Principles of Digital Communications
Prof. Abhishek Dixit
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 22
Modulation: Spectral Description of Sources (Part-1)

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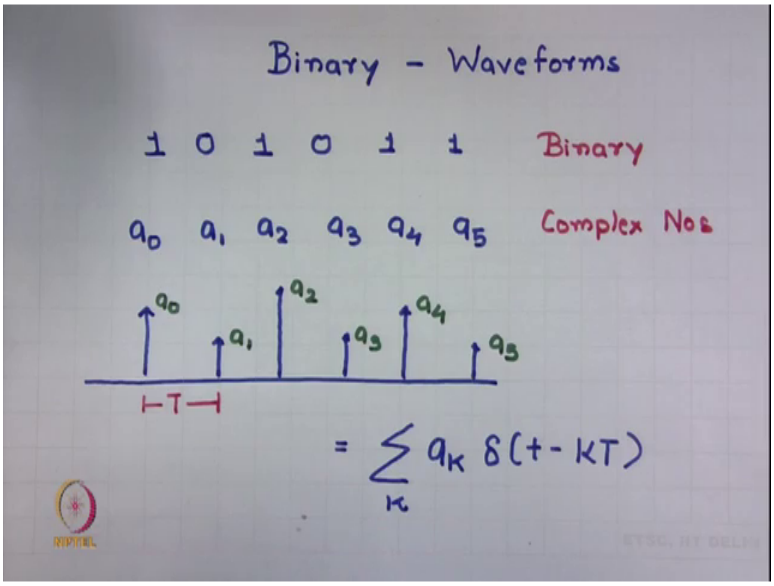

Welcome to new lecture on Modulation. We will start this lecture by revising some of the concepts. So, what we said that modulation basically deals with conversion of a binary sequence into waveforms ok. So, binary sequence means sequence of 1s and 0s. So, for example, we have a binary sequence and then what the first step we do in modulation is, we convert this binary sequence into complex numbers for example, here we have complex numbers a 1 a 1 a 2 a 3 a 4 and a 5 we can also map it into real numbers.

But for general I am assuming it to be complex numbers. Intuitively you can understand it as you are having train of impulses and the weights of these impulses is decided by these complex numbers. So, for example, this impulse has a weight a naught and a naught corresponds to this complex number. So, we have got the binary sequence and we have converted this into impulse train and the weights of these impulses are the complex numbers. Mathematically, we can understand it by this expression this just tells me that I have impulses which are T spaced.

So, T is the spacing between these impulses and the weights of this impulse is a Ks. So, a Ks means a 1 a 1 a 2 a 3 and so on so forth and I have this T shifted impulses K times where K goes from minus infinity to plus infinity so; that means, I am going to have unending impulse train ok. So, this is basic idea how you represent this weighted impulse train by this expression mathematically.

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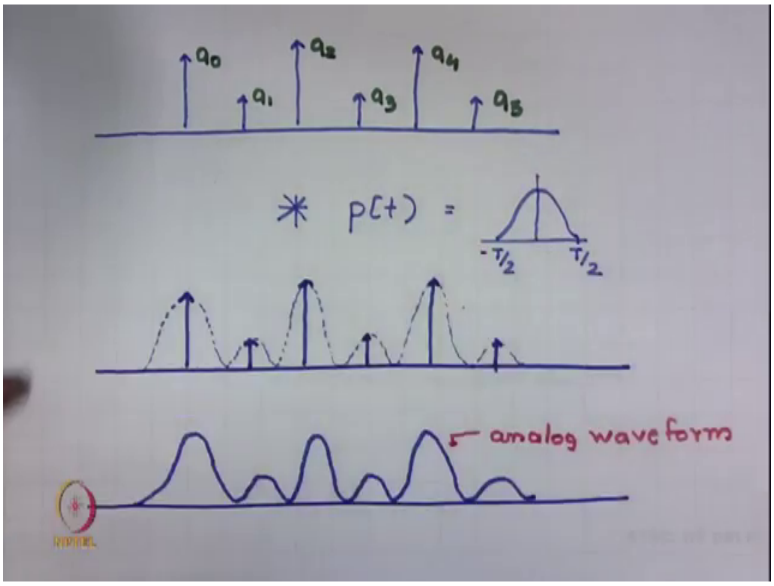

So, after we have got the impulse train, the next step that we do is we pass this through filter ok. So, we have assumed that I have a filter, which has got the impulse response P t. So, on passing this through a filter mathematically what would happen is that this impulse train would get convoluted with the impulse response of the filter which is P t.

And this impulse response of the filter I have assumed to have such a shape; that means, this is a kind of a pulse which goes from minus T by 2 to plus T by 2 ok. So, this is the impulse response of the filter and the effect of passing this impulse train to the filter is that you need to convolve this with such a pulse, convolution of this pulse where this impulse train can be simply achieved by just putting the caps of this pulses on these impulses. So, you take you pick this pulse, you put a cap of this pulse around this impulse, you get this waveform then you put this cap around this impulse you get this waveform you put a cap around this impulse you get this and so on and so forth.

So, basically what you would end up with is an analog waveform where this waveform is this cap and the height of this cap is decided by the weights of these impulses. So, after you have passed the sequence through a filter you will get an analog waveform. And this analog waveform will be a basement waveform meaning that most of its energy would be centered at t c.

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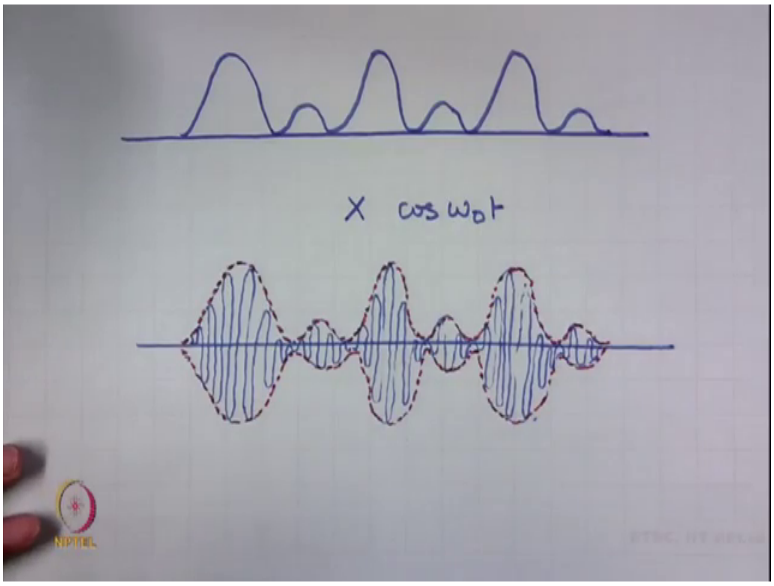

Then next after we have achieved analog waveform what we would do is, we would multiply it with the cos omega naught t; cos omega naught t on multiplication would induce rapid phase fluctuation. So, you would have a waveform like this where the envelope of this waveform is decided by this analog waveform and then this oscillates very fast at a frequency of omega naught.

So, what we are discussing is, the typical steps that happen in a modulator these are typical step steps not all modulators would do exactly like this, there might be some variations, but it is a good idea to start by thinking about a modulator along these lines. So, let us look at the block diagram of the modulator which we have seen couple of times.

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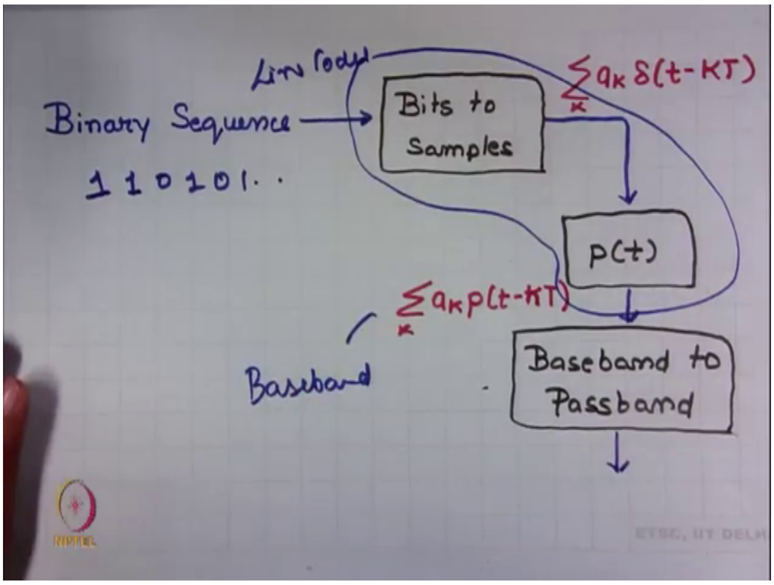

So, we have a binary sequence, remember that a binary sequence is a sequence of 1s and 0s and this binary sequence is then converted into samples and samples we know its nothing, but it is a weighted train of impulse. This weighted train of impulse then passes through a filter with impulse response P t, P t and we get a weighted pulse train instead of an impulse train and then this is a baseband signal and this baseband signal is converted to passband signal by using a multiplier.

So, what we are interested in this lecture is in understanding the impact of this mapping of binary sequence to these a Ks and what is the impact of impulse response of the filter on the spectral occupancy of various modulation escapes ok. So, remember what we have also said in one of the previous lectures that, if we focus only till this part we can also say modulator as line coder so in fact, in this lecture we will be dealing with these spectral description of various line coding schemes. So, line coding means you convert a binary sequence to samples and then you convert these way to train of impulse into weighted pulse train ok. So, this is line coder. So, line coder is same as modulator if we are ignoring this block.

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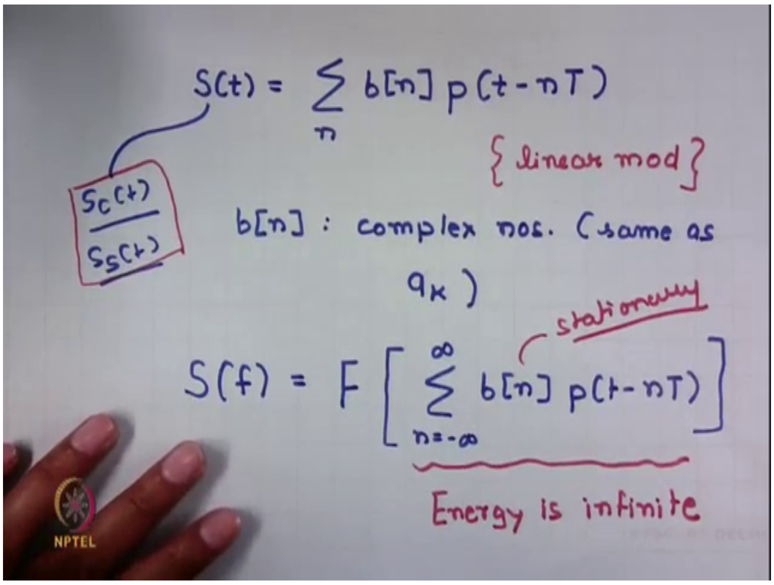

Let us see something more. So, we have said in the last lecture that we want to understand this waveform. So, S t is the waveform that is available here at this point and we can think about this as this. So, here this b n is the same thing as a case sometimes I am using a case and in this context I am using b n, so, b n a same things as a case ok. So, in general I can allow them to be complex numbers all right.

Now see this that this b n are multiplied by the common pulse shapes ok. So, each b n is multiplied by the same pulse shape and this kind of modulation is known as linear modulation. So, linear modulation is the kind of modulation where you have the same pulse shape ok. Now what is the impact of having this b n as complex numbers let us see it I have a slide for this.

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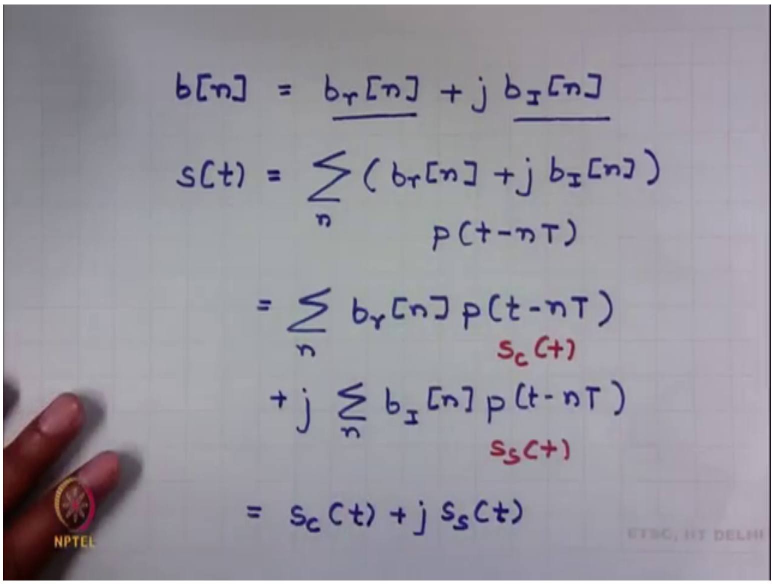

So, if I assume b n as a complex number, it can be thought as a sum of real part and an imaginary part. So, S t can be written down like this. So, I am just decomposing b n into its real part and its imaginary part, then you can multiply this with P t P t minus n t and we get 1 real waveform which I call S c t and you have another real waveform which I call as S s t and S t can be thought as just the sum of S c t plus j times S s t.

So, this is the impact of assuming b n to be complex, then S t is build up of 2 or real waveforms S c t and S s t S t then will be complex. After having understood let us come back to this, so, we want to take the Fourier transform of S t to calculate the bandwidth because typically that is used, but one problem that you would run straight away when you want to calculate the Fourier transform of this is, because we always assume this b n to be stationary.

Why do we assume to be stationary? We have talked about this when we have discussed random processes and we have said that the stationary wave forms by the practical model that we want to use because we do not want to say when this waveform stops and for what period it runs in ok. So, to avoid those kind of difficulties, we simply assume that the waveform is stationary. Now if this b n sequence is a stationary and if this runs from minus infinity to plus infinity, it needs to run from minus infinity to plus infinity if b n is stationary, then we know that the energy of the sequence is infinite with probability 1.

And then you would have problems in finding the Fourier transform of this waveform, because the Fourier transform of infinite energy waveforms do not exist normally right.

So, you are not sure whether the Fourier transform would exist or not sometimes it exist if we allow impulses. But in general you cannot say whether the Fourier transform of this waveform would exist because it has called infinite energy.

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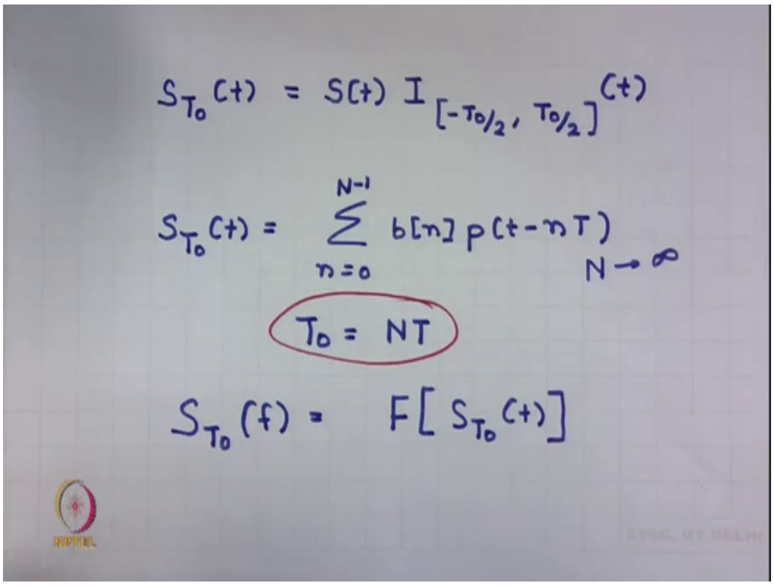

To avoid those kind of mathematical problems what we do is we define another signal S T naught t where S T naught t truncates S t. So, instead of S t going from minus infinity to plus infinity what we say that S t goes only from minus T naught by 2 to T naught by 2 ok. So, we are kind of making it a effectively stationary process if t naught is pretty large ok. The effect of truncation in this could be understood by limit N. So, instead of N going from minus infinity to plus infinity what we can assume that, n goes from 1 to N minus 1 where N is pretty large, but it is not infinity ok.

It is just dealing with mathematicians nothing else, we are just trying to make this waveform only exist for a finite duration. So, the total duration we said S T naught and if I have n samples and the difference between the 2 samples is t. So, the total duration would be N times T its a good idea to note this down that T naught N T. So, now, we can take the Fourier transform of this that happened for our good because now this S T naught t is having finite energy and if it has a finite energy then you can take the Fourier transform of this, now the Fourier transform of this is defined. So, this is S T naught f ok.

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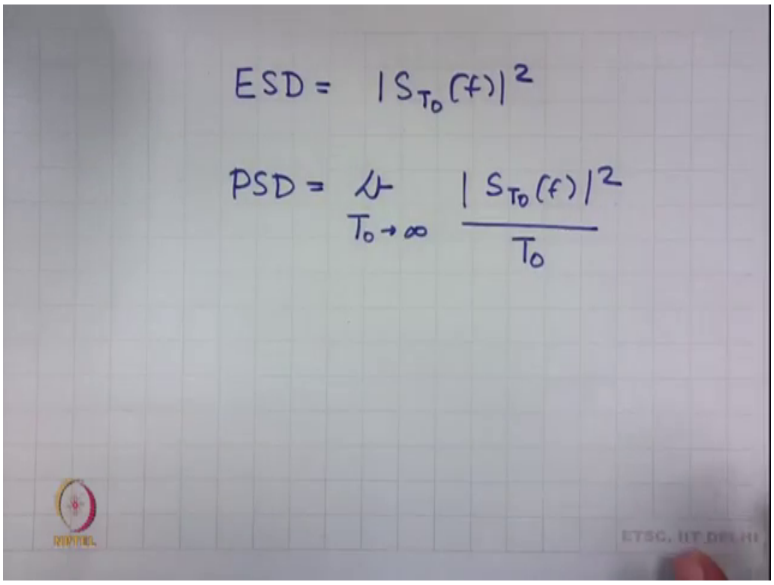

So, we did cover everything in the last lecture I am just revising giving it a better look probably. So, what we said in the last lecture that energy spectral density is nothing but it is the mod square of the spectrum of a signal that is the energy spectral density and power spectral density can be defined by taking the limit of this expression, for T naught tends to infinity and divide it by T naught. So, energy spectral density divided by the total duration of the signal and make the total duration of the signal tending to infinity, but not infinity ok. So, this is how we would define the power spectral density. So, we covered all of this in the last lecture and now the time to learn new stuff and the one thing that we will do is to find out what is this value for different kind of waveforms.

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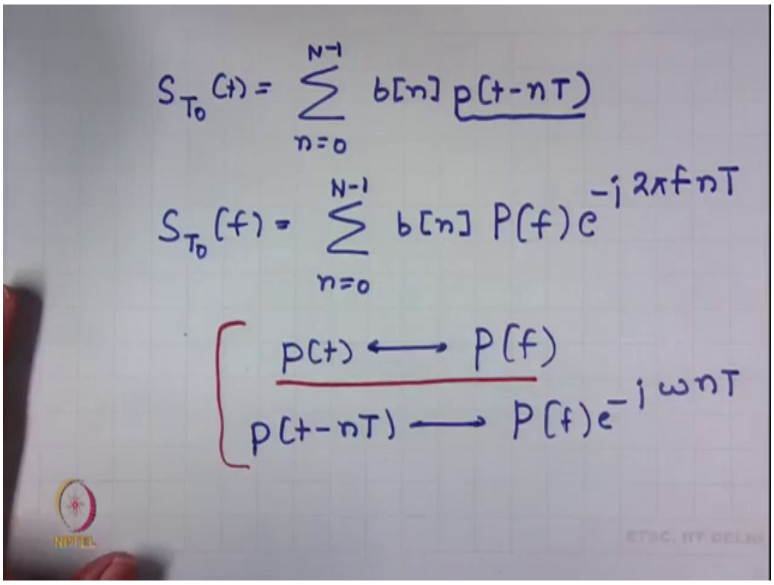

So, let us start with this is the same formula that S T naught t b n and this is P t minus n T and one thing that we need to do now is to take the Fourier transform of this, we can take the Fourier transform of this because N is finite, N is very large, but it still it is finite.

So, taking the Fourier transform of it b n we know its a constant with respect to time b n is not a function of time P t P t minus n t is a function of time. So, its a constant with respect to time. So, I have b n now I have to take the Fourier transform of this pulse; Fourier transform of this pulse I have assumed is P f into e to the power minus j 2 pi f n T. This is by the properties of Fourier transform. So, if we assume that the Fourier transform of P t is P f, then the Fourier transform of P t minus n T would be P f times e to the power minus j omega n T, so, omega is 2 pi f. So, using these properties of Fourier transform we can quickly calculate the Fourier transform of this expression its simple.

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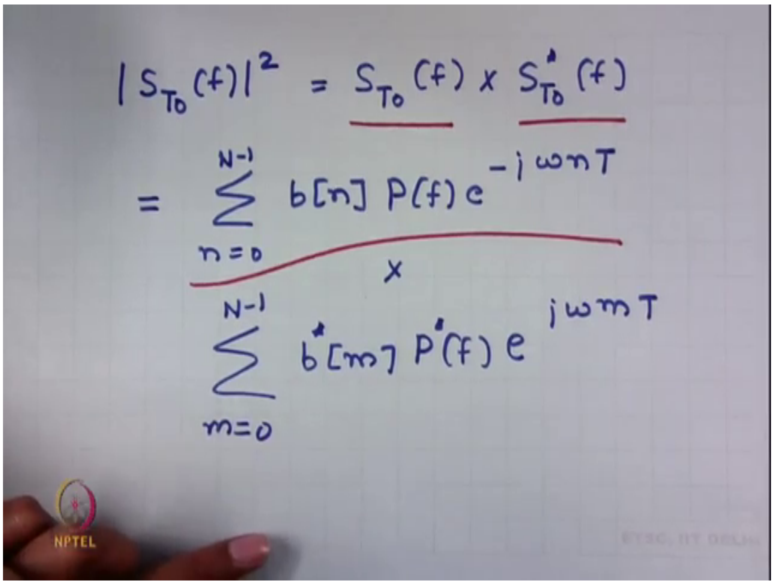

Now so, we have calculated the Fourier transform, now the task is to calculate energy spectral density which is mod of the spectrum of S T naught t square and we have seen it several times that the best way to compute this quantity is to multiply this quantity whether its conjugate. So, here we have to multiply S T naught f whether its conjugate ok.

So, that will give me mod square of S T naught f. So, its also easy and that is why we are not writing everything today. So, we have S T naught f which is this we derived in the last slide and then we have to multiply again with this, but we have to take the conjugation. So, if we have to take a conjugate here. So, it becomes b conjugate m conjugation here we got a conjugation here, when I am taking a conjugate of this quantity this minus will become plus. And everything else remains same, I have also introduced one more change that instead of n going from 1 to N minus 1 now we have m going from 1 to N minus 1 and while we do this we do it often and this is because we want to preserve the cross components. So, for example, in this multiplication you would have b 2 multiplied by b 5 right.

So, all these things become easy if you assume a different running variable this is commonly the case ok. This we do every time we need to multiply 2 summations ok. So, we have kind of trying to calculate the mod S T naught f square. Now one thing that you can see here before we move to the next slide then this P f is not a function of n. So, this is a constant with respect to summation. So, we can pull this out. Similarly this P f conjugate is not a function of m, so, this could also be pulled out. So, pulling this P f and P f conjugate out of this two summations what you end up with is mod P f square ok.

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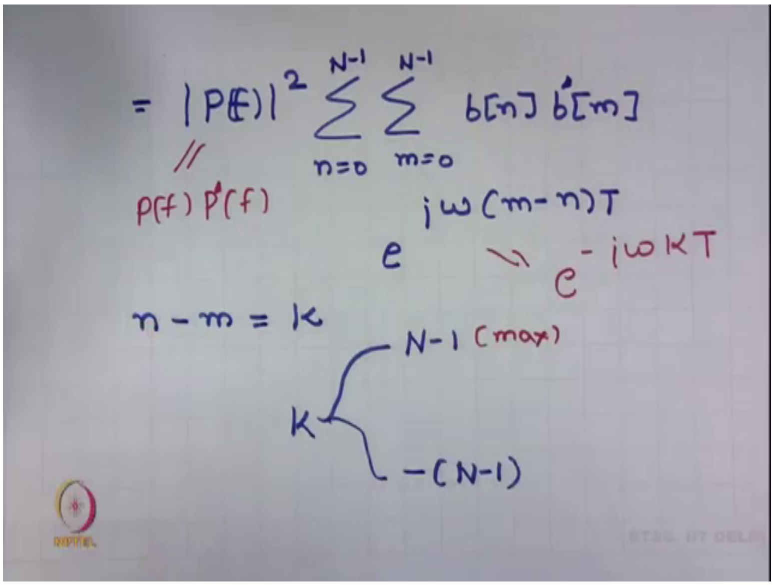

Mod P f square because this is nothing, but P f into P f conjugative ok; so, that is this part is taken care of. Now again from Fermats theorem we do not worry about the order of summation we simply club them together. So, now I have taken all two summations together what do you end up with is b n into b m conjugate into e to the power j omega and here I was having omega m and here I was having omega n within minus.

So, when I add these two things, I get omega m minus n. So, this is here j omega m minus n T. Now what we do is we try to substitute n minus m s K. So, first thing that we try to understand what is the limit of K from where to where K goes. And if you want to see that what should be the maximum value of K? Maximum value of K would happen when m is 0 because m is anyway positive right.

So, if I assume m is 0 I should get the maximum value of K which is the maximum value of n which is n minus 1. So, this is the maximum value of K. Similarly to find the minimum value of K, I need to assume n is 0 and then I find out the minimum value of K K which is the maximum value of m minus. So, minus maximum value of m minus N minus 1 because maximum value of m is N minus 1 right, we are running this summation from 1 to N minus 1. So, in short K should go from N minus 1 to minus N minus 1. Now one thing that we should do now is to replace this n minus m with K. So, this should become e to the power j omega minus KT.

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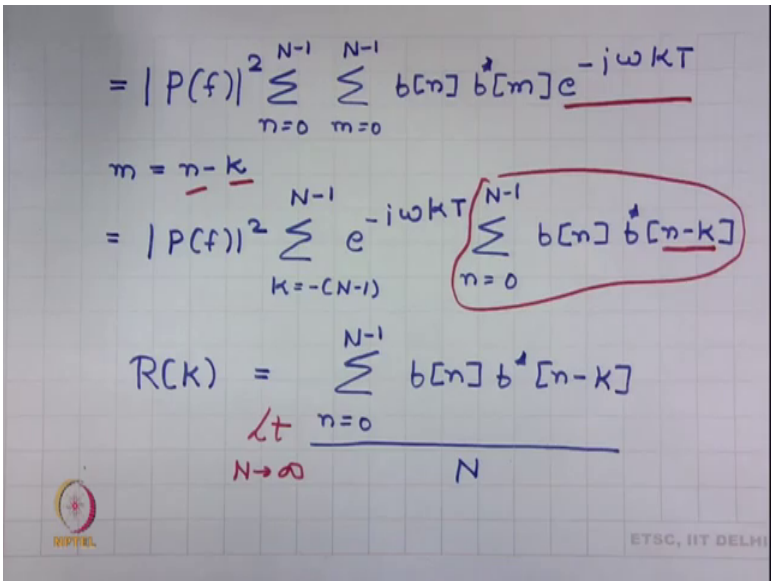

So, this is what I have here nothing difficult. So, mod P f square because it was already here the summations are intact have not still changed the limit or anything, b n into b m conjugate into e to the power minus j omega KT and we know that m is n minus K. Now we want to now change the limit. So, what we do if we see this is not a function of n. So, first what we want to do is to have 2 summations a summation with K and a summation with n as a running variable. So, instead of m because m we have substituted as n minus K. So, now, I have 2 variables n and K and we want to get rid of m we have got rid of m by using this change of variables. So, now, we have 2 variables K and n as you can see m is replaced by n minus K.

So, in this integration there was the m here we have replaced it with n minus K. And so, I have 1 summation that respect to k, k goes from minus N minus 1 to plus N minus 1 and have another summation with respect to n which as before goes from 1 to N minus 1. Now if you see here this was only a function of k. So, I have pulled this here and this I have combined all terms which has n as a running variable. So, I can reduce this summation in to this summation its trivial kind of its there is nothing difficult in here. Now autocorrelation function we define using this. So, this quantity should tell you that is in autocorrelation function we have divided by n and we are limit N tends to infinity.

Now, you should also see that when we are having these quantities like autocorrelation function or other things let me write.

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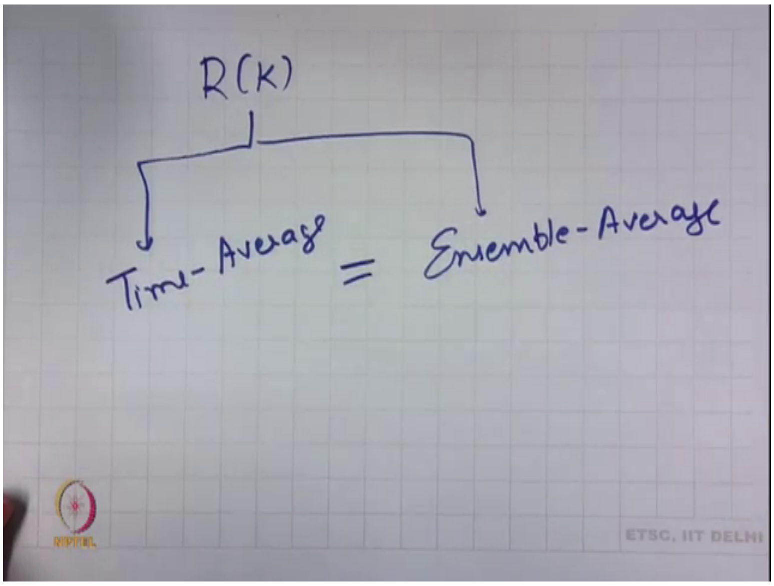

So, if you are having Rk or autocorrelation function there are 2 ways in which you can calculate this either by doing a time average operation or by calculating the ensemble average. So, we have seen all this before and for an ergodic process, this time average is same as ensemble average and we have also said all practical processes are ergodic processes and so, we can either calculate this autocorrelation function using a time average operation or we can find this using ensemble average ideas whichever idea is good we can use that.

So, currently I am finding autocorrelation function using time average idea and after a while we will do the same computations using this ensemble averaging or statistical averaging idea. So, right now we are trying to use this time average idea, which is more convenient at this point. So, autocorrelation function or time averaged autocorrelation function if you like to call it in that way is given by this quantity where you have to take the limit and tends to infinity. Now if you want to replace this thing you should realize that this is nothing, but R k times N because this whole thing is R k N is anyway very large, we are assuming N to be very large. So, this quantity is nothing, but R k times n.

So, substituting that in this expression what do we get is mod P f square everything remains same. So, let me put it here. So, this is same, this is same this quantity we have replaced with R k times N ok.

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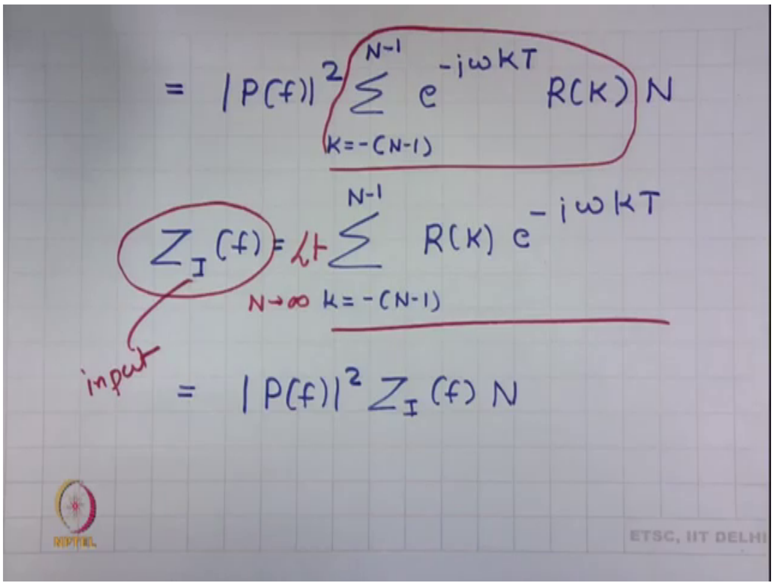

So, now you look at this what do you see? So, this i plug here. So, this is R K e to the power minus j omega K T summation K going from minus of N minus 1 and N minus 1 where N tends to infinity this is nothing, but it is the Fourier transform of autocorrelation function and Fourier transform of autocorrelation function is nothing, but it is the power spectral density ok. So, what you have is a power spectral density of input sequence, so, which we denote with Z I of f.

So, power spectral density I represents input sequence. So, finally, after all this maths what we get is mod P f square, this quantity we said is Z I f and anyway we have N here. So, we got this finally, coming back to what we were deriving and see what we have got now.

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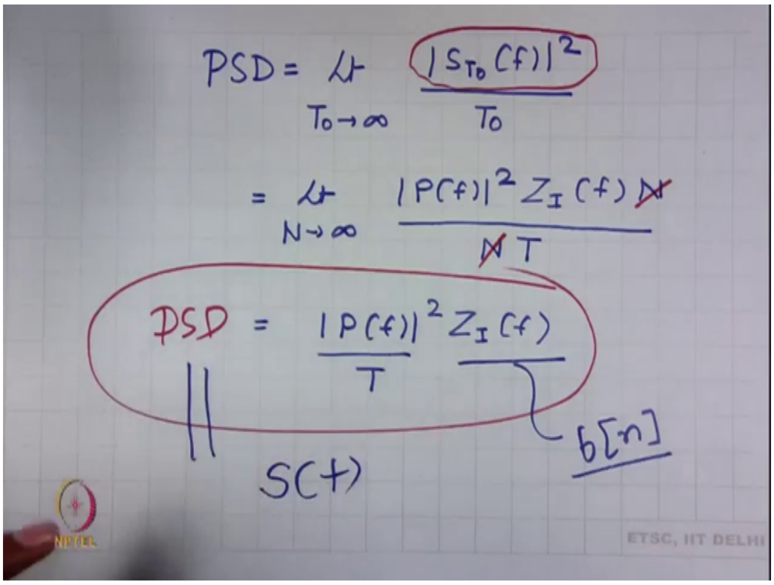

So, by definition we said power spectral density is this quantity will limit T naught tends to infinity and this quantity we have already evaluated now, we have evaluated the energy spectral density which we have derived it is nothing, but mod of P f square Z I f times N using this value of energy spectral density here.

And T naught as we have said is nothing, but N times T number of samples into the time difference between the samples and now instead of putting T naught as infinity we can put N as infinity right is one in the same thing. So, now, canceling N by N, what we get is a very neat formula it is power spectral density is mod P f square by T into Z I f where, Z I f is the power spectral density of the input sequence. So, this is one thing that you should know and remember its very useful formula. So, one big thing from here to see is that the power spectral density depends upon the a square of the magnitude of the spectrum of the modulating pulse; that means, which pulse you choose decides in some sense the power spectral density.

So, we have to be careful with the choice of our pulse. Secondly, it also depends upon the power spectral density of the input sequence and we will see some examples of how it influences the output power spectral density. So, just to avoid any confusion let me also say that this is a power spectral density corresponding to S t and this is power spectral density corresponding to b n ok. So, this we have to see carefully. So, this is b n this is power spectral density corresponding to b n and this is the power spectral density corresponding to S t and these are related by this. And this formula is also valid only for linear modulation. So, we are using just linear modulation and this you have to be careful with ok.

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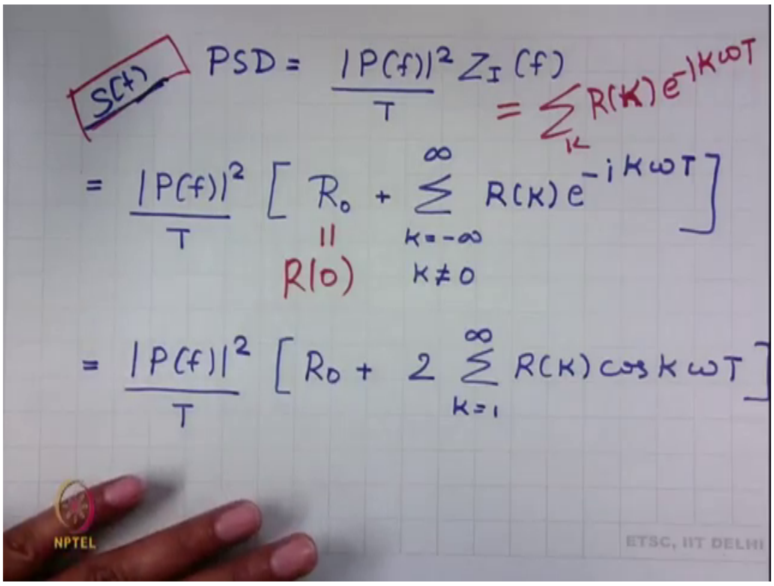

So, finally, P S D of the analog waveform. So, when I am seeing analog waveform I mean S t. Just keep this in mind otherwise it can be confusing sometimes because we have several power spectral density is going in here. So, we are saying the power spectral density of S t is this quantity mod square of P f where, P f is the Fourier transform of the modulating pulse divided by T, T denotes the spaces at which bits arrive or you want to create sequences and Z I f is the power spectral density of input sequence.

Now there are various ways to write it more powerful than this all these equations are derived simply from here. We know that anyways Z I f is what is it is the Fourier transform of we have already seen this, Z I f is a Fourier transform of the autocorrelation function of the binary sequence right nothing great in that.

So, instead of writing Z I f, I have written it in a somewhat a strange way in this because this is quite useful. So, what I have done is I have pulled out R 0, so, I have R 1 here. So, this you can interpret as R 0. So, I have pulled out R 1 then what I end up with is the summation except R 1 point. So, this is the summation this here K goes from minus infinity to plus infinity except K equals to 1 because that I have included here. For some reasons which will become clear the reason is simple that this is probably more useful than combining everything together ok. We will see some examples and then probably you will appreciate the use of this expression anyway its one in the same thing just its more convenient to use it sometimes.

So, K goes from minus infinity to plus infinity instead of that, I have assumed K going from minus infinity to plus infinity except K equals to 1 and K equals to 1 point I have pulled out. And this expression can also be converted into this expression again sometimes you will use this sometimes you will use this both are pretty useful. So, it is a good idea to see that there are various ways in which you can find the expression of the power spectral density. How we get this from this let me explain that as well its tedious, but its also trivial. So, let us just see we are trying to derive this expression from this expression.

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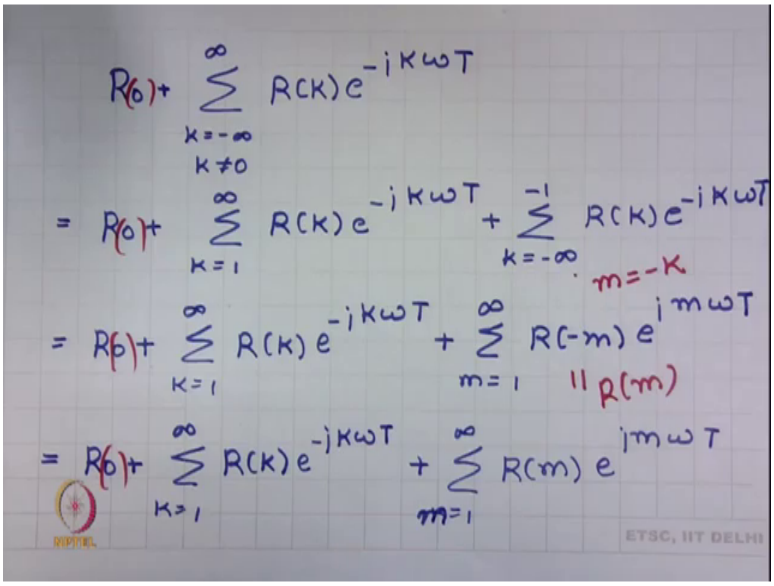

So, we have R 1 plus R K e to the power minus j K omega T. So, this is R 0 let me put it ok. So, that there is no difference between the 2 notations and now you see that K goes from minus infinity to plus infinity except K equals to 0. So, I can break this down into two summations where K goes from 1 to infinity everything same and K goes from minus infinity to minus 1 anyway K was not including 0.

So, I have broken down this summation into two summations that is good then I do not do anything, but here what I do is I introduce a change in variable. So, I have used m instead of K I am using m and I am saying m let m be minus K ok. So; that means, K should be replaced by minus m. So, I am replacing this K with minus m this K with minus m. So, this becomes positive and the limits should change instead of K going from minus infinity to minus 1 m should go now from 1 to infinity just normal exercise.

So, now I have this and now realizing that R of minus m is same as R of m because autocorrelation is an event function, I can rewrite this expression just by replacing R of minus m with R of m. So, everything remains same just I have replaced R of minus m by R of m, because they are same its an even function autocorrelation is even function. So, R of minus m is same as R of m ok, so, we have got this.

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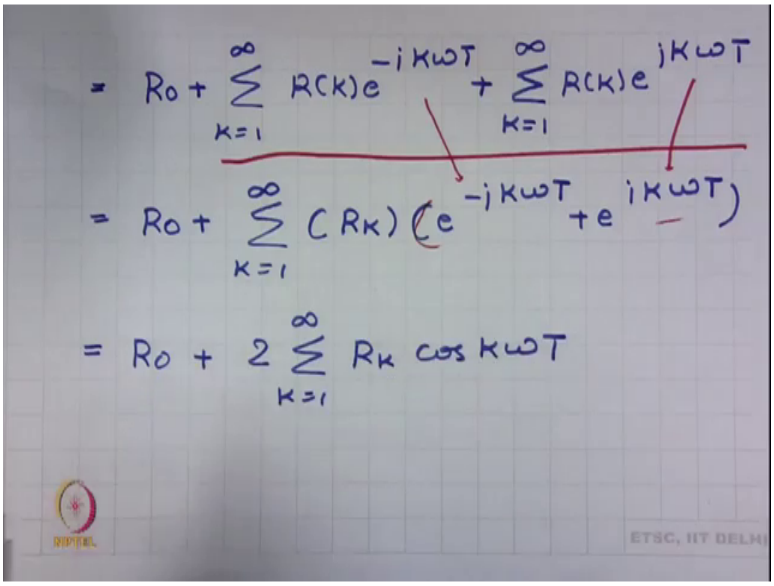

Now let us see what we do is now I do again something trivial because this is with m m going from 1 to infinity m is just a running variable. So, I can very well replace it with K nothing would happen. So, I am replacing now m with K and I am getting the same expression, let me try to put everything together. So, I had this. So, this is same as this just here I replaced m K nothing changes just instead of m I have got the K ok.

Now, you see everything has same almost other than these complex exponentials which are rotating in different directions. So, I can pull this out, I can take this summation R K common this I can multiply with e to the power minus j omega T from here and I have e to the power j K omega T from there and this you know is cos K omega T times 2 and this is what we have. So, it was a very simple proof nothing difficult, but it is a very convenient thing to realize to remember because it would be useful right.

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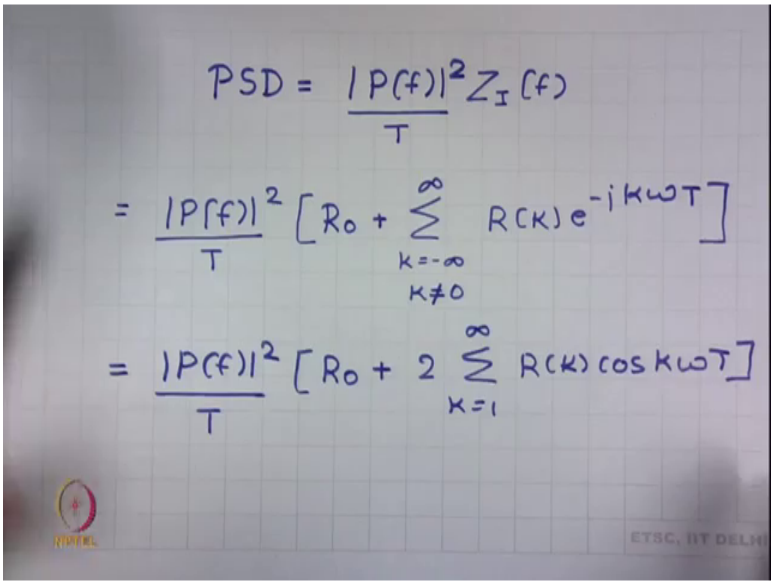

So, let me remind the three expressions of power spectral density that we will use this as simple to state and a convenient expression is this and a convenient expression is this ok. So, there are 3 ways to state the power spectral density.

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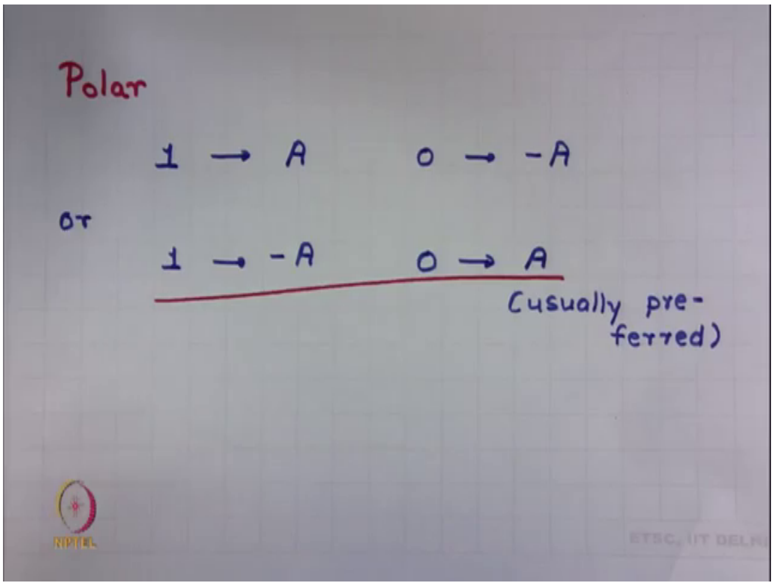

Let us now see some examples and the example of line coding mechanisms that I am starting with the first one is the polar signaling mechanism a polar line coding. So, in polar mechanism what happens is you can map A to 1 and you can map minus A to 0. So, we have a binary sequence 1 or 0 1 is mapped to A and 0 is mapped to minus A. There is nothing sacrosanct in this mapping what you can also do is you can map 1 is mapped to minus A and 0 is mapped to A and in fact, this mapping is usually preferred and what is the reason for this? The reason is that if you use this mapping the XOR of these binary sequence is same as multiplication of these numbers ok.

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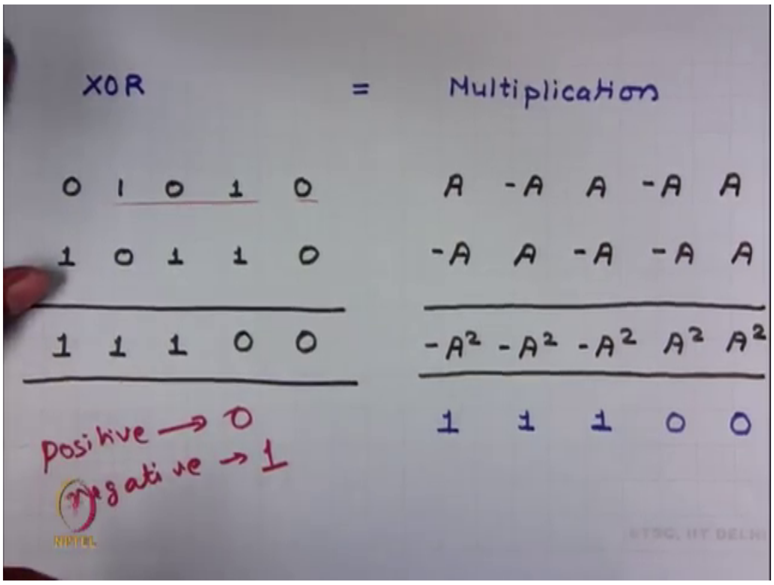

So, if you do the XOR operation on the binary sequence for example, if I have these two binary sequence and if I choose positive for representation of 0 and negative for the representation of 1. And if I see the XOR operation of these two binary sequence I obtain this sequence and in this case I am mapping positive voltage to 0, negative voltage to 1 positive to 0 negative to 1.

Similarly, I can have the representation for this binary sequence like this and if I multiply these numbers, I get some negative and positive numbers. If I assume negative numbers to be 1 and positive numbers to be 0 I see that the multiplication of these numbers is same as the XOR operation of the binary sequence and this equivalence is sometimes convenient in certain applications and though for our case it would not make a difference whether you use this mapping or this mapping.

But for practical systems this mapping is usually more preferred and the idea is that we want to have the equivalence between XOR operation for binary sequence and multiplication operation for real numbers and that is why this is usually preferred. But for this course we will continue using this mapping and this is because it would be more consistent when we steady line coding schemes. So, even though this is usually preferred, but for academic reasons I would continue using this mapping.

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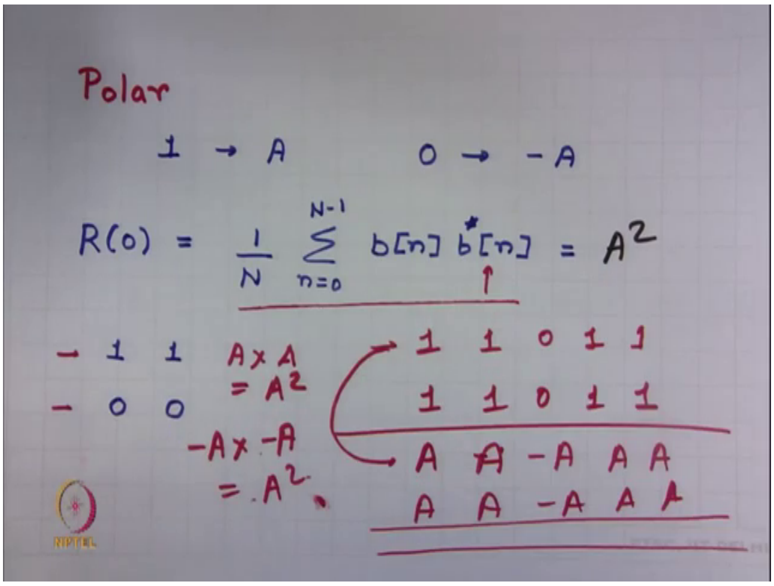

Let us see how we can find the power spectral density for polar signaling scheme and as I have said I will use this mapping where 1 is mapped to A and 0 is mapped to minus A and first thing that I need to find is R of 0 what is R of 0? By definition you have to compute this summation if you are interested in finding R of 0.

So, here the sequence is the same as the sequence, but it comes with the conjugation, but because I am assuming that these numbers A are real. So, if you take conjugation or if you do not take conjugation it would not matter ok. So, let us assume that I have a binary sequence some binary sequence does not matter and the next binary sequence that I have to take should be the same binary sequence as I have assumed here because this is same as this other than the conjugation, but conjugation really does not matter.

So, I will have the same binary sequence and then I can map this binary sequence to some voltages like this. So, here I am mapping these binary sequence to these voltages and you can see what happens is that if you have 1 here also you will have 1 because the sequence is same as this sequence.

So, if you have 1 in this sequence in this sequence also you will have 1. If you have 0 in the sequence in this sequence also you have 0. So, because these sequences are 1 and the same thing. So, I can have two situations either I will have 1 or I will have 1 if I have 1 in this sequence in the next sequence also I am going to have 1 ok. So, if I want to find out what is the result of this multiplication, here I am multiplying the numbers associated with these binary digits ok. So, if I am associating A to 1 this 1 will also be mapped to A. So, I will have A square for these 0s I will have minus A multiplied with minus A which again will be A square.

So, whether you have 1 or 1 every time we will be getting just A square ok. So, this summation you can think yourself will be nothing, but it will turn out to be as A square right. Because the product of this multiplication will always be a square whether you have 1 or you have 0 ok. So, the average will also be always A square.

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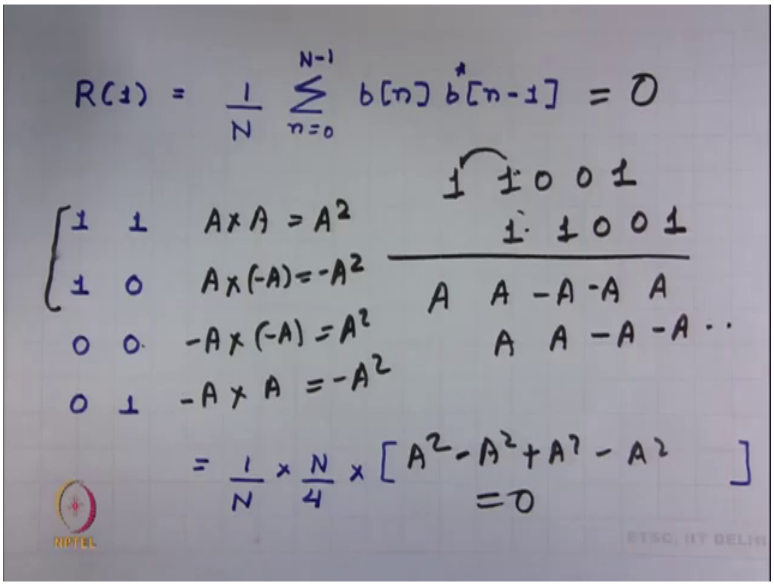

Let us now see what happens if I have R of 1. If I have R of 1 then this sequence would be delayed by 1 bit corresponding to this sequence. So, if I assume a binary sequence, this b n minus 1 will be a delayed version of this. But this b n and b n minus 1 are not the binary sequence they are the voltage corresponding to this binary sequence.

So, if I use the mapping which I have used before. So, 1 is mapped to A this one is mapped to A, this is minus A minus A A, so, I will have and so on so forth. Now if you have 1 here you can have either one. So, this situation or you can have 0 these bits are statistically independent of each other. So, if you have 1 here you can have very well a 0 or a 1 it is delayed by 1 unit. So, you can assume that if you have 1 here, you can very well could have had 1 or 1.

So, if I have 1 in this sequence I can have 1 or 1 similarly, if I have 0, you can have 0 or 1 and all these events will be equi probable if you assume that the probability of 1 is same as the probability of 0 ok. Corresponding to these binary digits the voltage mapping would be A and A and we have we are interested in finding their product.

So, this will be A square this will be A times minus A, this will be minus A square this will be minus A times minus A, this will be A square this will be minus A times A this will be minus A square and each of these pattern would happen and by 4 times if I am assuming N as the total length of the sequence if N is very large and all these 1s and 0s are equi probable I can assume that each of this pattern would happen for N by 4 times.

So, I have 1 by N each of this product would happen for N by 4 times and you have a contribution of A square minus A square plus A square minus A square and you can easily see that this will be 0 ok. So, R of 1 is nothing, but it is simply 0.

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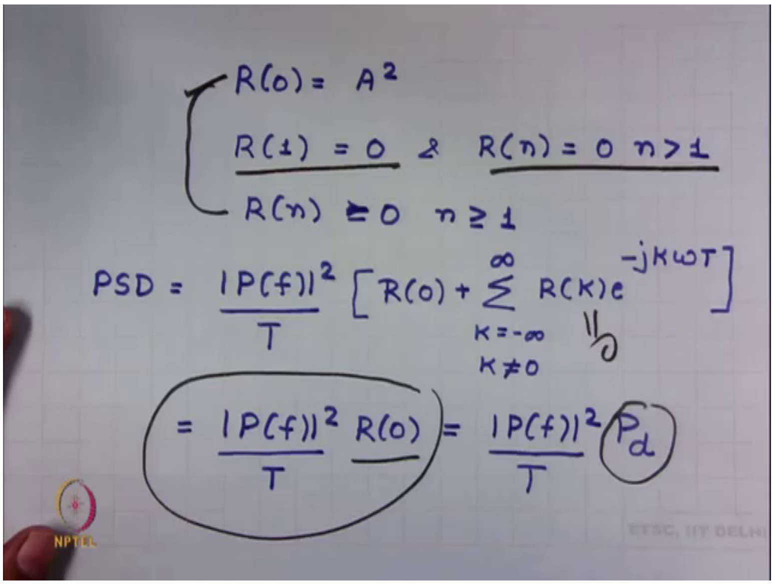

Similarly, you can find it yourself that R of n will also be 1 for n greater than 1 ok. In short I have already found that R of 1 is A square and R of n is 1 for n greater than or equals to 1 this we have already proven, this I leave to you to prove and in short I can arrive at these two conditions.

Power spectral density as I have already said can be given by mod square of P f divided by T R 0 and these terms and because all of them is 0 if K is not 0 then I simply end up with the power spectral density which is mod square P f by T times R of 0 and R of 0 is nothing, but it is the power of the data sequence binary sequence this we have already seen in one of the previous lectures that R 0 corresponds to the power of the sequence.

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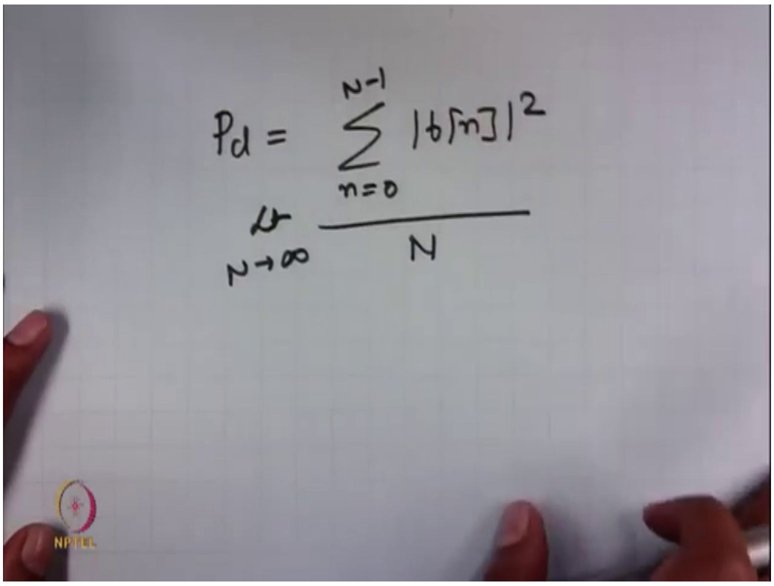

P d you can see is nothing, but this quantity all right and this is R of 0 ok. So, we have derived a very useful relationship of power spectral density for a polar signaling scheme.

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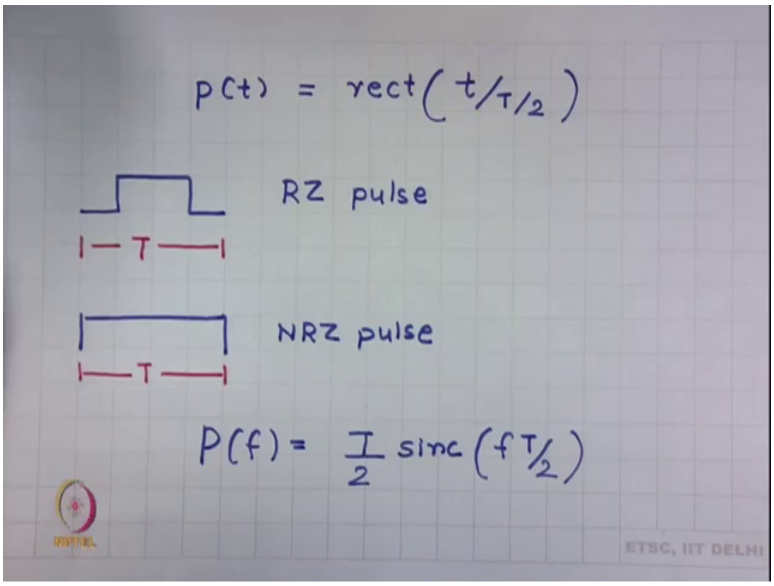

Now let me assume a pulse shape P t and the pulse shape that we are choosing is rect of T divided by T by 2 ok. So, if you see this what we are doing is we are saying that if I assume that these bit duration is T my pulse duration is only T by 2. So, the pulse duration is T by 2; that means, this pulse would take a value 1 or A for a certain duration and then it will drop down to 0 and such a pulse is known as written to 0 pulse; that means, it returns to 0 within a bit duration. You can also have a non return to 0 pulse; that means, if it is 1 it remains 1 for the entire bit duration it does not drop down to 0 within the same bit duration ok.

So, this is how we can have different kinds of pulses. So, here we have assumed R return to 0 pulse. The Fourier transform of this pulse P f you know from signals and system course its very easy T by 2 comes here and then you have sinc f T by 2 this is a Fourier transform of this pulse.

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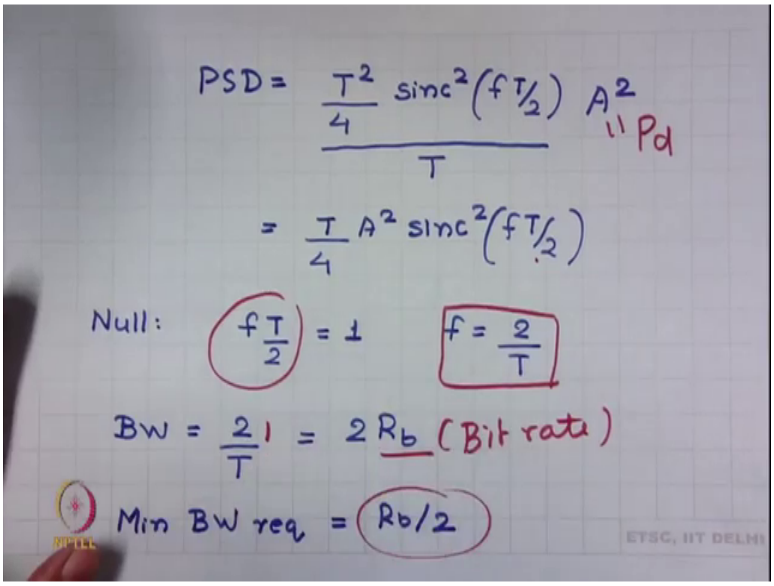

Now the power spectral density that we would have is mod P f square, P f was this. So, mod P f square would be this into A square corresponds to P d and we have T. We can simplify this to get say T cancels with T. So, we have T by 4 A square sinc square f T by 2; certain important things that you need to see is where the first null happens right because that decides the bandwidth of my waveform ok.

So, if I am trying to get the position of the first null, the first null will happen when this quantity or the argument of the sinc will become 1 and this will be the case when f becomes 2 by T. So, the bandwidth is 2 by T and 1 by T; 1 by T is bit rate which we say R b. So, R b is the bit rate. So, bandwidth is 2 times R b 2 times the bit rate and we will see short while from now or maybe in a next lecture, that the minimum bandwidth required is R b by 2. So, this is the minimum required bandwidth and this signaling scheme its taking a bandwidth which is 4 times b minimum required bandwidth. So, its wasting lot of bandwidth and this is the drawback of this signaling scheme.

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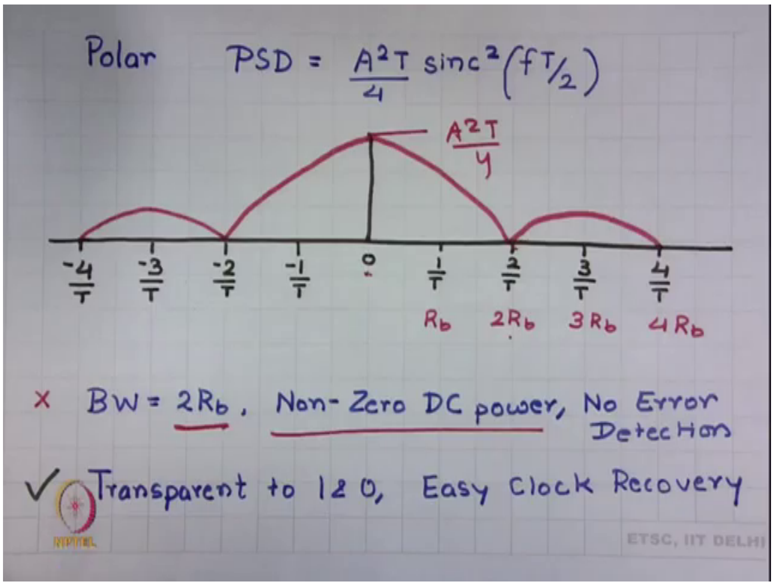

Let me plot the power spectral density of this signaling scheme the power spectral density as we have already derived is given by this expression which is this and its a sinc square. So, typically the shape is of that kind it decreases, with frequencies and so on so forth and this amplitude the highest amplitude here it will be A square T by 4 the first null happens at 2 by T; 2 by T corresponds to 2 times the bit rate ok. Let us see the disadvantages of this signaling scheme the bandwidth is 2 times R b.

So, that is wastage it has nonzero DC power. So, if we look at DC 0 frequency is having substantially high power that is a disadvantage; that is a disadvantage because of several reasons. One important reason is when we were using twisted pairs for echo cancellation etcetera you were using a transformer I have a picture for that.

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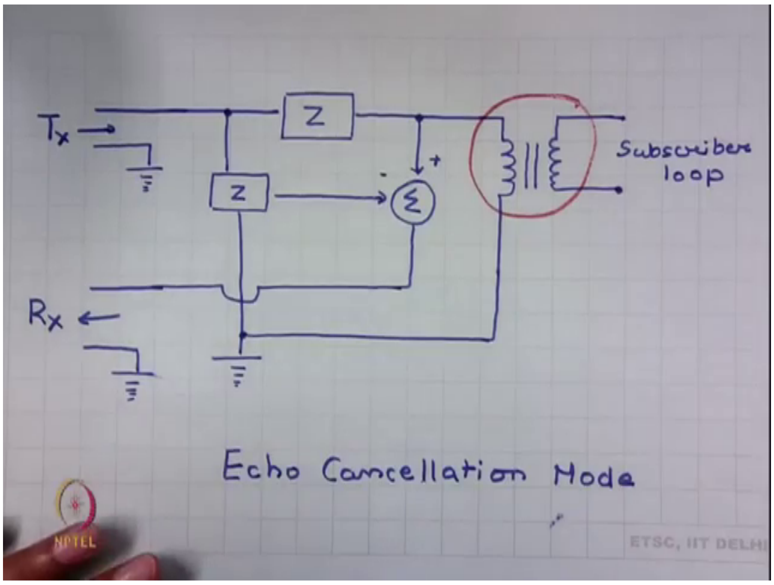

So, this is typical example of a echo cancellation circuit. So, echo cancellation circuit used to use a transformer. So, transformer does not pass DC right. So, if a signaling mechanism has lot of DC power this will go waste, otherwise you cannot use such a signaling mechanism wherever the transformers are used right.

And not just transformers we also use AC coupling circuits for impedance matching and so on and so forth and you will also lose power if you use such a signaling scheme ok. So, this is a disadvantage typically you do not want to have a large DC power ok. It does not do any error detection; that means, for example, if I have a binary sequence like this if I have a waveform like this and because of error if this part flips over then there is no way to tell that an error has happened.

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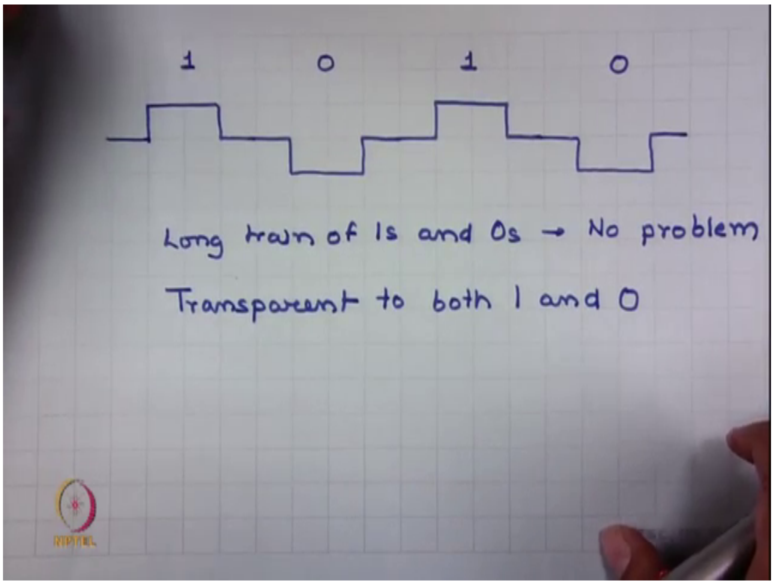

They are signaling schemes where you will be able to detect some errors if not called ok. We will see some examples of that, but polar signaling scheme as such does not do any error detection. Good point of this is that it is transparent to 1s and 0s, transparent to 1s and 0s means that suppose I come back to this diagram if you have a long trains of 0s.

If you have 0s continuously right even then you would have transitions ok. So, if you have a 0 then if you have a 0 there would be a transition. If you have transition then it is easy to keep your receiver synchronized with the transmitters ok. So, this for synchronization it is important that your signaling mechanism is transparent to 1s and 0s transmission. So, this is this advantage. The another advantage is easy clock recovery and this could also be seen very easily.

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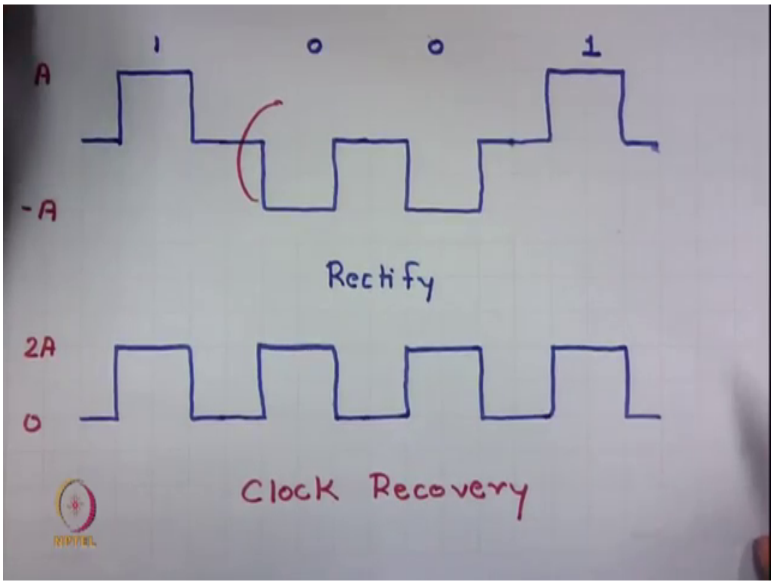

If you have a polar signaling scheme if you rectify this will flip over changes to this side. So, we have a clock and so, very easy clock recovery mechanism that you can use if we use a polar signaling scheme. So, these are the advantages of polar signaling mechanism.

So, with this we end today's lecture, today we have understood the important ways in which we can calculate the power spectral density. Power spectral density we said depends upon the two things; the one thing is that it depends upon the a square of the magnitude of this spectrum of the modulating pulse; that means, it depends upon the mod P f square and it also depends upon the input power spectral density ok.

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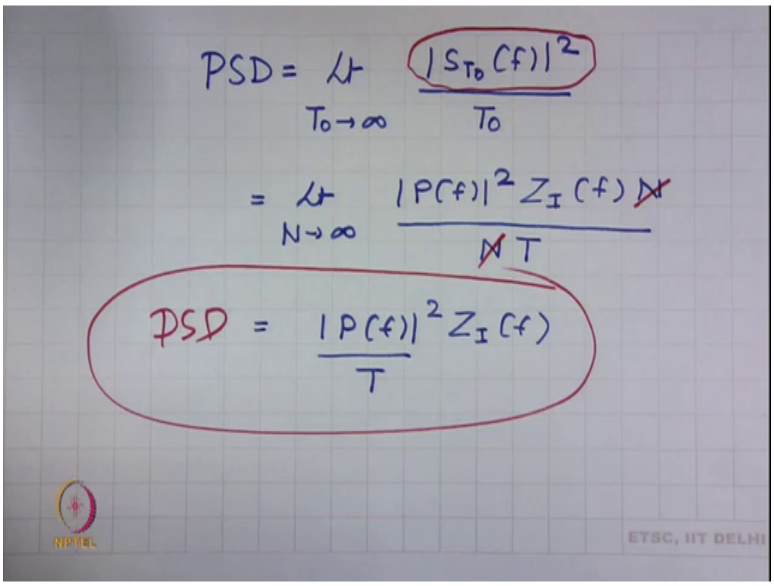

So, and then we have discussed an important example of a polar signaling scheme and we said that a polar signaling scheme has certain advantages and disadvantages and at this point it seems that it has some critical disadvantages particularly with respect to the bandwidth it takes the bandwidth twice of the data rate. So, that is too much because the minimum required bandwidth is R b by 2.

Then also it has nonzero t c power right and which would not be a good thing if you assume that you have a transmission over to strip a lines which uses transformer or we are using it over a digital communication system, which is using AC coupling for impedance matching. In the next lecture we will see important examples of other signaling mechanisms like uni polar signaling mechanisms or a bipolar signaling mechanism.

Thank you.