance, aerodynamic resistance, slope, and all the transmission losses. If we do that, then this value should meet the simulation.

(Refer Slide Time: 25:37)



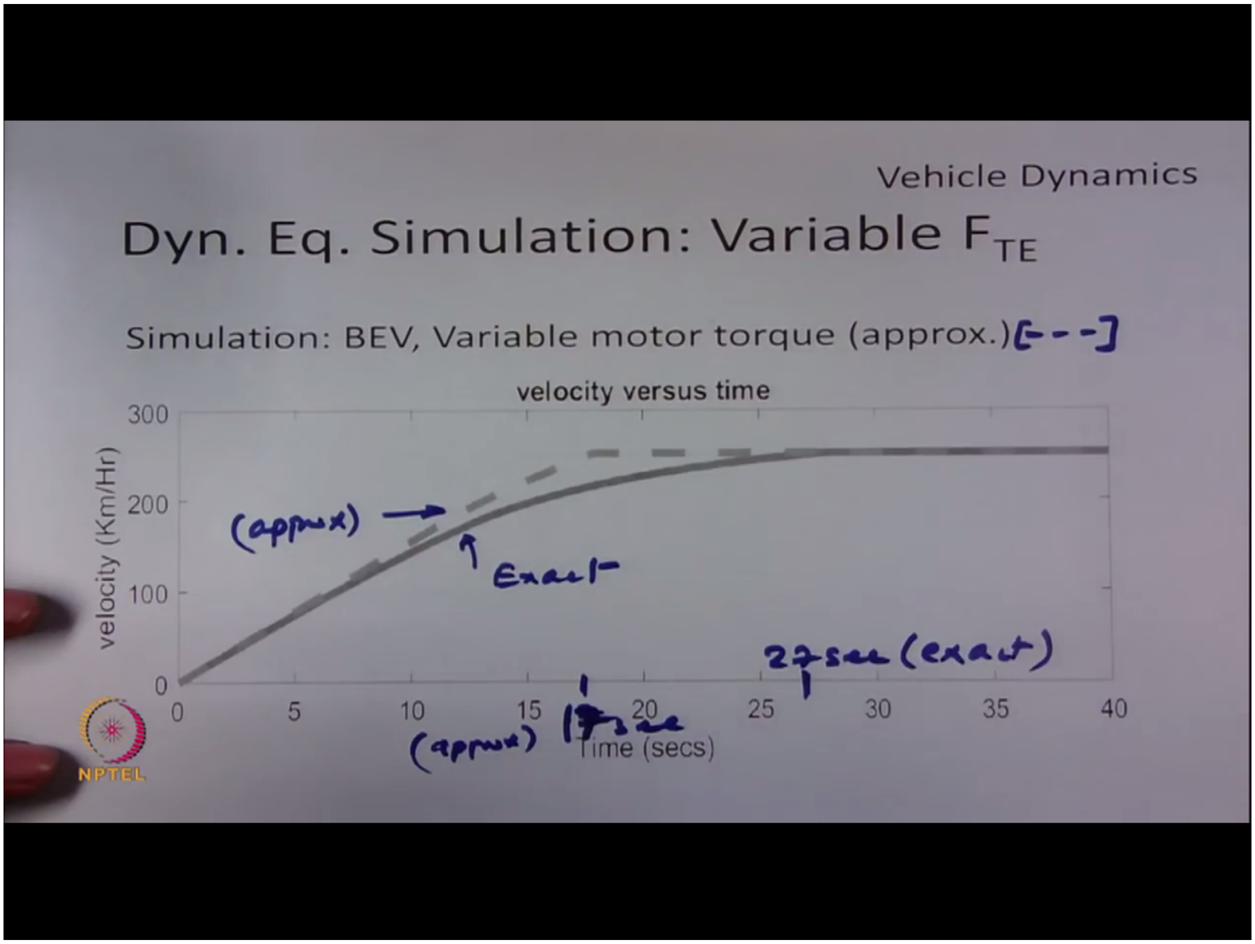
So let us see the graph of velocity with respect to time. So the bold graph is the exact graph that we have seen just some time back and this dotted graph is the approximate graph of velocity with respect to time. So we can see that the velocity comes to 0 in constant speed mode, why it is so, because there is no resistive forces and there is also no aerodynamic force, which was proportional to speed. So the torque requirement will be totally 0, so this kind of validates the approximate nature of this velocity variation. So this is basically exact graph and this is basically approximate variation of velocity with respect to time. So we can see that the time required for the approximate curve is 11.66. So this is equal to our calculation of TCT in approximation, so it is coming as per the calculation. Similarly the TCP is coming roughly as 5.6 seconds, which is again meeting the calculation. So our calculations can be also validated using this kind of graph. But we can see that the approximation and the exact curves are quite not matching, especially in constant power mode and constant speed mode. So it shows that the effect of resistive forces suggest rolling resistance force , gradient force, and force due to aerodynamic drag force cannot be neglected for approximations.

(Refer Slide Time: 28:37)



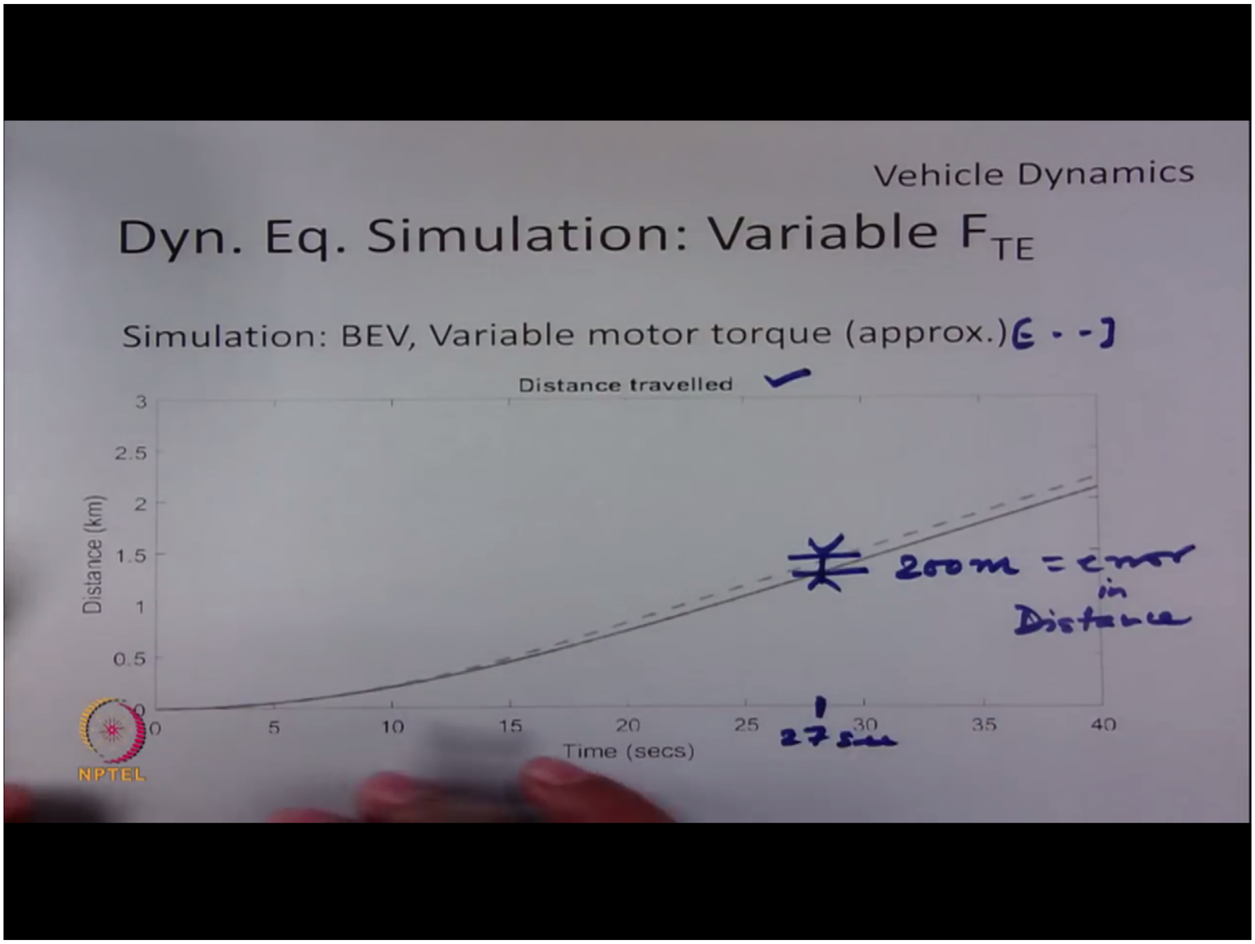
We can also see the variation of velocity with respect to time for both exact curve and so this is exact curve and this is approx, so approx is basically shown in doted curves. So again we can see that here the constant speed mode starts very early compared to the exact mode. So why it is so? Because the system is able to, you know, accelerate very fast because it doesn't have to support the… the resistive forces. So we have seen that the constant speed mode starts somewhere around 17 seconds, while the… so… for the approximate, while actually it starts at around 27, so… so 27 seconds for exact curve.

(Refer Slide Time: 29:55)



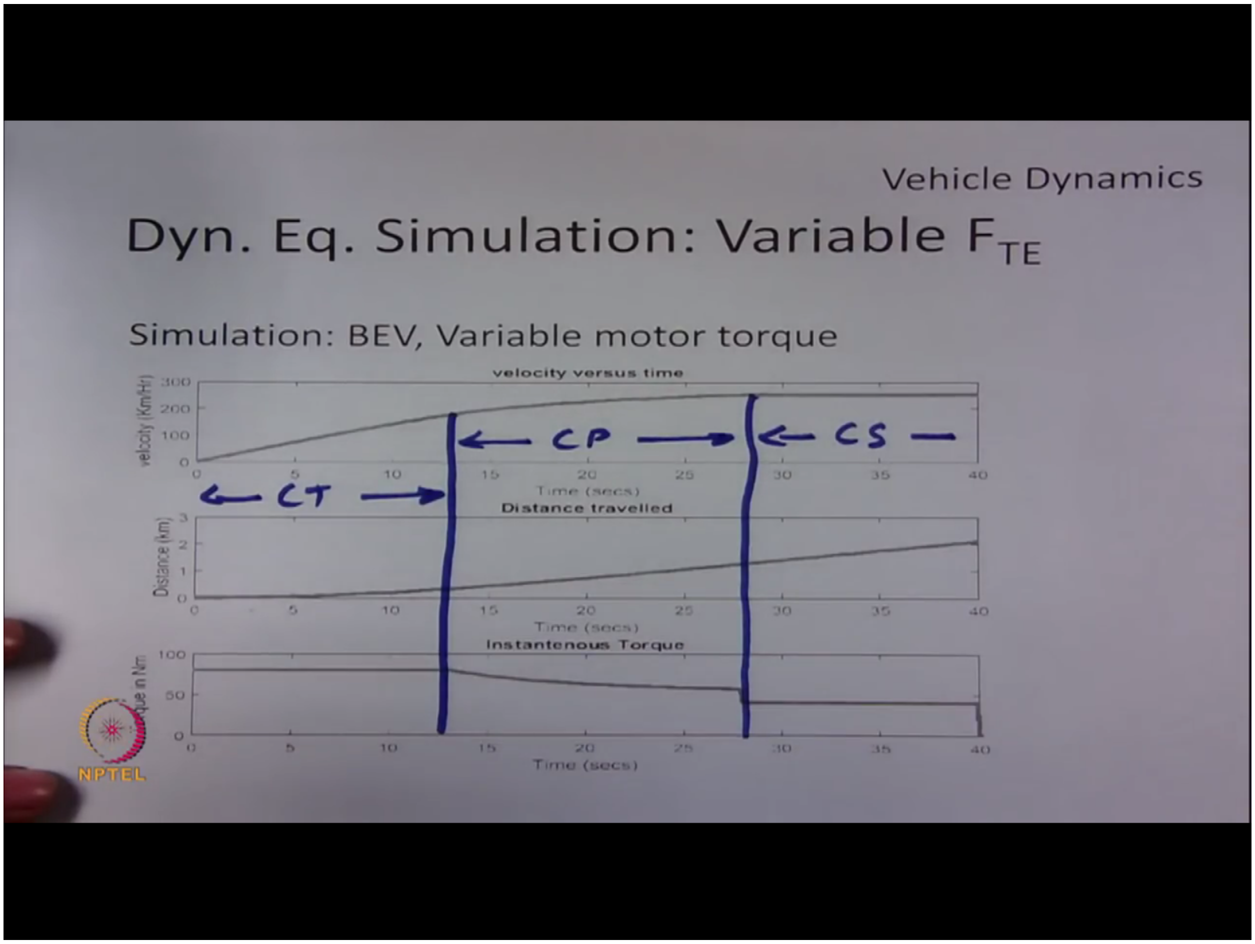
Similarly if we see the distance variation of the vehicle with respect to time, so the variation is not very high and its only in terms of few 100 meters. So at the end of you know, constant power region, which is around 27 seconds. So you can see the... the error is basically, you can say may be 200 meters. So this is the error in distance at vehicle velocity equal to V max.

(Refer Slide Time: 30:48)



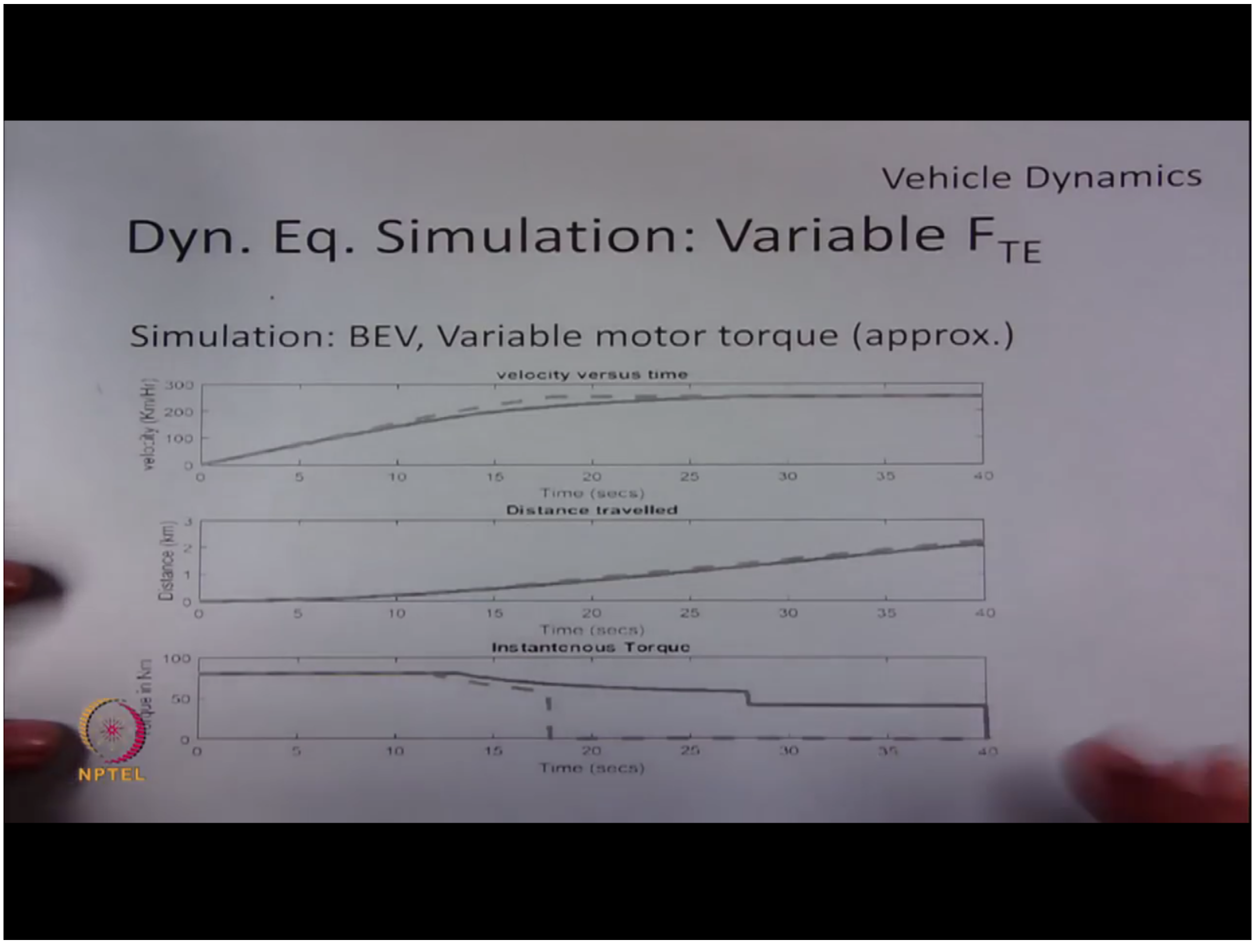
Now let us try to quickly see all these variation, there is velocity variation with respect to time, distance, with respect to time and torque variations with time, in a same graph, so where we can appreciate the variations. So basically, so this is basically constant torque region, this is constant power region, and this is constant speed region.

(Refer Slide Time: 31:35)



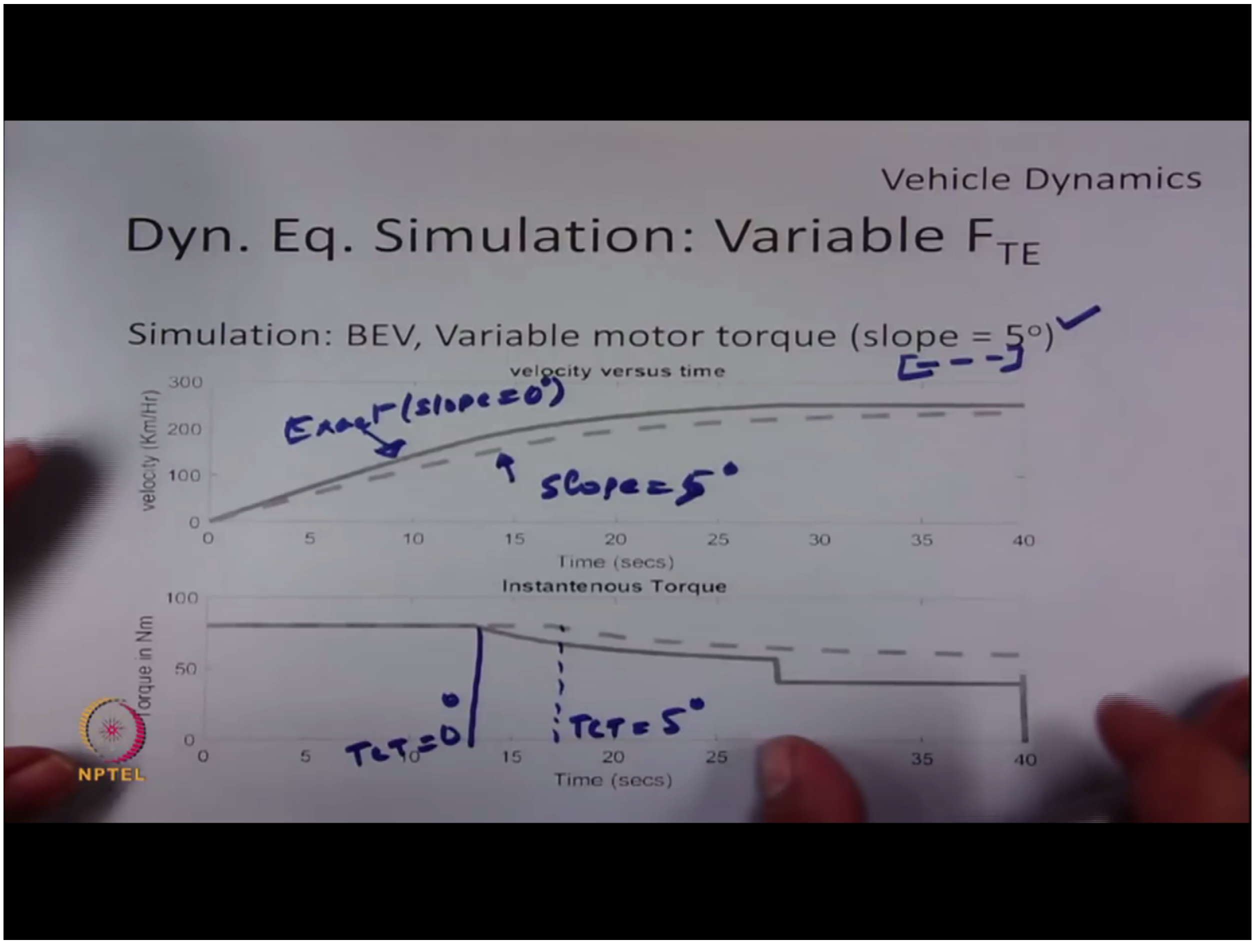
We can also combine the approximate variation also in the same graph as exact graphs, so we can see that the constant power region is very low in comparison to constant power region of the exact curve.

(Refer Slide Time: 31:58)



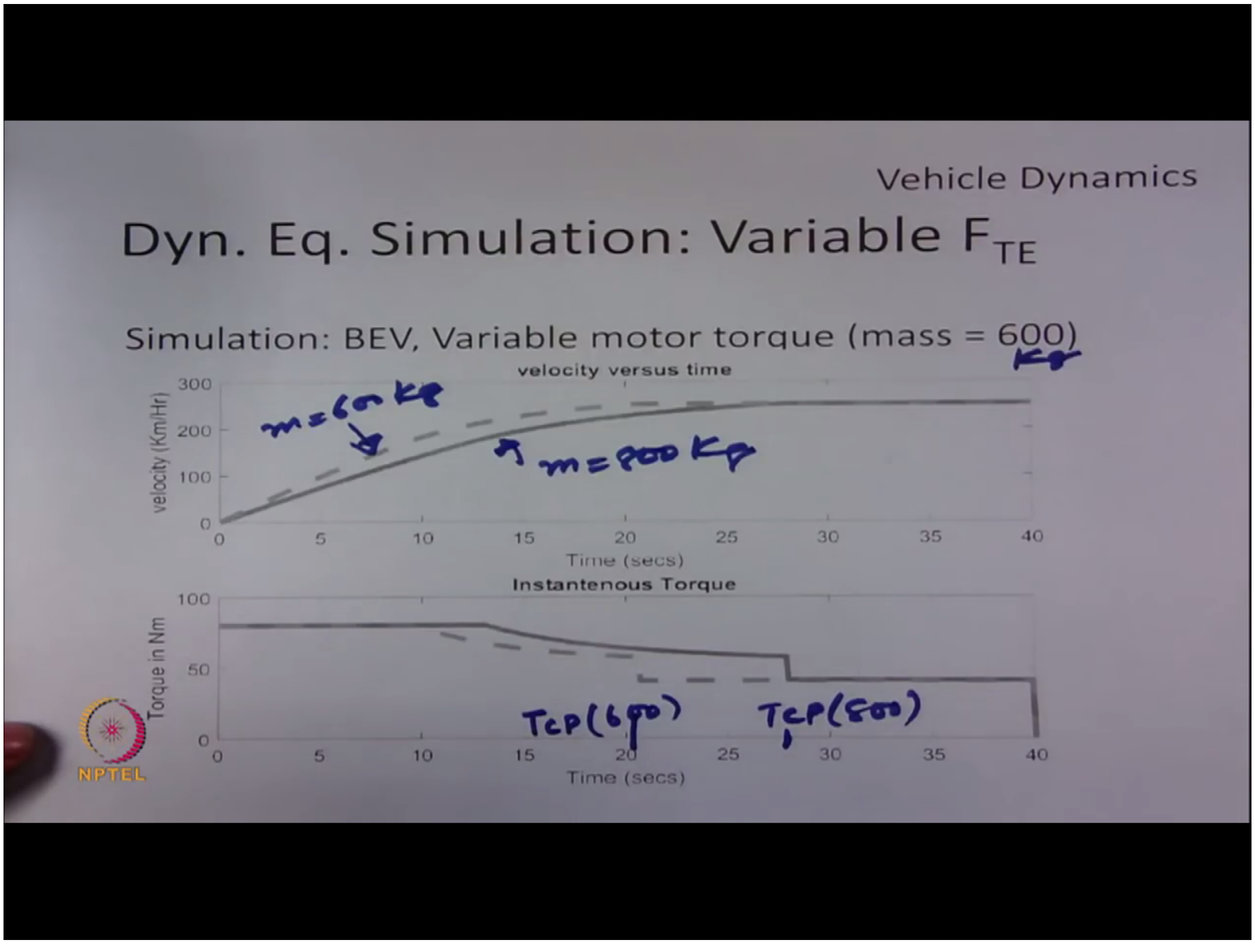
So once we have the simulation with us with exact variation of velocity with respect to time, distance with respect to time, as well as torque variation with respect to time, we can do some performance analysis for different kind of conditions. So let's say, if we assume that the vehicle is driving on a slope of constant 5 degrees, then what will be the vehicle performance in different regions of operation. So again this will be plotted in the dotted curves. So the… this is exact curve and this is the condition of, with slope equal to 0 and this is with slope equal to 5 degrees. So we can understand that if there is a slope, we now have a gradient force also to cater and therefore the acceleration has become slower, compared to the case where the slope is 0 degrees and we can also see that since the acceleration has become slower, it will take more time to reach the rated speed, so rated speed is reaching at around 17. So this is the TCT for 5 degrees slope and this is TCT for 0 degree slope, so you know, the constant torque region has become more, because of the low acceleration due to slope.

(Refer Slide Time: 34:11)



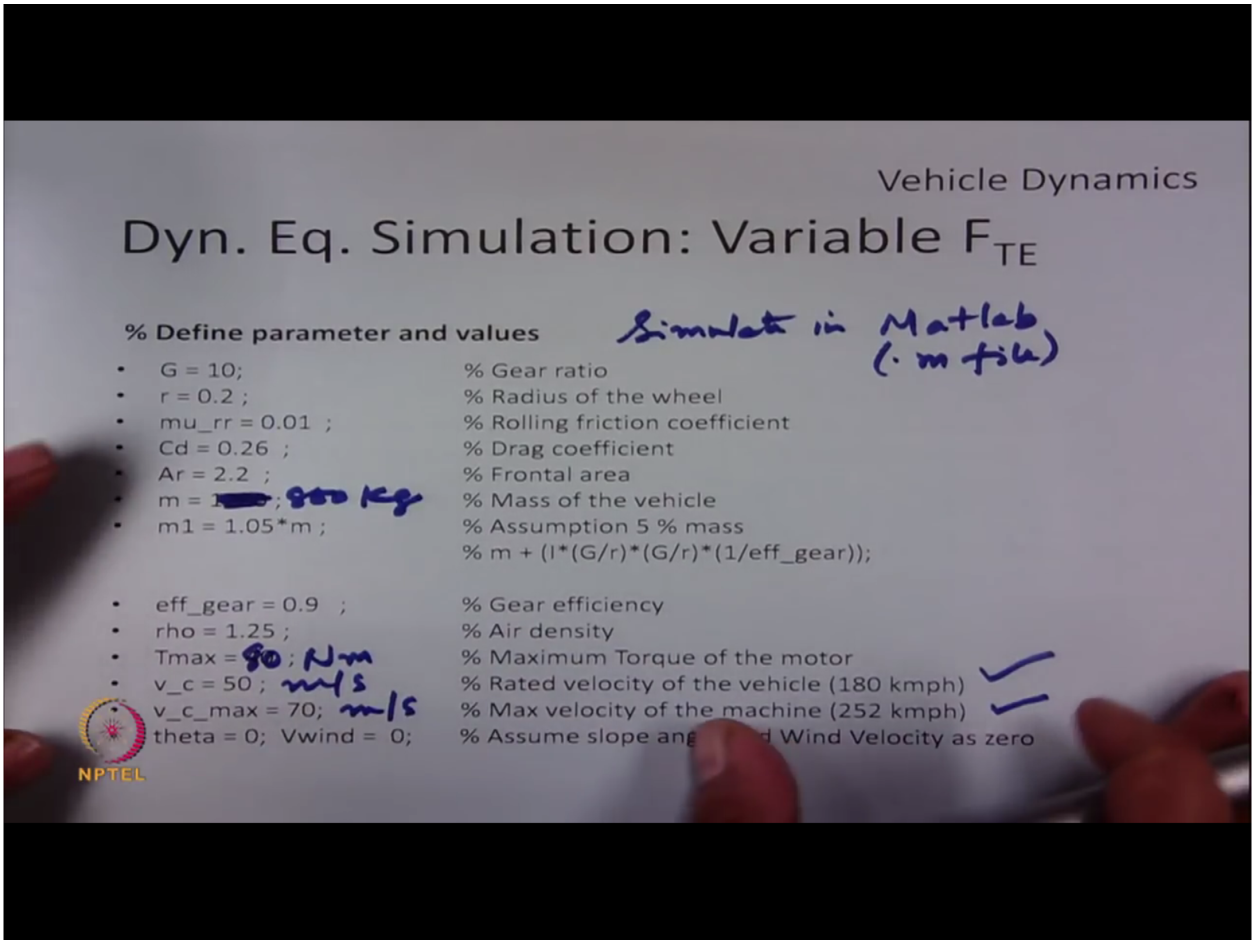
We can also see the variation in performance of the system when the mass is reduced from 800 kg to 600 kg. So if we reduce the mass of the vehicle, we expect a faster acceleration. So we can see that also in that graph. So this is basically the case where mass is 600 and this is mass is equal to 800 kg. So when we reduce the mass, the acceleration becomes faster and we can reach the V max velocity earlier. We can see that the V max reached in, so this TCP for 600 kg is around 20 seconds and it is TCP for 800 mass. The system is able to accelerate faster.

(Refer Slide Time: 35:35)



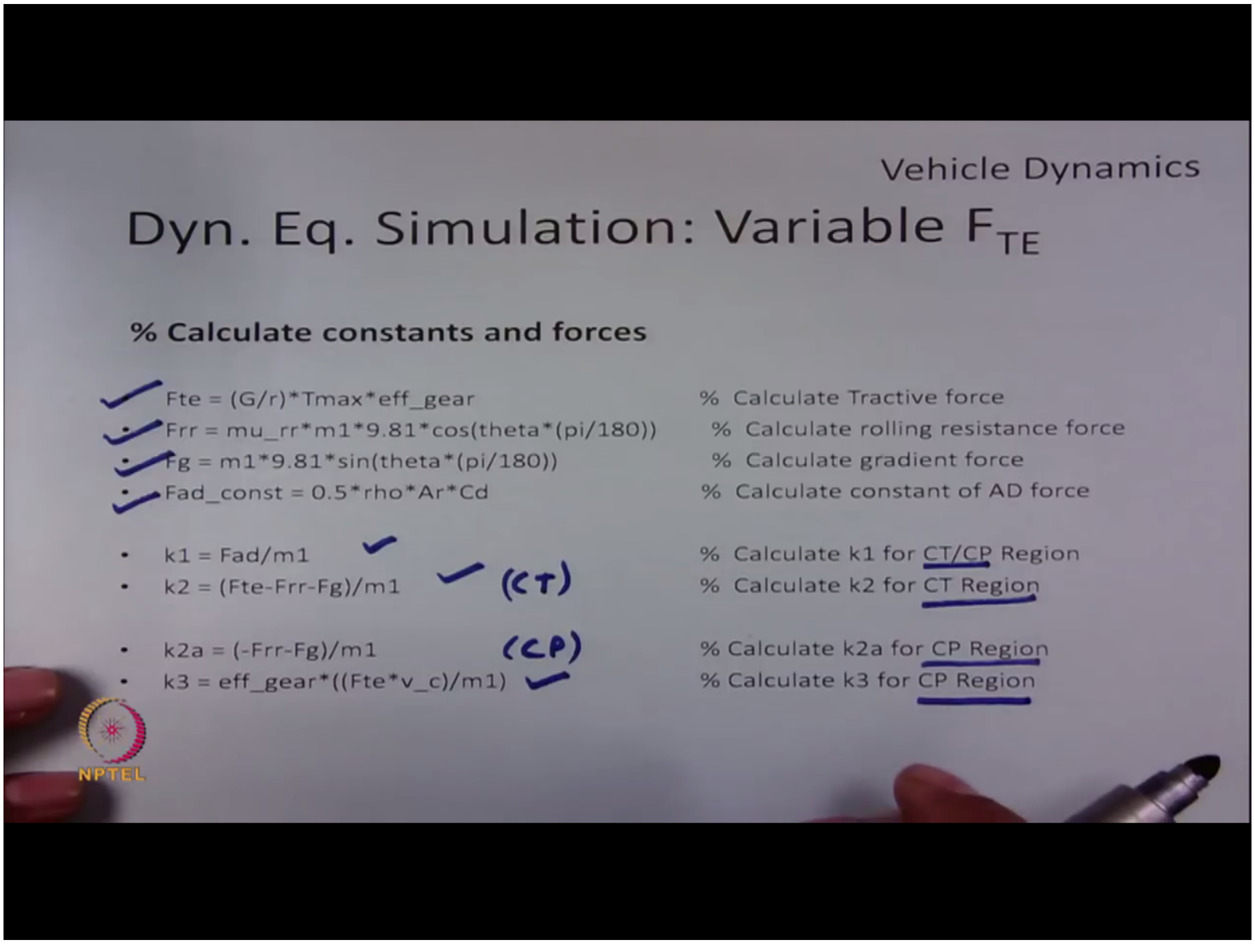
So this was you know, some of the performance checks we can do when the simulation is with us. Now let us try to see the kind of simulation file. So if you want to simulate in MATLAB, basically in dot M file, we can do this kind of parameter definition and locating values very similar to what we have seen in the analytical calculation. So we will just put all the values of gear, radius, Mu RR, CD, AR, mass, efficiency of gear, Rho, Tmax. So this is basically 80, so 80 Newton meter, VC of 50 m/sec and 70 m/sec. We know that 50 m/sec means 180 km/hr and 70 m/sec is 252 km/hr. So the mass is basically 800 kg in this case. So as I said the mass is 800 kg and the torque is 80 Newton meter.

(Refer Slide Time: 37:02)



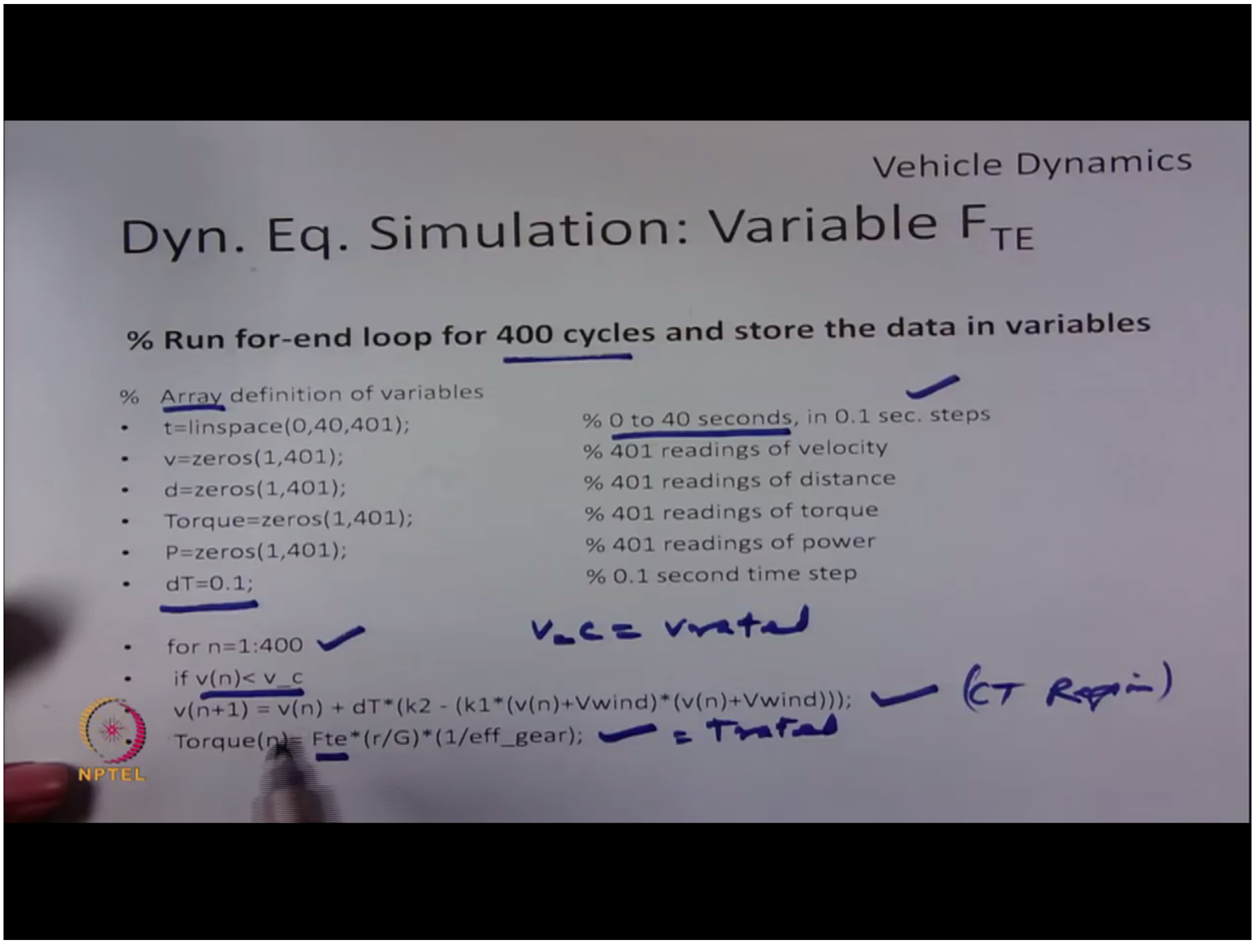
Once we define the parameters and the variables, we can start with the calculations of forces, so we can calculate the tractive effort, the rolling resistance force, gradient force, and the constant term of the aerodynamic force. And so once we have this all forces and constants with us, we can calculate the K constant, so K1 K2 K3 and we have a new constant, which is K2A. So K2, which is FTE – FR – FG/M1 in, so this is constant torque region K2 and this is basically K2 for a constant power region. So K1 is constant for both these things, but K2 is different for constant torque and constant only comes at a picture in constant power region.

(Refer Slide Time: 38:16)



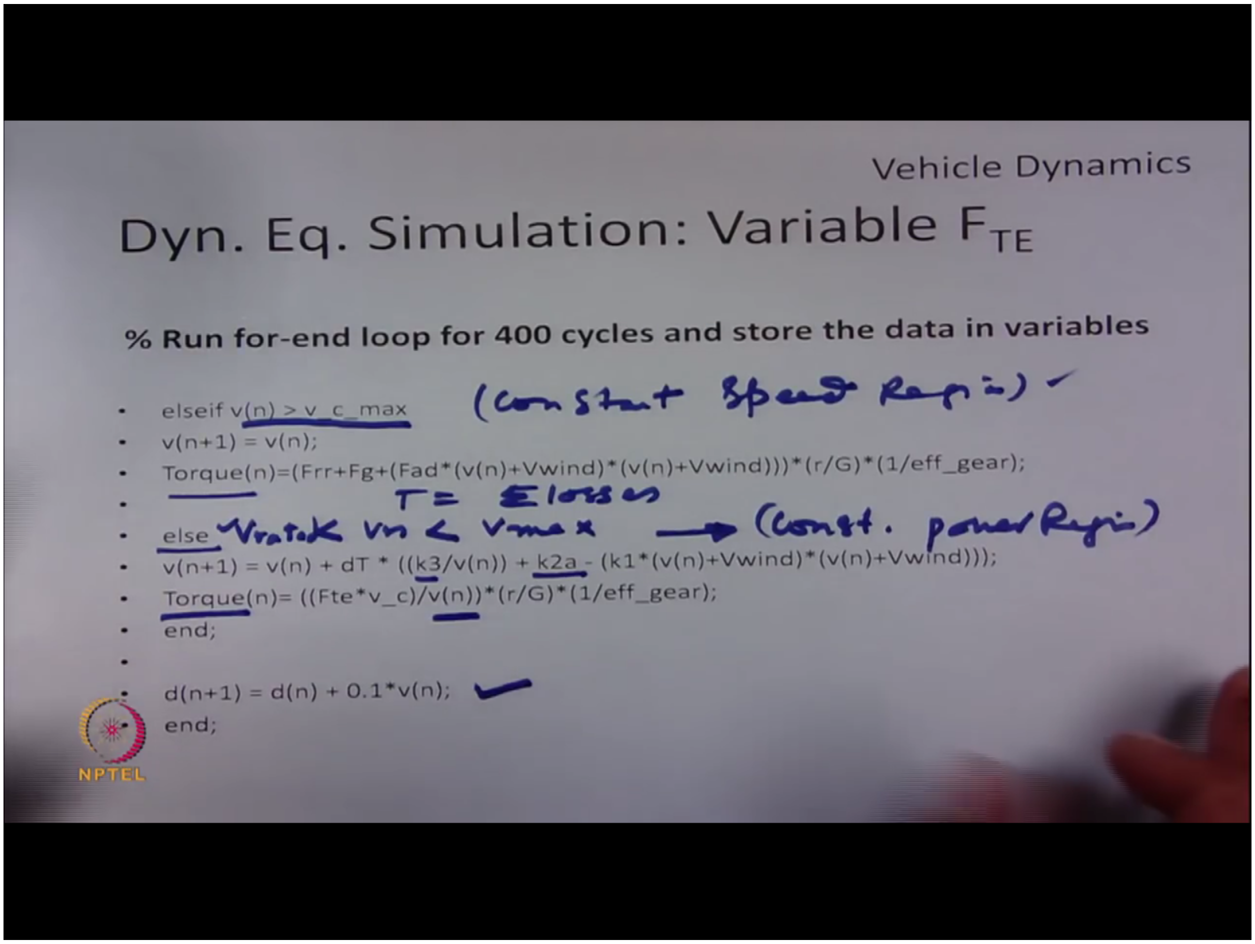
So once we have defined the variables and calculated the basic forces and the constants, we can write the actual for loop. So as we have seen in previous simulation file, we have to define the arrays, so in this case since the TCT is around 13 seconds and TCP is around 27 seconds, so we thought it is reasonable to do the simulation for 40 seconds and with the time step of 0.1 seconds. So DT is 0.1. So we have to define, you know, velocity array, distance array, torque array. And so we will simulate for 400 array number, since 40 second with the time step of 0.1 means 400 cycles. So we have three regions to simulate. So we have to simulate the constant torque region, the equation is different, in constant power region the equation is different. So we have different equations for constant torque, constant power, and constant speed. So when the vehicle velocity is less than VC, so VC is our V rated. So when the velocity of the vehicle is less than V rated, we have to write the equation pertaining to constant torque region. So this is constant torque region velocity equation in terms of VN+1 and VN and the torque will be constant, so FTE is constant and torque will be equal to T rated. So this calculation has to be done in constant torque region.

(Refer Slide Time: 40:32)



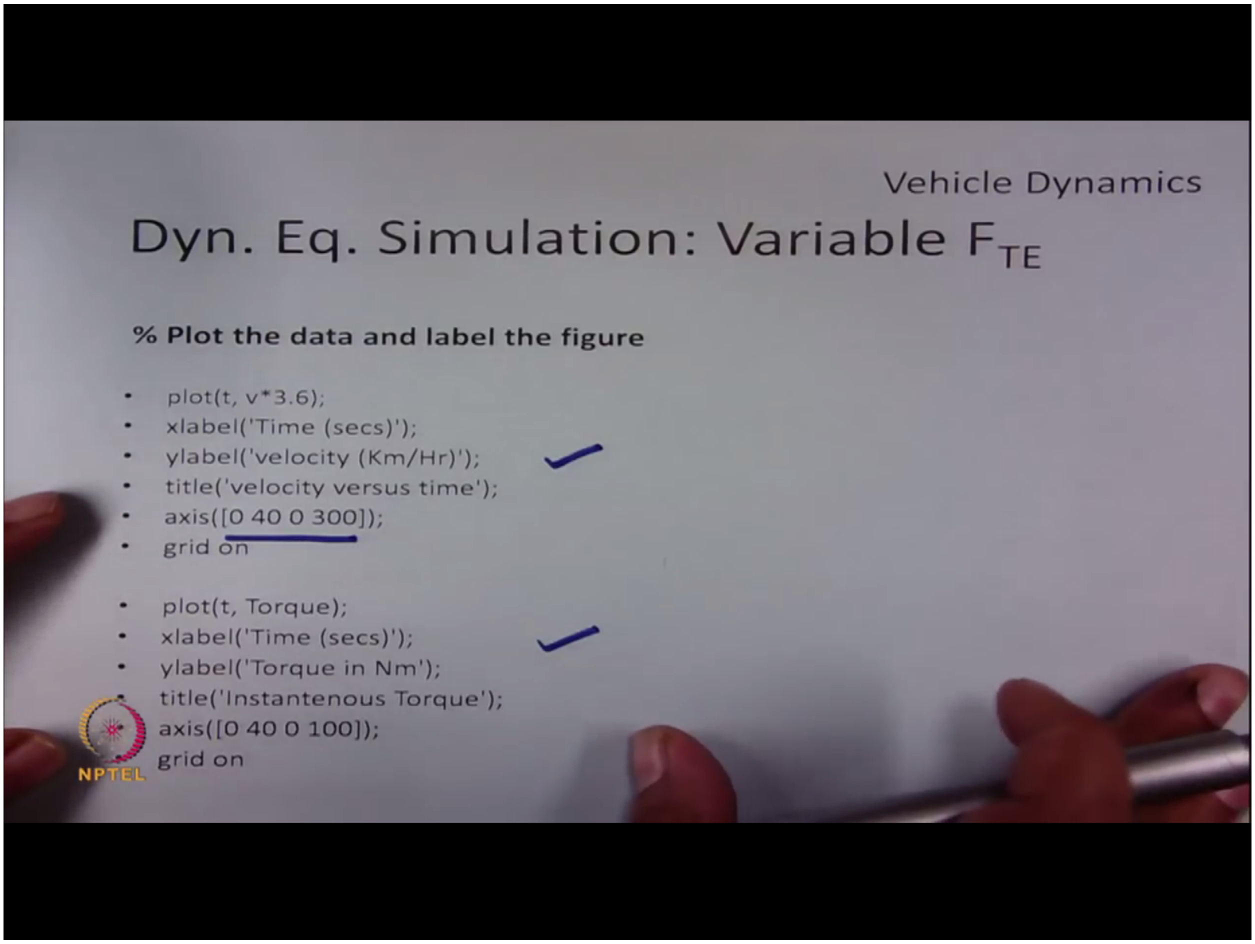
When the velocity of the vehicle is greater than VC max, we basically are in constant speed region. So the speed will be constant and there is no acceleration. So torque is equal to just all the losses. Else, else means the condition where VN is between V max and V rated. So this is basically a constant power region. So it... so where we have to write the velocity equation in terms of K3, K2A, and K1, and the torque will be inversely proportional to vehicle velocity. So all these regions we have to define and they will be you know, toggled with respect to the variation of vehicle velocity. So when we are less than V rated, it is constant torque, when we are between rated and max we are in constant power mode and beyond V max, we are in constant speed mode. We can also calculate the distance as we have done in the case of constant FTE.

(Refer Slide Time: 42:25)



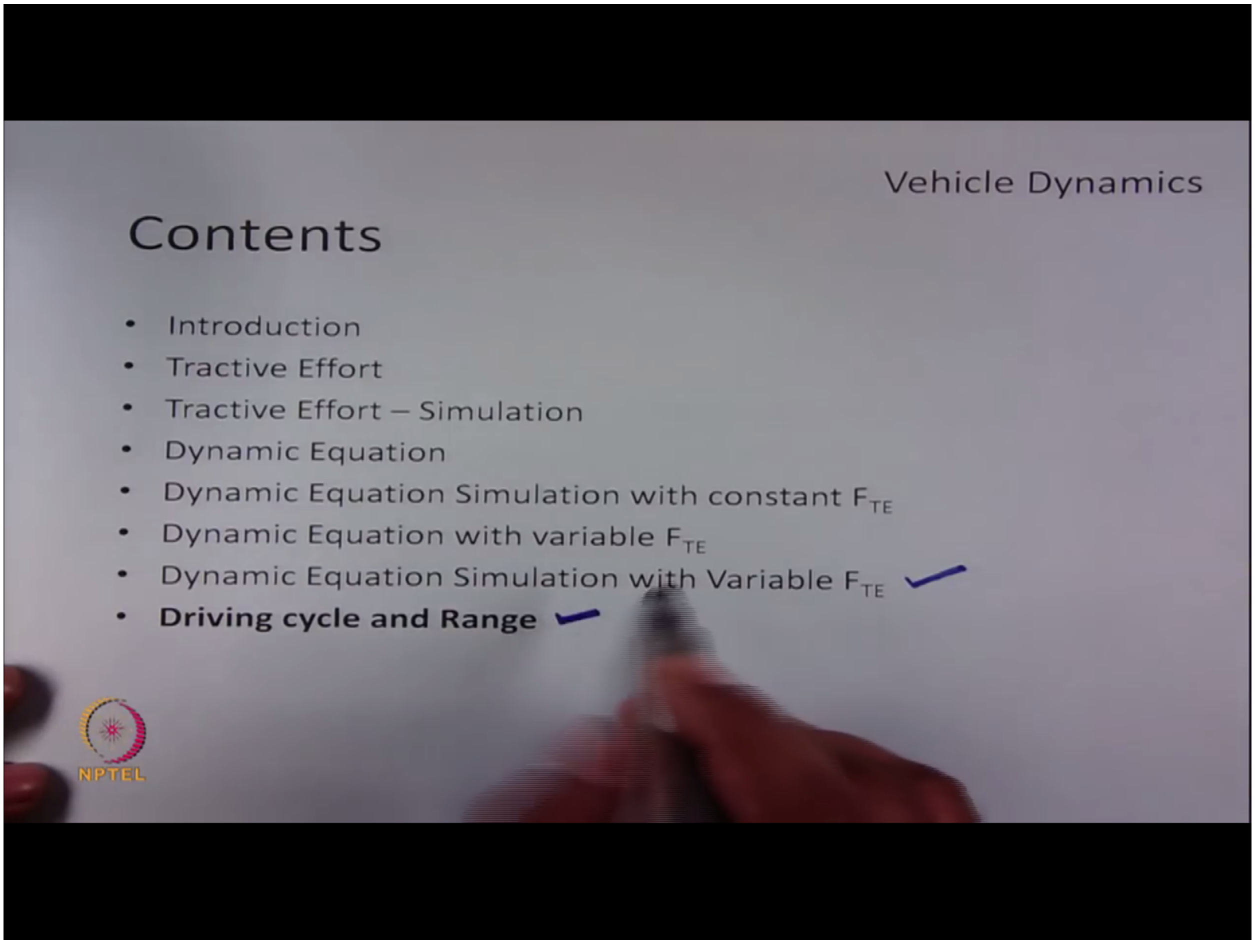
So once the whole program is written, we can plot this in simple plots, we can label them and put a proper access so that the clarity of the graph is maintained.

(Refer Slide Time: 42:42)



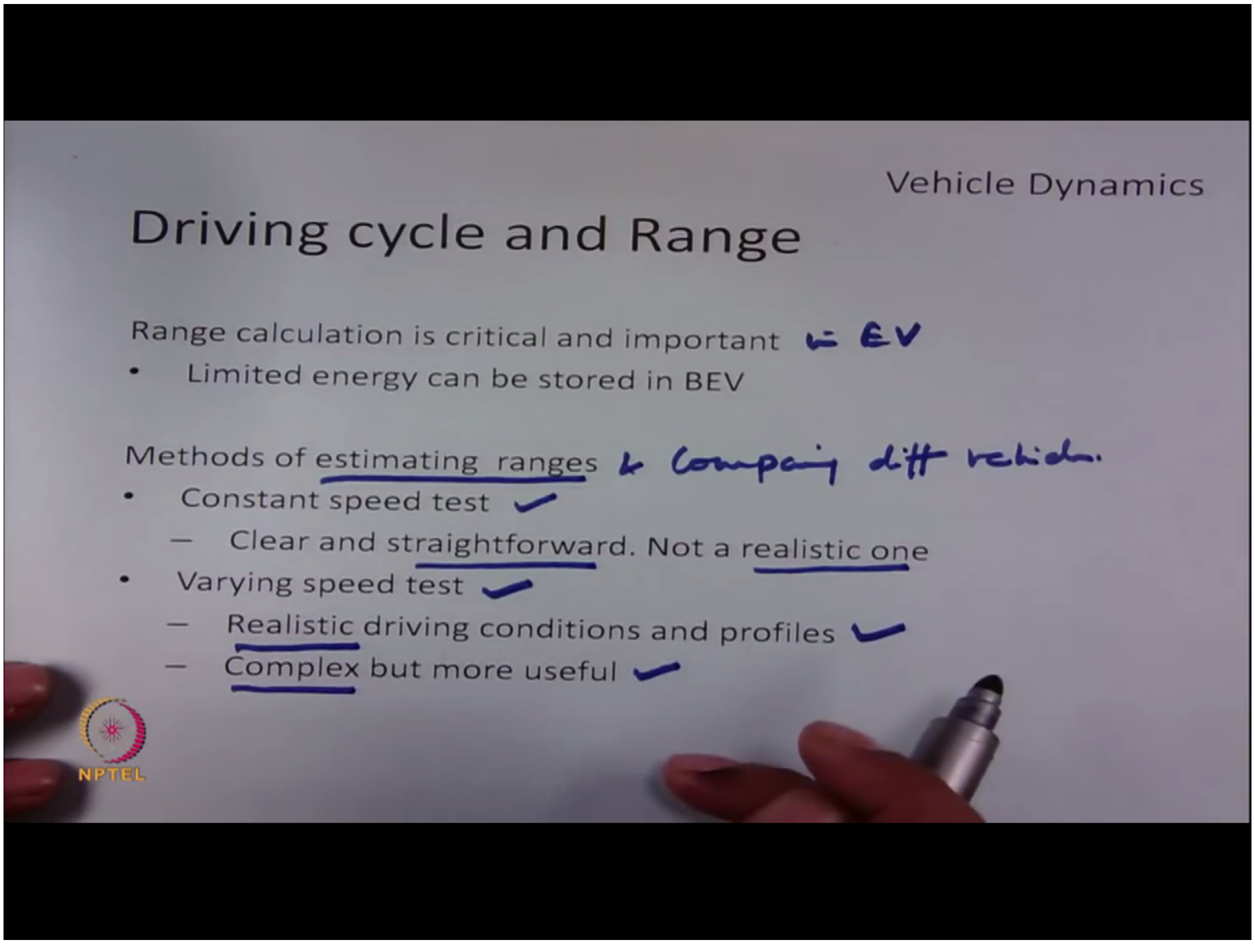
So this is the end of the discussion under the topic dynamic equation simulation with variable FTE. Now let us go to the next topic, which is understanding of different driving cycles and range.

(Refer Slide Time: 43:07)



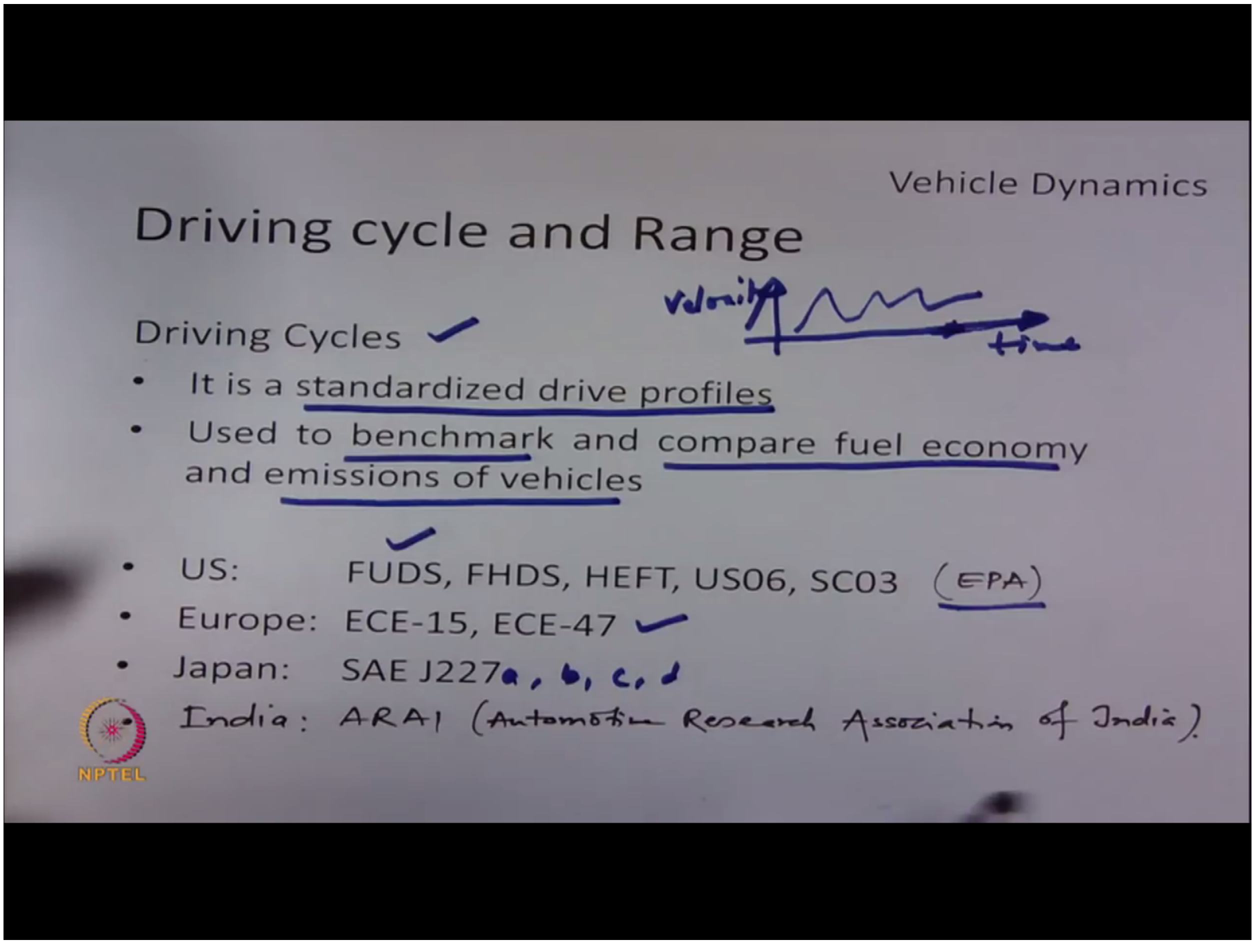
So range calculation is very critical in important in EV application because we have a limited energy that can be stored in batteries. So what are the different methods of estimating ranges and also comparing different vehicles. So one is constant speed test. We operate the vehicle in constant speed and try to understand the range. So this method is very straight forward, but it is not a realistic one, because a typical vehicle will not operate in constant speed mode all the time. So the practical condition is or the realistic condition is varying speed. So we do mostly varying speed test to test the range of a vehicle. So we have to define different driving conditions and profile. So we have profiles for urban transport, profiles for highway driving, we can have profiles for you know hill type of terrain etc. So this kind of test is complex, but it is useful and can be applied for vehicle range performance testing.

(Refer Slide Time: 45:02)



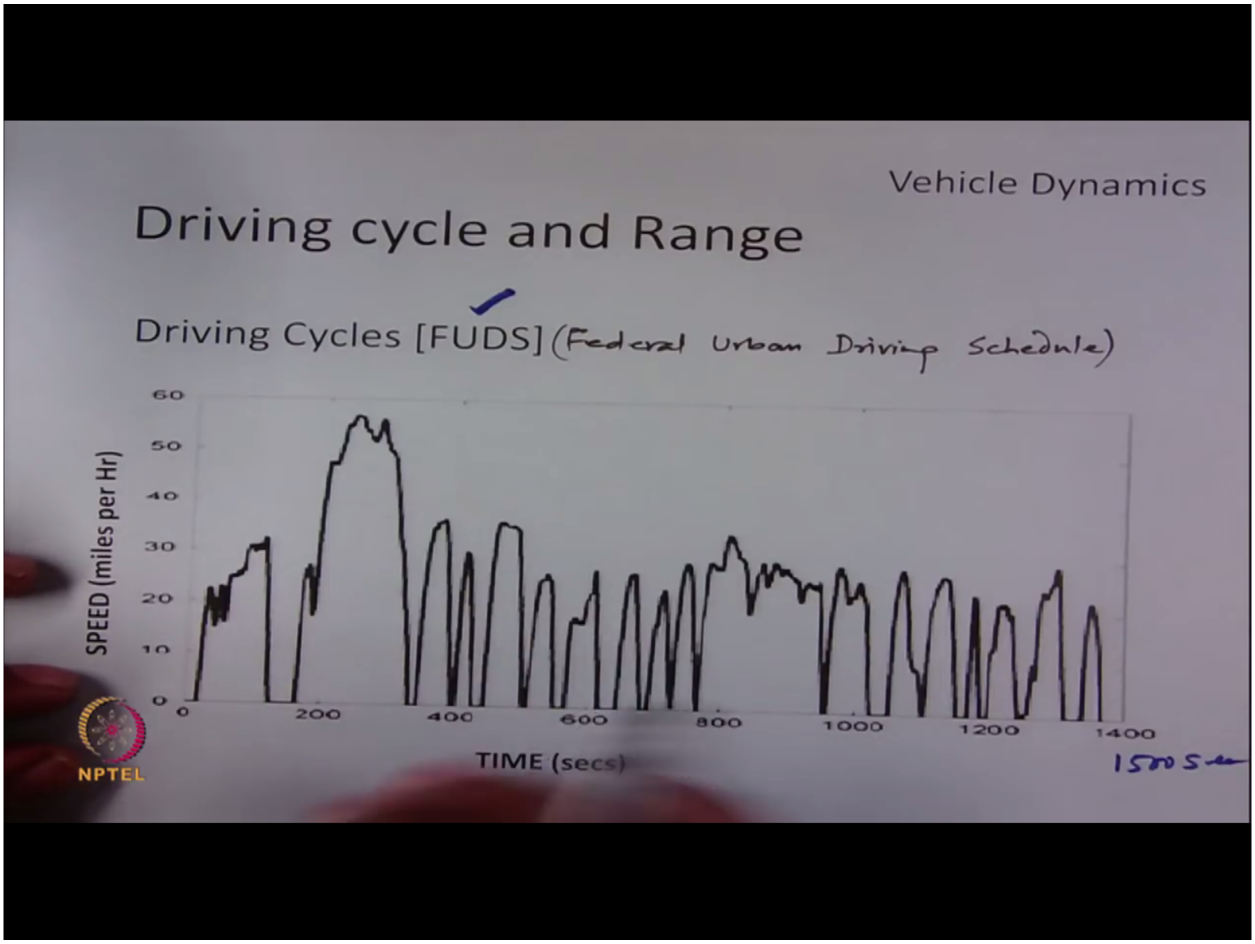
So driving cycles is one of the ways to taste the range. So different regulation agencies have defined standard drive profiles, means they will define you know, graphs of velocity with respect to time. So it can be any variation. So this... and so vehicle range estimation will be done based on this different kind of profiles. So this range calculation will be used to benchmark and even compare the fuel economy as well as emissions of the vehicle. So it has multiple purposes. So in United States, there is agency called EPA, this is environmental protection agency. So it has defined various kind of driving cycles and profiles. So FUDS, which is federal urban driving schedule, this is federal highway driving schedule, this is you know, high energy and fuel efficiency kind of graphs. In Europe we have curves or driving cycles such as ECE-15, ECE-47. In Japan there is a driving profile, which is SAE J237 and it has all the variation, A, B, C, D, based on the different time that is involved. In India also the agency known as ARAI, which is basically in Pune, so it is automotive research association of India, so it defines lot of profiles for Indian urban and highway driving conditions.

(Refer Slide Time: 47:38)



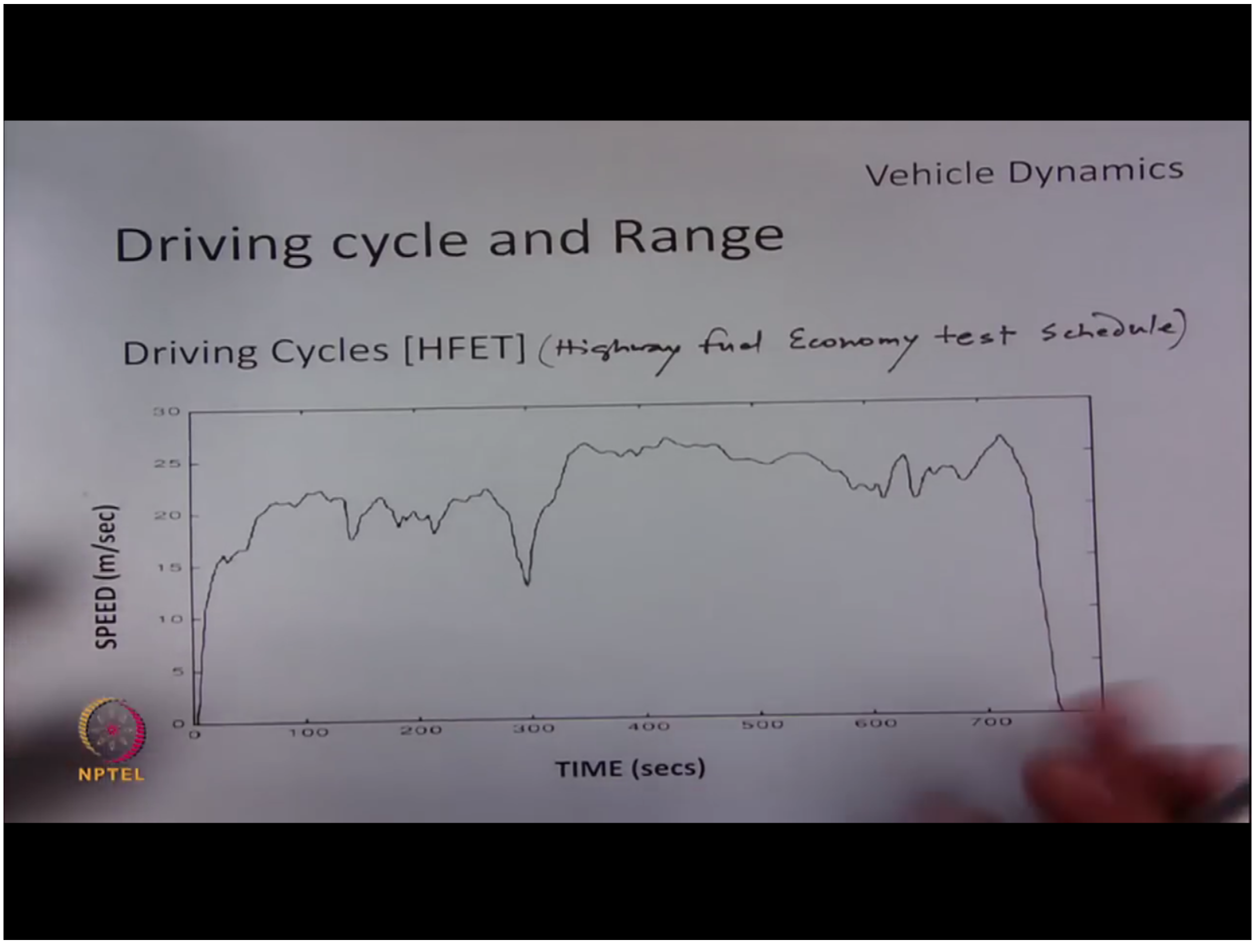
So let us see some of these driving profiles. So in FUDS, basically this kind of variation of speed versus time is given so it is basically for 1500 seconds. So it has very high variation of speed with respect to time. So this is a typical, basically urban kind of transport, where you have frequent start-stops.

(Refer Slide Time: 48:10)



So let us see the driving profile of a HFET driving profile. So it is highway fuel effi… economy test schedule. So we can see that the variation is not high here and mostly you are operating at higher speeds most of the time.

(Refer Slide Time: 48:33)

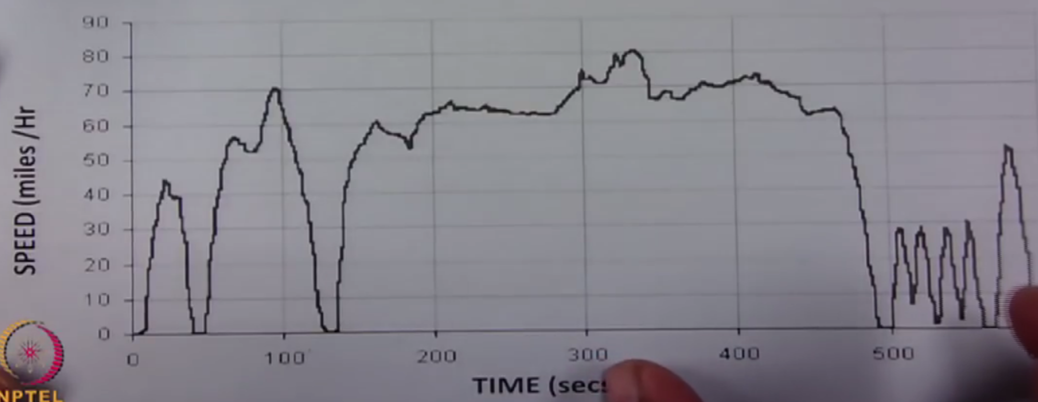


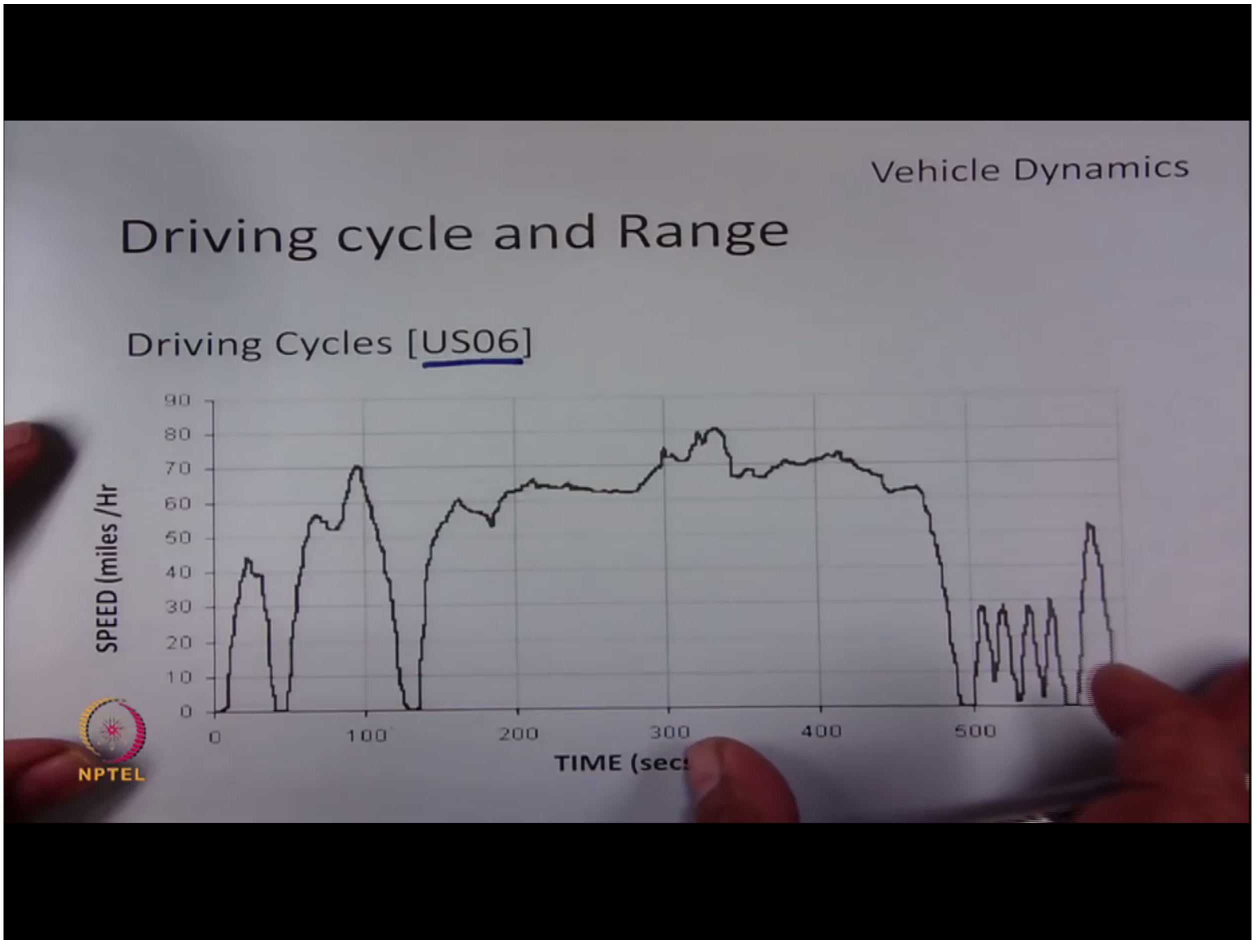
There is another driving cycle, which is known as US06. So it takes care of highway conditions and even some of urban conditions and it has very high variation of DV/DT, so this acceleration is very high. So it's quiet rigorous driving cycle as per vehicle performance is concerned.

(Refer Slide Time: 49:02)

Driving cycle and Range

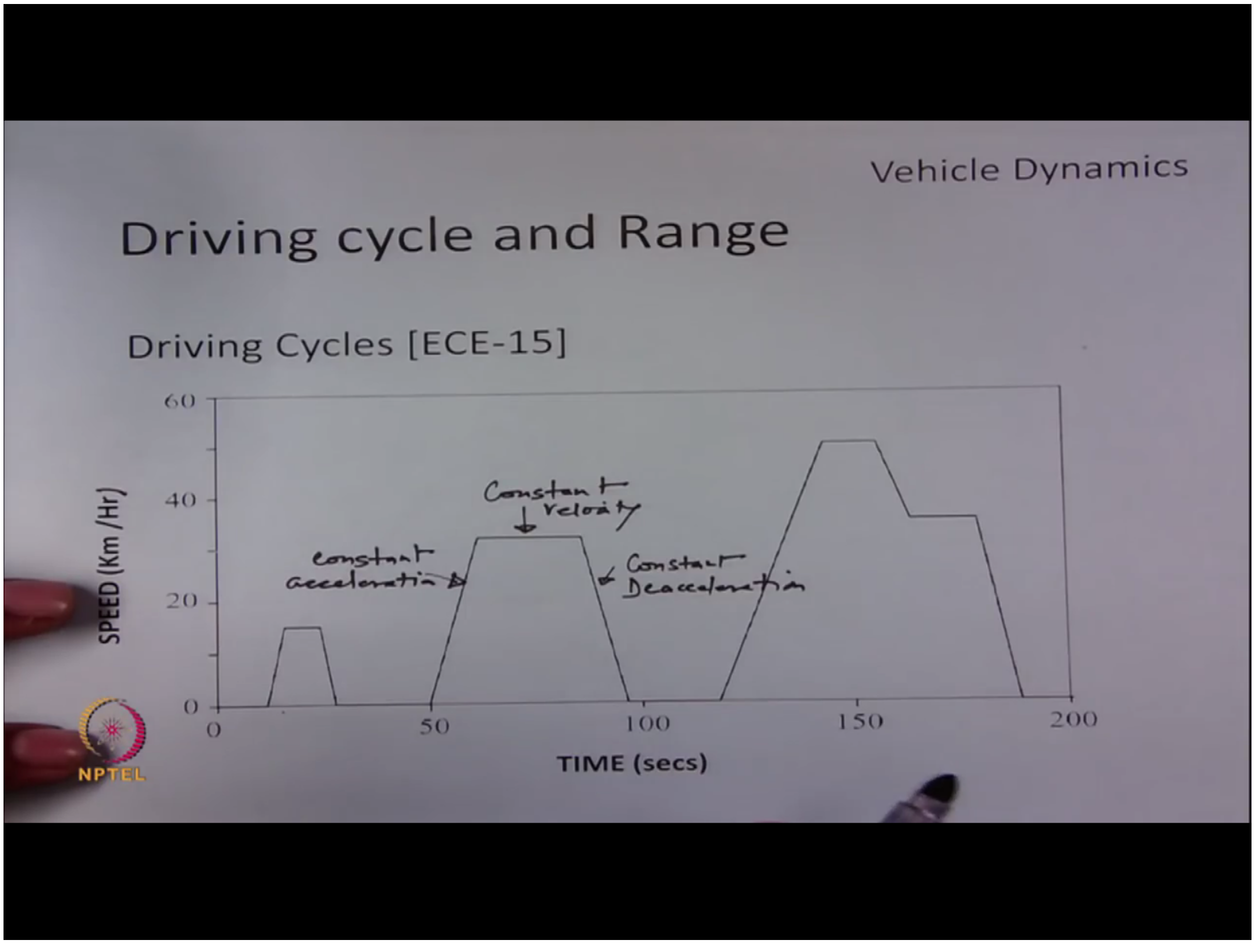
Driving Cycles [US06]





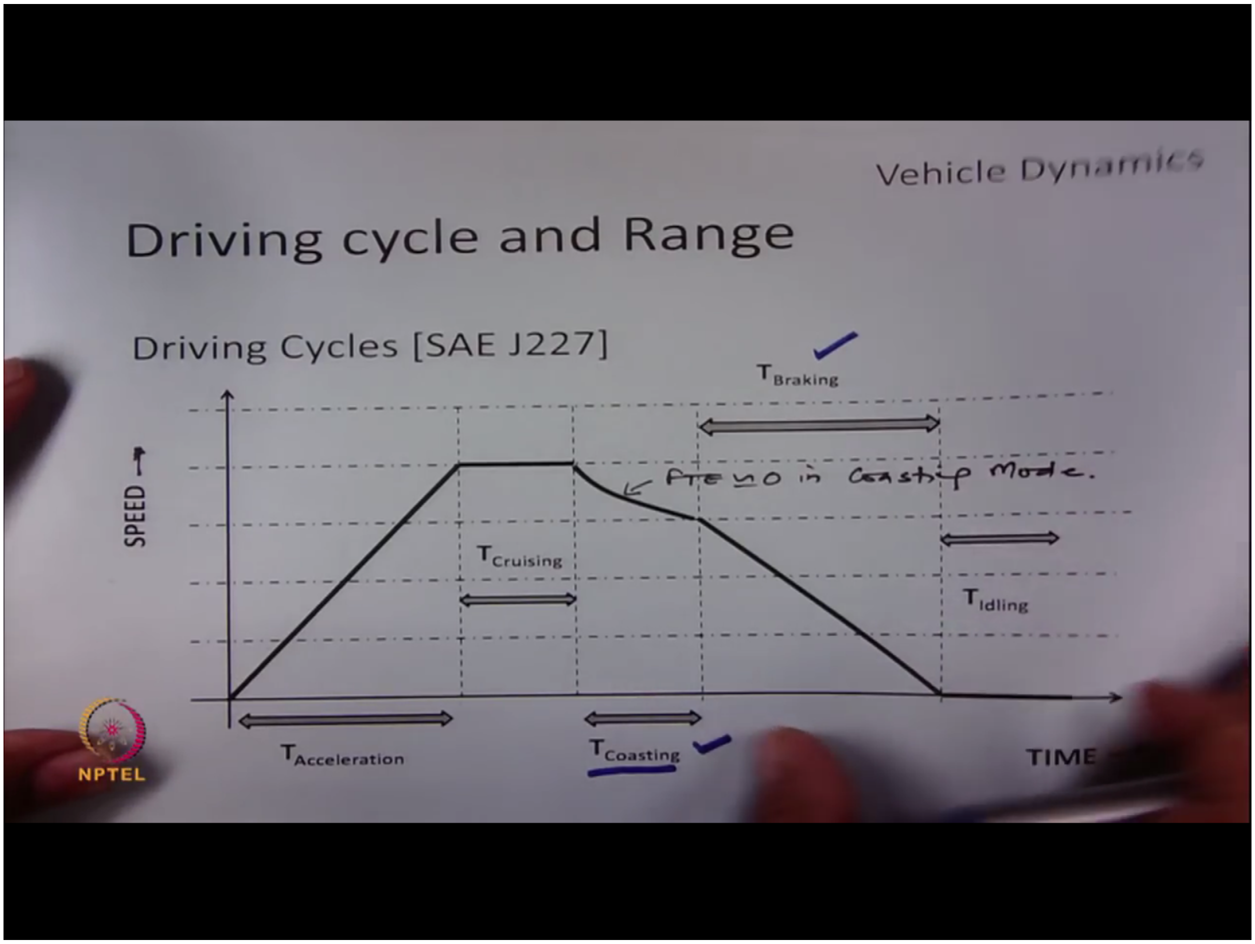
So this is a European driving cycle, which is known as ECE-15. So this driving cycle, you know, mostly have very simple driving profiles. So it has constant acceleration, V rate, constant velocity parade and constant de-isolation parade. So it has different magnitudes, so... of, so this acceleration is for less time, this acceleration is for more time, this acceleration is for still more time. So such kinds of variations are defined to test the vehicle range performance.

(Refer Slide Time: 49:43)



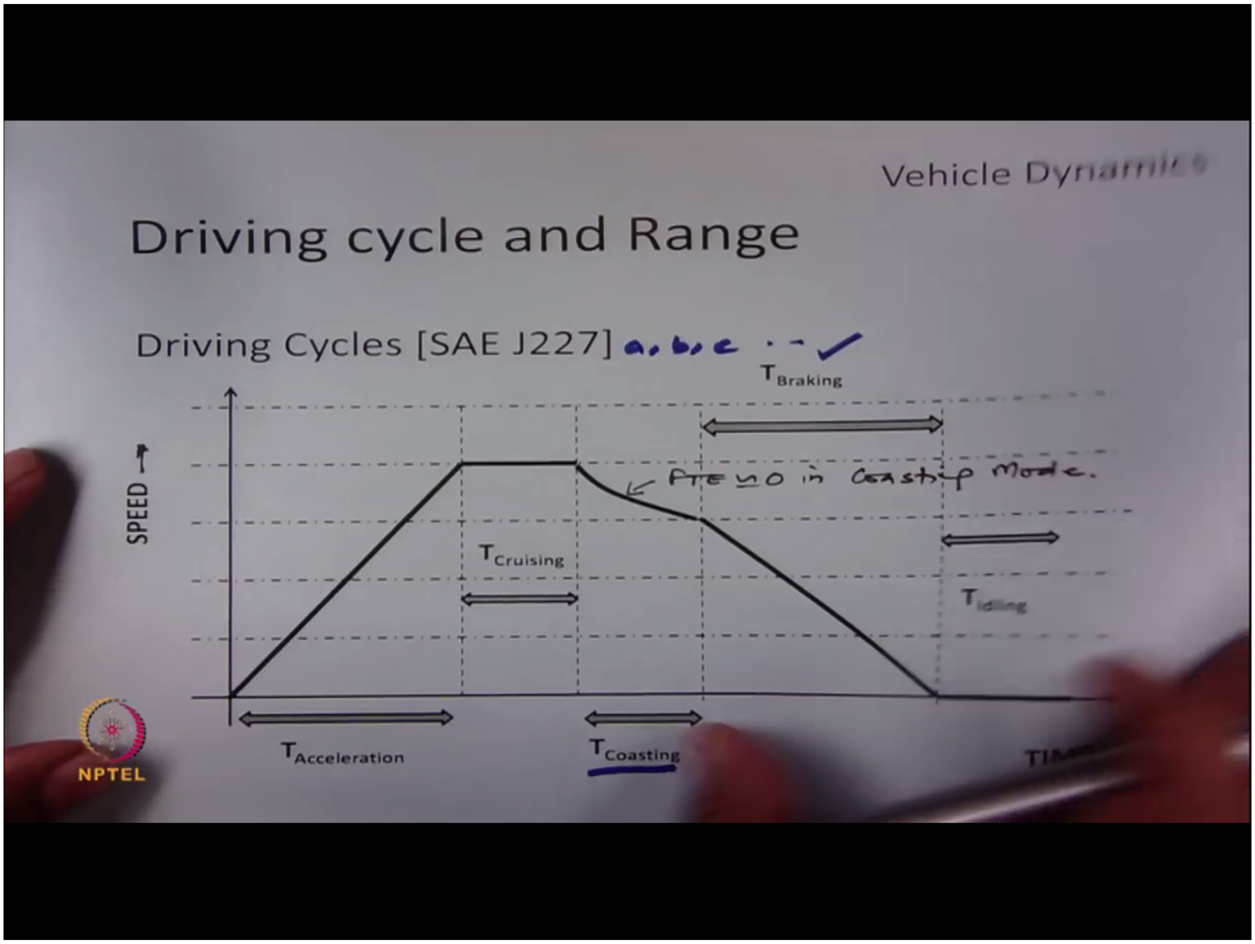
So we have talked about a driving cycle which is known as SAE J227. So in this a driving cycle, we constructed as a addition of different acceleration and constant speed. So we can see that we have a acceleration period, which is defined as T acceleration, we have a high speed cruising for period of de-cruising a de-isolation or T breaking here, a idealing period of where there is no speed and there is a coasting. So T coasting is a period, where the tractive effort is reduced to 0, so it is kind of vehicle is floating and inertia and road conditions.

(Refer Slide Time: 50:42)



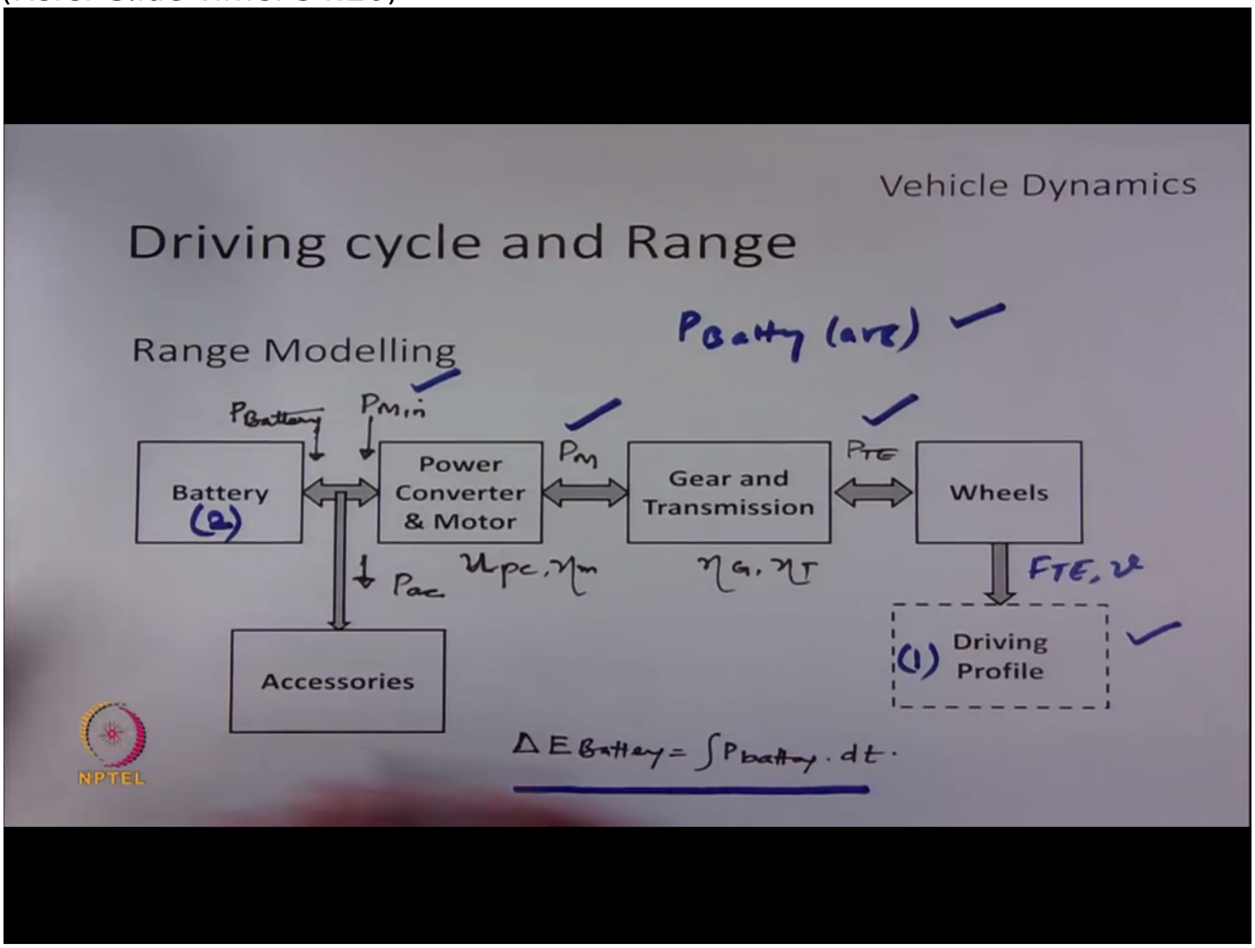
So different variation of this is available in terms of J227A 227B, where this different time such as T isolation, T cruising are varied for different A B C.

(Refer Slide Time: 51:15)



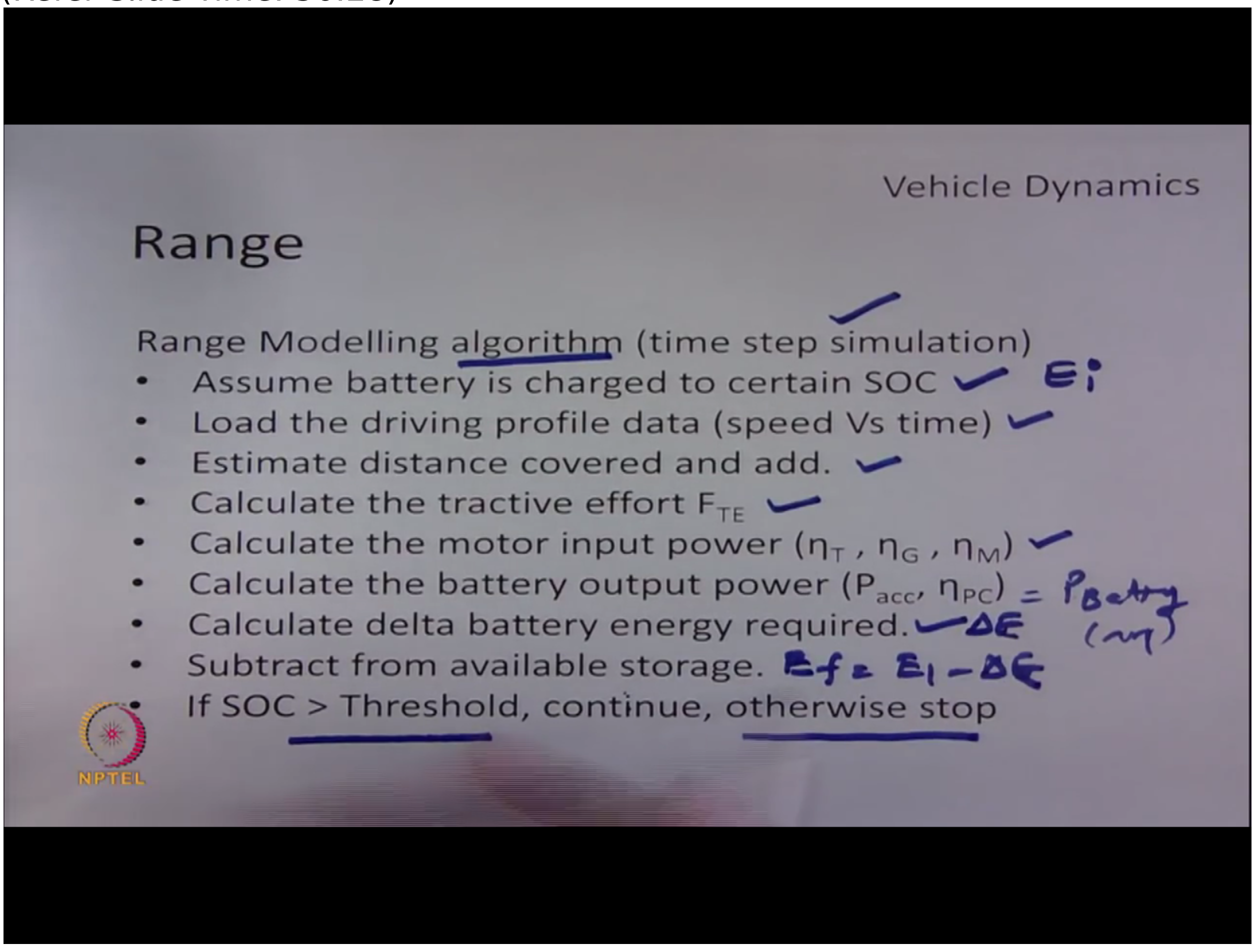
So how to calculate range of a vehicle. So when we want to calculate the range of the vehicle, you know, the first thing we have to define is the driving profile, so this is the first thing? On what driving conditions you are checking the range? Secondly, what is the battery capacity? So these two things are important to start with. So these two things should be known initially. Afterwards for this driving profile, so which is a basically variation of velocity with respect to time, we have acceleration, de-acceleration, constant speed etc. The vehicle tractive effort and velocity will be known to us and we can calculate the power that is required by the vehicle in a time span. So these all are function of time. So once we know the efficiencies of gear and transmission, we can calculate the power that is required on the motor shaft. Similarly when we know the efficiency of motor and power converter, we can know the power required at the input of the power converter. We also have power required for different accessories such as basically steering, AC, and other loads, so this has to be added to accessory loads and together we can get the battery power. So this all things can be calculated as the function of time once we know the driving profile. So if we have the P battery available, we can calculate the average. So P battery average can be calculated. Once we know that and the total time elapsed, we can calculate the energy that is required from the battery. So you can see that when we are operating the

vehicle on a driving profile, the vehicle will run as long as the energy required by the vehicle is equal to the energy stored in the battery. So once the battery energy is all exhausted, vehicle has to come to rest and the distance travelled in that duration is the range. So it's very straight forward. So this calculation goes as long as the battery capacity comes below a threshold level, where we say that we cannot supply any more energy. So the distance travelled is the range.
(Refer Slide Time: 54:20)



So let us see the algorithm we do for range calculation. It is a time step simulation. So we have to model all the systems, so we have to model the transmission system, we have to model the gears, we have to model the motor, we have to model the power converter and even the power required for the accessories. So we have to have all these models with us. So we will start with a battery, with a certain accessory, then we will load the driving profile data. So for each variation of velocity versus time, we can calculate the distance, it is speed into time. Then we have to calculate tractive effort and we have to take care of all kind of efficiencies of different systems like gear, transmission, motor, power converter, and power due to accessories. If you have that, we know the total power that is drawn from the battery. Once we know that, we can calculate the P battery average and calculate the energy required. So we have to subtract the energy required that is delta energy from the energy initially. So we have to E initial – delta E we have to do. So once this becomes negative, we have to stop this simulation. So subtract from the available storage and if SOC is greater than threshold, we can redo the equations and we have to stop where the E final, which is E initial – Del E becomes negative. So there we have to stop this equation.

(Refer Slide Time: 56:28)



So that is all for today's interaction and we will continue discussion on the topic of vehicle dynamics in our next interaction. So thank you for listening the lecture.