Principles of Digital Communication
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Detection
Lecture – 35
Performance of M-ary Signaling Schemes

Good morning. Welcome to a new lecture in detection, in this lecture we will talk about Performance of M-ary Signaling Schemes. In the last lecture we discussed the performance of binary signaling schemes.

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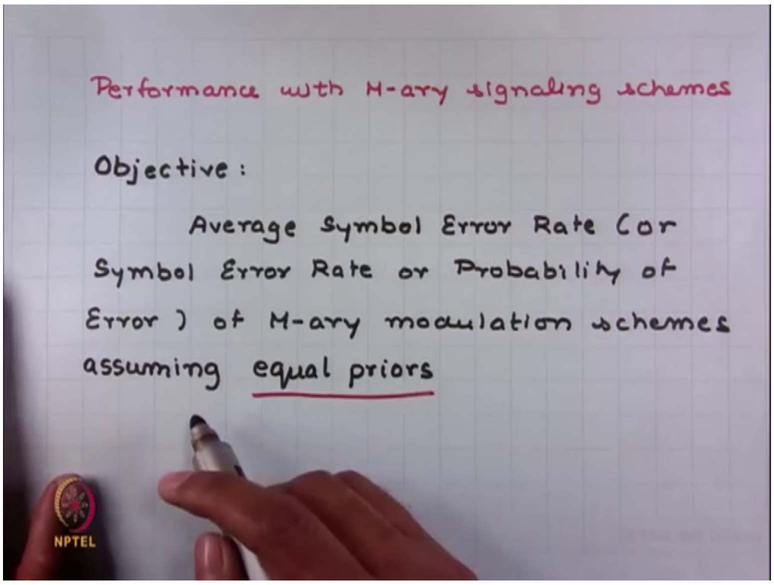

So, what is the objective of this lecture in this lecture we will learn how to calculate the average symbol error rate of M-ary modulation schemes and we assume that these signals happen with equal priors. We also sometimes call this every symbol error rate as simply symbol error rate or probability of error. So, I will use these words interchangeably ok.

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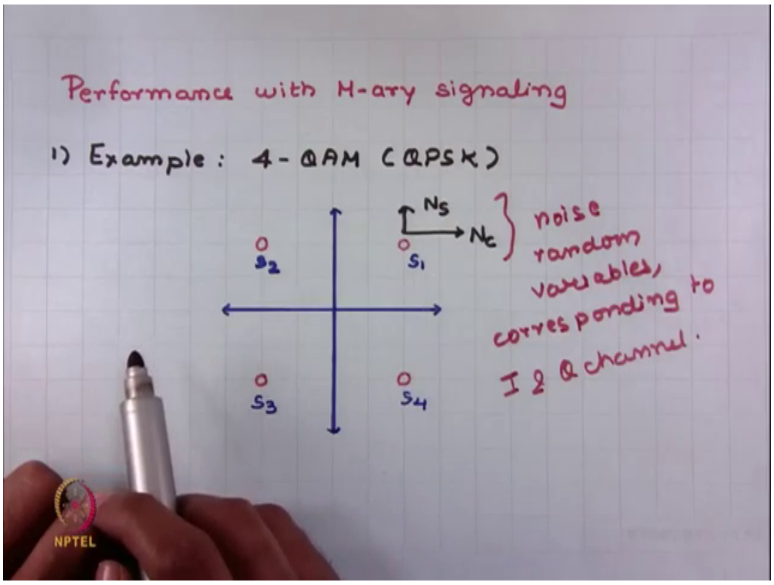

So, to start looking into this performance of M-ary signaling schemes we will take the 4 QAM as the running example. So, in this 4 QAM we have 4 signal points S 1, S 2, S 3 and S 4, to start calculating about this probability of error we will first calculate what is the probability of error given that we have transmitted signal S 1 ok. So, suppose we have transmitted signal S 1 what is the probability of error in that case.

Also you can look in this picture we have shown N C and N S these are the noise random variables corresponding to I and Q channel ok. So, N C is the noise random variable in the I channel and N S is the noise random variable in the Q channel and we have assumed that is this noise rights on this signal S 1 when signal S 1 is transmitted.

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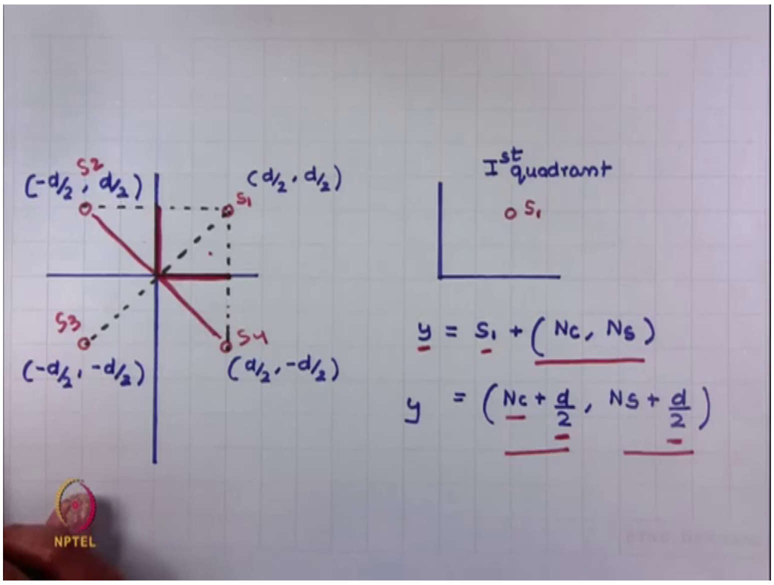

So, first let us look into the decision region of the signal S 1 how do we calculate the decision region of the signal S 1; so, to calculate that you have to draw lines from S 1 connecting all other signal points. So, we have this is signal S 2, this is signal S 3 and this is signal S 4. So, first step is that you draw lines from S 1 to S 4 to S 3 and to S 2 ok, once you have made these lines so, the next step is to draw perpendicular bisectors for this line. So, this is the perpendicular bisector to the line connecting S 1 and S 4, this is the perpendicular bisector for the line connecting S 3 and S 1.

And this is the perpendicular bisector for the line connecting S 1 and S 2 right and decision regions is governed by these perpendicular bisectors. So, now you can see clearly that the decision region of signal S 1 is simply the first quadrant, this perpendicular bisector is not influencing the choice of decision region of S 1. So, decision region of S 1 is simply the first quadrant and it is also very simple to see this for example, if this if a received point lies in the first quadrant this received point will be closest to S 1 and does the detector will decide for S 1 right ok. So, why let us say is the received quantity and this is S 1 the signal transmitted plus noise riding on the signal.

What is the location of S 1? S 1 is located at d by 2 and d by 2. So, it is d by 2 in the x direction and d by 2 in the y direction and this random variable n c adds to this d by 2 in the x direction or in the I channel and this random variable n is adds to this d by 2 in the y direction or in the Q channel. Now when will an error happen and error will happen when the received number will slip out of this first quadrant. So, it might go either to second third or fourth quadrant, if the received number slips out of the first quadrant then an error will happen and when will the received number or observation slip out of the first quadrant.

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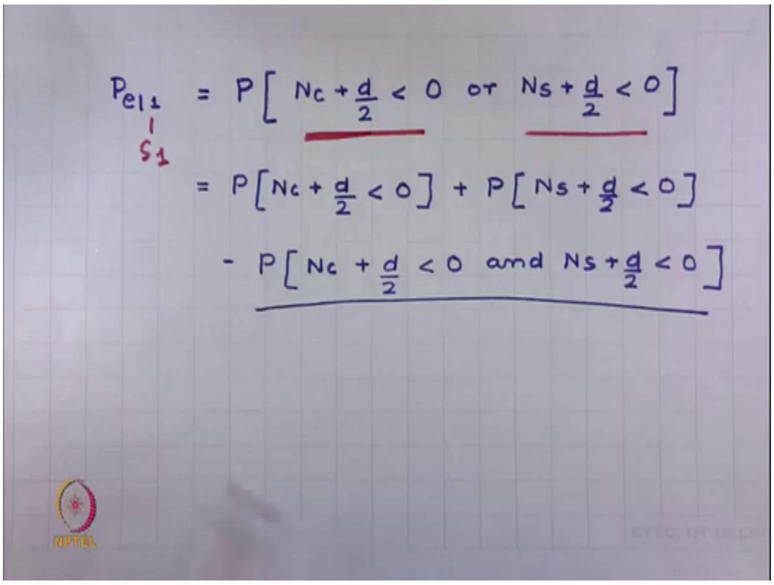

This will be when N C plus d by 2 that is the value of the received observation in the x direction is less than 0 or N S plus d by 2 becomes less than 0. So, probability of error given that we have transmitted signal S 1 there will be an error when the received observation will slip out of the first quadrant and this will happen when this is true or this is true.

And hence the probability of error given that we have transmitted S 1 is simply obtained by calculating the probability of these two events. So, probability of N C plus d by 2 less than 0 or N S plus d by 2 less than 0 is simply probability of N C plus d by 2 being less than 0 plus probability of N S plus d by 2 being less than 0 minus probability of N C plus d by 2 being less than 0 and the probability of N S plus d by 2 being less than 0. Let us understand, why is this?

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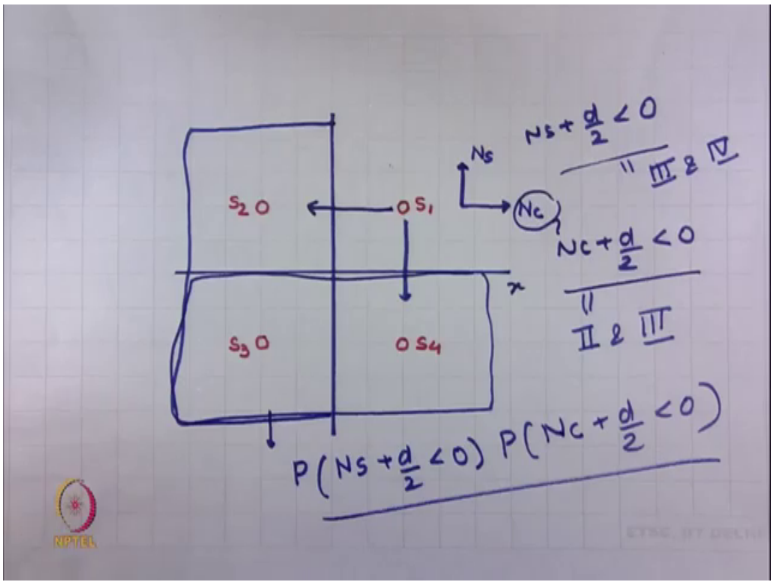

So, look at this constellation again. So, what we are saying is we are transmitting signal S 1 and if this observation lies in this first quadrant then the receiver will detect signal S 1. This received value will slip out of this first quadrant when the noise amplitude N C becomes less than d by 2 when this will happen the received value will lie to the left of this y axis ok. So, when N C plus d by 2 is less than 0 the received value will slip to the left of this axis. Similarly when n is plus d by 2 is less than 0 the received value will slip below this axis below x axis.

And thus you see when this event happens the received value will lie either in the second quadrant or in the third quadrant, when this event happens the received value will lie either in the third quadrant or in the fourth quadrant. So, this corresponds to the received value lying either in the second or third quadrant, this event corresponds to the received value lying either in the third or fourth quadrant. Now, we are calculating the probability of the received value being in third quadrant twice and thus you have to subtract the probability that received value is in the third quadrant and what is that probability.

The probability that received value is in the third quadrant is simply the probability that N S plus d by 2 is less than 0 and at the same time N C plus d by 2 is also less than 0. When these two events happen at the same time than the received value will lie in the third quadrant and thus to calculate the probability of error given S 1 from these two events we have to subtract this probability alright.

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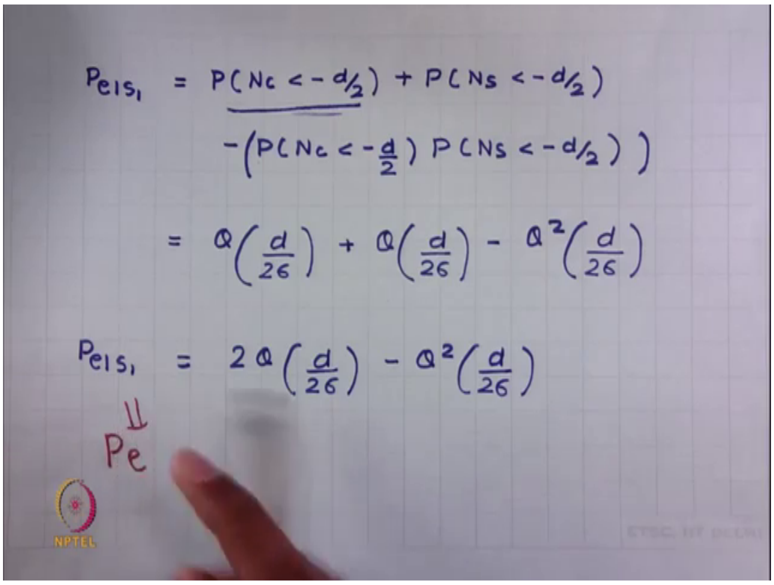

So, we get that the probability of error given that S 1 is transmitted is probability that N C is less than minus d by 2 and N S is less than minus d by 2 minus probability that N C is less than minus d by 2 and probability that N S is less than minus d by 2. Let us see: what is this probability in terms of Q function so, first let us look at a picture.

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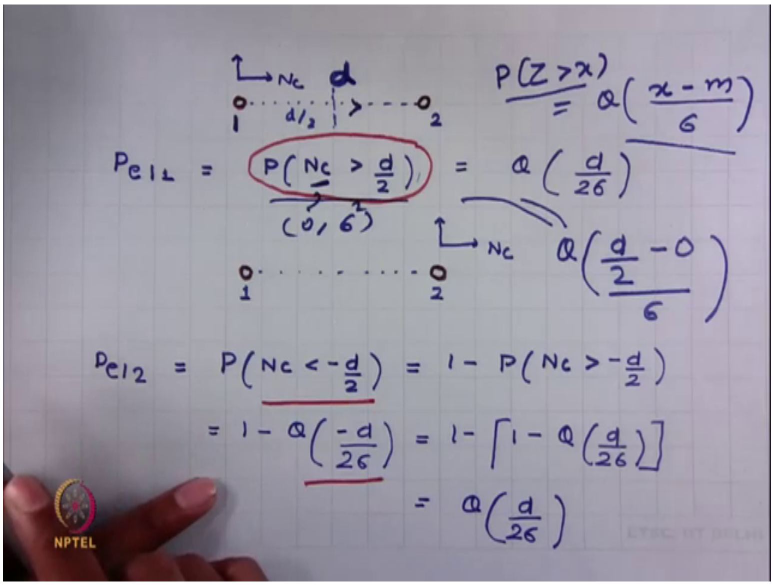

So, let us simply assume that we have 2 signals 1 and 2 and these two signals are separated by a distance of d and let us assume that we are transmitting signal 1 and when will in error happen error will happen when the noise amplitudes become larger than d by 2 ok. So, a transmitting this signal perpendicular bisector will be at d by 2 from this place, if the noise amplitudes become larger than d by 2 then the received value will lie to the right of this perpendicular bisector and the detector will assume that the transmitted hypothesis is 2.

So, an error will happen when N C becomes larger than d by 2 and we know that this random variable is a Gaussian random variable with mean 0 and a variance of sigma square and we know how to calculate the probability of this event in terms of Q function. So, we have seen before that if you have to look about the probability of a Gaussian random variable taking a value greater than x then you can easily calculate this probability in terms of Q function and this will be Q of x minus mean of this random variable divided by a standard deviation of this random variable.

We have seen in one of the lecture that this can be simply calculated in terms of Q function like this. Here N C is also a Gaussian random variable so, to calculate this probability it will be Q of what is x in here x is d by 2 minus mean what is the mean of this is 0 and what is the standard deviation is sigma ok. So, the probability of this event is simply Q of d by 2 sigma, why we are a stressing so much on this, because we will get these kind of expressions and from now onwards I will simply write that probability of N C greater than d by 2 is simply Q of d by 2 sigma so, now, you must know this.

Similarly if I have to find what is the probability of N C less than minus d by 2 so, the probability of this event can be understood as 1 minus probability of N C greater than minus d by 2, because either the numerical value of random variable will be less than minus d by 2 or it will be greater than minus d by 2 and the sum of these two probabilities should be 1. So, probability of N C being less than minus d by 2 is simply 1 minus probability of N C greater than minus d by 2 and what is this by using the same logic I can write that this is simply 1 minus Q of minus d by 2 sigma x is minus d by 2 here.

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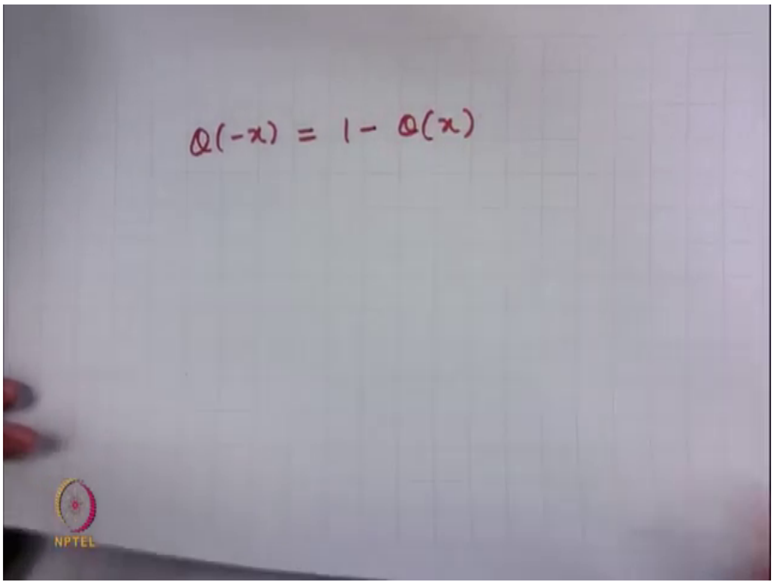

And we can use one property that Q of minus x is simply 1 minus Q of x ok. So, using this property I can write that this is 1 minus Q of d by 2 sigma and this is simply Q of d by 2 sigma. Hence, in nutshell probability of N C greater than d by 2 is Q of d by 2 sigma and probability of N C being less than minus d by 2 is also Q of d by 2 sigma and why is this so, because these random variables are Gaussian random variables and if they are 0 mean they are symmetric about 0 and hence the probability of noise amplitudes being less than minus d by 2 or being greater than d by 2 must be same.

So, let us now get back to this equation. So, we have in investigating the probability of error given S 1 and now we clearly understand how to calculate this probability, this is simply Q of d by 2 sigma and what is this, this is also Q of d by 2 sigma because this random variable is also with 0 mean and has variance of sigma square and what is this probability it is nothing, but it is Q square of d by 2 sigma and hence probability of error given that you have transmitted S 1 is simply this ok. Now, but we have to find probability of error right average symbol error rate now we have found out what is the error rate given that we have transmitted S 1, but from here we have to go to find out the average symbol error rate.

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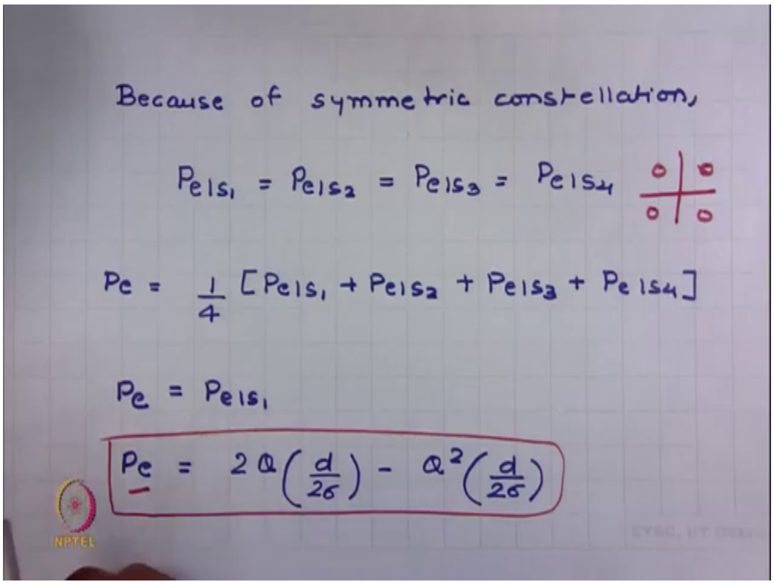

Now, because this 4 QAM is a symmetric constellation so, you can easily work out that probability of error given S 1 will be simply probability of error given S 2, will be same as probability of error given S 3, will be same as probability of error given S 4 right all signals are symmetric to each other right. The average probability of error will simply be 1 by 4 times the sum of these probabilities and because all these probabilities are same you can simply get that the probability of error is simply probability of error given S 1.

And, hence the overall probability of error is also 2 Q of d by 2 sigma minus Q square d by 2 sigma ok. So, average symbol error rate is the same as the error rate when you have transmitted one of the signal.

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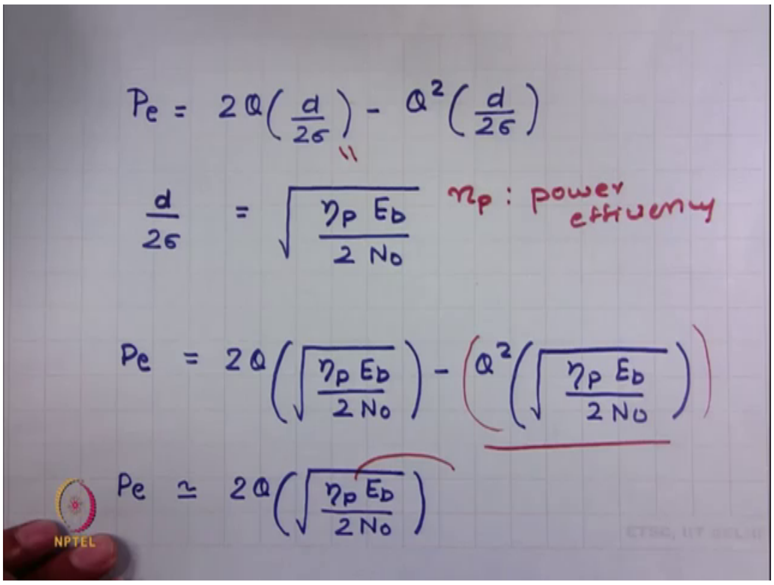

So, we have got this expression of average symbol error rate in terms of Q function and d by 2 sigma and as before now the idea is to express this in terms of root of Eb No right we are interested in doing so. And d by 2 sigma same as in the case of binary signaling schemes is simply a square root of eta P, where eta P is power efficiency times Eb No by 2 ok. So, d by 2 sigma is a square root of power efficiency multiplied by Eb No by 2. So, if you have to write this in terms of Eb No it is simply 2 Q function of root eta P Eb No by 2 minus Q square of root of eta P Eb No by 2. And you know that this is a pretty small number because the probability of errors that we are interested in is 10 to the power minus 8 or 10 to the power minus 3 or a number like that.

And, hence this Q function typically should give us a value like 10 to the power minus 5 4 or something in that order and Q square will does be a pretty small number compared to Q and hence you can safely ignore this and you can arrive that the probability of error is approximately 2 Q of under root of eta P Eb No by 2. The next job remains is to calculate the power efficiency for M-QAM modulation scheme that is what we add up to. So, now, we will be finding out the power efficiency of M-QAM modulation scheme.

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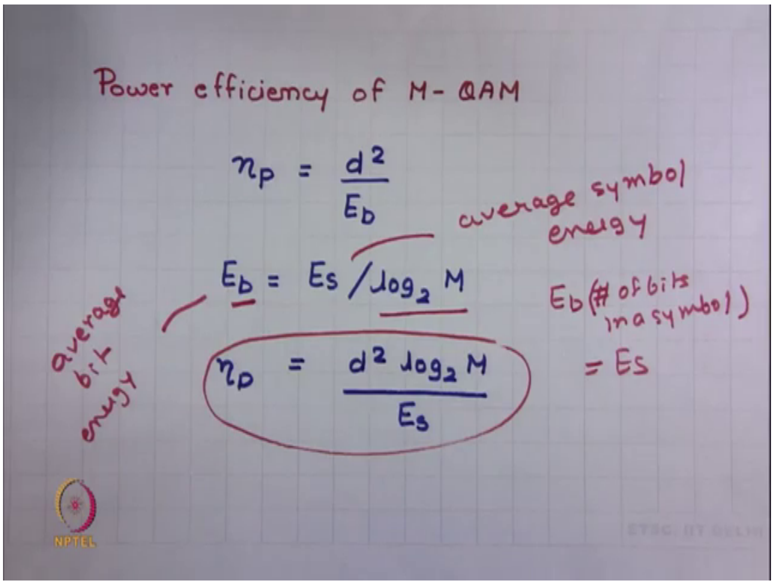

Now, this power efficiency is simply d square by Eb right this we have seen in the binary signaling schemes and what is this Eb, Eb is Es by log 2 M. So, Es is the average symbol energy and Eb is the average bit energy, sometimes I also call this average bit energy as bit energy because it is obvious that we are only interested in average quantities right.

So, if this average bit energy should be number of bits in a symbol. So, average bit energy multiplied by number of bits in a symbol should give me average symbol energy or signal energy and this average bit energy is average symbol energy divided by number of bits in a signal and number of bits in a signal or a symbol is simply log 2 M right.

If M is the number of signals that we have the number of bits that are required to represent these M signals is simply log 2 M. So, average bit energy is average symbol energy divided by log 2 M so, eta P power efficiency is simply d square times log 2 M divided by average symbol energy and now we are good to go. Also remember this formula because we will be kind of using it several times in this lecture.

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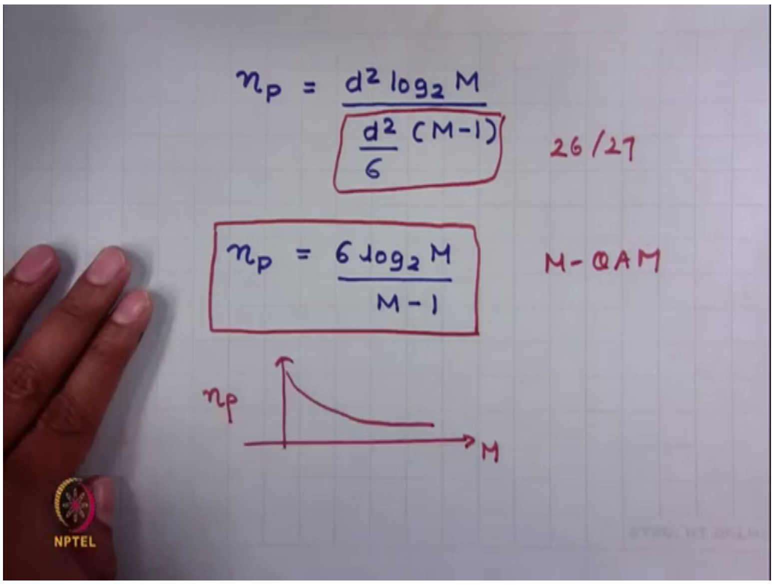

So, power efficiency is d square log 2 M divided by average symbol energy and for QAM we have already calculated the average symbol energy I think in lecture 26 or 27 and we can use that average symbol energy relationship now to calculate this power efficiency in case of M-QAM which is 6 times log 2 M divided by M minus 1. One immediate thing that you should notice now and we will talk about this later as well, that this power efficiency decreases with M. So, if we are interested in the power efficiency for a given constellation this power efficiency decreases with M does the modulation schemes becomes less and less power efficient as you increase the number of symbols in the constellation ok.

So, for M-QAM power efficiency decreases with M, but if you remember the spectral efficiency of M-QAM increased with M. So, if you wanted a modulation scheme with a larger spectral efficiency you should increase M, but then what it will do is, it will reduce the power efficiency of your modulation scheme alright, we will talk about this point in lot more detail later on in this lecture.

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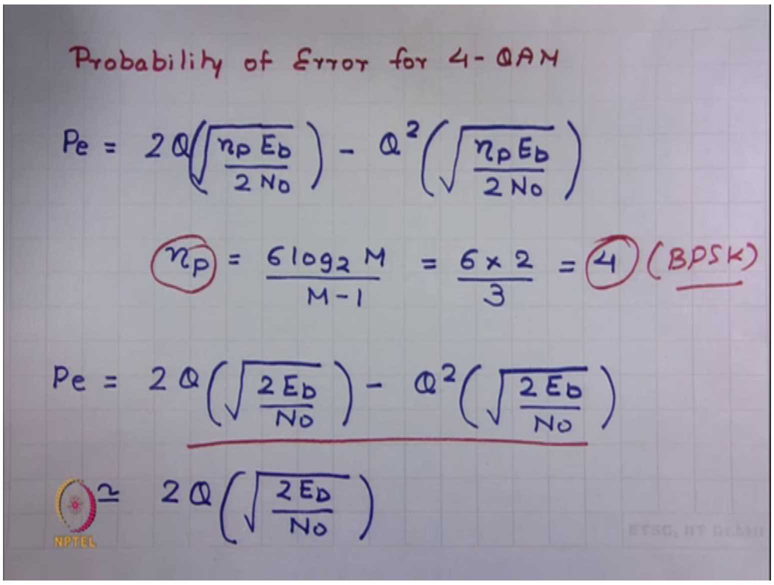

So, probability of error for 4 - QAM we have calculated this as this and now the job remains to plug in this value of power efficiency which you can easily calculate by using this relationship. So, this will be 6 times 2 divided by 3 which is 4 and now you can get easily the formula for probability of error for 4 - QAM which is this.

So, you plug the value 4 in this relationship and you can also approximately calculate it like this. Important point, that you have to notice now as well that the power efficiency of 4 - QAM is same as the power efficiency of BPSK alright. So, BPSK has the same power efficiency as 4 - QAM alright. So, we have been able to calculate the probability of error for 4 - QAM right and we have understood few concepts while doing this evaluation, now the question is what happens as you increase the number of symbols in the QAM.

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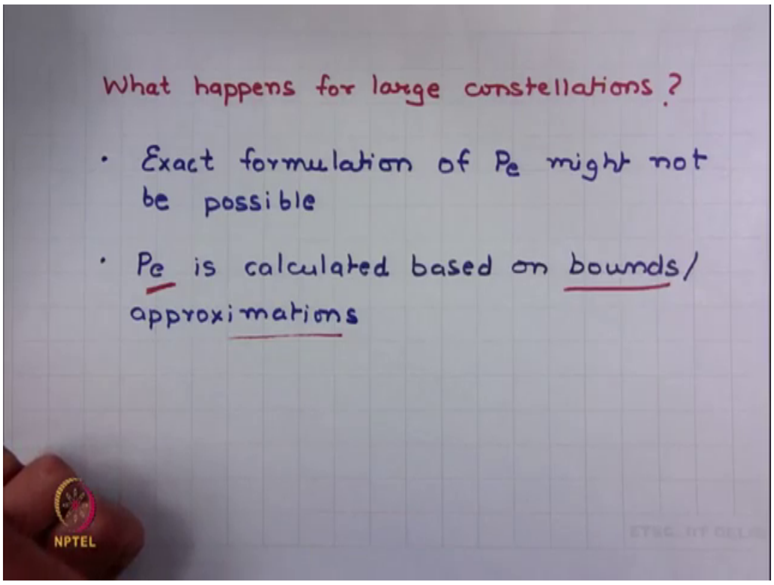

So, we still will continue with QAM and we will like to understand what happens when you have large constellation for example, what happens when you go to 16 QAM or to 64 QAM or to 128 QAM what is so ever. So, in case of large constellations exact formulation of probability of error might not be possible and what will then be possible is to calculate this probability of error using some bounds or using some approximations, but exact calculations of probability of error will not be possible and now we will learn about these bounds in approximations that you can use to calculate the probability of error for large constellations.

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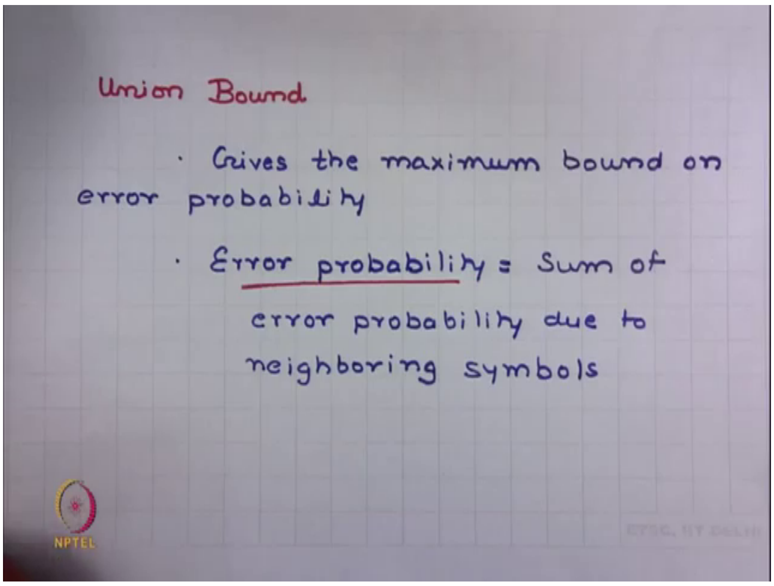

And the first bound that we will study is union bound, union bound gives us the maximum bound on error probability and this union bound is simply calculating the error probability by summing up the error probability due to all neighboring symbols and it is best to look at an example to understand what I mean.

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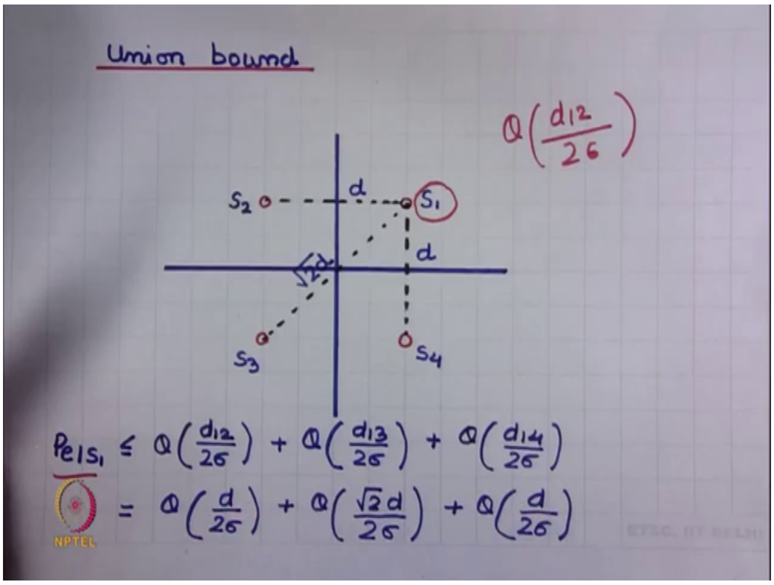

Suppose you want to use this union bound approach to calculate the probability of error given that you have transmitted S 1. So, given that we have transmitted S 1 what is the probability of error in that case. So, to estimate this probability of error you have to look at neighbours one by one. So, first you look at the neighbour S 2 and you find out what is the probability of error due to this S 2 and this probability of error is simply Q of d 12 divided by 2 sigma, where d 12 represents the distance between 1 and 2 of course, we know that this is d, but we are trying to generalize the stuff.

So, remember if the 2 signals are separated by a distance d, for the probability of error the noise amplitude should be larger than d by 2 and we have already evaluated that this will be Q of d by 2 sigma right and that is why we work for a minute or so, in deriving this so, we can use this expressions now seamlessly ok. So, probability of error because of this S 2 is simply Q of d 12 by 2 sigma, probability of error due to this S 3 is Q d 13 by 2 sigma, probability of error due to this S 4 is Q d 14 by 2 sigma.

So, what we are doing is we are looking at a neighbour one by one and we calculate the probability of error due to each neighbour and then we sum up these probability of errors and then we get probability of error given that we have transmitted a signal and what is d 12, d 12 in this case is d, d 13 is root 2 d and d 14 is d for 4 QAM regular 4 QAM constellation. And hence we have been able to predict probability of error given S 1 by summing up these probabilities. And remember this gives us an upper bound on the probability of error meaning that the probability of error cannot exceed this right, but it will be less than this ok. So, it is a bound it is it is not giving you and exact formula for probability of error.

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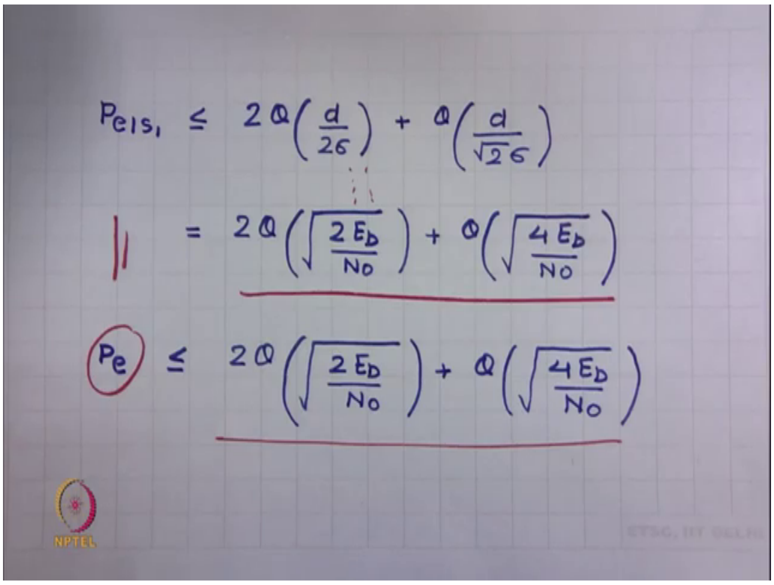

And we can collect these two terms and we can write that this probability of error given S 1 is simply less than 2 times Q of d by 2 sigma plus Q of d by root 2 sigma and of course, you can also write this in terms of m knows and then you get probability of error given S 1 simply by this. So, this is the probability of error given S 1 and how can I think about probability of error average probability of error and this will be simply probability of error given S 1 because again we are interested in or a regular constellation.

So, probability of error given S 1 will be same as probability of error given S 2, will be same as probability of error given S 3, will be same as probability of error given S 4 and does the average probability of error will be same as probability of error given S 1. So we have got the upper bound for this probability of error by using union bound before proceeding further let us stop for a minute and let us try to identify why this union bound gives us an upper estimate of the probability of error and to think about that issue.

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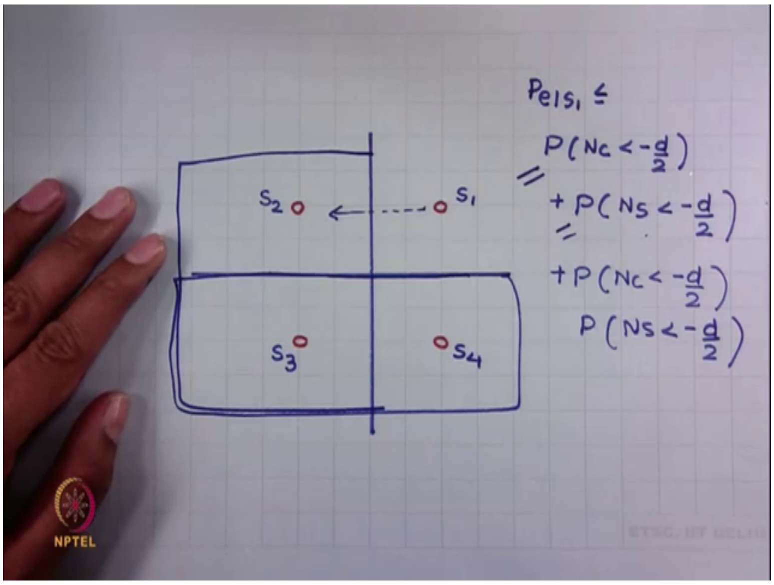

Let us see what is happening, when we are calculating the probability of error due to S 2 alone we will require that the N C should be less than minus d by 2. So, when we are saying that N C should be less than minus d by 2 we are covering the second and third quadrant ok. So, probability of error due to S 2 would be given by this condition and probability of error due to S 4 will similarly be given by this condition and when we use this condition we are already covering the third and fourth quadrant.

And now when we think about this probability of error due to S 3 actually you want that this N C should be less than minus d by 2 and N S should be less than minus d by 2, but you have already covered third quadrant twice and you are adding again the probability of received value to lie in the third quadrant and thus you are recounting events and thus the union bound gives you an upper estimate of the probability of error ok.

So, just revise this, this will give me the probability that received value lie in either second or third quadrant this will give me the probability that received value will lie either in the third or fourth quadrant and this will give me the probability that the received value will lie in the third quadrant and you know that you have calculated the probability of received value to lie in the third quadrant way too many times and thus this probability of error will actually we much more than the actual probability of error. And thus union bound is not at all a tight bound right and you can use a better bound than the union bound which we call as the intelligent union bound.

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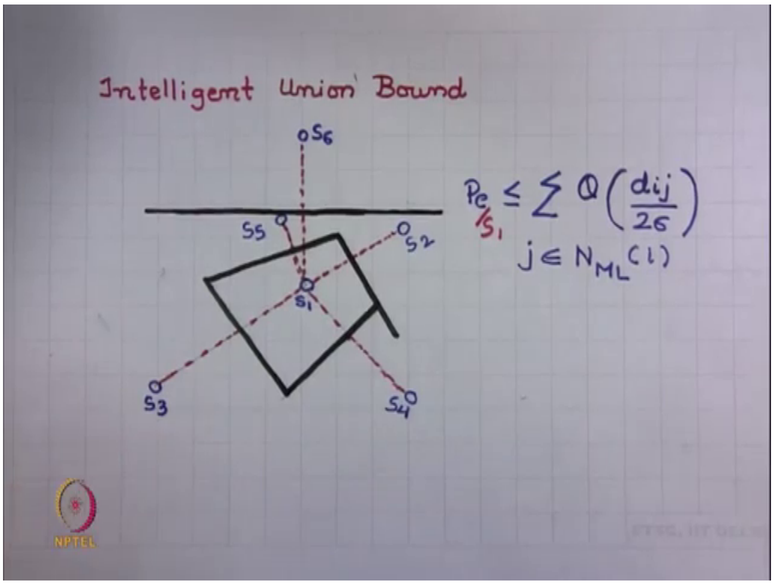

So, let us look at this intelligent union bound and what does this intelligent union bound do is this makes the bound little bit more tighter. So, what it says is in this summation you should only include the signals which are influencing the decision region of the considered signal. For example, if we are considering a signal S 1. So, we are assuming that signal S 1 is transmitted and we want to calculate the probability of error in that case, then you should only consider the signals contribution to the error as the signals which are influencing the decision region for the signal S 1.

So, the first question is how do we calculate the decision region and the idea is same as before you start by drawing lines to all signals from S 1. So, we are in now calculating the decision region for S 1. So, we have to draw lines from S 1 towards all signal and then the second step is to draw perpendicular bisectors to those lines. So, this is the perpendicular bisector for the line connecting S 1 and S 2, this is the perpendicular bisector for the line connecting S 1 and S 3, this is the perpendicular bisector for the line connecting S 1 and S 4 and this is the perpendicular bisector for line connecting S 1 and S 5 and this is the perpendicular bisector for the line connecting S 1 and S 6.

Now you know that this perpendicular bisector is not influencing the decision region of S 1 and hence this signal S 6 can be ignored when you are calculating the probability of error when S 1 is transmitted that is it ok. So, in this summation you should ignore or you should only consider the signals which are influencing the decision region of the considered signal. Let us quickly see this with an example.

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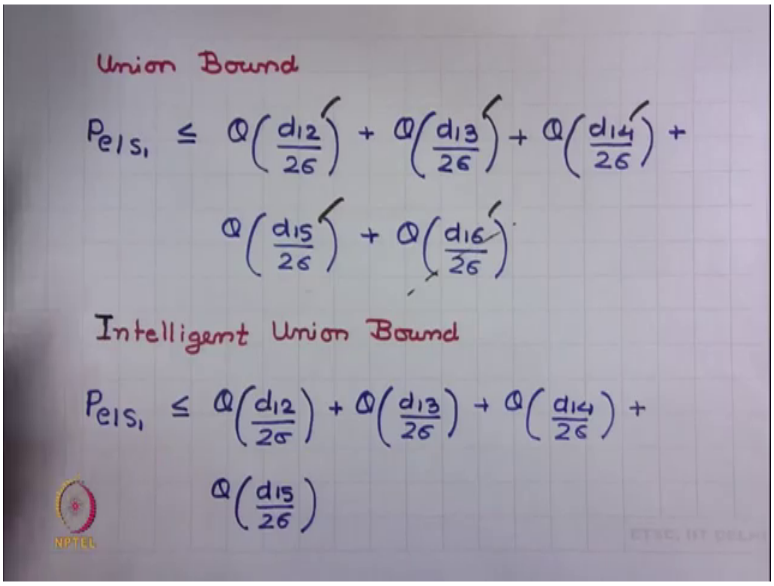

So, if we have to evaluate the probability of error given S 1 and the constellation is this, then when we are doing the union bound approach we have to have the contributions from all neighbours. So, from 2, from third, from fourth, from fifth, and from sixth, what when you are doing intelligent union bound because the signal 6 was not influencing the decision region of S 1 you can ignore this term in intelligent union bound.

So, intelligent union bound makes the bound little bit more tighter by pruning the terms which are irrelevant. So, it has got rid of this term and thus the probability of error given S 1 will be lesser in this case compared to this case ok, is there still an upper bound right. So, the probability of error will still be less than this because you will be counting few terms twice again in here ok.

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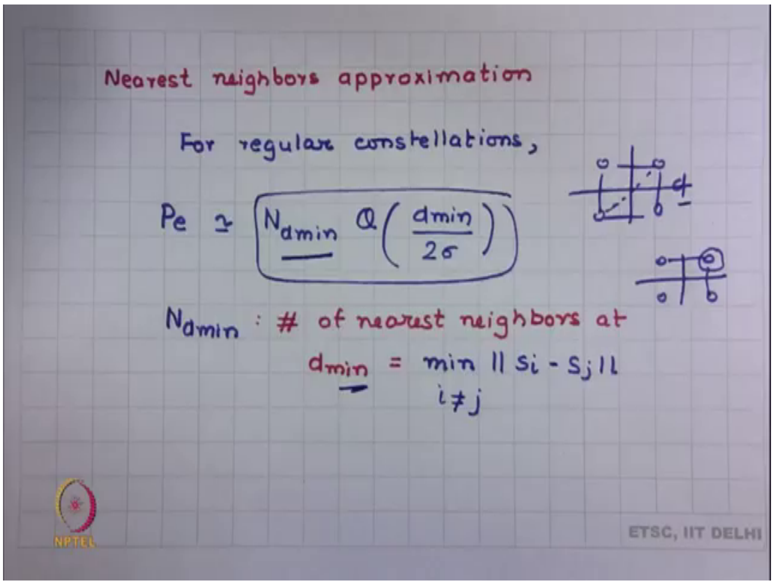

So, now, from bounce we are going to approximation and this is nearest neighbours approximation probably one of the most widely used approximation when you are calculating the probability of error and what is this, it can be used for regular constellations. So, normally we will deal with regular constellations only right the signal set will be regular constellation. So, for PSK the symbols will lie on the circle for QAM the signals will lie on a rectangle ok. So, probability of error is approximated as N d minimum times Q of d min by 2 sigma.

So, first let us define d min, d min is the minimum distance between 2 signals in a constellation for example, if I consider 4-QAM we know that these signals lie at the minimum distance of d and the distance between these two signals is larger root 2 d. So, there are some signals which lie at a minimum distance we call that minimum distance as d min and the number of neighbours of a signal at the minimum distance d min is referred to as number of nearest neighbours ok.

So, for example, in this 4-QAM: how many nearest neighbours to this symbol? There are 2 nearest neighbours one this and one this ok, hopefully now you have understood the meaning of what d min which is the minimum distance present in a constellation and N d min is the number of neighbours at that minimum distance d min and probability of error is simply obtained by this relationship.

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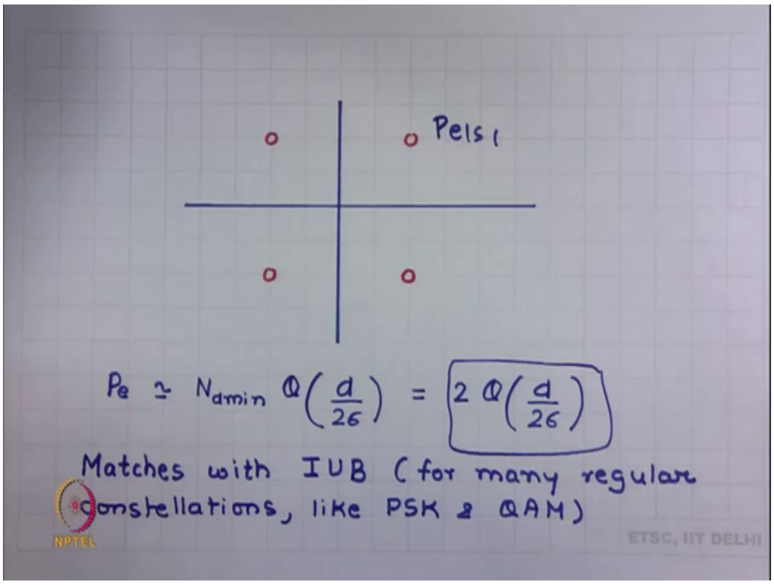

So, let us work out this 4 QAM. So, let us consider again the probability of error given S 1 how many neighbours, 2 neighbours and what is the d min in this case is simply d. So, probability of error is this, now this matches with intelligent union bound that we have used and this will be the case for many regular constellations like PSK and 4 QAM.

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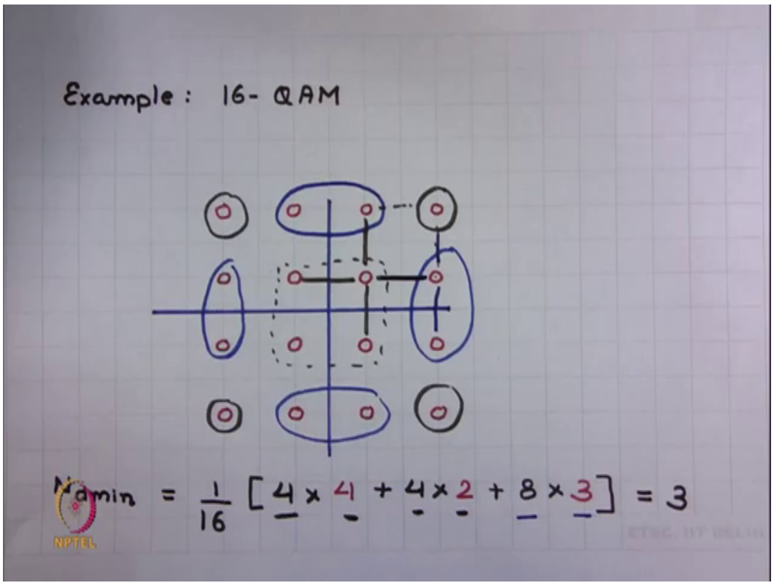

So, let us use this nearest neighbours approximation to calculate the probability of error in 16 QAM and the idea would be to first calculate the number of nearest neighbours or in fact, the average number of nearest neighbours. So, let us first consider these 4 signals in this constellation and how many nearest neighbours the signal has ok. So, this signal has 4 nearest neighbours 1, 2, 3 and 4 and similarly this signal will also have 4 nearest neighbours, this signal will also have 4 nearest neighbours, this signal will also have 4 nearest neighbours. So, there are 4 signals with 4 nearest neighbours.

Now, you consider these signals this, this, this and this, these signals on the boundaries will have only 2 nearest neighbours. So, there are 4 signals which have 2 nearest neighbours and now we have 8 remaining signals. So, 2 here 2 here, 2 here and 2 here if you look at these 8 signals each one of them will have 3 nearest neighbours. So, this will have this, this and this. So, there are 8 signals with 3 nearest neighbours. So, if we want to find the average number of nearest neighbours this will be 3 and now the probability of error is straightforward.

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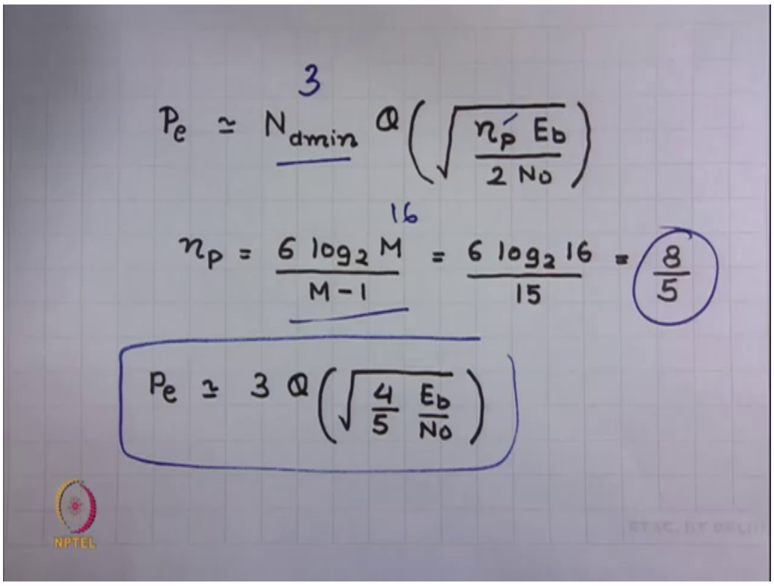

We know N d minimum for 16 QAM which is 3 and we have already calculated the power efficiency which is expressed by this relationship and for M being 16 this power efficiency turns out to be 8 by 5. And thus the probability of error is approximately 3 times Q of root of 4 Eb by 5 N naught ok. So, calculation was really trivial you can easily calculate the number of nearest neighbours and you can easily calculate the power efficiency and you get an approximated value of probability of error for 16 QAM.

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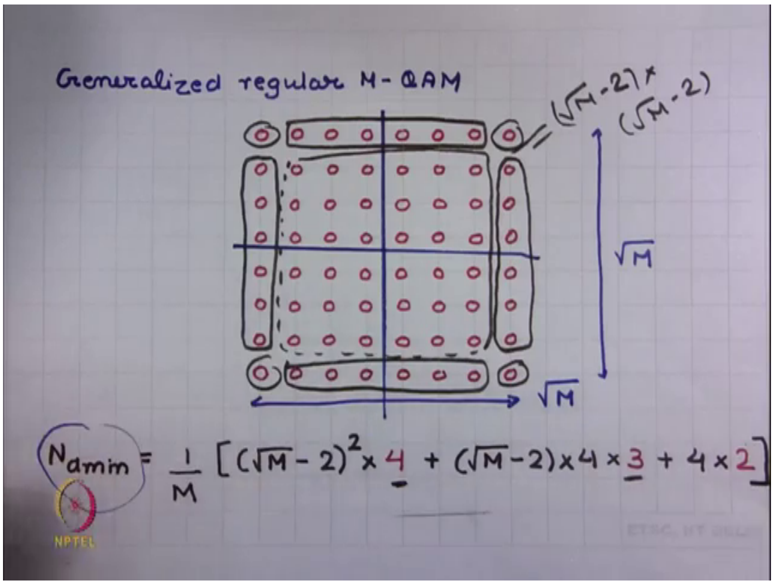

Can we generalize this for any M- QAM we have already generalized the power efficiency for any M – QAM, now we want to find or to generalize the N d minimum for any M-QAM and of course, we always assume that we have regular QAM constellations. So, if we assume that this has M symbol points and let us assume that is the square grid constellation that is usually when we talk about regular QAM structure. So, how many columns we have in here, we have root M columns and we have root M rows.

So, if I consider this group, this group of points all will have 4 neighbours and how many points do we have in this group. So, in this group how many rows we have, we have root M minus 2 rows and we have root M minus 2 columns. So, number of points in this group is simply root M minus 2 square and all these points have 4 nearest neighbours. Now you can think about these points how many points here, again we have root M minus 2 points in here and there are 4 such groups. So, one here, one this, one this, and one this so, you have 4 such group and each group has root M minus 2 points and each member of this group will have 3 nearest neighbours.

So, we have 3 here, remaining are these 4 points the points which are lying at the edge these 4 points will have only 2 nearest neighbours. So, average number of nearest neighbours can be calculated by carrying out this arithmetic and you can easily do this yourself.

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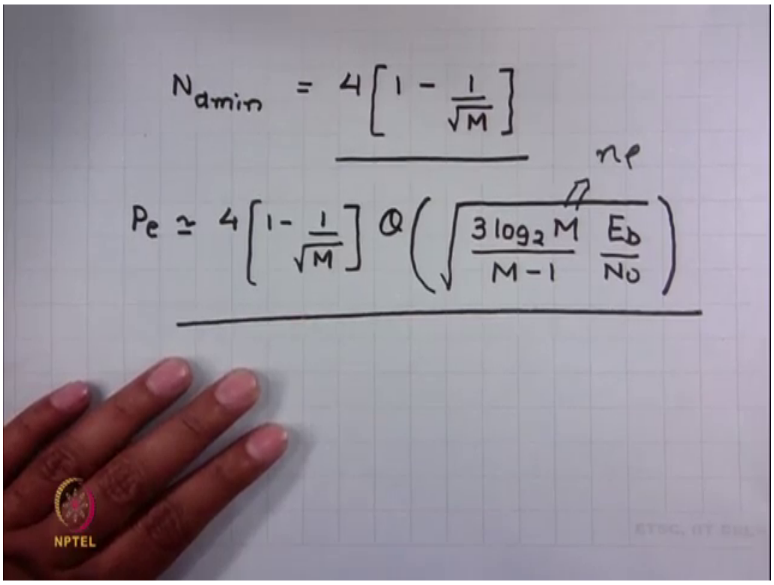

And we get average number of nearest neighbours 4 times 1 minus 1 by root M and that is the probability of error at M- QAM where we are assuming square grid constellation. So, the number of bits in a symbol must be even for a square grid constellation. So, we assume that M- QAM the probability of error can be calculated like this. So, here we have just substituted for the value of power efficiency as well. So, with this what we have done is we have been able to complete the probability of error expression for M-QAM and we have also derived the power efficiency for M-QAM.

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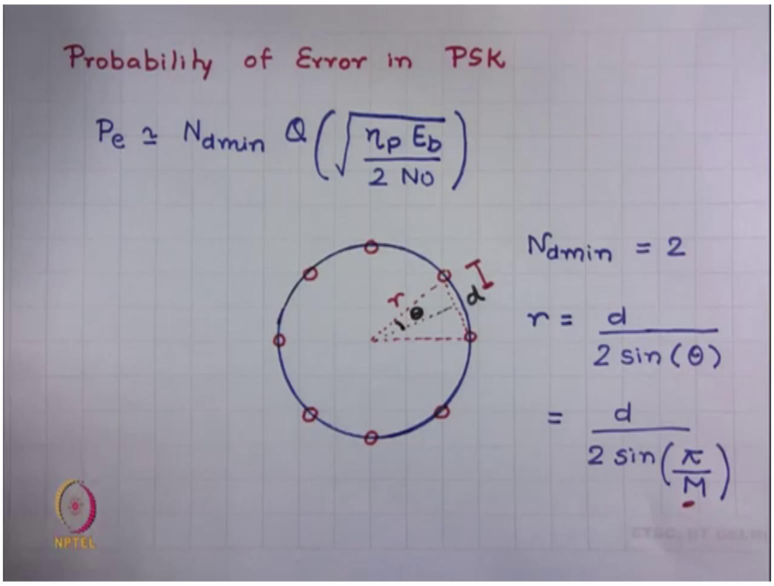

And next is to quickly see the probabilities of errors for PSK and PAM and then we will make the comparison and we will conclude this lecture. So, probability of error in case of PSK using nearest neighbours approximation is again given by this relationship in PSK the task is easier, because average number of nearest neighbours are 2. In fact, every symbol has 2 nearest neighbours so, one this and one this if you consider this signal, that is easy.

What is power efficiency? Power efficiency can also be calculated easily so, just see that this radius of the circle though we have done this calculation, but I think it will be worthwhile to relook at it. So, if I consider that the radius of the circle is r, this distance is d by 2. So, r sin theta is d by 2 and sin theta is sin pi by M where M is the number of points.

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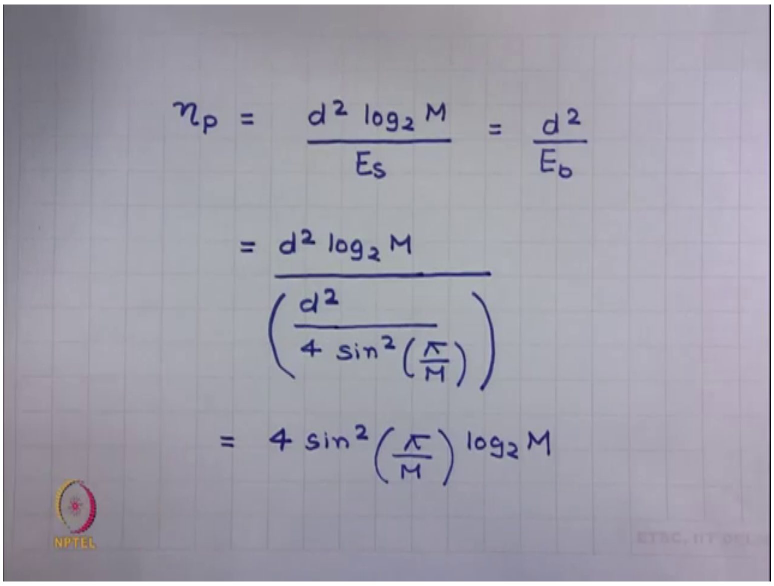

Then we know that the power efficiency is d square log 2 M by Es or d square by Eb you can think in any way as you want we are thinking in this term and we have already derived this relationship today and this Es is simply r square right. So, energy of each point is simply r square so, we are squaring this up we get Es and from this we get the power efficiency as this 4 times sin square pi by M log 2 M alright.

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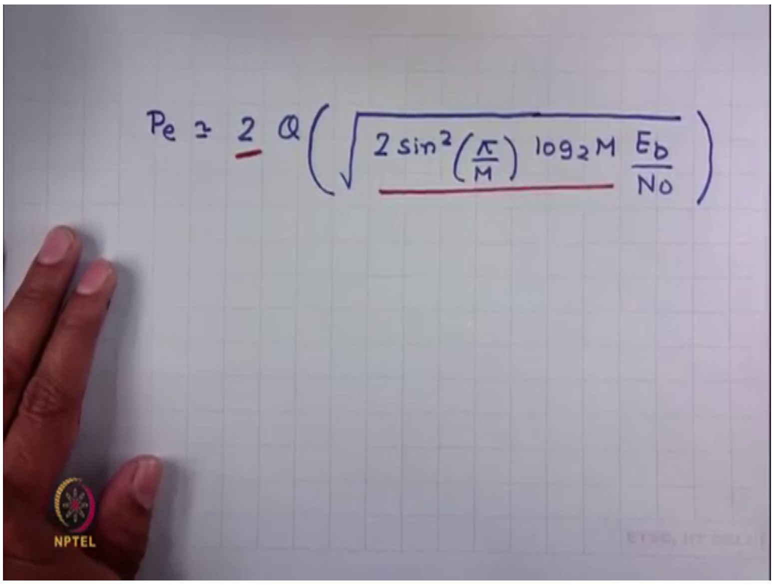

And from this we can calculate the probability of error by substituting the value of N d minimum and substituting the value of power efficiency. So, this is there is power efficiency by 2 and hence the factor 4 has become a factor 2. So, calculating the probability of error for PSK was rather trivial simple right. In the same spirit let us calculate the probability of error in M - PAM again it is simple.

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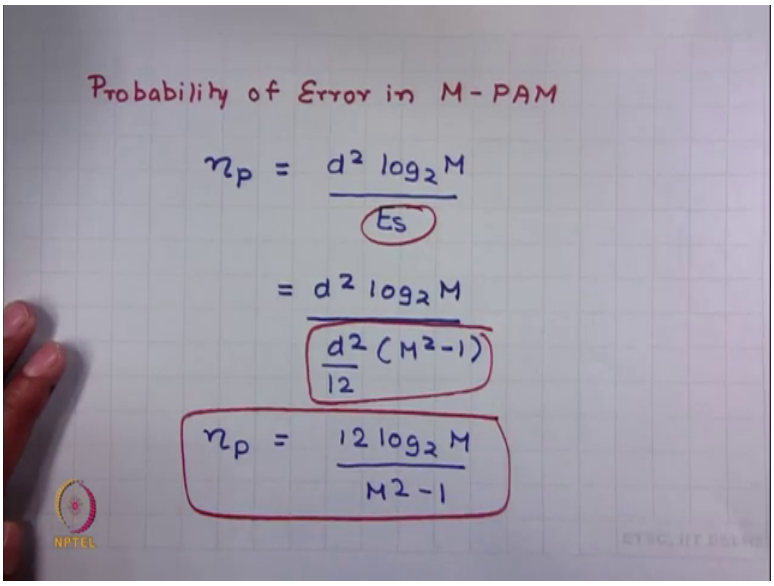

So, power efficiency is d square log 2 M by Es, Es we have calculated I think in lecture 26 which is this and power efficiency thus can be easily calculated like this ok.

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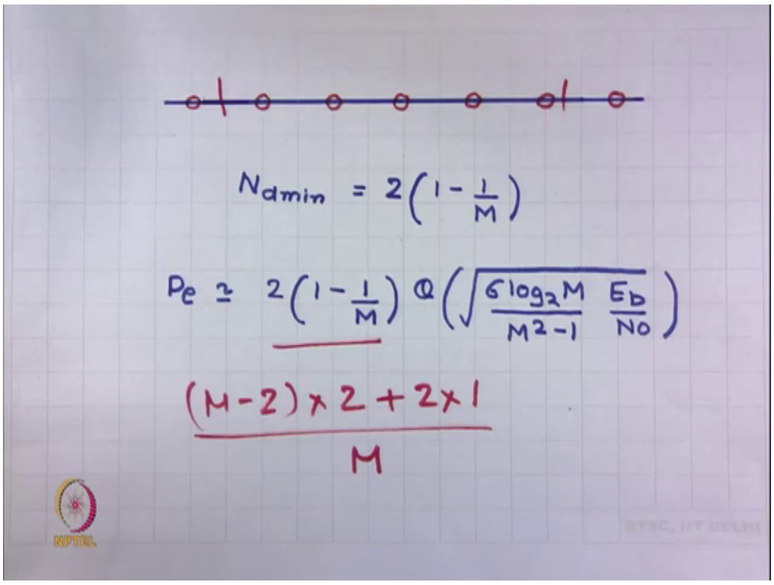

And how to calculate this N d minimum in the case of M - PAM N d minimum can be simply given by this expression ok. So, you can think that M minus 2 signals will have 2 neighbours and the signals at the end of this line there are 2 signals which will have one neighbour. So, average number of neighbours is given by this which simply simplifies to this thing ok.

So, probability of error is simply this N d minimum times Q function of a square root of eta P Eb No by 2. So, we have already calculated power efficiency just plug this in here and you can get the probability of error expression for M- PAM alright. So, after understanding the probability of error in case of QAM the probability of error calculation for PSK and for PAM is fairly simple.

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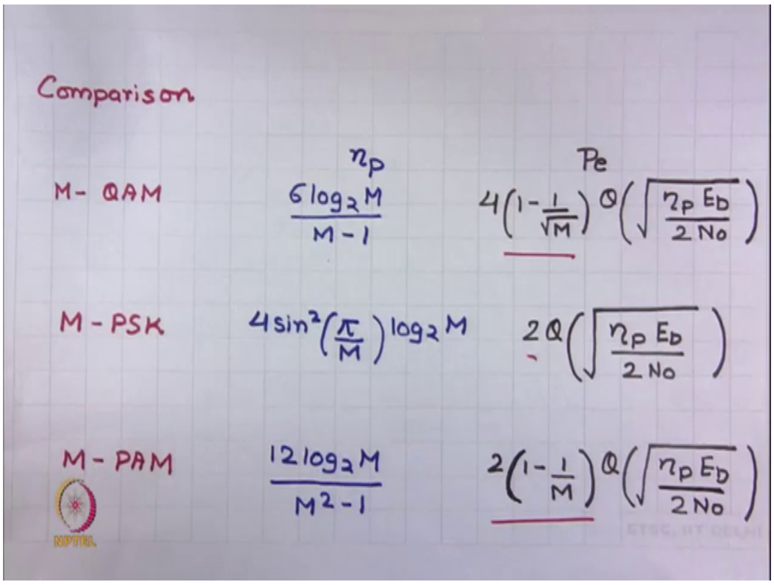

So, finally, we compare the performance of these 3 modulation schemes that we have seen M-QAM, M-PSK and M- PAM. So, these are the power efficiencies of these modulation schemes and then if you plug in this N d minimum that we have got multiply where this Q of root of either P Eb No by 2. We can also arrive at the probability of error expression for these modulation schemes ok. So, this just summarizes whatever we have discussed so far ok.

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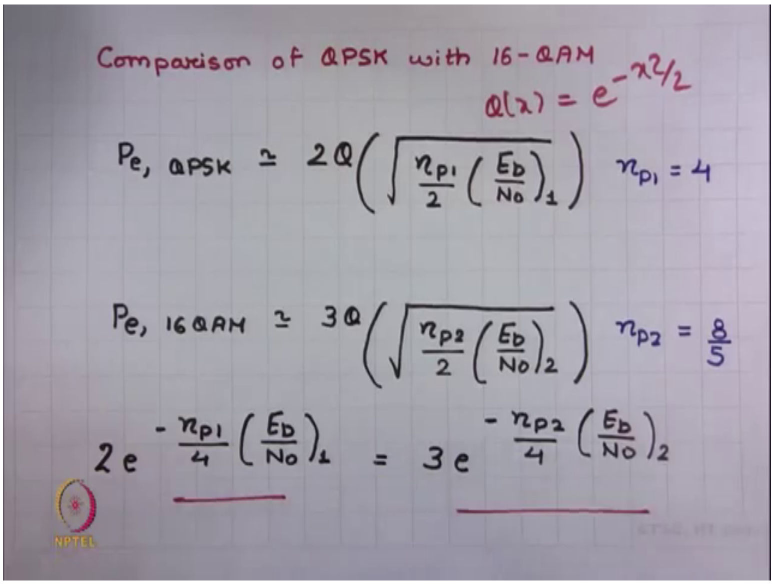

Now, what I would like to do is let us compare QPSK with 16 QAM and let us see what happens. So, in QPSK we have got this as the probability of error the number of nearest neighbours in QPSK is 2 right. So, QPSK is same as 4 QAM number of nearest neighbours was 2 and then we have this factor in here eta P 1 represents the power efficiency of QPSK and Eb No 1 represents the Eb No of QPSK. Similarly in case of 16 QAM the number of nearest neighbours was 3 and we are assuming that it has got a power efficiency of eta P 2 and it has got an Eb No of 2 and we know what is this eta P 1 and eta P 2.

What we want to do is, ask the question that when these two probabilities of errors should be same, what is the difference in their Eb Nos requirement that is what we want to see and we have done such an exercise also for the binary signaling scheme. So, if we want the probability of errors to be same and this happens given my Eb No is suitably large. So, I can approximate this function as this function we are same as in the last lecture I am using the relationship that Q of x is simply e to the power minus x square by 2 for large x and thus I can convert this to this expression for large Eb Nos. And, thus for these two modulation schemes to have the same probability of error this should be same as this.

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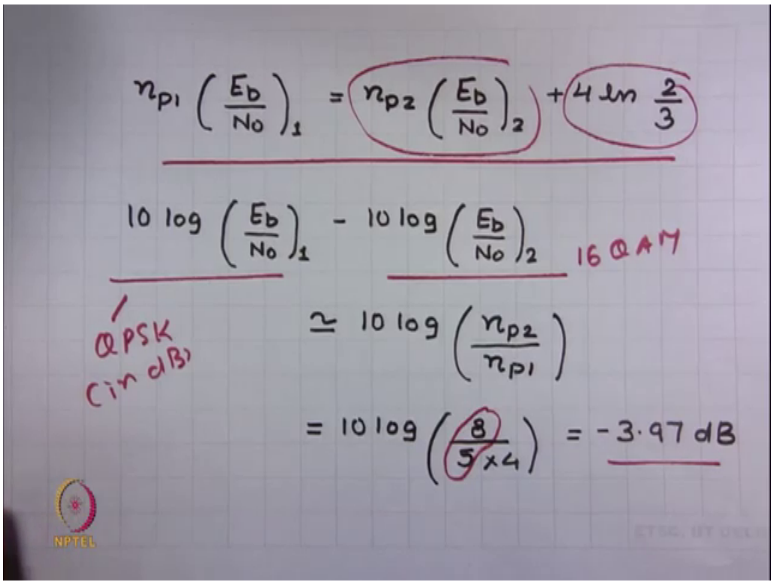

And if you want this should be same as this what we want is, this expression should be same as this expression ok, I have just done some arithmetic have taken log on both side and from that I have got this relationship. Now, this quantity will be much smaller compared to this quantity if I assume Eb No 2 be larger and thus I can ignore this thing and thus again taking the log I get 10 log Eb No 1 minus 10 log Eb No 2 is approximately 10 log eta P 2 by eta P 1. So, this is the Eb No requirement in QPSK in dB scale this is the Eb No requirement in 16 QAM in dB scale and this difference in Eb No requirement should correspond to the ratios of power efficiency in dB scale.

So, eta P 2 is 8 by 5 16 QAM power efficiency and power efficiency of QPSK is 4 thus taking this ratio and trying to express this in dB scale I get a number minus 3.97 dB, thus Eb No requirement in QPSK systems is minus 3.97 dB smaller than the Eb No requirement in 16 QAM systems thus QPSK is more power efficient than 16 QAM that is one thing.

Second thing is and quite useful thing that you can quickly calculate this number by just the ratios of power efficiency and that is why this power efficiency is so important, calculation of power efficiency is trivial is easy just straight forward formulas for that. Once you know these power efficiencies you can easily calculate their ratios and you can easily obtain a number which tells you how much one modulation scheme is more power efficient than the another modulation scheme and this is quite useful.

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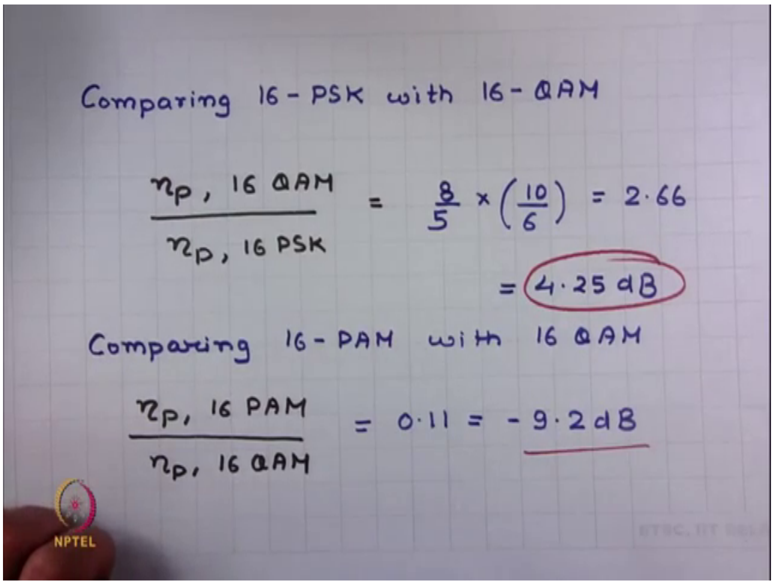

For example, you can compare 16 PSK with 16 QAM, calculate the ratios of their power efficiency, 16 QAM again has power efficiency of 8 by 5 16 PSK you can work out by using the formula it has got a power efficiency of 0. 6, you easily calculate the ratio 2.66 and you easily arrive at a number 4.25 dB; that means, in 16 PSK the Eb No requirement is 4.25 dB larger than the Eb No requirement in 16 QAM so, you can easily calculate that.

Similarly you can calculate the 16 PAM efficiency with 16 QAM efficiency and by using the same arithmetic you can arrive to the result that 16 QAM requires minus 9.2 dB smaller Eb No than 16 PAM, it is quite useful. Let us look at the same thing graphically.

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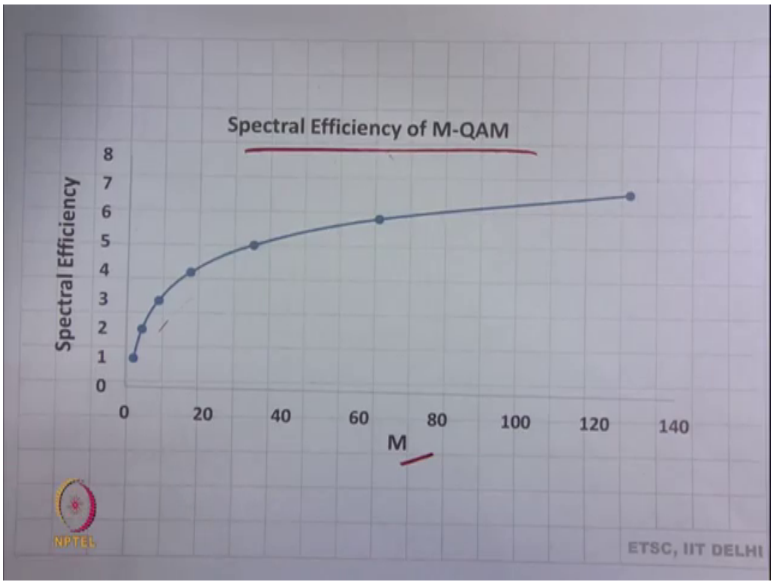

So, first this we have seen before in the modulation lectures we see that the spectral efficiency of M- QAM increases with M and that is why we love this M-QAM because, if you want to pack more bit rates per given bandwidth you should use larger Ms.

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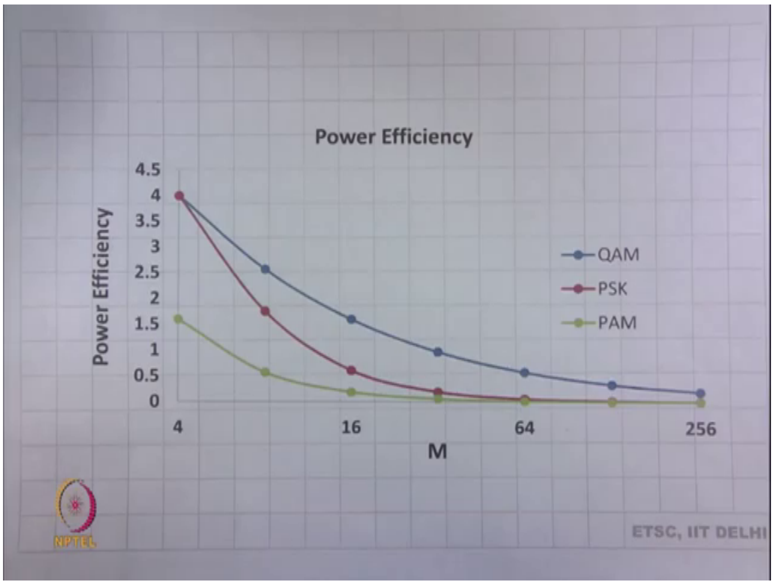

But as soon as I increase this M what happens in terms of power efficiency power efficiency starts to decrease. So, power efficiency of QAM, PSK and PAM all these modulation schemes decreases as you increase M of course, for QAM the decrease is not so much as in the case of PAM or PSK and thus we like to use QAM. Though I did not mention the spectral efficiency of PSK and PAM in this case, but the spectral efficiency of PAM and PSK at passband will be same as the spectral efficiency of QAM ok.

All these 3 modulation schemes have got the same spectral efficiency, but in terms of power efficiency QAM is best, QAM has got the largest power efficiency for given M and thus QAM will require the least Eb No when you are comparing this QAM this PSK and PAM.

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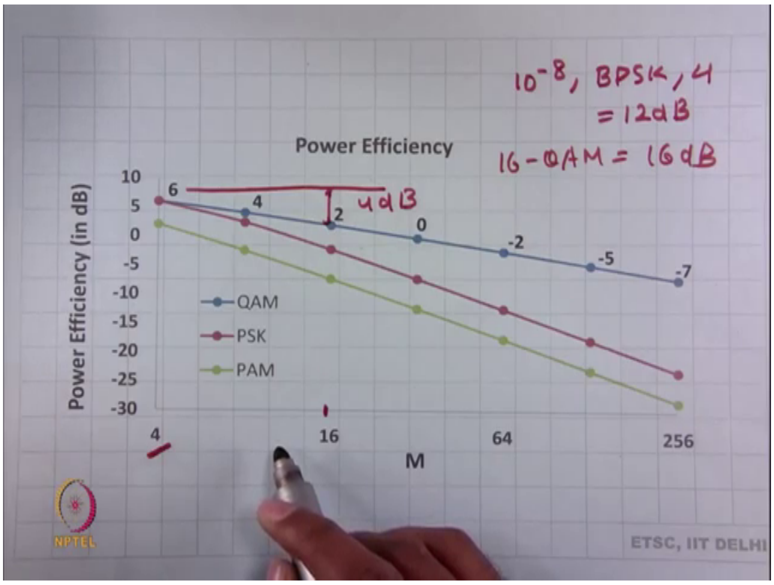

We can also plot this power efficiency in dB scale because this tells you quickly how efficient is one modulation scheme compared to another modulation scheme. For example, in the last lecture on binary modulation scheme we said that for a bit error rate of 10 to the power minus 8 BPSK requires Eb No of 12 dB and BPSK has got a power efficiency of 4 - QAM also has got a power efficiency of 4 when you are considering 4 QAM.

Now so, if we want to find out what Eb No 16 QAM requires for a bit error rate of 10 to the power minus 8 you can easily deduce this from this figure. So, you see that 16 QAM has 4 dB is smaller power efficiency then 4 QAM. The 16 QAM we will require 4 dB larger Eb No then what 4 QAM required. So, Eb No requirement in 16 QAM is 16 dB. So, you can quickly calculate and tell these numbers.

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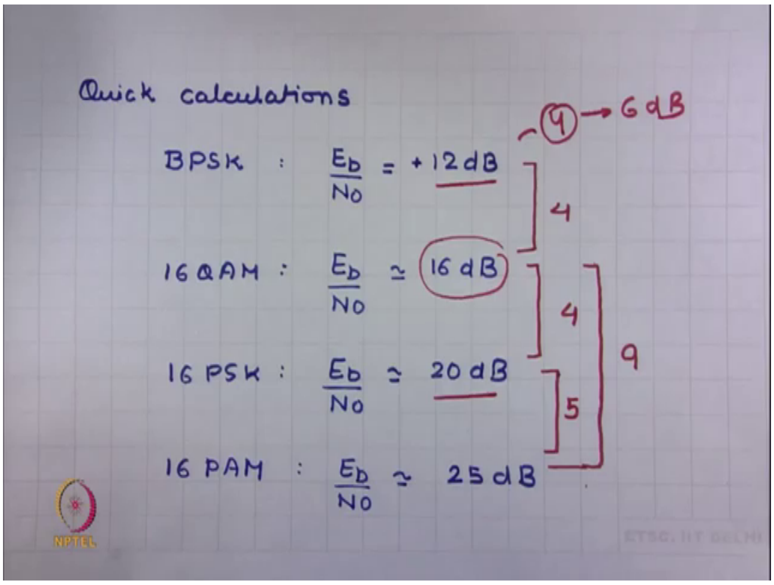

Let us look at some other calculations quick calculations for example, BPSK we said required Eb No of 12 dB it has got a power efficiency of 4 or 6 dB in dB it is 6 in numbers is for 16 QAM has got a 4 dB lower power efficiency and thus it will typically require 16 dB Eb No to have a bit error rate of 10 to the power minus 8.

16 PSK has again a 4 dB approximately these are approximate numbers 4 dB is smaller power efficiency than 16 QAM and hence it will require and Eb No of 20 dB to have a bit error rate of 10 to the power minus 8. 16 PAM power efficiency was 9 dB smaller than that of 16 QAM and does it will typically require and Eb No of 25 dB to have a bit error rate of 10 to the power minus 8.

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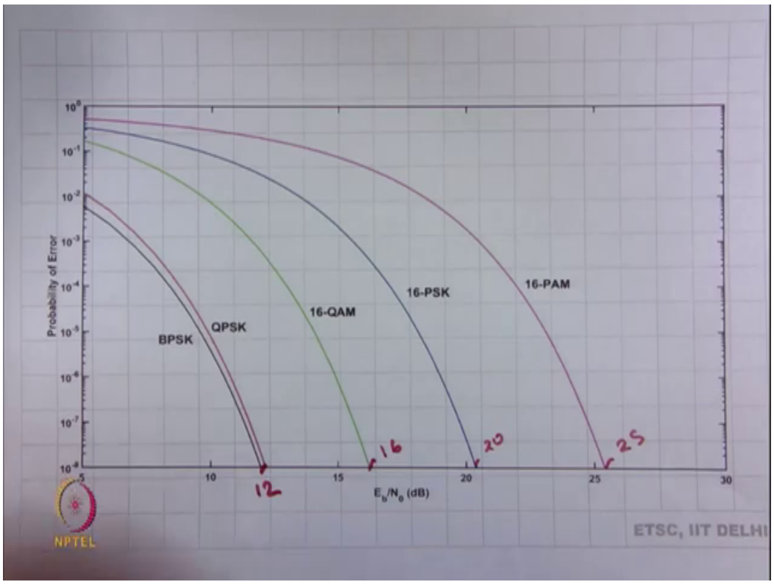

And, if we look at this probability of error graphs was this Eb No you get the same numbers almost. So, QPSK, BPSK are around 12 dB 16 QAMs 16 Db, 16 PSK little bit higher than 20 dB and 16 PAM little bit higher than 25 dB, but what Eb No it will require for probability of error of 10 to the power minus 8 can be easily calculated by this ratio of power efficiencies and this is what we are stressing on.

So, in this lecture we have completed the analysis of probability of error for spectrally efficient modulation schemes like QAM, PAM and PSK and in the next lecture we will look into the probability of error for orthogonal modulation schemes. Remember the main difference between orthogonal modulation schemes and the modulation schemes that we have seen today is that in orthogonal modulation schemes the power efficiency increases as we increase M right because orthogonal modulation schemes are optimized to harness power efficiency.

Thank you.