**Principles of Digital Communication**
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**Lecture - 19**
**Modulation Complex Baseband Representation of Passband Signals (Part-1)**

Good morning. Welcome to new unit today and in this unit we will be starting with modulation. So, let us see what we will cover in this unit.

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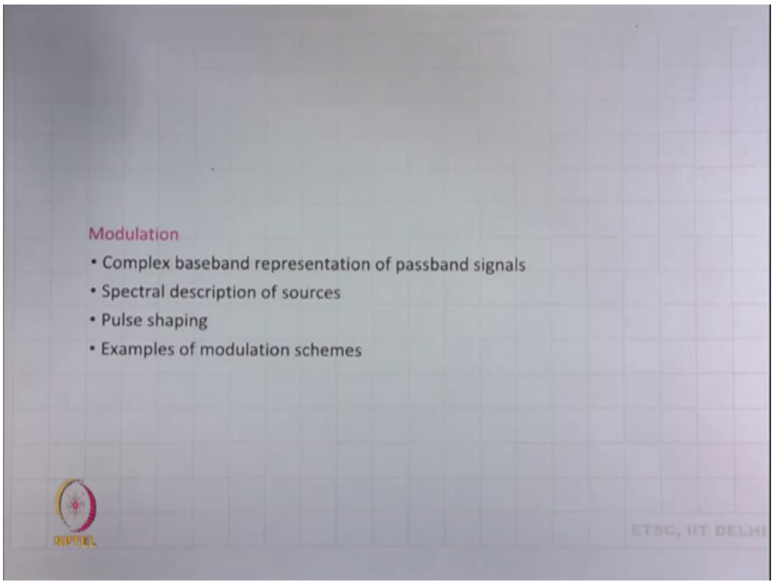


We will first talk about complex baseband representation of passband signals. So, in this lecture we will start with this today, then we will talk about a spectral description of sources, then we will introduce the issues of pulse shaping and then, we will see some examples of modulation scheme ok.

So, this we will cover over let us say next few weeks ok. So, we are in the word of modulator and let us first introduce what a modulator does.

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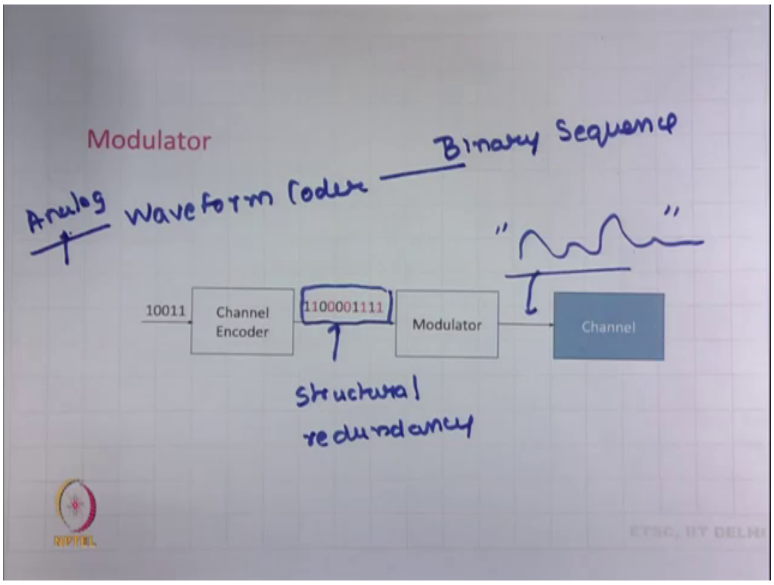


So, you know that we already have looked into the issues of waveform coding. So, waveform coder takes in analog waveform and this analog waveform is converted into binary sequence, then this binary sequence goes to channel encoder, channel encoder introduces some structural redundancy, introduces some structural redundancy in this binary sequence and modifies this binary sequence and this structural redundancy is introduced so, that the binary sequence is made more robust to channel errors

And after this channel encoder, we have modulator which takes in this binary sequence and converts this binary sequence to analog waveforms and these analog waveforms then go through channel. So, the basic purpose of modulator is to convert binary sequence into analog waveforms which can travel over the channel efficiently.

As we have already introduced in the first lectures that, first we have this waveform coder which converts this analog waveform into binary sequence and the job of the modulator is to convert this binary sequence back into waveforms. And though it looks like we are duplicating our efforts, it is not the case, because these waveforms that the modulator produces are efficient to travel over the channel whereas, these waveforms that waveform coder deals with these depends upon the source characteristics.

So, let us see in detail now the Block Diagram of a Modulator.

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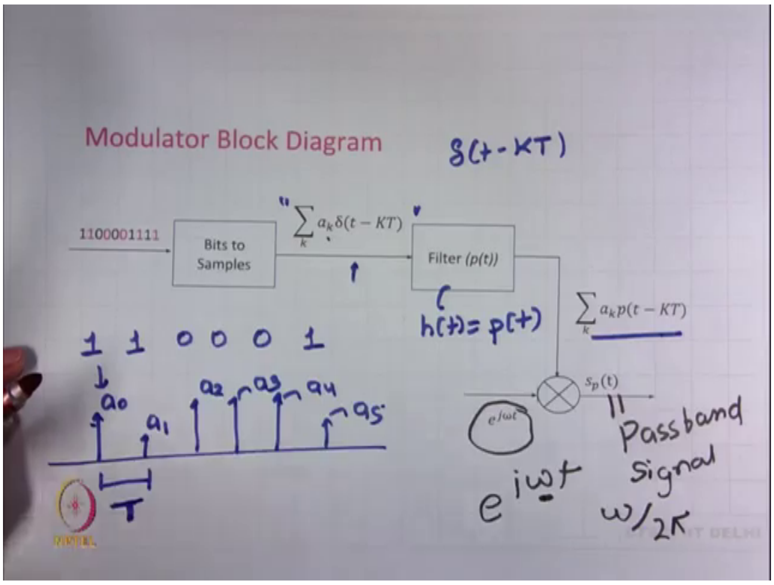


So, overall purpose is to take in the binary sequence and produce some analog waveforms. So, the first step is that you have this binary sequence or a sequence of bits and these sequence of bits are converted into sequence of samples. For example, if we have these bits what you can do is, you can have an impulse train. Do not worry about the correctness of this mapping. The only thing that I am emphasizing at this moment is what typically happens in this bits two samples converter.

So, you have an impulse train. So, this is these are impulses, these impulses let us say separated with a duration of T seconds and the weights of these impulses namely a naught a 1 a 2 a 3 a 4 a 5 is decided based on what you have in here. So, we will see a lot about this later on, but at this point the only thing that I am emphasizing is these binary sequences are mapped to some numbers and these numbers decide the weights of the impulses in the impulse train and these impulses are separated with let us say a duration of T seconds. So, this is what we have. So, delta t minus KT represents that you have impulses separated with a time of T seconds and a k's are the weights of these impulses.

So, the first step is corresponding to a binary sequence decide on these a case. That means, at this point I would be transmitting these impulses at a duration of T seconds and the weights of these impulses would be varied in accordance with the binary sequence that we have in here. So, that is the first step that happens in a modulator, then this weighted impulse train passes through a filter which has an impulse response of pt.

So, the impulse response of this filter is p of t. Now, we already know what happens.

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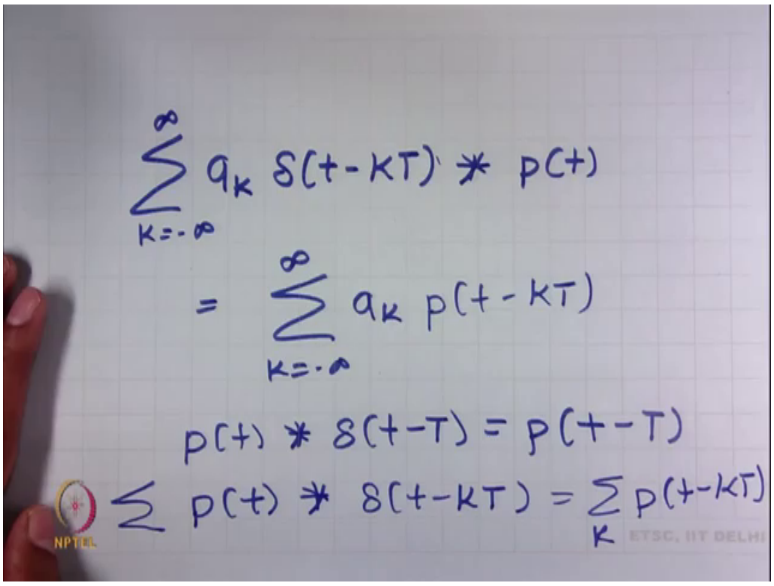


So, if you have a weighted impulse train and you want to find out what happens to this weighted impulse train when it passes through a filter with impulse response pt, you can easily obtain the output by convolving this weighted impulse train with the impulse response of the filter which is pt in this case. The output of this convolution operation is simple.

So, a k is not a function of time. So, it is constant. So, net effect is that you are convolving this pt with this weighted impulses. So, what you get is nothing, but weighted pulse train, ok. So, you must know that if you have a signal convolving with and impulse let us say this you simply get pt minus T and similarly if you convolve this with train of impulse what you get is nothing, but train of pulses. So, at the output of this filter we have weighted pulse train. Let me also draw this.

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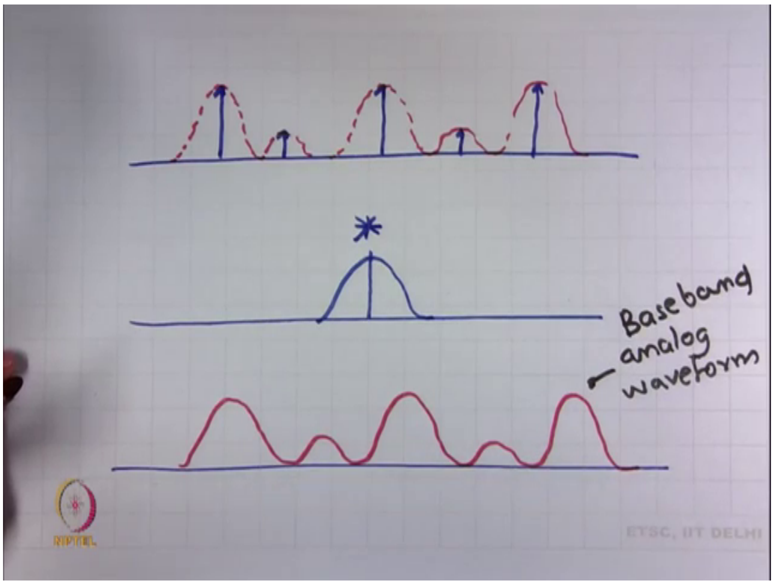


Let us say we have weighted impulse train and we are convolving this with the filter.

Let us say whose impulse response is like this. What happens simply is that this signal begins to write on each position of an impulse or you can simply understand this as this is some kind of a cap and you put this cap on these impulses. So, something like this. So, putting this cap here, then I put this cap on this impulse. The higher of the cap is decided by the weights of these impulses, then I am putting it here, here and here. Output at the filter would be some waveform like this ok.

And this is an analog waveform moreover this is a baseband analog waveform. Baseband means that most of its energy would we sanded at around dc. So, let us go back to the picture that we are trying to understand. So, at this point I am having a weighted pulse train and weighted pulse train is nothing, but it is a baseband analog waveform. Now, what we do to this weighted pulse train is we multiply this with the rotating complex exponential e to the power j omega t is a rotating complex exponential. We multiply this with a rotating complex exponential and what we get is a passband signal.

Passband signal means it is a signal which has its frequency centered at around some higher frequencies. In this case the spectrum would be centered around omega frequency, omega in radians per second or if you are interested in frequencies around hertz you just have to divide this omega by 2 pi. Anyway the main idea is that this will be a passband signal which would have most of its energy centered around omega by 2 pi hertz. This is a baseband signal. So, these steps happen typically in a modulator. Not all modulators would have the same steps. There would be some variation, but it would be good idea to start thinking about a modulator in terms of these blocks.

So, let me remind you have a binary sequence. This binary sequence is converted into a weighted impulse train, this weighted impulse train passes through a filter with impulse response pt, we get a weighted pulse train, this weighted pulse train is baseband analog waveform, this multiplies with rotating complex exponential to get a passband analog waveform.

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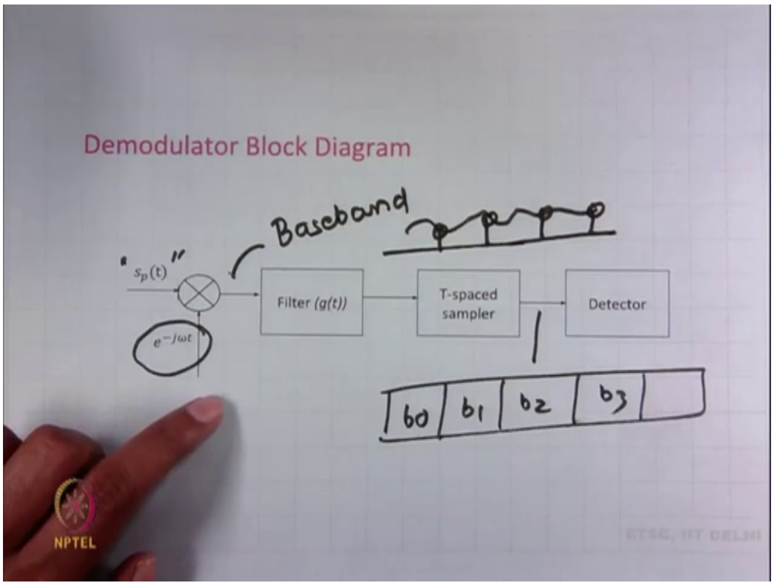


Let us look at the demodulator. So, now we have this passband waveform Sp t. Here I am showing that I have the same passband waveform which was available at the output of the modulator, but in reality channel will corrupt this. At this point I am assuming that I have the same passband waveform available at the input of a demodulator that was available at the output of a modulator.

Then, the first step should be that I convert this passband waveform to a baseband waveform and again multiply this with a rotating complex exponential, but now this rotating complex exponential would be rotating in a different direction that is it, ok. So, here it was rotating in positive omegas and here it would be rotating in a different direction. So, if it was rotating in clockwise, you can assume it will be rotating in anticlockwise or other way around right, then this baseband waveform passes through a filter. This filter has a different impulse response which we call as gt and you expect some analog waveform. How to decide about this filter, this we will see later on.

And then you pass this through a T space sampler, which samples this waveform at a duration of T seconds and it picks in these numbers. So, at this point you have a vector available and vector contains the sampled values of this waveform. So, you have a collection of some numbers and looking at these numbers the detector have to decide about the transmitted binary sequence. So, let us say I have some numbers b naught b 1 b 2 b 3 and so on so forth. Looking at these numbers detectors job is to decide the binary sequence.

So, we will a steady detector in the last unit at this point what we are interested in just focusing our attention to this modulator, ok. So, trying to understand how should we map a binary sequence to a weighted impulse train, how should we decide the impulse response of this filter, how should we understand this operation. Though this looks really trivial and simple, the lot of things that we have to understand in here and this is where we will start into this lecture trying to understand the relationship between this passband signal and this baseband signal, ok.

So, we will look into these issues in next couple of lectures, ok.

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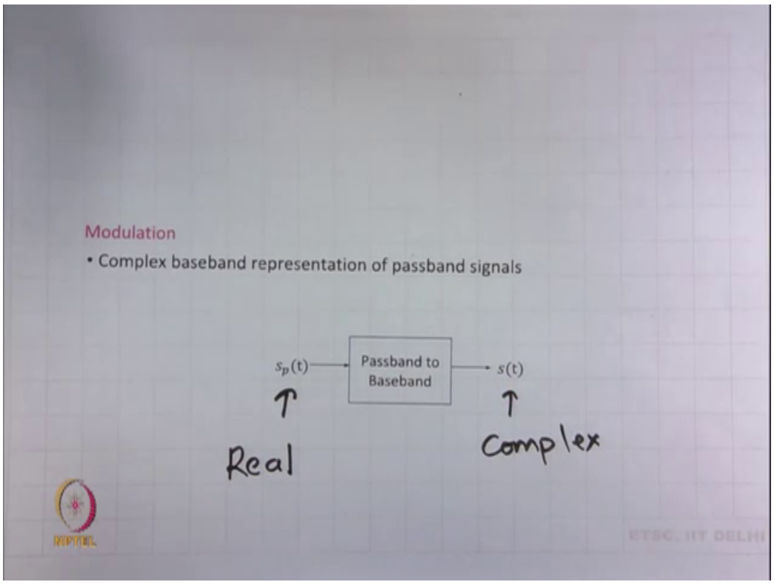

So, the theme of today is to understand the relationship between the passband waveform sp t and the baseband waveform which we call as s of t. Remember first that this passband waveform travels over the channel, right. So, between modulator and demodulator there is the channel. So, this is a physical waveform and if it is a physical wave form, this passband waveform will always be real and this baseband waveform that we want to think about we really allow this to be complex for analytical ease.

So, whenever you are thinking about a complex waveform, a complex waveform is nothing, but it is a sum of two real waveforms or when you want to talk about two real waveforms, at the same time you can simply talk about it as one complex waveform, so that your analysis becomes easy and this is the approach that we will take. We will restrict this to be a real waveform because this has to be a real waveform.

If it has to travel over the channel, but this waveform really resides at the modulator and demodulator at signal processing unit and this we will allow to be complex. These things will become eventually clear as we will talk about the relationship between the passband waveform and the baseband waveform. But before starting with that let us first understand this little bit more carefully.

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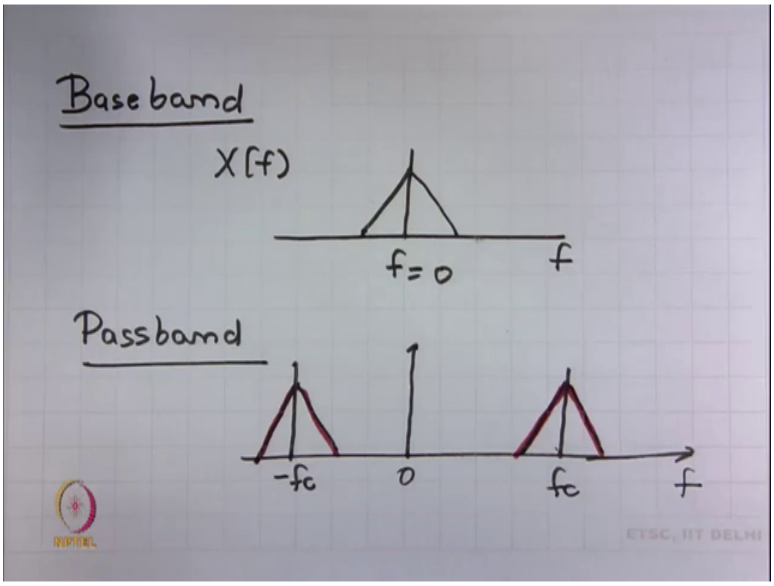


Let us first start by revising the definitions of baseband and passband signal. So, remember we have already talked about this. I am just revising it. So, baseband signal is a signal whose spectrum is centered at around dc. That is the baseband signal and we have a passband signal and a passband signal is a signal whose spectrum is centered around some higher frequencies typically we call that higher frequency fc and if it is symmetric you would have also a spectrum at around minus fc, ok. So, these bias I have just used to denote the location of fc and minus fc a spectrum is just this.

So, this is the spectrum of the signal ok. This point let us mark as 0, here we have frequencies and this is the spectrum of the signal.

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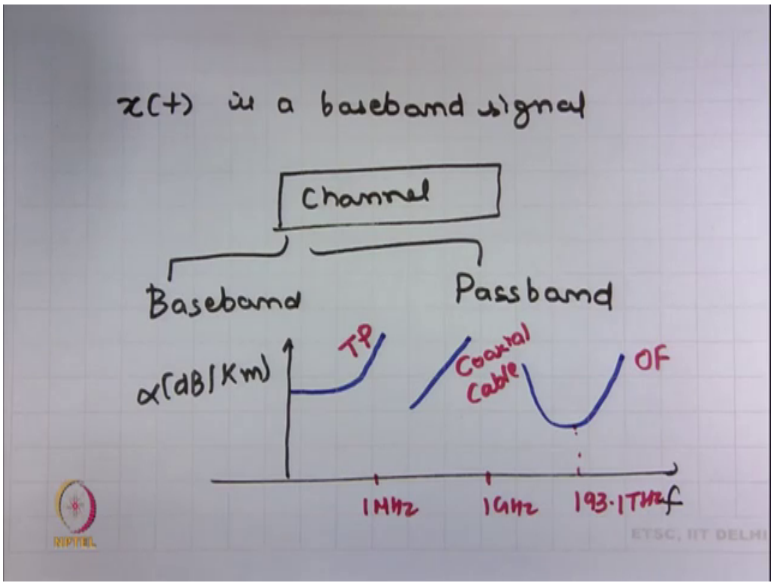


So, the information signal xt is a baseband signal and the channel can be a baseband channel or it can be a passband channel. We have also seen these issues in lecture 6. For example, in the case of wired we have already plotted the attenuation was this frequency curve. Let me plot it again.

So, we have seen three kinds of wide channels. We had this two state pair, coaxial cable and optical fiber. Let me also label frequency axis let us say this point is around 193.1 terahertz. So, this is optical fiber coaxial cable and twisted pair. So, if you look at the attenuation versus frequency profile of three important wire channels, you will see that optical fiber has a low loss at around 193.1 terahertz. That means, it is a passband channel because it only allows frequencies around this range to pass through the coaxial cable. On the other hand is also a passband channel because it has also a low attenuation for frequencies around 1 megahertz to 1 gigahertz.

That means it can also efficiently transmit these frequencies. Twisted pair, on the other hand has been used both as a passband and as a baseband channel. It can transmit the frequencies at around dc as well and thus this is a baseband channel, but it also supports ideals like discrete multi tone modulation where it has been using the ideas of passband communication. So, the bottom line is that you have both kinds of channels baseband and passband. Passband channels are becoming more important because they support higher data rates and thus most of the communication is becoming a passband communication.

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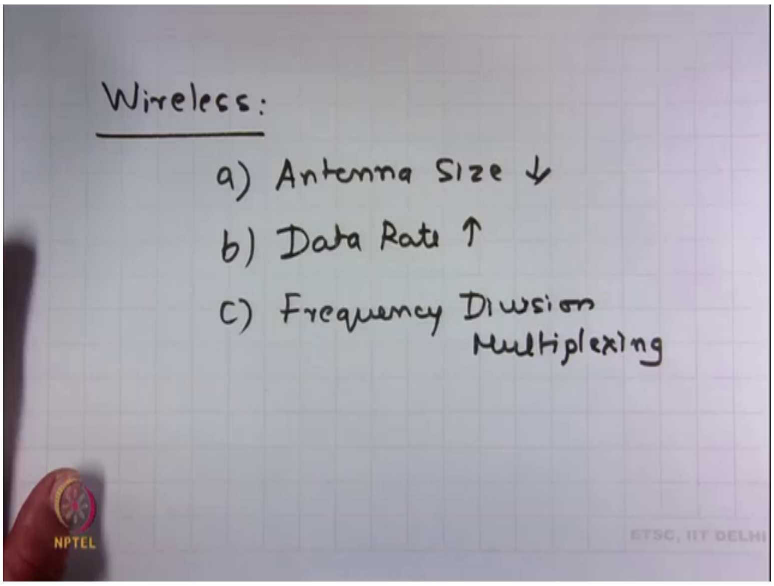


On the other hand wireless communication is purely passband right and we have seen several reasons for that. We have seen that antenna size is reduced if you use higher frequencies and we want to have communication receivers with small antennas, then if you have higher frequencies your data rate also increases, then we have seen that if you use this passband communication, you can explore the ideas of frequency division multiplexing and so on so forth which allows a better sharing of the spectrum amongst operators.

So, in short wireless communication is predominantly passband communication. On the other hand in wired communication as well the trend is to go towards passband channels. So, what I would like to believe is all future communication would happen based on this optical fiber, which is a passband channel signal. On the other hand is a baseband signal and thus if you want to transmit this baseband signal over channel, you need to up convert this signal to the frequency supported by the channel. Let me try to understand this better.

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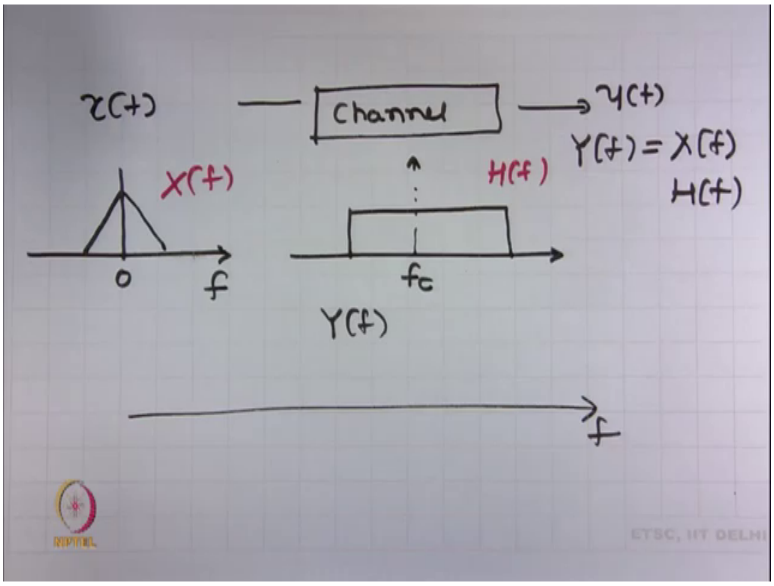


So, if I have a signal X t and let us say this signal has got a spectrum like this. So, centered at 0 and let us say that this I am trying to pass over a channel and channel has the spectrum centered at around f c what happens what is the output. So, you must have seen from signals and systems that the output spectrum is nothing, but it is the multiplication of input spectrum and the channel frequency response.

This is except this is a spectrum of the signal. Let me write it and this is H f, this is the frequency response of the channel, this is centered at around 0, this is centered at around fc and there is no overlap in these two spectrums if fc is pretty large. So, Y f would be flat 0. That means, you cannot transmit a baseband signal over a passband channel and thus we need to do something different to enable the transmission of baseband signal over channel.

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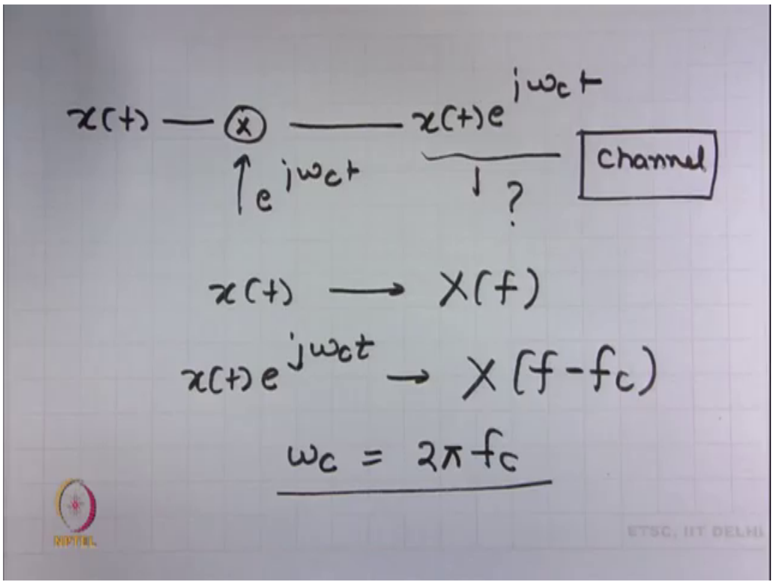


And the idea is that you take in this baseband signal, you multiply this with rotating complex exponential, you get x t e to the power j omega c t. Let us now first investigate what is the spectrum of this signal. So, from the codes in signals and systems you must have seen that if x t has a spectrum X f xt times e to the power j omega ct would have spectrum at around fc X f minus fc. So, the spectrum is translated to fc where omega c is 2 pi f c ok. This is typically the notation that we use sometimes even without mentioning that there is a relationship between this omega c and fc and this relationship is omega c equals to 2 pi f c.

So, the spectrum of this signal would be centered at around fc and now if you want to pass this through a channel let us see what happens.

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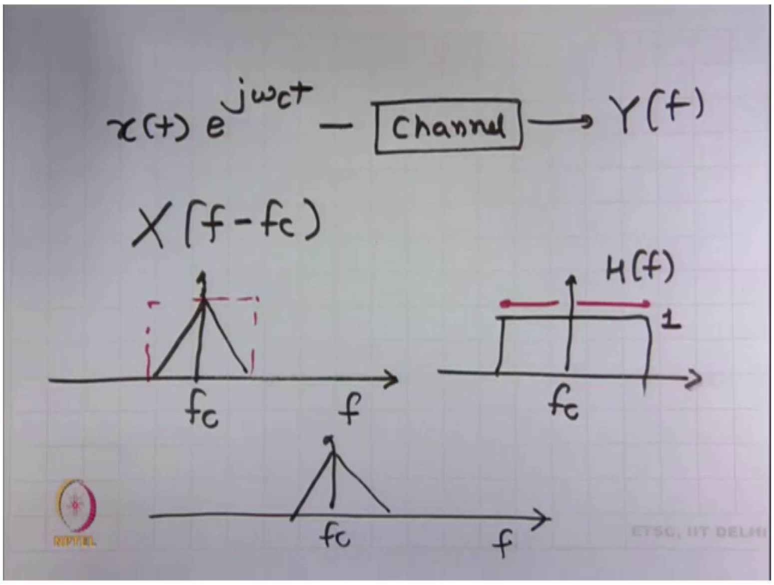


So, now if you want to pass this x t e to the power j omega c t through channel so, this has got a spectrum X f minus fc. So, its centered at around fc channel spectrum as before is centered at around fc. So, this is H f. Now if you want to find output which will be the multiplication of this with this and I assume that this is two wide at least this width is much larger than this width, then what you get is so you have at the output exact replica of input available especially if you want to believe that you have a flat H f with magnitude of one for a wide range of frequencies. Of course, we are not assuming any noise.

So, the bottom line is that before transmitting a baseband signal over a passband channel, we need to multiply that baseband signal with e to the power j omega c t.

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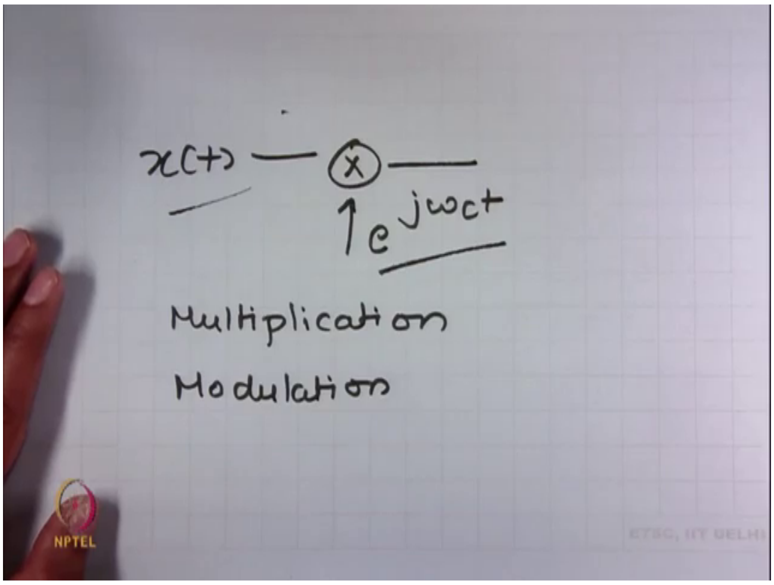


Now, some notations this multiplication of a signal with e to the power j omega ct is referred to as multiplication of course, but this is also known as modulation and it is confusing because we define modulation as the process of converting this binary sequence into waveforms and now we are saying that, this modulation is also that you multiply baseband signal with rotating complex exponential and the reason is this is normally the definition of modulation that you encountered in analog communication, right.

So, sometimes we loosely say this multiplication as well as modulation certain in certain books you also find this operation refer to as modulation, but mostly what we will try to mean with this modulation in digital communication context is the conversion of this binary sequence to analog waveforms ok, but do not get confused if you also find certain books mentioning this operation is as modulation. This is for historical reasons.

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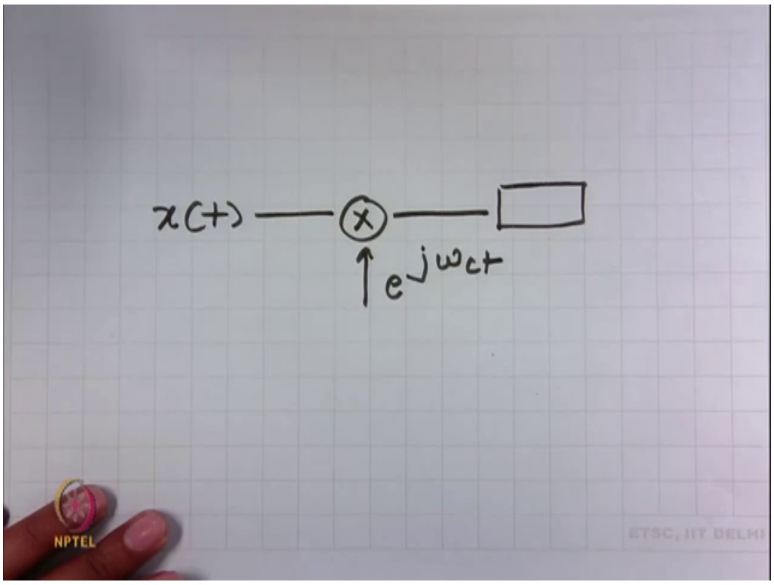


One thing that you should notice that when I draw these block diagrams for example let me draw this. I normally do not indicate the arrows when the signal moves only in the forward path. So, it is clear it should be clear that if you have such a situation, the signal is getting multiplied with a this multiplier and there is no need of having an artificial arrow in here. So, sometimes when you see situation like this, you should always interpret that the signal is moving in forward path. If it is in, so I will indicate that with arrows explicitly.

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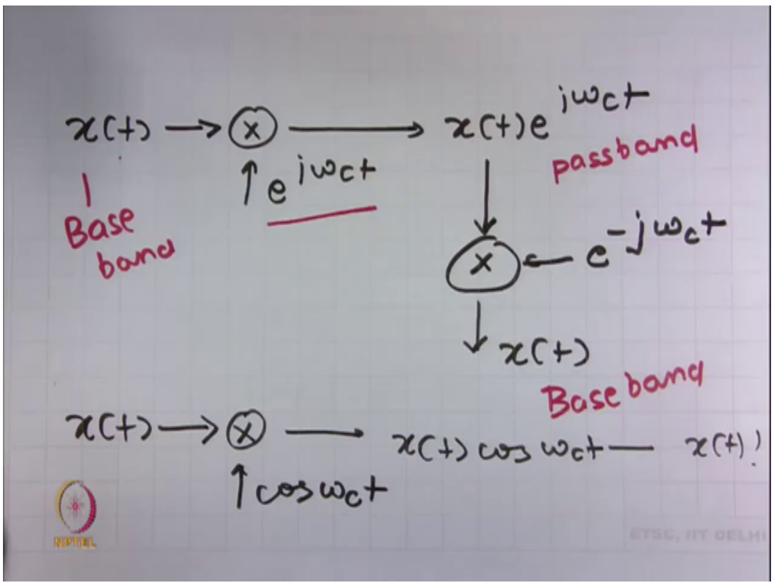

Now, let us see one more thing that if you have this xt and you multiply this with e to the power j omega ct, we get x t into e to the power j omega ct and the idea would be how can I get this xt back from this passband signal. It is very simple. You need to multiply this with e to the power minus j omega c t and what we get then is xt. So, this is a passband signal and this is the baseband signal and this is also baseband signal. So, going from baseband to passband to passband to baseband is easy. The only issue here is that you also have to appreciate is this is a complex signal and complex signals do not exist in reality, ok.

They are fictitious signals, analytical signals, but they are not physically realizable. So, when I am saying that xt is multiplied with e to the power j omega c t is only for analytical ease because now you see that these conversions are almost trivial is a straightforward one line step to go from passband to baseband and baseband to passband.

So, in reality what happens is instead of having this e to the power j omega c t what you would do is, you would multiply this with cos omega ct. Cos omega ct is a real signal. It has almost the same effect as this signal in translating the spectrum because this signal can be understood in terms of this signal it is nothing, but it is e to the power j omega ct plus e to the power minus j omega ct by 2.

So, we do not worry about this at this point. The point that I am just trying to emphasize is that instead of multiplying this with e to the power j omega ct in reality we will multiply this with cos omega ct and now it will be little bit harder to think about how can you go from x t cos omega ct to x t,. Little harder, but it is not that hard. We will see how can we do this.

Once we go in detail in this passband to baseband conversion at this point, it suffices to say that whenever you see a complex signal, it should ring a bell in your head that this is only for analytical ease right. For practical purpose what we do is all operations are carried out with real signals, ok.

All physical operations of course if you are carrying out some operations in a processor, in a computer or using a digital signal processing unit, there you can have complex numbers, but as that physical operations are concerned like for example if you have a physical modulator, then you can only feed this modulator with a real signal all right.

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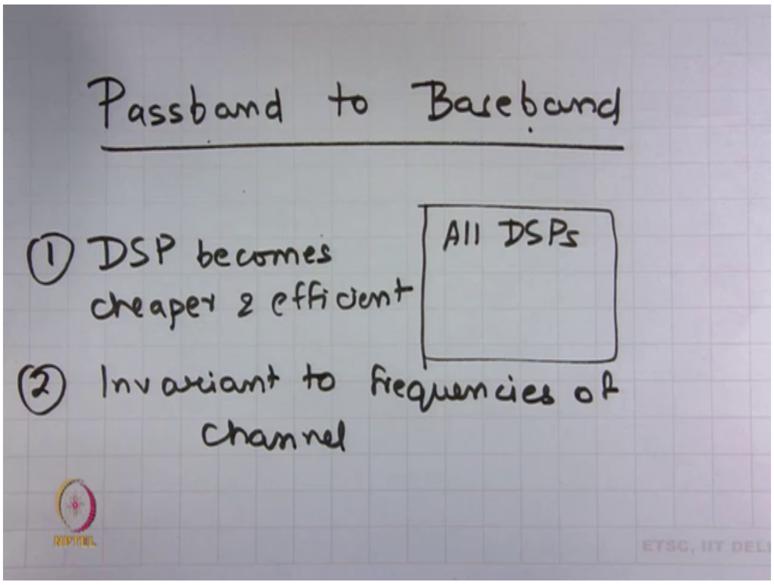


So, let us now summarize what we have been saying so far. We are saying that this issue of passband to baseband conversion is important because of the two reasons; first all information as a baseband signal and most of the information travels as a passband signal does. This conversion from passband to baseband has to be clearly understood, then the question would arise can I not do all operations in the passband domain itself. For example, let us go back to this picture of demodulator that we had. So, let us look at this picture of demodulator.

So, the question that you can ask is do I need necessarily to do this down conversion from passband to baseband domain and the answer is yes because all digital signal processing units there you find do processing in baseband domain. So, all detectors and so on so forth operate in the baseband domain and the reason is very simple, because the baseband signal has lower frequencies contained present than the passband signal and thus, the sampling rate required to process a baseband signal is much smaller than the sampling rates required in the passband signal and does the DSP becomes cheap and simpler because sampling rate requirements reduces.

Thus once you do the passband to baseband conversion, the requirement of sampling rates reduces which makes DSP cheaper. So, that is the first reason DSP digital signal processing becomes cheaper and efficient and the second important reason is you are making this processes for baseband domain. They remain invariant to frequencies of channel.

So, whether your channel is an optical fiber channel or it is a microwave channel or it is a wireless channel, all these channels operate in different frequency bands, but once you convert the data of these channels to pay spent domain, the DSP algorithm remains invariant to the physical characteristics of the channel namely to the frequencies at which these channels were operating and thus, you can have or you can use the same DSP processor to process the data of all kinds of channels, right.

So, the inventory costs reduces and the design becomes more scalable, ok. So, that is the second reason why we want to do DSP in baseband domain. It is cheaper, it is invariant, it can be used for different kinds of channels and so on so forth and most important we do not lose any information. When you do this passband to baseband conversion, what you lose is a predictable phase, right. The passband signal is just oscillating with higher frequencies. The phase is rapidly fluctuating, but it is also predictable. Checking out a predictable quantity does not lead to any loss in formation and thus when we do this passband to baseband conversion; we do not lose out on the information, right.

So, we only gain by this idea of passband to baseband conversion and this is how most communication systems are implemented,. So, we have even though a passband signal at the input of a receiver, the first thing that you need to do is to convert it into a baseband signal. So, here in today's lecture what we will try to understand the relationship between this passband and baseband signal because we know that even though we receive the passband signal, we only process the baseband signals, right. So, in this lecture we will try to develop this idea of establishing a link between a passband and baseband signal, ok.

So, let us start. This topic is really important though is simple is really important and it is it is somehow very confusing for the students. So, we will cover this topic very leisurely at a at a very slow pace, so that you can grasp the concepts and if you have understood this, then most of the problems and an nitty gritties of digital communication system would be sorted out.

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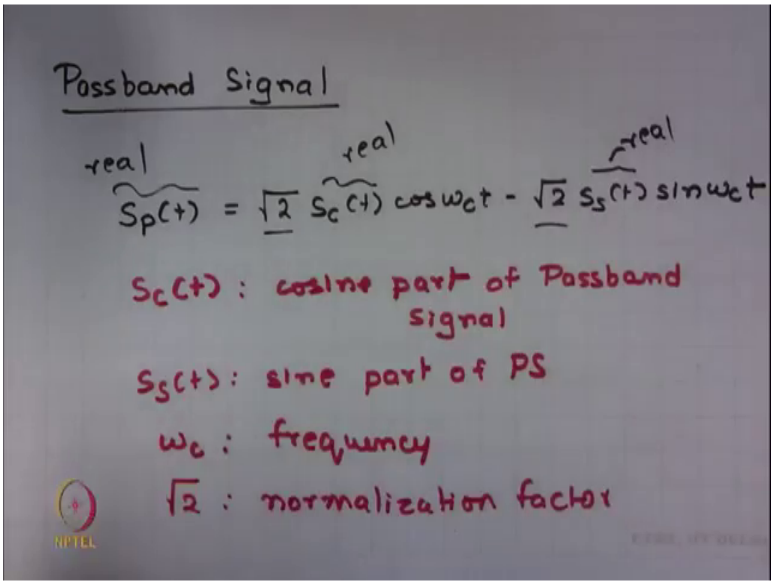


Let us look at a passband signal. So, when we are talking about a passband signal, let us write equation of a typical passband signal ok. So, here you see certain things. The first is this sct. We say it is s cosine part cosine part because this is multiplied by cos omega ct cosine part of passband signal, then we have sst which is the sine part of passband signal omega c is the frequency at which you have a passband general. Of course, it is an angular frequency in radians per second. Why do we have this root 2 here? So, root 2 is a normalization factor.

So, we write the equation with root 2 because of a certain reason and this reason will eventually become clear, however there are many textbooks on digital communication systems which do not use this factor of root 2, ok. So, there are many books. So, do not get confused. They simply do not have this root 2 factor. There are certain books which have instead of root 2 a factor of two. So, different books are using it different normalization factors. Why we are using this root 2 what eventually become clear because anyway this is going to make some equations messy. You are going to have lot of root 2s in the equation.

So, there is a certain price that we pay for having this root 2, but it gives us also some advantages which will eventually become clear, ok. So, this is typically how a passband signal we will look like it would have a cosine part, it would have a sine part and this cosine part is multiplied by cos omega ct and the sine part is multiplied by sine omega ct, and we would see later on why we have this 2 cos and sine part. In this lecture the focus would be in understanding the relationship that exists between the passband signal and its equivalent baseband signal. This is the focus and we would try to keep that focus,. Now just a reminder. So, Sp t is a passband signal, it is a real signal ok. There it cannot be complex because it is a physical signal.

We have said any physical signal has to be a real signal. So, Sp t is a real signal and we will always assume it to be a real signal, right. Sc t is also a real signal. So, that is we will assume it also to be a real signal because it is multiplied with a real signal cos omega ct is a real signal. So, it has to be a real signal. If you want to have Sp t as a real signal similarly Ss t is also a real signal. So, everything is real in this equation. There is no complex quantity involved here, ok. So, this is how a passband signal equation looks like.

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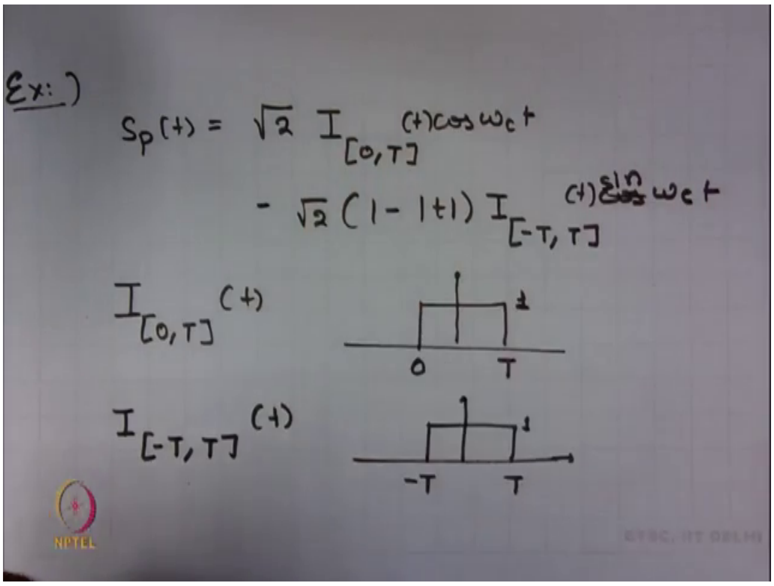

Now, let us take one example. Let me take Sp t as root 2 times I 0 T cos omega ct minus root 2 times 1 minus mod t. I going let us say from minus T to plus T cos omega ct. Now it should be sin omega ct. So, sin omega ct ok. So, this is a equation of a passband signal which looks exactly like this we have a cos omega ct and we have sin omega ct and instead of Sc t we have this function and instead of Ss t we have this function. So, what is this I? 0 to T T let us also put a t here.

So, what is this quantity let us define this. Let us say that this is a rectangular function which goes from 0 to T with let us say some amplitude 1, Ok. Similarly this I minus T to plus T. T would be a function which goes from minus T to plus T. So, it is more convenient it is more convenient and clear what these rectangular functions are if I use this notation. So, this is just a notational that I represent this signal with this function I represent this signal with this function. So, these limits minus T and T to T tells me where my gate lies. So, it lies between minus T to plus T. So, if we have look this signal, this is a passband signal.

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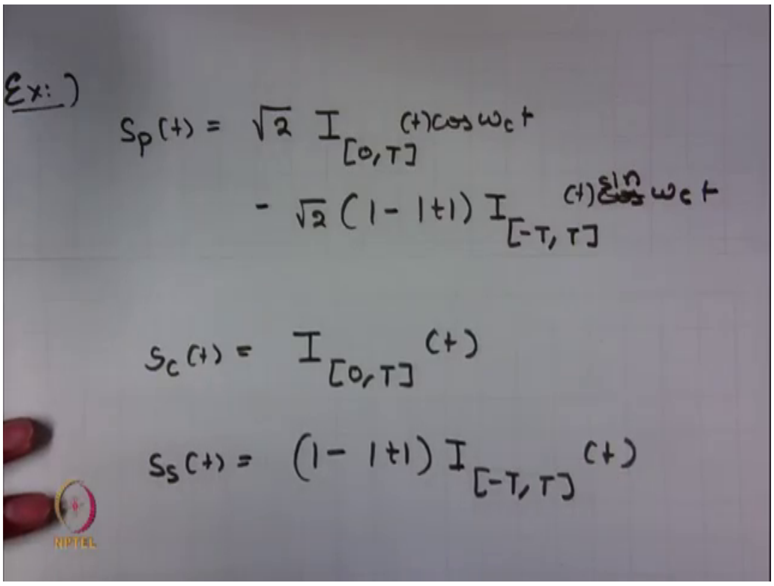


And if I am interested in the cosine part which is Sc t the cosine part would be I 0 to T t and the sine part would be 1 minus mod T I going from minus T to T t. So, this is the sine and cosine part of this passband signal.

Now if I look at these signals because these are rectangular functions, rectangular functions have the spectrum which is sink and sink is closed from minus infinity to plus infinity, right. So, this signal has the frequency spectrum which spans from minus infinity to plus infinity, but as we know that a sink function though it goes from minus infinity to plus infinity, most of its energy is confined around dc and thus we consider this to be a baseband signal. Similarly this signal is also a baseband signal. So, when we are saying baseband signal, we simply mean where most of the energy is concentrated on. We do not worry about whether all its energy is confined or not because anyway every time limited signal will have a frequency spectrum which is spend some minus infinity to plus infinity you cannot avoid that,.

So, just remember that even though we have rectangular functions as the cosine part of passband and sine part of passband, they are very well baseband signals.

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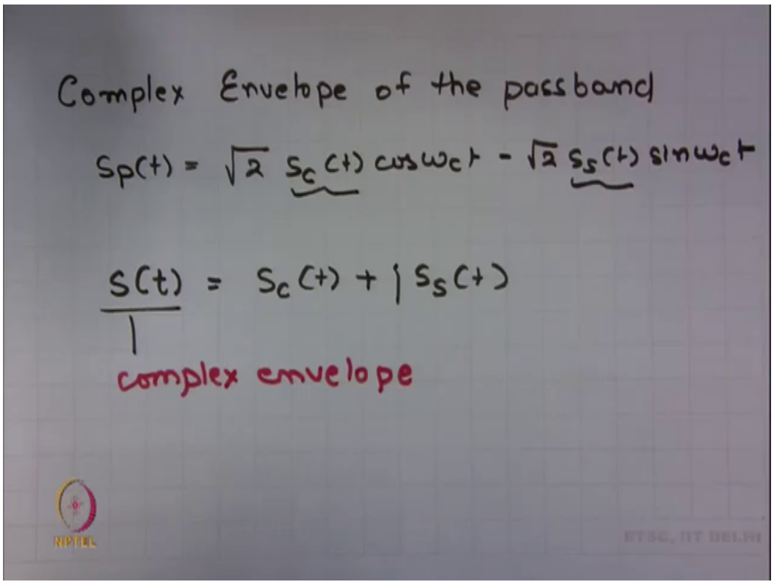


Now, I can define a quantity known as complex envelope of the passband signal. So, let me write this passband signals equation again. So, if I have this as an expression of my passband signal I can define a quantity S t by taking in the cosine part S ct plus j Ss t and this S t is known as complex envelope of the passband signal. So, complex envelope of the passband signal can be obtained by simply taking the cosine part and the sine part and adding them by having this j here.

So, in this way and then you have a complex signal. As I have said before that because it is a complex signal we know that it is just for analytical ease. It does not exist physically. It is just we are using this notation, so that some of the computation or analysis can be made simpler. So, this is how we define the complex envelope of the passband signal.

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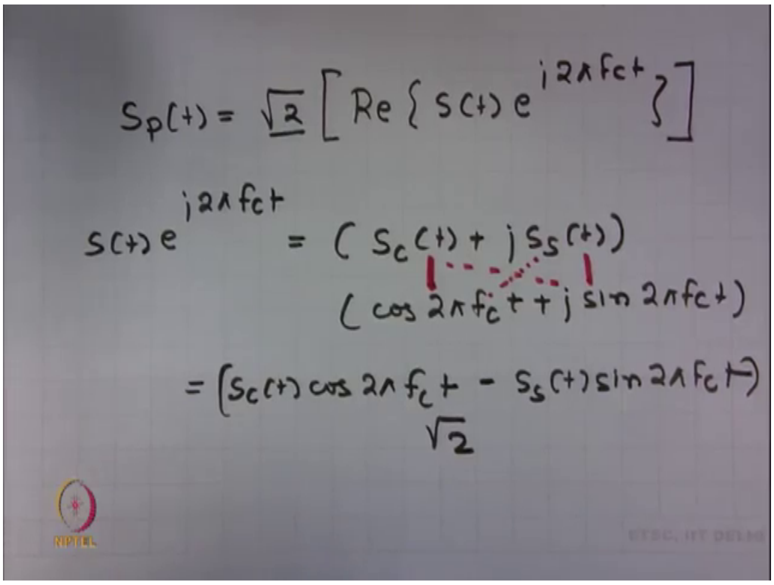


Now, let us see how these complex envelope and a passband signal is related. So, passband signal can be obtained by taking the real part of this signal. So, here S t is a complex envelope of the passband signal, Sp t is a passband signal. If I multiply this complex envelope with this rotating complex exponential take the real part of the entire quantity multiplied with its root 2, then we get a passband signal. Let us see.

So, what we have is S t times e to the power j 2 pi fct. So, S t is Sc t plus j Ss t. You multiply this with this. So, from Eulers theorem we have and now when I multiply this quantity with this quantity, you see that this can be multiplied by this, but once you multiply this with this if you multiply this term with this term, what you end up with is a complex term and because we are taking only the real part of this product, this term will anyway we loss.

So, we do not worry about this, ok. So, this multiplication of this with this would be lost. Similarly when you multiply this with this it will be lost because it is anyway giving us a complex term and when we take real part of the product again this will be lost.

So, what you would have is this multiplied by this and this multiplied by this. So, I can write this as Sc t cos 2 pi fct, then I multiply this with this, I have Ss t sin 2 pi fc t. So, this is the real part and then, I have to multiply this with root 2 and this is indeed the expression of the passband signal. So, idea is that passband signal can be represented using this relationship before I draw a picture for this because it is very important to see the picture concerning this operation, so that it stays with you forever. Let me do a trivial thing.

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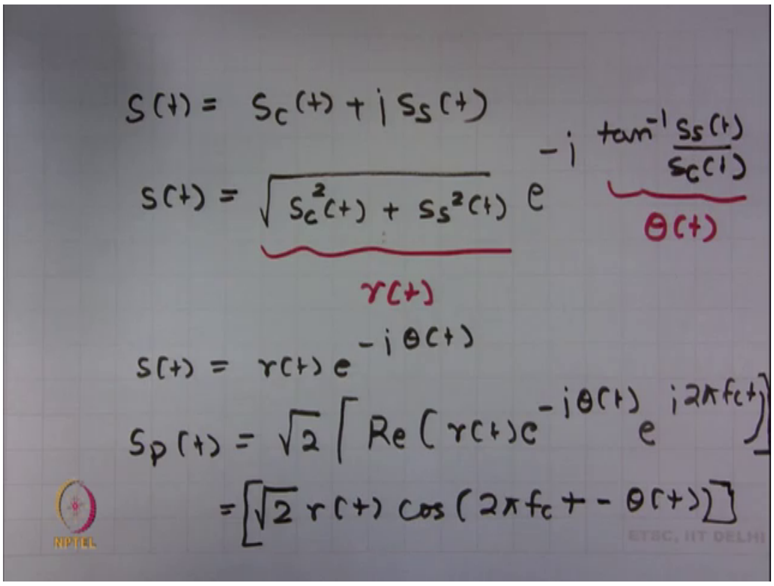


So, let me write this complex envelope question again which is this. Now, you see something important that I can write this as in this. So, this is in Cartesian coordinate I can represent this is in polar coordinate like this. So, I call the envelope as rt and I call this as theta t. So, S t can be written as rt times e to the power minus j theta t. So, if I use that expression Sp t is root two times real part of s t which is rt times e to the power minus j theta t into e to the power j 2 pi fct.

If I see this, this is a real quantity, right there is nothing complex. So, it can be pulled out. So, we get root two times r t. If I am taking real part of this quantity, what I will get is cos 2 pi fct minus theta t. So, this is the expression for a passband signal, ok. So, passband signal can also be represented in the amplitude of this complex envelope and the phase of the complex envelop, ok.

So, this is another representation. This is not used a lot. So, let us just look at it and forget it. The most important expression for us is this quantity this one. So, let me now draw a picture for this.

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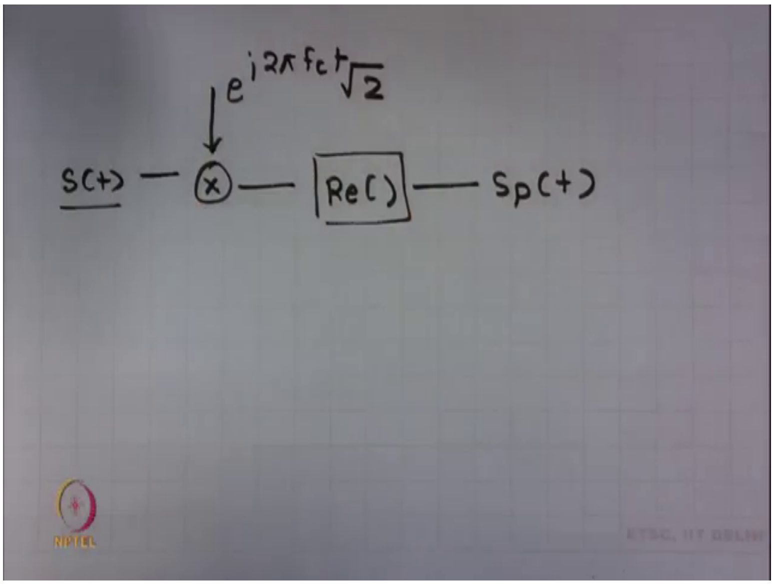


So, what I am saying is I have s t which is the complex envelope of the passband signal. I multiply this with e to the power j 2 pi fct and let us also take this root 2 into account. So, let me multiply this with root two times e to the power j 2 pi fct. What I have to do? Then is to take the real part of this quantity and then, I get Sp t ok. So, this picture tells us the relationship between the passband signal and its complex envelope, ok. This is also known as the complex baseband representation of this passband signal, ok.

So, you can find the complex baseband representation of the passband signal if you look at this diagram. So, this is the relationship. This diagram defines the relationship between the complex baseband signal which is S t and its equivalent passband signal which is S pt and you can obtain this by doing this operation.

So, this is all analytical because this is a complex quantity, this is complex quantity right. So, this is all for analytical tool, right and we will also see how we do things in reality, but this picture should be remembered and it should be there in your head whenever you are trying to establish the relationship between passband and baseband signals.

Now, let us see can we go back from Sp t to S t. If I want to go from passband signal to baseband signal, what is that representation?

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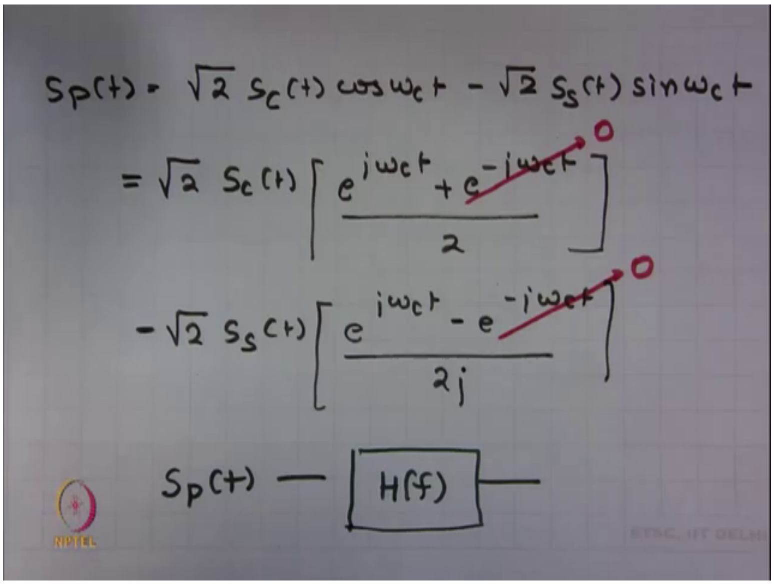


So, let me write Sp t again which is root two times Sc t cos omega ct minus root two times Ss t sin omega ct. Let me use Eulers expression here. I can write this as, so what I have done is I have just use the Eulers expression and I have expanded these terms.

Now, let us try to think what happens if I pass this Sp t. If I have Sp t and pass it through a Hilbert filter let us see does something interesting happen.

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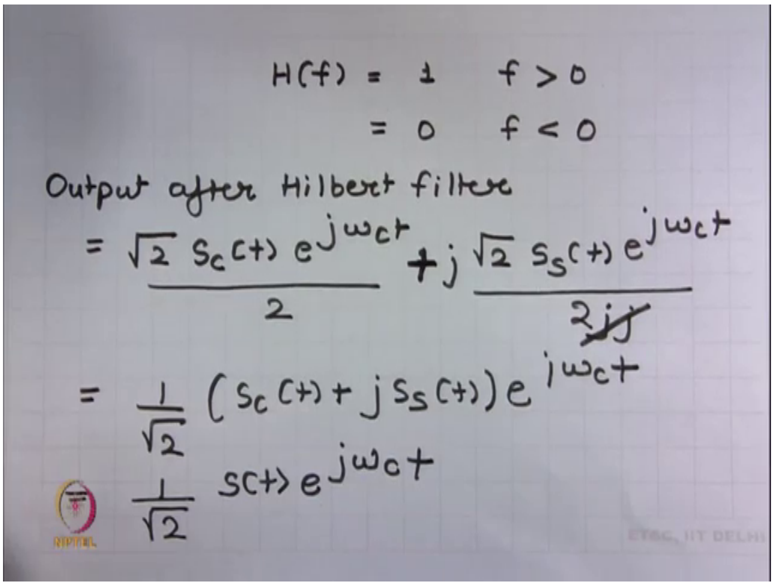


Heilbert filter we define as a filter whose frequency response is 1 for f greater than 0 and it is 0 for f less than 0, right.

So, it is kind of a unit to step in frequency. So, very simple expression for Hilbert filter and it is very useful. So, what it does is it simply pass the positive side of the spectrum and it blocks the negative side of the spectrum. So, if I pass this Sp t through Hf what happens is this would this term would vanish because this part of the spectrum is around minus omega c right and once it is pass through Hilbert filter, it would vanish. Similarly this term will also go to 0. So, what you would have is output after Hilbert filter and this would be root 2 s ct e to the power j omega c t by 2 minus root 2 Ss t e to the power j omega c t by 2 j.

I can have j in here and j in here and this j square cancels with this negative. So, I have a positive sign here, then I can write this as 1 by root 2 Sc t plus j times Ss t e to the power j omega ct and this is nothing, but it is 1 by root 2 S t e to the power j omega ct. So, let me draw this picture again.

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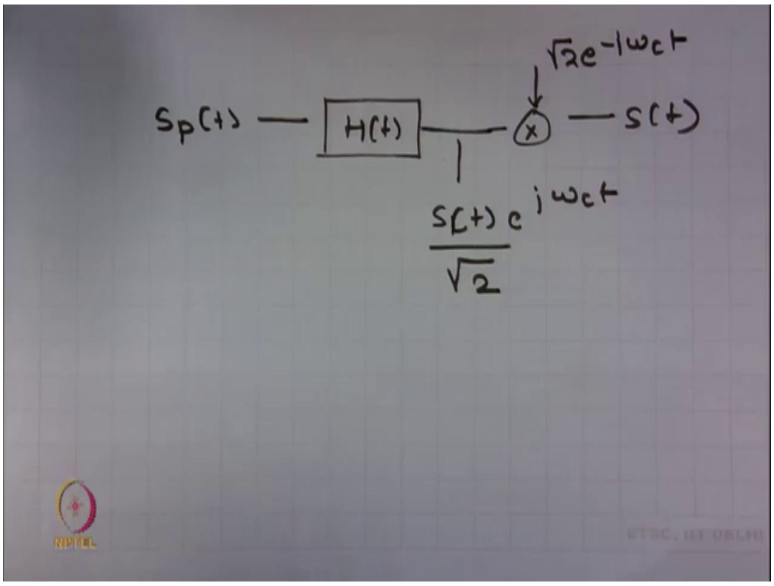


So, if you have Sp t you pass it through Hilbert filter Hf what you end up with is S t times e to the power j omega ct by root 2 and of course, now we want to get S t from this. So, I can simply get it by multiplying it with root 2 times this and I get S t. This completes our idea to get back S t from S pt. So, let me draw the complete picture again.

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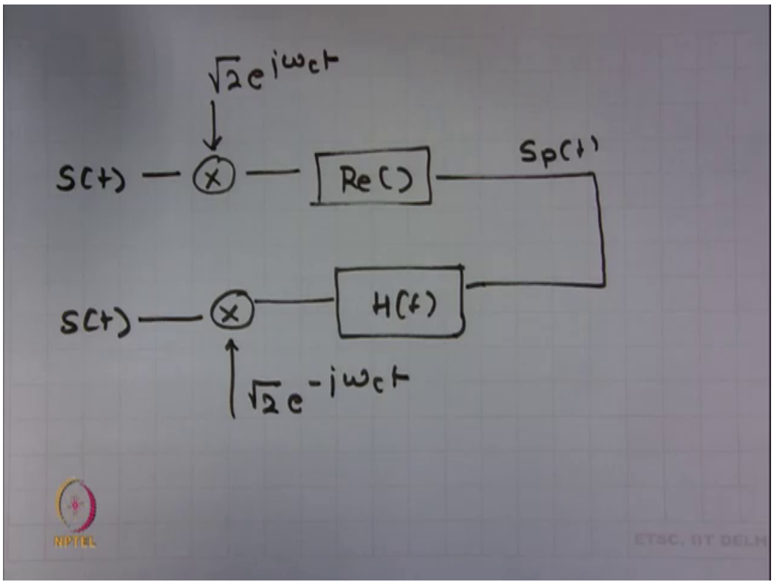


So, if we have a complex baseband signal let me multiply this with root 2 times e to the power j omega ct. Take the real part I get Sp t, Sp t I pass it through Hilbert filter which is like a unit to step in frequency. I multiply this with root two times e to the power minus j omega c t. I get S t.

So, this diagram is very important because it establishes the relationship between the complex baseband signal and it is equivalent passband signal and how can you think about going from one domain to another domain now let us see and complete. So, this is we have said is for analytical ease. Let us see how do we do things in practice.

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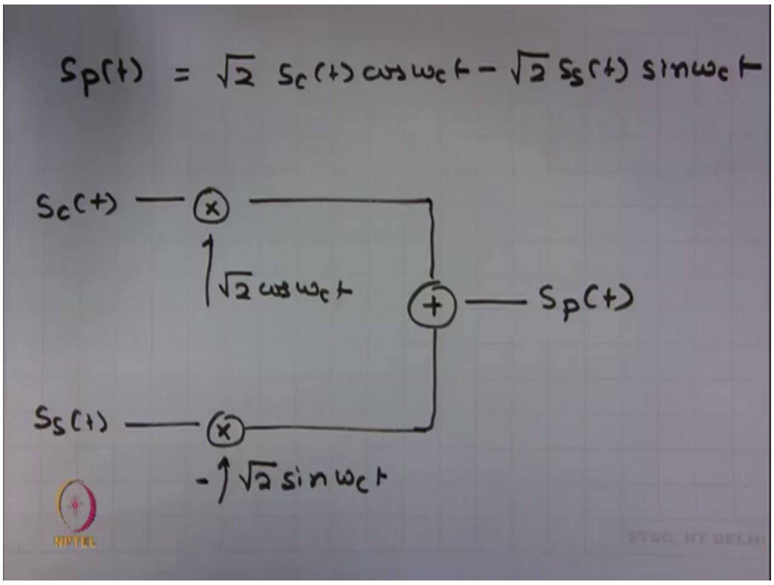


So, let us write that expression again. Sp t as root 2 Sc t cos omega ct minus root 2 Ss t sin omega ct.

So, in practice what you can do is, you can take this Sc t which is a real signal multiply this with root 2 cos omega ct, you take Ss t multiply this with root 2 sin omega ct. The minus and you add these two things what you get is Sp t. So, this is how you can go from two real baseband signals to a passband signal, Ok. You produce them individually, multiply this with cos omega ct, multiply the sine part with sin omega ct with proper normalization factors, add them up and you get Sp t. So, this is very straightforward how can we go from Sp t to Sc t and Ss t let us see it.

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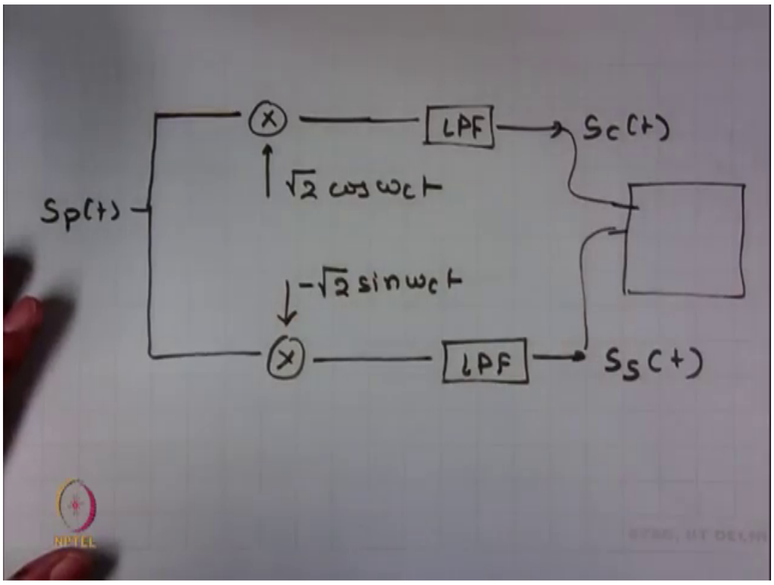


So, if you have Sp t what we need to do is multiply it with root 2 cos omega ct. Let us see what is the end result first and then, we will complete this picture.

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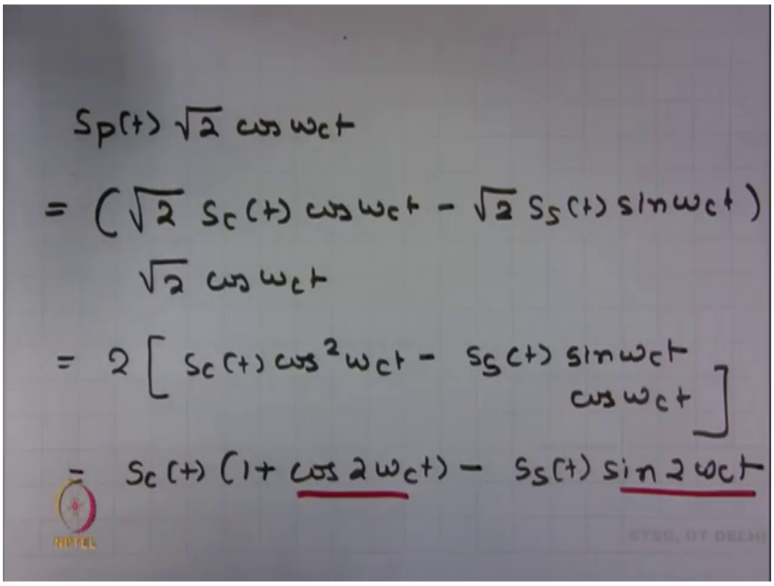


So, if you have Sp t multiplied by root 2 cos omega ct let us see what we will have we will have. So, first we can write the expression for Sp t multiply this with root 2 cos omega ct. We can pull two out, we will have Sc t cos square omega ct minus Ss t sin omega c t into cos omega ct, then I can write this as Sc t 1 plus cos 2 omega ct minus Ss t sin 2 omega ct.

Now, if you look at this very clearly you see that these are high frequency components and you can kill these high frequency components by passing through a low pass filter. So, if I pass this through a low pass filter if I have a low pass filter here what we will get is just this part Sc t times 1 which is Sc t and similarly you can multiply this with minus root 2 sin omega ct, pass it through a low pass filter and you get Ss t. So, practically if you want to generate a passband signal from the baseband components, you can use this theme and you can generate the baseband components from the passband components using this receiver architecture.

Now, in digital signal processor what you would feed is this component. So, our signal processor would take Sc t and Ss t. It would convert these waveforms into numbers, it would convert this into a sequence of real numbers this into a sequence of real numbers. If it wants it can take these two sequences of real numbers together treated as complex numbers. It can do all processing in complex domain and so on and so forth. So, DSP can do processing in complex domain. It is just for the transmission we require only real signals.

So, today's lecture what we have basically talked about this big theme, the relationship that exists between a complex baseband signal and a passband signal. So, this passband signal is a real signal and this baseband is a complex signal. It is for analytical ease and this complex baseband signal can be thought in terms of the two real signals Sc t and Ss t and from the passband signal, you can derive these two individual real signals, ok.

So, tomorrow we will carry forward this idea and we will like to talk about some more interesting relationships that exist between these passband and baseband signals.

Thank you.