Principles of Digital Communication
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Lecture – 27
Modulation
Pulse Amplitude Modulation & Quadrature Amplitude Modulation (Part-2)

Good morning welcome to next lecture on Modulation. In the last lecture we started with Pulse Amplitude Modulation and Quadrature Amplitude Modulation. And we looked into some key aspects first we discussed about this M PAM where; M represents the number symbols that you have in the modulation scheme.

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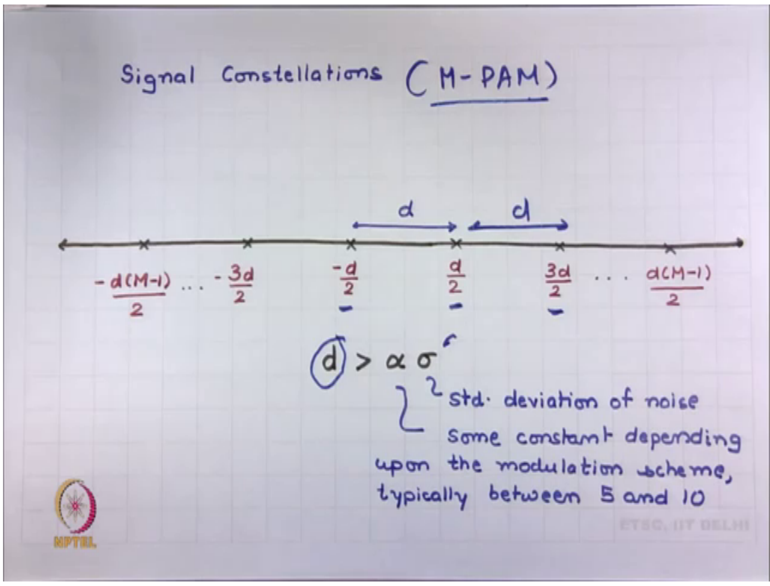

And we said that in M PAM we restrict the signals to be real signals and thus you can arrange these signals only on the real line. And typically we have understood that the distance between the 2 symbols at least should be greater than certain threshold that you choose depending upon the standard deviation of the noise ok.

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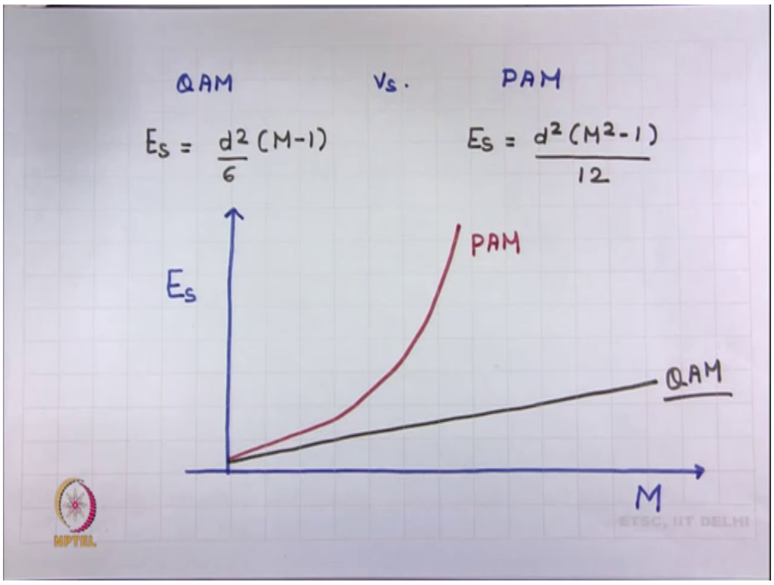

And we looked into another modulation of scheme which is quadrature amplitude modulation scheme. And there we have seen that in quadrature amplitude modulation scheme you use the complex dimension. So, you can have the complex signals and once you allow the complex signals and you use this quadrature amplitude modulation; the one straightaway advantage that you get is you reduce the bandwidth inflation that happens when you do the baseband to pass band conversion. More importantly we have also seen that if we look at the average energy per symbol.

In QAM versus PAM in PAM the energy per symbol increases with M square whereas, in QAM the energy per symbol increases with M and thus in QAM average energy per symbol is much smaller than what you would require in PAM; and thus for higher M this PAM should never be used we should go with QAM. So, as M increases energy per symbol increases but what is the advantage of M? That we will see in this lecture why we like to have a large M if at all that is required, but before answering this question let us try to understand some other modulation schemes.

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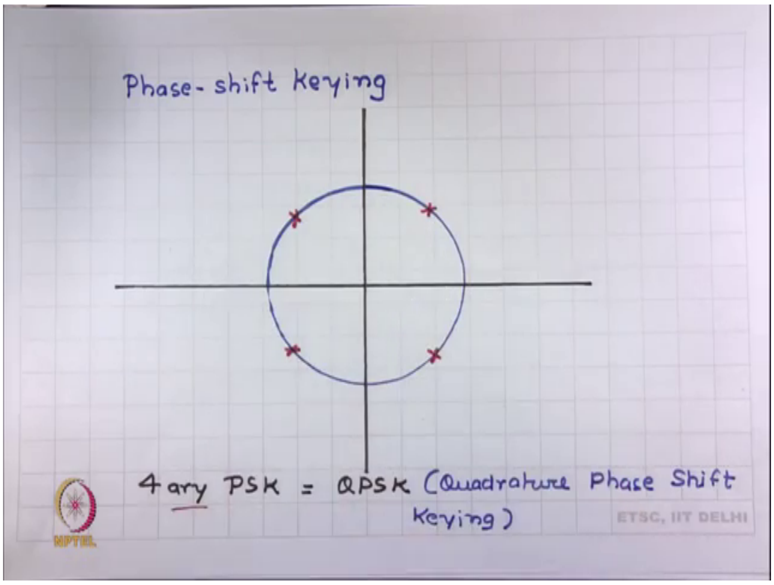

And this is phase shift keying, phase shift keying is also QAM just the difference is, now you can only arrange the symbols on a circle. So, in QAM we have seen that the standard QAM modulation scheme uses a square grids whereas, in phase shift keying we use circular grids. So, the symbol can only lie on a circle. Once the symbols are lying on a circle we know that the energy of this symbol is same is not it. So, each symbol has the same amplitude and the same energy. The only difference is in the phase of these symbols, the phase of these symbols vary and that is why the name phase shift keying that is.

So, that is the idea behind phase shift keying, we also have a very popular 4 ary phase shift key where you have 4 symbols. So, this ary also represents the number of symbols that are there in a modulation scheme and this 4 ary phase shift keying is also known as Quadrature Phase Shift Keying or QPSK where there are 4 phases alright.

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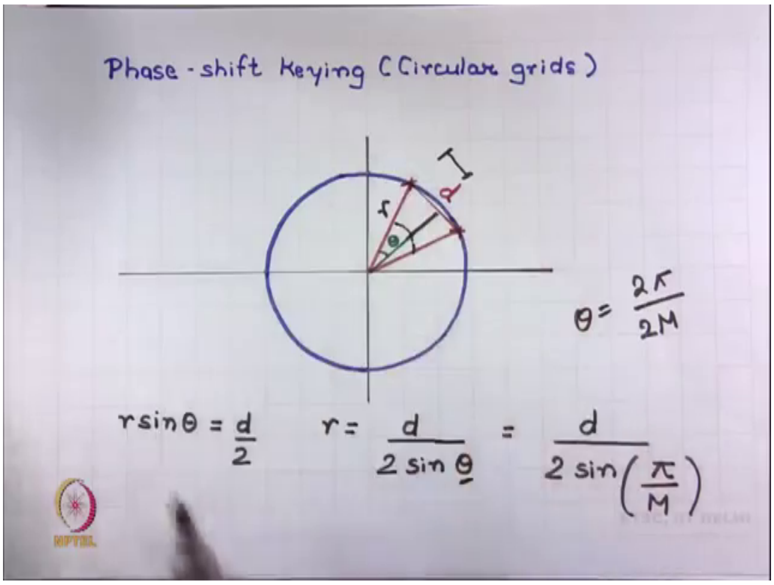

So, let us look at what happens if you allow the symbols only to lie on a circular grid as against the square grid that is used in QAM. So, by using very preliminary geometry we can see that if this distance is r or the radius of the circle is r, then we know that r sin theta is d by 2 where, d we are assuming to be the distance between 2 symbols.

So, r sin theta can be given as d by 2. So, from this we get r as d by 2 sin theta and what is theta? Theta is nothing, but it is 2 pi so the total phase is 2 pi divided by the number of symbols M. So, that will give me this angle and for theta is half of this. So, we divide it by 2 we get theta as pi by M where M is the number of symbols. So, we get the radius as d by 2 sin pi by M symbol. Energy is r square right energy is just given by the radius.

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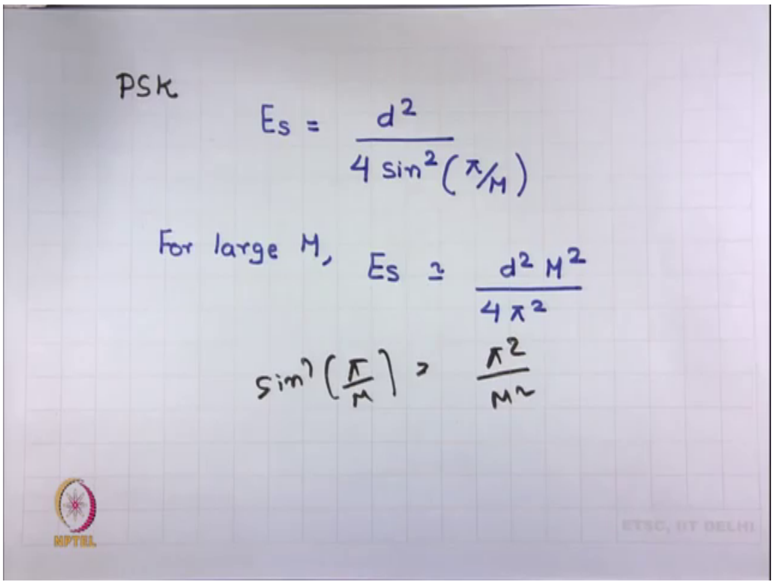

So, its r square so we just square this thing up and we get average energy per symbol which is d square by 4 sin square pi by M. We can see that for large M sin square pi by M can be approximated as pi square by M square of course, this is only true if M is pretty large. We get average energy per symbol as approximately d square M square by 4 pi square. Important thing is average energy again grows with M square for very large M. And, thus you must have understood; that PSK base system should also not be used for large M right.

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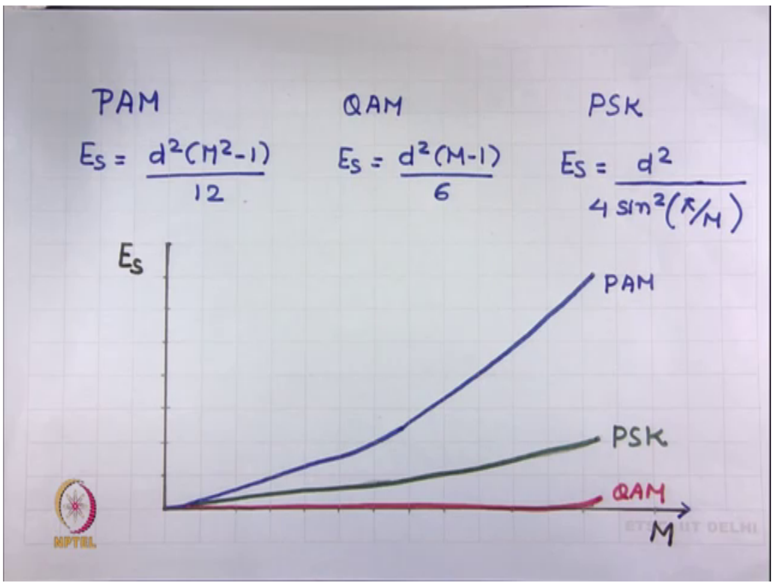

So, this is how the average symbol energy varies with M. So, this is how the PAM average energy will scale as M square PSK also scales as M square for large values of M for small values of M it is not scale as M square. QAM has much smaller average symbol energy compared to PAM and PSK of course; it looks like as it is constant because for PAM and PSK energy grows too much. So, that you do not see linear increase in case of QAM.

But the thumb rule that we have developed now is that this QAM modulation scheme should only be used for large values of M and that is why in this course would you focus on QAM because it is only of practical use. This PSK and PAM does not work for large values of M and that is why all modern communication systems have either use this QAM or other modulation schemes, but PSK based or PAM based modulation schemes are not very useful ok. let us try to focus now on PAM and QAM and let us forget this PSK based modulation schemes, because as we have said they are not very useful modulation schemes.

And for this PAM we want now to talk about these degrees of freedoms which we have introduced in lecture number 6 of this course. So, if you have forgotten about that please relook at lecture number 6 we worked a lot on developing the basic so that we can work freely when we are discussing the main stuff.

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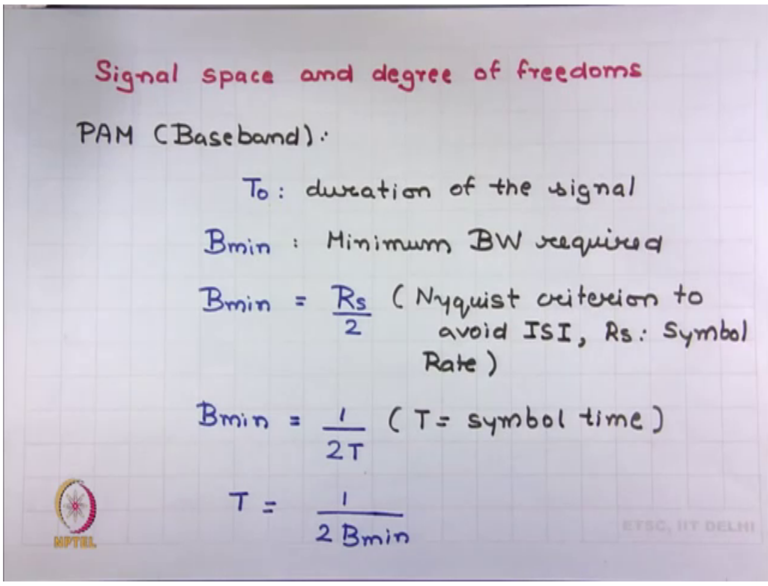

So, let me revise, that if I assume T 0 as the duration of the signal. So, I have a signal continuous time waveform with a total duration of T 0, and if I assume B minimum as the minimum bandwidth required. Then in the lecture 25 in Nyquist pulse shaping we have seen that minimum bandwidth required is nothing, but it is R s by 2 where R s is the symbol rate right. If you are generating the symbols at the rate R s then minimum bandwidth that you required is R s by 2 to avoid inter symbol interference. So, this is absolute minimum bandwidth that we would require.

So, B minimum is one by 2 T the symbol rate is one by T T usually denotes in our course is the symbol time. So, rate at you are spitting out the symbol or your transmitter is spitting out the symbol is 1 by T. So, from this we get T as 1 by 2 B minimum.

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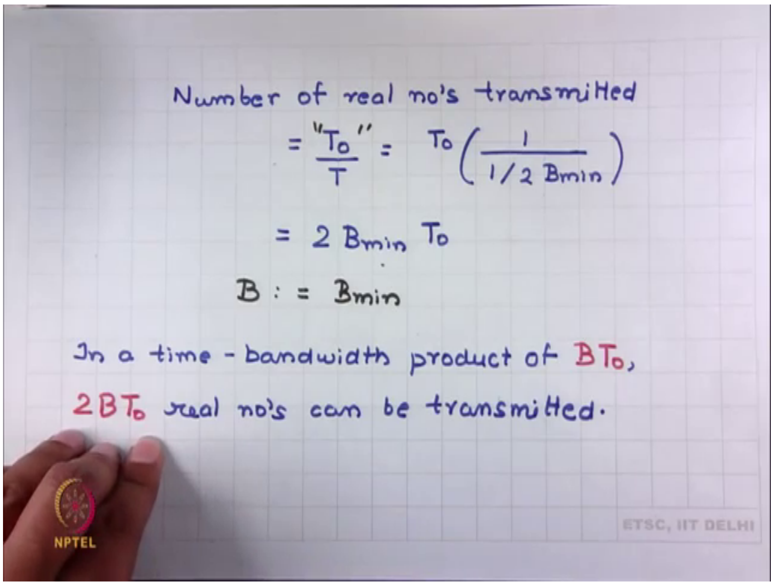

And So, if you look at the number of real numbers that are transmitted it will be the total duration. So, we are having the total duration as T 0 and in PAM we are only generating real numbers, you are generating real numbers at every T seconds ok. So, the total number of real numbers that you would transmit would be T 0 by T and T is 1 by 2 B minimum

So, from this we get number of real numbers that would be transmitted is 2 B minimum times T 0 right is always very hard to talk about this B minimum B minimum. So, we would replace B minimum with B. So, B you should understand as the minimum bandwidth that would be required in our modulation scheme all right. So, if we consider a signal with the bandwidth B and with the duration of T 0 how many real numbers will represent that signal we would have 2 B T 0 real numbers to represent that signal ok. So, hence the degree of freedom of a signal is 2 B T 0 what is the degree of freedom degree of freedom as we have introduced before is just the length of ary that you would require to represent a continuous time signal.

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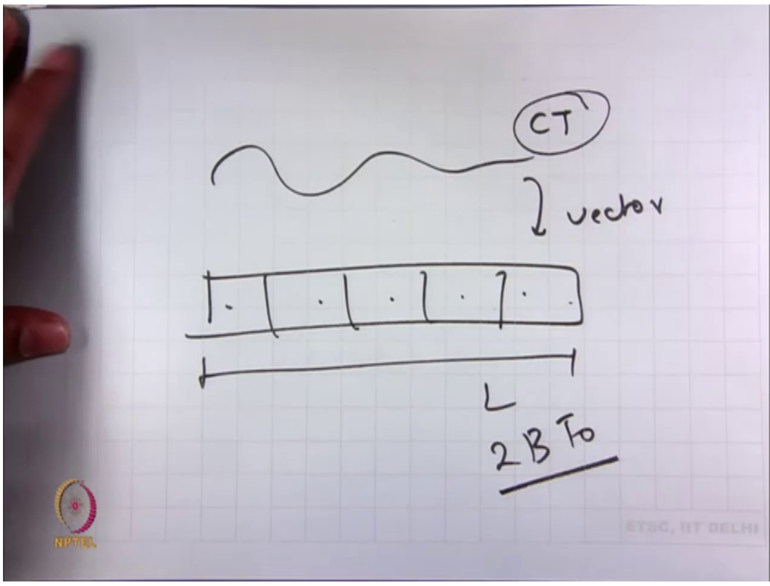

So, let me introduce that again. So, if you have a continuous time signal we have said several times that we can convert this into a vector or an ary where you have certain real numbers filling in, and the length of this ary or vector that you need to have would be 2 B T 0 where B is the bandwidth of this continuous time signal and T 0 is the duration of this continuous time signal. Of course we are only allowing real numbers to be filed in this array and hence the number of real numbers that you would require 2 B T 0.

And, hence the degree of freedom of this signal is 2 B T 0 degree of freedom is nothing, but it is the number of real numbers that you would require, number of real numbers correspond to real degrees of freedom and number of complex numbers that you would require would correspond to complex degrees of freedom ok. So, you can have both real degrees of freedom or complex degrees of freedom is very simple if you allow the complex numbers to be filled in actually you are talking about the complex degrees of freedom. If you allow only real numbers to be filled in you are talking about real degrees of freedom alright.

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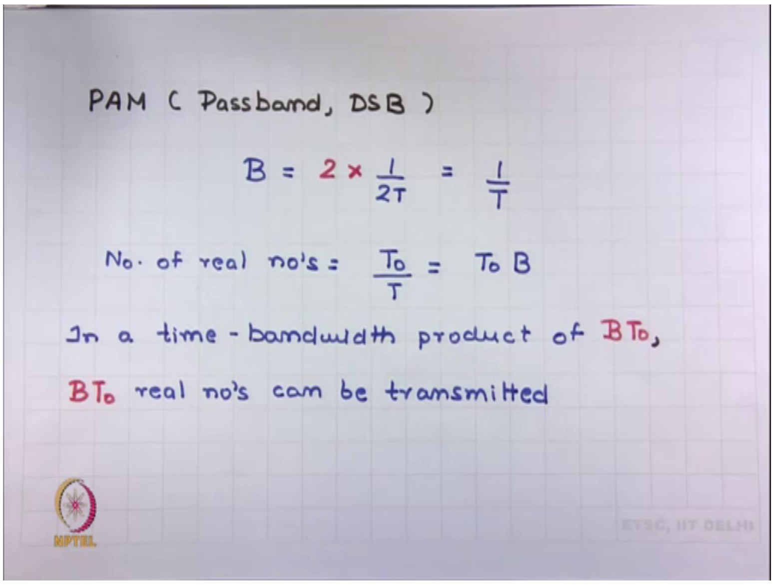

So now, what happens in the pass band PAM. So, we have been talking about the baseband PAM. So, far in pass band PAM what would happen the minimum bandwidth that you would require would be 2 times one by 2 T; because in double sideband modulation scheme bandwidth inflates with a factor of 2. And hence B is 1 by T is not it. So, number of real numbers in case of passband PAM would be T 0 by T and T is 1 by B ok.

So, number of real numbers that you would have in passband PAM would be T 0 times B, thus if you consider a passband PAM signal with a time bandwidth product of B T 0 time bandwidth product means that the underlying signal is of bandwidth B and duration T 0 that is the time bandwidth product of the signal. If you are talking about a time bandwidth product of B T 0, then you need to have B T 0 real numbers representing that signal ok. And these degrees of freedom are important this will become clear eventually wise that in passband QAM. Now, we are talking about the QAM and let us see what changes passband the bandwidth is always 1 by T.

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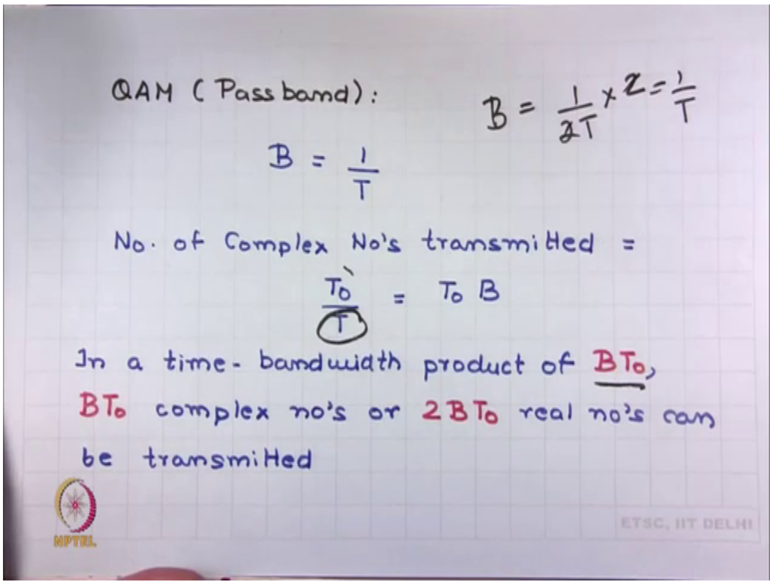

So, remember in the baseband bandwidth required is 1 by 2 T, but if you are talking about the passband the bandwidth scales with a factor of 2. So, thus the bandwidth that you would have is 1 by T, but now what happens is QAM is a complex signal that is different. So, now we are not talking about the real degrees of freedom we are talking about the complex degrees of freedom because the underlying signal QAM is a complex signal. So, total duration of the signal is T 0 the symbols are generated at a rate of T. So, total number of complex signals that you would have is T 0 times B.

So, in a time bandwidth product of B T 0 you would have B T 0 complex numbers generated or 1 complex number is 2 real numbers. So, if I say that I have B T 0 complex degrees of freedom, its simply means that I have 2 B T 0 real degrees of freedom. So, that is what happens this is important that in QAM passband in a time bandwidth product of B T 0 again I end up with 2 B T 0 real degrees of freedom. So, QAM at passband is same as PAM at baseband in PAM at baseband also we had this 2 B T 0 real degrees of freedom I will summarize this in a while, but let us first cover this QAM at baseband.

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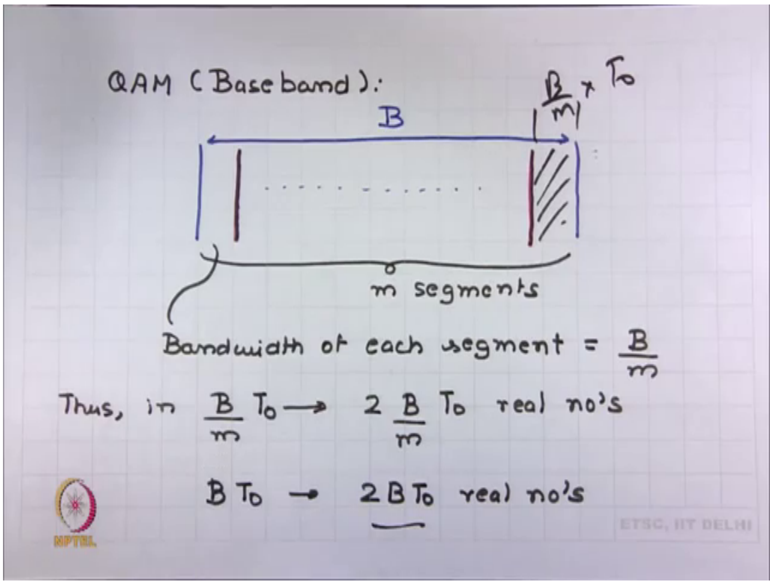

In QAM at baseband you can use the simple idea, let us assume that I have the bandwidth of B I can divide this bandwidth of B in M segments. So, bandwidth of each segment is B by M. Now, this is a passband signal even though I am considering as a baseband signal, but if I consider the total baseband bandwidth is B and then I consider different segments each segment can be thought also as a passband signal is not it.

So, this is the idea that we are using bandwidth of each segment is B by M and in B by M T 0 time bandwidth product. So, if the bandwidth of 1 segment is B by M and the total duration is T 0 I can have 2 B by M T 0 real numbers or 2 B by M T 0 real degrees of freedom. Thus, by using the same idea if I consider time bandwidth product of the signal as B T 0 I can have 2 B T 0 real numbers or 2 B T 0 real degrees of freedom.

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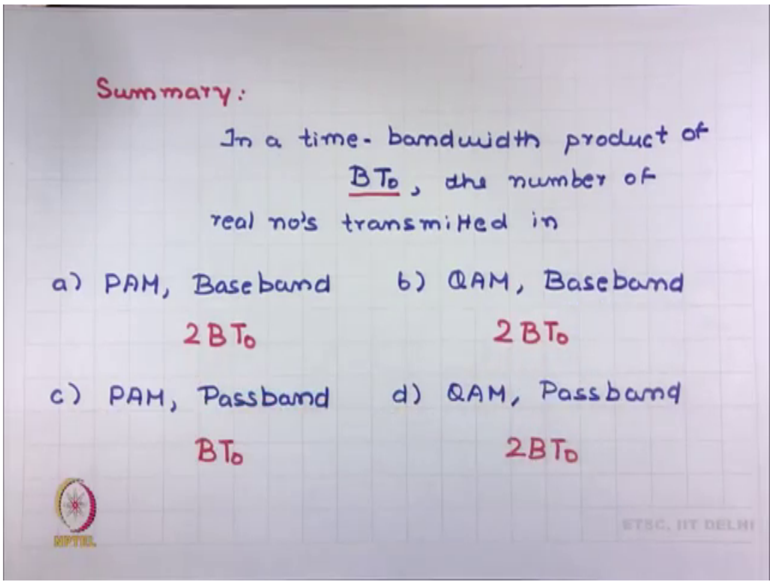

Let us summarize the comparison between PAM and QAM. So, if I consider a signal with a time bandwidth product of B T 0. Let us see how many real numbers can represent that signal faithfully in case of PAM at baseband we have seen that we require 2 B T 0 real numbers PAM at passband we require B T 0 real numbers, QAM at baseband we require 2 B T 0 real number QAM at passband we require 2 B T 0 real numbers ok. So, as you can see that the QAM does not reduce in the degrees of freedom it has when you go from baseband to passband ok

So, once we have understood this then we can talk clearly about a spectral efficiency of a modulation scheme. Spectral efficiency is really important because there you want to see how many bits you can have over a channel what is the bit rate that a channel offers. And, to understand that spectral efficiency clearly you need to understand the degrees of freedom that a modulation scheme offers and the time bandwidth product tells me about the degrees of freedom; this will be clear when we talk about the spectral efficiency ok.

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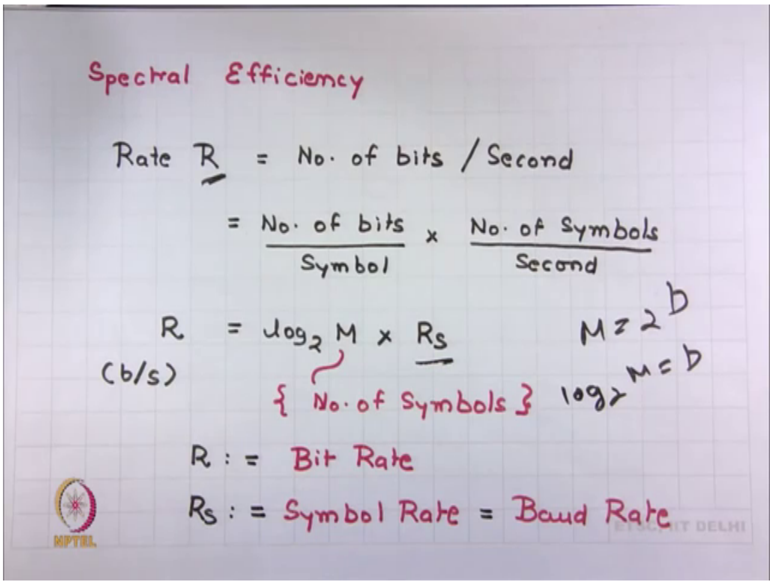

So, first let us see spectral efficiency, in a simpler way in the way that you can probably appreciate quite easily. So, the spectral efficiency depends upon the rate and what is the rate? Rate is number of bits that a transmitter spits out per second ok. So, that is the bit rate or simply the rate. So, we are using the letter R to represent the bit rate number of bits per second and I can understand this number bits per second as number of bits per symbol and then number of symbols per second and number of bits per symbol is nothing, but it is log 2 M if M is the number of symbols the number of bits would be log 2 M we have already seen M is 2 to the power b. So, log 2 M is b where b is the number of bits per symbol.

So, number of bits per symbol is log 2 M number of symbols per second is R s that is the symbol rate. So, we are using the notation R s to talk about that. So, bit rate is nothing, but it is log 2 M times symbol rate. Symbol rate is also known as baud rate its important term that we use in digital communication context baud rate is also known as symbol rate or symbol rate is also known as baud rate. So, this is an important expression relating the bit rate to symbol rate.

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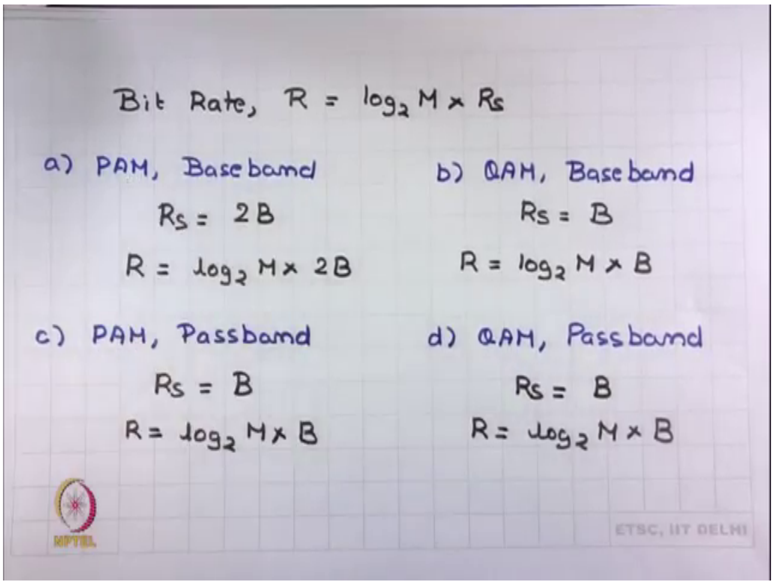

Now, we know that bit rate is log 2 M times symbol rate; what is the symbol rate for baseband PAM?

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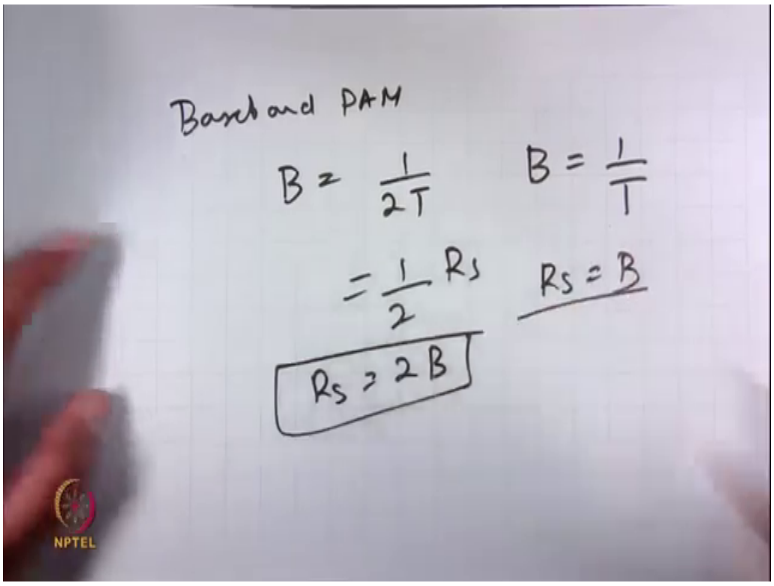

We have seen that in baseband PAM B is 1 by 2 T and this is 1 by 2 R s. so R s is 2 times B. So, if you have the bandwidth available to as B the symbol rate that you can have is 2 times b. So, the maximum symbol rate that you can have is 2 times the bandwidth that the channel offers ok. You cannot have a higher symbol rate than twice the bandwidth available. So, in baseband PAM R s is 2 times b. So, rate becomes log 2 M times 2 B in passband PAM R s is B because for passband B is 1 by T. So, R s becomes B. So, for passband PAM R s becomes B bit rate becomes log 2 M times B for QAM R s is B bit rate becomes log 2 M times B for QAM at passband R s is B and this r becomes log 2 M times B.

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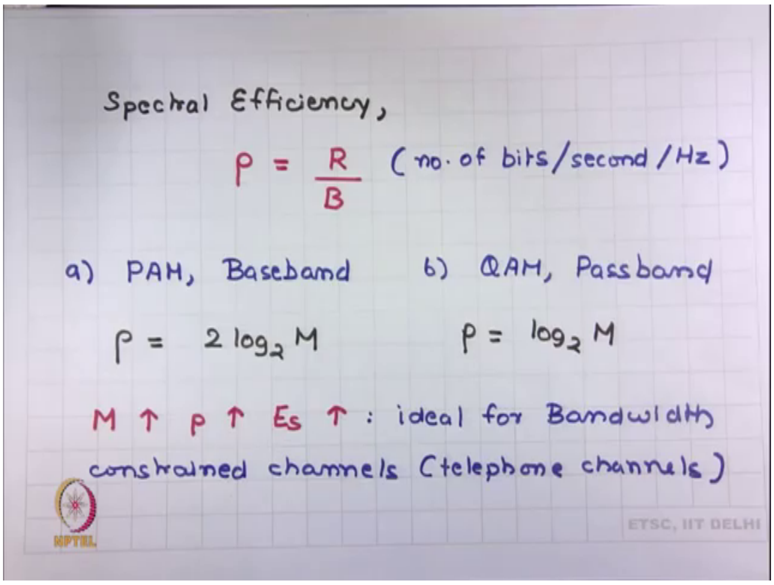

Now, once you have defined for the bit rate, you can talk about the spectral efficiency. The spectral efficiency is nothing, but it is the bit rate divided by the bandwidth that is it that is the spectral efficiency the unit will be number bits per second per Hertz; number bits per second because this is a bit rate divided by the bandwidth the unit of bandwidth is Hertz.

So, the unit of a spectral efficiency is number of bits per second per hertz and two most practical case that we need to consider is PAM at baseband and QAM at passband. PAM at baseband you already have calculated the bit rate divided by a bandwidth.

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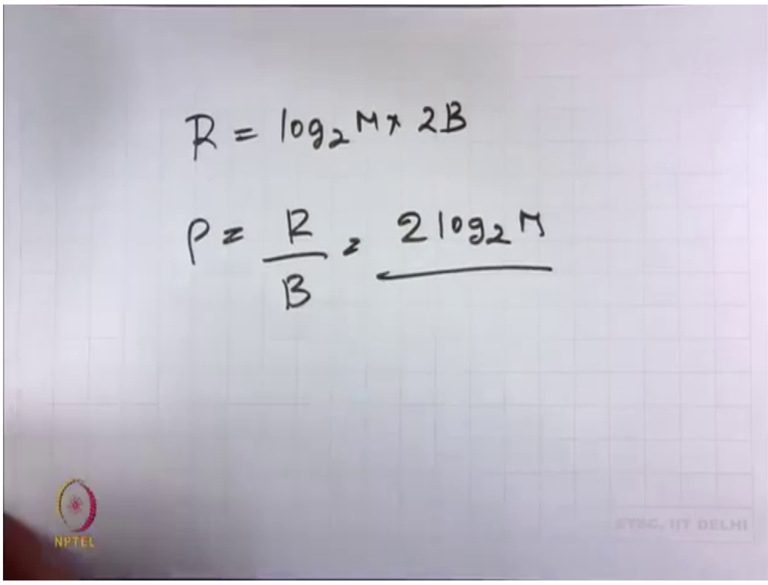

So, once I have bit rate, log 2 M times 2 B the spectral efficiency is R by b. So, the spectral efficiency would be 2 times log 2 M. So, this is the spectral efficiency of a baseband PAM 2 times log 2 M. Similarly, for passband QAM you can have the spectral efficiency of log 2 M of course, for passband PAM the spectral efficiency would reduce by a factor of 2 for baseband QAM the spectral efficiency would be same as log 2 M.

Important point is as you increase M the spectral efficiency improves; that means, you can pack in more bits per second over that channel. But also the symbol energy increases we have seen as M increases e s increases. And that means, there is a trade off between the symbol energy and the spectral efficiency, if you want to have higher bit rates over the channel what you want to do is you need to have the modulation schemes with higher M. And once you do that you need to pump in more symbol energy as well and increasing M would be important in bandwidth constrained channel.

So, we have seen in lecture one that the communication channels are of basically 2 kinds; certain channels are bandwidth constrained channel; that means, the bandwidth is the more precious resource for those channel. For example, telephone channels are bandwidth constrained channel you can always pump in more power in telephone channels that is not the primary concern. But over telephone channel what we wanted to have always is more and more bit rates that is the primary source and thus for bandwidth constrained channel you want to have a larger M; because that will increase the spectral efficiency of your scheme, though it will punish you by making you to invest more symbol energy ok.

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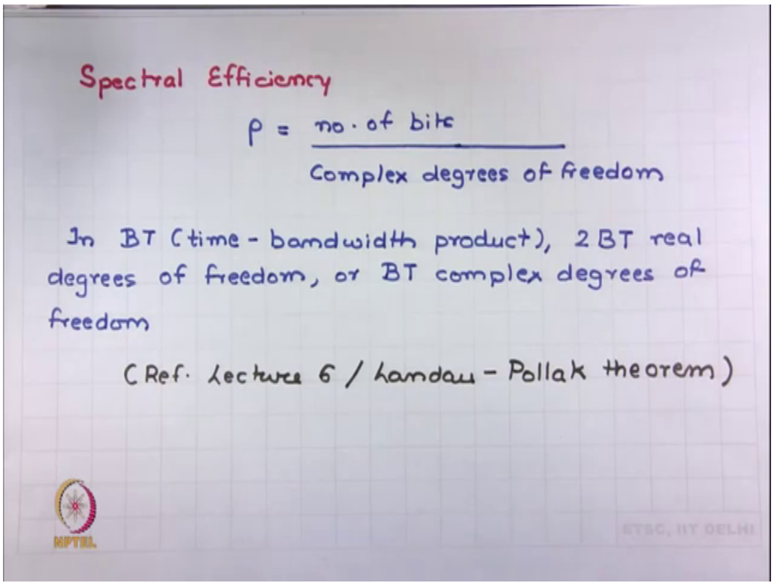

Let us now try to look at the definition of a spectral efficiency, from another context and this definition is also useful we will get to the same numbers, but we will define it in a different way that is it. So, spectral efficiency can also be defined as number of bits divided by number of complex degrees of freedom, this is an important idea and this is useful particularly in the context of error control coding where we want to talk about the spectral efficiency in terms of degrees of freedom.

So, we have seen it several times that if I have a time bandwidth product of BT I can have 2 BT real degrees of freedom or I can have BT complex degrees of freedom this you can refer to lecture 6. Or, if you want to look at this very rigorously you can look at Landau Pollak theorem which states or make these things more rigorous alright.

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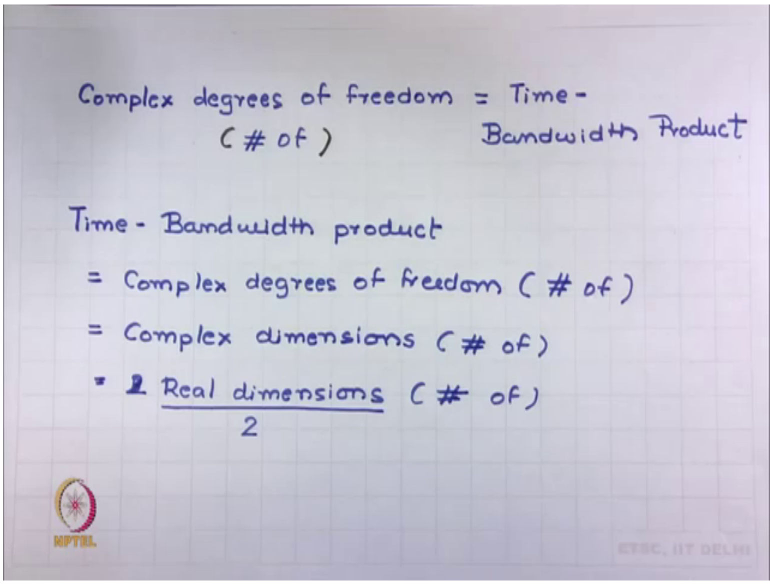

The idea is that number of complex degrees of freedom is nothing, but it is the time bandwidth product. And there various ways in which I can understand this time bandwidth product which is the number of complex degrees of freedom. The degrees of freedom are also known as dimensions of a signal they are interchangeable words; sometimes you talk about dimension, sometimes you talk about degrees of freedom, complex degrees of freedom or complex dimensions are one and the same thing.

So, time bandwidth product in certain books is also written as number of complex dimensions. And it would be same as number of real dimensions by 2 because number of real dimensions is 2 times time bandwidth product. So, time bandwidth product would be half of the number of real dimensions ok. Now, let us look at this.

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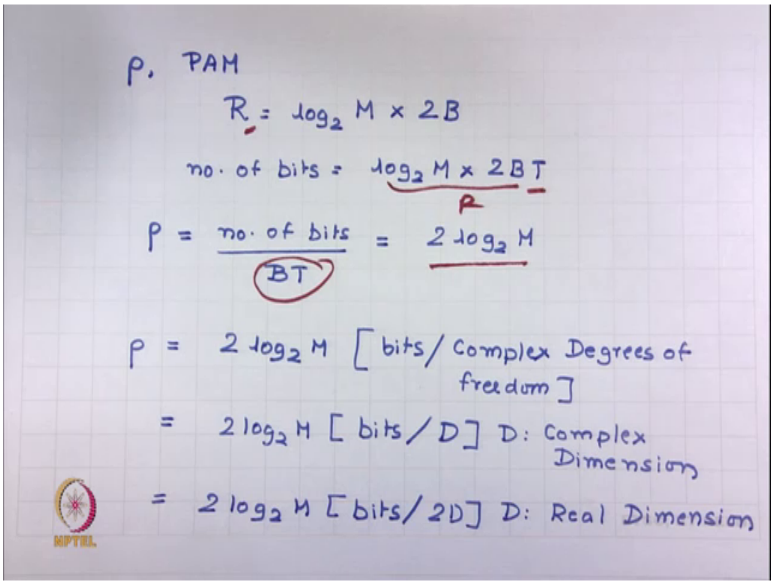

So spectral efficiency, to calculate first thing that I need to calculate is number of bits; number of bits would be R times R is the bit rate number of bits would be R times the total duration. So, this is R and this is the total duration T. So, number of bits would be this divided by B T would give you the spectral efficiency 2 log 2 M. Now let us look at the units 1 unit of course, we have seen is bits per second per hertz or I can write the units is 2 log 2 M bits per complex degrees of freedom because PAM bandwidth product is complex degrees of freedom or its sometimes written as 2 log 2 M bits per D where D represents the complex dimensions or you can write this as 2 log 2 M bits per 2 D where D represents real dimension.

So, it depends upon context to context and book to book. So, once you see d you have to ask the question whether it is your complex dimension or it is a real dimension if the underlying definition of d is real dimension then you would see the 2 d in here if you see d as complex dimension then you would just have d. So, this definition of a spectral efficiency is also useful and is used in several context ok. So, what we are doing in this lecture is we are developing this ideas, we have looked again at this degrees of freedom, complex dimensions, complex degrees of freedom how is it related to time bandwidth product and things like that. And we are developing the notation of a spectral efficiency for PAM and QAM and we have seen very interesting thing then as M increases the spectral efficiency increases; that means, you can have more and more bits per second per Hertz.

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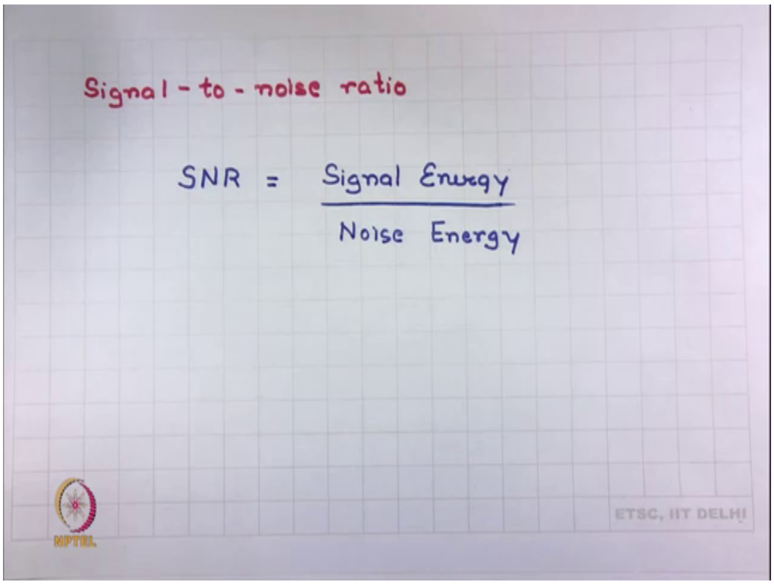

Now, let us define signal to noise ratio, it is also an important metric and when we do detection we will see that, the error performance is the very strong function of the signal to noise ratio ok. And thus it is important to understand what is this; signal to noise ratio is signal energy divided by noise energy or we can talk about the signal energy per dimension noise energy per dimension and things like that. This will become clear, but first let us start by revising some basics.

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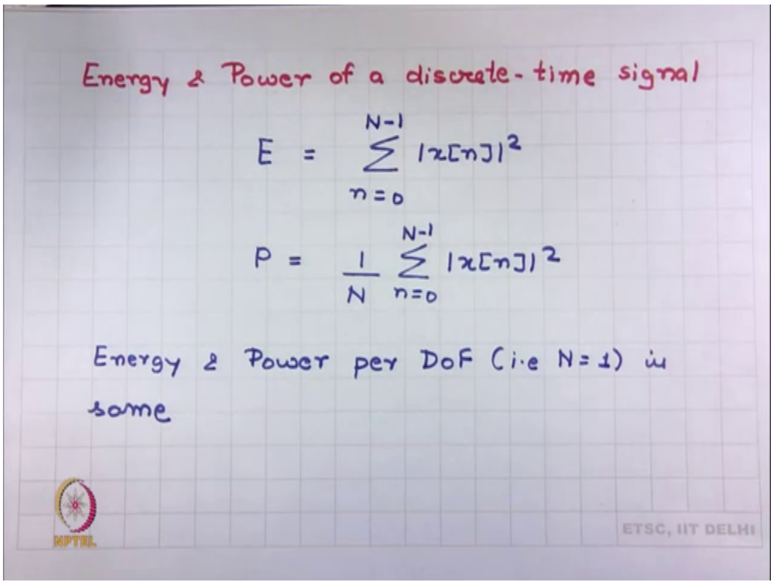

And the basic is how do we define the energy and power of a discrete time signal, you can refer to book open ham if you have forgotten how the energy and power of a discrete time signal is defined. Energy of a discrete time signal is defined like this, it is same as how we define the energy of a continuous time signal; just integration is replaced by summation that is the only difference when you go from continuous time to discrete time.

And for the power we divide by the number of samples that you are considering. And this is the power of a signal, now one thing that you need to know is when we are talking about per degree of freedom; that means, per sample if you are talking about per sample then N is 1. And the energy and power per degree of freedom is same ok. In discrete time signal we do normalization to get a discrete time signal the time duration of the sample is considered to be unity when we are talking about degree of freedom we invariably mean that we have got the discrete time signal probably derived from the continuous time signal. So, that is one thing that we have to keep in mind. The second thing is what we have discussed about white Gaussian noise.

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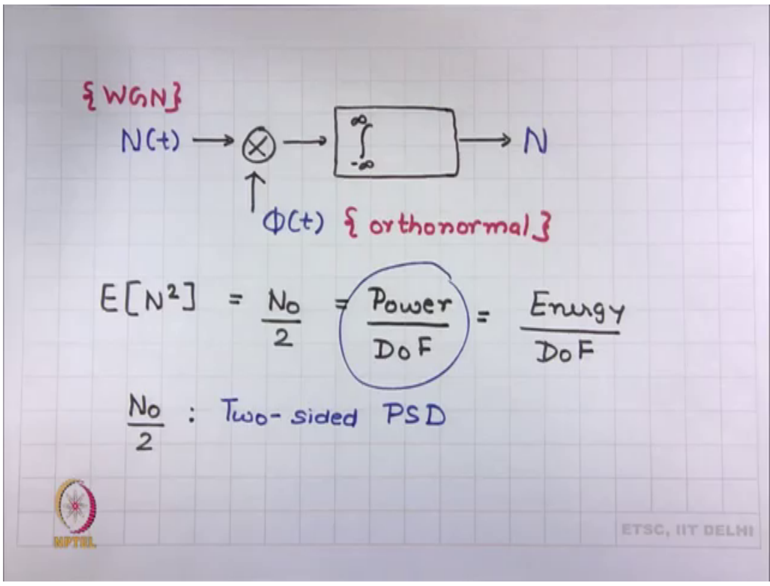

We have seen that, if you have a white Gaussian noise N t, passed through a correlator correlator is fed with this orthonormal function, at the output of the correlator you get a Gaussian random variable denoted by N. And when we talk about the expected value of N square we get N 0 by 2 all this we have done in lecture 17 of this course. So, expected value of N square is N 0 by 2, where N 0 by 2 is the power spectral density; that means, the power per degree of freedom of noise is N 0 by 2 or you can say it as energy per degree of freedom of noise is N 0 by 2. Let me draw a diagram and maybe it will become then more clear. So, if I have a white Gaussian noise what this power spectral density? It is constant.

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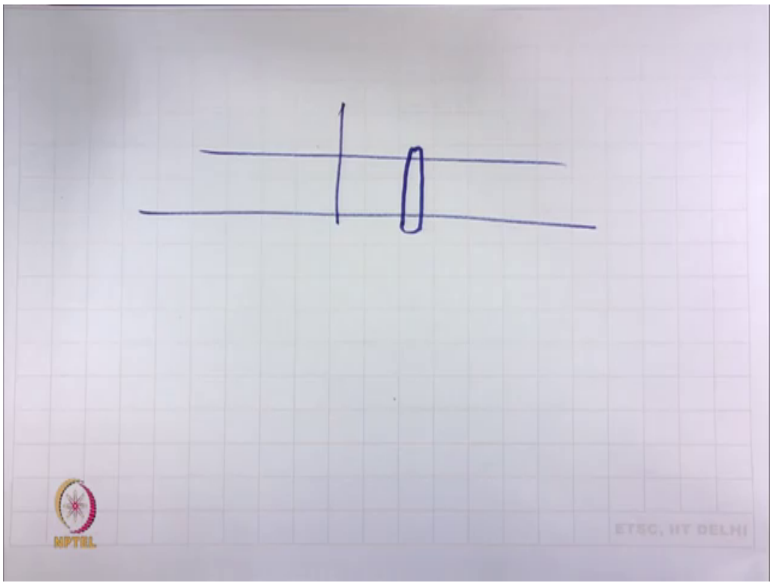

Now, from this power spectral density you are taking a chunk of this power out this power is available only where this spectrum of this orthonormal function is. And so the power that you get out from this random variable corresponds to the power that is available per degree of freedom ok. And power is same as energy when we are talking about degrees of freedom. So, these 2 things you have to remember that power per degree of freedom of noise is N 0 by 2 energy per degree freedom of noise is also N 0 by 2 ok.

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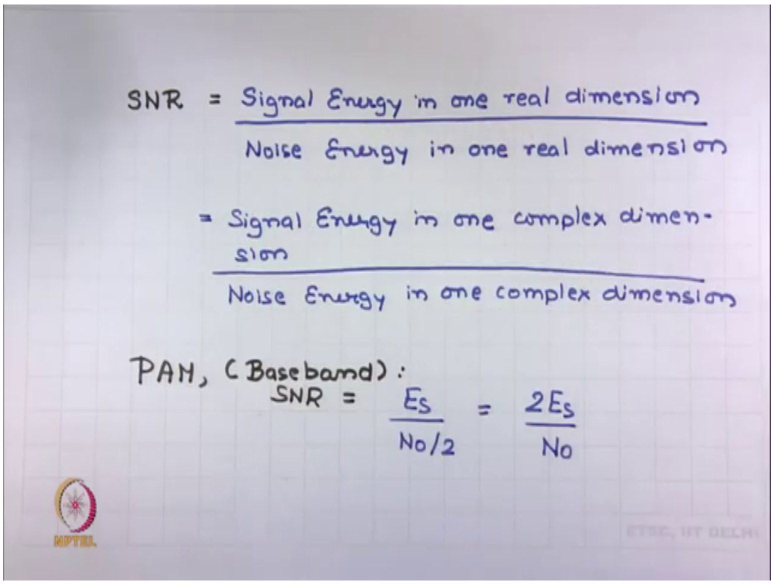

So signal to noise ratio we can define now more specifically as signal energy in one real dimension. And noise energy in one real dimension or degrees of freedom interchangeably. Or if you want to consider complex dimension then you should consider the signal energy in one complex dimension divided by noise energy in one complex dimension we have to be fair. If for signal you are considering real consider the dimension also for noise as real, if you are considering for signal a complex dimension also consider for the noise the complex dimension just be consistent.

Using these ideas let us see; what is the signal to noise ratio for baseband PAM, signal energy in one real dimension is E S right. You are representing one signal while just one number and that number corresponds to the energy of the signal was the noise energy in one real dimension N 0 by 2. So, signal to noise ratio becomes 2 E S by N 0 ok.

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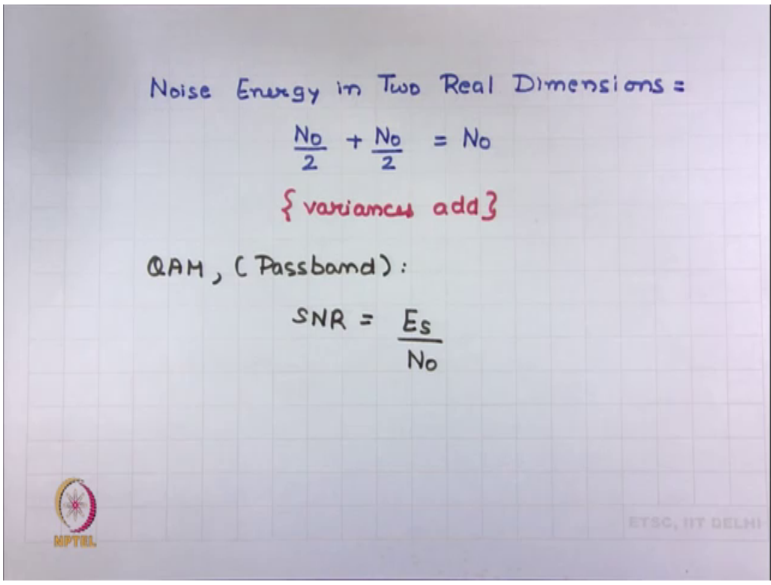

Let us now see for QAM, but QAM is a complex signal; that means, it occupies a complex dimension. So, the energy of QAM E S is in complex dimension. So, for noise also we have to consider the noise energy in complex dimension. So, noise energy in complex dimension is nothing, but the sum of noise energy into real dimension complex dimensions to real dimension the variances of noise at this also we have covered; if the noise random variables are independent, then the variances of the noise N ok.

So, the variance of this noise in one dimensions is N 0 by 2 the variance of the noise in another dimension is N 0 by 2, the total noise variance would add and it would become N 0. So, when we are defining the signal to noise ratio of QAM passband it would be E S where this E S is the energy in complex dimension divided by the noise energy in complex dimension which is also N 0 ok. So, this is the signal to noise ratio of QAM.

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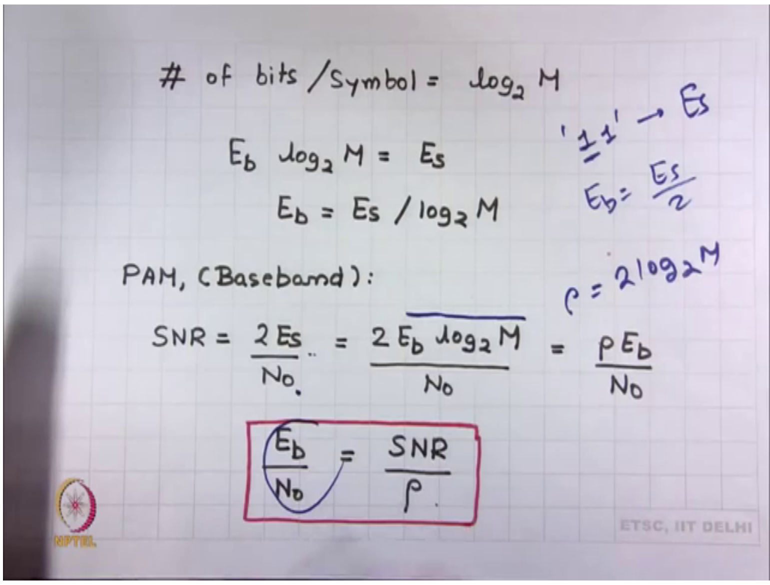

Let us now see, can we have some other ways to talk about the signal to noise ratio. So, one way is we know that number of bits per symbol is log 2 M. And from this we get the energy per bit. So, if you are investing E S energy in a symbol per bit energy would be obtained by dividing the total symbol energy by number of bits, that you have and number of bits in a symbol is log 2 m. So, bit energy is E S divided by log 2 M ok.

So for example, if you have 2 bits in a symbol, you get the energy of a symbol as E S per bit energy that you need to spend is E S by 2. The total number of bits, so E S is a symbol energy divided by the number of bits which is log 2 M. So, we get bit energy as E S by log 2 M. Let us look at this signal to noise ratio for baseband PAM again, we have obtained signal to noise ratio is 2 E S by N 0 E S is E b times log 2 M and we have already seen that the spectral efficiency of baseband PAM rho is 2 log 2 M.

So, we can write signal to noise ratio is rho times E b by N 0. E b by N 0 is also known as E b N o right. So, instead of saying E b by N 0 all the times its more convenient to say it as E b N o. So, signal to noise ratio is rho times E b N o E b N o is signal to noise ratio divided by rho and in digital communication systems we talk more about E b N o. This is more interesting, because it takes into account the spectral efficiency of the modulation schemes some modulation schemes has a higher spectral efficiency than the other modulation schemes. And thus to be fair to those modulation schemes rather than comparing modulation schemes based on SNR a signal to noise ratio it is more convenient to compare modulation schemes based on E b N o's. So, that is also another metric that is used E b N o is SNR by rho right.

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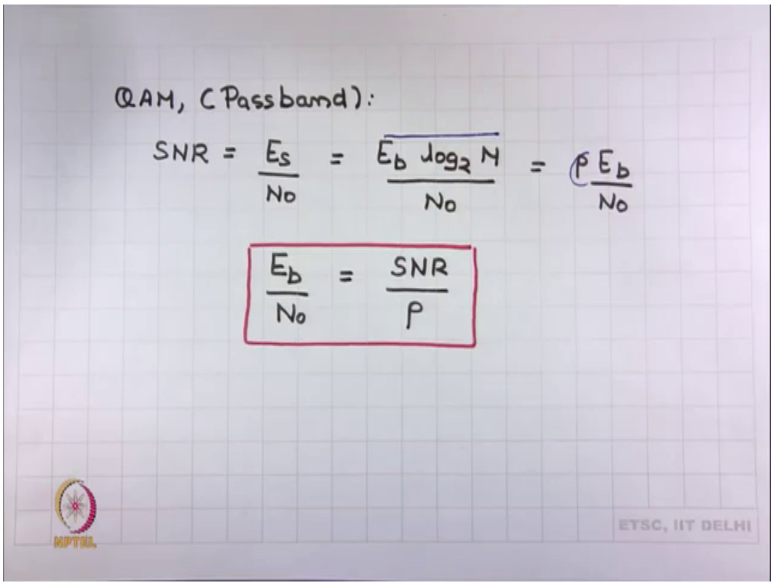

Let us see what happens for QAM at passband. So, SNR we have said is E S by N 0. E S is E b times log 2 M and for QAM log 2 M is rho. So, again for SNR at QAM we get SNR is rho times E b N o or E b N o is SNR divided by rho. Thus we have got the same relationship between SNR and E b N o for QAM and PAM all right.

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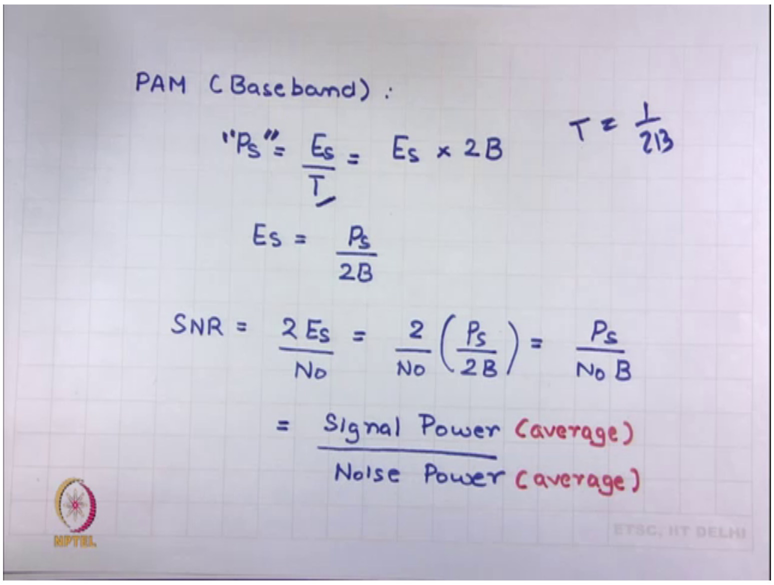

Let us derive another relationship, so power of the symbol is E S by T energy divided by the total duration that is the power. And T is for PAM at baseband T is one by 2 B, B is the band with that is required. So, power symbol is E S times 2 B so E S is P S by 2 B. So, for PAM and baseband signal to noise ratio is 2 E S by N 0 E S can be substituted as P S by 2 B from that we get this.

So, signal to noise ratio is P S by N 0 times B P S is signal power or average signal power, and what is this N 0 times B it is the noise average power. This we have seen that if you pass a noise with the filter with the band with B the output noise power is N 0 times B. So, signal to noise ratio can also be understood as average signal power divided by average noise power; maybe this is more convenient to use it in this way ok. When you do not have this factor of 2, let see whether we get the same expression for signal to noise ratio also for QAM.

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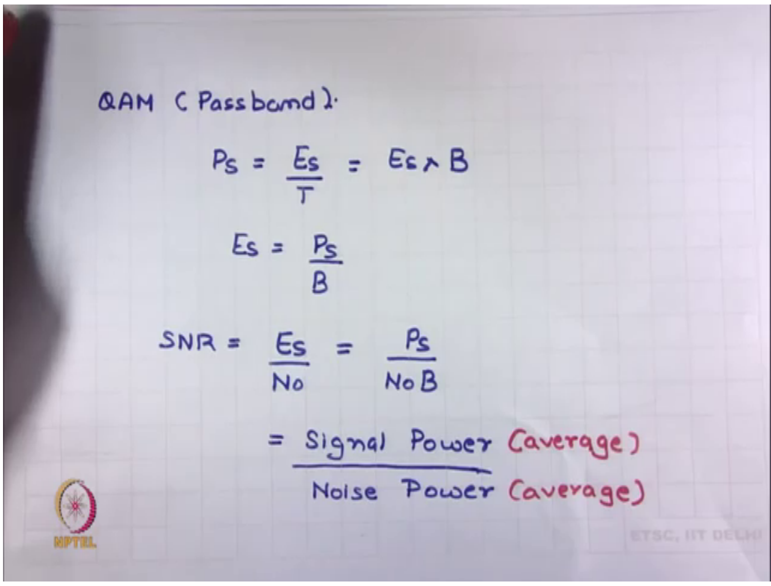

For QAM, the average symbol power is E S by T it is the same thing as for PAM, but for QAM this T is 1 by B. So, from this we get E S is P S by B and now substituting this average symbol energy in terms of average symbol power, we get the SNR for QAM also exactly the same as we got for PAM. So, average SNR for QAM is also average signal power divided by average noise power. So, with this we have come to the conclusions of this PAM in QAM still we have to do their error performance calculation, but this we will do after we have finished with detection. And there we will compare a how this

QAM and PAM and BSK performs in terms of bit errors, but. So, far what we have been seeing in this modulation is how allowing the complex signals helps us in reducing the bandwidth inflation.

That is the one thing that we have seen in context of QAM. Then we have seen the impact of increasing M increasing M invariably increases the average symbol energy that is required and this grows with M square in case of PAM this grows as M in case of QAM. And this for very large M also grows as M square in case of consolation schemes using circular grids. For example, BSK based modulation schemes and thus from the context of this average symbol energy, the com looks to be most favourable modulation scheme at least for large values of M and that is the cases ok

The second thing that we have started looking into is the spectral efficiency of these modulation schemes. And there we have seen that the spectral efficiency increases as log 2 M for both PAM and QAM. And thus, if you want to pack in more bit rates per channel you need to use large M and that is what we do for bandwidth constrained channels ok where the bandwidth is a more sacred resource then the power. And telephone channels are the examples of that we have also looked into how can we define the spectral efficiency in terms of in terms of bits per compacts dimension.

And that is also sometimes a useful definition; and finally, we have developed the expression for relationship between SNR and E b N o and we have seen that E b N o is SNR by spectral efficiency and finally, we have seen that SNR can either be interpreted as average signal energy in one real dimension divided by average noise energy in one dimension or by average signal power divided by average noise power. Now, we have to move to the modulation schemes, which will be more useful in power constrained channels. So, communication channels are basically of 2 kinds.

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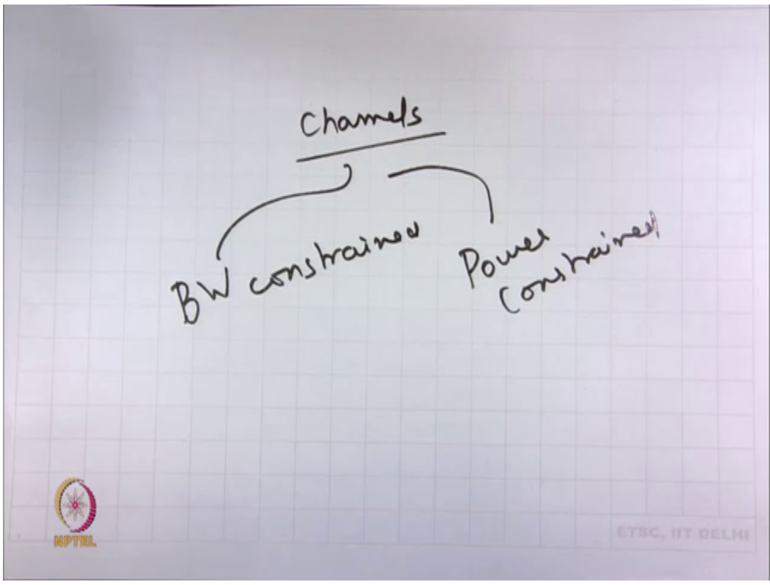

Either they are bandwidth constrained or they are power constrained, example of a power constrained channel is a satellite channel, satellite communication systems basically work on batteries. And you want to use modulation schemes which minimises the use of batteries. So, that the battery replacement does not happen to frequently you can imagine that, the cost of replacing a battery of a satellite is really an expensive operation. And therefore, for satellite channels we want to use modulation schemes which used as little power is possible and you do not care about bandwidth at all, these schemes may make a very bad use a bandwidth, but you want to have a schemes which use little power.

Where is in telephone channels which we have just seen you can pump in lot of power by having a device at a central office or at a telephone exchange and you do not really worry about power so much. Of course, if you use more power that creates nonlinearities and other issues, but primarily you do not worry about using lot of power. What you want to do is you want to extract more bit rates over that ok.

So, now the modulation schemes that we will like to focus will be the modulation schemes, which will be beneficial from the point of view power constraining channels like satellite channels. And these modulation schemes are orthogonal modulation schemes we will start with this orthogonal modulation schemes in next lecture. But let us try to understand this trade of between the power requirement and bandwidth from the point of view of Shannon’s capacity formula which is quite useful and insightful.

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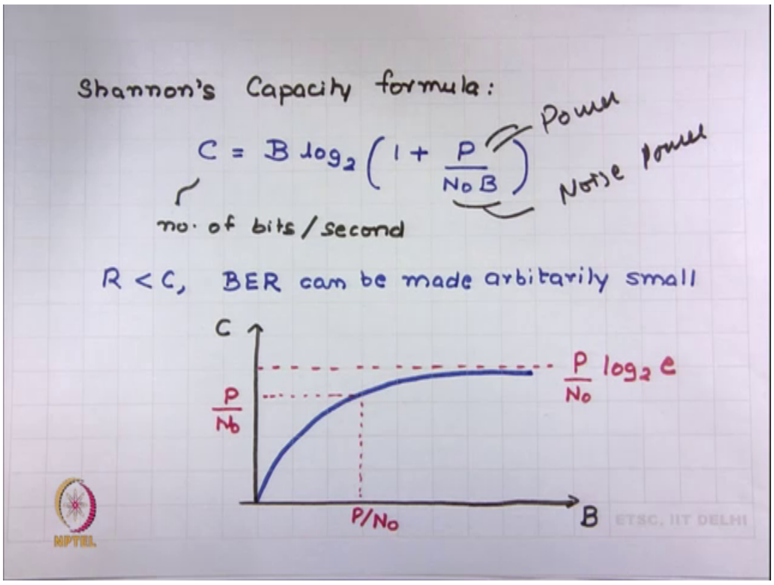

Shannon's capacity formula, as you must have seen is C times B log 2 of 1 plus P by N 0 B. So, P is the power or average signal power N 0 is average noise power ok. So, basically it is a SNR: Signal to Noise Ratio. What is B? B is the bandwidth of the channel C is the number of bits per second that you can transmit over that channel. So, Shannon's capacity formula says that if you have a transmitter which is operating at a bit rate of R.

And if this bit rare is less than C, where C is the channel capacity you can make bit error rate very small arbitrarily small ok. So, the upper limit of ray that you can have over a channel is governed by this C. And this graph compares very beautifully the relationship between channel capacity and bandwidth as you increase bandwidth channel capacity increases of course, the noise power also increases. And thus the channel capacity saturates to a value of P times log 2 e divided by N 0 for very large values of B.

So, its simply says that having infinite bandwidth does not serve you too well because then you also end up with signal to noise ratio of 0 which is not good. So, you can increase the capacity by having large P, but not 2 infinity and this is the upper limit.

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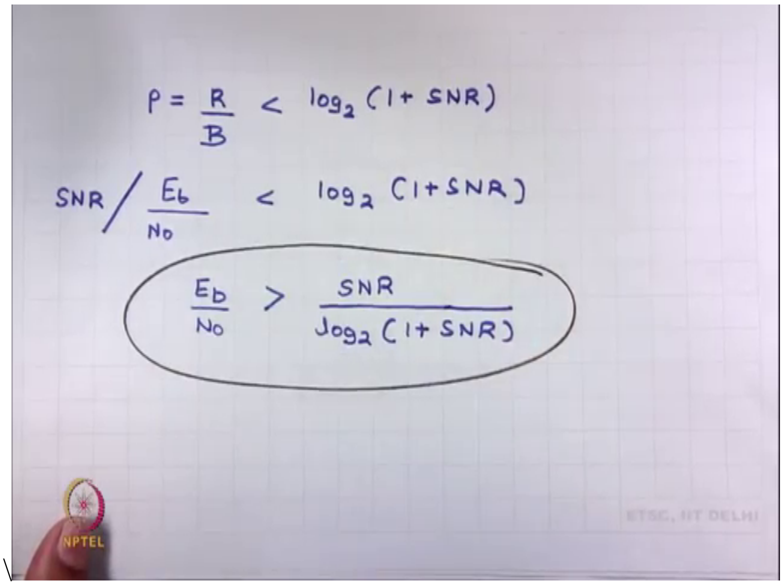

Then we see that, the spectral efficiency we already have seen is rate divided by B. So, its simply says that a spectral efficiency should be less than log 2 1 plus SNR this is this ratio signal to noise ratio. So, what we are saying is spectral efficiency should be less than log 2 1 plus SNR. And what is the spectral efficiency? Spectral efficiency is SNR divided by E b N o. So, SNR divided by E b N o should be less than log 2 1 plus SNR that simply means that E b N o should be greater than SNR divided by log 2 1 plus SNR ok. This is useful relationship; that means, the minimum E b N o that you need to have is dictated by this expression.

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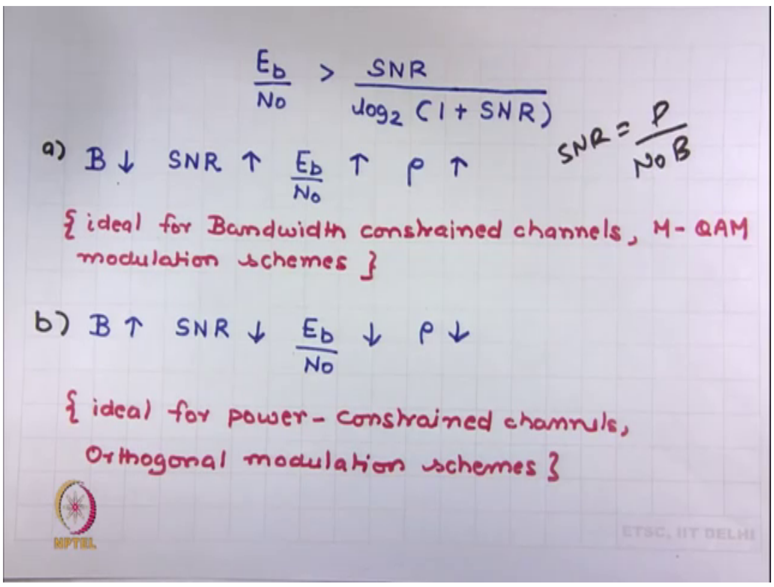

Let us see it more clearly, so we have said that E b N o should be greater than SNR divided by log 2 1 plus SNR. If we want to reduce bandwidth, what happens to SNR? SNR increases because SNR is P by N 0 B, if you reduce bandwidth SNR increases if SNR increases that means E b N o requirement increases; that means, you want to have a larger E b N o. But reducing bandwidth also increases your spectral efficiency is not it.

So, once we want to reduce bandwidth which we want to do in bandwidth constraint channels the penalty that you have to pay as you need to have large E b N o. And M QAM based modulation schemes are best to do that, because you can have a larger and larger spectral efficiency by having larger M's without increasing E S too much. And you can pack more data over a given bandwidth ok.

So, we have seen the modulation schemes which are ideal for bandwidth constraint channels. The other strategy could be to have larger B's larger B is allow for smaller SNR, SNR reduces then from this expression you can see that E b N o requirement also reduces. However, when we are having large bandwidth the spectral efficiency also reduces. And this thing we want to do over power constraint channels, and the best modulation schemes for power constraint channels that forms the substratum of next lectures are the orthogonal modulation schemes. Orthogonal modulation schemes are best suited for power constraint channels like satellite channels.

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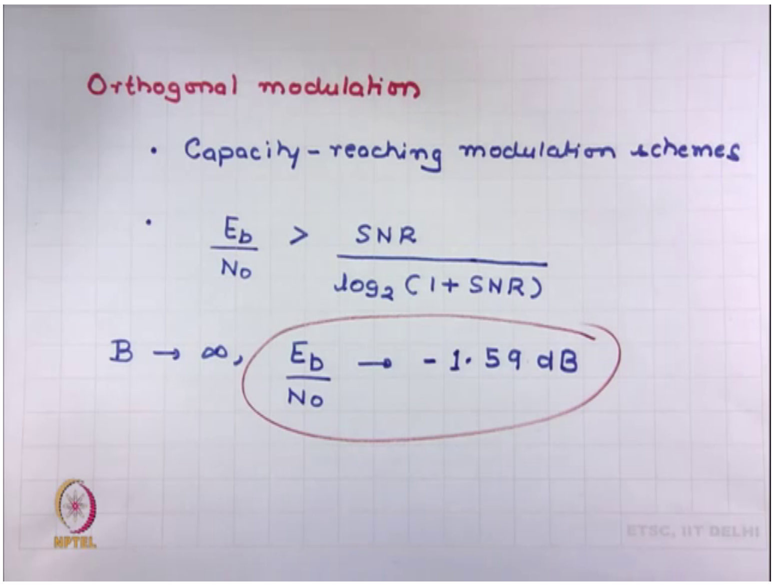

We were see that orthogonal modulation schemes are capacity reaching modulation schemes. Minimum E b N o that you would need is when bandwidth tends to infinity and at that time by substituting, in this expression only you can find that the E b N o should be larger than minus 1.59 d B and this is a golden number which you must remember. So, golden number golden rule in communication systems and error control code. That means, if you do not care about bandwidth at all if you assume that the bandwidth that is available to you is infinity, you can have a bit error rate arbitrarily small 10 in to 0 by just affording an E b N o of greater than minus 1.59 dB that is the channels capacity ok.

And we will see that orthogonal modulation schemes allows us to do so by assuming that you have infinite bandwidth, you can reduce the E b N o requirement to as small as 1.59 d B. And still get a better rate of 10 to the power minus 9 or even smaller than that as small, so that it could be considered as 0. So, we will look at this orthogonal modulation scheme in the next lecture.

Thank you.