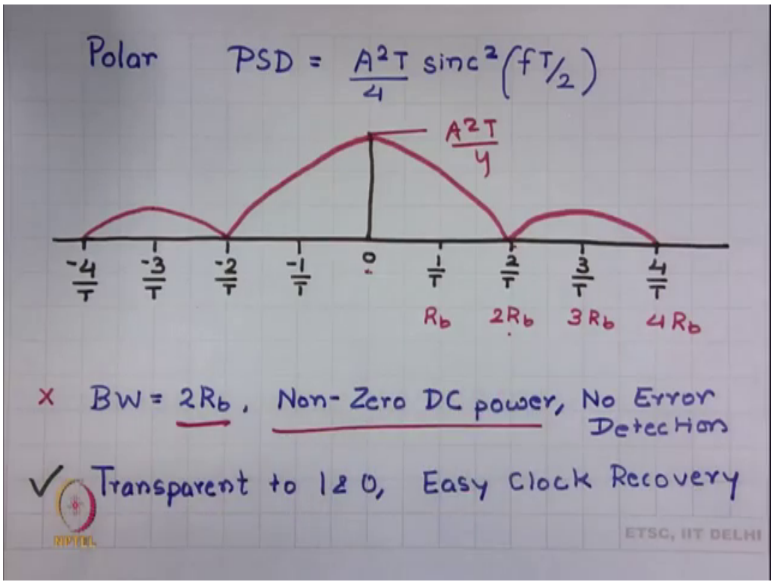
Principles of Digital Communication

Prof. Abhishek Dixit
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 23
Modulation: Spectral Description of Sources (Part-2)

Welcome to the new lecture of Modulation. So, in the last lecture we started with power spectral density calculations and we calculated the power spectral density of the analog basement waveform. And we said that the power spectral density is a function of the square of the magnitude of these pattern of the modulating pulse, it is a function of the power spectral density of the input sequence. And we also looked at the power spectral density of the polar signaling mechanism and we got a power spectral density function like this.

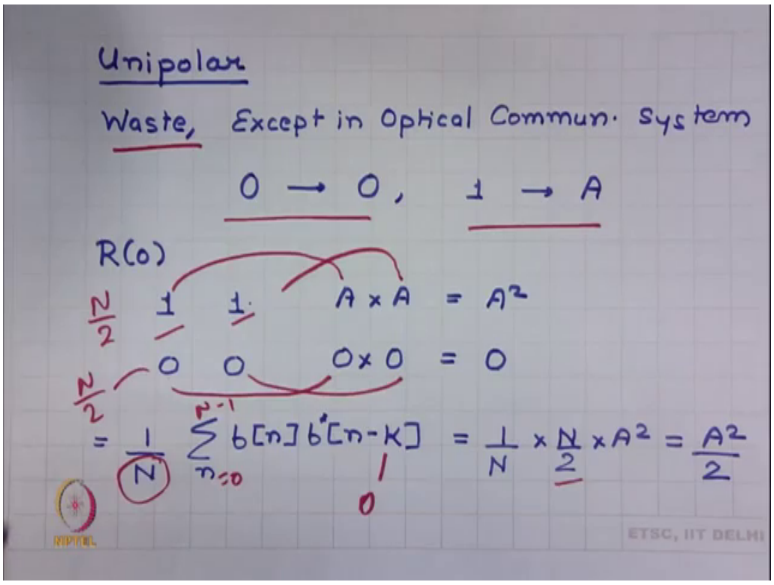
(Refer Slide Time: 00:55)



The important points that we can look in this power spectral density is there the power spectral density has a bandwidth of 2 times R b where R b is the data rate. It has a nonzero DC power which transform it is not like AC coupling circuits does not like and it would create lot of loss if this signaling mechanism is used. Then it cannot detect any error, it is transparent to 1 and 0’s because it keeps on making transitions even if there is a long trains of 1’s and 0’s, the number of transitions does not decrease with the long trains of 1’s and 0’s.

So, it is good for synchronization that is typically required in a digital communication system and we can easily recover the clock by just rectifying the pulse ok. And today we will look at the other signaling mechanism particularly unipolar, bipolar and some other interesting signaling mechanisms.

(Refer Slide Time: 01:55)



So, let us start with unipolar: so unipolar signaling mechanism is a waste, it does not have useful properties except in certain communication systems like optical communication systems, because in optical communication systems you cannot use something like polar signaling mechanism. Because in polar signaling mechanism we are transmitting the amplitude of A and of minus A right. In optical communication systems basically the communication happens not always, but mostly by modulating the intensity of the light and intensity of the light cannot be negative, it is always a positive quantity so right. So, there you cannot have a positive and negative intensity.

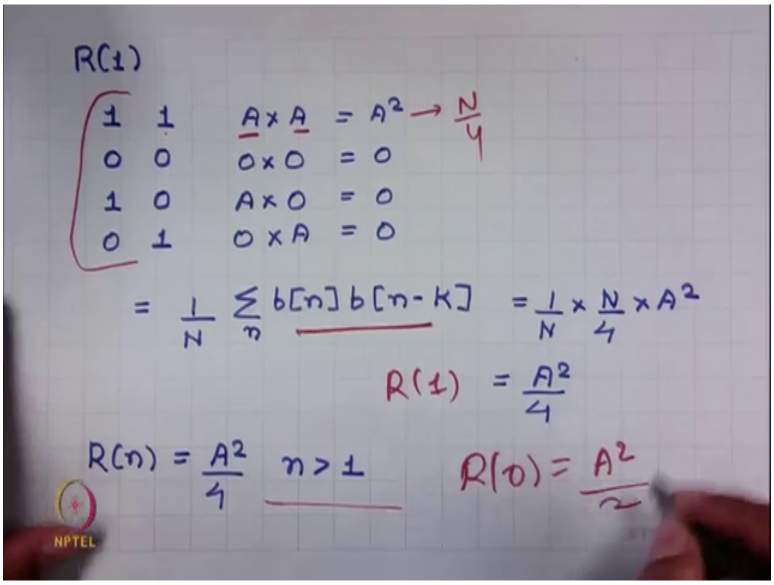
So, you cannot use a polar signaling mechanisms in optical communication systems mostly, in optical communication systems also there are various techniques by which you can even transmit or detect phases, but mostly optical communication system is built or thrives on the transmission of intensity ok. So, say unipolar signaling mechanism is a waste, except in certain communication systems where polar signaling mechanism cannot be used ok. So now, let us see R 0. So, to calculate the power spectral density we need to calculate R 0 and R m in general; and how we did it in polar? So, we assume that we have a 1 and the another sequence is also 1 because there is no delay in the sequence.

So, whatever you have here you should have it here or you can have 0 or 0, when you have 1 you transmit an amplitude A, when you have 1 you transmit an amplitude A, when you have 0 unlike in polar signaling mechanism where you were transmitting minus A in unipolar u transmit 0 ok. So, this is the mapping that we use in unipolar. So, for transmitting 0 we use 0, for transmitting 1 u transmit amplitude A. Again there is no sacrosanct about this mapping, it could have been used in other way around as well for 0 we could have transmitted A and for 1 we could have transmitted 0 ok.

But this is more natural that you transmit 0 for 0. So, for 0 we are transmitting 0, for 0 we are transmitting 0. So, when you have this what we get is A square and when you have 0 into 0 you get a 0. So, it is little bit easier in unipolar signaling scheme because sometimes we get 0 and then you do not have to account for this ok. So, R 0 is 1 by N summation n b n into b n conjugate minus k where k is 0 ok.

Now this would happen, if I have a sequence for n where n goes from 0 to N minus 1; that means, if you are considering N samples capital N samples and being pretty large then this event would happen with N by 2 times and this event will also happen N by 2 times. Because we are assuming that occurrence of 1 is as likely as the occurrence of 0 ok. So, this event happens N by 2 times when it happens what we have is A square. So, R 0 is A square by 2 ok, this is easy.

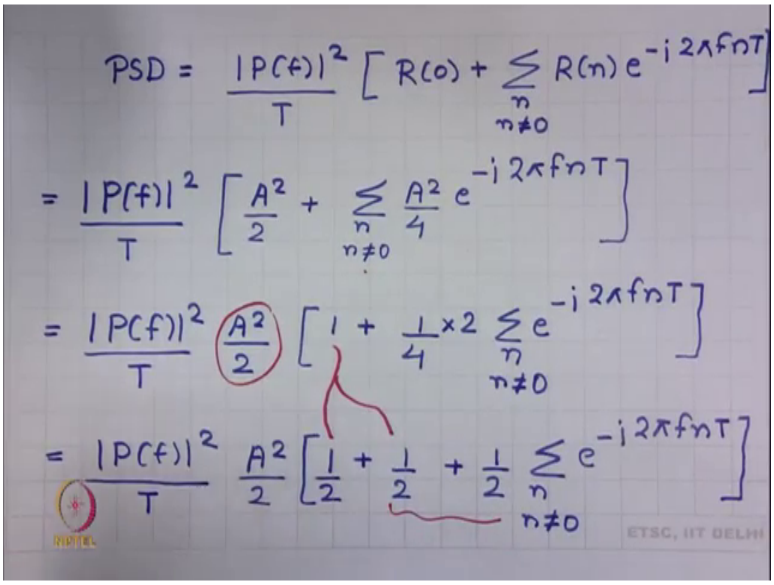
(Refer Slide Time: 05:23)



Now, let us see what is R of 1. R of 1 again we have to assume the four possibilities, we have 1 the bit in the sequence delayed by 1 unit is also 1, we have 0 0, we have 1 0, we have 0, we have 1. So, there can be four combinations possible. Again when it is 1 we use A, 1 we use A and we multiply because here we have multiplication. So, we get A square wherever we have 0 output would be 0 because 0 multiplied by anything would be 0 ok.

So, we have A square and this would happen N by 4 times right because there 4 possibilities and these events are equiprobable. So, each event would happen N by 4 times if n is pretty large. So, we have 1 by N only this event contributes and it happens N by 4 times it contributes a value of A square to the summation in the end what we get is A square by 4 ok. So, R of 1 we have derived is A square by 4, R of n similarly by using the same logic you can prove that it is A square by 4, it would be same and R of 0 we already derived, it is A square by 2.

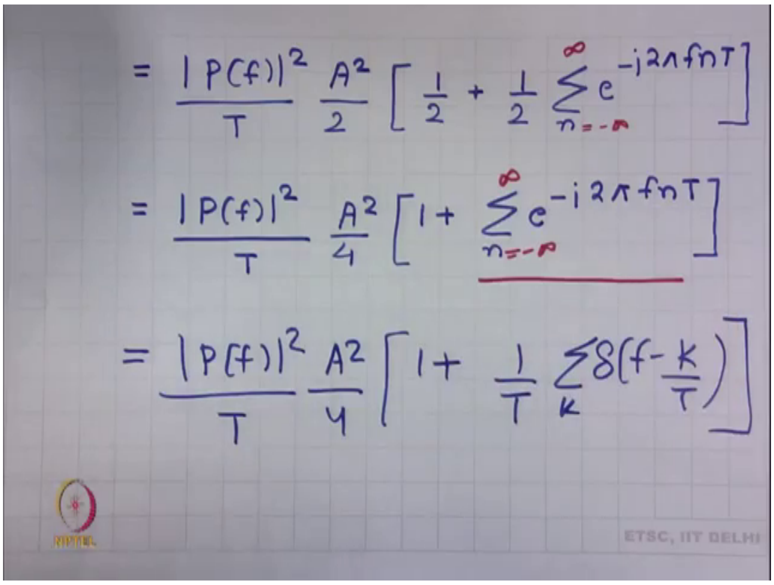
(Refer Slide Time: 06:43)



So now, what we do is power spectral density is mod of P f square by T R 0 plus summation R n into this quantity n not equals to 0 because n equals to 0 has already been taken into account ok.

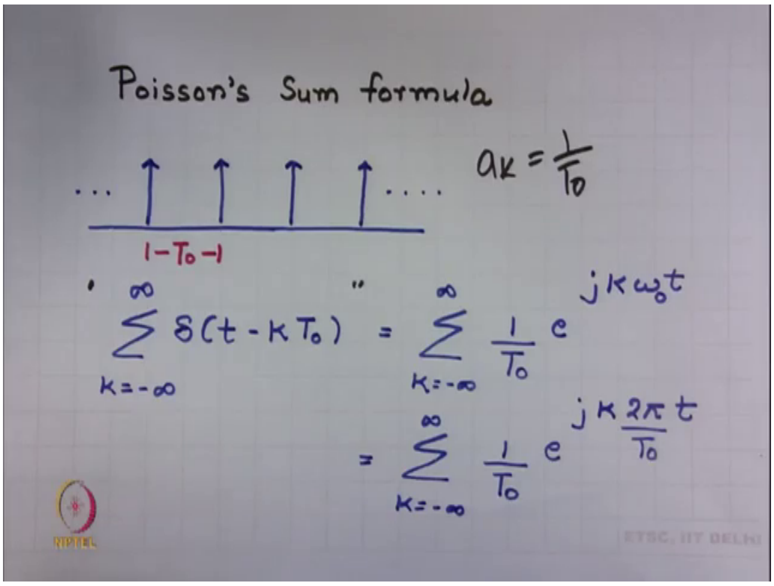
Now, R 0 we said it is A square by 2, R n is A square by 4, n is not 0 so, this point is not included and the summation. Now we take A square by 2 out or we end up here is 1, here you end up with half by taking A square by 2 out you get half and this summation. Now we do something strange, but useful that this 1 I split it into half and half because I have a half here, and what I want to do is I want to include this n equals to 0 point as well. So, I want to include this half into this summation so, that my summation can run from minus infinity to plus infinity ok. So, that is why I break it into 2 parts and I want to plug in this half to this side of the summation ok. So, let us see if it gives something useful.

(Refer Slide Time: 08:01)



What do we have is everything remains same, just half I have plugged to this. So now, my n goes from minus infinity to plus infinity. And now, half I am taking outside so, this becomes 4 1 plus summation this n goes from minus infinity to plus infinity ok. Now this is something that you might have seen before what this summation is if we have not, it belongs to a class of Poisson's sum formula.

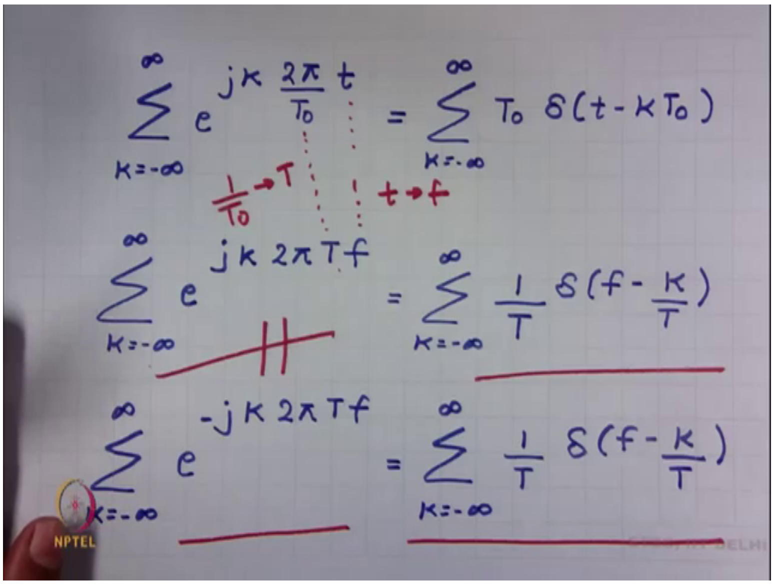
(Refer Slide Time: 08:37)



To find out that summation let us look into this Poisson's sum formula and Poisson's sum formula can be easily understood if you remember a basic identity from the course in signals and systems. So, in signals and systems you must have studied that if you have an impulse train and the impulses are separated by time of T naught, then this impulse train would have the Fourier series coefficients a k's as 1 by T naught.

That means this impulse train can be written in terms of its Fourier series where a k's are nothing, but 1 by T naught and omega naught is nothing but it is 2 pi by T naught. So, this you must have studied in signals and systems that an impulse train has the Fourier series coefficients which are constant and the values of these constants is given by 1 by T naught, where T naught is the time difference between the 2 impulses in the impulse train. So, I can say that this is nothing but this and this is same thing as this ok.

(Refer Slide Time: 09:57)



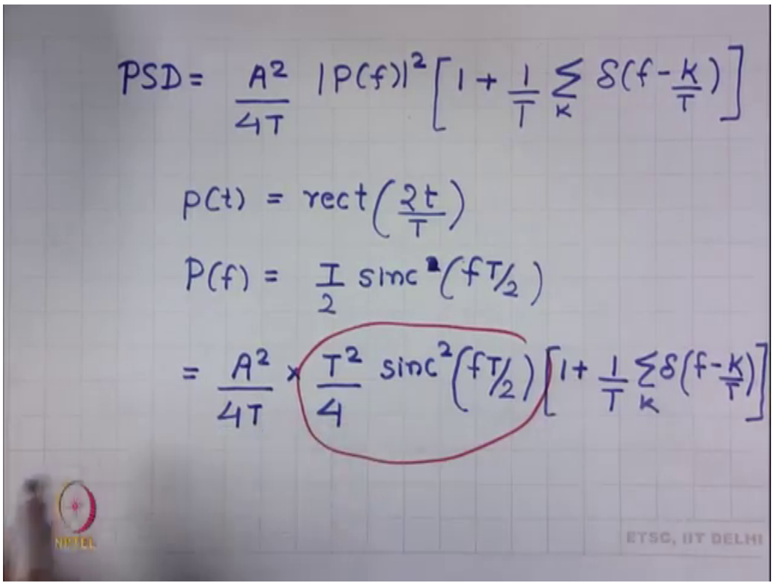
So, this would be than this what I have done is I have just shifted this T naught to this side. So, it follows from the previous equation and now what I do is I map 1 by T naught to T and I map t to f ok. So, I know I have been playing this trick for a while.

So, you must now know that this is useful. So, I am mapping 1 by T naught to T and I am mapping small t to f. So, here this T naught will be mapped to 1 by T. So, this T naught is mapped to 1 by T and this t will be mapped to f. So, I can say that this would be same thing as this. Now because k goes from minus infinity to plus infinity whether you have positive here or you have negative here does not matter. So, these two things are same because k takes in all values from minus infinity to plus infinity.

So, this summation is same as this. So, from here I can prove that this summation is nothing but this and this identity goes by the name of Poisson's sum formula and this is what we will use now. What we do next is we want to put in this result into that expression that we had. So, we had this expression and we were trying to investigate this.

So, we have calculated it. So, let us put it here, we get A square by 4 1 plus this quantity is 1 by T summation delta f minus k by t for all case and what this tells me is. Now in the power spectral density function I would have impulses as well and these impulses would be shifted by 1 by T ok. So, I am expecting in power spectral density some impulses.

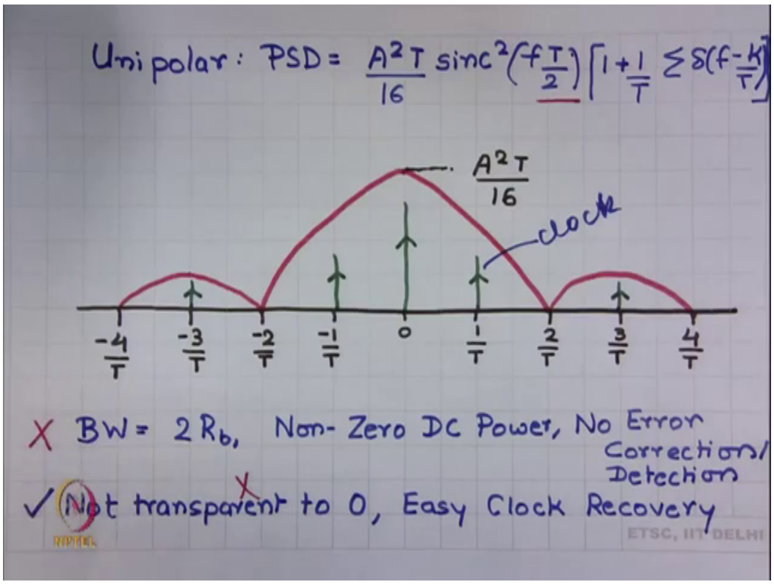
(Refer Slide Time: 12:13)



So, let us see. So, this is what I wrote in the last page, again assuming p t to be written to 0 pulse the same pulses we have used. So, there is no confusion rect 2 t by T. So, Fourier transform of this pulse is this as before nothing changes and now I have to substitute this in place of this I get this. This is mod P f s square 1 plus 1 by T the same thing as this. Now you can cancel T by T and you can have 1 by 16 and so on so forth.

So, the resultant power spectral density looks like this. So, what you see now is where is the first null? The first null would happen at the same position as before because we are having the same expression in the case of the polar signaling mechanism. So, let me see if I have that slide. So, look at the polar one where we were having sinc square f T by 2, we are having this and the null happened at 2 by T.

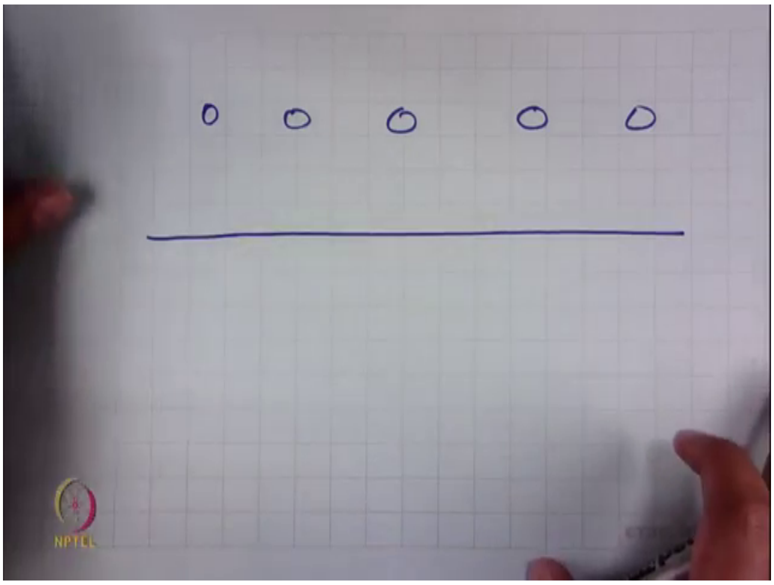
(Refer Slide Time: 13:29)



Similarly, the null here would happen at 2 by T, everything remains same just now the peak value of power spectral density is A square T by 16 ok. Now what do you see more is we are getting the train of impulses and these impulses are located at 1 by T. So, there would be at 0 1 by T 2 by T, but because it is multiplied with this it would be 0 3 by T and so on so forth.

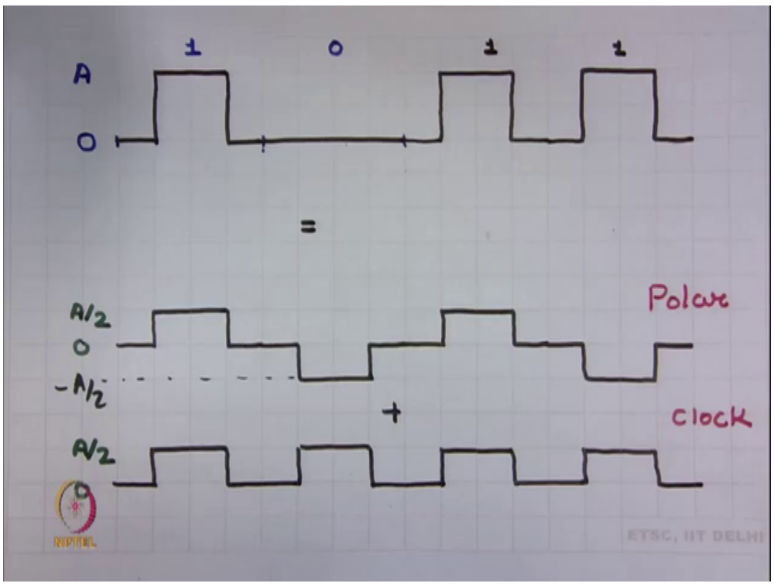
So, this is the power spectral density of unipolar signaling scheme or you have a train of impulses, what you see here the bandwidth requirement is 2 times R b same as before it also has non-zero DC power again like polar signaling scheme it does not have any error correction or detection possibilities. Is also not transparent to 0 so that is also a disadvantage right compared to polar signaling scheme, because if you have long trains of 0’s and for 0’s what we are having is nothing. So, there is no pulse used for 0’s. So, let me try to draw that.

(Refer Slide Time: 14:39)



So, if I have long trains of 0’s I get a signal like this, there are no transitions in this signal and hence the synchronization where the transmitter would be difficult if you have long trains of 0. So, it is not transparent to 0 so, it is transparent to 1. Easy clock recovery and this we can see from this is spectrum only that you have a impulse at 1 by T, if you have impulse at 1 by T this impulse corresponds to the clock. So, clock recovery should be easy, we will also see this in a different way ok. So now, you might be wondering that unipolar and polar signaling mechanisms they have such a different spectrum.

(Refer Slide Time: 15:39)

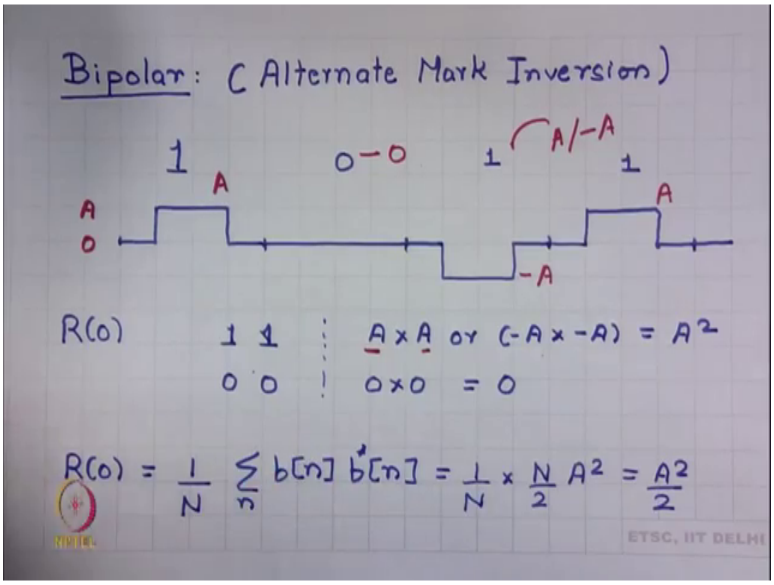


In unipolar now, what we are seeing is we are having impulses present in polar signaling a scheme there were no impulses; and why is this so? Intuitively is there a reason for this? Yes, there is a reason if you see I have a unipolar signaling mechanism. So, one I am using a R z pulse with amplitude A, 0 I am using nothing and for 1 I am using this pulses. Now I can break down this waveform into a polar waveform and a clock ok, it is easy. So, wherever there is 0, I can create this 0 by having minus A or minus A by 2 to be precise. So, this would be minus A by 2 so, I can have a minus A by 2 and in the clock I would have A by 2. So, these 2 things will cancel. So, I can recreate 0 by having this pattern.

So, unipolar signaling scheme can be broken down into a polar signaling scheme plus a clock and clock is a periodic signal and you must have studied in the course in signals and system that any periodic signal has a discrete spectrum right. So, you have impulses present in this spectrum of a periodic signal. So, that is why in the unipolar signaling scheme in addition to a spectrum like that of a polar signaling scheme you have impulses present because of the presence of a clock, and because there is inbuilt clock in this unipolar signaling scheme what you can say is clock recovery is easy ok.

So, we have finished with unipolar signaling scheme again we see that there are some disadvantages to this unipolar signaling scheme, it has all disadvantages of polar signaling scheme and it has one additional disadvantage that it is not transparent to 0. And we will see also that there are some other disadvantages of unipolar signaling scheme compared to polar signaling scheme and hence this is not very popular signaling mechanism.

(Refer Slide Time: 17:51)

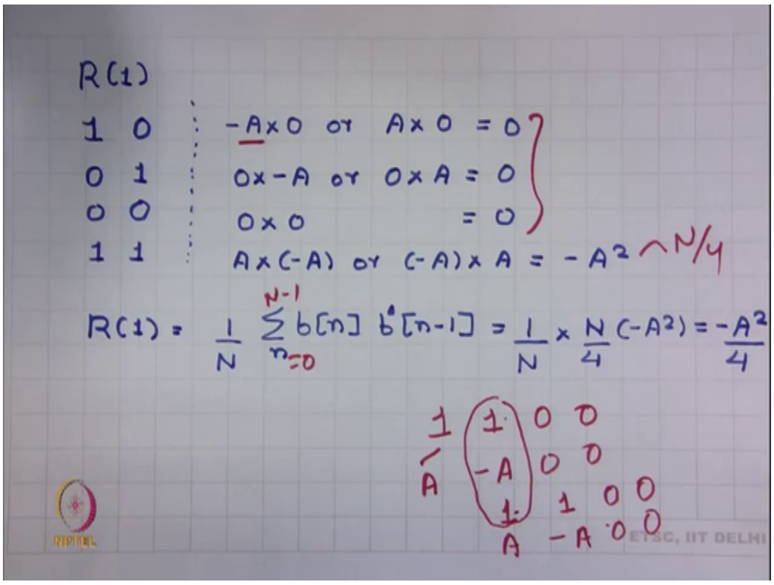


Next let us study the third one bipolar signaling scheme. And this has been used or a version of this has been used in telephone systems, this is also known as alternate mark inversion signaling scheme.

So, if we want to see this, what you see is for 1 I am using a pulse, for 0 I am using no pulse. So, let us say this is 0, this is A and for next 1 I am using a pulse of opposite polarity then of this pulse. So, I am using a pulse A or minus A for 1 and for 0 I am using no pulse and I should choose a pulse of opposite polarity for the 1 that comes after a 1. That means, if I have a 1 and I have chosen a pulse with amplitude A, for the next 1 I should use the pulse with amplitude of minus A ok. For next 1 again I should flip the amplitude and I should choose the pulse which is a flipped version of the amplitude of the pulse which I have chosen for the last one ok. So, this is a bipolar signaling scheme.

So, remember in a bipolar signaling scheme what we do? We choose a 0 for 0 and for 1 we choose either A or minus A and we choose a pulse amplitude for a 1 of the opposite polarity of what we have chosen for the last one. So, this is the idea in a bipolar signaling scheme. R 0: R 0 is easy you can have 1 1 or 0 or 0, 1 you can have either A or A, if you have chosen A or you can have minus A into minus A. So, you get A square and for 0 you get 0 right. So, R 0 is A square by 2, same as that of unipolar signaling scheme using the same ideas. What about R 1? Let us see what happens in the case of R 1; R 1 again the 4 possibilities 1 0, 0 1, 0 0, 1 1 right.

(Refer Slide Time: 20:13)



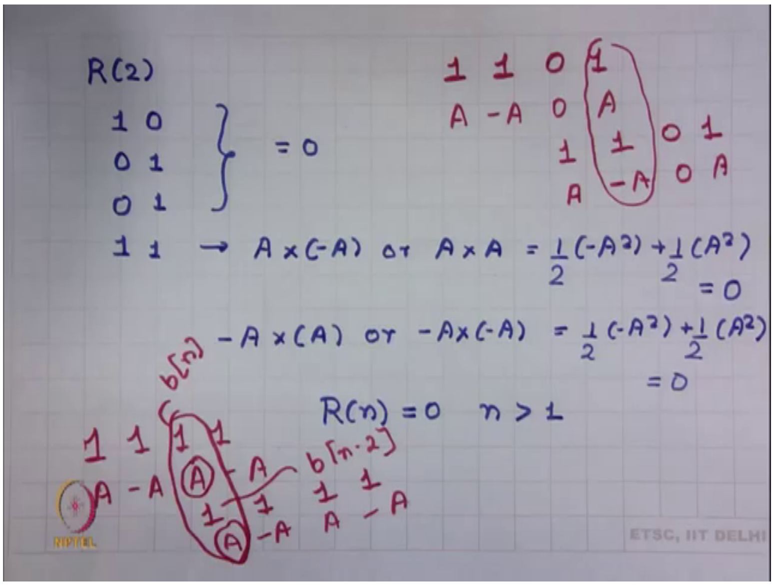
Now, if I have 1 I have chosen minus a let me assume, for 0 I would choose a 0 I could have chosen A for 1, for 0 I am choosing 0, but this is less important because in the end I am going to have 0. Similarly because there is a 0 I am going to have a 0, if there is a 0 I am going to have a 0. So, these three are less important we do not have to worry about because we know the final answer would be 0. This is interesting 1 1; what would happen here? So, let us say if I have chosen A, now this 1 means that this should be the amplitude of the sequence when it is delayed by 1 unit. So, for example, if I have a sequence 1 1 let us say 0 0. Now I may have chosen A for this; if I have chosen A for this then I have to choose minus A for this and 0 and 0.

Now, if I delay it by 1 unit, I get this and I am choosing A for this minus A for this and 0 for this and 0 for this; that means, if I have 1 1, if I have chosen minus A then for this one I should have chosen a right because for consecutive one I should choose the amplitude of the opposite polarity. Thus if I have chosen A in the last bit I should have chosen minus A, if I have chosen minus a in the last bit I should have chosen A right. So, in the end I should get minus A square this will happen N by 4 times if I am considering a sequence of length N 0 to n minus 1, then this event would happen N by 4 times right. So, 1 by N this happens N by 4 times and when it happens it has a contribution of minus A square.

So, in short I am going to have minus A square by 4 as my R 1, it was not too difficult, but it was relatively difficult to calculate then polar and unipolar signaling scheme. Interesting thing R 2 is different in this case, because this scheme is having some kind of memory right. In polar and unipolar signaling scheme there was no memory right, you are choosing an amplitude level independent of what you chose in the past. Here you are choosing an amplitude level depending on what you chose in the past right, if you have chosen A for the next one that is following up you would choose minus A right.

So, here you have some kind of a memory and so in this case R 2 will not be same as R 1, where as in previous cases R 2 R 3 everything was same as R 1, because there was no memory.

(Refer Slide Time: 23:23)



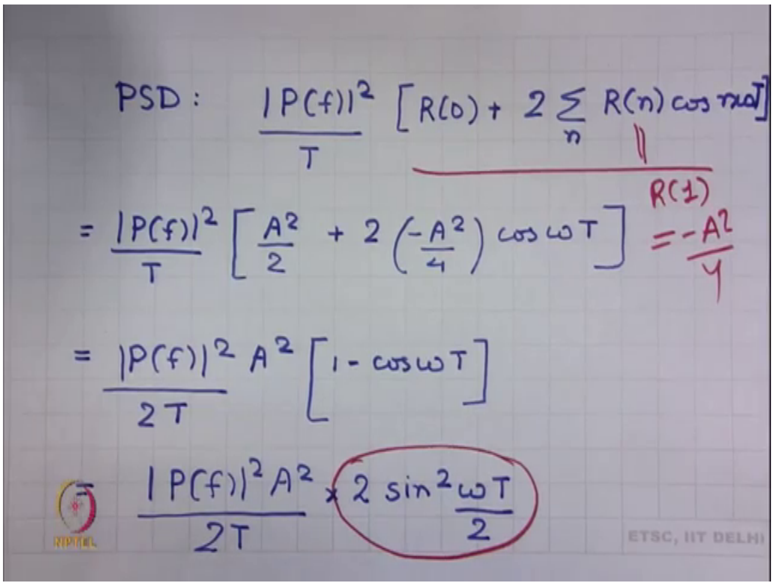
Let us see R 2 here; again for these three sequences because they involve 0 the calculation is easy we get 0, for 1 1 now let us see. So, let us see an example. So, let us say if you have 1 1 0 and I have 1, and let us say that we have chosen these amplitude levels. If I delay the sequence with 2 units of time I can get A, minus A, 0 and A.

So, I am focusing; if I am focusing on this let us say as an example and let us say I have fixed A then it can happen that I had get minus A, this can also happen that if I have chosen A here I could have got A again here, if there was 1 in between. For example, if I had a sequence like this then my mapping would become like this and let us see now if I delay this by 2 units I get A minus A A minus A. And let us see now what happens is I am getting A and I am getting A. So, if I am looking at this. So, this is let us say is b n and this would be b n minus 2. So, both situation can happen, if you have A you can get minus A as in this case, if you have A you can get A and these 2 events would happen with equal probability.

Because this depends whether you have 0 in between or you have 1 in between, if you have 0 in between you would have minus A if you have 1 in between you would have A and the occurrence of 0 and 1 is same so, they happen with a equal probability. So, let us say this would happen with the probability of half and when it happens it takes a value minus A square, it has a probability of half when it happens it takes a value of A square in short the resultant would be 0.

So, here we have assumed that we are having A, you could also start with minus A and the same logic flows and you get a 0; that means, R of n is 0 for n greater than 1, but R of 1 was not 0. So, I am just proving it for the case of n equals 2, you can prove it for other ends do this exercise and I request you to come to the conclusion that R of n is 0 for n greater than 1.

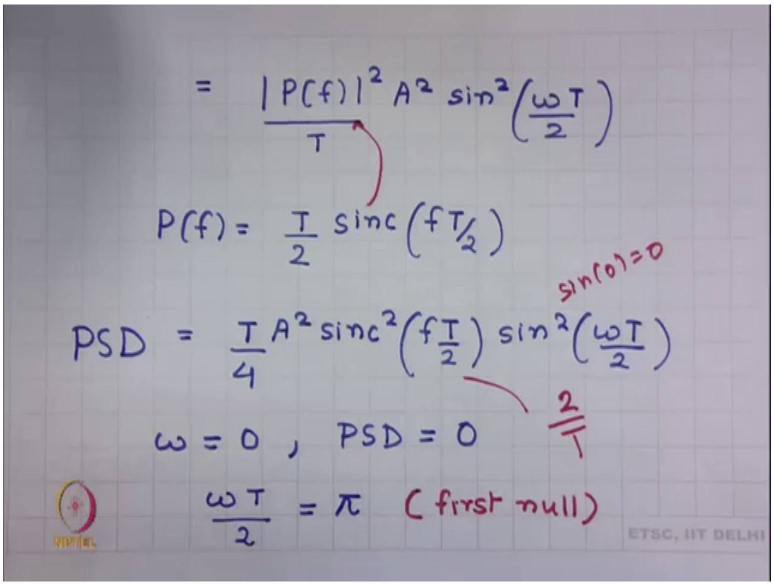
(Refer Slide Time: 26:25)



Let us see now the power spectral density.

Let us see now the power spectral density. And now I am using this expression for power spectral density I said that all three expressions are useful. So, if I see this R of 0 we calculate it to be A square by 2. So, there is only R of 1, rest are 0 and R 1 was minus s square by 4. So, we have minus s square by 4, 2 times minus A square by 4 cos omega T corresponding to n equals to 1. Then it becomes a trivial exercise, I pull this A square by 2 to this side, what I get is 1 minus cos omega T and 1 minus cos omega T I can write as this 2 times sin square omega T by 2.

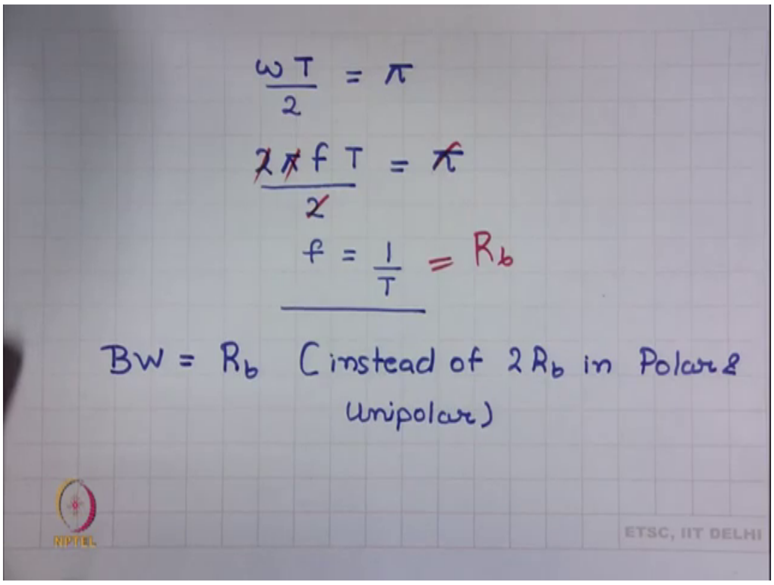
(Refer Slide Time: 27:15)



Simplifying this further, we get this just cancelling 2 by 2. Now assuming that P f is T by 2 sinc f T by 2 as we have been doing for a while, we get power spectral density. So, plugging this into this expression we get T by 4 A square sinc square f T by 2 sine square omega T by 2 ok. Interesting things to note when omega equals to 0 something wonderful happened that the power spectral density becomes 0 because sin 0 is 0. Now this is great, this what; and this is what we have been looking for. We wanted to have a power spectral density is 0 for omega equals to 0 and we have got this because of this function here.

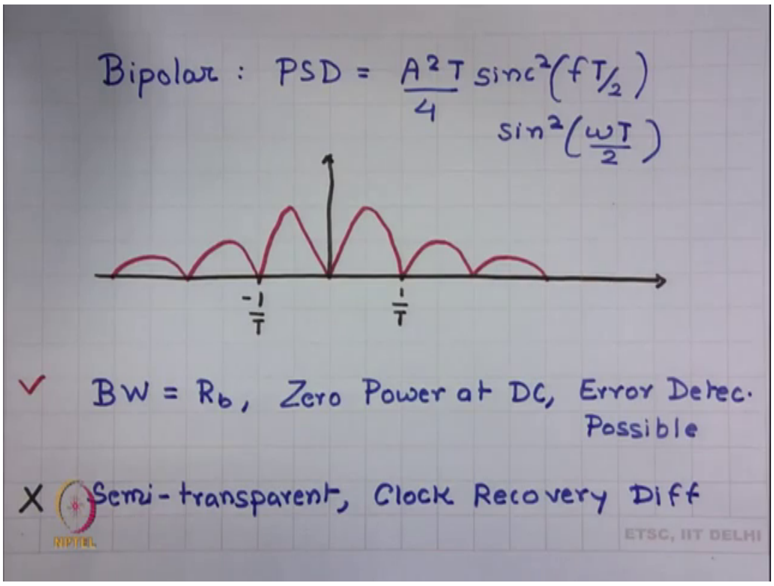
Second thing let us see where we get the first null. So, I get a one null because of this at 2 by T, this is obvious this we have done. So, let us see whether this changes the location of the null because of this function null will happen when omega t by 2 becomes pi here investigating for the first null.

(Refer Slide Time: 28:27)



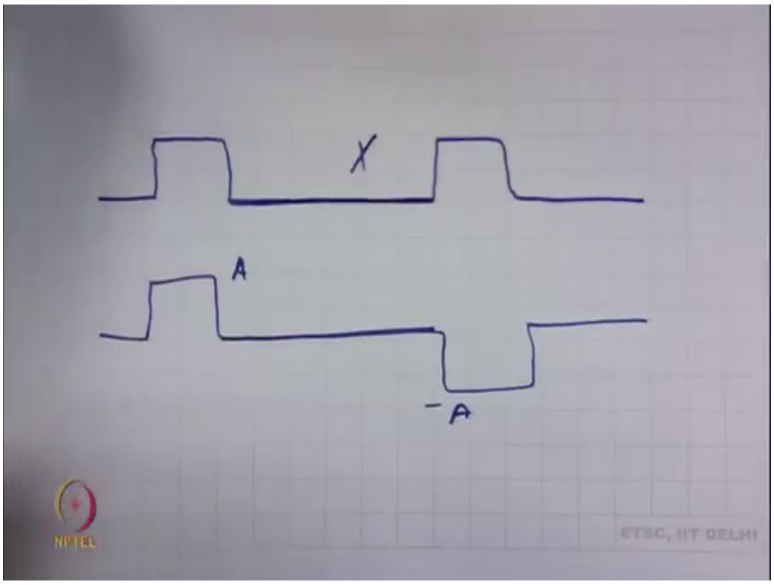
And if we look at this omega T by 2 pi 2 pi f T by 2 equals 2 pi we get f equals to 1 by T. So now, the first null happens at 1 by T and 1 by T corresponds to R b; that means, in this case the bandwidth that is required is R b instead of 2 R b as it was required in polar and unipolar signaling scheme, this is great. So, this has achieved 2 important advantages it has got a bandwidth same as R b.

(Refer Slide Time: 29:01)



Even though not R b by 2 but it is better than unipolar and polar signaling mechanism, it has got 0 power at DC right.

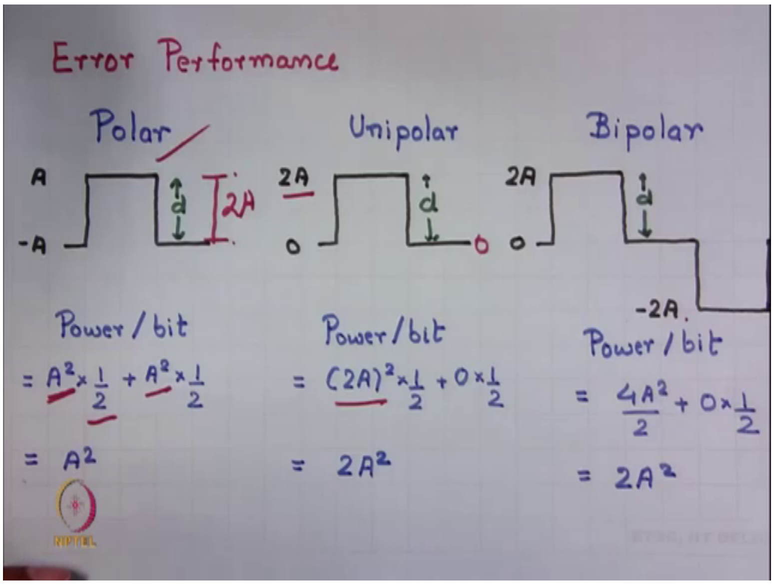
(Refer Slide Time: 29:21)



So now, you transformer and AC couplers would like this, it has been error detection possibility. So, it can detect one error. So, you are using bipolar and you have got this waveform you know instantaneously that there is some error here, because what you are expecting is if you have got an amplitude of A you know next time either you should get 0 or you should get minus A. Because if it is one we would have used minus A, if you are getting this you know that an error has happened somewhere. So, at least you are able to detect errors.

So, this is an advantage of bipolar signaling scheme disadvantage is its semitransparent so, again if you have long trains of 0's. So, if it is pretty long let us say this is not correct so, if it is pretty long, then you have no transitions, if you have no transitions your synchronization circuit will not like it and your receiver would get out of synchronization with your transmitter so that is its disadvantage. Clock recovery is also difficult because you have null at 1 by T, if no power at 1 by T where clock lies. So, intuitively clock recovery is difficult in bipolar signaling scheme.

(Refer Slide Time: 30:45)



Next, we can also compare which with the signaling schemes based on their error performance. Everything about this slide will become clear when we study detection, but intuitively it would be a good idea that I can introduce the error performance at a very naive level ok.

So, error performance typically depends upon the distance between the amplitude levels when you are transmitting 1 and 0. So, in this case this is the distance. So, for these signaling schemes to have kind of the same error performance you want to preserve this distance. So, you want to keep this distance same, I am talking about this distance. This distance has to be same; this is the distance between the amplitude of 1 and 0.

Now this distance in polar signaling scheme is 2 A the difference in the amplitudes if I want my unipolar signaling a scheme to have the same distance and if this is 0 then this amplitude should be 2 A. In the bipolar if I want to have the same distance then this should be 2 A and this should be minus 2 A. So, in this way these signaling schemes would have the same error performance, but let us see what is the power requirement in the same case, power requirement per bit. So, when I am transmitting 1, I am using a power of A square and I transmit 1 with the probability of half. When I transmit 0 the power is A square and I transmit 0 with the probability of half. So, power per bit becomes A square for polar signaling mechanism. If I do the same exercise for unipolar when I transmit 1 I am spending 2 A square and I use this much power half the time when I transmit 0 I do not use any power.

So, power per bit in unipolar signaling scheme comes out as 2 A square in bipolar again by doing the same exercise power per bit would turn out to be 2 A square. So now, you see that if I compare polar, unipolar and bipolar signaling scheme in terms of the power requirement per bit for the same error performance what I see is polar signaling scheme requires minimum power both unipolar and bipolar signaling scheme require twice as much power as the polar signaling scheme and this is an advantage of using polar signaling scheme. So, bipolar loses out on that, if your channel is not power constraint; that means, if you can pump in more and more power in the channel then there is no way you can use bipolar signaling scheme ok.

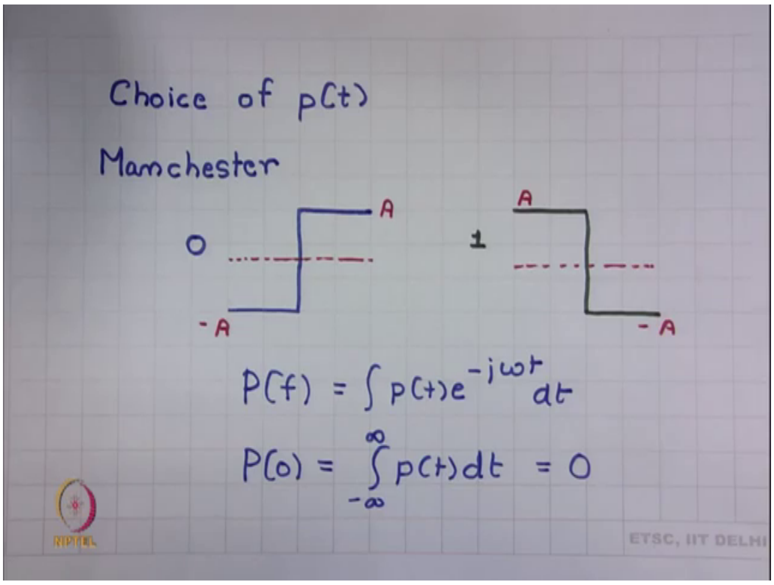
So, we have looked into three kinds of signaling mechanisms polar, unipolar, bipolar and I hope you can now take up other kind of signaling mechanisms yourself. So, some important examples of signaling mechanism at least one that I like you to read yourself is HDB 3.

(Refer Slide Time: 34:09)

Because this has also been used in practical digital communication systems try to read about HDB 3 by yourself, this is some kind of a variant of a bipolar signaling mechanism making it also transparent to long trains of 0's ok. So, so far I have been trying to motivate you how does the choice of mapping of these 1's and 0's to ak's influence my power spectral density.

But I could also influence my power spectral density by choosing p t, choice of p t is also important.

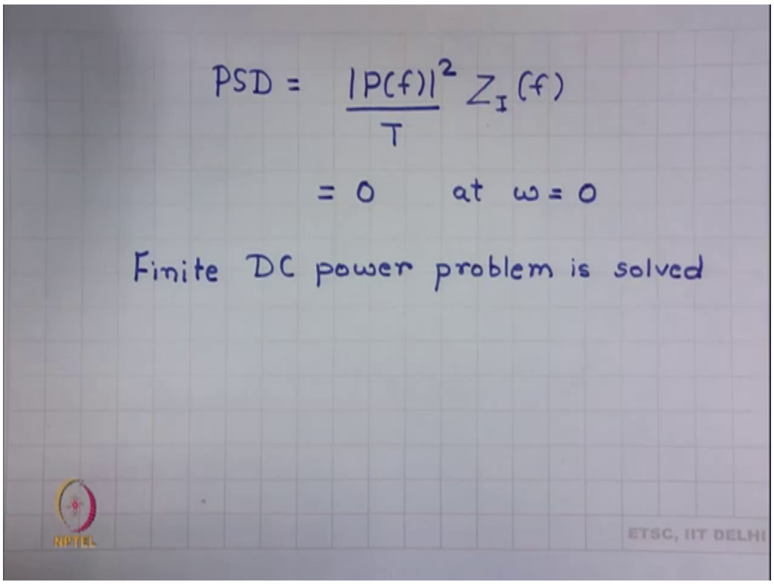
(Refer Slide Time: 34:45)



And one popular example of line coding scheme which takes the advantage of the pulse shape is Manchester coding. So, Manchester coding has such a pulse for transmission of 0 and such a pulse for a transmission of 1. Now you know that an immediate advantage of Manchester coding is that it has transitions even if you have long trains of 0's or long trains of 1's right. So, it transparent scheme long trains of 0's or long terms of 1's are not going to impact the synchronization performance. So, it is good in that sense.

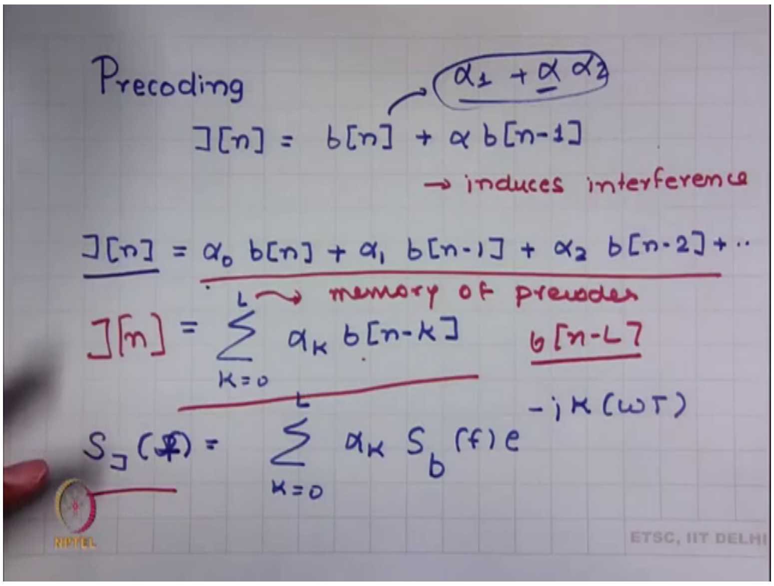
The second important advantage of this Manchester coding could be understood in terms of the Fourier transform of the pulse. So, Fourier transform of the pulse can be obtained by using this Fourier transform equation. And if I want to look at P of 0 this is a moments theorem which we have used. So, in moments theorem you put f is 0 then if f is 0 this becomes 1. So, P of 0 becomes the area of the pulse and the area of the pulse is 0, because it takes in a negative value for half the time and it takes a positive value for half the time. So, overall area is 0; that means, P of 0 is 0. If P of 0 is 0 without doing any complications what I have done is I have made power spectral density 0 at omega equals to 0.

(Refer Slide Time: 36:15)



So, this is one way in which finite DC power problem is solved. So, Manchester coding looks good in sense that it is transparent to a long sequence of 1's and 0's and it also has 0 DC power and thus there would be no problem when this signaling scheme is used over digital communication systems which are using transformers and AC couples.

(Refer Slide Time: 36:55)



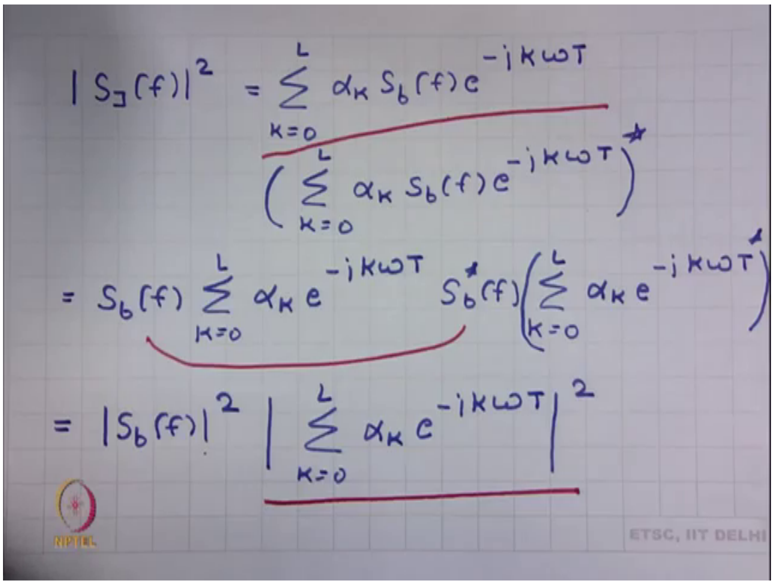
Next we are going to introduce a big idea in case of spectrum shaping and this idea is of pre coding ok. So, pre coding uses the idea that if you have a sequence a b n I am producing another sequence J n where this sequence J n is produced by adding a delayed version of the sequence to b n and I am using some fraction of this alpha is some constant.

So, I am adding some fraction of b n minus 1 to b n and I am using a resultant sequence which I call as J n. So, idea is simple. So, for example, if you have b n as some number alpha 1 and b n minus 1 is some number alpha 2, you add some constant to this you get a new number alpha 1 plus alpha alpha 2 which is store in J n. So, you have modified your sequence and this idea is known as pre coding, you are doing some pre coding before transmission. I could generalize this; I could say that I create this J n by adding alpha naught times b n plus alpha 1 times b n minus 1 plus alpha 2 times b n minus 2 and so on so forth ok.

So, I am generalizing how I am producing my J n. In short I can compactly write this by this expression where L is the number of delayed sequences that I want to have in my summation. So, L is also denotes the memory of your pre coder so; that means, it would take L previous samples; that means, that denotes the memory of a pre code. So, now, what I am doing is I am producing a new sequence J n from b n and their delayed version. Now if I want to find the Fourier transform of J n, the Fourier transform of J n is simple, this is not a function of n. So, we use this notation to tell that this is a Fourier transform of J n.

So, we use this is a Fourier transform of J n this is the summation times alpha k and the Fourier transform of this sequence I am saying it is S b f times e to the power minus j k omega T. Now what we are interested in, we are interested in the power spectral density that we will get after pre coding. And to get power spectral density first we have to calculate the energy spectral density.

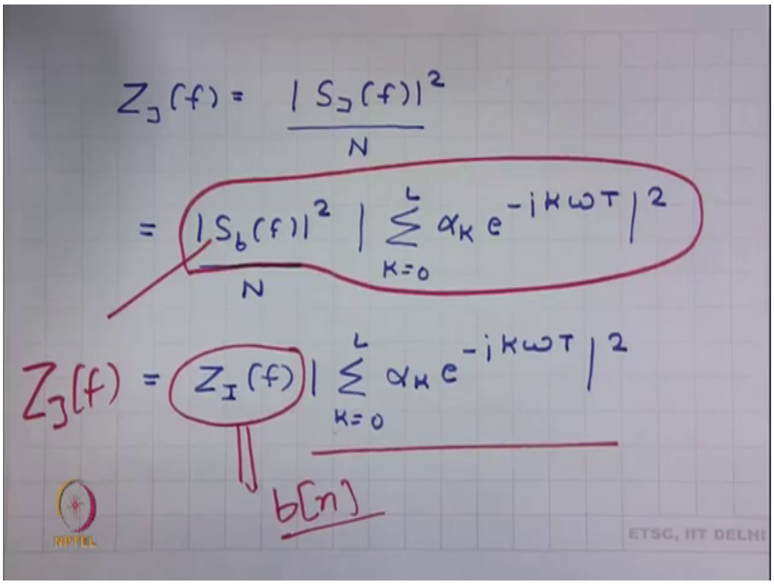
(Refer Slide Time: 40:03)



So, energy spectral density is nothing but it is the magnitude of the Fourier transform square. So, if we do this what we get is I have to take the Fourier transform which I have calculated here and I have to multiply with the conjugate of this right. So, we have been doing this for a while, take the Fourier transform multiply this with the conjugate of the Fourier transform ok.

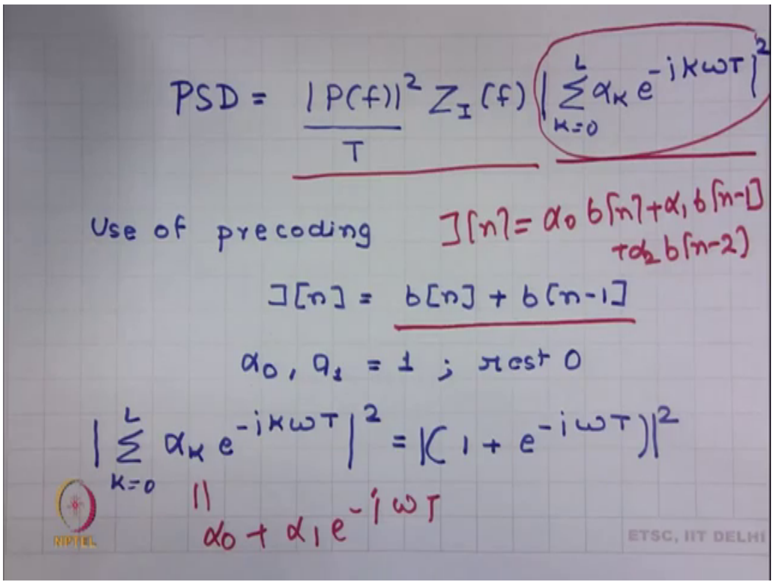
Now, I have pulled out this S b f because it is not a function of k pulling this out what and what remains is this summation that alpha k into e to the power minus j k omega T. what we have now is we have conjugation everywhere, but I am just interested in this S b f conjugating this, again this S b f is not a function of k I have pulled out. Remaining things I keep with a conjugation sign. Now I am pulling this S b f to this side which gives me mod S b f square and what I end up with is this quantity. So, this times conjugate of this which gives me mod square. So, this is the energy spectral density of J n.

(Refer Slide Time: 41:33)



Now, power spectral density of J n can be obtained by having the energy spectral density which we calculated dividing by N. So, energy spectral density that we calculated is this, we just calculated in the last page dividing this with N. Now dividing this with N; that means, I am dividing this, this is the energy spectral density of the original sequence. So, dividing this energy spectral density by n gives me the power spectral density of the original sequence which we called as Z I of f ok, we call this quantity Z I of f previously so, I am using the same notation. This gives me the power spectral density of b n and here I am finding the power spectral density of J n ok. So, power spectral density of J n is power spectral density corresponding to b n times this quantity.

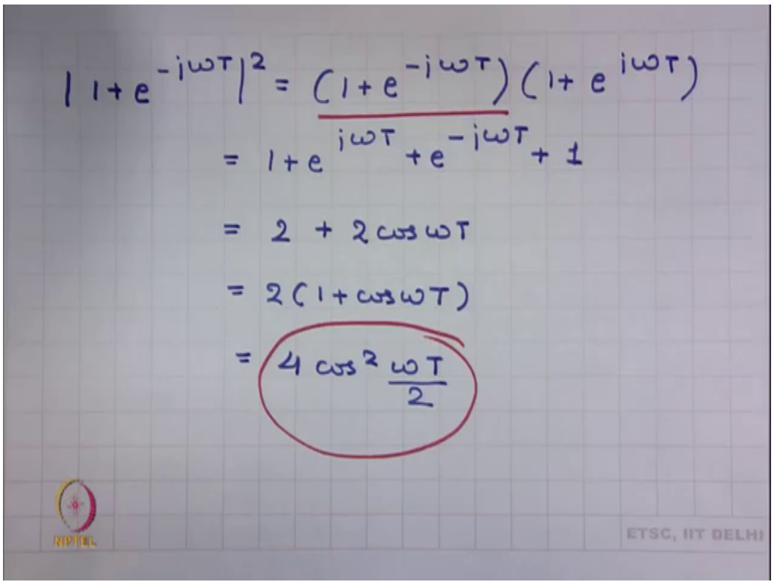
(Refer Slide Time: 42:43)



So, result in power spectral density now would have this extra term in there as well. So, up to now the power spectral density was just this, but because of this pre coding I have now this extra term in the power spectral density ok. Let me explain the use of pre coding by just taking a very simple example. So, I have J n b n plus b n minus 1. So, I am using as simple pre coding as this. So, I said J n is alpha naught times b n plus alpha 1 times b n minus 1 alpha 2 times b n minus 2 and so on so forth. So, alpha naught is 1 alpha 1 is also 1, other alphas are 0. So, in that case if I look at this, what is this? This will be simply so, expanding this I have alpha naught plus alpha 1 e to the power minus j omega T other alphas is 0 so, we do not have to consider them

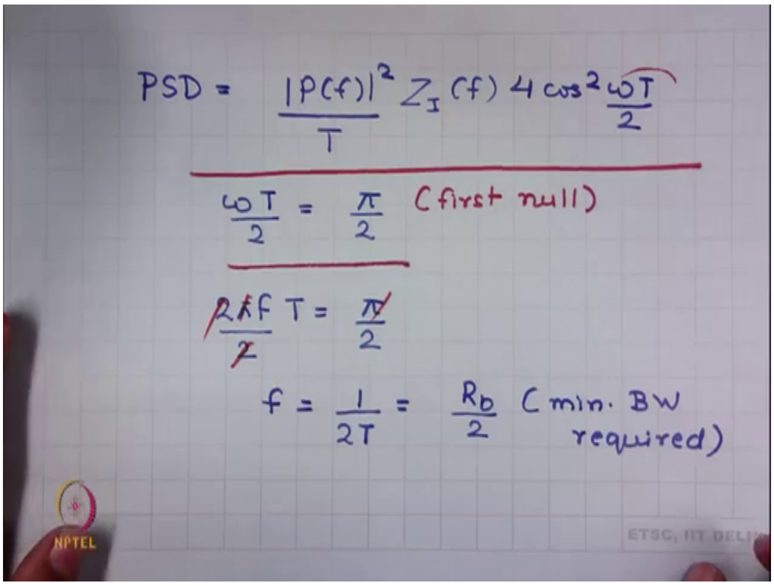
And now, when alpha naught and alpha 1 is 1, what I have this 1 plus e to the power minus j omega T mod square, I guess it is simple. Now let us see does all this math's give us something it should give us something otherwise as a waste.

(Refer Slide Time: 44:01)



So, this square I can write as this times its conjugate. So, taking a conjugation of 1 gives me 1 taking a conjugation of this changes the sign of this. So, I get e to the power j omega T. Now I multiply term by term I get 1 plus e to the power j omega t plus e to the power minus j omega T plus 1 and this is simply in the end 4 times cos square omega T by 2 great.

(Refer Slide Time: 44:39)



So, let us see the resultant power spectral density in this case becomes this. Let us look where has my first null gone to; the first null will go where this omega T by 2 is pi by 2 the first null will go to 0 when omega T by 2 is pi by 2. So, this decides the location of the first null. So, putting omega is 2 pi f 2 by T 2 cancels with 2 pi cancels with pi, what I get is f is 1 by 2 T which is R b by 2 the minimum bandwidth required in a digital communication system.

So, with using the simple pre coding mechanism it cannot be anything simpler than this what we are doing is simple pre coding mechanism, what I am doing is I am reducing the bandwidth requirement of my digital communication system it has dropped down to R b by 2.

So, pre coding mechanisms in general gives us some interesting results and thus they are used a lot in a spectrum shaping. So, today we have completed basically different kinds of line coding mechanisms and we have also seen how we can use this pre coding. It looks very powerful it is actually very powerful in shaping the spectrum of my signal. In the next lecture we will see some other ways in which we can calculate this power spectral density. We will see how can you calculate the same power spectral density by using what is known as Markov chains.

Thank you.