**Principles of Digital Communication**
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**Lecture - 33**
**Detection**
**Sequence Detection**

Good morning, welcome to a new lecture in Detection. And to today's lecture what we will do is first we will complete what we were discussing about waveform detection and then we will start looking into performance of some signalling schemes. So, remember in the last lecture what we have said is that we have this random process, which was infinite dimensional random process and this infinite dimensional random process was projected on the signal space; signal space is the space which is formed by these orthonormal functions.

And when infinite dimensional random process passes through this signal space it becomes a finite dimensional random process. And by looking at this finite dimensional random process, we can do detection optimally that is whatever we lost how because of this finite dimensional signal space was irrelevant what detection right. So, hence when you go from waveforms to vectors, you lose out on something and whatever you lost out is not important for your detection it was really irrelevant for the detection. So today what we will do firstly, as we will form the optimum rules for waveform detection and before doing that we have to revise some properties ok. So, that will help us in getting started alright.

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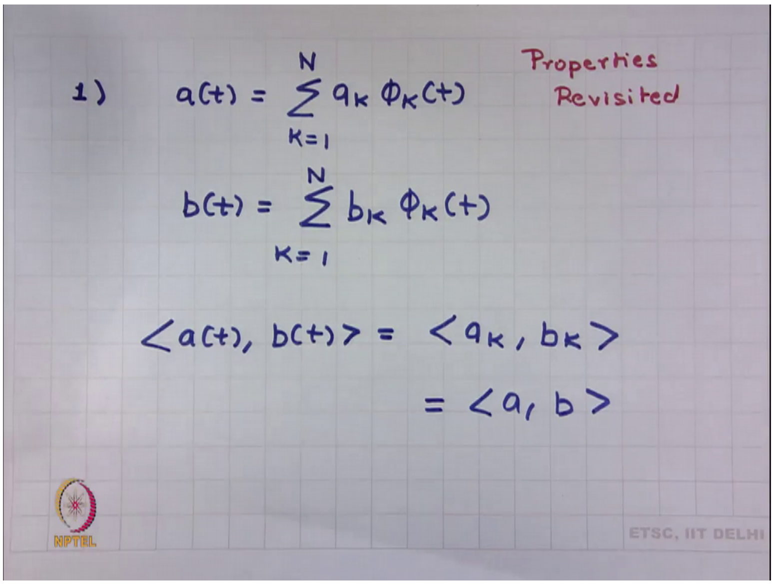


So, these properties we have looked in several times and those I will just state them and if needed we will also look at their proof. So, if we have two signals a t and b t and a k and b k are the coefficients of the signals along orthonormal functions, then if you want to take the inner product of these two signals, this inner product is say as the inner product of the coefficients ok.

And these coefficients also are the elements of the vectors. So, if I want to think about the inner product of the signals, this is the same thing as inner product of the coefficients, this is same thing as inner product of the two vectors a and b alright so, that is the first thing.

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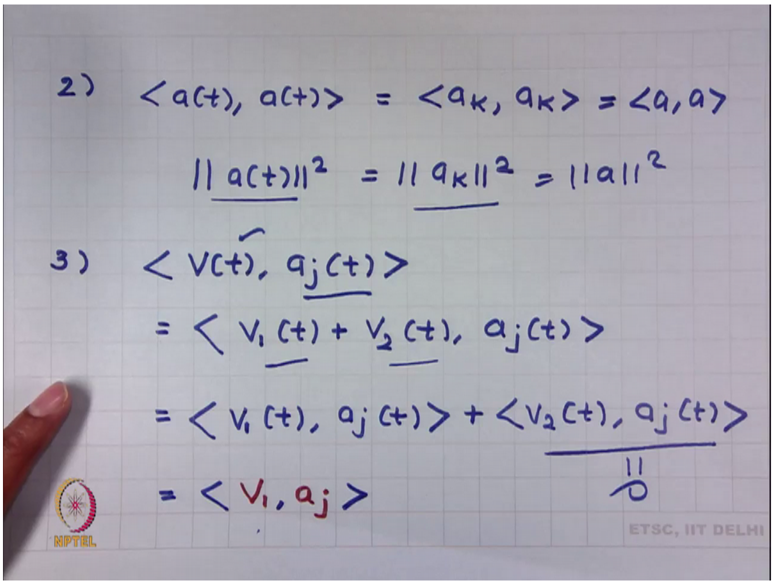


Second is if you want to take the inner product of the signal with itself, this would be same as taking the inner product of coefficients with itself and this thing is same as taking the inner product of vector with itself and so the energy of the signal is given by this. This is same as energy in the coefficients and this is same as energy in the vector ok.

So, all these things are equivalent that is to say. Now, let us take the inner product of V t with a j t what is V t? V t is the infinite dimensional random process that was falling on the receiver, a j t is one of the signal from the signal set. So, we know that this V t can be decomposed into two parts one is V 1 t and V 2 t in the last lecture we established that this is really irrelevant for detection and this is only important for detection alright.

So, you can take the inner product of this with V 1 t and inner product of this with V 2 t and this is really 0 right. This we have seen in the last lecture and thus thinking about inner product of V t with a j t is same as taking the inner product of V 1 with a j or V 1 t with a j t its 1 and the same thing ok.

So, these are the waveforms and these are the vectors corresponding to these waveforms and we have already seen whether you take the inner product of the waveforms or the wave form vectors corresponding to these waveforms its one and the same thing alright.

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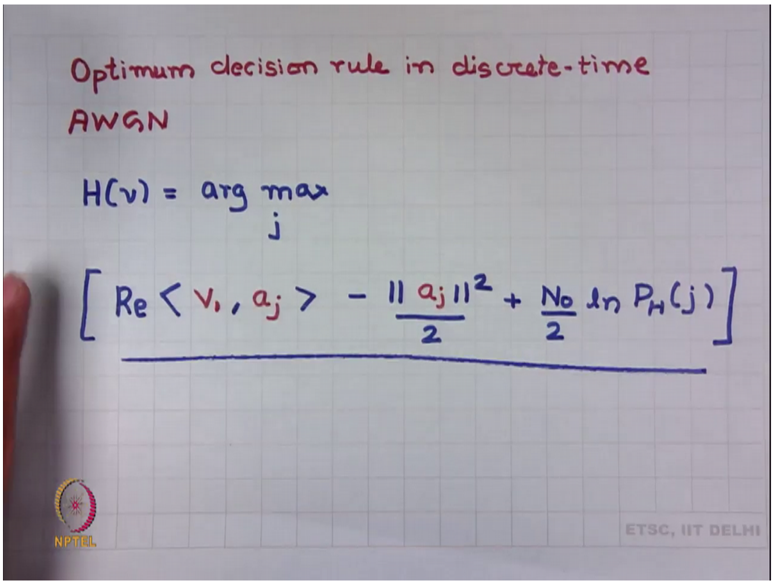


So, let us look at some optimum decision rule in discrete time additive white Gaussian noise channel. So, this we have seen before. So, for discrete time was covered, I am just restating them so that you can see that the rules for continuous time AWGN channel are same as the rules for discrete time AWGN channel ok. So, there we have seen that the receiver decides for the hypothesis j and this hypothesis j is the one which maximizes this thing, this we have seen in the lecture.

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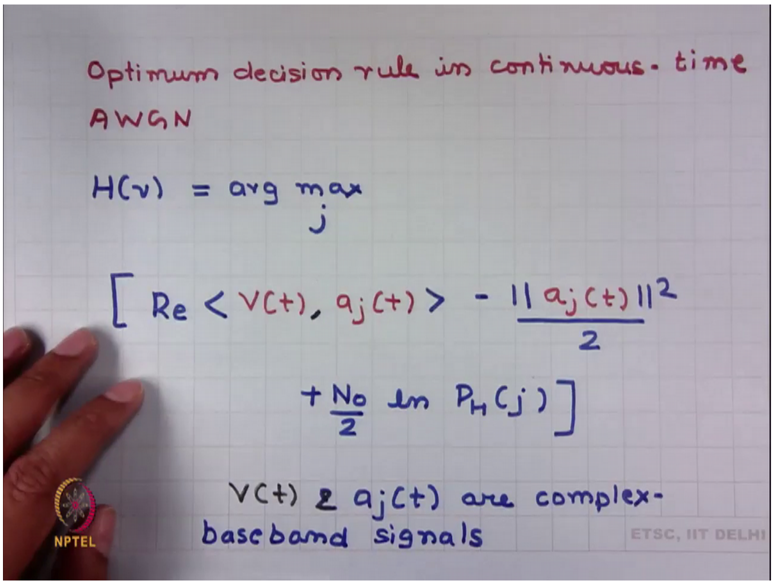

Now, what happens in continuous time case? Again is exactly same so I will just mention the changes; the changes are that in case of vectors we are thinking about the inner product of the vectors, but we have seen that this is same as taking the inner product of wave forms right. We have derived that V 1 inner product with a j is same as V t inner product with a j t ok. And we have seen that whether you want to think about the energy of a vector, this is the same as thinking about the energy of the waveform.

So, the rules for continuous time AWGN channel are exactly same as the rules for discrete time AWGN channel. So, in this case what we assumed is that these signals are complex signals and when is the case, when we consider the signals to be complex baseband signals right. So, we cannot transmit any signal which is complex. So, we can only consider complex signals in base pen domain right, these signals are equivalent half the pass band signals this we have seen.

So, whenever you are taking the real part of the signals and if the underlying signals the complex, it means that they are complex baseband signals. How can you think about pass band signals?

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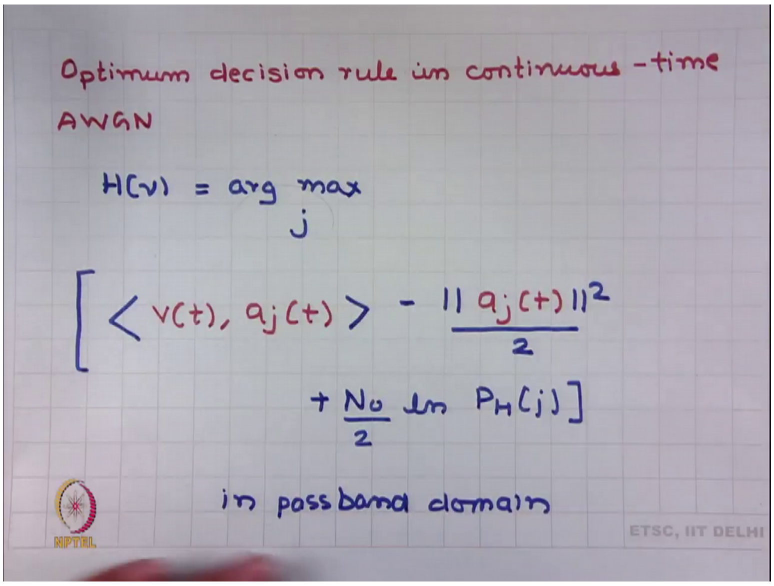


For pass band signals exactly the same decision rule follows, but just simply instead of having real part you do not have any real part because these signals are anyway real signals.

So, what we can see is that these rules are exactly same. Let us look at something which is slightly different, mathematically slightly different. So, let us consider the case for m l detection. In m l detection what happens is you do not have this term because priors are equa, if the priors are equal you really do not want to consider this term right.

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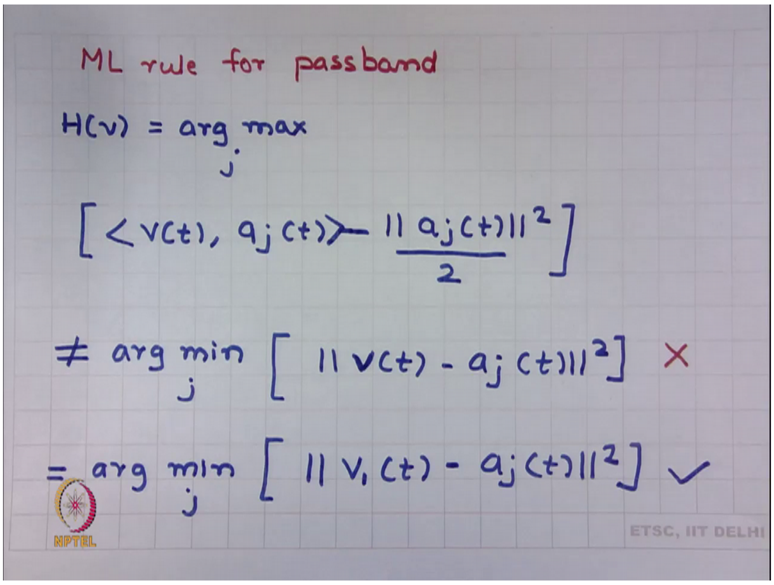


So, in m l the rule is simply this you want to choose j for which this quantity is maximum. In case for vectors we said that this is same as minimizing the distance, maximizing this thing is same as minimizing the distance of V t from a j t.

But you cannot state this in this case, because this distance is infinity right because V t is actually infinite dimensional signal alright. If you want to consider this distance, this distance would be infinity and thus it is mathematically imprecise to find out the j for which this distance is minimum, because all these distances are actually infinity. So, you cannot state that m l in case of waveforms is really minimum distance decoder.

However, if you want to think about minimizing the distance then you have to think about minimizing the distance of V 1 t from a j t; remember V 1 t is a signal which is after the projection of V t on the signal space. So, that is really a small issue mathematical interest case otherwise everything remains same ok. So, now let us look into this successive transmission.

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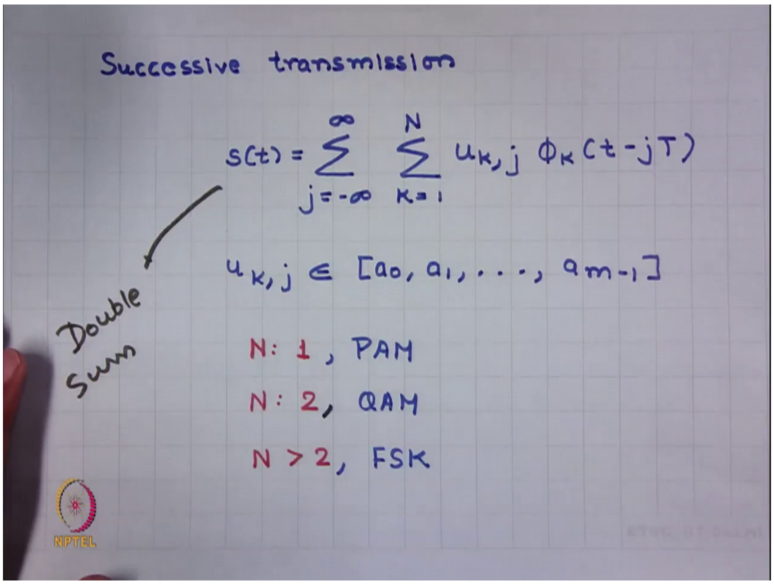


So, in case of successive transmission now, we have these symbols and these symbols are riding on these orthonormal functions and we also have orthonormal functions which are T spaced orthonormal functions in this signal expansion. So, what we are doing is, we are having bunch of n orthonormal functions.

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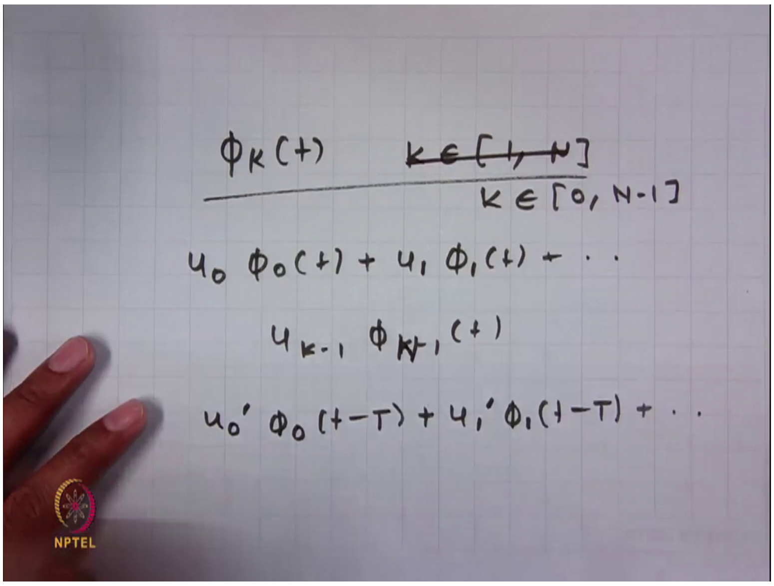

So, let us assume that we have n orthonormal functions. So, we would be having the cymbals which will be manipulating these orthonormal functions. For example, you may have a symbol u 0 which might ride on phi 0 t and you might have u 1 which will write on phi 1 t and you might have u k minus 1, which is writing on phi k minus 1 t or rather phi n minus 1 t if you are having n orthonormal functions right. So, in this case k might go from 0 to N minus 1.

So, you have symbols writing on these orthonormal functions, and then you have next set of symbols, which will be writing on the orthonormal functions which are derived by t spacing these orthonormal functions. So, in the next timeslot you would again be having some other symbols which would be writing on the orthonormal functions which are T is spaced orthonormal functions.

So, this is in general the equation for successive transmission, we have seen such equation before for example, in lecture 5 when we have done double sum expansion using for is series and DTFT actually this is the basic equation that we had there ok. So, the equation is simple you try to understand that you have bunch of orthonormal functions, these functions are orthonormal to each other and when you T space these orthonormal functions the new set of orthonormal functions that we derive are also orthonormal to these basic set of orthonormal functions.

And this u k j denotes that these are the choices that we make from this signal set. So, we have m signals and we make it choice and accordingly we modulate these T spaced orthonormal functions. If number of orthonormal functions that we have is 1 then we are in the regime of pulse amplitude modulation, if number of orthonormal functions that we have is 2 if n is 2 we are in the regime of calm, if N is larger than 2 then we are talking about orthogonal modulation schemes like FSK ok.

So, this is equation for any general modulation scheme. So, how is successive transmission different from a 1 shot detection problem that we have been focusing so, far?

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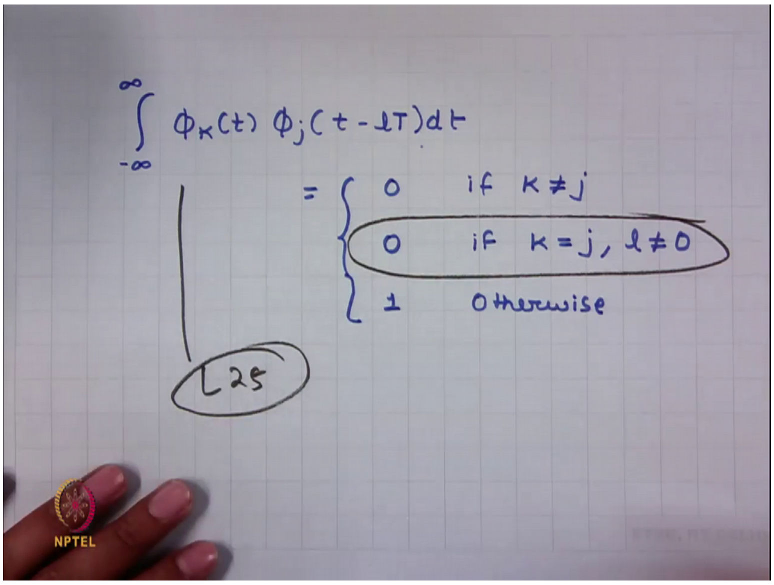


So, in one short detection problem what happened is we are transmitting a symbol we are receiving the symbol and then we are bringing down the communication system. In successive transmission what happens is, we are transmitting a sequence of symbols ok. And the first question that we have to worry about is whether these symbols are interfering with each other or can they possibly interfere.

The answer is no because we have already solved this problem. So, we have said if these T spaced orthonormal functions are orthonormal to these basic orthonormal functions then you can safely avoid inter symbol interference. So, this we have seen in lecture 25. So, for avoiding inter symbol interference this equation must be satisfied. Once you have satisfied in this equation then you cannot worry about inter symbol interference anymore.

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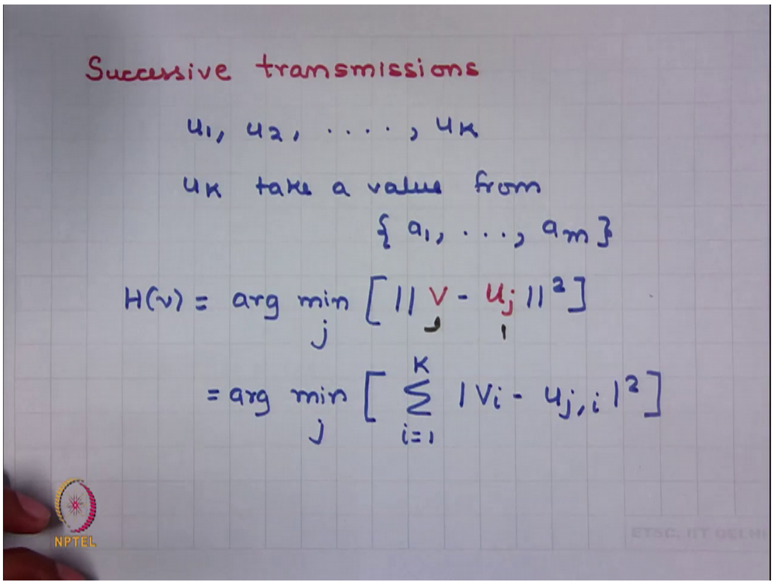


The second thing is that we are more interested in this detection issue is that, if we observe these case embers at the same time are we any way better off then by just looking a symbol at a time. So, for example, if you are transmitting k symbols and if your detector focuses on one symbol at a time, he has got some probability of error, but what happens when he looks at the group of these k symbols at the same time is he anyway doing better or is there some possibility of doing better from just observing one signal or a symbol at a time.

So, we have these bunch of k symbols and as before we are assuming that this choices that we make s from this signal side. So, the rule remains same, you want to choose the hypothesis which minimizes the distance from the received vector. So, V is the received vector and u j corresponds to the hypothesis, this equation is bit complicated and does let us try to work this out.

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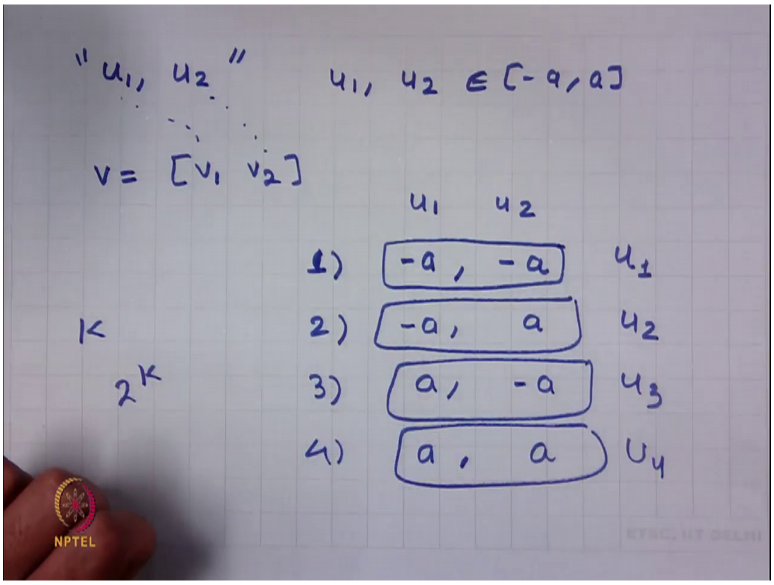


So, what we are saying is let us concentrate on a simple case when we have just two symbols ok. So, we are considering a group of two symbols, let us try to understand this issue. And to simplify this further let us assume that these symbols that we have can draw values from a binary set. So, either you an can be minus a or plus a u 2 can be minus a or plus a. Suppose we are receiving two numerical values, V 1 corresponding to this symbol that was transmitted and V 2 corresponding to this symbol that was transmitted.

Now, if we are considering two possibilities of u 1 and u 2, then we have 4 combinations possible because u 1 and u 2 can draw values from binary set. So, we have first possibility when u 1 can be minus a, u 2 can be minus a we have a second possibility when u 1 can be minus a u 2 can be plus a we have a third possibility when u 1 can be a and u 2 can be minus a we have fourth possibility, when you an can be a and u 2 can be plus a. So, we have 4 possibilities and each of this possibility corresponds to a hypothesis. So, we call this as u 1 u 2 u 3 and u 4. So, these are 4 possible hypothesis; and what is this minus a? This is the first element correspond to this hypothesis ok.

So, just trying to understand the notation. So, if I am considering K symbols at a time and if these symbols are drawing values from a binary set, then the total number of hypotheses that we run into is 2 to the power K alright. So, when we are transmitting a group of symbols and when we are trying to detect a group of symbols at the same time then the definition of our hypothesis changes right. So, each possible group corresponds to a hypothesis and if you allow these symbols to take any value without any constraint and if these symbols are taking values from a binary set, if you are considering K symbols at a time then you have 2 to the power K possible hypothesis.

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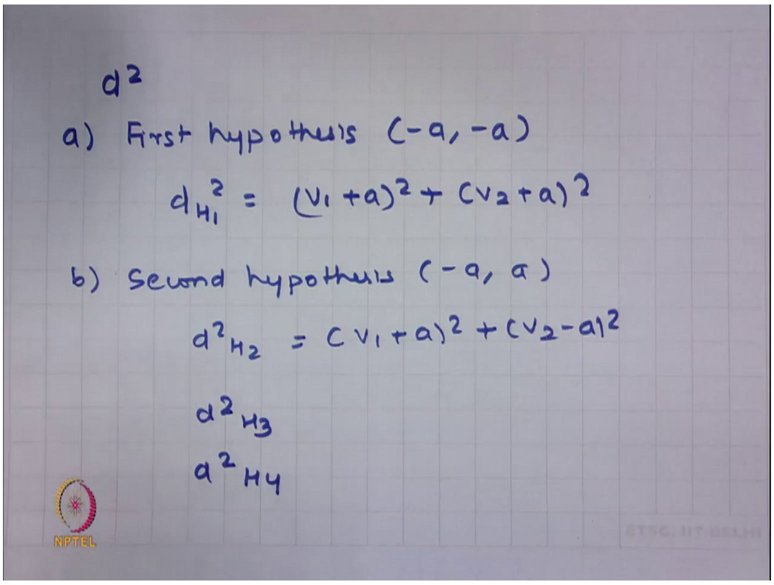


Now, what we do is, we calculate the distance square from each possible hypothesis for example, let us say we consider first hypothesis and what I the first hypothesis in this case its minus a and minus a. So, we have to calculate the distance square from this first hypothesis and this will be V 1 plus a whole square plus V 2 plus a whole square. What are these V 1 and V 2? These are the received numerical values right we will be receiving two numerical values each numerical value corresponding to a transmitted symbol.

Similarly we can consider the second hypothesis and the second hypothesis is minus a and a and we can calculate distance squire from this second hypothesis, which will be this. And similarly you can calculate the distance square from third hypothesis you can calculate the distance square from fourth hypothesis and what is your map rule then?

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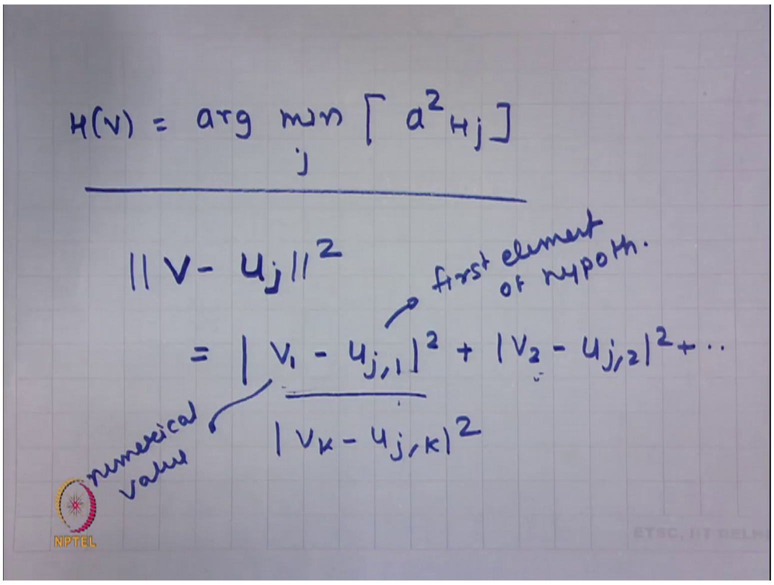


Your map rule is simply trying to find out j for which this t square H of j is minimum. So, this is your map rule. So, conceptually everything looks simple and similar there no differences. Here we are considering the group of symbols as a vector, we are forming a received vector by collecting the numerical values that we have got corresponding to each symbol and then everything remains same we are calculating the distance of the received vector from all possible hypotheses. We are trying to identify a hypothesis or rather choose a hypothesis which has minimum distance or distance of square, if distances minimum distance the square is also minimum.

So, that is what is happening here. Now you can also express this in another way for example, I can talk about this norm square of V minus u j, u j is the vector corresponding to the hypothesis j and V is the received vector and this is nothing, but its V 1 minus u j 1 mod square plus V 2 minus u j 2 mod square V k minus u j case mod square ok.

So, what we are saying? We are saying that the norm square of V minus u j where V is the receipt vector and u j is the vector corresponding to hypothesis j, the norm square of u minus u j is simply the sum of mod square of V 1 minus u j 1 plus mod square of V 2 minus u j 2 and so on so forth. What is this V 1? V 1 is the numerical value corresponding to the first transmitted symbol and what is this u j 1? u j 1 is the first element of the hypothesis ok. So, we now know that the norm square of V minus u j is simply mod of V 1 minus u j 1 square plus mod of V 2 minus u j 2 square and so on so forth.

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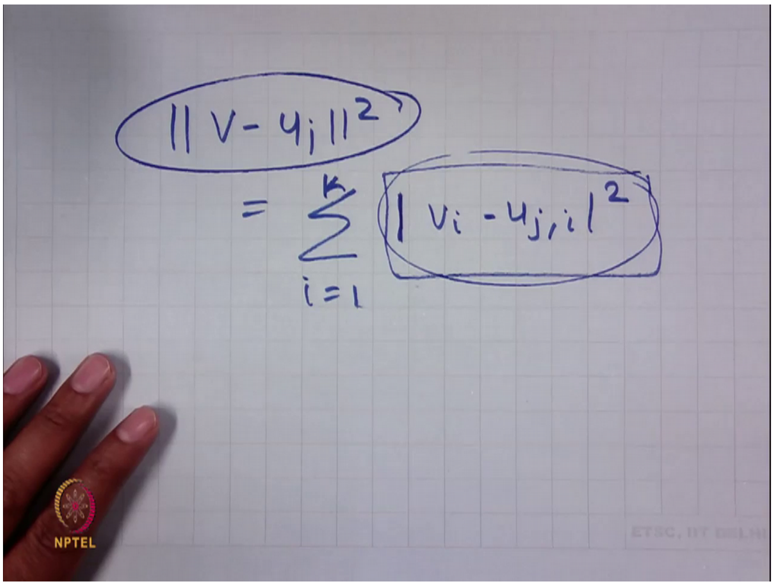


So, in short I can write that norm of V minus u j square is simply sum of mod of V i minus u j i square, where I will go from 1 to k if you are considering k symbols at a time alright. So, what happens in one short detection? In one short detection what we are trying to do is, we are trying to minimize this. We are looking at one element of a hypothesis at a time. So, we are just concentrating on one symbol at a time, not a group of symbol and you want to find out the hypothesis or in this case rather the element of the hypothesis which has the minimum distance ok.

So, what you are trying to do in short is, you are just trying to minimize this one term. And if you continuously use your one short detector and if you assume that these elements of these hypotheses are statistically independent, we will see what happens when they are statistically dependent, but let us first concentrate what happens if the elements are statistically independent.

So, if we are using one short detector, we are trying to find out the first element which minimizes this distance or rather distance square, then we concentrate on the second element of the hypothesis, we try to identify the second element of the hypothesis which minimizes this distance is square. Then subsequently detector will look at each element of the hypothesis and it would try to do its best to minimize the distance or distance square from the received numerical value right. So, what would it do is, when it minimizes this mod square for each element, it will also result in minimizing this term ok.

This is the idea right that you have to carefully understand. So, what we are saying this is some of these terms. One short detector focuses on one term at a time, tries to minimize that term and then it looks at the second term tries to minimize this and so on so forth. That means, it chooses one term minimizes it, choose the second term minimizes it and so on so forth and it goes up to k terms look at the k term tries to minimize it. There is a theorem rather a trivial theorem that if you do your best all the time, you must have done your overall best.

So, if one short detector tries to look at the term, tries to minimize it, it will also make sure that it minimizes overall this quantity and thus what we have seen is, one short detector is not any way different from a detector which is focusing on looking at k symbols at a time. So, this is rather the conclusion in the case of sequence detection or symbol detection.

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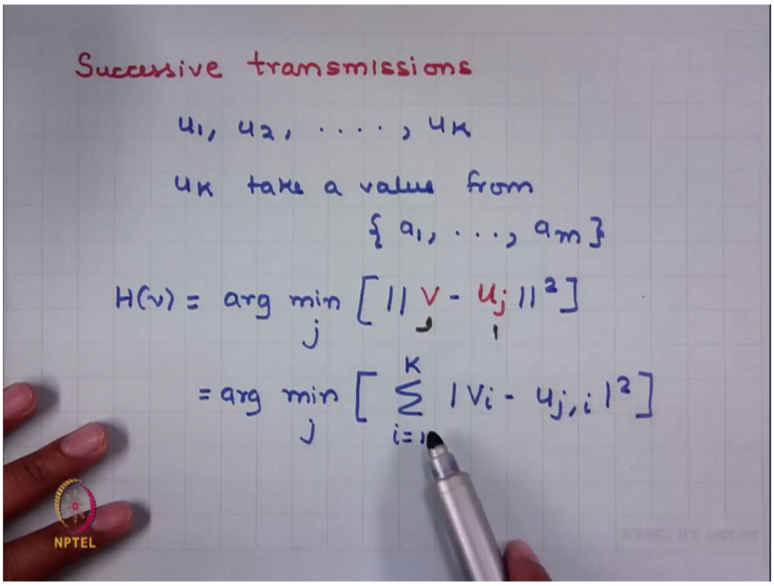


So, what we are saying is, the big goal is to find out hypothesis which minimizes this and we have seen that this problem is same as trying to minimize this. Once short detector looks at a term tries to minimize it, it does its best to minimize one term it continue to do its best to minimize k terms individually. Once it would minimize all k terms individually it will also make sure that it has overall minimize that stuff ok. So, one short detection is not any way different from sequence detection, if the symbols that are transmitted are statistically independent.

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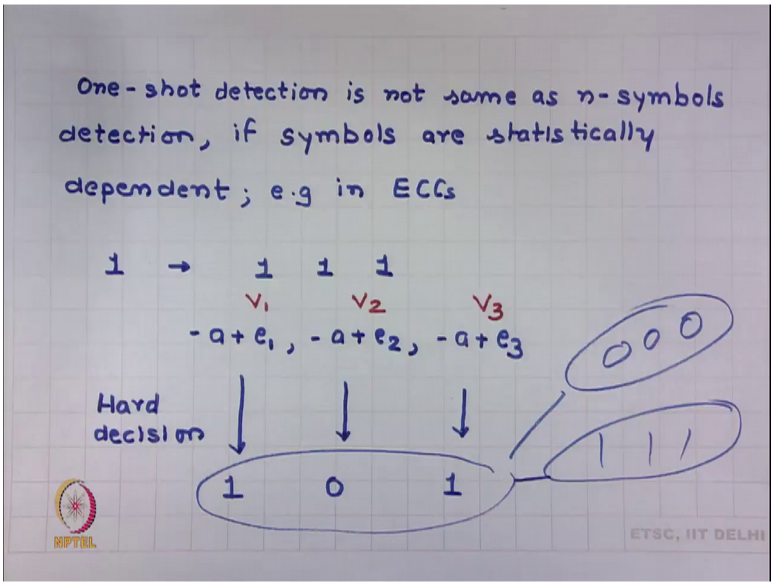


So, we can conclude that one shot detection is same as n symbols detection, but there is one catch this will not be same, if symbols are statistically dependent. If symbols are statistically dependent then one shot detection will not be same as n symbols detection, n symbols are statistically dependent in error control codes.

So, we have seen some times back that in error control codes, we try to introduce a structural redundancy or dependency so that you can correct errors. That means, one symbol is a statistically dependent on another symbol and we try to make use of this statistical dependence, to reduce error possibilities. And except in that case one short detection is same as n symbol detection

So, let us see in case of ECC what happens, and let us take the example of error control codes known as the repetition code. So, instead of sending 1 bit you are sending 3 bits. So, in repeating 1 bit 3 times and so, if suppose in this case an error has happened. So, a transmitter is transmitting 1 1 1, receiver because of some error has decoded this down to 1 0 1.

Now, receiver gets alarmed because it knows that an error has happened, because it knows that it should find either three 0's or three 1's, it cannot find 1 0 1 that is not a possible sequence of symbols in repetition coder. So, repetition coder either transmits 1 1 1 or 0 0 0 because whatever bit it sees it repeats to spit 2 times, a transmits 3 same bits at a time. And those there is some structural dependency, receiver can look at the sequence in total and it knows that an error has happened.

Receiver will not be able to look 1 bit at a time and tell whether an error has happened that cannot be the case. So, receiver has to look in totality, it has to look at these 3 bits at the same time and when it looks and the 3 bits at the same time it can tell you whether an error has happened or an error has sent happened ok.

So, let us now see that we have got 3 received numbers and these received numbers are minus a plus e 1, minus a plus e 2, minus a plus e 3; e 1, e 2 and e 3 are the error magnitudes and because of this error magnitudes receiver is receiving something different from what is transmitted. So, when it is one receiver is transmitting minus a, but it is receiving minus a plus e 1 because of this error and once this error magnitude is too large, namely if it is larger than a then an error will happen ok.

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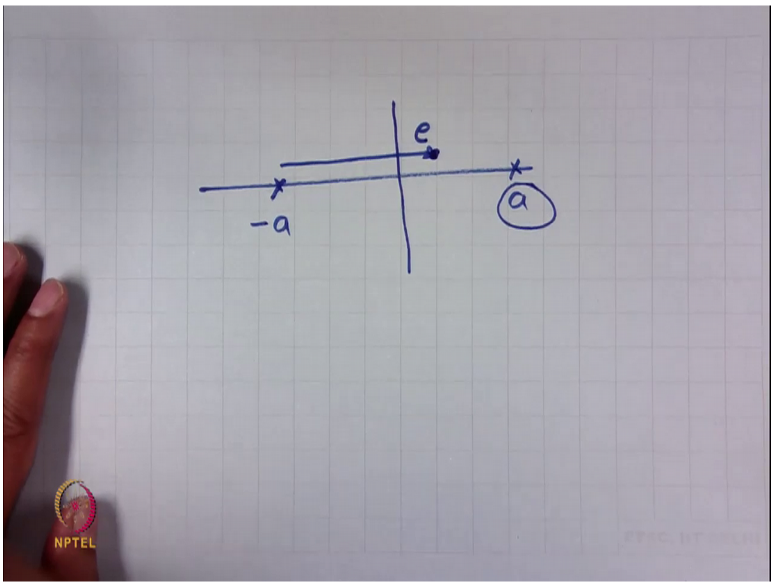


For example so, whatever we were transmitting is minus a and if the error magnitude is larger than a, then your received number would lie to the right of this perpendicular bisector and then you would decide that a possible hypothesis is 0 because the received number lies to the right of this perpendicular bisector and that will create an error.

So, error magnitude or a large error magnitude can create errors. So, there are two strategies basically to decode these possible sequence of symbols in case of ECCs in two ways, one is what is known as hard decision. So, you look at a bit at a time. So, you ignore these two bits, you take a decision on a bits and you take is what is known as a hard decision that is you cannot change your decision ok.

So, we are looking at a 1 bit at a time we are taking a hard decision we might be deciding for this bit and in this case we have decided that this is 1. You look down this bit at a time you take a decision for this bit, you look down this bit at a time you take a decision for this bit and then after this you look at what you have got and now, because you know that you should be getting three 1's or three 0's by looking at this 1 0 1 you decide that the possible transmitted sequence of symbols is 1 1 1.

So, by looking at this you have you can make two guesses, that is what you have to guess. You know that either you can have three 0's or three 1's, by looking at this you better decide for three 1's because ones are in majority right you better guess that the possible transmitted sequence of symbols is 1 1 1. What is the probability of error in this case?

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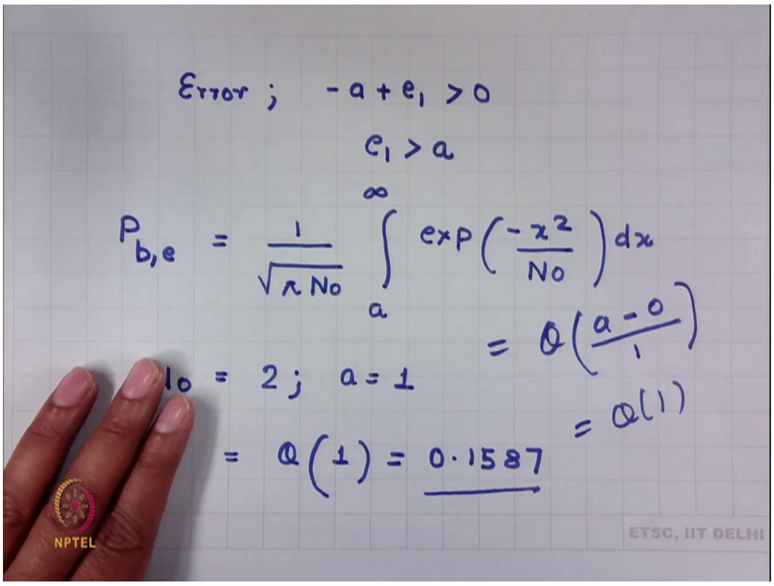

So, we have already seen that an error happens, when the magnitude of error is larger than a, that will slip down your received symbol to the error plane to 1 in which it should not be lying. So, probability of bit error is simply obtained by integrating the p d f of noise. So, noise as usual we assume to have a standard deviation of N naught by 2 mean to be 0 and we are just investigating the situation when this error or noise amplitudes become larger than a.

That means, you have to integrate this p d f from a to infinity. And let us assume N naught to be 2 and a to be 1 to get some numbers, you can easily see that this will integrate down to. So, Q of a minus mean is 0 the standard deviation is 1 and a is 1. So, this will boil down to Q of 1 and Q of 1 is 0.1587; that means, about 16 percent of the time a receiver might make an error alright.

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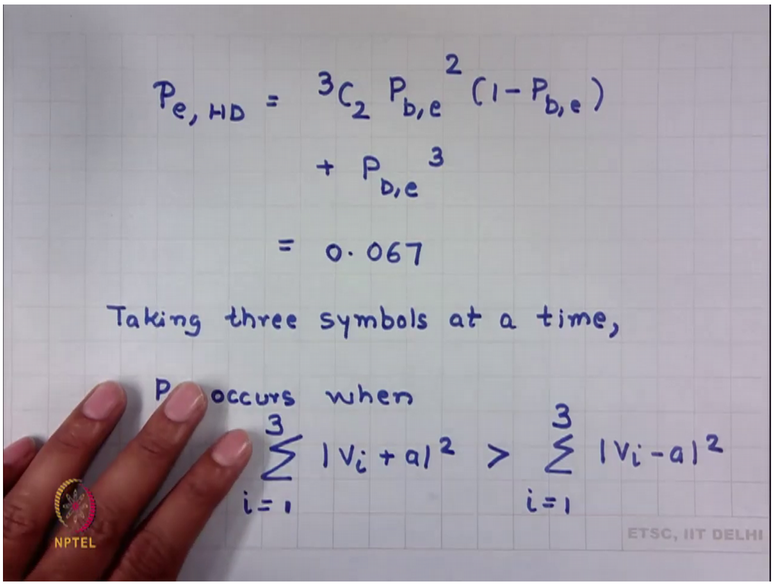


But now, you can make use of this structural redundancy and you can use majority rule which says that you can always correct an error if only 1 bit is an error. For example, in this case if you use the majority rule. So, here only 1 bit is an error and you can always correct this to 1 1 1, because you decide spaced in the majority if you see more ones, you decide that possible transmitted signal set as 1 1 1. An error will only happen if 2 bits or 3 bits are in error, if 1 bit is an error that will not create errors actually.

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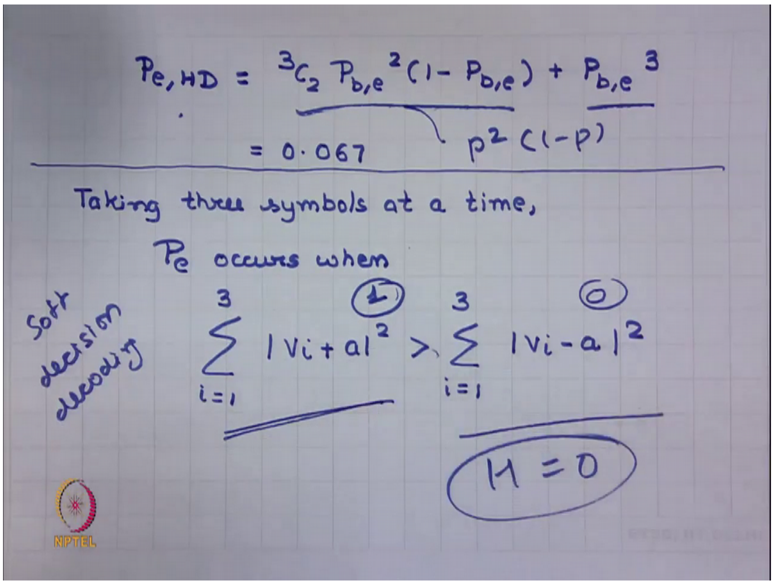


So, let us see what is the probability of error in this hard decision decoding right so when will ever happen, when either the 2 bits are in error. So, this corresponds to the possibility of 2 bits in error. So, if 2 bits in error this possibility is given by P square and 1 bit is correct. So, that is 1 minus P and you have 3 see 2 possible choices ok.

Because you want that any 2 bits out of the 3 bits can be in error. So, this term corresponds to the probability of error when the 2 bits are in error and to this we need to add the possibility of 3 bits to be an error and this possibility or probability is simply given by P cube where p is P b e ok. Working this out what we have got is probability of error in hard decision decoding is 0.067.

So, what it is doing is, its looking at this group of 3 bits, it is trying to use the structural redundancy that is inbuilt and using that structural redundancy it is trying to correct the code or the received symbols ok. And so, it will be able to correct until and unless in this case for example, 2 or 3 bits are in error and this probability is 0.067. So, it has reduce the probability of error which was around 0.16 to 0.067. So, that is some saving.

Now, let us see what happens. So, this was in hard decision decoding, let us see what happens when you are looking at the group of 3 symbols at a time. So, now, we are not doing hard decision decoding, we are doing something which we call as soft decision decoding. Soft decision decoding is more optimum than the hard decision decoding and this we will see. So, when we look at 3 symbols at a time probability of error occurs when the distance of the received vector from the hypothesis 1. So, this corresponds to situation when hypothesis 1 is transmitted let us look at this.

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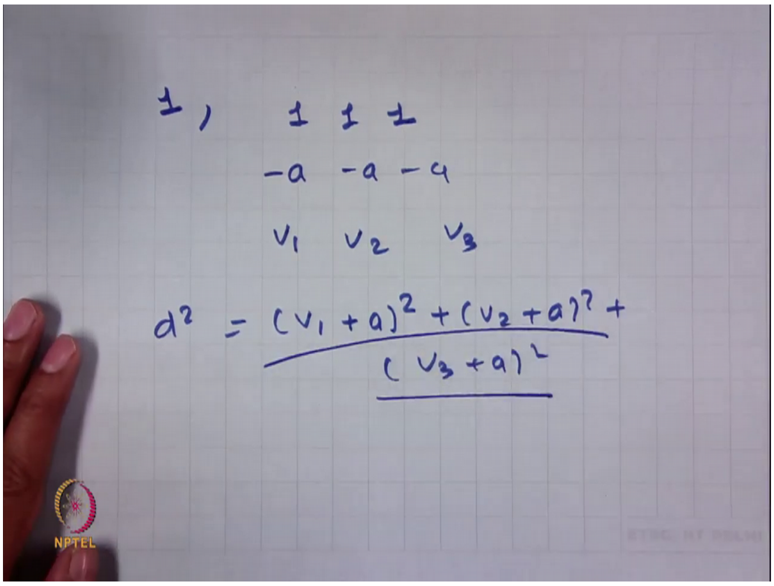


So, when hypothesis 1 is transmitted, you are transmitting three 1's and when you have three 1's you have 3 minus a; and let us say the numerical values that we have is V 1 V 2 V 3. So, what is the distance of squire from this hypothesis 1? The distance is square from this hypothesis 1 is V 1 plus a square plus V 2 plus a square plus V 3 plus a square ok.

We are assuming everything to be real. So, there is no need of more and so and so forth. So, this distance square corresponds to the case when hypothesis 1 is transmitted. So, this situation corresponds to the case when hypothesis 1 is transmitted and this is the distance of square when hypothesis 1 is transmitted. Similarly this is the case when hypothesis 0 is transmitted and this corresponds to the distance of square when hypothesis 0 is transmitted. And probability of error will occur when this distance square is larger than this distance square.

So, why is this? Because we have transmitted this hypothesis 1. So, given that we have transmitted this hypothesis 1, this distance square should have been smaller than this distance square, because my detector would anyway choose the hypothesis for which it has got the minimum distance or distance of square. If this distance square is larger than this distance square then detector would choose hypothesis 0.

And this will create an error situation because we have transmitted hypothesis 1, that is the premises we have transmitted hypothesis 1 then what is the probability of error to take place. And that would happen when detector choose hypothesis 0 and it would choose hypothesis 0, when this distance square is a smaller than this distance square, alright. So, let us expand this and see to what this condition translates to.

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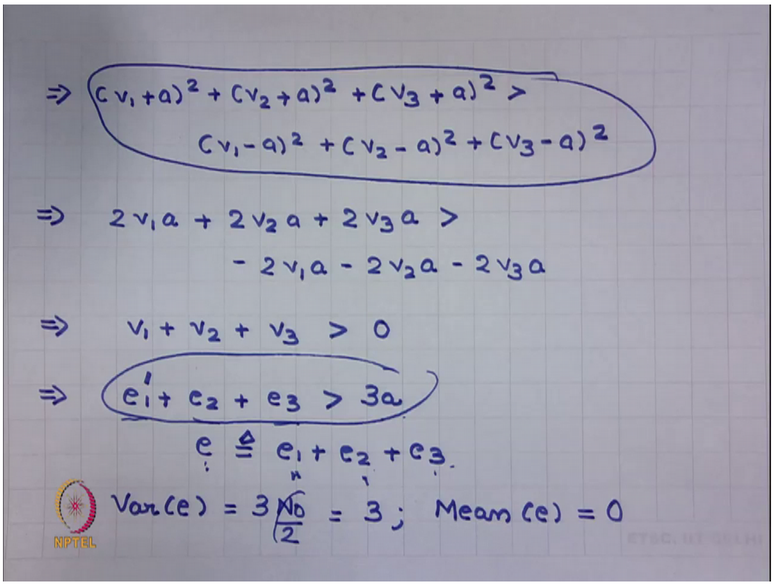


So, just expanding terms assuming everything to be real; so I will not read it out you can stop the video and work out these automatic yourself. So, this simply translates to the condition that e 1 plus e 2 plus e 3 should be greater than 3 a for error to happen. And was this e 1 e 2 and e 3 these are the error magnitudes corresponding to the symbol 1 symbol 2 and symbol 3. So, nitration remains same.

So, what I am saying is that if we start from this basic equation cancelled some terms, reduce this equation, you can simply find that this condition simply translates to the condition when some of these errors; that means, e 1 plus e 2 plus e 3 should be larger than 3 a for error to happen. And to simplify stuff let us define error as e which is sum of e 1 plus e 2 plus e 3. If you are summing the bunch of independent Gaussian random variables each error corresponds to a Gaussian random variable, these errors are also independent error happening in one symbol is complete independent of error that will take place in second symbol.

So, these corresponds to independent Gaussian random variable. So, e is also a Gaussian random variable; what is the variance of e? So, variance of e is just 3 times variance of each error right. So, if we assume that e 1 e 2 and e 3 has a variance of N naught by 2 this is what fear always assumed, then the variance of e is simply 3 times n naught by 2 right. Variance is add if the underlying random variables are independent. And if we assume n naught as 2 then the variance of e is simply 3 what is the mean of e at 0? And hence we can model e as a Gaussian random variable with variance of 3 and mean as 0 is not it.

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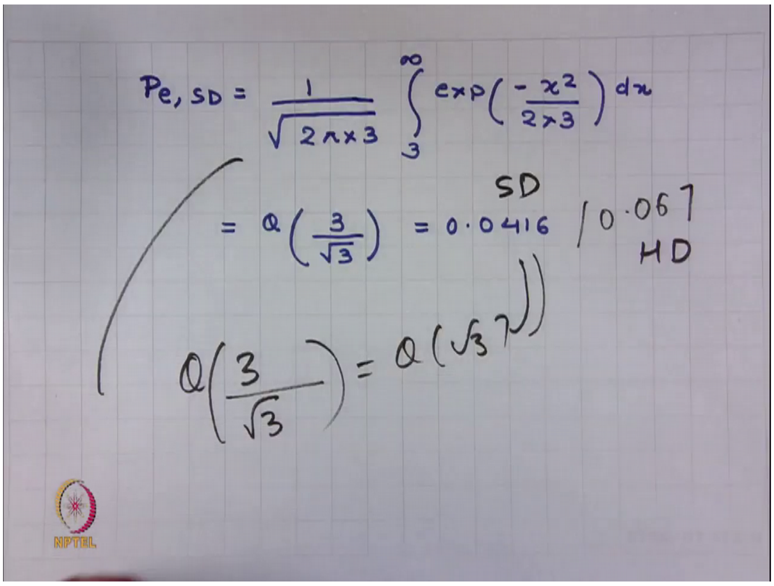


Now, to estimate the probability of error, you need to find out what is the probability of error when the error magnitudes becomes larger than 3. So, it goes from 3 to infinity; however, you have to substitute the new variance new variance is 3. So, I have to have 3 in here 3 in here and what is this integration? So, this is Q of 3 minus mean is 0 divided by standard deviation which is root of 3. So, this is simply Q of root of 3 which is 0.0416, let me look at what was the result in hard decision decoding. So, let me write down this here.

So, this was in hard decision and this is in soft decisions and you have got that when your are considering 3 bits altogether, you have been able to reduce errors. When you consider 1 bit at a time and then you use the structural redundancy of the code to reduce errors you could reduce it to a number like 0.067, it is also small error probabilities. If you consider that you have dropped this down from 0.15 to 0.067 there is lot of good to even a smaller number 0.416.

Hence we sort of have got this idea that looking at the 3 bits at the same time it is much better when the bits and the symbols are statistically dependent and this is the idea. We will talk more about soft decision decoding and hard decision decoding later in the course and this time I am just trying to establish the fact that, if you are transmitting n symbols and you want to look down these n symbols at the same time its only useful when these symbols are statistically dependent. If these symbols are statistically independent whether you minimize the distance for each symbol or you minimize the distance for n symbols at the same time its one and the same thing. If you do your best each time you would be able to do your best collectively.

But when these symbols are statistically independent, we can use this statistical dependence to look down this collection of symbols at the same time and we can reduce the probability of errors. So, with this we have come to the conclusion of this lecture, and in this lecture we have looked into the rules for detection of waveforms and then finally, we have looked into detecting n successive symbols. We have seen that detecting n successive symbols is no different from detecting our symbol at a time, when the symbols are statistically independent.

However, when these symbols are statistically dependent it is a good idea to look at these symbols collectively and we have seen a strategy which reduces the probability of errors when the symbols are looked at the same time holistically. So, in the next lecture we will start by looking into the performance of m l signalling schemes and we will start with binary signalling schemes.

Thank you.