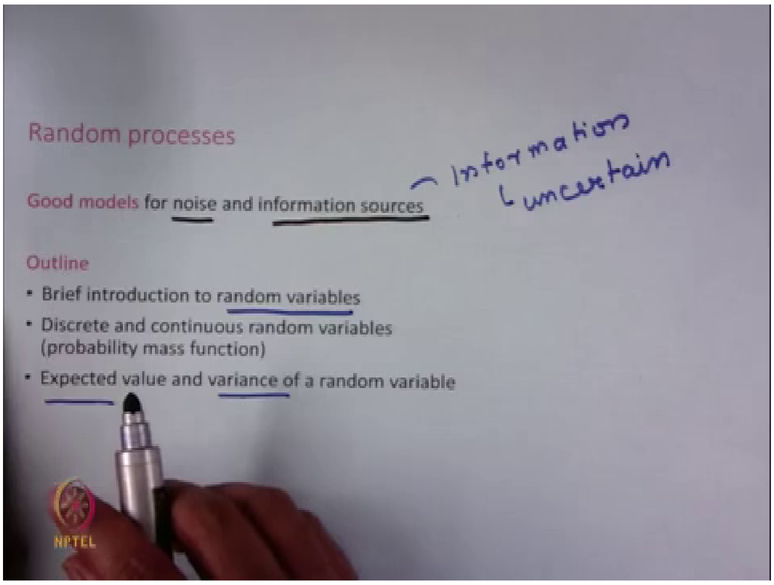
Principles of Digital Communication
Prof. Abhishek Dixit
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 7
Random Variables & Random Processes: Discrete Random Variable

So, good morning, welcome to this new unit on Random Processes. And why do we study about this random processes? Random processes are really central to the understanding of digital communications and systems.

(Refer Slide Time: 00:35)

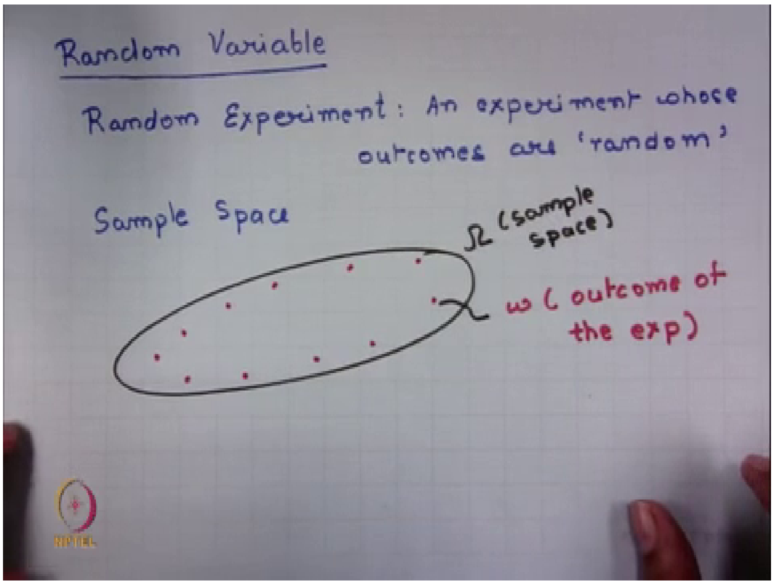


As you can see that they provide a good models to study noise and information sources. So, as you can guess that noise is anyway a random signal and random processes are use to model that and information sources transmits information and information is in the uncertainty right.

So, any information is uncertain and any uncertain uncertainty also translates to randomness. So, random processes are used to model both noise and information sources. So, what we have caught for you in this lecture is you will start looking into random processes my first understanding about random variables right. So, to study random processes, we have to first discuss this random variables. Then we will talk about two kinds of random variables: discrete and continuous random variables. Today in this lecture, we will talk about probability mass function and then we will talk about expected value and variance of a random variable.

So, these are the really central concepts in understanding of digital communication. So, I will like to caution you. So, in this course what I would expect is that you already have some exposure to probability theory and you have seen these random variables before, but I will believe that you might have forgotten about what you have studied in course in probability. So, we will provide a short recap of the concepts that are required to understand process in no way I am trying to teach everything that is there in the random variables. So, we will just only focus on what is important and central for understanding of a random processes.

(Refer Slide Time: 02:38)



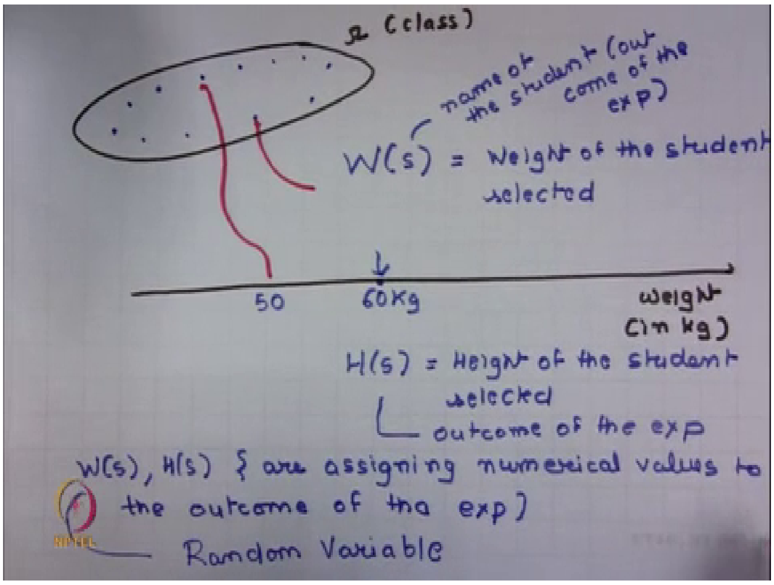
So, the first question and the most important question is what is a random variable? So, the first thing that we will talk like to discuss is about what is a random variable. Now to understand this the concept of random variable, I will like to first define a random experiment. So, random experiment is an experiment whose outcomes are random.

For example, if you flip a coin right you know that either you get head or tail. So, the outcome of flipping a coin is uncertain; it is random and such an experiment is known as random experiment. Now the second thing that we will like to define is sample space. So, the sample space I draw a big circle and I use a capital omega to denote a sample space.

Sample space contains all possible outcomes of an experiment. So, if I do an experiment I random experiment, for sure I get various outcomes right and sample space contains all possible outcomes of that random experiment. So, this is omega denoting the sample space which is a collection of all possible outcomes and this outcome we usually denote with little omega and little omega here for example, denotes the outcome of the experiment.

Ok. So, now, we have understood what is a random experiment and what is a sample space. Sample space is just collection of outcome of the random experiment. Now to discuss about random variable let us choose one example. Let us take sample space as a class.

(Refer Slide Time: 05:15)



So, this is a class; let say class of all students doing this online course and digital communication and the elements of the sample space are the students. So, in a class you have several students and all these students are the elements of this sample space. Now do I do a random experiment right?

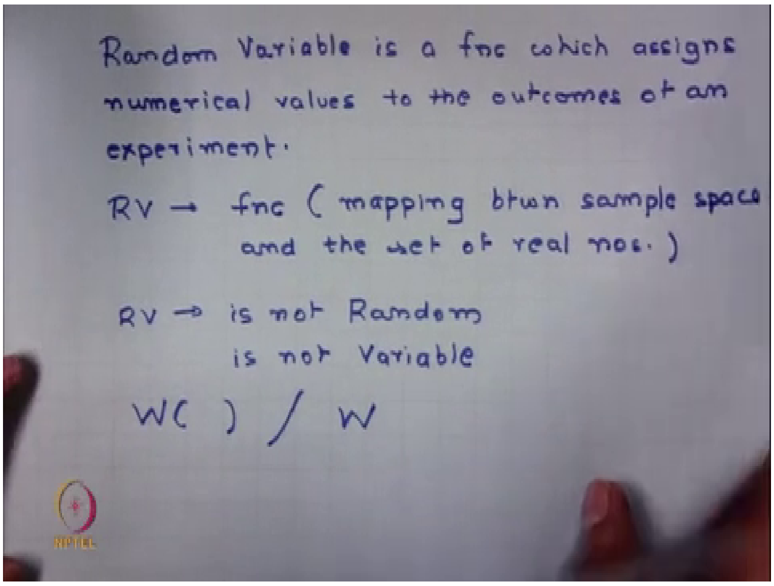
So, the random experiment is randomly I select one of the students. So, let me select this is a student and then I pass the name of this student through a function W s. So, I introduce a function W s where s is the name of the student and W s essentially gives me the weight of the student selected. So, for example, my randomness is a selecting a particular student, I could have selected any student, but randomly I selected this student and then I pass the name of this student to this function which returns me the weight of the student and I draw this one a real line. So, on this I have weight in kilograms.

I find the weight of a student; let us say the weight of the student is 60 and I put this on a real line ok. I do this experiment again and then I select it randomly another student and let us say this student name is again passed onto this function which returns me the weight of that student and let us say the weight of that student is 50. So, what is this function doing? This function is taking the outcome of the experiment. So, here s is the outcome of the experiment and for a given outcome of the experiment, it is finding a numerical value for that outcome.

Here in this case the numerical value is nothing, but is the weight of these students selected, but in general this function provides me a numerical value to the outcome of the experiment. For example, I could have chosen another function, I could have another function H of s; H of s might give me the height of the student selected. So, again for every outcome of an experiment so, s is the outcome of the experiment and to a given outcome of an experiment I pass it to this function H of s and I get a numerical value corresponding to that outcome of the experiment.

So, both W s and H of s both these functions are assigning numerical values to the outcome of the experiment. And interestingly both these functions or any function with assigns a numerical values to the outcome of the experiment is known as a random variable. So, what is a random variable?

(Refer Slide Time: 10:06)

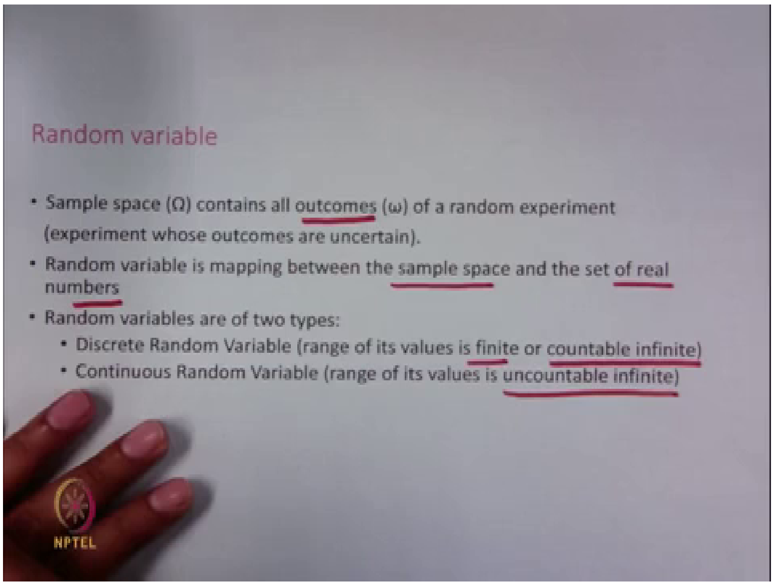


Random variable in short so, random variable is a function which assigns numerical value to the outcomes of an experiment. So, as you can see now, this random variable is a function, that provide mapping between that is another definition another way to think about the same thing. It provides a mapping between sample space and the set of real numbers.

So, the function it is a function that maps the sample space to the set of real numbers. Random variable is not random because there is nothing randomness there is no randomness involved here. Randomness is which outcome turns up, but for a given outcome you have a perfectly deterministic mapping all right. So, random variable is not random and is also not available. So, as you can see that we try to make things extremely complicated for you by given a name which is not what the name suggest it is a function it is neither random nor variable, but on the other hand it is a function which provides mapping between sample space to the set of real numbers.

So, if it is a function usually functions are donated. So, we use this parenthesis to denote a function, but as a notation the random variables are usually denoted with the capital letter, but without any parenthesis. So, we have introduced the idea of random variable; let us see. So, these are the basic things that we have discussed so far.

(Refer Slide Time: 12:53)



First we have talked about the sample space. The sample space contains all outcomes of a random experiment. Random experiment is an experiment whose outcomes are unceertain and random variable provides a mapping between the sample space and the set of real numbers.

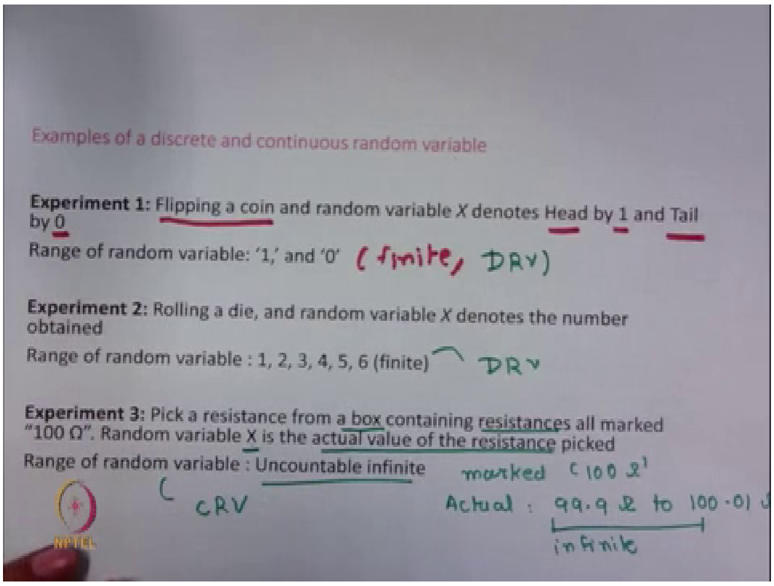
Random variables can be of two kinds: the first kind of a random variable that is easy to understand is a discrete random variable and the another kind of a random variable is a continuous random variable. So, when we are talking about a discrete random variable, the range of the random variable is either finite or countable infinite. For a continuous random variable, the range of the values is uncountable infinite ok. So, we have already introduced these words countable infinity and uncountable infinity. So, you distinguish means infinity is of different orders, you have a countable infinity which is exactly like as if the set of integers. And uncountable infinity is like when we are considering the set of real numbers.

The basic difference between the set of integers and the set of real numbers is between any 2 integers, you can have only a finite number of integers whereas, in the case of real numbers between any 2 real numbers, we have infinite number of real numbers right. So, the set of integers and the set of real numbers is both infinite, but there is a difference in the kind of infinity that they depict. The set of integers is countable infinity is less severe as the set of real numbers where the infinity nature is uncountable.

So, again I repeat if the range of a random variable is finite or if it is infinite, then the infinity is nature is countable infinity, then we call that random variable as a discrete random variable. And if the range of values of a random variable is uncountable infinite, we call that that as continuous random variable.

Let us see some examples of that examples of discrete and continuous random variable. We have some trivial examples the examples that you might have seen before.

(Refer Slide Time: 15:22)



Let us say I flip a coin and flipping a coin as we know as a gives me head or tail and because it is a random variable. I choose a random variable X, random variable assigns numerical values with the outcome of the experiment.

So, I choose to assign numerical value 1 to head and a numerical value, 0 to tail all right. So, I did this assignment I could have chosen any other numbers, it does not matter. So, I flip a coin and either I get a head and tail for which I choose 2 numerical values either I will get a one corresponding to head or I will get 0 corresponding to a tail.

So, the range of random variable in this case is 1 and 0 and there is no surprise that this range is finite and this random variable is a discrete random variable ok. I can do another trivial experiment of rolling a die and then the random variable X can be used to denote the number obtained and in that case the range of random variable can be either 1, 2, 3, 4, 5, 6. And again as you can see that the range is finite and this is also an example of a discrete random variable because range of the random variable is finite.

I can take a more trickier example of suppose I have a box containing resistances and all resistances are marked as 100 ohm resistance ok. So, I have a box and this box contains 100 ohm resistances and I pick a resistance randomly from the box and I choose a random variable x which tells me the actual value of the resistance picked.

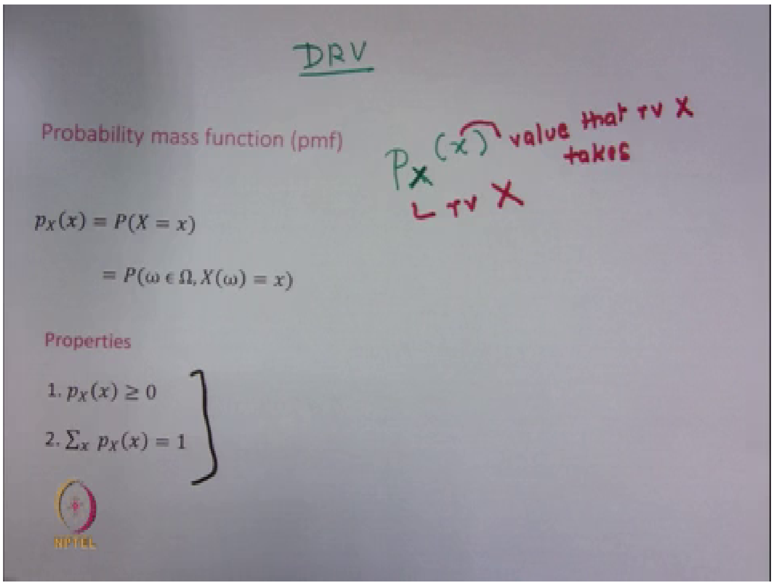
So, X denotes the actual value of the resistance picked be careful that is the difference between marked value. Mark value is always 100 ohm, but actual value might be different from this marked value because you might have studied in basic electrical engineering that the actual value depends upon the tolerances right; manufacturing tolerances.

And so, actual value let us assume might vary from 99.9 ohm to let us say 100.01 ohm and so, you see here that actual value lies in this given range 99 ohm to 100.09 ohm and there are infinite number values in this range; there are infinite values in this range. So, actually this range of random variable is uncountable infinite because it is like the real numbers.

So, the actual value can be anything it can be it is a real number if I assume that I have an ohmmeter with infinite precision, then I will get your real number and the set of real numbers belongs to this uncountable infinity. And so, this random variable x in this case is a continuous random variable ok. So, we have defined these two kinds of random variables discrete random variables and continuous random variable. And if I ask you a simple question which do you think is easier to deal with, the answer is obviously, discrete random variable.

So, basically in this lecture we will focus on discrete random variables and the first concept that we have to study concerning this discrete random variable is probability mass function.

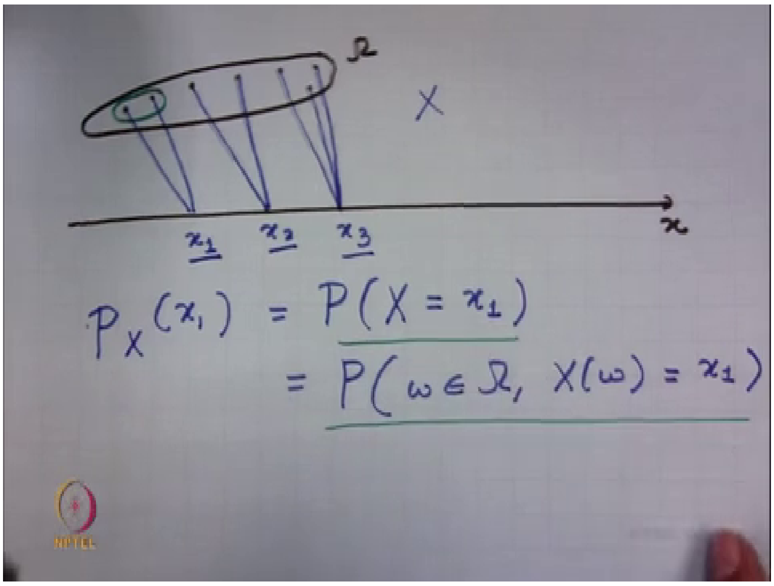
(Refer Slide Time: 19:08)



So, what is the probability mass function? So, first let me introduce the notation that I have used for probability mass function. So, you see that there are 2 x in this probability mass function. One x is where the capital letter X and this denotes the random variable X. This notation is really important and it is very important that you sort this notation clearly in your head because this might confuse you and this is small x is the value that random variable X takes.

So, we have 2 things going in here: one is the capital X which denotes the random variable X and I have a small x which denotes the value that random variable X takes.

(Refer Slide Time: 20:26)



Let us make it slightly more clear and let us go back to an example of the sample space. I have a sample space and this sample space contains some possible outcomes and again I have a real line. So, what happens is what might happen is, we might end up with these outcomes. And I say that these outcomes when these outcomes happen, these outcomes are map to a real number x 1. If these outcomes are mapped; if these outcomes happen and they are mapped to a real number x 2, these outcomes are mapped to a real number x 3. So, that is a random variable.

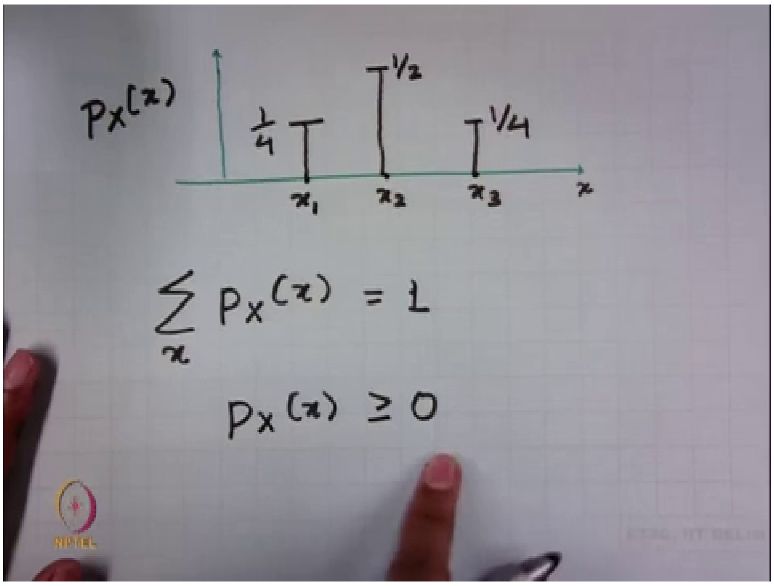
So, I have a random variable X which assigns numerical values to this outcomes to to these 2 outcomes it assigns a numerical value x 1, to these 2 outcomes it assigns a numerical value x 2 and to these 3 outcomes it assigns a numerical value x 2. Now if I ask, what is the p m f P x of x 1. So, it just tells me is what is the probability for a random variable X taking a value x 1 this is the interpretation of the p m f.

So, here what I am saying is to this random variable X can take 3 numerical values what is the probability that this random variable X takes in a value x 1 all right and this is denoted by probability mass function. As you can see that probability that random variable X takes in this value x 1 is same as the probability that these 2 outcomes turn up.

So, this probability is same as probability of omega. Omega remember is used to denote the outcomes of an experiment belonging it to the set of the sample space omega lies in the sample space and what is the probability that X omega. So, random variable takes in the independent variable as omega. So, it takes in the outcome of an experiment and what is the probability that outcome of this experiment is assigned to a numerical value x 1 ok.

So, essentially these 2 are exactly same just we have written this in bit more formal way. So, what I we are introducing is the concept of probability mass function with simply it tells me is what is the probability that, this random variable X takes in this value x 1 all right. So, let us make one more picture corresponding to this.

(Refer Slide Time: 23:43)

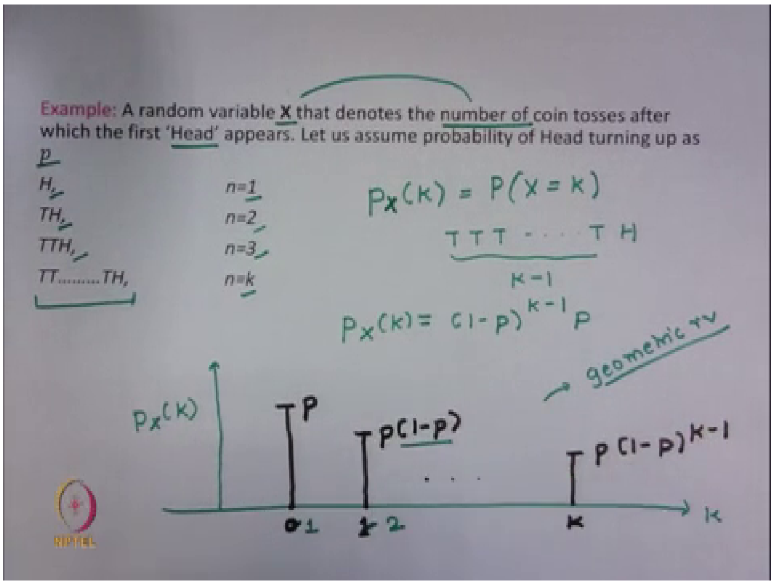


So, let us have a picture. So, on x axis I have x and x can takes in 3 numerical values x 1, x 2 and x 3 and corresponding to these 3 numerical values, I am calculating is probability mass function. And let us assume that probability that x 1 happens is 1 by 4. Let us assume that the probability that x 2 happens is half and let us assume that the probability that x 3 happens is again 1 by 4 ok. So, probability mass function basically tells me what is the probability with which x 1 happens x 2 happens or x thre3 happens right and because these are just probabilities and you might have learned it already that the sum of probabilities is 1.

So, I can easily write that P x of x for all values of x is nothing, but one why is this so? Because some of probabilities is 1 and you can also understand quite easily the second property of this probability mass function which is that probability mass function is always non negative right. So, it is always strictly greater than 0 or equals to 0. It can never be negative because the probabilities can never be negative. So, that is the another second property of probability mass function.

So, let us go back to the slide and see if we have covered all the concepts. So, we have introduced the idea of probability mass function. I have said that probability mass function p x of x just denotes probability that random variable X takes in the value x and it satisfies these 2 properties that p m f is always a non negative is greater than or equals to 0 and if you sum p pmf for all possible values of X, you end up with one because sum of probability is so always 1.

(Refer Slide Time: 26:02)



Let us do one example to sort things clearer in your head. And this example is simple example where this random variable X denotes the number of coin tossed ok. So, I am interested in number of coin tosses after which the first head appears that is good and I assume that the probability of head turning up as p. So, p is the probability with which you get a head.

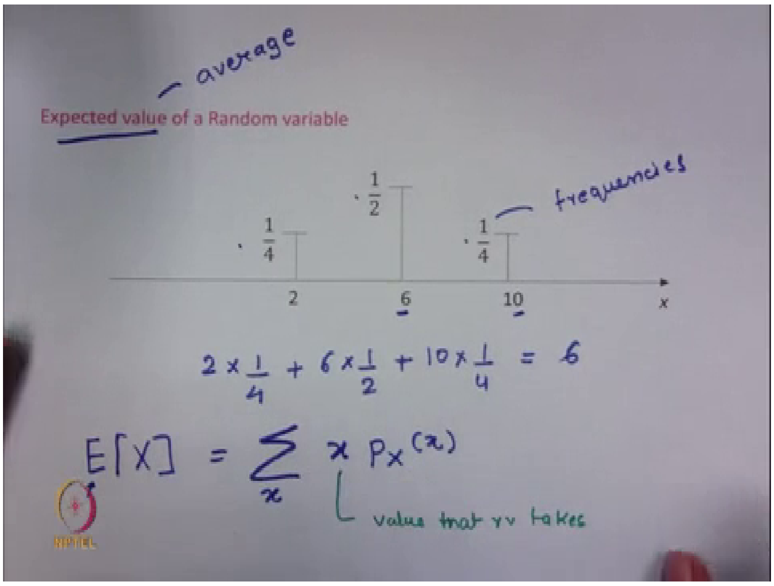
So, what I am saying is this random variable is the number of coin tosses after which I get a head. For example, if you get a head at the first toss the number is 1; the random variable takes the value 1. If you get a head in the second toss the random variable takes the value 2. You might get a head in the third toss where the random variable takes the value 3 and or you might get it in kth toss where the random variable takes the value k.

So, if I am interested in the probability mass function which is what is the probability that, X takes in the value k. So, what is essentially this is say what is the probability that you get a head in the k'th toss. So, this will be the case when you have k minus 1 tails and on k'th toss you get a head right. So, if I am interested in writing what is the probability for this event, you can see that the probability of getting a tail is 1 minus p and there are k minus 1 tails. So, you multiply 1 minus p k minus 1 is times assuming that they are independent and so and so forth and the probability of getting a head is we know p.

So, this is the pmf. This is the probability where X takes a value k and it would be better if I can draw this for you. So, if I have on this axis, I have k and on this axis I have probability mass function and let us draw various points. Let us say k equals to 0, the probability of that is p you just plug it into this expression, for k equals to 1, I get the probability as p into 1 minus p and so and so forth for k equals to k I get p into 1 minus p k minus 1. So, this has to be there is slight mistake this has to be 1 and this has to be 2 all right.

So, now what you see here is that the height of the bars decreases geometrically. So, they decrease with this factor of 1 minus p and hence this is known as geometric random variable ok. So, what is the geometric random variable? If the height of the bars the height of the probabilities decreases geometrically, you get a geometric random variable. This example is just to tell you how to calculate the probability mass function and then we introduce the important concept of expected value of a random variable.

(Refer Slide Time: 29:44)



So, let us look at this concept of expected value of a random variable and this is also really central if you want to understand random variable or random processes. What this expected value of a random variable tells us?

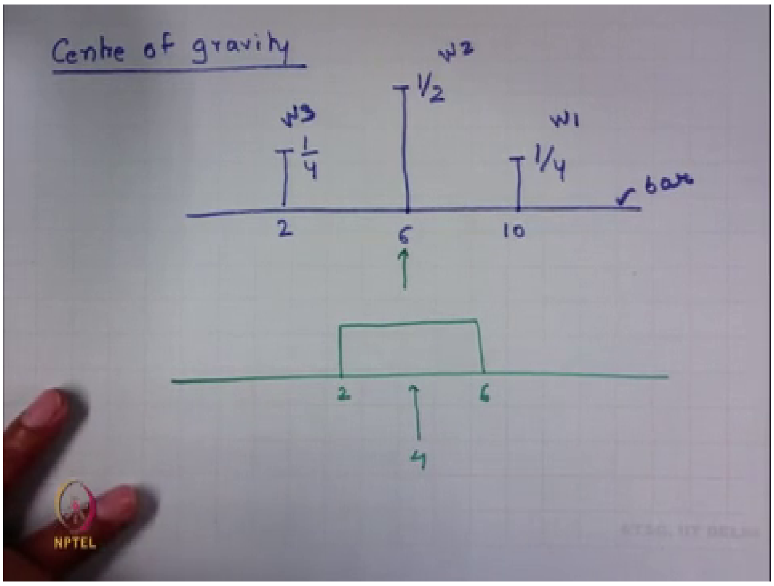
This expected value is a some kind of average value of the random variable. So, for example, let us say let us look at this picture and let us say this x takes a value 2 and the probability as 1 with a probability of 1 by 4, it takes a value 6 where the probability of half, it takes a value 10 with the probability of 1 by 4 and I ask you what is the average value of this X ok.

So, you can think about the average value if you start thinking about this probabilities as frequencies. So, so you can think or interpret this probability as the frequency with which you get this value and then getting thinking about the average value of X is trivial it is a straight forward, it is just you have to take 2 multiply with this probability. You taken 6 multiply with half you take 10 multiplied with 1 by 4 and final answer that you get on solving this is 6.

So, expected value of a random variable is exactly this what we have calculated formally or mathematically, I can define expected value of X. So, this is used to denote expected value of x as; so, what we what we did is just we have taken a value that a random variable takes small x. So, this is the value that random variable takes and then you multiply this with the probability of the occurrence of this value and then you have to sum it up for all possible x.

So, you take in x 1 multiply with probability of x 1, you take in x 2 multiply with the probability of x 2 happening, you take in x 3 you multiply with the probability of x 3 to happen and finally, you calculate the expected value of a random variable X ok.

(Refer Slide Time: 32:32)

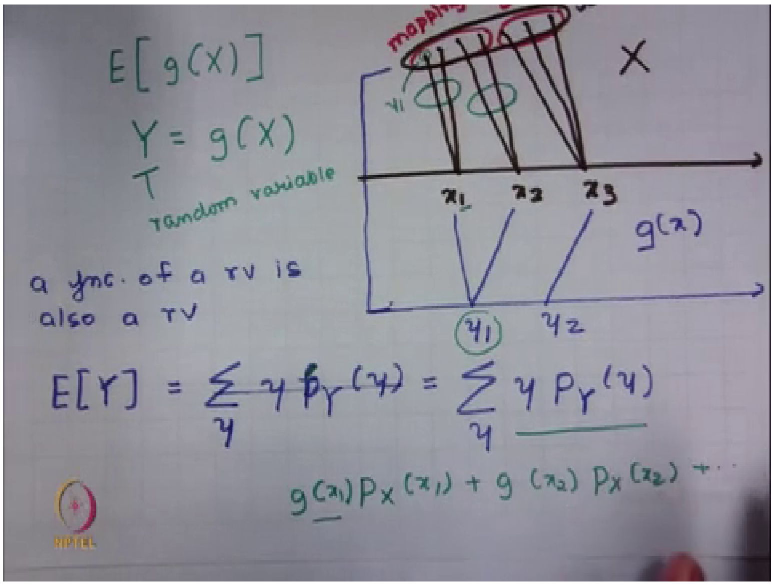


So, one good way in which we can interpret this is by thinking about this as centre of gravity. Thinking about expected value as centre of gravity for example, let us look at this is structure again. So, I have 2 and the value of probability was 1 by 4. I had 6 the value of probability was half I had 10 the value of probability was 1 by 4; so, this just repeating the diagram that we had.

So, if you think as if this is as a bar and this as some kinds of weights. So, I have a weight W1, W2 and W 3and this 1 by 4 tells me what is the weight that you have put on this bar. So, weight is the centre of gravity of this structure as you can see that clearly the centre of gravity would be at this point. If you keep your pen below this point, this bar can balance itself because the weight on this side and this side is at the same distance and it is of the same value. So, it will perfectly balance itself.

So, thinking about the expected value of a random variable as a centre of gravity helps us in some cases. For example, let us assume that I have another structure I have block of uniform density and this block is kept between 2 and 6 right. So, if I ask you where is the centre of gravity centre of gravity, you can immediately think will be at the point 4 all right. So, thinking about the expected value of a random variable as centre of gravity in certain cases eases out the computation and you can quickly arrive at the answer. Let us look at slightly involved concept um. It is nothing is involved if you have understood this, but this is little bit more trickier. What we are doing is what we are interested in finding is what is the expected value of a function of a random variable.

(Refer Slide Time: 34:34)



So, I have another random variable Y which is some function of random variable X ok. Let us understand this pictorially what happens pictorially in such cases. And to understand pictorially we start with our sample space, we put in some dots which denotes the outcome of the experiment and then I have a real line and then I map these outcomes to some numerical values. So, I have been assigning some numerical values to these outcomes right. And who does this for us is a random variables. So, I can choose a random variable X which provides me a mapping between this sample space omega and this set of real numbers.

Now let us say that, I have another function g which takes in these numerical values. So, I draw another line. So, this is a function which takes in this numerical values and mapped with different numerical value ok. So, there is a sum function acting on x which takes in these values of x and provides bit different numerical values. So, if you see from end to end from here to here, what you have got is you have got some numerical values assigned to the outcome of the experiment.

For example this outcome this experiment, this outcome has been assigned a numerical value y 1, this has been assigned a numerical value y 1 and these also are assigned a numerical value y 1. So, what we are doing is again we are assigning these outcomes of this experiment some numerical values.

So, this is also a random variable right. So, y is also a random variable. So, in conclusion what I am trying to stress is a function of a random variable is also a random variable that is important right. So, X was a random variable a function of X is also a random variable and taking the expected value make sense in this case. I am just trying to say that expectation operator make some sense here all right.

So, now we have to find what is this expected value of g of X. So, if you look it, I am trying to find expected value of y and expected value of Y by definition is nothing, but y and then you multiply it with the probability; this has to be probability mass function with which y happens; so, this. So, let me write it again. So, expected value of Y is nothing, but y and the probability with which this y happens and then you have to sum it for all y's.

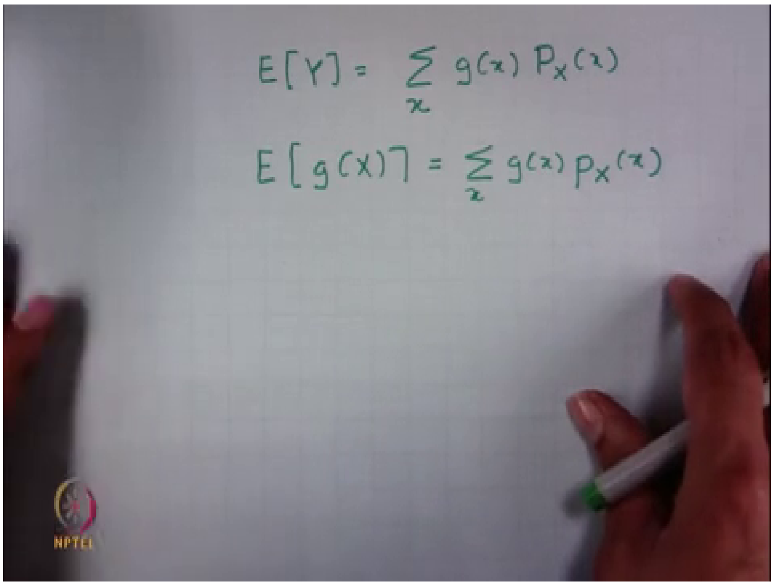
Now, pictorially what does it mean is that you take in a value, let us say y 1 in this case. You take in this value y 1 and you find out what is the probability with which this y 1 happens and if you see carefully the probability with which this y 1 happens is this. So, whenever these outcomes happen, y 1 happens.

So, you take in this y 1 and you multiply with the probability with which these outcomes happen ok. You take in y 2 and you multiply with the probability which is the probability of any outcome within the set to happen right. So, it taken y 1 multiply with the probability of any outcome within the set to happen, you take in y 2 and you multiply with the probability of any outcome within the set to happen.

Now, this will make things complicated for us. So, we try to propose another way in which you get the same average right. So, you can get the same average if you start thinking about this. So, let us say this happens and this happens with a probability of this. So, this has a probability of this event and when this happen? What is the value that it takes? So, if this happens, what is the value it takes? It takes the value g of x 1 this is the value. So, y 1 is g of x 1. So, it takes in the value of g of x 1 and this happens with a probability of p of x of x 1 right. Similarly you can say, what is the probability with which this item happens? This item happens with a probability of p x of x 2 and when this happens, it takes a value g of x 2 and so on so forth.

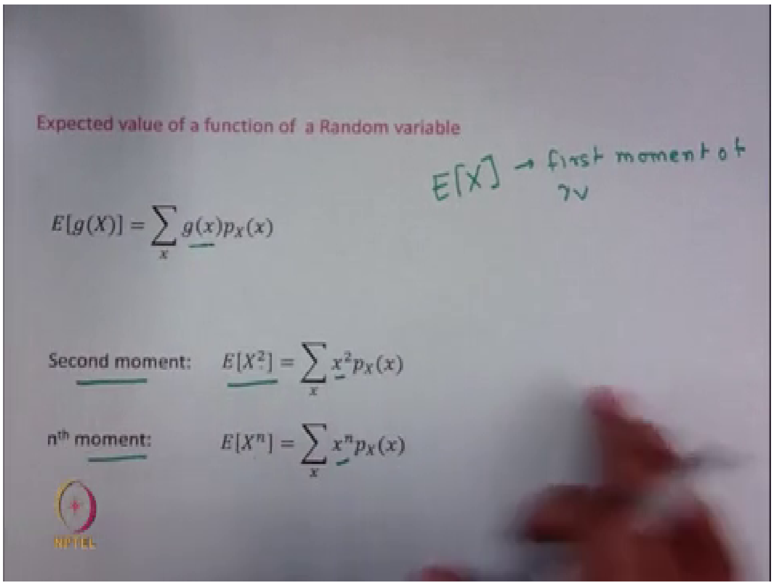
So, this will give you the same average as this expression is one and the same thing only the interpretation is different. So, in the end what you would end up with is.

(Refer Slide Time: 40:46)



So, if I start with this expression this above expression, what we would get is the expected value of Y is nothing, but summation g of x P of x in x probability mass function of x for all possible values of x ok. So, the way to think about this is finally, we have concluded that expected value of g of X remember, y was g of X is nothing, but summation x g x into P x of x. So, what we have said is, you can interpret the expected value of g of X without thinking about the probability mass function of random variable y; you can still get the expectation by thinking in terms of probability mass function of x. So, you do not have to do an extra step in trying to find the probability mass function of y, you can do it straight away ok.

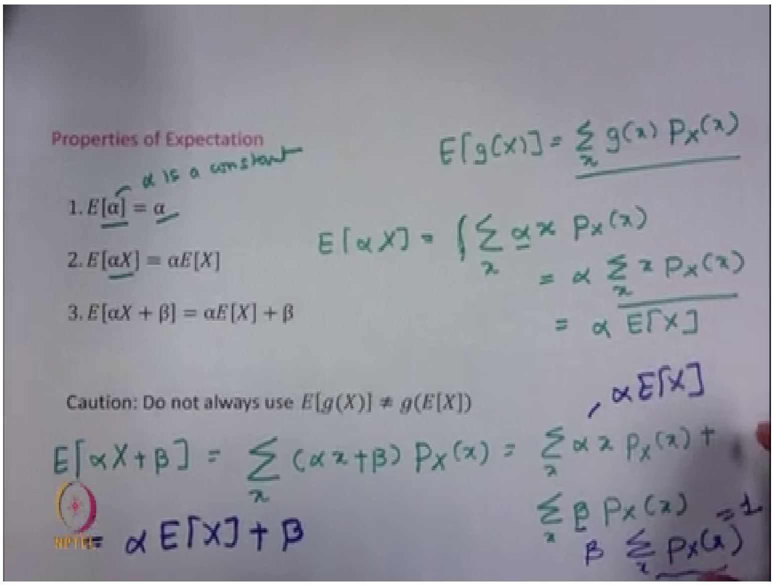
(Refer Slide Time: 42:01)



So, if this definition is clear, let us make use of this make use of the hard work that we did and let us try to define second moment of a random variable. So, what is the first moment was? So, first moment is nothing, but expected value of X is the first moment of the random variable. What is the second moment? Second moment is E X square. So, if you are trying to find the expected value of X square that is known as a second moment. If you want to find the expected value of x n that is known as n'th moment.

And if you believe this definition which we have worked out so, what you can do is you just have to replace. So, X square and X n are functions of X and if you just have to a find the expected value for a given function of X, you can still interpret this in terms of probability mass function of a x; just you have to replace in place of x g of x; g of x in this case is x square g of x is in this case is x to the power n. You can find the various moments of a random variable X ok. So, this is one way in which we can find moments and in next lecture of probably a lecture after, we will look into a different way in which you can compute the moments of a random variable.

(Refer Slide Time: 43:25)

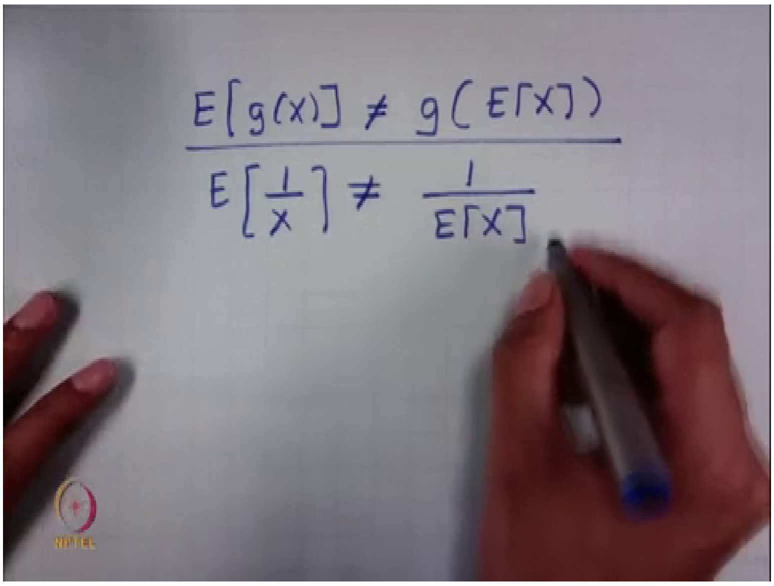


Let us now look at the properties of expectation operator. So, what is the expected value of alpha where alpha is a constant. So, if you treat alpha as a constant; that means, random variable always takes in the numerical value alpha. What is the average value of that random variable? Its alpha there is no nothing difficult here maybe the second one is more interesting. So, what is the expected value of alpha X and you can look at the expected value of alpha X. So, just use this property which we derived. So, expected value of alpha X is summation alpha times x p x of x and alpha is not a function of x, it could be pulled out of summation and this you know is alpha times this is E X.

So, what we are saying is expected value of alpha times X or a constant times a random variable is nothing, but it is that constant times the expected value of that random variable. Let us look at this as well what is the expected value of alpha X plus beta. Plug it in the equation that we have this one and working out some steps. So, that extremely clear, we get this and if you look carefully, this beta can be pulled out because it is not a function of X. So, I can have beta times and this term is this quantity is one because the sum of probabilities is always 1. So, essentially and this quantity is nothing, but alpha times E X and this quantity will then be just beta.

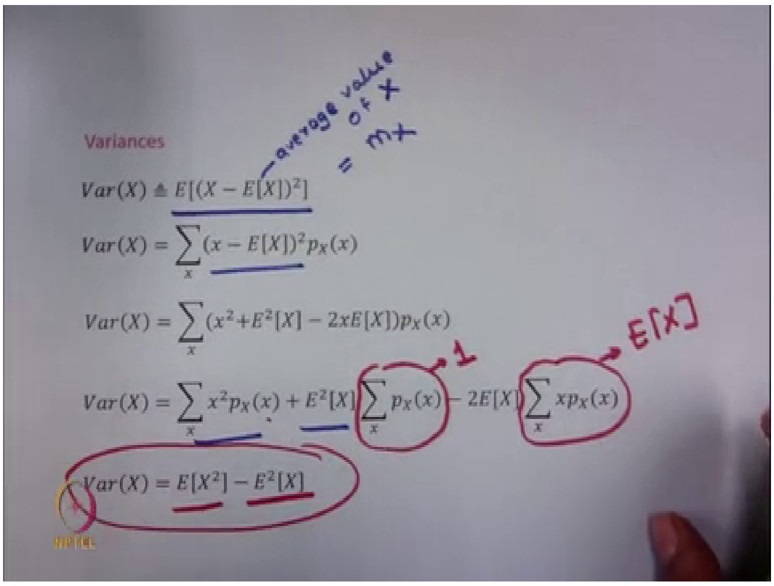
So, expected value of alpha X plus beta is nothing, but alpha times expected value of X plus beta ok. So, these are some basic properties of expectation operator that might help you in doing certain things quickly, but I advise you do not get excited with this and do not use it in all situations.

(Refer Slide Time: 46:49)



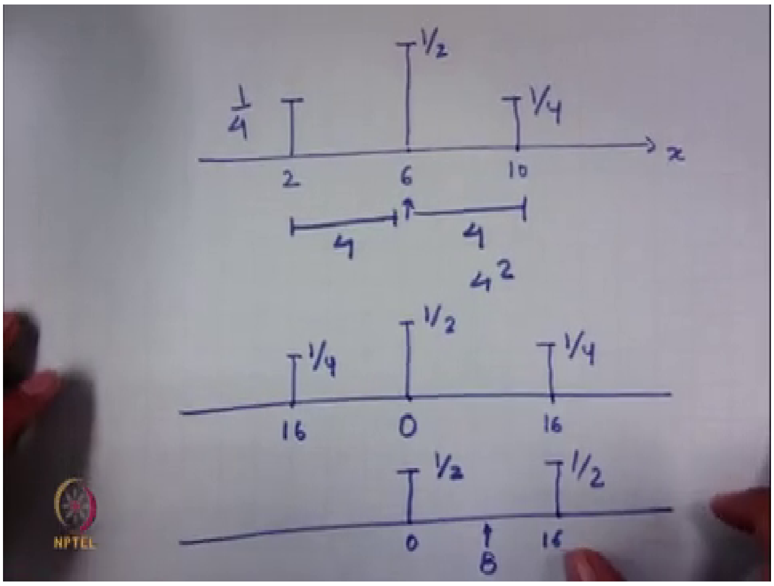
For example, remember that expected value of g of X is not same as g of expected value of X; it is not same. These two things are not same. So, what I am saying is that expected value of a function of a X is not same as function of expected value of X. For example, you work this out that expected value of 1 by X is not same as 1 upon expected value of X; it is not same as. So, do not get carried away with these properties and do not try to use them in all situation, you have to be bit careful. So, what I would advise you is to start with definition have sufficient practice and then it will be clear to use when can you use that all right.

(Refer Slide Time: 47:48)



So, let us now introduce the last concept of this class that is about variances. So, what is a variance? And you can see here as it is defined that variance is you take expected value of X minus E of X which is the average value of X or sometime this is also known as mean of X mean of a random variable X which is an average value a mean of the random variable X and then you square this and find the expectation.

(Refer Slide Time: 48:34)



For example let us see this, the same cases we did before. So, let us say that x takes this value 2 with the probability 1 by 4. It takes in a value 6 with a probability half, it takes in the value 10 with the probability 1 by 4 and what I am looking for in this expression is to see what does this mean. So, we have already calculated that the mean of this random variable was here. So, try to think it in terms of centre of gravity and this will be extremely clear that the mean of this structure will lie here, then I take the distance from this mean. So, in this case, I take this distance which is 4; in this case, the distance from the mean is this and I and then I do this square of these distances.

So, I do two things. First I calculate the distance of the values that this random variable takes with respect to mean and then I square those distances and then I find the expected value of that quantity. So, basically what we are trying to do is we are trying to give more weightage to the distances that are longer. So, if I do that, this point would convert to a point 0 and this would happen with a probability half. So, I have subtracted the 6 from the mean, mean is also 6. So, I get 0 and then I square 0, I get a 0 and this point happens with the probability half and if I want to interpret this point. So, this is with a distance of 4 from the mean I square this and I get 16 and this happens with the probability 1 by 4.

And this point again translates to 16, this happens with a probability 1 by 4. So, I take distance from the mean, square them and I finally, get this because that two 16 and each happening with a probability 1 by 4. I can translate this to this case, there you have 0 happening with a probability half and you have a 16 all so, happening with a probability half. And what is the expected value of this quantity? Expected value of this quantity is 8 ok. So, this is one way which you can interpret variance.

So, let us try to have after understanding this equation, let us try to have another meaning of this. So, I can plug this into this definition. So, I use the definition of expected value of a random variable which is defined like this. So, I replace X with the value of that random variable; E X is a constant. So, it is not a random variable anymore. So, this comes out like this square into probability mass function of X right, then I expand this. So, this is in the form a minus b whole square. I can explain this is as a square plus b square minus 2ab times this quantity. And if you work out this algebra, what you get is, you get this plus this and if you are careful again then this quantity is nothing, but it is 1.

Why is this 1? Because the sum of probabilities is always 1 so, this quantity is 1. And if you look at carefully this quantity is also E of X. So, this is E square X minus 2 times E square X. So, what we get is E square X with a negative sign is plus 1 minus 2, I get minus 1 and this quantity as I have already defined is the second moment of this random variable E X square. So, variance of a random variable can also be thought as you take a second moment of random variable and subtract from this the square of the first moment of the random variable.

So, this is how you can interpret this variance. So, we will finish this lecture here. So, in this lecture, what we have basically achieved? We have started to look in to random processes and we have said that. We will like to do first by understanding about random variables and then we have also introduced discrete and continuous random variable. We are just looked into probability mass functions, which is used to characterize discrete random variables and we have introduced two important quantities with respect to random variable and that is the expected value of random variable and a variance of a random variable. We will try to build upon this and in the next lecture, we will like to have more interesting things when we will discuss about continuous random variables.

Thank you.