Hello everyone welcome to NPTEL online course on electric vehicles. So let us continue the discussion on the topic of vehicle dynamics. So today we will discuss the next topic, which is deriving of different dynamic equations with variable FTE. So this is normally the case. So electric vehicle will be given variable tractive effort forces and different agents of vehicle states. So we will see this derivation of different dynamic equations in today's lecture.
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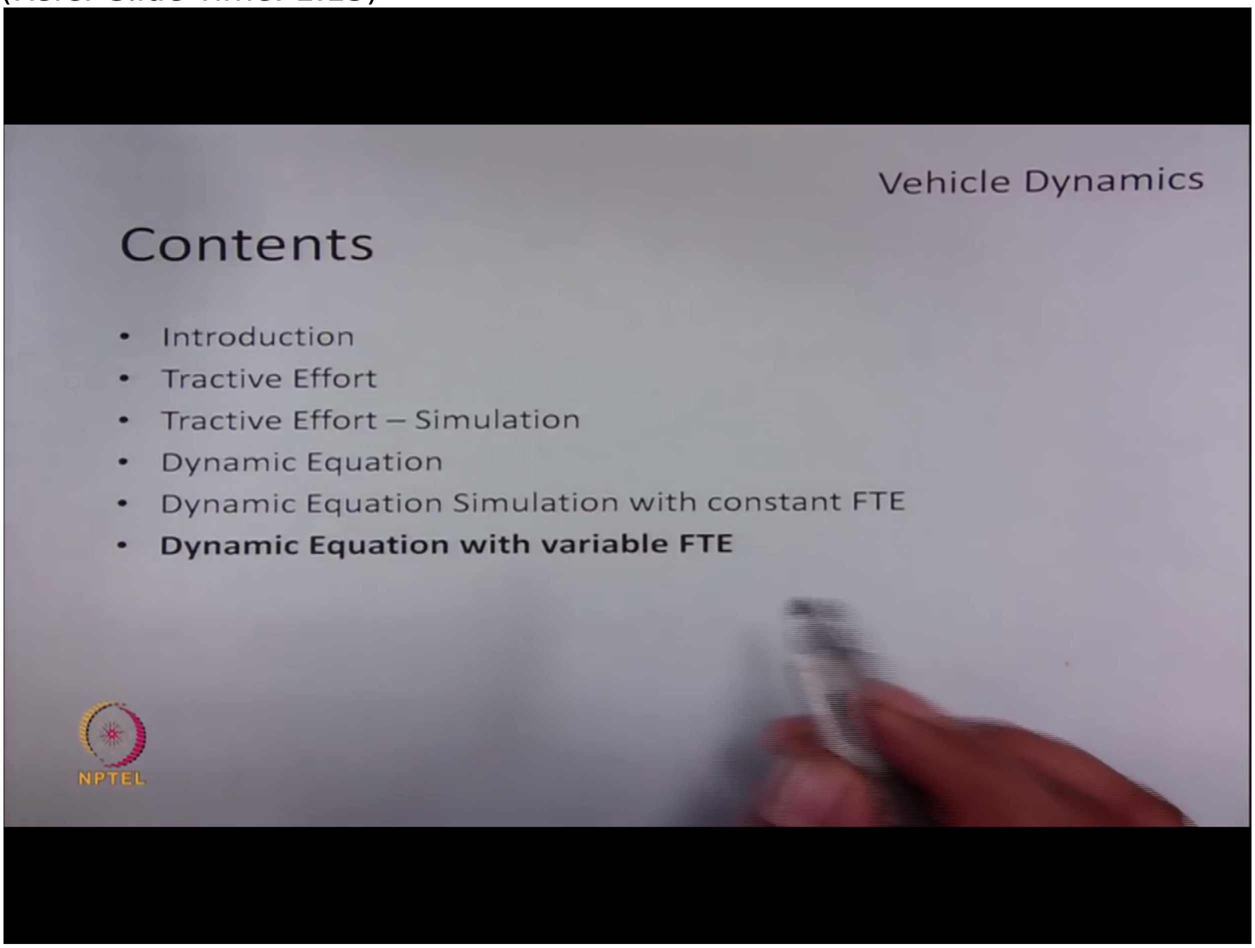

In a practical scenario, an EV needs to operate in three modes. So these three modes are with respect to the vehicle speed. In the first mode a initial acceleration is required for quick pickup. So as most of us know that when we start the vehicle, we would like to speed up the vehicle as fast as possible. So the initial pickup by means of a high acceleration is an important requirement. So in this mode, generally the motor will be asked to deliver the maximum torque capability of the machine. So as the vehicle speed increases because of acceleration, this mode can go up to the related speed of the machine. At rated speed and if the maximum torque is given, the machine will reach it's rated power capability and therefore this mode needs to be stopped when the vehicle speed is closed to rated speed. So this mode is known as wide open throttle mode in IC engine vehicles. So this IC engine vehicle, the throttle or the supply of the fuel will be given the maximum, such that the maximum acceleration can be obtained initially. So when we want to go for high speed operation of an vehicle, which is beyond the rated speed of the motor. The motor needs to operate in field weakening region or costume power region. We will see the details later. Since we are going beyond the rated speed of the motor, now the full torque cannot be given. Therefore, the acceleration or the torque in this mode is limited by the machine power rating. So they will be given within the limits of machine power rating. So when the required speed is reached in this mode and the vehicle need not be accelerated any more, we reach a region which is called as constant speed operation. So the speed will be operated at high speed without any acceleration. So it is a constant speed operation. So the torque from the motor will be required only to support the opposing forces due to rolling resistance, aerodynamic drag, or gradient force if we say slope, and all kind of losses due to transmission losses, due to gears, or machines, or power converter.

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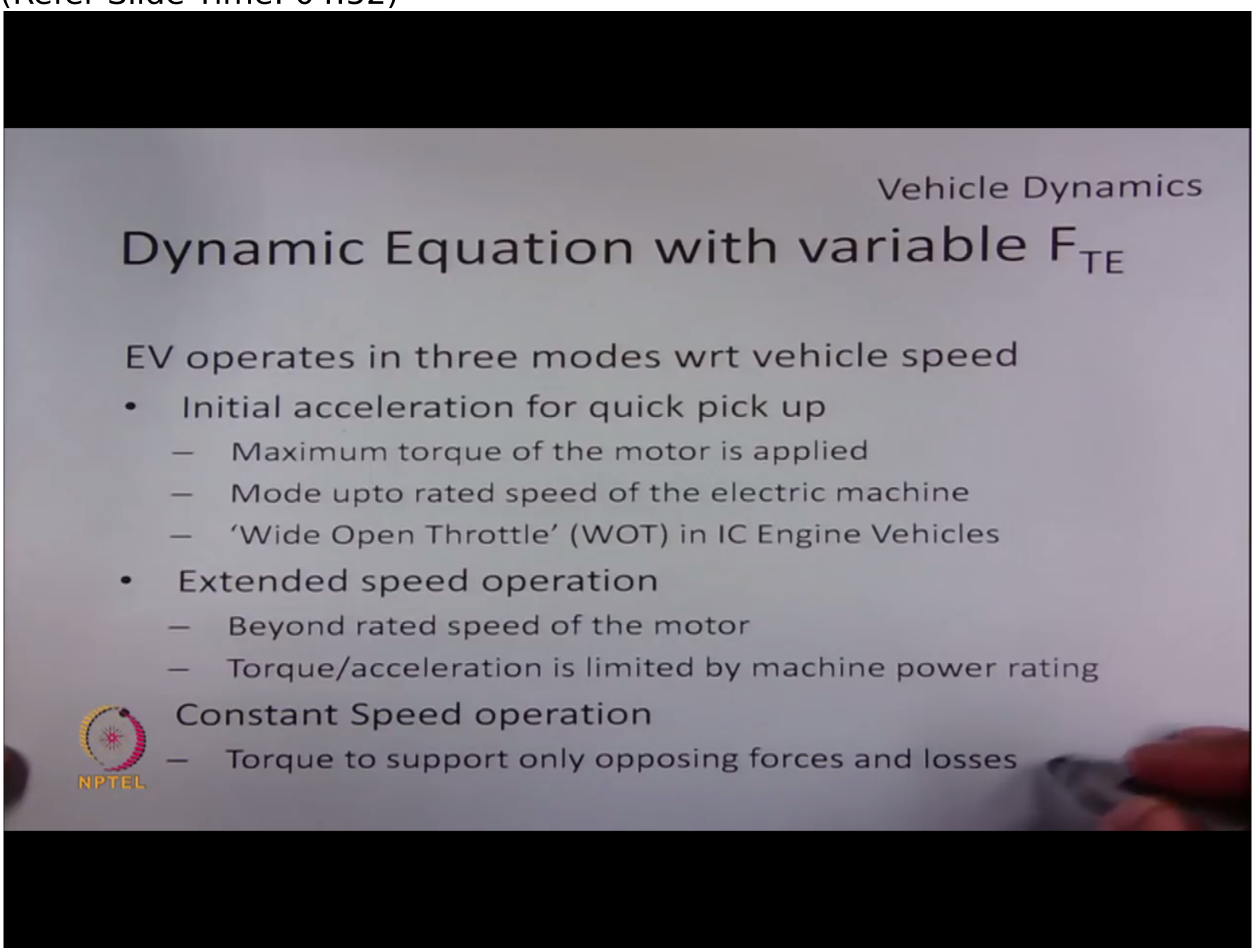

So what are the names of these three modes. So they are generally known as constant torque mode for initial acceleration, constant power mode when we want to operate the vehicle greater than the ideal speed and constant speed mode where no more acceleration is required. So generally when we buy any vehicle, we see what is the acceleration for first few seconds. So generally this is key performance indicator of any vehicle. So you can see the claims such as the vehicle reaches 80 km or 100 km/hr in 10 seconds or 20 seconds, so the faster the acceleration, the better the pickup of the vehicle is. Secondly, if the vehicle needs to be operated on highways, we also see the highest vehicle speed that is possible in the vehicle. So a high speed operation is not required if the vehicle has to operate within the urban driving.

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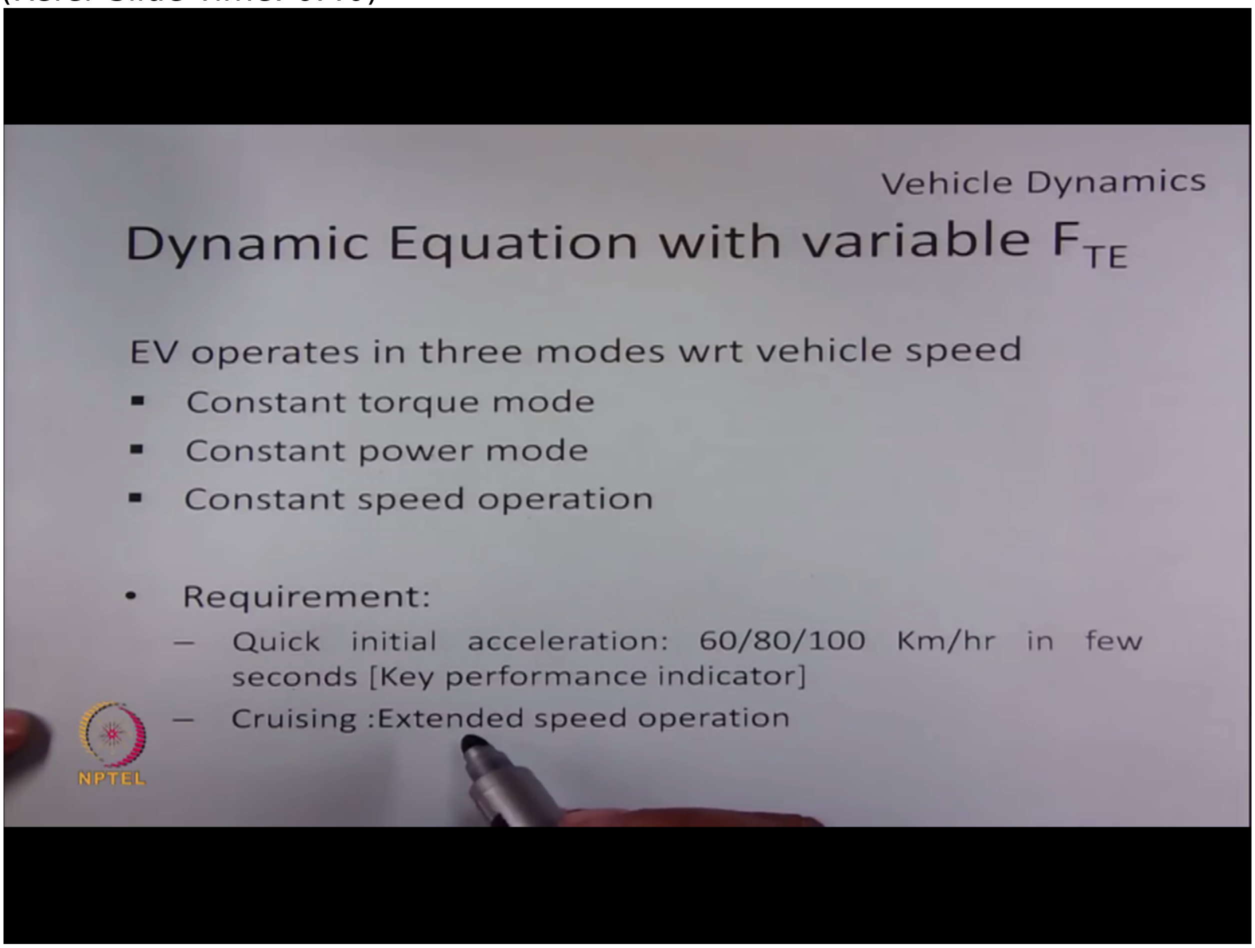

So let us see this three modes of operation on a torque speed graph. So typically, let's say when the vehicle is at rest, so we are 0 km/hr. The first thing the vehicle does is it tried to accelerate as fast as possible. So that can be done if we give a rated torque from the motor, such that the rated or the maximum tractive effort is available on the wheels. So this is a constant torque mode. Now you give TM rated to the motor shaft and the vehicle speed will increase. So you can see that this is the power curve and this is the torque curve. So during this mode since the torque is constant and the speed is increasing, the power will increase linearly in this region. So power is precaution to velocity because the force is constant. So when the vehicle speed reaches the rated speed of the motor, we reach the maximum power rating of the machine. So this point is basically rated power of the machine and you cannot give the same torque at speed higher than this, because it will violate the rated power condition of the motor. So after this omega rated, of the machine has reached, any further increase in speed is achieved by going in constant power mode, so this is the constant power mode now.. So in constant power mode we are not violating the rated power rating of the motor, such that we will apply the torque proportional to rated power by speed of the vehicle and this is the maximum acceleration that is possible in this mode. So torque will be function of… basically torque will be function of inverse of vehicle velocity in this region. So after operating the vehicle in this mode for a definite speed, so let's say this is the maximum speed, the vehicle needs to be operated, the system will go in constant speed mode. In constant speed mode, we don't need much torque because we only need to support the losses and acceleration is 0. So generally a very small constant torque is required and the power will increase linearly proportional to torque into the increase in velocity, so it will be almost… So these are the three regions of operation, constant torque mode to rated speed, constant power mode up to the let's say this, the maximum speed, and constant speed mode, beyond the maximum speed.

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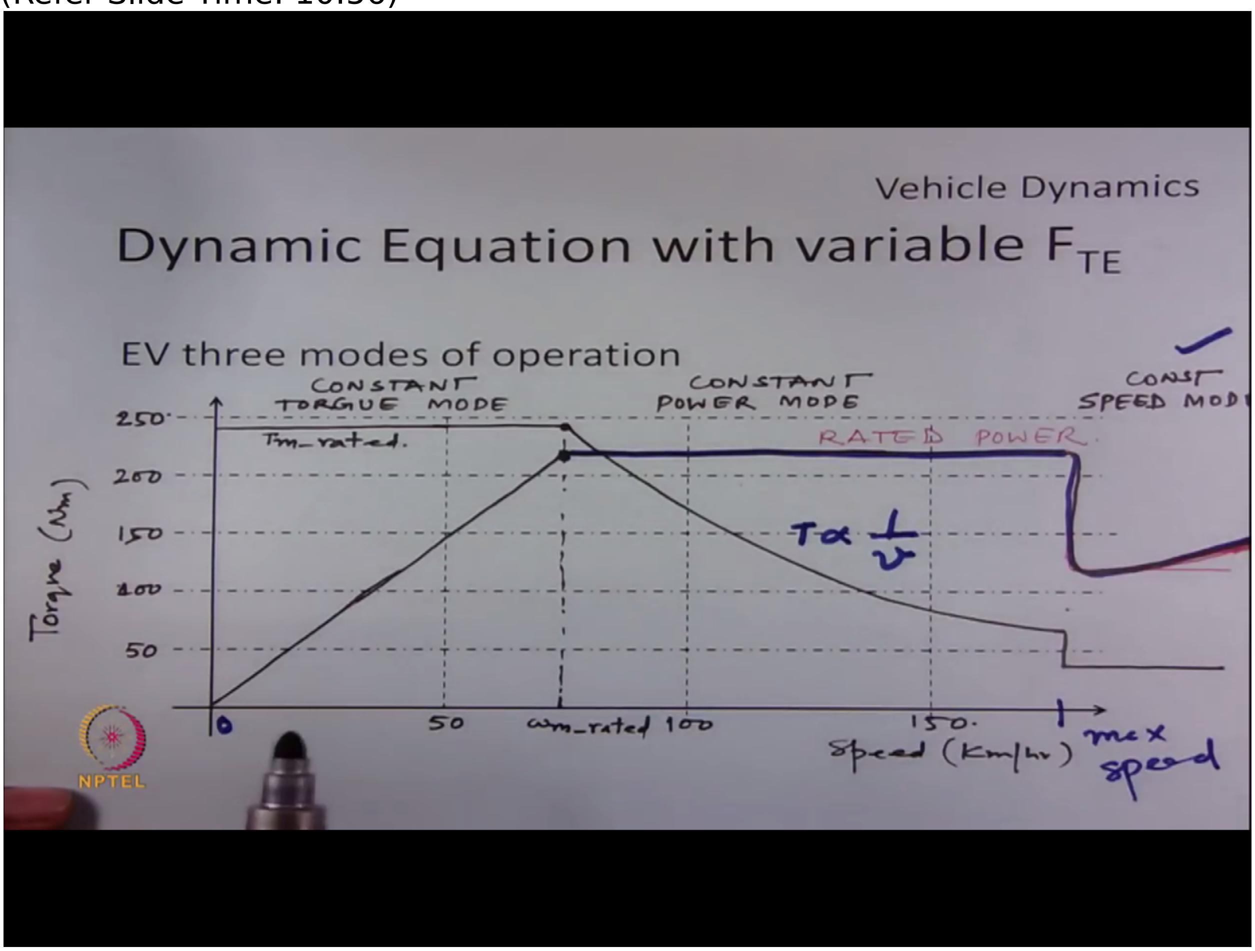

So let us try to understand these different modes in more detail. So in a constant torque mode, we are operating at low speeds and high torque, so we need in the initial acceleration, the machine outputs, constant output torque, generally equal to the rated of the machine, the output power increases proportional to speed, since the torque input is constant. So what are the limit on this torque, power, and the speed operation is... So T can be maximum T rating of the machine. The power can be maximum up to the power rating of the machine and the speed can be maximum up to the maximum speed rating of the machine. So it has to operate within these constraints.

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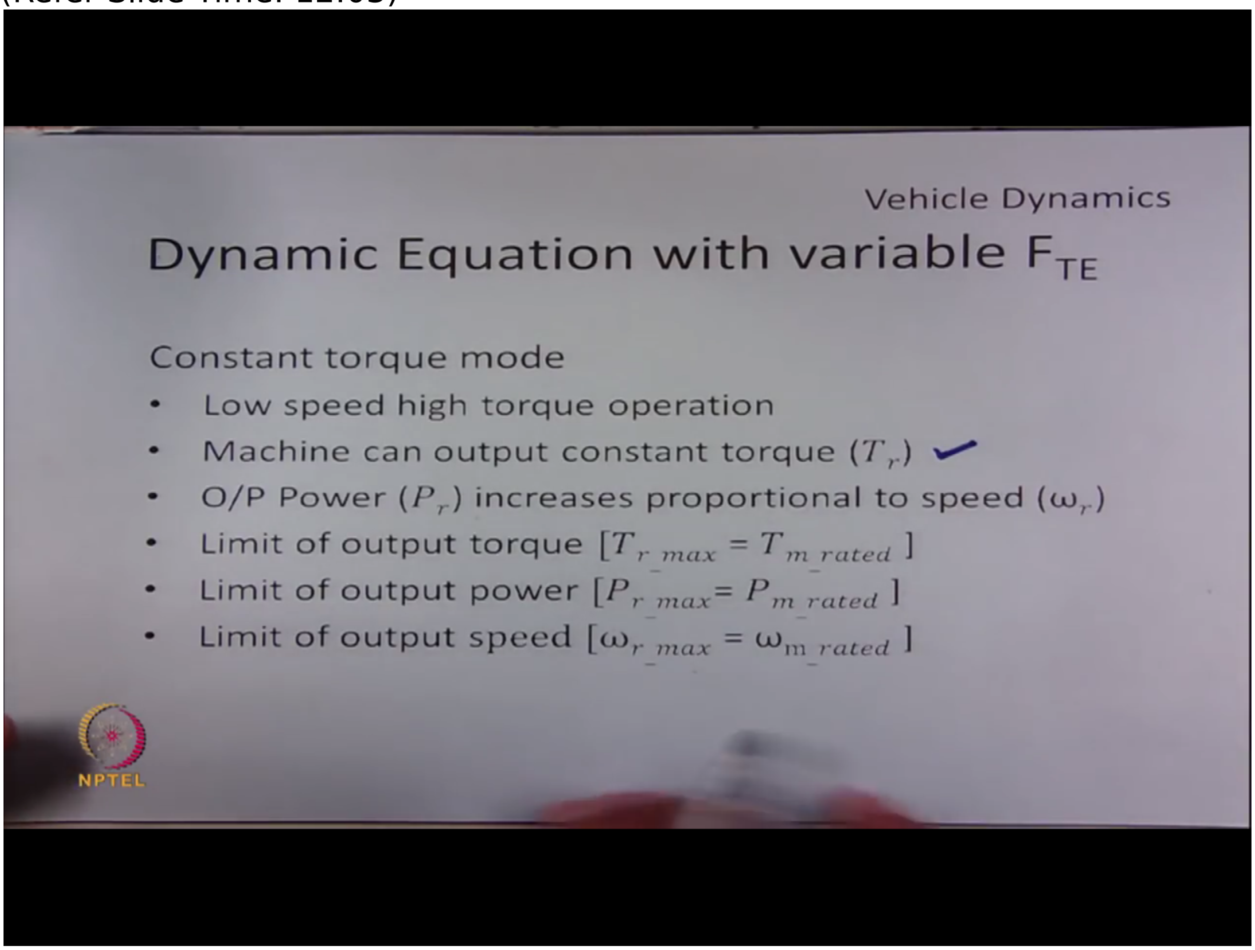

So we have discussed already the derivation of dynamic equations when the input torque is constant. So this mode is very similar to that and the same understanding can be used to know this mode of operation. So we know that this TM is constant, generally it is equal to TM rated. We know it can be simplified as DV/DT = -K1 V square + K2.

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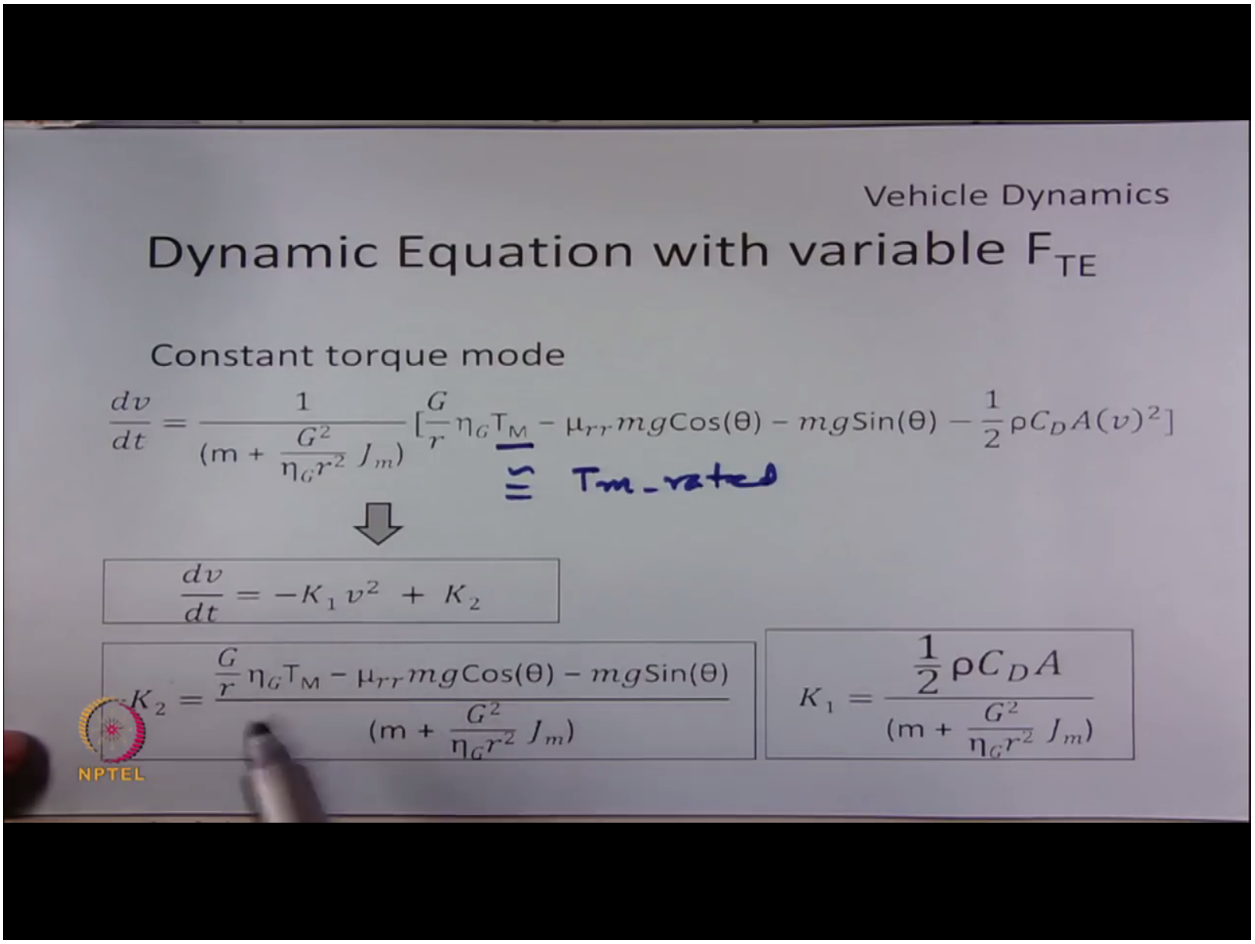

And which can be solved and it's possible to understand the variation in velocity with respect to time using this equation. Secondly we can also find out the variation of distance with respect to time in this mode. We can also find the time that will be required to go to a particular speed. So if somebody tells, I want to know the time that is required to reach a velocity VF, we can calculate using the time formula, which is inverse of this velocity formula.

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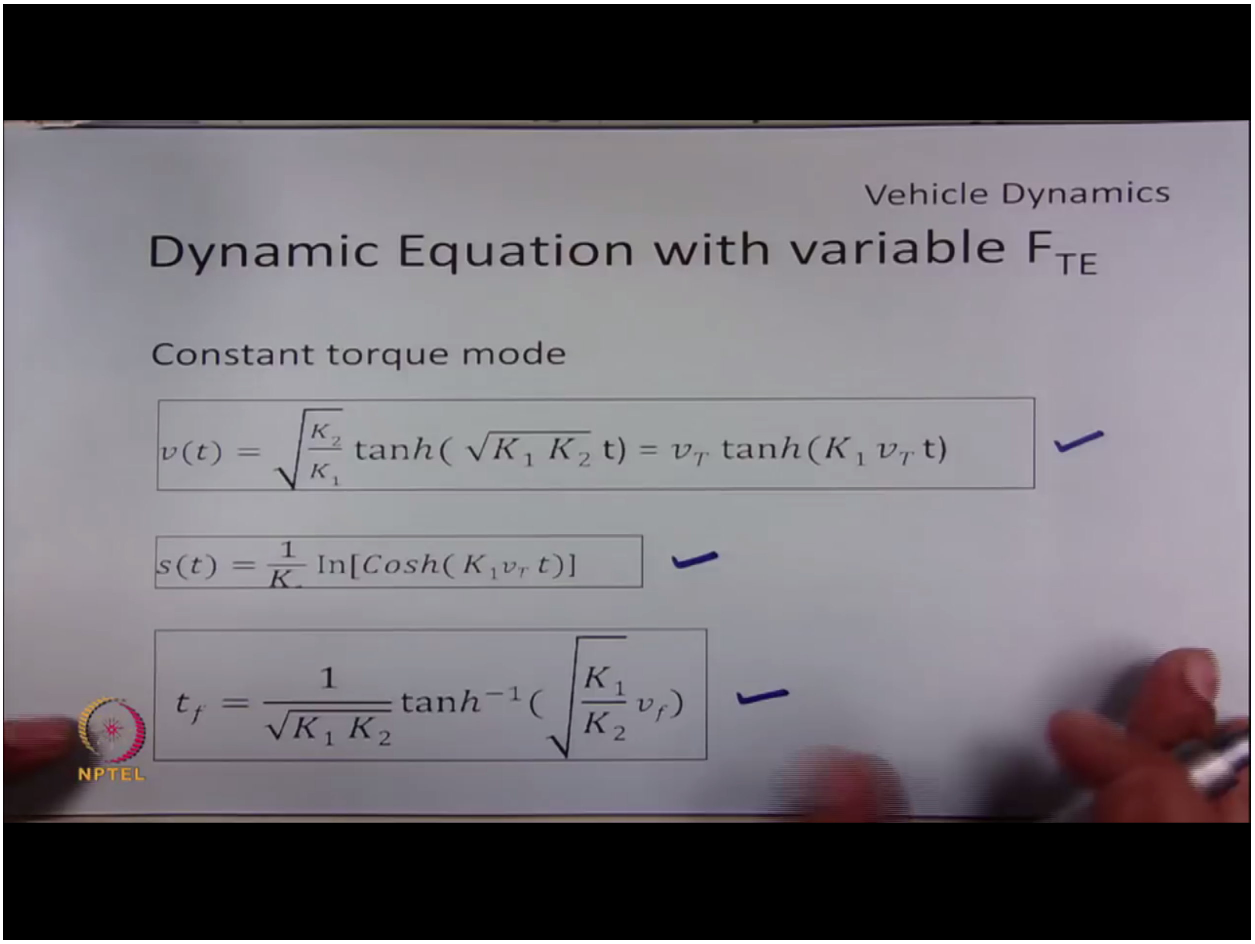

So we have also seen the expression for average power. So since the power variation is not constant in constant torque region, we need to calculate the average of the power that is required from the motor, so we can do that by using this expression and this calculation of average power will be used to calculate the energy that needs to be supplied from the battery. So this is a simple product of average of power into the time elapsed.

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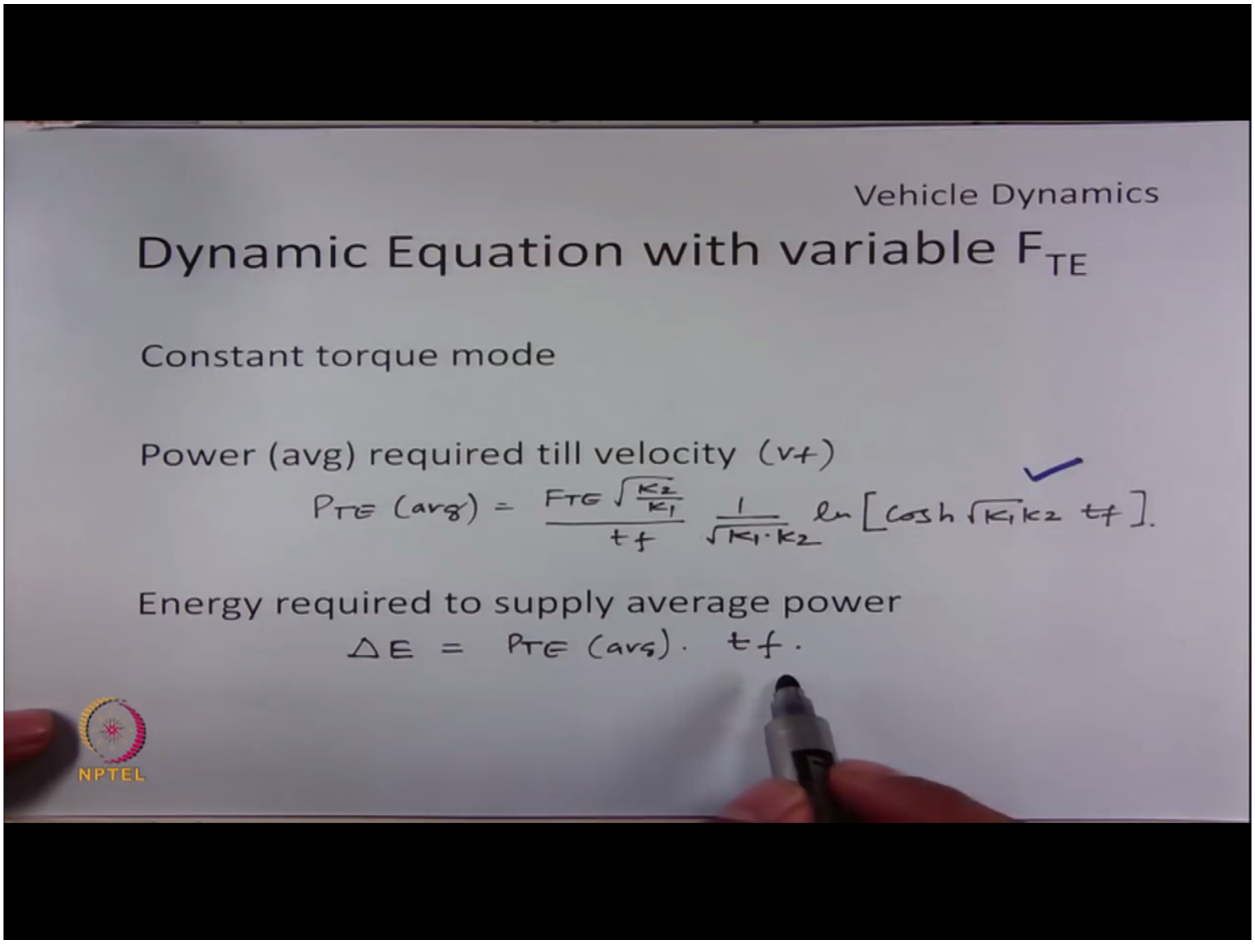

The second mode of operation is known as constant power mode. So this mode is generally a high speed, low torque mode of operation. So this mode comes into existence when the speed of the vehicle reaches the rated speed of the motor. So this is… so this mode is generally started when omega R is greater than omega M rated. So in this mode generally the machine outputs a constant power, but since this power is generally equal to the rated power condition of the machine, the output torque can only vary as inverse proportion to speed. So what are the limits on value of torque, power, and speed is? So the torque is proportional to the rated power by the current omega R, which is greater than omega M rated. The Pmax can be maximum up to PM rated, and the speed, which the omega R can go is limited by the safe limit of the vehicle for which it is designed.

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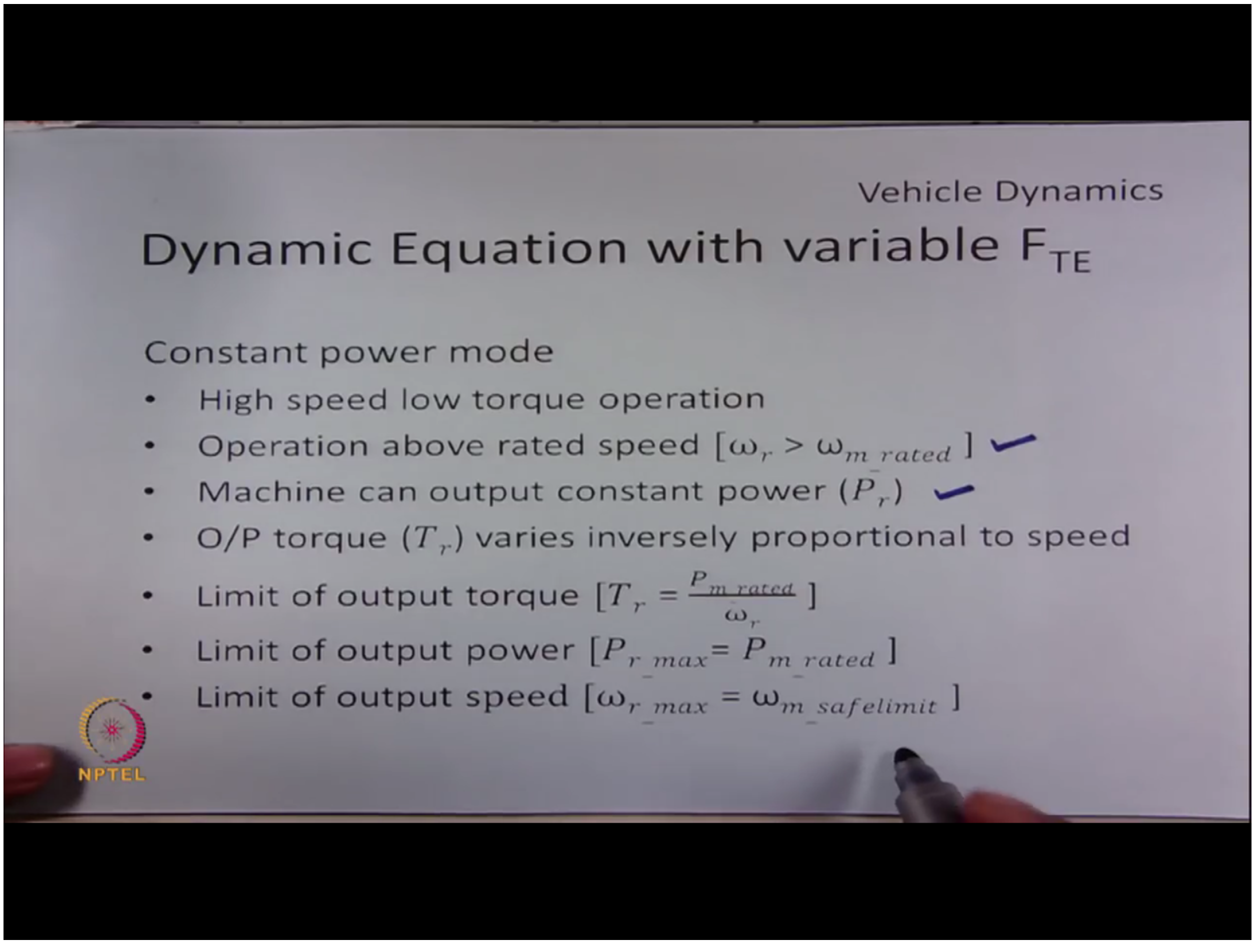

So as seen the expression of motor torque as a function of velocity, so this TM is not constant, so it will be a function of velocity as a rating of the machine by omega R. So for any omega R, which is greater than omega M rated, expression of TM is PM rated by omega R. This omega R can be written in a… as a function of vehicle velocity as PM rated by velocity into R/G.

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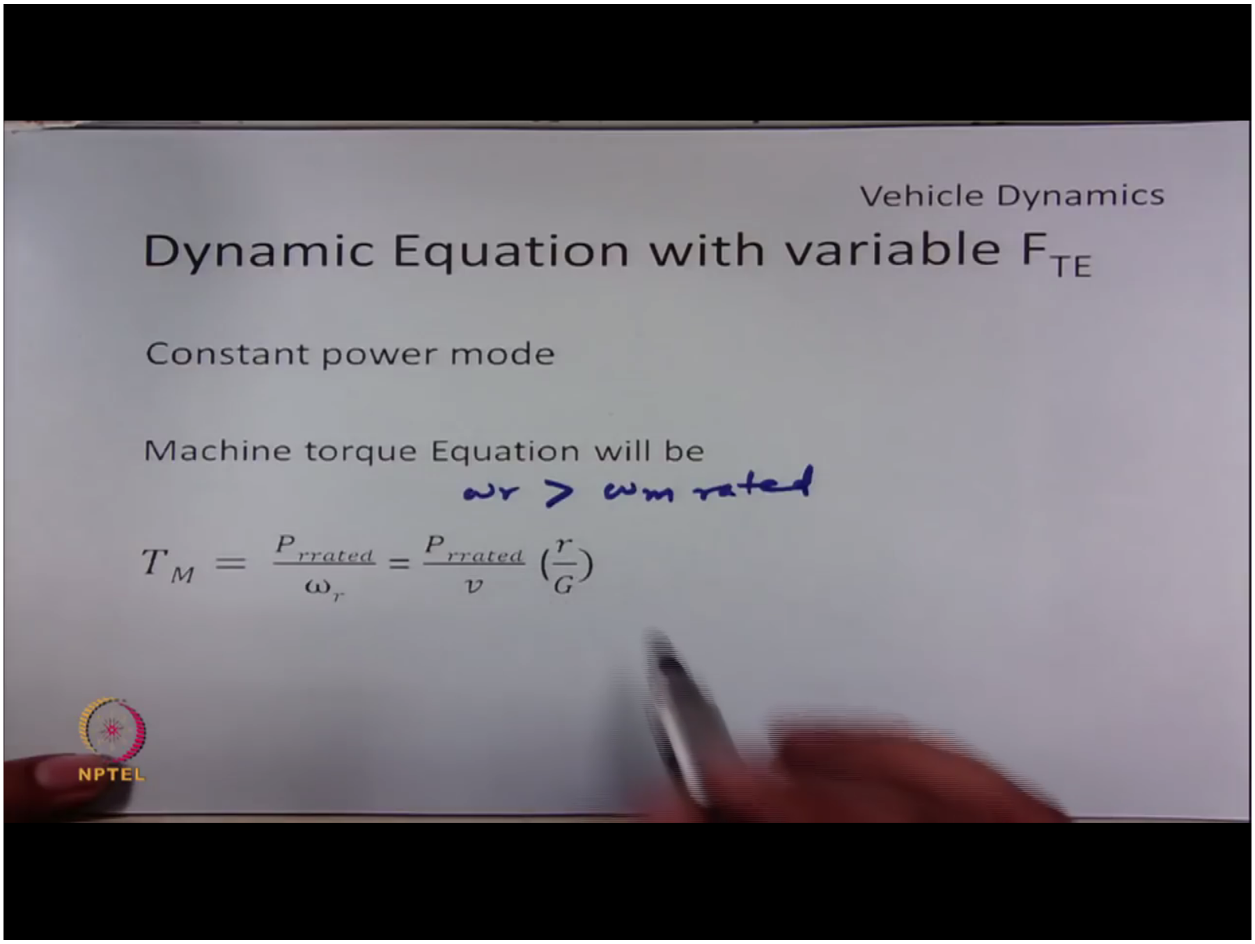

So this expression of torque can now be introduced in the velocity equation as this. So we can see that now the DV/DT equation is a function of three constant. So the K1 is the constant of V square term, K2 is a pure constant term independent of velocity, and a new term is added now, which is K3/V. So this K3 is the constant for 1/V term. So this is a more complex equation compared to constant or mode.

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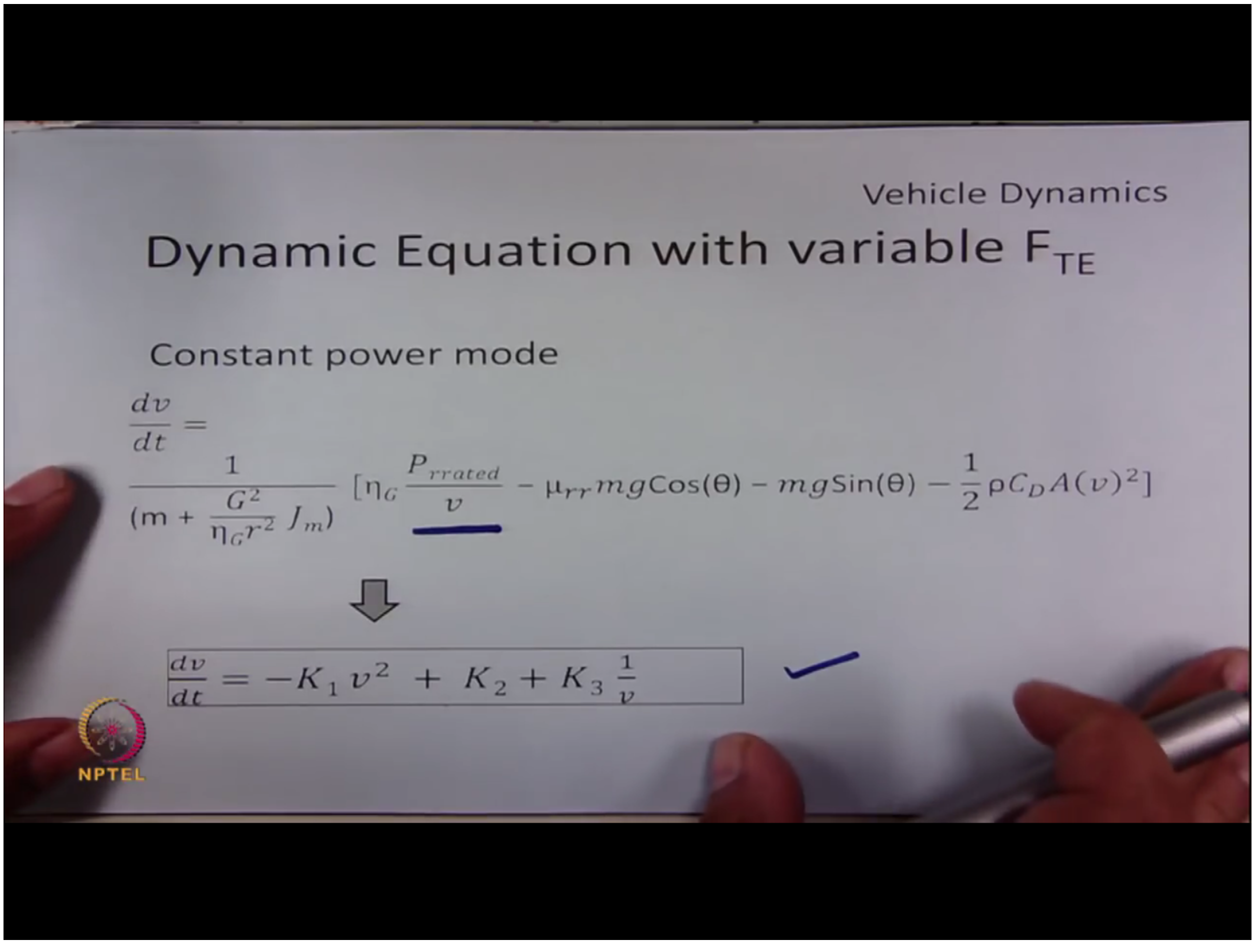

So let us see the third mode of operation, which is constant speed operation. So as we have discussed, when the vehicle velocity reaches its maximum limit or we can say that this is safe limit of operation, we start this mode and the vehicle operates at constant speed. So when it is constant speed operation, there is no requirement of any accelerating torque and the machine only needs to supply the power proportional to losses due to load forces, roll resistance, aerodynamic drag, and gradient force. Thus other losses in the system, such as that present in transmission system. So the now value of torque is proportional to addition of all the opposing forces, so rolling resistance force, gradient force, and aerodynamic force.

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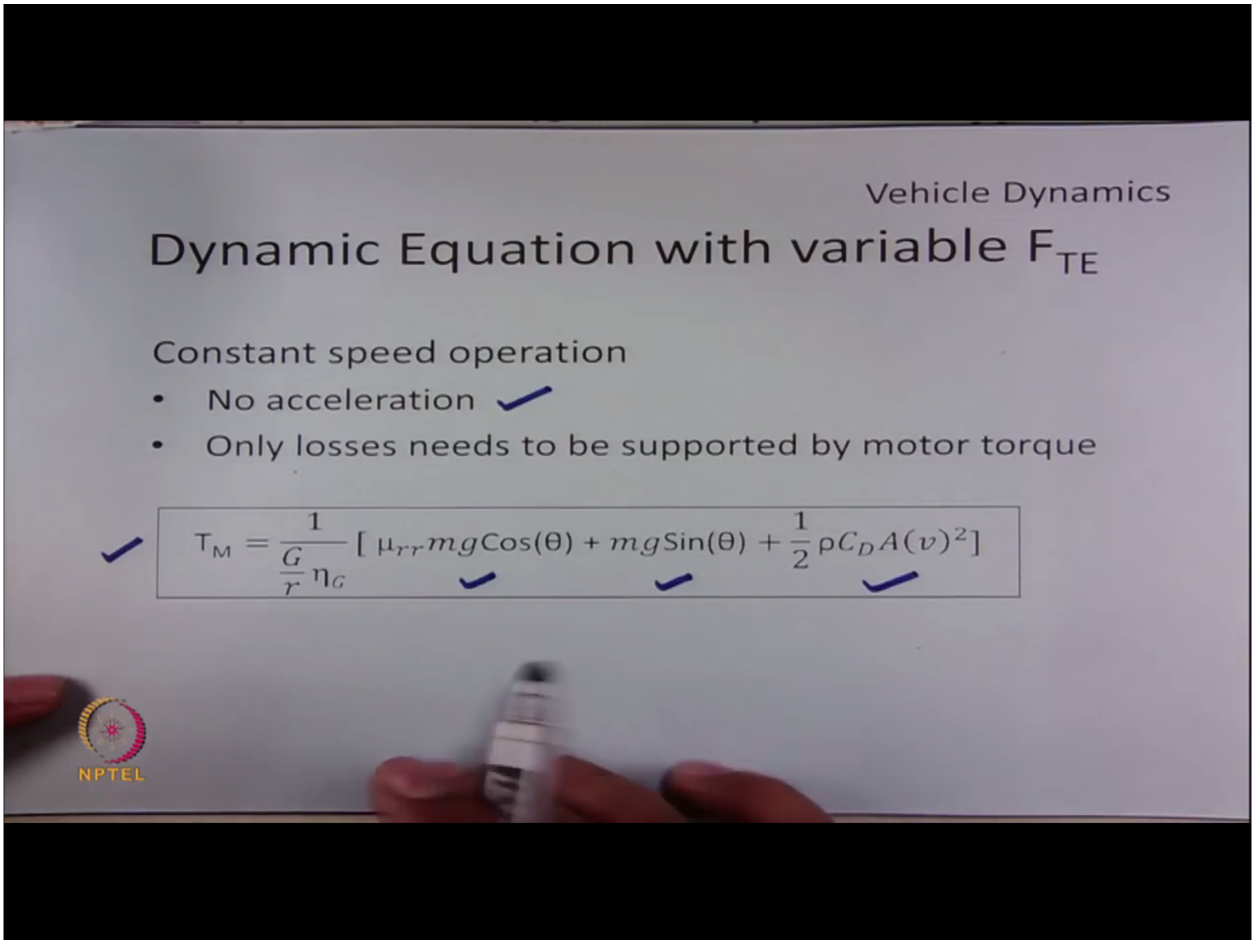

So we have seen the expression of force for all these three modes of operation. So now let us see the simplified equation with respect to three modes. So it is possible to get some simple equations, if we ignore the load forces and inertia, if we do that the system equation becomes very simple and straight forward. Secondly, this assumption of ignoring this process is quite valid, since the magnitude of this process is low compared to the accelerating forces. In constant torque region the expression of DU/DT will be just 1/M G/R Eta G into TM, because the rest of the load forces is now assumed to be 0. The value of TM is equal to T rated in constant output. So this is a fairly simple equation to solve. If we integrate DV over time function, we can do that in this way, so we can take DT on other side and integrate both side. So let us assume that at time initial the velocity is V initial and at time of T final the velocity is V final.

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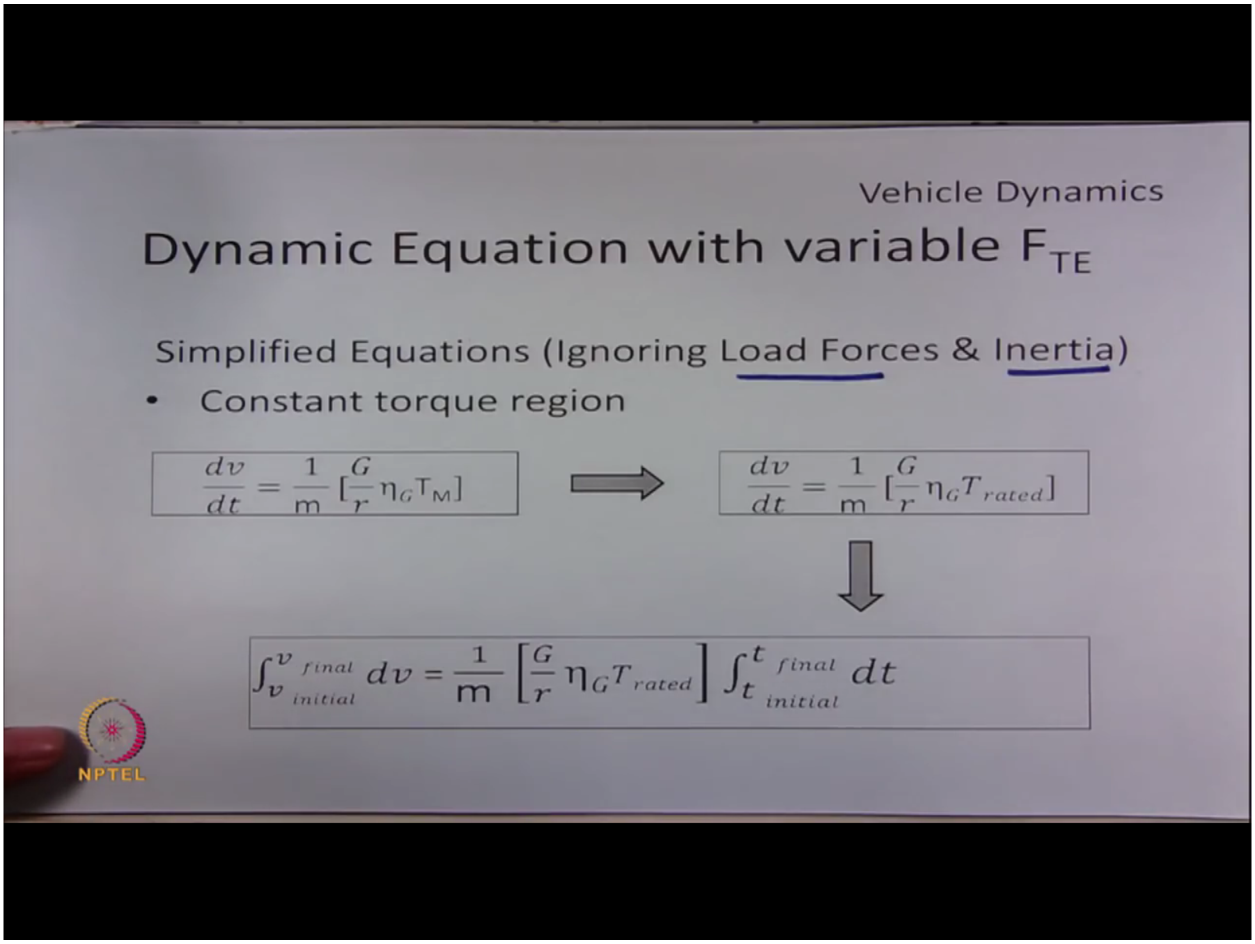

Solving the equation we will get this expression, so V final – V initial is this term into T final – T initial. So if we assume that the V initial is 0 at T initial = 0 seconds and we know that in constant torque mode, the V final is the V rated of the vehicle. So if we substitute this on the first expression, we can get the time required for the vehicle to reach the rated velocity, given a constant torque of V rated. So this is a very simple expression, which can be used to calculate the time elapsed.

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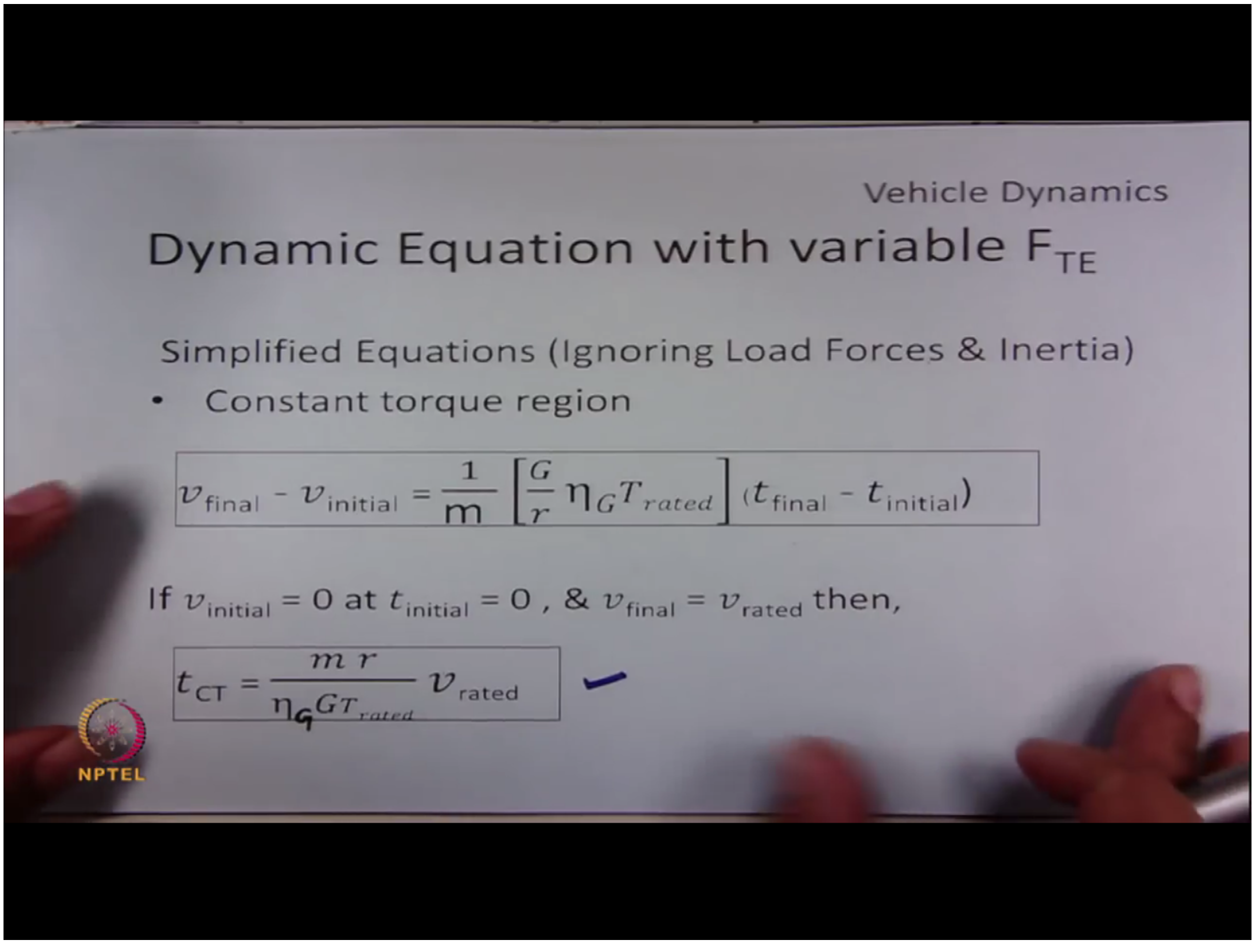

We also know that the variation of power in this mode is linear. So in constant torque region the variation of power is linear in, and then it become constant in constant power region. So if we want to calculate the average power that is required in constant torque region, it is just P rated by 2. So since we know the initial and final values, we can say the P average in constant torque region is half P rated. So once the average power is known, we can also calculate the energy that is required in this mode by just multiplying the P average by the time that is left in constant torque region. So it is half P rated into T seat.

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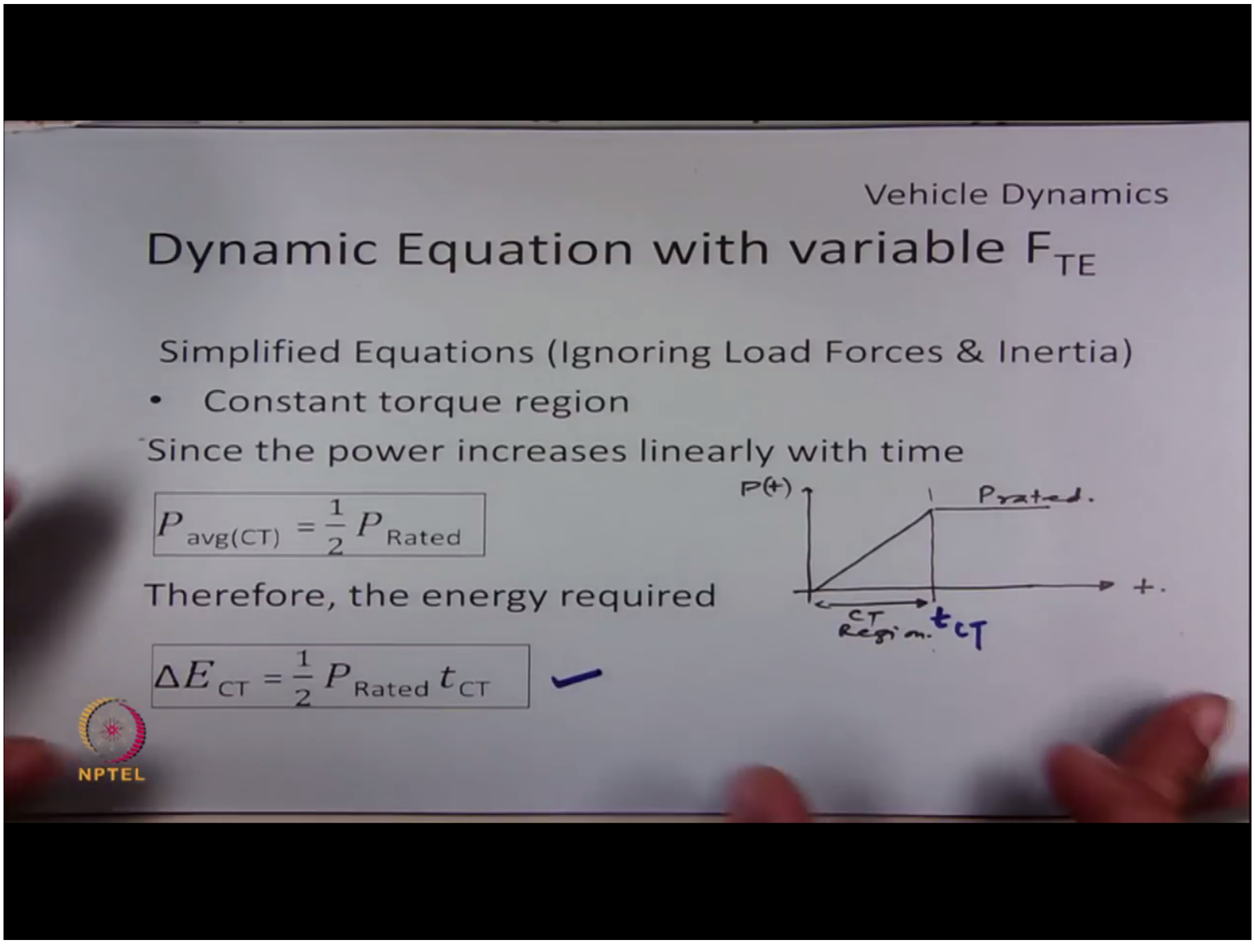

So we have calculated the expression of time that is required in constant torque region, the average power, and the energy required. Now going to the second mode of operation, which is constant power region and again ignoring all the load forces and inertial forces, the expression of DV/DT will be the same as what we have seen earlier. So what is the motor torque in this mode of operation. So we know that motor torque is not constant and it’s a inverse function of vehicle speed. So motor torque is P rated by omega M or current speed. So omega M is greater than omega rated in this mode of operation. So this P rated can be re-written as T rated into omega rated. So we also know that the ratio of omega rated by omega M can be re-written as V rated by V, because the ratios are same. So this is the expression for variation of torque with respect to the vehicle velocity. So this expression of TM can be substituted in this equation and we will get expression of DV/DT in terms of vehicle velocity.

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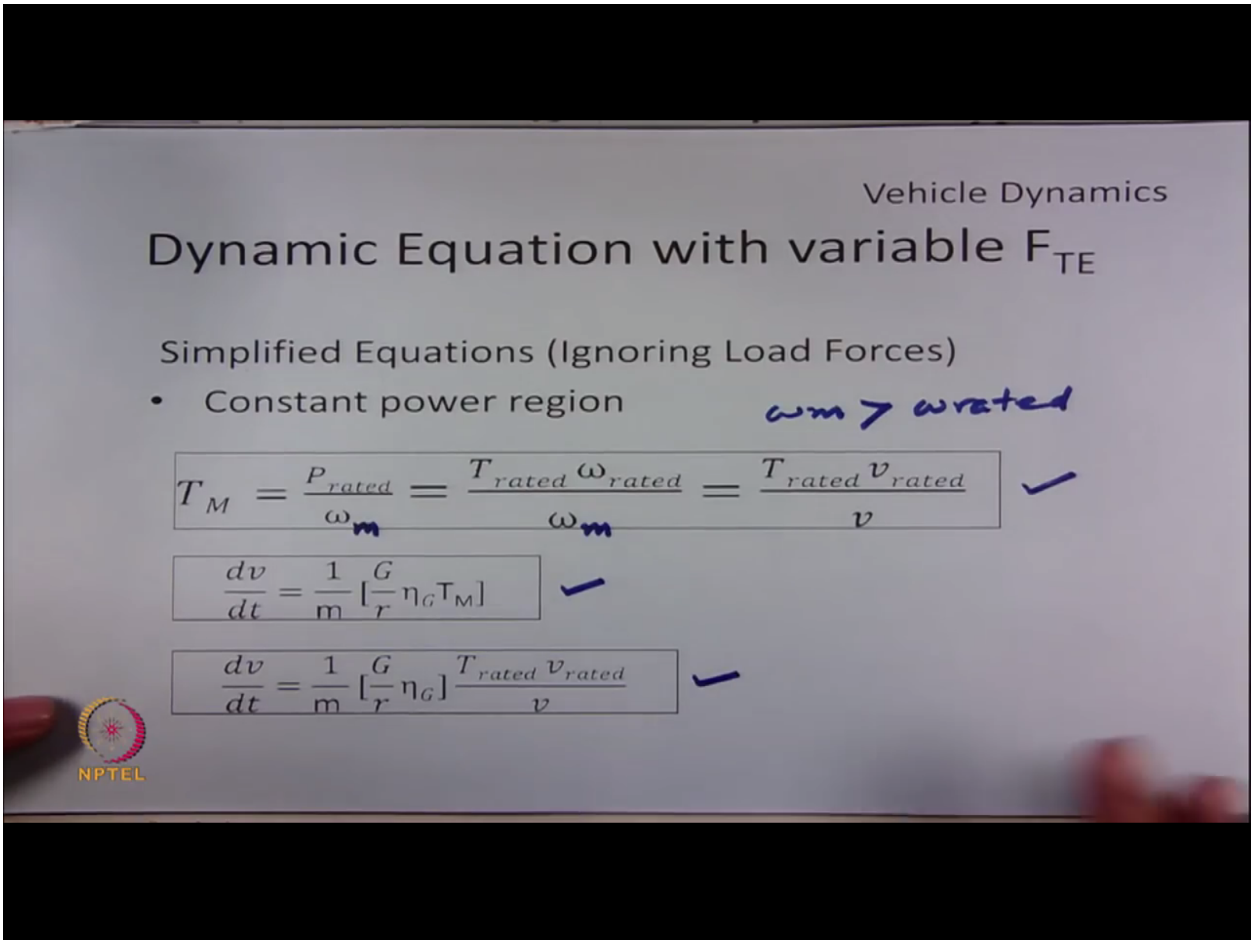

So now this equation needs to be solved as function of time. It can be done by multiplying the inverse expression of V on the… this side, so we will have V DV/DT expression where the DT is now transferred to the other side. So this expression can be integrated as a function of time similar to the method we have adopted earlier. So we will assume a T initial and T final and V initial and V final. So T initial in this case is basically TCT and V initial in this case is V rated, that we know. So on doing the integration and simplifying, we will get this expression, so this will become half VF square – V initial square and on this side we will get T final – T initial into this constant term.

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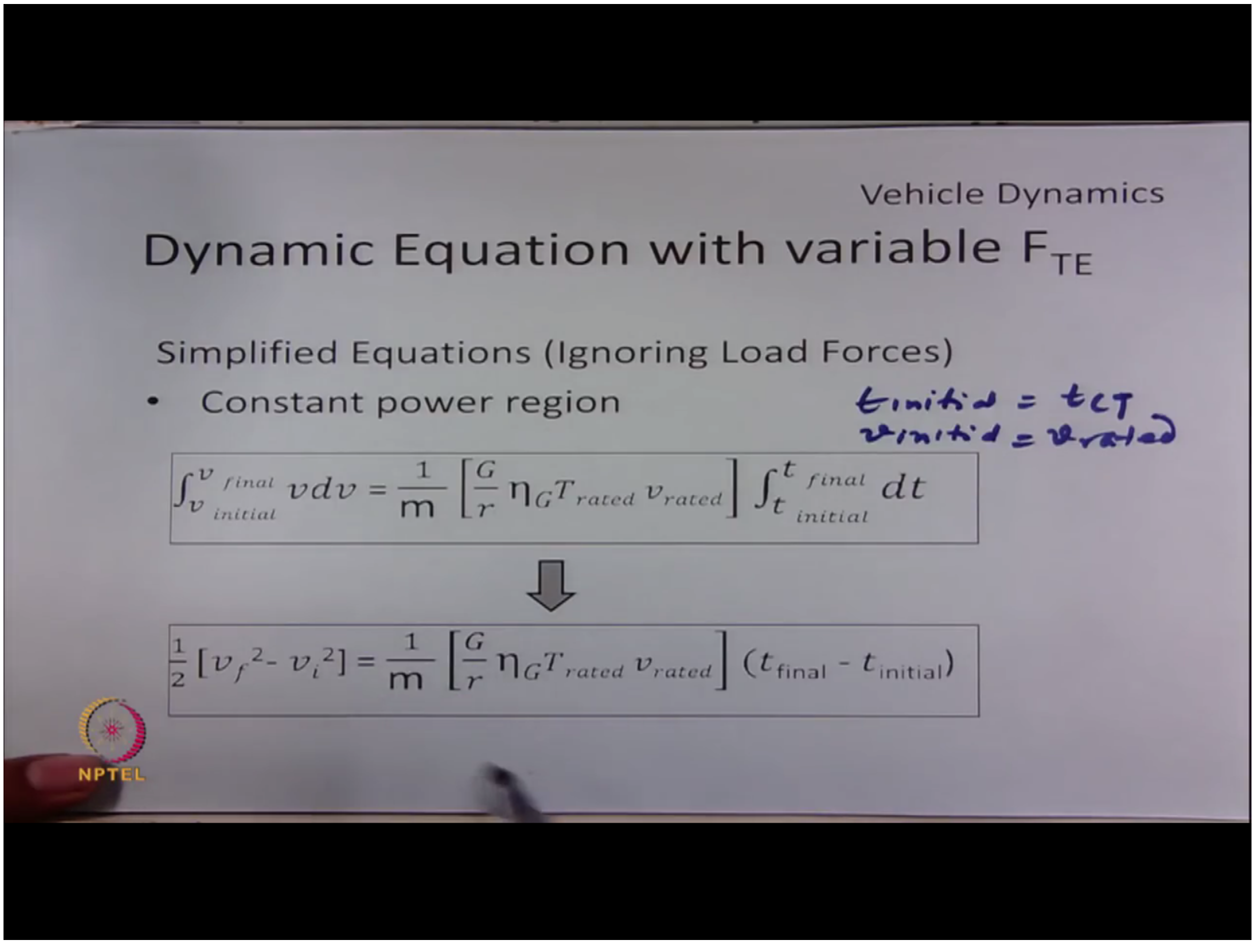

So on solving this expression the value of time elapsed in constant power region will come equal to this.

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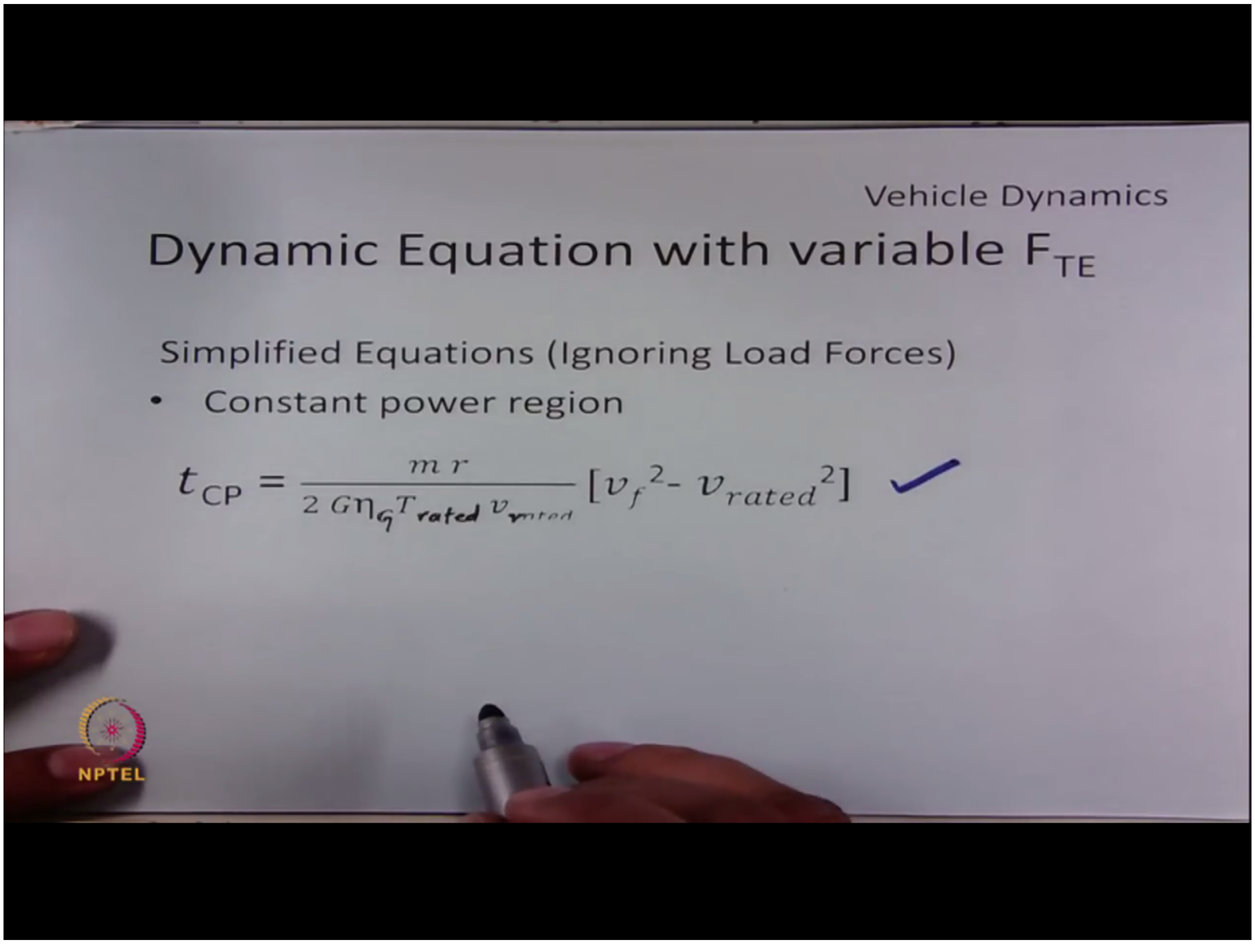

We also know that in constant power region, the power that is outputted from the motor is constant = P rated. So since in this whole region, the power is constant, we can say that the average power in constant power region is equal to P rated. So using this average power in constant power mode, we can also find the energy that is required to be delivered, the Vs will be equal to average power into the time elapsed. So it will be also equal to P rated =… so this is T final, this is TCT and therefore TCP=T final – TCT.

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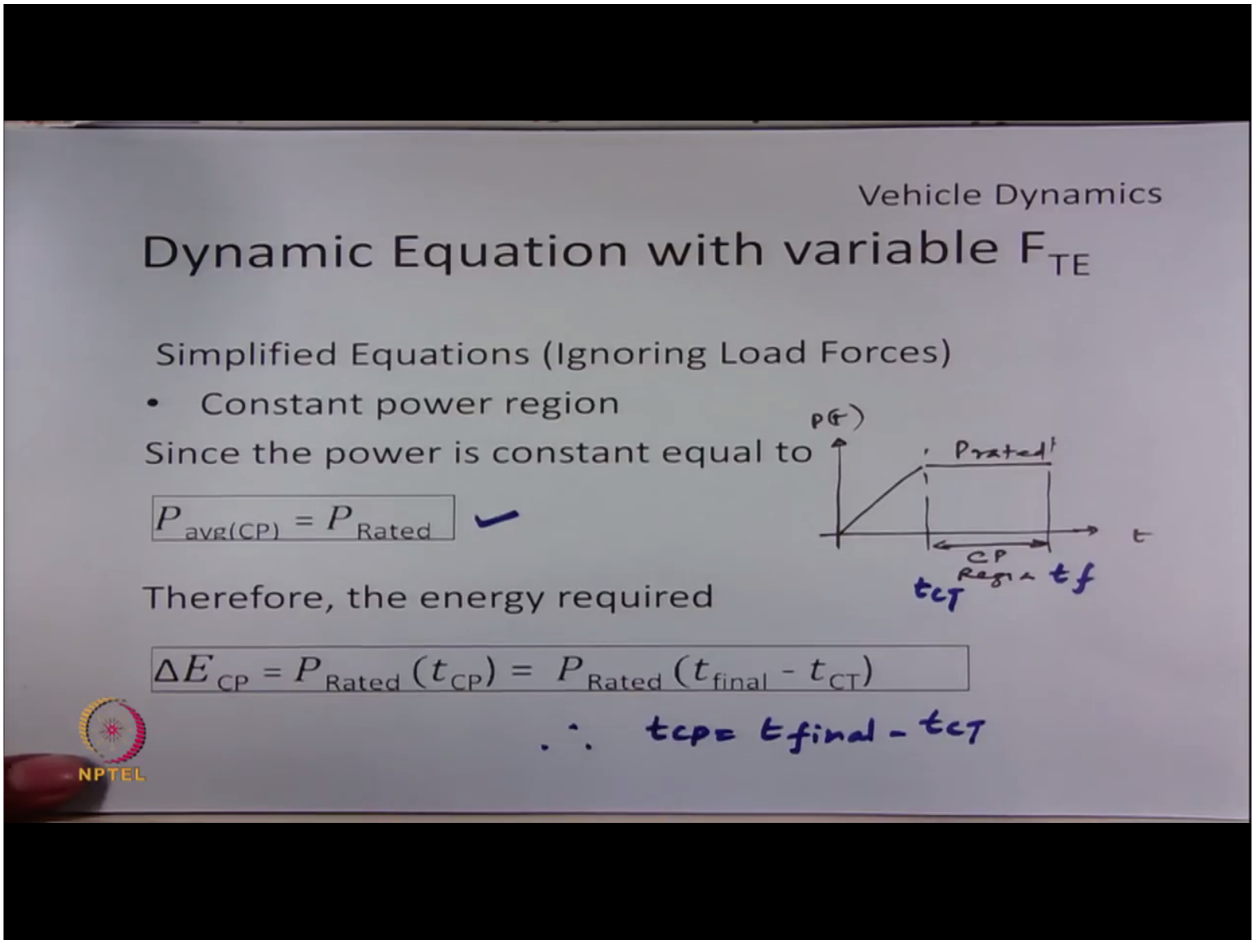

So now we have the expression of time that will be required to reach greater speed. So this is TCT, the time elapsed in constant torque region and the time elapsed in constant power region. So once we have this, we can calculate the T total, which is nothing but TF. So if we add this two terms, we will get a very simple equation. So this is the total time the vehicle will take to reach a speed of VF, which is greater than V rated. So the vehicle will operate in constant torque region for speed lesser than V rated and it will operate in constant power mode, when the vehicle velocity is greater than V rated.

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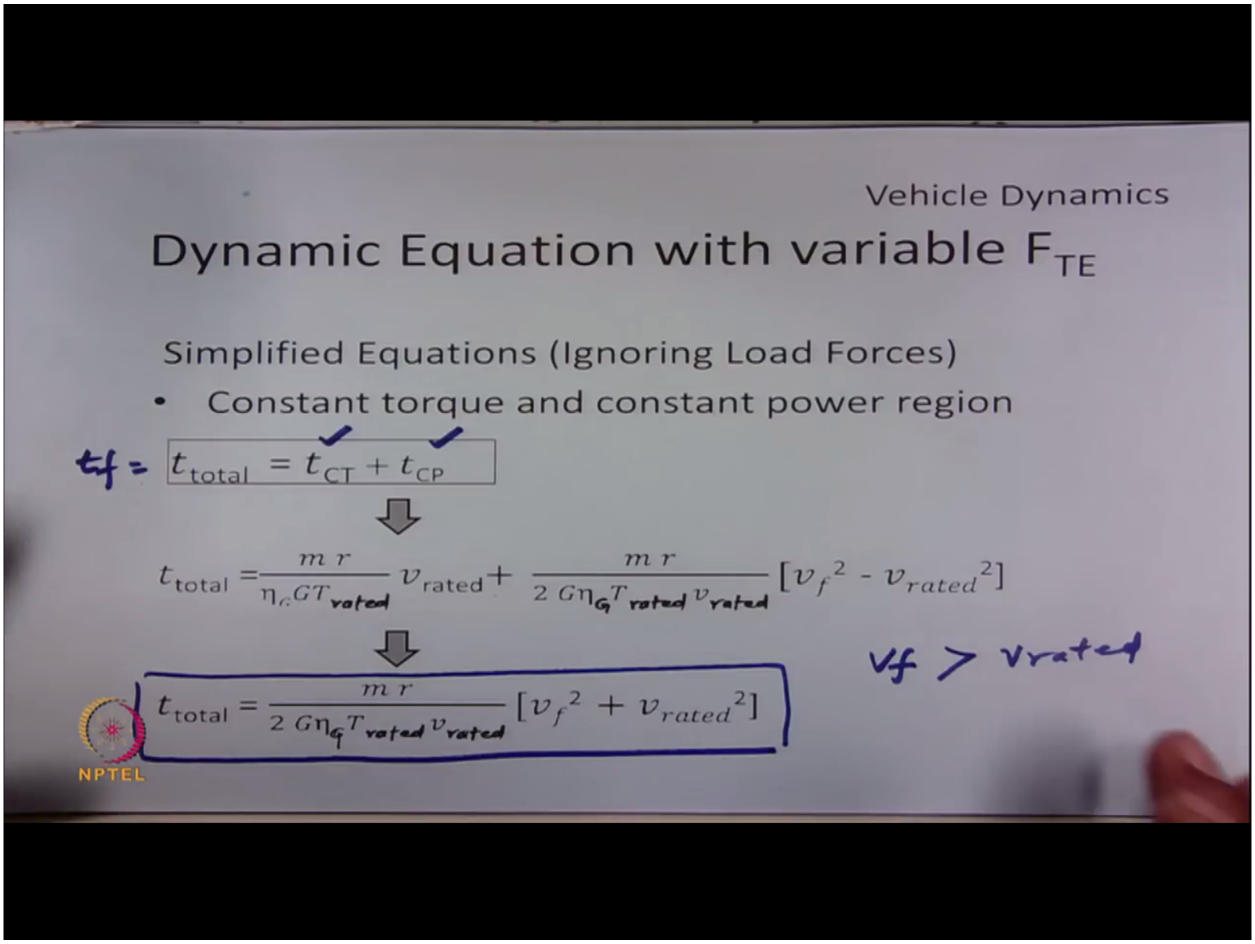

Similarly we can also calculate the total time that is required for the vehicle to reach the speed of VF greater than V rated as the addition of the energy required in constant torque mode and constant power mode. So this expression we know, now substituting this we will get Del A total = P rated T final – TCT by 2. So you can see that by using simple assumptions, it is possible to calculate the time required to reach the maximum speed, the average power, and the energy that is required to achieve this kind of operations.

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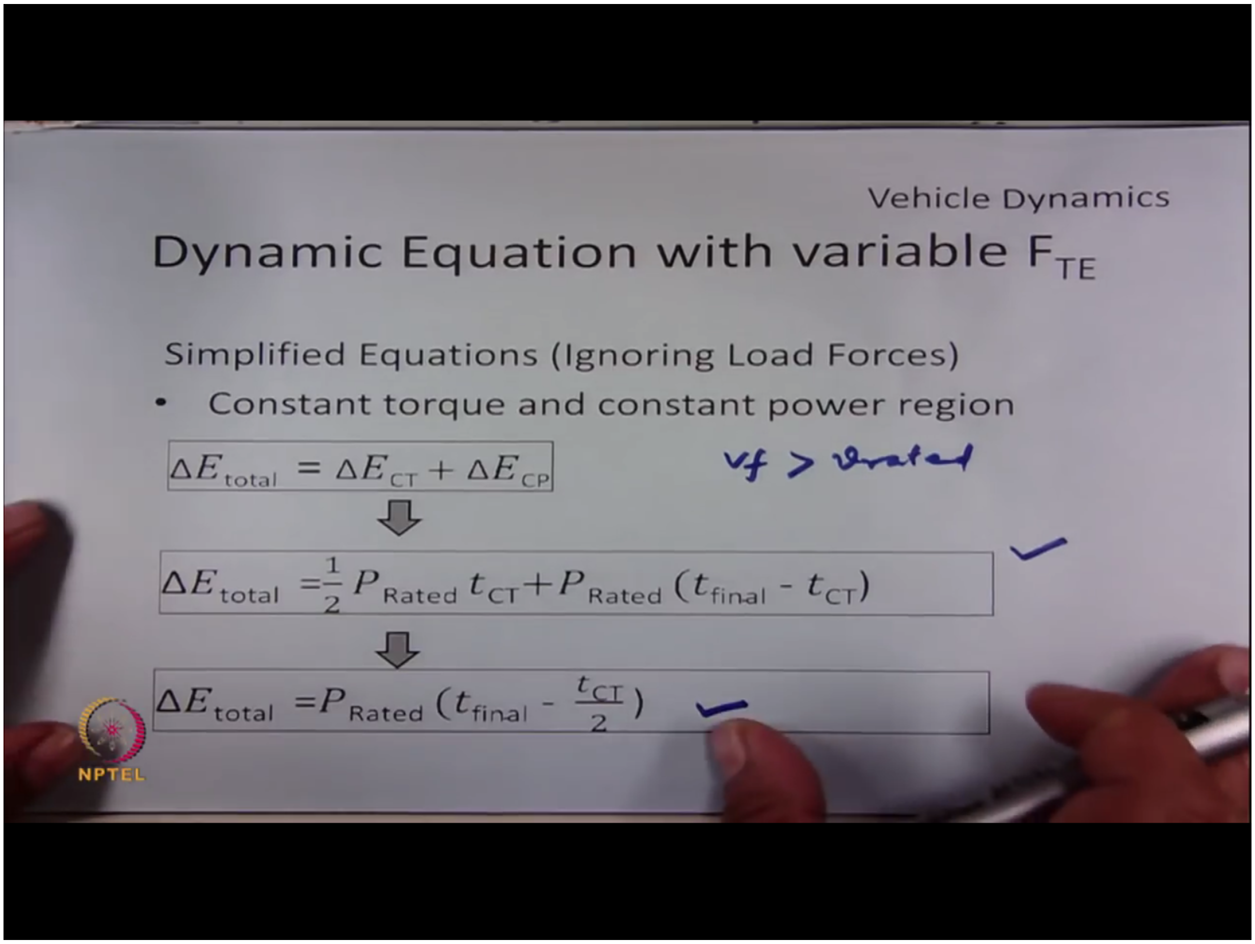

So that is all under the topic of dynamic equations with variable tractive effort input. So in our next interaction, we will try to see some quick calculations using the formula we have derived today as well as MATLAB based simulation graphs, which will further enhance our understanding on this topic. So thank you for listening the lecture.

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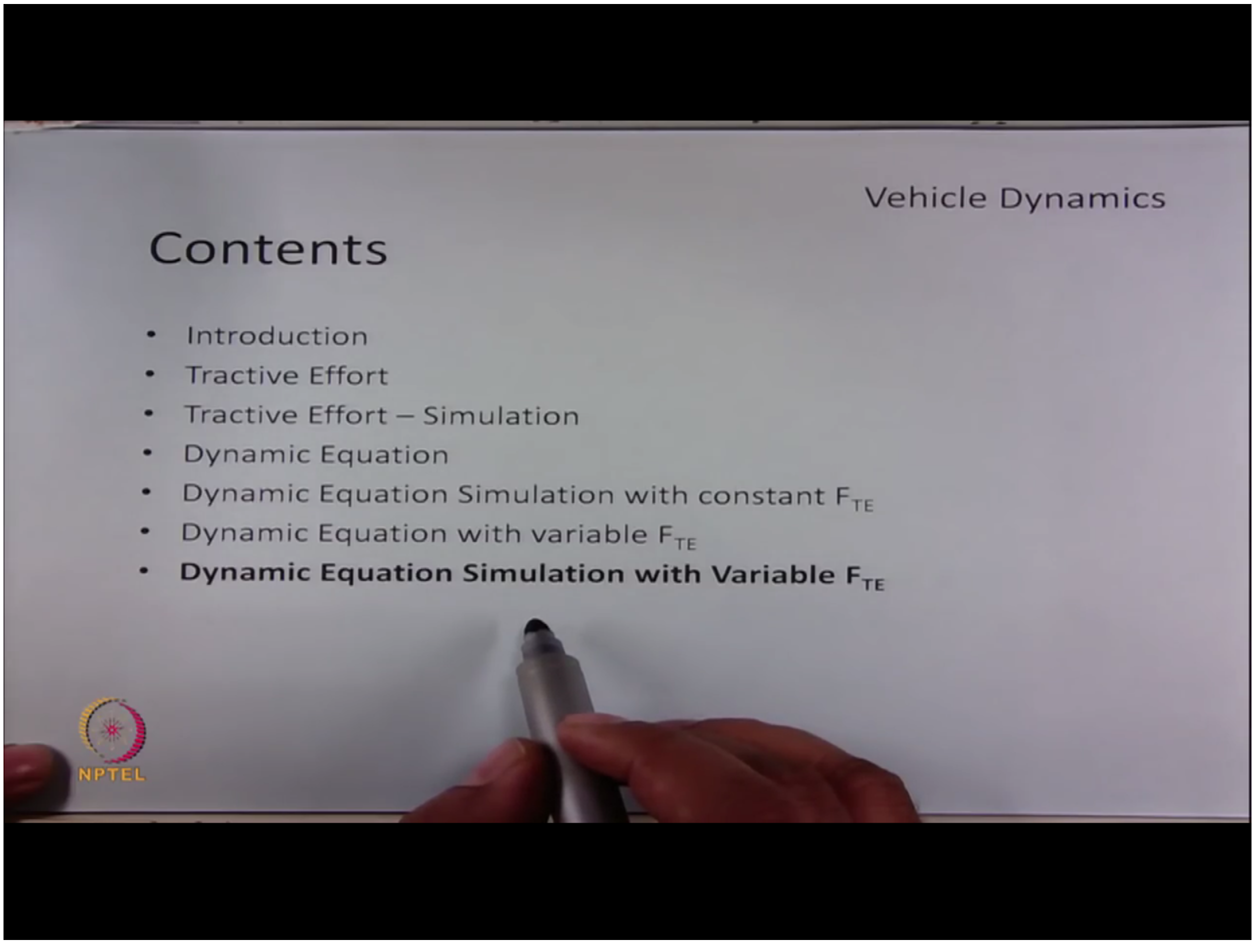
Vehicle Dynamics
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