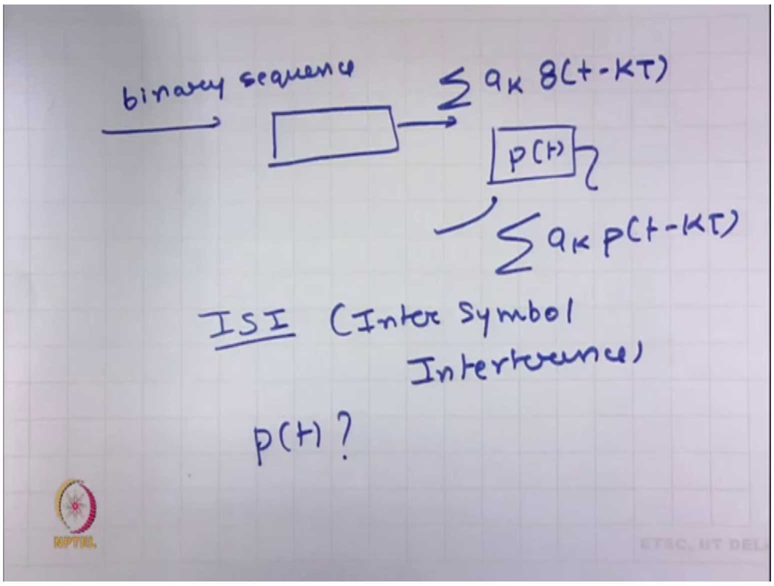
Principles of Digital Communication
Prof. Abhishek Dixit
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture - 25
Modulation
Nyquist Pulses

Good morning welcome to new lecture on Modulation and in today's lecture we will talk about issues of pulse shaping.

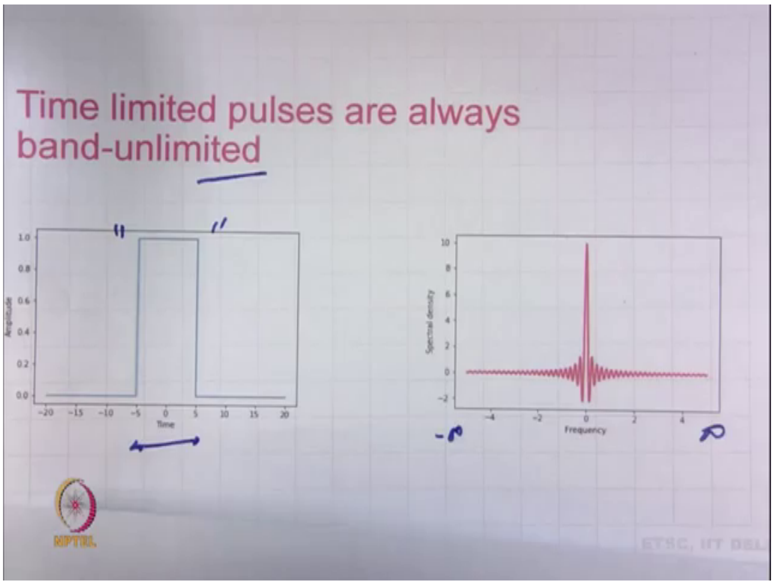
(Refer Slide Time: 00:33)



So, let us revisit the modulator and we have seen that in a modulator first we get binary sequence and this binary sequence is converted to weighted train of impulses and then this weighted train of impulse passes through a filter to get a weighted pulse train. And today we will investigate what is the good value of this p t then pulse response of this filter so, that we can avoid things like inter symbol interference.

So, this lecture will be dedicated to understand the good values of P t. So, in previous lectures we have seen how the choice of P t influence the bandwidth occupancy and today we will see that, if we want to avoid inter symbol interference you need to use little bit more bandwidth than t theoretically possible and we will also understand in the process what is this inter symbol interference ok.

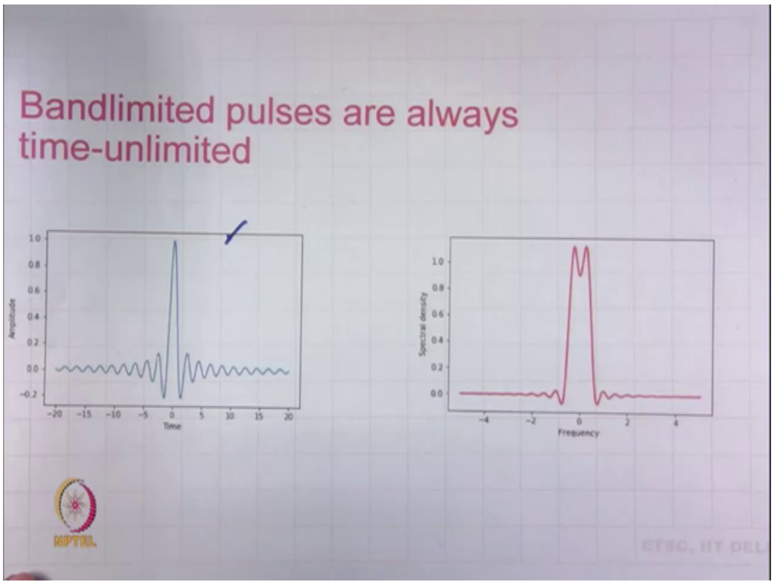
(Refer Slide Time: 01:58)



So, let us get started and the first thing that we would see is really a recap of few concepts that we have already seen before. And the first idea that we have seen in one of the lectures in the first week is that this time limited pulses.

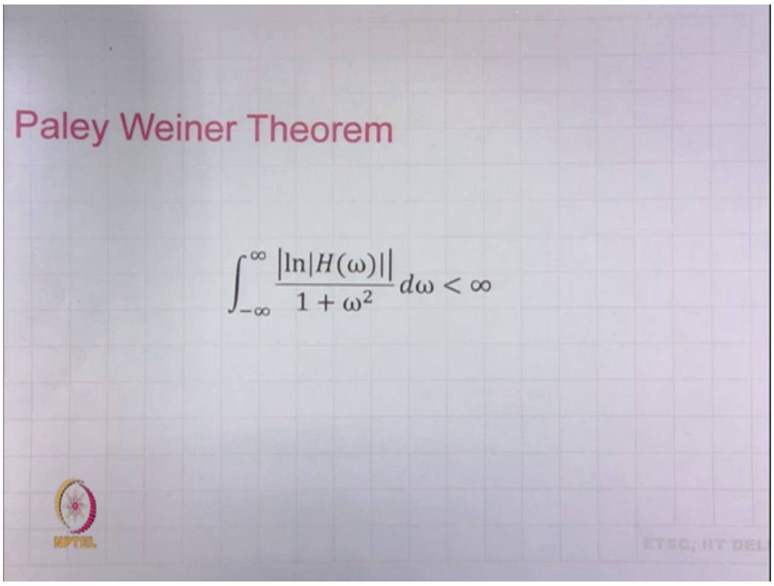
So, this pulse is a time limited pulse, that means it has a duration only for 10 seconds and then there is no amplitude in the pulse right. So, this is a time limited pulse, it can finds itself to this duration. Now if you take a time limited pulse its spectrum would spend from minus infinity to plus infinity; that means, the time limited pulses are always band unlimited right, this we have seen before as well. And we had also seen previously that if you try to band limit a pulse for example, this spectrum span from minus infinity to plus infinity if I try to band limit this is spectrum, what would happen is that this pulse will spread in time.

(Refer Slide Time: 02:52)



Because band limited pulses are always time unlimited, you cannot have the best of both words. So, for example, you cannot have a pulse which is time limited, at the same time band width limited that is not possible. So, if we try to limit in bandwidth this pulse will spell in time.

(Refer Slide Time: 03:19)

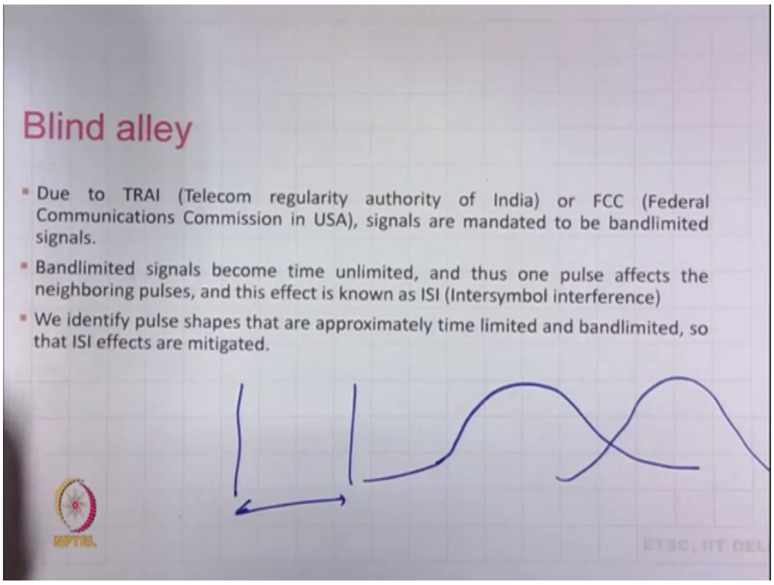


We can also understand this from Paley Weiner theorem, as we have already seen that the physically realizable systems has to be causal systems because only causal systems are physically realizable right and why are only causal systems are physically realizable?

Because causal systems are non anticipatory systems and physically realizable systems would be non anticipatory systems. And then we have also seen that if the system is a causal system is frequency response should satisfy Paley Weiner theorem.

That means that we have seen I think lecture 6 that if the frequency response has to satisfy Paley Weiner theorem, it simply means that frequency response cannot be 0 for continuous range of frequencies right. And if the frequency response cannot be 0 for a continuous range of frequency, it simply means that the signal or the system has to be band unlimited right. So, this is also what we have seen that physically realizable systems are band unlimited systems. So, this is like the problem right.

(Refer Slide Time: 04:34)

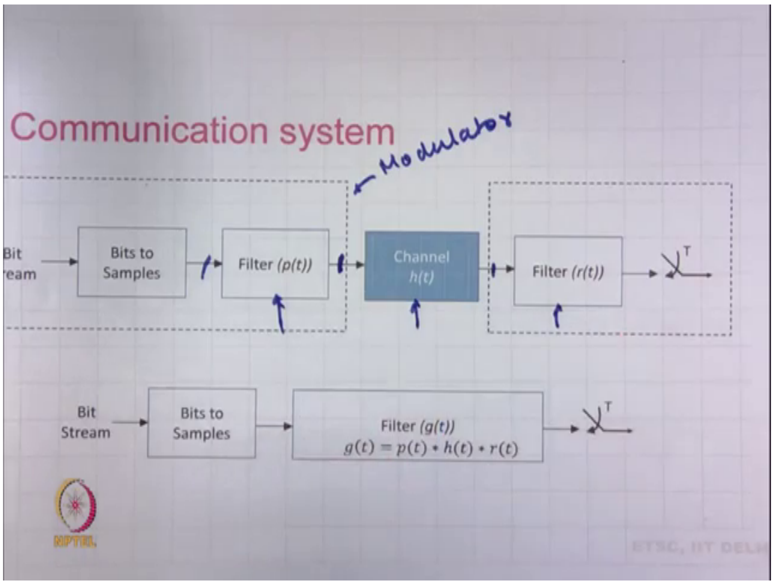


So, we get into kind of Blind alley and this blind alley is that these regularity authorities like TRAI in India or FCC in USA they mandated the signals to be bandlimited signals right Why do we want to have bandlimited signal? So, that your signal should not influence the signal of the another operator right. So, every operator is given a frequency band to operate and the signals of those operator should be confined within that frequency band.

So, these regularity authorities of India mandates the signal to be band limited, but as soon as you try to make signal band limited, these signals will become time unlimited. So, if I want to see the signal in time domain there will be time unlimited and if they become time unlimited what would happen? This pulse will interfere with this pulse and this causes what is known as inter symbol interference. And we have to deal with this issue of inter symbol interference.

So, in this lecture today we will see and identify the pulse shapes that are approximately time limited and band limited. So, that these ISI effects or Inter Symbol Interference effects are mitigated. So, in short what we have said so, far is that because of regulations, you have the signals have to be band limited and as soon as you want to make them band limited or approximately band limite, they also spell time and when they spell in time they begin to interfere with other pulses and causing what is known as inter symbol interference.

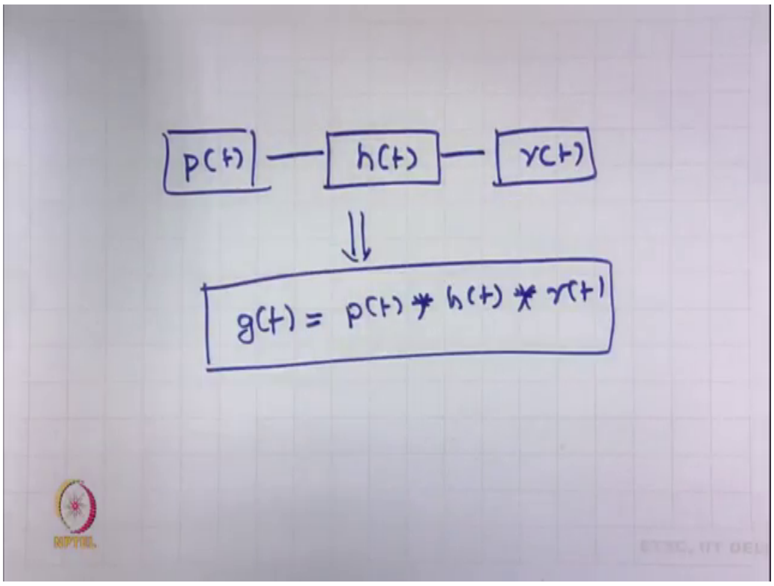
(Refer Slide Time: 06:12)



So, to understand this issue clearly, let us look at the communication system. So, we have here in the modulator and we have the bit stream; bit stream is converted to samples or a weighted impulse train and then you pass it through a filter. Also at this point we have frequency up converter, but we do not want to go to the pass band domain because the space band signal has the complete information right. So, we have already seen what happens when you go from basement to pass band domain, we are not having this frequency up convertor or frequency down converter at this point because those issues have been address separately right.

Here we just want to focus on baseband right. So, that is a modulator we have seen it before and we have also seen the typical design of a receiver; receiver has a filter with an impulse response r t followed by a sampler, which is sampling at an integer multiples of T. So, this is typically the design of a communication system. And first we are assuming all this filters and the channel and this filter to be linear time invariant system.

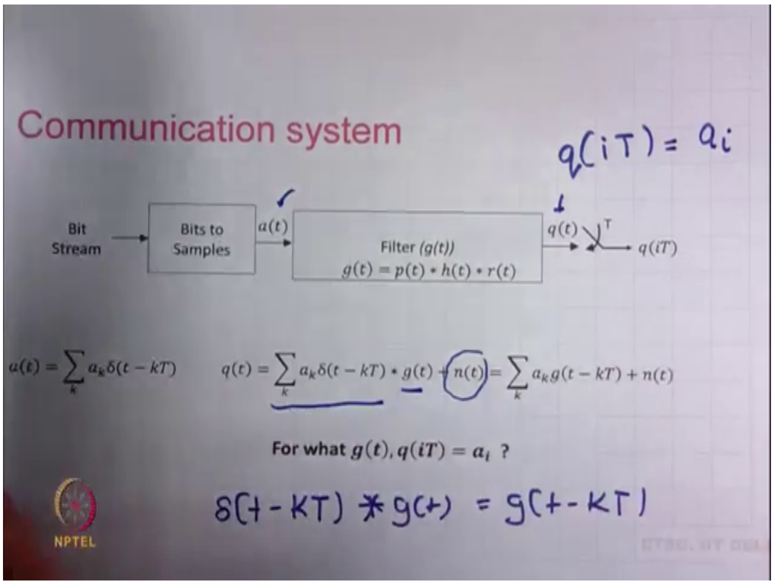
(Refer Slide Time: 07:53)



And you must have see it from the course in signal sense systems, that if you have 3 LTI systems with impulse response p t, h t and r t these 3 systems can be converted into one system, which has got an impulse response formed by the convolution are these 3 impulse responses ok.

So, namely g t which is the impulse response of the equivalent system is simply p t convolution h t convolution r t ok. So, if the systems are linear time invariant systems, you can replace this 3 systems with one in a time invariant system which is got an impulse response the convolution of the impulse responses of these 3 systems. So, here we are using the same idea that, we have these 3 systems this filter has got an impulse response of p t channel also we are assuming to be linear time invariant system and then we have a filter with an impulse response of r t at the receiver, these 3 systems can be replaced by one filter which has got an impulse response p t convolution h t convolution r t. So, this is typically the communication system.

(Refer Slide Time: 09:23)

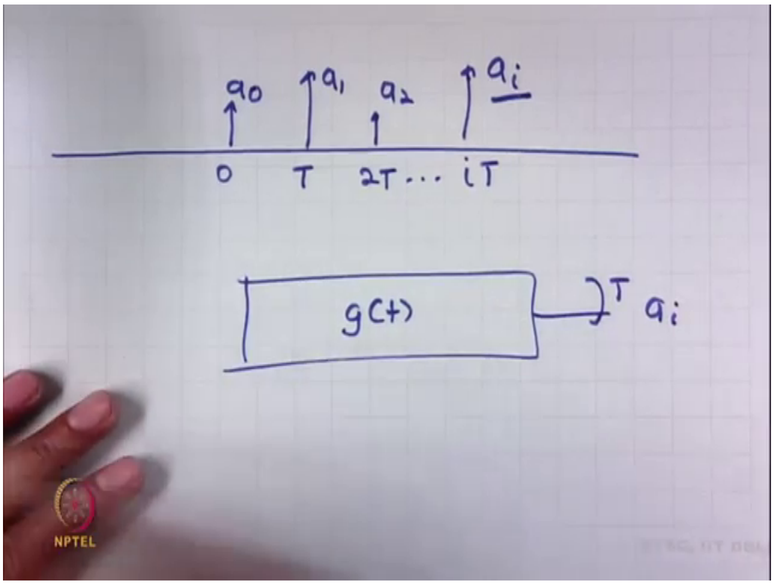


Then a question that we are asking is, if let us say at the input of this filter we have got a signal a t, where a t is given by this. So, its nothing, but it is a weighted impulse train. At the output of the filter this filter let us assume that we have a signal q t, where q t is given by input convolution g t. So, input is this, this is the input convolution with g t. So, this is the output of this filter and we also assume that some noise has added in at the output. So, we have the signal convolution with impulse response of the filter plus some noise addition right.

We have seen that noise is additive, so, it simply adds at the receiver. Now you know that till the t minus k T convolution with g t is simply g t minus k T ok. So, this is convolved with this is not a function of time. So, a ks are constant with respect to time. So, this simply becomes a k times g t minus k t summation for all possible values of k plus n t.

And the question that we want to ask is if I sample this output let us say at i T say if we sample this q t at i T we get q of i T and we want this q of i T to be same as a i why is this so? Because at i T time instance we are transmitting a i the symbol with the value of a i and at the receiver what we want to receive is a i faithfully ok. So, let me draw this.

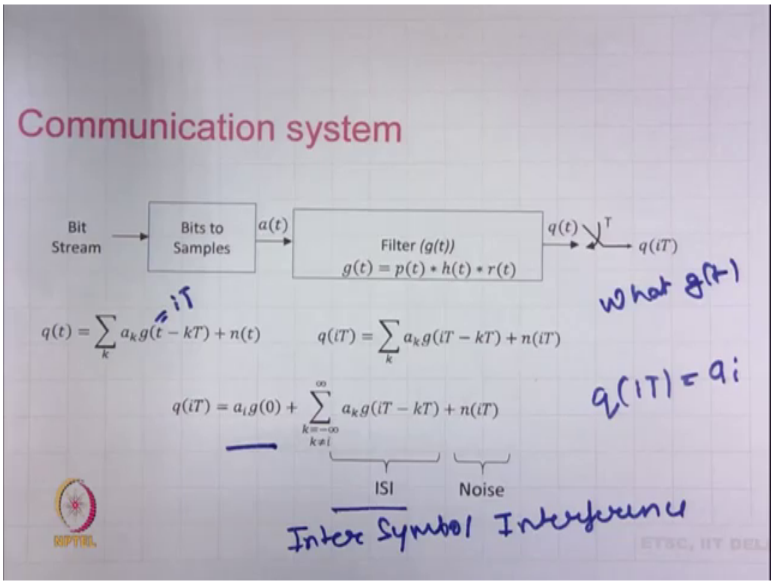
(Refer Slide Time: 11:36)



So, let us say at the input of this filter a t; a t is nothing, but it is a weighted impulse train. So, let us 0 T 2 T and let us say i T and it has got the weight a 0, a 1, a 2 and a i. So, at i T time instance we are sending an impulse with the weight a i and at the output of the filter g t after sampler what we want to receive is, a i faithfully right that is the job of a communication system whatever you are transmitting at the output of a filter and the sampler you want to receive faithfully that.

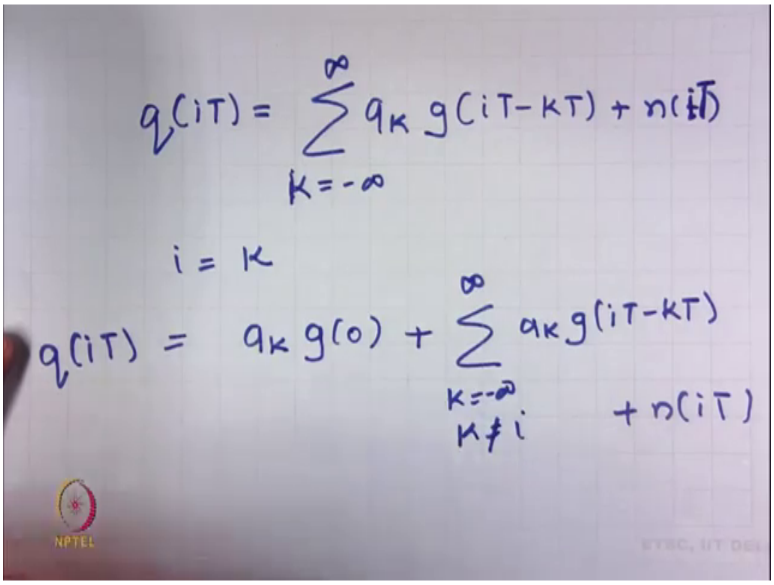
So, that is the question that we are asking we want that q i T should be same as a i and we are asking other question that for what g t for what impulse response this q i T is same as a i.

(Refer Slide Time: 12:48)



And let us see this. So, we have already said that q t is a k times g t minus k T summation for all values of k and then there was some noise addition. Now we want to sample this output at time instances with integer multiples of T. So, I get q i T is a k. So, this t is replaced with i T and this t is also replaced with i T. So, this is the q i T this is the values of the samples of this output q t. If we look this carefully let me first do it separately.

(Refer Slide Time: 13:33)

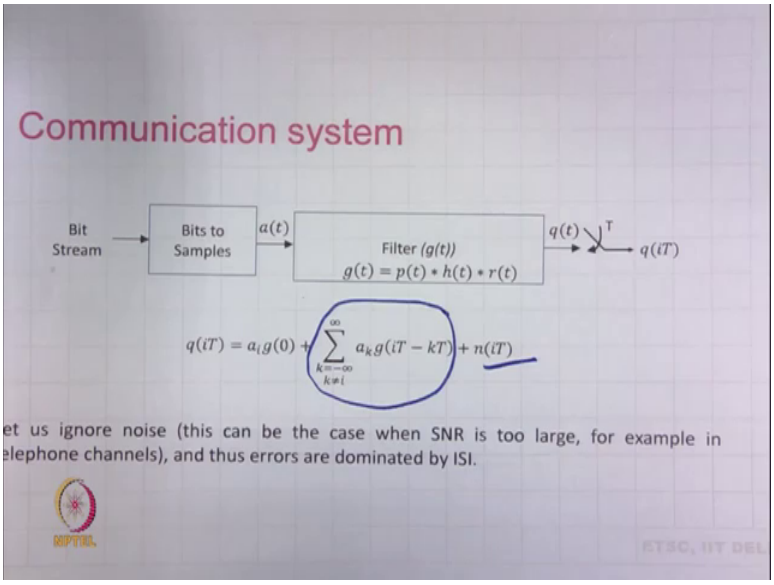


So, what we are saying is q i T is summation a k g i T minus k T plus n t. K takes n all values from minus infinity to plus infinity this is also i T. Now, let me assume that i is same as k or k is same as i. So, that is a k times g of 0 and I can collect all other terms other than when k is not same as i, so, this is q i T.

So, this summation contains all terms going from minus infinity to plus infinity what I am doing is, I am just pulling out the term corresponding to the situation when k is same as i. So, that term is a k times g of 0 and all other terms are contained in the summation except when k is same as i and then also we have n i T. So, this is what we have. So, q i T is represented in this term plus this term and noise. So, what we wanted is q i T should be same as a i.

So, this is the information that we want and this is inter symbol interference. So, this is inter symbol interference and this term corresponds to the noise and we want that this inter symbol interference to be 0. So, that is the objective because what we want is q i T should be just same as a i nothing else there should be no contribution from this summation this summation corresponds to inter symbol interference. So, what is the question that is running on? The question is what is the value of g t? So, what is the value of g t for which q i T is same as a i. So, that is the question that we are asking.

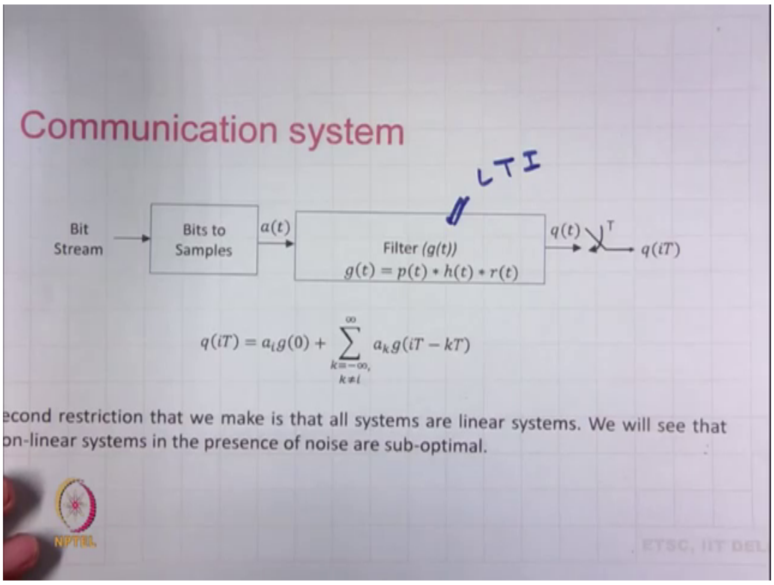
(Refer Slide Time: 16:18)



But we will ask this question making two assumptions, first assumption that we make now is that noise is 0. We do not want to study at this moment impact of noise, we assume that noise is 0. So, that we just handle inter symbol interference at this moment ok. And this will be the case when the signal to noise ratio is very large and signal to noise ratio is large in channels like telephone channels.

So, this is fairly a reasonable assumption to make. And if signal to noise ratio is pretty large; that means, this noise power is a small or that simply means that noise can be ignored. So, in the absence of noise most errors happens because of this inter symbol interference and that is what we want to focus at this moment.

(Refer Slide Time: 17:17)

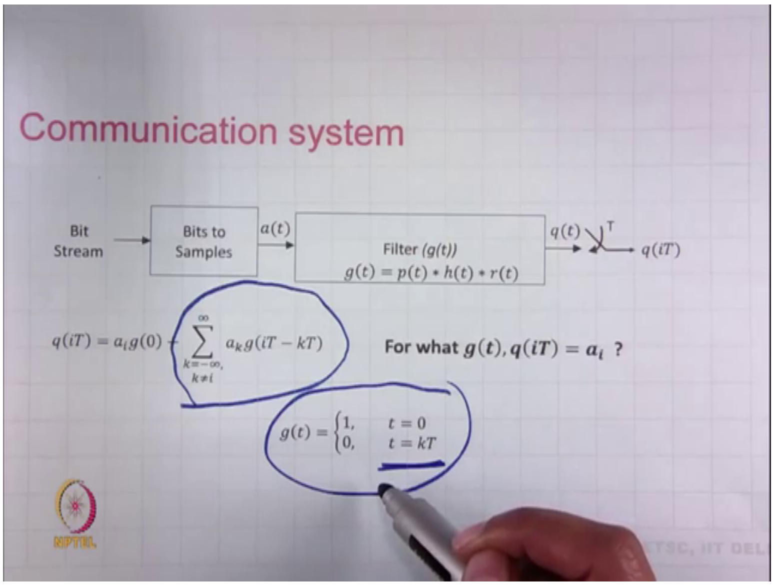


The second thing that we are assuming is that this filter g t is a linear time invariant filter. And before we have assume that this p t h t and r t all these are linear time invariant systems. Can we have a filter which is not linear time invariant or can we have a system which is a non-linear system? Could we have use the systems replacing these filters with impulse response of p t and impulse response r t and the channel with an impulse response of h t with non-linear systems, can then we not get q i T same as a i? Why we are restricting our self to linear time invariant systems only?

The answer is that if we assume non-linear systems, non-linear systems are not optimal in the presence of noise ok. So, when we are restricting our self to linear systems actually we are not losing out anything right. Linear systems are well behaved even in the presence of noise and this is a good assumption to make first it makes analysis simple and second is linear systems are well behaved and optimal in the presence of noise and that is why we are asking this question that for what g t q i T is same is a i by assuming all systems to be linear time invariant systems only ok.

So, these are the two assumptions that we have made. We have made that all systems or channel is LTI system, the receiver filter is LTI system, the transmitter filter is an LTI system as second assumption that we have made is that the noise is ignorable ok. So, signal to noise ratio is pretty large ok.

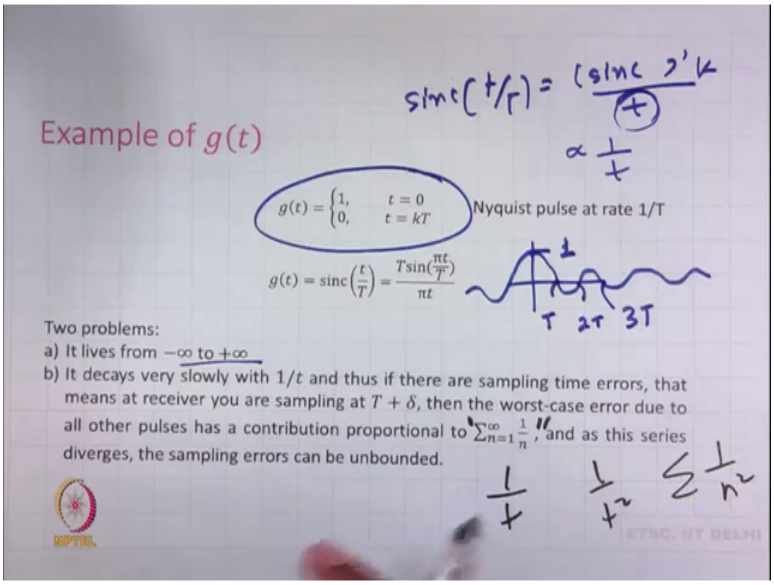
(Refer Slide Time: 19:19)



So, let us go back and look at this question again. So, we have been asking this question several times. So, we are saying q i T is this. So, now, we do not have any component corresponding to noise and if you ask for what g t q i T is same as a i answer is pretty simple if you look at this. What we want g of 0 to be g of 0 should be 1 right.

That means g t should be 1 for t equals to 0 and what we want here? We want that g t should be 0 at t equals to k T. So, if g t is 0 for t equals to k T, there will be no contribution from this term and inter symbol interference will be 0. So, if I use a filter whose impulse response satisfies these two condition then what we can see is that the inter symbol interference will be 0 ok.

(Refer Slide Time: 20:19)



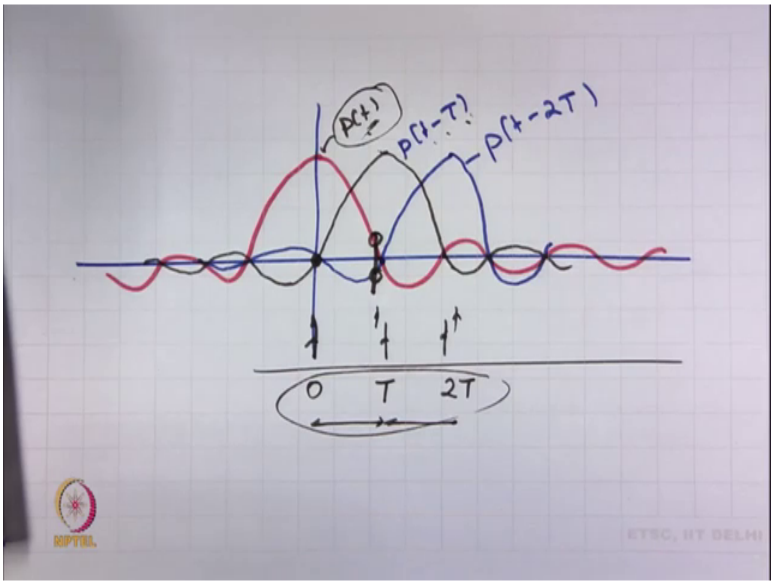
So, what are the examples of g t? A sinc function. So, for example, we know that a sinc function sinc t by T is a function which has a t equals to 0 value of 1 and at all integer multiples of t it has 0. So, sinc function is a function that satisfies Nyquist criteria. And this is in fact, the Nyquist criteria even though we have not mentioned it.

So, this is Nyquist criteria that g t should be 1 for t equals to 0 and for t equals to k T g t should be 0 and if you have a function which satisfies this than inter symbol interference is 0 and this function is known as Nyquist pulse at rate 1 by T and sinc function is an example of the Nyquist pulse because it satisfies this condition, so, its easy.

So, what is the problem with this pulse? The problem with a sinc pulses that the sinc pulse extends from minus infinity to plus infinity and we have seen that this sinc pulse sinc t by T which can be approximated as some sine by t. So, we are not writing on the exact values, the only thing that we need to appreciate is that sinc t by T translates to some constant time sine of something divided by t this is an oscillatory function.

It goes up and down and so, for large values of t, this function decays as 1 by t and this is a slow decay. And because of this slow decay in practical situations this is not a good function to use even though theoretically its fine. So, what we are saying is the sinc function decays as 1 by t and thus for practical applications, this is not a nice function to use because what would happen is in practical situations let us say we are using a sinc pulse.

(Refer Slide Time: 22:52)



So, one sinc pulse is like this and let us use another. So, this is p of t and then let us draw p of t minus t which would be something like this. Let me draw one more something like this something like this. So, this is p t this is p t minus T, this is p t minus 2 T. And where we are sampling? We are sampling at these instances. So, we are sampling here we are sampling here and we are sampling here and so on so forth.

So, this is 0 T 2 T and so on so forth. So, now, what you see is that when you sample this you just have contribution from p t. The contribution from p t minus t and p t minus 2 T is 0. When you are sampling at t you just have contribution from p t minus t, but contribution from p t and p t minus 2 T is 0 and hence this looks like its avoiding inter symbol interference.

But what happens in practices that, your clock does not sample exactly at the duration of t seconds right this pulse is created at the transmitter this clock is at the receiver. So, there might be a mismatch between this pulse duration of T and the sampling duration of T, because this clock is produced at the receiver. Moreover for the same clock there is a always some jitter. So, sampling instances not always perfectly happen at the duration of T. So, there might be jitter in the clock and sometimes you may sample here sometimes you may sample here. And once there is a clock jitter what happens if you sample at this time instance, you begin to have contribution from this pulse and this pulse and many other pulses. And in the presence of this clock jitter this might become pretty large value and hence it will create an error.

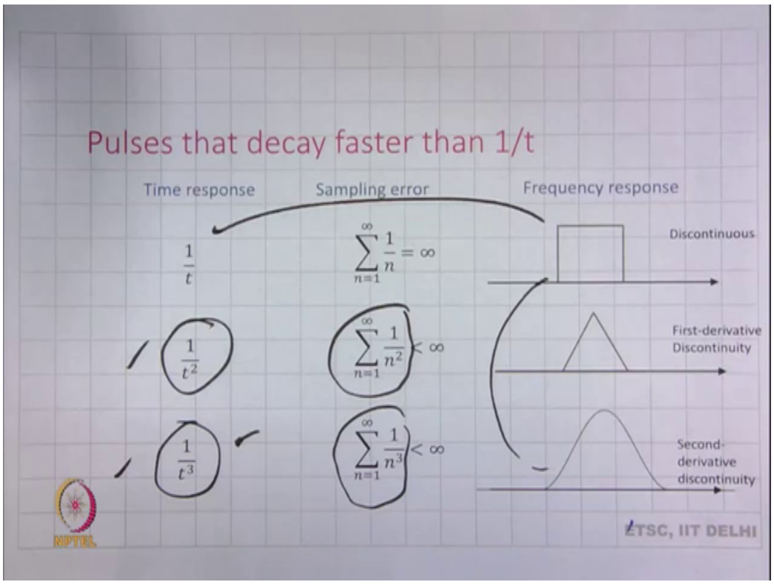
So, what we are saying is if you can have a clock, which can perfectly sample at a duration of t seconds and this duration of t seconds is match to the duration of the sinc pulses produced by the modulator, then there is no problem right, but impact is what happens is that there is clock jitter. So, the sampling time instances are not perfect, they do not always happen at integer multiples of t, but there is some (Refer Time: 26:18) sometimes it happens, earlier sometimes it happens later.

And once there is a clock jitter whenever you sample this pulses, there might be contributions from (Refer Time: 26:29) pulses. And this contribution might sum up and may become a pretty large number and it might create an error ok. So, that is the problem with the sinc pulse. So, what we are saying is let us go back. So, because the sinc pulse decays too slowly, it has a decay of 1 by t when you take into account the sampling time errors, that say instead of sampling at t which is fine.

You begin the sample at T plus delta, where delta corresponds to the jitter in the clock then you begin to collects samples from all pulses then you begin to have contributions from all pulses and this contribution might add up and may become unbounded. In fact, it can be shown that the worst case error for a signal which decays as 1 by t is given by this series. So, if you pulse decays as 1 by t, this is the contribution from all other pulses and this series diverges and does the sampling errors can be unbounded. So, what do we want is, we want the signal to decay faster we wanted to decay with 1 by t square because when it decays as 1 by t square, you can see that the worst case error due to other pulses has a contribution proportional to this series.

And this series converges and thus if you want pulses which decays faster than the sinc pulse. So, sinc pulse though theoretically is good practically it will have issues right.

(Refer Slide Time: 28:26)



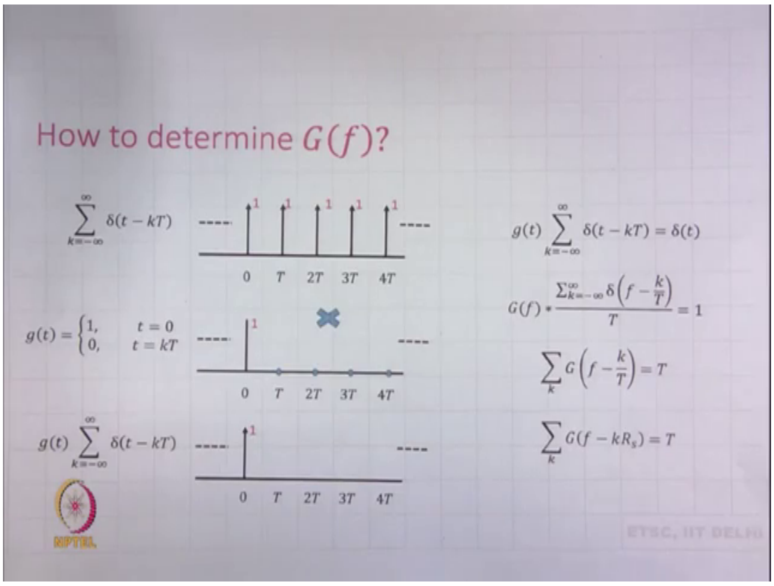
So, practically what we want is, we want the pulses that decay faster than 1 by t. So, we want them to decay with 1 by t square or 1 by t cube and so on so forth. And you can show that easily that the sampling error when you have a pulse with this decay is given by this series and this series converges the sampling error for the pulses which has this kind of decay is given by this series and this series also converges. So, we want pulses that decay like this or like this.

In terms of frequency response what do we want? So, you can see also from basic course sinc pulses systems, that if you have a frequency spectrum like if its a discontinuous spectrum whenever you have these discontinuities, the pulse in time domain decays as 1 by t. If you have first derivative discontinuity, so, if you differentiate this signal you end up with signal like this is not it? So, when you have a first derivative discontinuity or slope discontinuity in the spectrum, this translates in the time domain to a pulse or to a signal which decays as 1 by t square. If you have a frequency response which has second derivative discontinuity. So, if you differentiate it twice, then you get to a signal like this.

Then in time domain this will correspond to a pulse which decays by 1 by t cube. So, you can understand the examples of good pulse from good time domain, where you just have to see whether the decay is proportional to 1 by t square or 1 by t cube or higher. If you want to understand this good pulses in terms of frequency response, you can also understand this by ensuring that the frequency response should have either first derivative discontinuity or should have second derivative discontinuity, it should not be discontinuous function discontinuous function is bad ok.

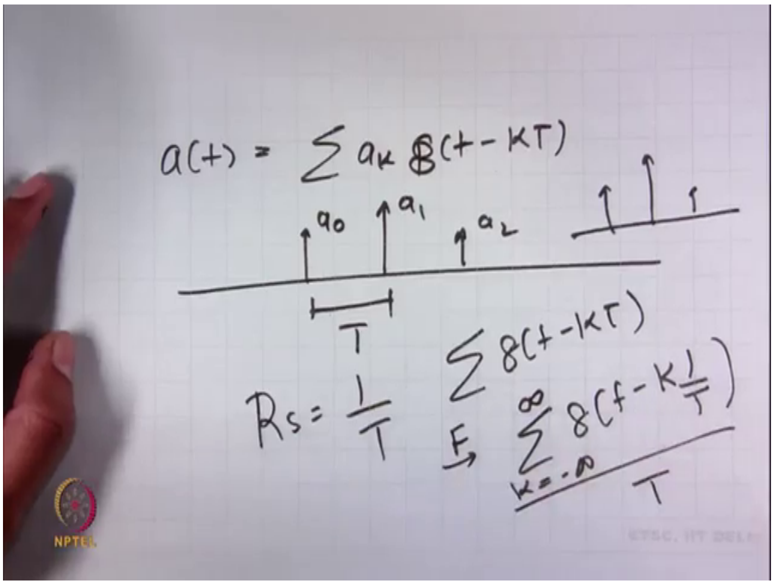
So, we want the pulses which satisfies either this condition or this condition or even better conditions in terms of inter symbol interference and reducing sampling errors and that is it should have second derivative discontinuity ok.

(Refer Slide Time: 31:11)



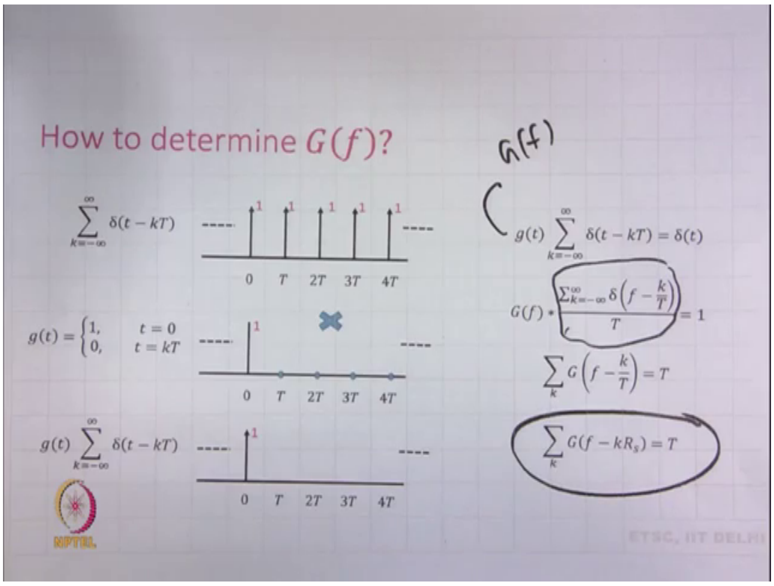
So, now we have to determine the frequency response of the good pulse. So, to determine G f corresponding to good Nyquist pulses, we again make use of this impulse train we taken this impulse train. This impulse train has the impulses which have separated by duration of T seconds, where T corresponds to the rate at which you are producing those aks.

(Refer Slide Time: 31:51)



So, if you see we have this a t which is a k p t minus oh sorry delta t minus k T. So, what is this? So, you have impulses which were separated by a duration of T seconds and they carry the weights a 0 a 1 a 2 and so on so forth and what this a 0s a 1 and a 2? If we are in the regime of digital communication these are quantized real numbers and actually these are symbols. So, we had a symbol rate R s which is 1 by T. So, at every T seconds your modulate is spits out a symbol. So, symbol rate is 1 by T and this a naught a 1 and a twos are symbols quantized real numbers.

(Refer Slide Time: 32:49)



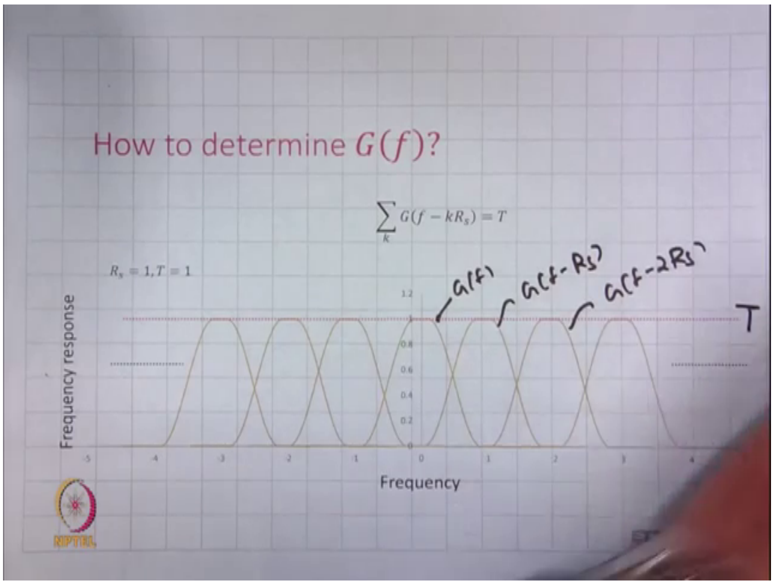
So, this T is the same T at which we were producing symbols and we know that what is g t? The two conditions for g t that must be satisfied as it should be 1 and 0 and for all this timing instances which corresponds to integer multiples of t, g t should be 0.

So, if I multiply this impulse train with g t, what I end up with is an impulse. So, this is what I am saying g t multiplied with an impulse train is nothing, but an impulse. Now we know that multiplication in time domain is convolution in frequency domain. So, if g t has a frequency response G f what is the frequency response of this impulse train? Is an impulse train given by this relationship. So, this you must know from before and this we have also derived in one of the lectures. If you have this impulse train is Fourier transform is nothing, but delta f minus k 1 by T divided by T where k goes from minus infinity to infinity. So, an impulse train has frequency response which is also an impulse train.

Is impulse train is also known as picket fence right. So, because you have this pickets, pickets are soldiers. So, these impulses looks like soldiers pickets. So, we have this impulse train which is also known as picket fence has a Fourier transform which is also a picket fence. So, leisurely we say that picket fence is Fourier transform which is picket fence anyways. So, we have an impulse train the Fourier transform of this impulse train is also an impulse train. Of course, it has a frequency of 1 by T and then there is an extra factor of T in here and what is the Fourier transform of delta t is 1? So, what we can say is G f convolution this thing should be same as the Fourier transform of delta t which is 1.

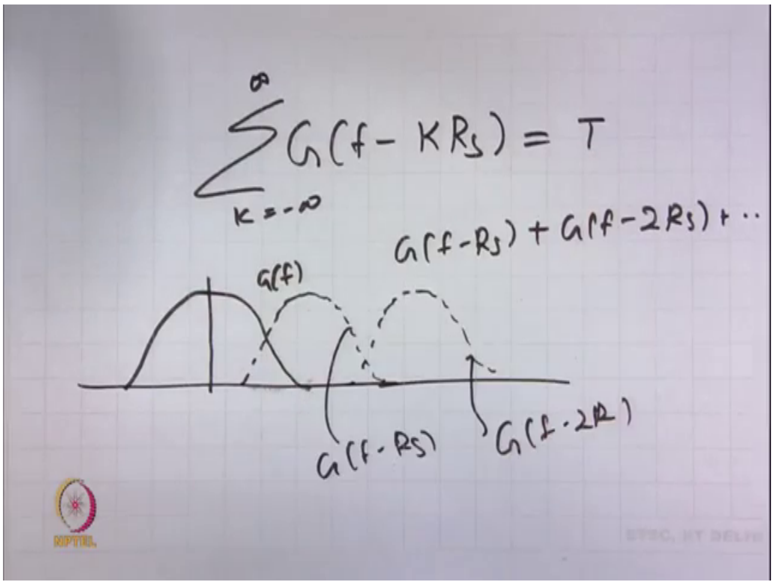
And we know that G f convolution with impulse gives us this. So, I have taken T to this side. So, we have seen this several times that if you take a signal convert this with an impulse you just get the same signal, but it is shifted to the point where impulse has an effect. So, delta is simply replaced by G that is it; that is the fact of convolution of a signal with an impulse and then I write 1 by T as Rs; where R s is the symbol rate, so, we get this condition. So, the frequency response of a Nyquist pulse must satisfy this condition.

(Refer Slide Time: 36:16)



So, let us look at this what does this mean? This simply means that.

(Refer Slide Time: 36:24)



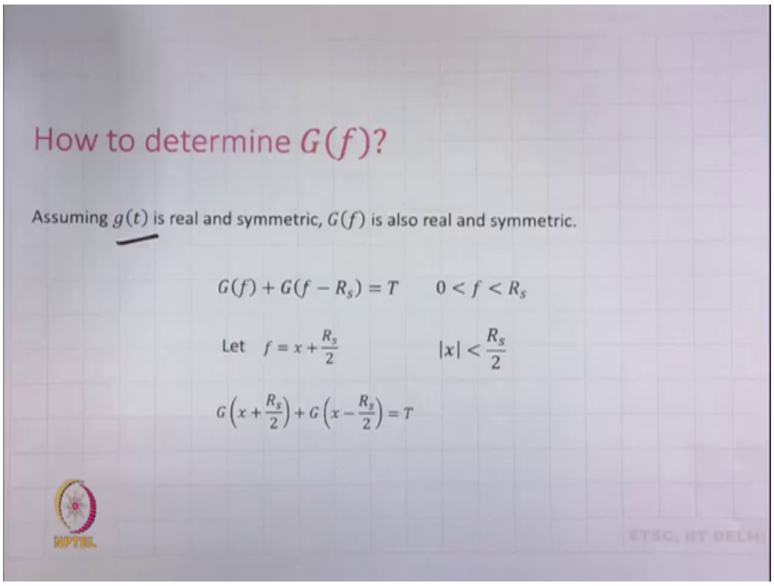
So, let me write this. So, if I have G f minus kR s is T; k going from minus infinity to plus infinity it simply means that if I have a G f let us assume some G f. So, let us say G f and then I have G f minus R s plus G f minus 2 R s and so on so forth all these terms. So, if this is G f, G f minus R s would be let us say this G f minus 2 R s let us say would be this. So, let us say this is G f minus R s this is G f minus 2 R s and so on so forth. So, I have a spectrum and you have various shifted spectrums with the spectrums are shifted at

R s 2 R s 3 R s 4 R s and so on so forth and you add up all these spectrums then what you should get is some constant.

So, that is the Nyquist criteria in frequency domain. So, if G f is the frequency response of the Nyquist pulse, you take that spectrum you shift this spectrum at integer multiple of sample rates then you sum of all these spectrums what you should get is a flat constant. So, that is this picture is saying we have this G f. So, this is G f minus R s and so on so forth G f minus 2 R s.

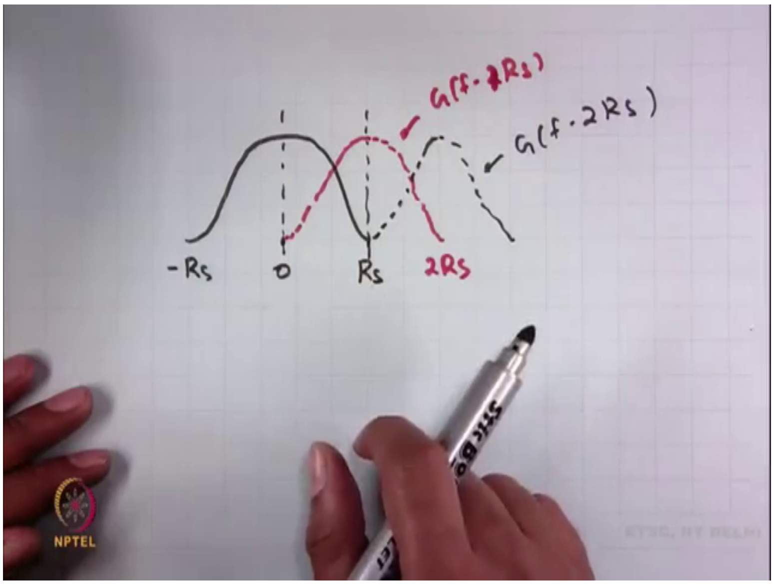
So, we have this main spectrum we shift this by R s this by 2 R s and so on so forth then I add all of them I should get a constant. So, T; in this case for this diagram what I have assumed is R s as 1 and T is also 1 ok. So, that is the interpretation of this equation and this should help me in identifying what are possible solutions for this G f.

(Refer Slide Time: 38:44)



So, let us start by making some assumptions. The first assumption that we make is g t is real and symmetric that would imply that G f is also real and symmetric. If a time domain signal is real and symmetric is frequency spectrum is also real and symmetric and if you want to look at this equation for frequency is between 0 and R s what happens? So, let us make diagram for this.

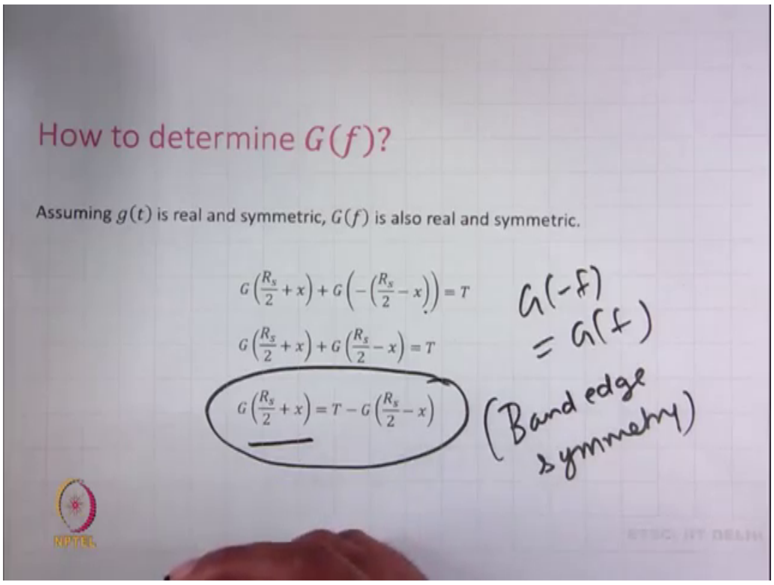
(Refer Slide Time: 39:28)



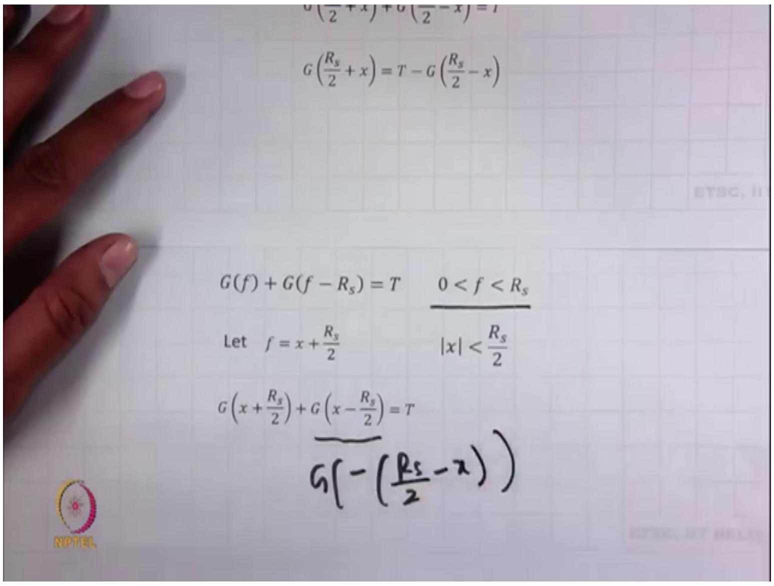
So, if we have G f which let us assume is confined between minus R s and plus R s and we take a shifted version of that like this, let us assume that this is G f minus 2 R s this is 2 R s, so, sorry G f minus R s. And let us then consider third spectrum which is G f minus 2 R s, then what you can see is that G f minus 2 R s and spectrum which are shifted beyond this 2 R s, would have no impact information for frequency range between 0 and R s.

Because they are shifted at 2 R s and if G f is limited between minus R s and R s, G f minus 2 R s and spectrum beyond that spectrum shifted beyond 2 R s would have no impact in the summation. So, between this frequency range; that means, the frequency between 0 and R s, I will have the impact of G f and G f minus R s. Let us now assume that f is x plus R s by 2. So, there is a change of variable and assume that mod of x is less than R s by 2 so, that this s satisfied. So, when I assume that f is x plus R s by 2, I simply substitute that in here and I get this.

(Refer Slide Time: 41:38)

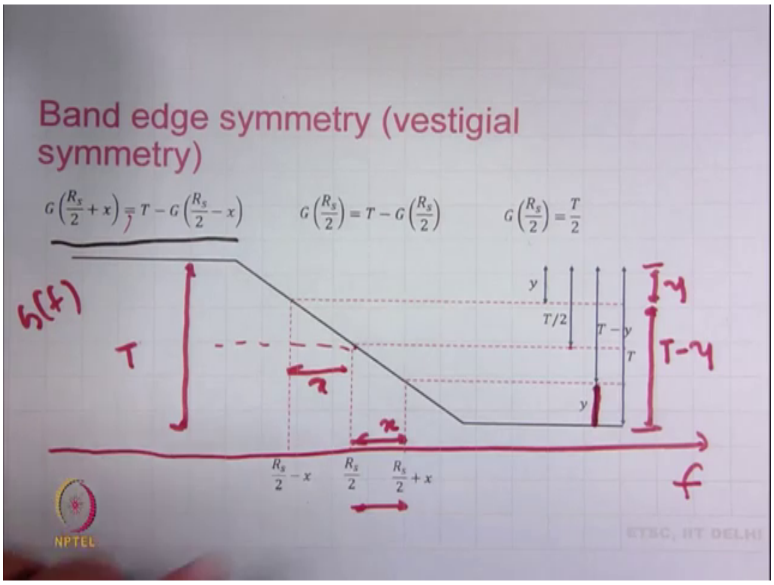


(Refer Slide Time: 41:44)



And then what we can do is, we can just write this as G of minus R s by 2 minus x. So, from this we have got this question and because G f is symmetric, G of minus f is same as G of f. So, this is nothing, but this. So, from this we can get G of R s by 2 plus x is T minus G of R s by 2 minus x and this condition is known as band edge symmetry condition or vestigial symmetry condition.

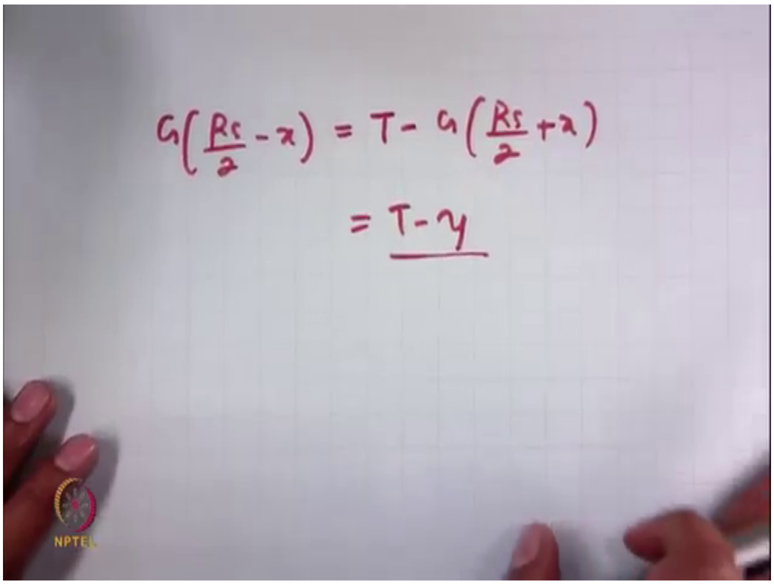
(Refer Slide Time: 42:36)



That means the spectrum of the Nyquist pulse must satisfy this band edge symmetry or vestigial symmetry by which I mean that this should be satisfied. And the first thing that you can understand is just let us assume x to be 0; if x is 0 then g of R s by 2 should be same as T minus G of R s by 2. From this we get G of R s by 2 should be same as T by 2. So, that is what we have got.

So, let us assume that this is T this is what we have plotting is G f. So, let us assume that is T. So, on this axis I have f. So, r frequency corresponding to R s by 2 I should have a value of T by 2. So, this value is T by 2 and as I shift x from this point this distance is x. So, let us assume that the value of G R s by 2 plus x is y, so, this is y. So, what should be the value of G R s by 2 minus x let us work this out.

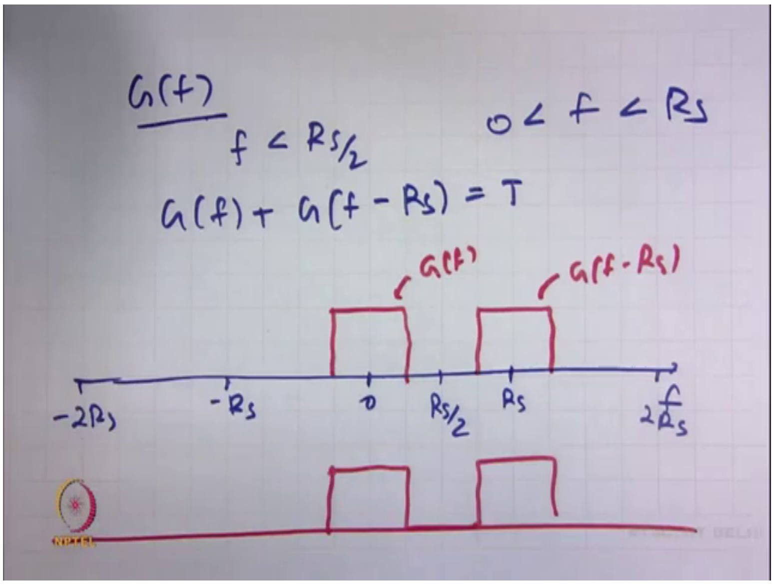
(Refer Slide Time: 44:00)



So, G R s by 2 minus x should be nothing, but T minus G R s by 2 plus x and this we have said is y. So, G R s by 2 minus x should T minus y. So, this value should be T minus y and this total duration is T right, so, this is T. So, what should be this? This should be y and this is the idea behind vestigial symmetry if you move x from the centre of frequency. So, centre of frequency I mean R s by 2, where the value is exactly half of this total value. So, if we move x to this side and x to this side if this value is y this value is also y. So, G f should have a spectral which should have this band edge symmetry ok.

So, this is the solution following from very simple ideas right and what is the idea? Idea is simply that g t should be a function which should be 1 at t equals to 0 and it should be 0 at the sampling instances, which are integer multiples of t ok. So, we now know what or how should the spectrum of Nyquist pulse should look like. There are one to small points that are remaining about this spectrum G f, let us look at this.

(Refer Slide Time: 45:46)

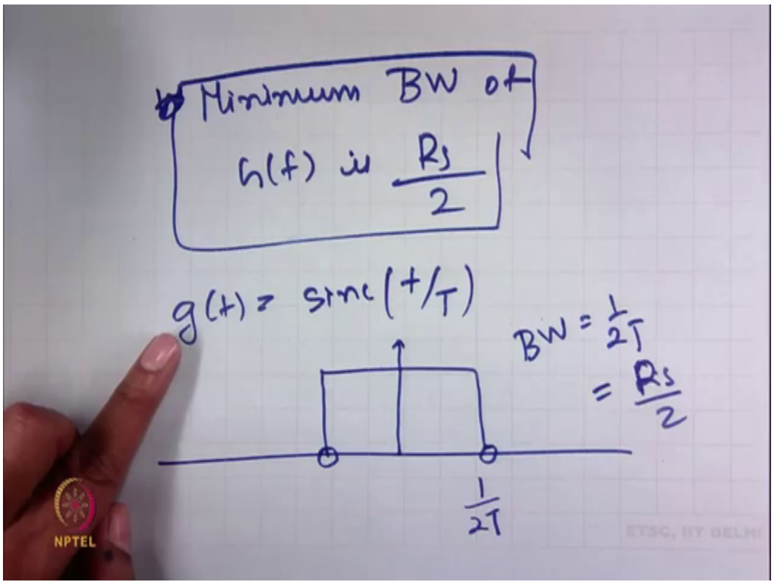


So, one thing is let us assume that G f is restricted to frequency is less than R s by 2 ok. So, let me first try that equation G f plus G f minus R s should be constant. Again we are looking only at the frequencies between 0 and R s. So, if let us assume that G f let me make one scale. So, let us assume 0 here and R s is here, 2 R s is here minus R s and minus 2 R s, so, this is R s by 2. So, let us assume that I have a G f which is like this and now this is my G f and so, I also have to have G f minus R s which would be this. So, this is G f minus R s.

And if I add these 2 things up what would I get? They are not overlapping. So, what I will get is exactly this and this is not a constant and so, what we learn from here is, if G f is restricted to frequency is less than R s by 2 then this condition can never be satisfied. So, this can only be satisfied if G f has the support at least greater than R s by 2 support I mean one sided support.

So, G f should have one sided support at least greater than R s by 2, then only this condition can be satisfied. So, this condition will not be satisfied if one sided support of G f is less than R s by 2. So, this has to be kept in mind.

(Refer Slide Time: 48:16)

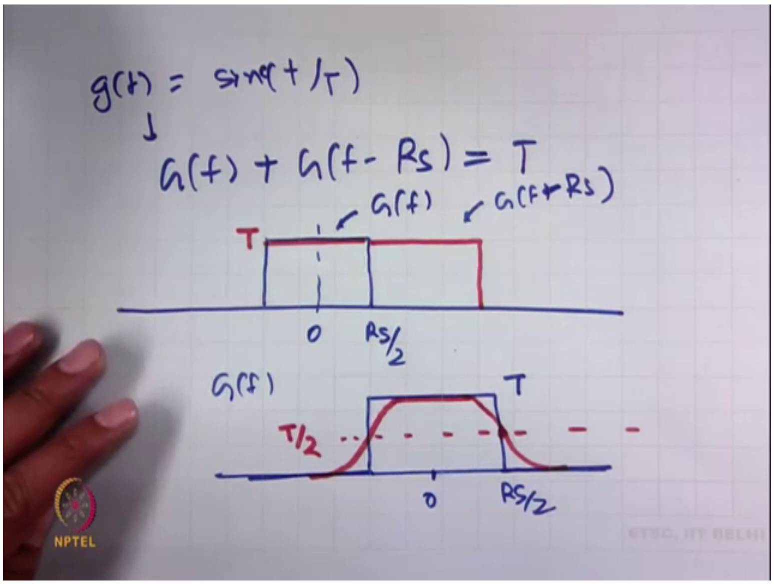


So, from this we have got very important idea that minimum bandwidth of G f is R s by 2 this is absolute minimum bandwidth because if bandwidth is less than R s by 2 then you can never satisfy Nyquist criteria. And s o, we have looked into the sink pulse g t sink t by T what is the spectrum of this sink pulse? The spectrum of this sink pulse is erect and where does you have a null in here at 1 by 2 T. So, what is the one sided bandwidth of this signal? The one sided bandwidth of this signal is 1 by 2 T and what is 1 by T it is R s. So, the bandwidth is R s by 2.

So, if I choose the sinc pulse as the Nyquist pulse, then the sinc pulse translates to rectangular spectrum and the one sided bandwidth of this function is R s by 2. Thus if I choose the sinc pulses as the Nyquist pulse then what I am doing is, I am getting an advantage that this pulse requires minimum bandwidth and it satisfies Nyquist criteria to avoid inter symbol interference.

But the problem would be that this is spectrum as we have seen is a discontinuous spectrum and when you take into account the practical situations of Georgian sampling clock and so and so forth, the sampling errors might become unbounded and that is the reason why we want to avoid this pulse. However, this pulse is best in terms of the bandwidth requirement that it offers. And so, the idea is I need to use the pulse this which may be required little bit more band than R s by 2, but it offers me a smooth decay from t to 0 ok.

(Refer Slide Time: 50:58)



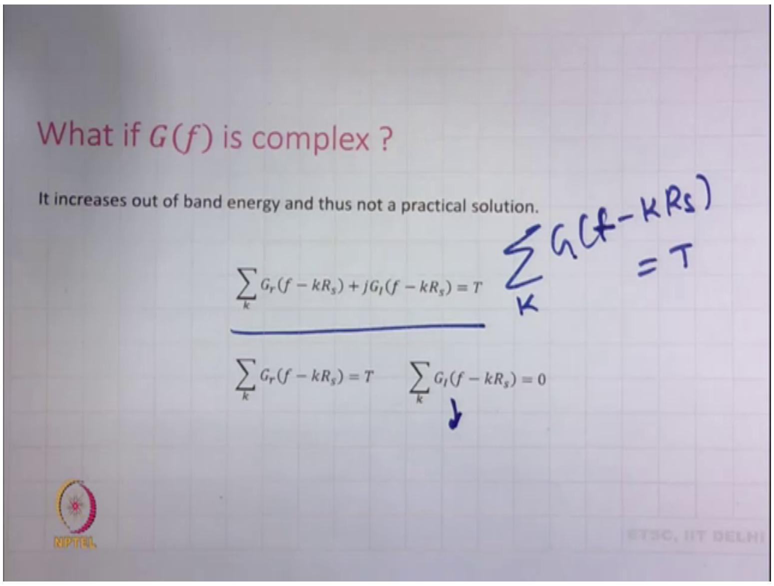
So, if I let us say I have a sinc pulse. So, this is sinc. If I have sinc pulse and if it has to satisfy Nyquist put in T which is this, then you see that this would have a spectrum. So, this will be G f 0 R s by 2 and you will have G f. So, G f minus R s and this should be T and because these are non overlapping. So, this amplitude should also be T. So, the spectrum, so, G f should be spectrum like this which is T between 0 to R s by and then this is 0 ok.

So, this satisfies band is symmetry this also satisfies this condition only problem it has is the sharp discontinuity. So, to avoid sharp discontinuity what we want to do is, to have little bit more bandwidth. So, we have seen that for R s by 2 my value should be T by 2 and then it should have some band edge symmetry. So, I can have a some pulse like this. So, what is this pulse doing in? It is having little bit larger band width in R s by 2 and assuming that it is satisfying band is symmetry, if it satisfies band edge symmetry then it would satisfy this condition.

And if this decrease from T to 0 is a smooth, this might be a good example of the spectrum of the Nyquist pulse. So, we will see what are the good examples of the spectrum of the Nyquist pulse? By that this moment I am trying to highlight two important ideas one important idea is that to satisfy Nyquist criterion, the minimum bandwidth that G f should have is R s by 2. Second thing that we have understood is if you want to have minimum bandwidth of R s by 2, the solution is the same pulse that same pulse has a discontinuous spectrum and there becomes all troubles related to sampling errors and so on so forth.

So, what you would like to do is to span little bit more on bandwidth. So, you want to have band with little bit more than R s by 2 and using this little more bandwidth what you would like to do is, to allow your spectrum to make a smooth transition from t to 0. Because a smooth transition would allow it to have sampling errors bounded and as long as it satisfies banded symmetry and as long as it satisfies Nyquist criterion tan inter symbol interference can be avoided ok.

(Refer Slide Time: 54:50)

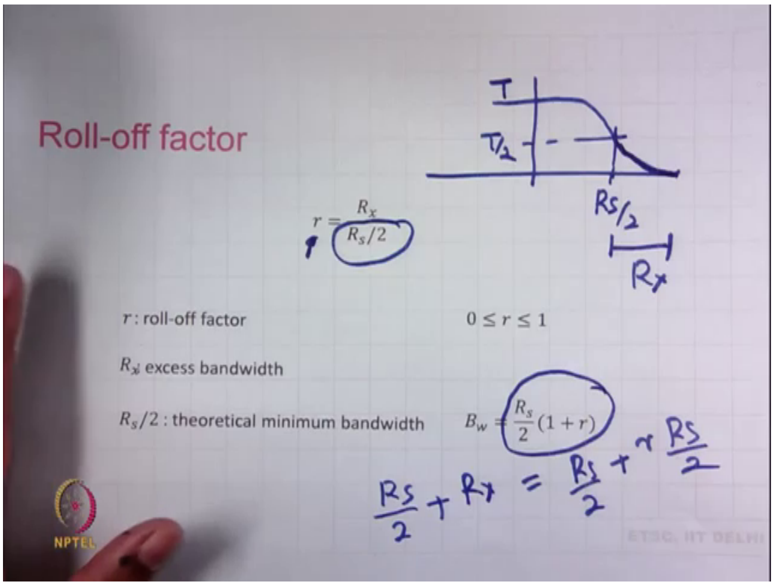


So, for all this discussion what we have assumed is that G f is real and symmetric; the question that we ask is what if G f is complex. So, we get any more benefit. If G f is complex then instead of this condition, it should satisfy this condition you can break this G into 2 parts real parts and imaginary part and this should be T. So, from this we get that this should be satisfied and this condition should be satisfied.

Now by having the imaginary part, you do not have any different condition than the condition that we had before. So, the real part should satisfy the same Nyquist criterion as it had to satisfy before and that is by having imaginary part, we do not improve in facilitating a smoother transition from t to 0. So, this does not help us in any way in making the spectrum to move smoothly from t to 0.

And thus this is redundant and we might face some more energy by having an imaginary part. So, what I am trying to point out is by having G f to be complex, we are just wasting energy in having some imaginary part and having an imaginary part is helping us no way in having a spectrum which goes more smoothly from t to 0. So, thus we restrict G f to be real ok.

(Refer Slide Time: 56:58)

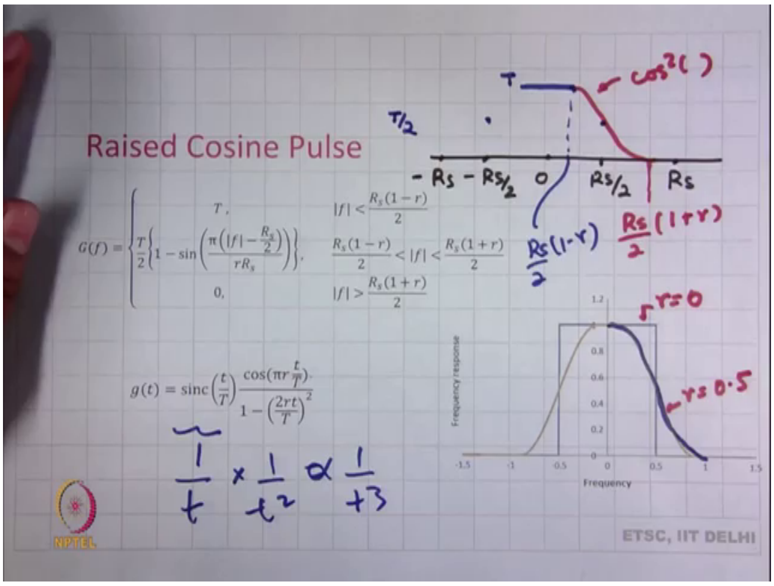


Then now next we want to develop is some examples of G f that I used in practice and to understand them we have to understand this roll off factor. So, the roll off factor is defined in terms of this ratio of excess bandwidth divided by R s by 2. Remember R s by 2 is the minimum required bandwidth. So, we have said that this is minimum what we require.

So, if I look at G f suppose G f is like this and we know that this must be R s by 2, at this point this should be T by 2 this is T and so, this requires little bit more bandwidth. So, this part this is referred to as excess bandwidth. So, roll off factor is nothing, but it is the ratio of this excess bandwidth divided by theoretical minimum bandwidth and we can find this roll off factor to be between 0 and 1. So, you do not want to have lot of excess bandwidth because then it will lead to band wastage.

So, what we want to do is we try to keep excess bandwidth at most as R s by 2. So, the theoretical bandwidth than would be required would be R s by 2 plus excess band width which is R x and R x is nothing, but it is r times R s by 2. So, the bandwidth requirement is R s by 2 times 1 plus r where r is the required roll off factor. So, let us look at the examples of G f that I used in practice.

(Refer Slide Time: 58:53)



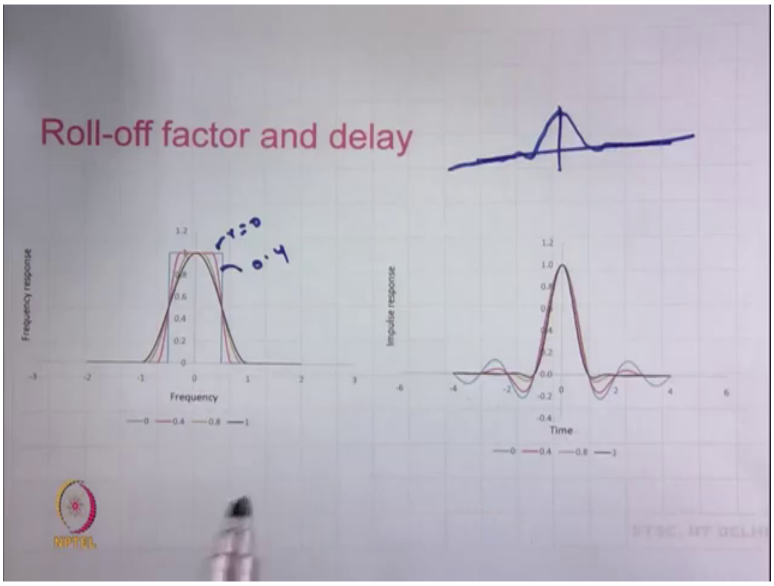
So, here we have example of raised cosine pulse raised cosine pulse has such an spectrum ok. So, what it simply says is that. So, what are the key points? So, we have 0 R s by 2 and we want to restrict ourselves to R s ok. Similarly you will have minus R s by 2 and minus R s. Now what we see is that here we should have a value of T and at R s by 2 I should have a value of T by 2 ok.

This is to satisfy Nyquist criterion and this we have already seen before. Now what should happen is you should have to satisfy banded symmetry which for the raised cosine pulse we choose this function as a cos square function the exact expression is given in here we are not worrying about that but in general this has to be some cos square function.

And this would take some excess bandwidth and this point is R s by 2 1 plus r. This point similarly is R s by 2 1 minus r. So, this is how the spectrum of a raised cosine pulse we will look like it should be constant for mod frequency is less than R s by 2 1 minus r for frequencies between this and this it should be given by cos square function this is a cos square function and for frequencies greater than this it should be 0 right and here I have the exact spectrum drawn in for you for 2 roll off factors of course, r equals to 0 is the frequency response of this impulse right.

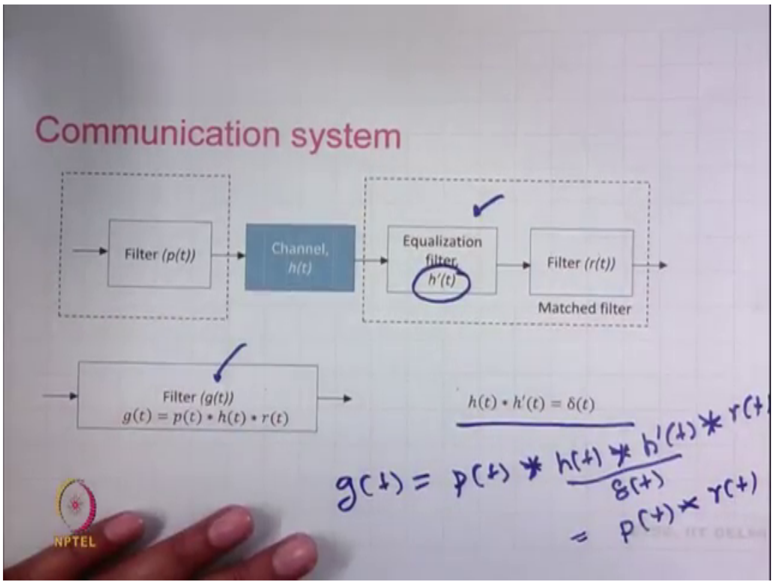
And for r equals to 0.5 some excess bandwidth is incurred and the spectrum becomes continuous right. If I take the inverse Fourier transform of this pulse, we get this pulse and if you see carefully this decays by 1 by t. So, I have cos pi r t by T. So, this is basically an oscillatory term, it does not decay with t and this term for large values of t decays as 1 by t square. So, g t decays approximately as 1 by t cube. And this also is clear from the spectrum because the spectrum has second derivative discontinuity and this is a very popular example of the pulse that is used it goes by the name of raised cosine pulse.

(Refer Slide Time: 62:14)



I have the picture corresponding to the frequency response of various roll off factors. So, this one is r equals to 0.4 and so on so forth the basic idea is simple as roll off factor increases excess bandwidth requirement increases, but in time domain time domain signal becomes more and more confined right. For example, at r equals to 1 I have a signal like this it becomes more and more time limited as roll off factor increases. It would be a good idea for you to plot these frequency response and impulse response yourself by changing the value of R.

(Refer Slide Time: 63:10)

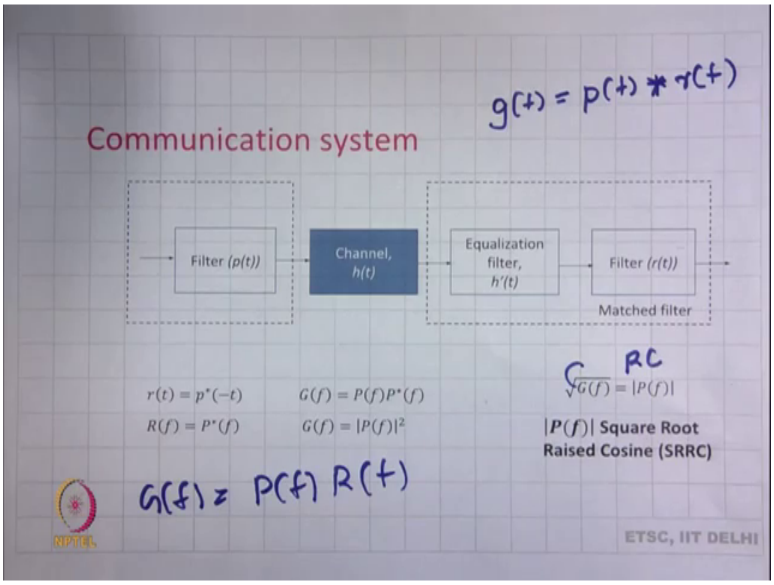


Finally, let us come to the last point here we have talked everything with respect to g t. Where g t has an impulse response obtained when the convolution of impulse response of this filter channel and this filter. Now what we have done is we have introduced one more block which goes by the name of equalization filter and we are saying that this equalization filter has an impulse response h dash t and this equalization filter is to inverse the effects caused by channel.

So, if channel has an impulse response of h t if I use a filter with an impulse response of h dash t and if I choose this equalization filter say impulse response such that h t convolution h dash t is delta t, then you know that this channel impact would be negated by this equalization filter.

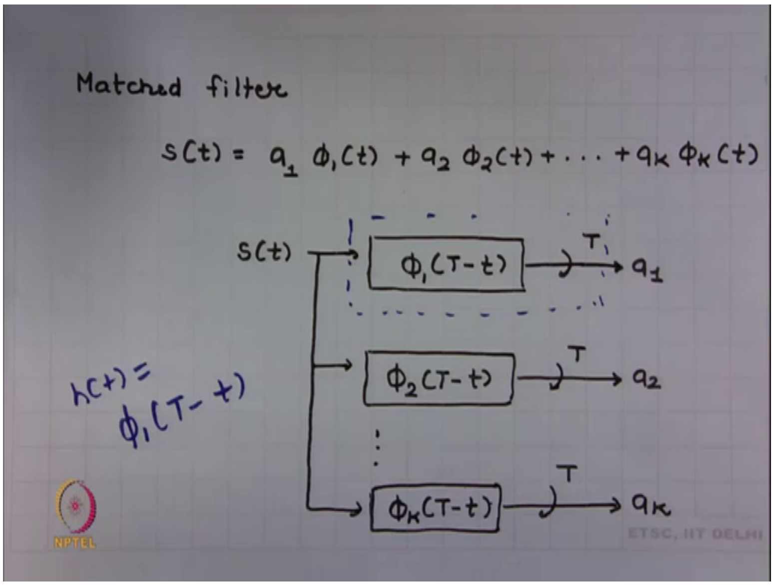
Because now what would happen is, g t would be p t convolution h t convolution h dash t convolution r t and this is delta t p t convolution delta t is nothing, but p t convolution of r t convolution with r t. So, g t is nothing, but p t convolution with r t, so, channel effects are negated.

(Refer Slide Time: 64:53)



So, in the presence of equalization filter g t is given by p t convolution with r t and how should we select this r t, remember that the r t is the impulse response of the matched filter.

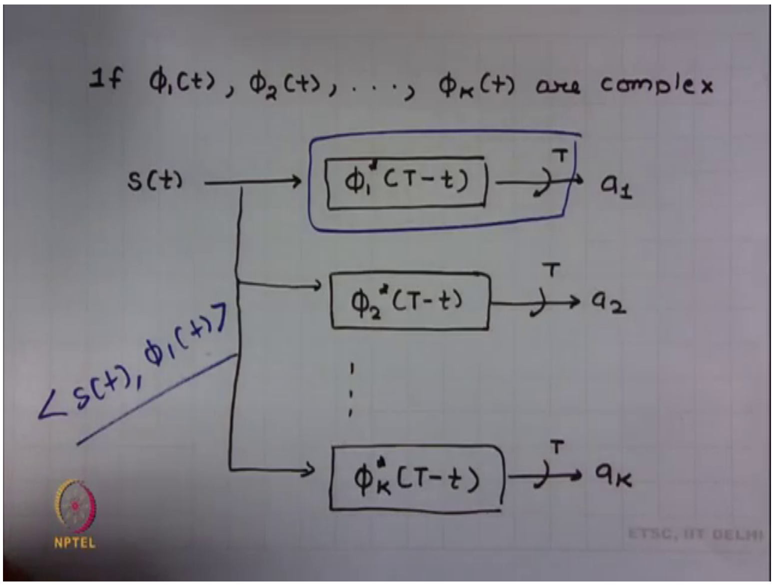
(Refer Slide Time: 65:13)



So, let us see what we have seen about this matched filter, we have seen about this matched filter in lecture number 4. So, we know that if we have to receive a signal s t, which is expressed in terms of orthonormal functions phi 1 t phi 2 t and phi k t, then the job of the receiver is to extract the coefficient of the signal along this orthonormal functions.

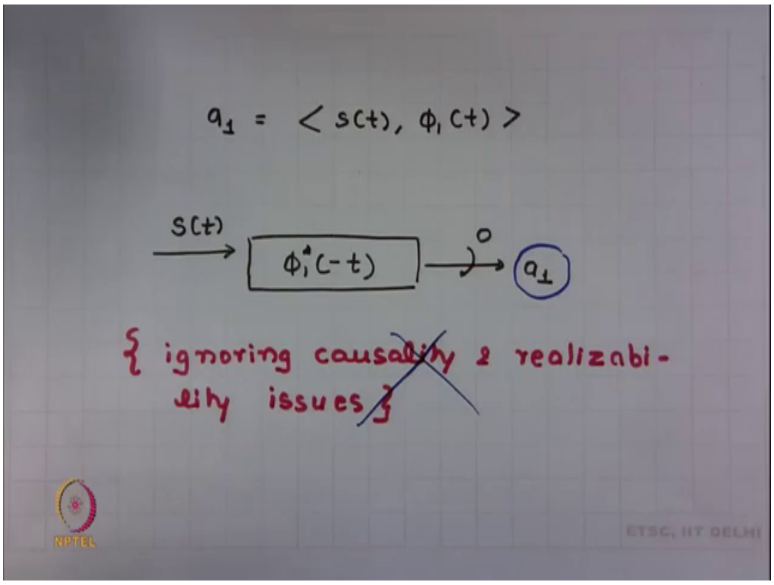
And the way we can extract the coefficients of the signal along this orthonormal function is to use what is known as a matched filter ok. And matched filter simply has an impulse response which is matched to one of this orthonormal function. So, namely if this matched filter extracts the coefficient a 1 of the signal s t along phi 1 t, then the impulse response of this matched filter is phi 1 T minus t and then there is a sampler which samples the output at T ok. And similarly you can have other matched filters and you can extract the coefficients a 1 a 2 and a k we have seen all of this in lecture number 4.

(Refer Slide Time: 66:33)



So, we have also seen that if these phi 1 t, phi 2 t and phi k t are complex functions then the impulse response of the matched filter also need to have this conjugation because the job of a matched filter is simply to calculate the inner product. So, it would be calculating the inner product of s t with this orthonormal function phi 1 t. And you can prove it yourself that if this receiver has to carry out this inner product operation then the impulse response of this filter should have a conjugation if these functions are complex functions alright.

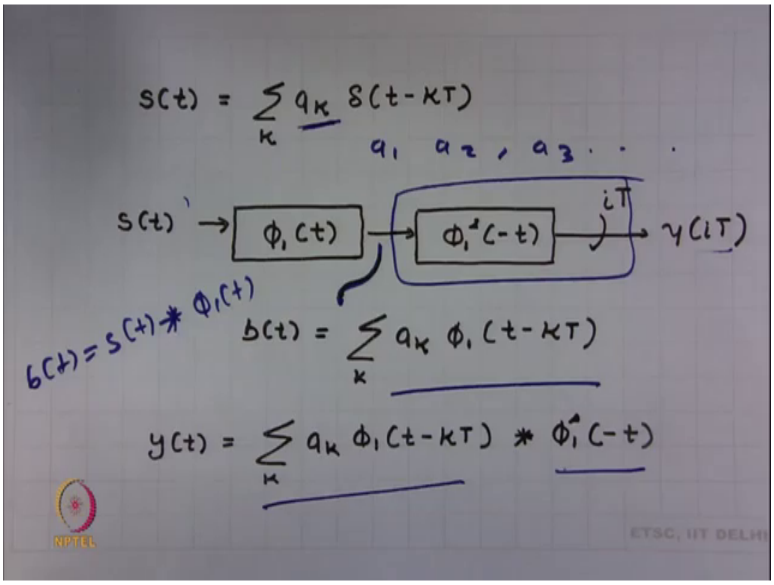
(Refer Slide Time: 67:16)



Now, let us see something else here we have assumed that d is some constant and the sampler also needs to sample at this time T alright. We can assume the value of t also to be 0. So, if we can have the impulse response which is phi 1 minus t conjugate and you can sample at time instance 0.

Of course, this will create causality and realizability issues which we have seen in lecture number 4, but let us forget these causalities and realizability issues for simplification and let us simply assume that the impulse response of the matched filter is simplify phi 1 minus t conjugate. And what we will get is the coefficient a 1 right, but the job of a communication system is to continuously receive signals right it would not stop by just getting the signal a 1.

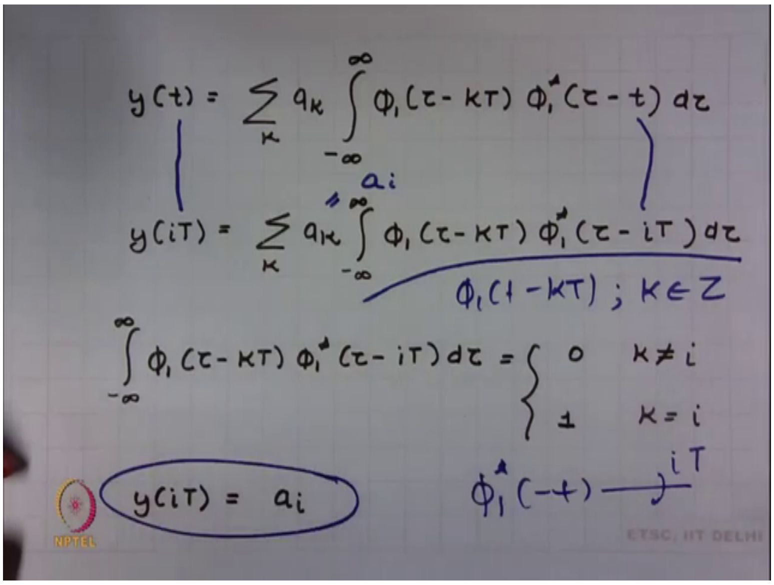
(Refer Slide Time: 68:16)



So, in general the signal does not just transmit one coefficient it transmit a series of coefficients. So, it transmits a 1 a 2 a 3 and so on so forth and it transmit these coefficients or these symbols at integer multiples of T alright. So, let us see what would be the design of a matched filter in that case when the matched filter should receive these symbols a 1 a 2 and a 3. So, to understand that let us assume that I have a signal s t and first I pass the signal s t through a filter phi 1 t and then let us assume that I have a matched filter which has this impulse response and then I am sampling at integer multiples of T.

And let us assume that I have got some output y i t. So, b t which is the output at this filter is simply a k phi 1 t minus k t and why is this so? Because this b t is nothing, but s t convolution with phi 1 t and we have seen this several time that in that case b t will be simply this function. This output at this place would be b t convolution width phi 1 minus t conjugate and let us solve this out.

(Refer Slide Time: 69:47)

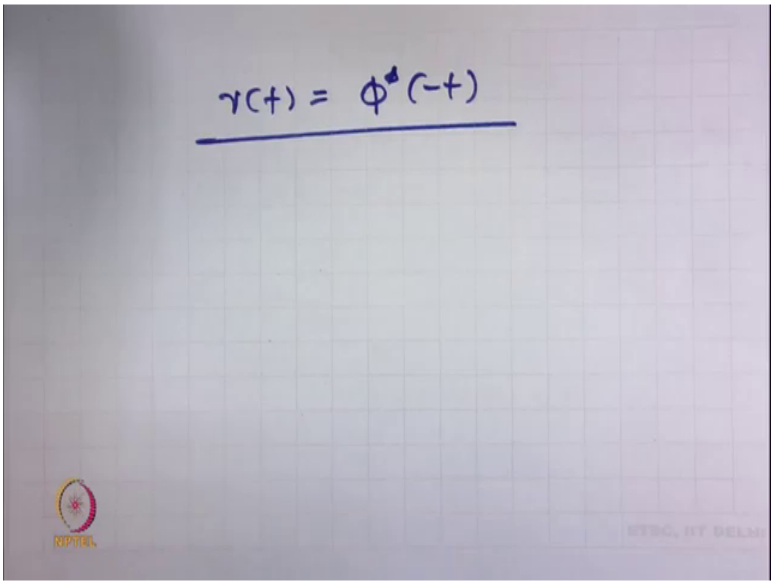


So, then we can say y t is this function. So, what we have just done is, we have carried out the convolution operation. Now when I have to find the output at i T time instances I just have to substitute t as iT. So, substituting t as iT we get this. If this phi 1 t minus k T is an orthonormal set for k belonging to the certain set of integer, then I can easily appreciate that this would be 0 if k is not same as i because its an orthonormal set and if k is same as I then this will be 1 ok.

In that case if this phi 1 t minus kT are orthonormal set for k belonging to set of integer then y i T is simply a i ok. So, this summation would have contribution only when this is 1 and this will be 1when k will be same as i. In that case a k will have contribution only for a i and rest terms will go to 0 because of this function.

So, y i T will be a i. So, what we learn from this is, that if you want to make a receiver which is receiving the sequences continuously, the sequence is a transmitted at integer multiples of t then the matched filter should look like this and here we are ignoring any causality and realizability issues ok. So, in short we can simply assume the matched filter response as phi 1 minus t conjugate followed by sampler, which is sampling at i T time instances and this matched filter will give us sequences which are transmitted at integer multiples of t ok.

(Refer Slide Time: 72:03)

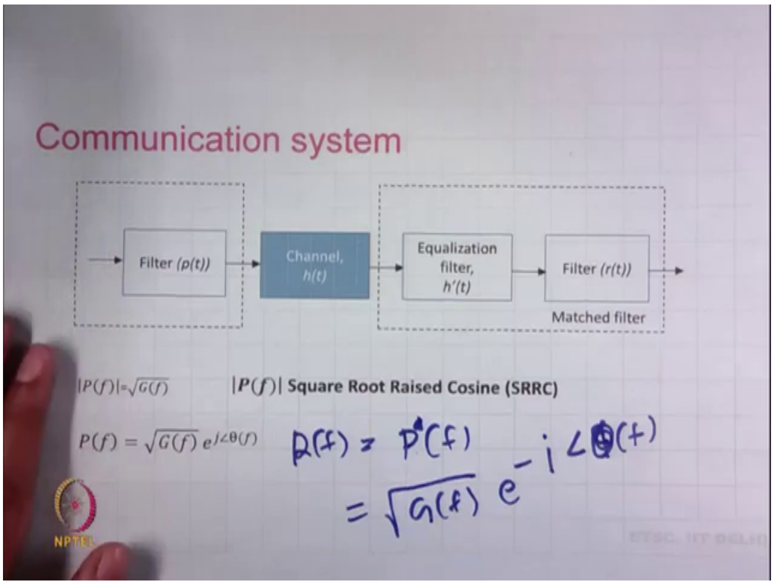


So, in short I can say that the impulse response of a matched filter is phi conjugate of minus t ok. So, this is what we are saying in here. So, I am having a matched filter, matched filters impulse response is matched to the filters impulse response. If I take the Fourier transform of this from this I get R f is p conjugate of f using the properties of signals and transforms.

Now what is G f? G f is nothing, but P f times R f and R f is P conjugate f. So, G f is P f times P conjugate of f and this is mod of P f square. So, from this we get mod of P f should be square root of G f and what is G f? Its a raised cosine filter that we have seen and so, the mod of P f should be square root of raised cosine. So, the filter that you have to use at the transmitters should have an impulse response, which is a square root raised cosine that is important ok.

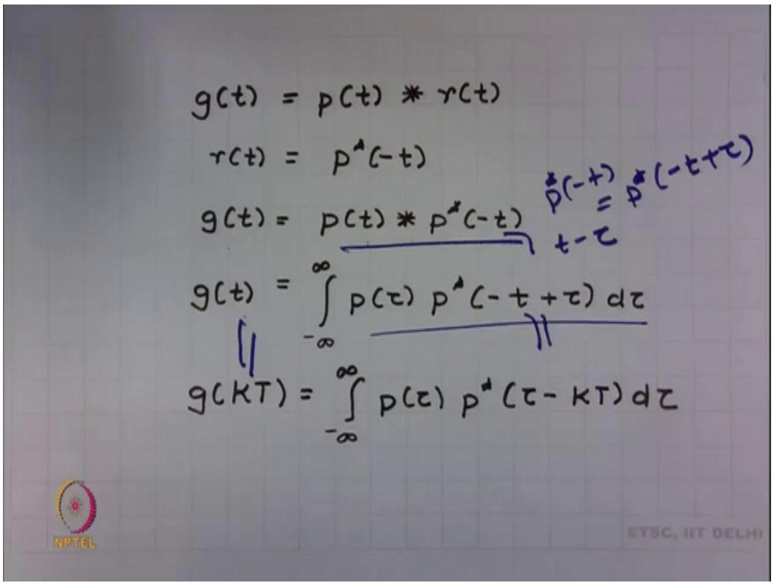
So, whatever we discussed before what is for g t and G f; G f has to be raised cosine, but the P f a mod P f has to be a square root of raised cosine. So, this caused with an abbreviation of SRRC.

(Refer Slide Time: 73:45)



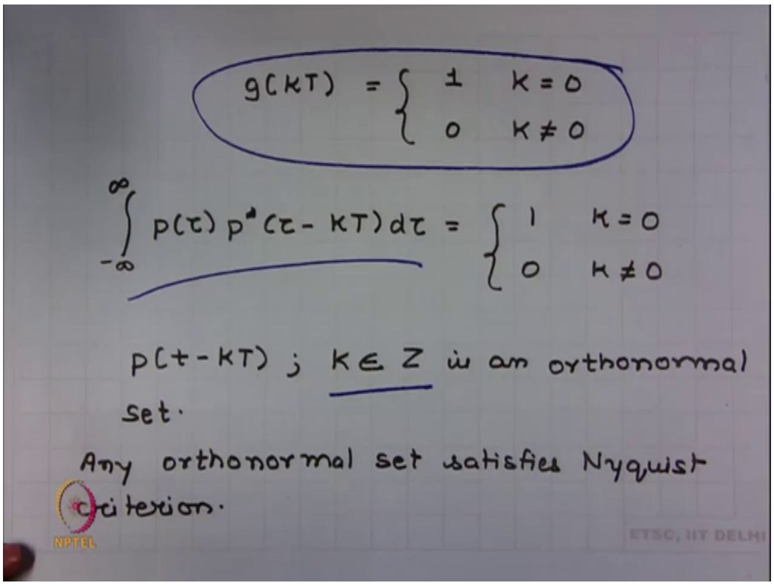
So, what we know is mod of P f should be square root of G f and P f in general is square root of G f into some angle. So, we can choose this theta f our self and there is no restriction; there is only restriction on what should be the amplitude of this P f. So, if you choose this P f with this angle, you have to choose R f. So, Rf because R f is P f conjugate, so, then this R f is this theta ok.

(Refer Slide Time: 74:26)



And one last issue that we have to see is that this g t is nothing, but it is p t convolution with r t and we are assuming that channel has been compensated and has been accounted for by using this equalization filter. And we have seen that r t is p minus t conjugate in that case, g t is p t convolution with p minus t conjugate and just writing out this convolution operation we get g t is this function ok. So, simply replacing this t by t minus tau. So, p of minus t conjugate will become p conjugate minus t plus tau alright. Then putting this t as kT we get g of kT is so, we have to substitute t as kT we get this.

(Refer Slide Time: 75:34)



Now, what should be this g of kT? We know that g of kT should be 1 if k is 0 and if k is not 0 then this should be 0 this is from the Nyquist criterion to avoid inter symbol interference and we have seen this before. So, from this we get this should be 1 for k equals to 0 and it should be 0 for k not equals to 0 and thus we have saying that this p t minus kT should be an orthonormal set for k belonging to a set of integer. So, what we say is if we have p t which satisfies Nyquist criterion, then p t minus k t should be an orthonormal set. You can also conversely prove that any orthonormal set will also satisfy Nyquist criterion ok.

So, we know that we can easily generate several Nyquist pulses and this is an easy way also to generate several orthonormal sense alright. So, in this lecture today we have clearly understood how should we choose this p t and this r t as well this matched filter response, to avoid inter symbol interference of course, we have not considered noise at this moment.

But later when we will see noise we will also see that this is an optimal thing to do even in the presence of noise. And what we have identified it is that this filter response should be belonging to raised cosine family where you end up with different functions for different values of roll off factor as you increase roll off factor you span little bit more bandwidth than the minimum required bandwidth and the advantage of doing that is to make the frequency spectrum having a smooth transition from t to 0.

And we have seen that we can have the spectrum which has the second derivative discontinuity and because of the second derivative discontinuity, the time domain equivalent of that spectrum would decay with 1 by t cube and that will allow the sampling errors to remain bounded and this is important from practical point of view. So, in the next coming lectures what we will do is, we will look at some examples of modulation schemes particularly we will start by looking at pulse amplitude modulation and quadrature amplitude modulation and then we will define bandwidth efficiency and other parameters for those modulation schemes.

Thank you.