Principles of Digital Communication
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Lecture - 17
Random Variables & Random Processes: Spectral description of Random Process

Good morning, a warm welcome to the next lecture on Random Process. And hopefully today will be the last lecture on random process. So, let us start by looking at what we have discussed in the last lecture. So, in the last lecture, we covered different kinds of random processes.

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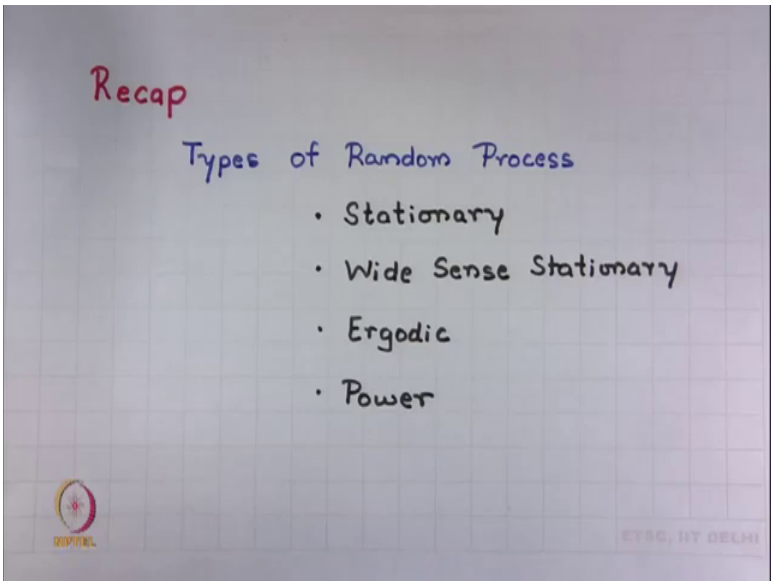

So, we looked into stationary random processes which are also known as a strictly stationary random processes; we looked into wide sense stationary processes, ergodic processes, power processes.

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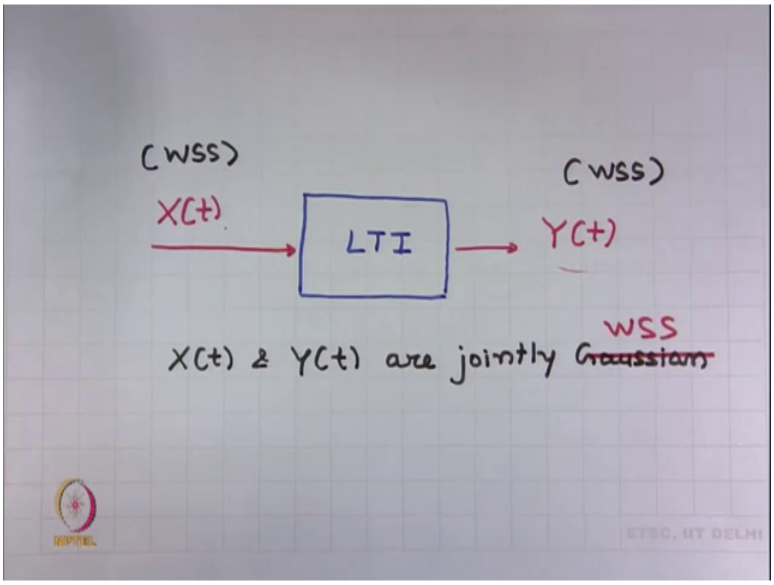

And we looked into what happens when a wide sense stationary process let us say x t passes through a linear time invariant systems, we said and we have seen that the output process also remains wide sense stationary. And we have seen that these two processes are also jointly wide sense stationary processes ok. So, this is what we discussed in the last lecture. Let us see what we have to cover today.

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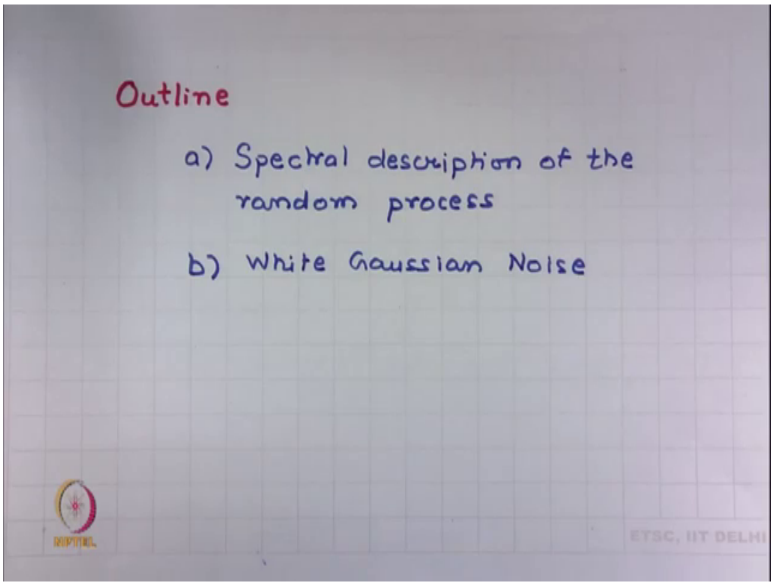

So, today we will talk about this spectral description of the random processes and we will see this white Gaussian noise. So, these two are the big concepts that we will introduce today ok. So, let us start by looking into the spectral description of a random process. And the way I like to do it is by thinking in terms of linear functionals ok.

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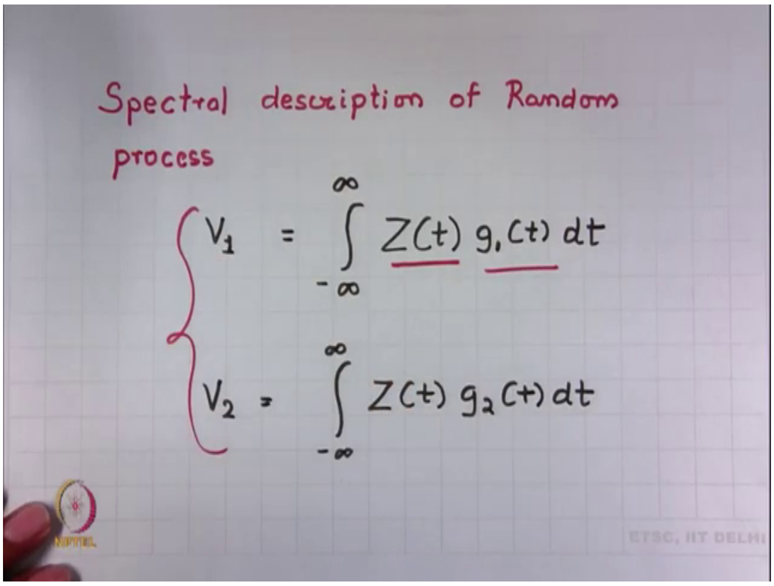

So, we have already defined what are these linear functionals. For linear functionals, we have to take the inner product of a process. So, this is a process Z of t with our deterministic function g 1 t ok. So, if you take the inner product of a process with a deterministic function, you get a random variable. And this random variable we call as the linear functional of the random process. Remember we are considering this function to be a real function, and that is why we do not have any conjugates here.

Similarly, I can take the inner product of a random process with another deterministic function. So, I will end up with a different random variable. And this random variable I call this as V 2. So, I am collecting two linear functionals of the random process by thinking about the inner product of the random process were two different deterministic functions ok.

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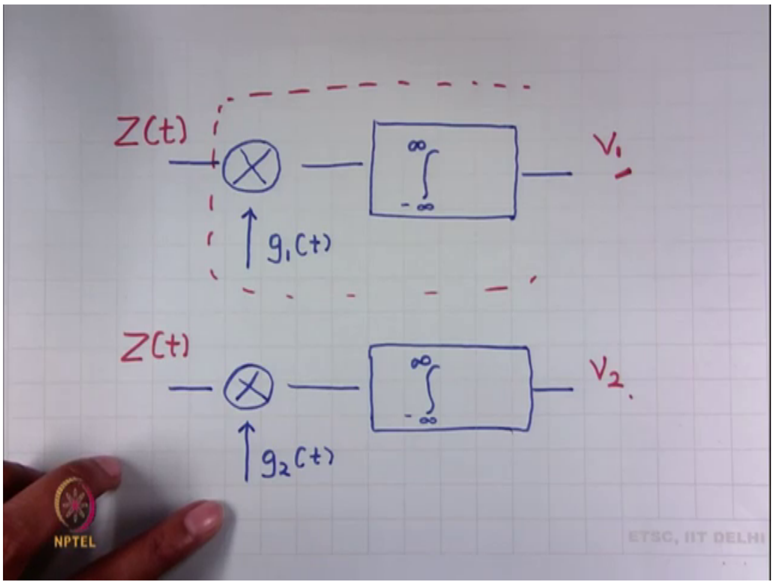

Why are we carrying it this out because if you remember the receiver architecture, then you know that in receiver we have this correlator. And what does this correlator do the correlator takes the inner product of the received waveform with a deterministic function. So, here we are assuming that at the input of this correlator, we have the noise, and we want to understand what happens when this noise passes through a receiver ok.

So, when this noise passes through this correlator structure this does the inner product of this noise process or a random process with this g 1 t, and what we end up with is this linear functional V 1. If this noise passes through a different deterministic function g 2 t, we end up with another linear functional that is V 2.

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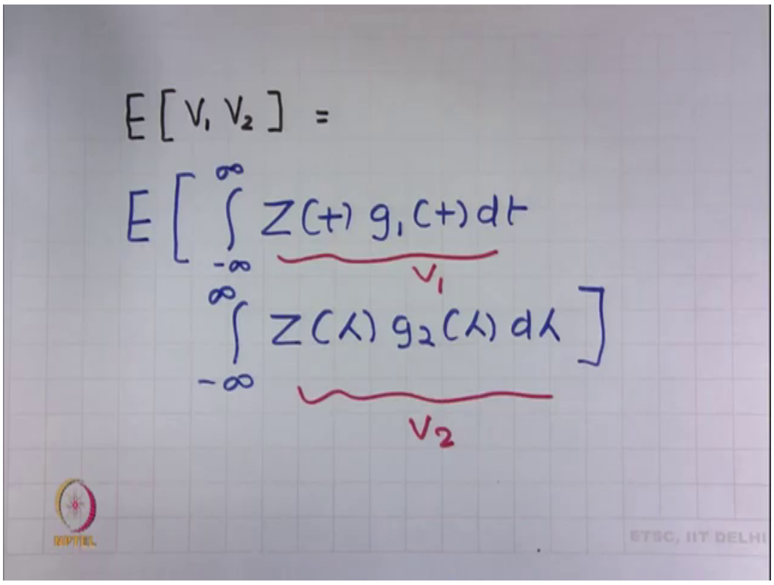

So, the question that we asked now is what is the expected value of V 1 and V 2, and this we have to carry out ok. So, what we are interested in is the expected value of V 1. So, I write expression for V 1 which is take the inner product of Z of t the random process Z of t with the deterministic function g 1 t. So, this is V 1. And what is V 2, V 2 is nothing but this quantity ok. So, I have changed the independent variable from t to lambda, it does not matter right. So, this is my V 1, and this is V 2. And I am interested in calculating what is the expected value of V 1 times V 2. So, now, what we have to do is to rearrange this integration.

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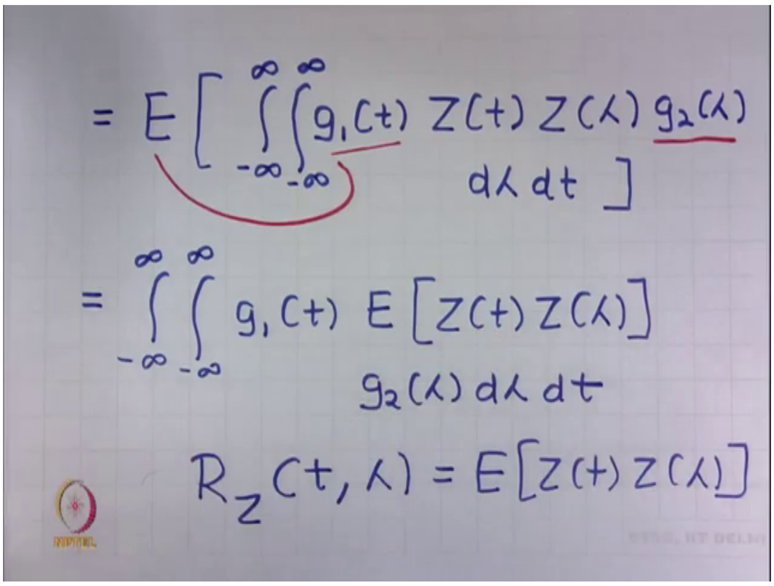

And when I rearrange this integration, what I get is so there are two independent variables hence it has to be double integration ok. So, what I did is I have converted this expression into this expression. And as I have said we can always do things like this ok.

Now, what we want to do is to pull this expectation operator inside the integration. And as you know that this is a deterministic function, this is also a deterministic function, so expectation operator will only operate on this term ok. So, let us do this. From this what we will get is ok. Now, we already know what is this quantity. So, we know that this quantity is the autocorrelation function. So, this is the autocorrelation function of the random process Z, where I am collecting the two random variablesZ of t and Z of lambda.

So, this denote that I am collecting the random variable from the random process Z of t, and the time instants t and I am collecting another random variable at the time instants lambda ok. So, I know that I can convert this or I can represent this into this form, and this is known as autocorrelation function.

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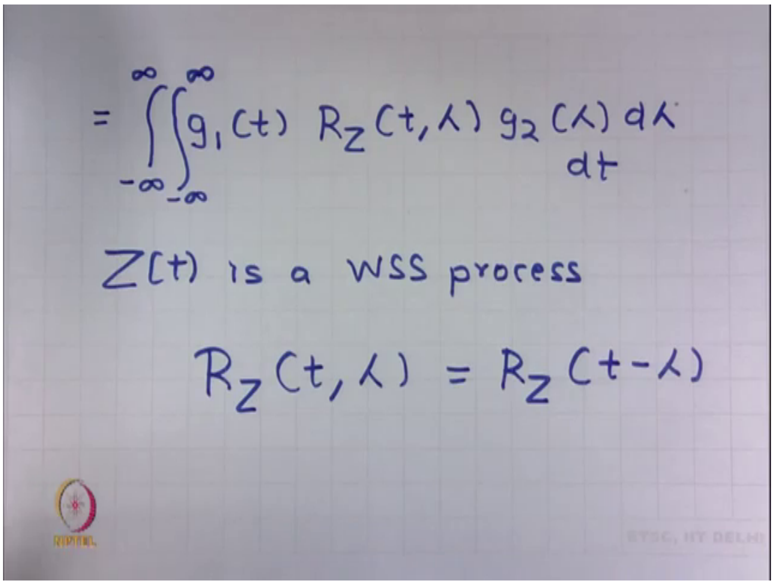

So, substituting this in place of this, we get ok. Now, let us make some assumptions to simplify this further. And the assumption that I make is Z of t is a wide sense stationary process. So, this is the assumption that I make. If Z of t is a wide sense stationary process, we have already seen that this autocorrelation function based on two arguments can be converted into the autocorrelation function based on single argument, where the single argument is obtained by taking the difference between these two arguments. So, for a wide sense stationary process, this autocorrelation function can be conveniently represented in this form ok. So, now, let us substitute this value of autocorrelation function in here.

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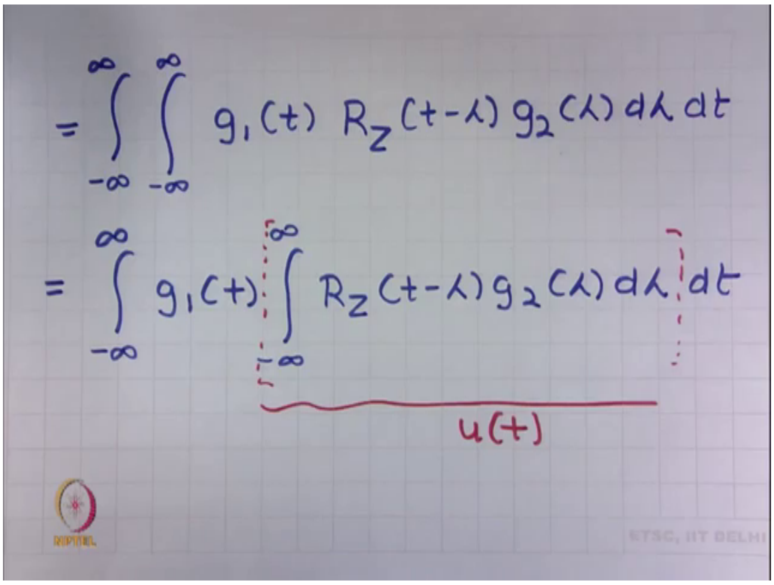

And let us see what we get ok. So, from this we can obtain this and let us now rearrange the order of integrations. So, let me convert this into this form. Let us look at this function. I call this function as u of t. So, let me see what is this u of t.

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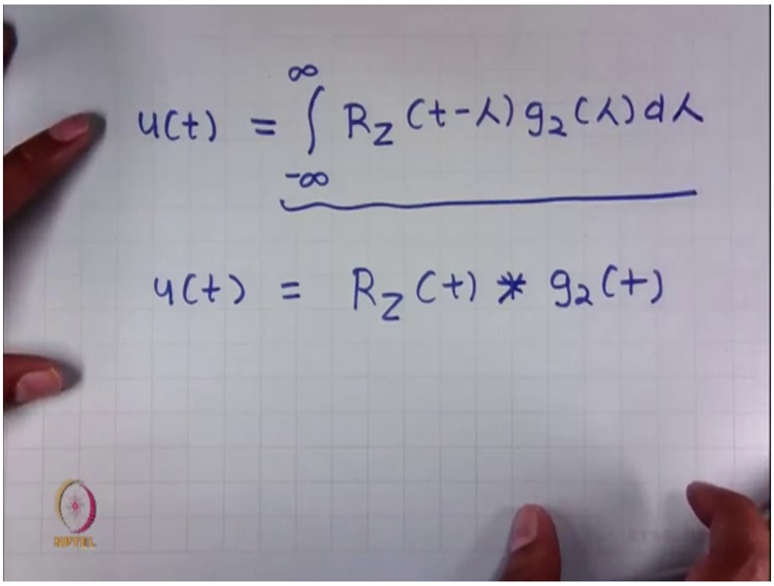

So, I have defined a function of u of t which is nothing but first of all as lambda is a running variable, this will not be a function of lambda, but this will be a function of t. So, this is a function of t. And we call this function as u of t. Now, you can also realize that this is a very famous integration formula, and this represents the convolution of the two functions ok.

So, u of t is nothing but it is the convolution of autocorrelation function with deterministic function g 2 ok. So, I can write it like this. And when I write it like this, I realize that this is nothing but it is the convolution between these two functions ok. So, let us now go back to that expression which we obtained.

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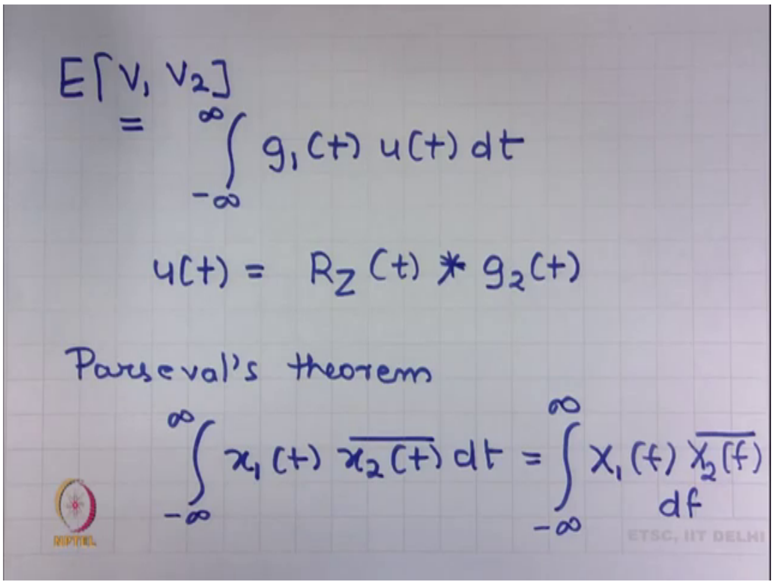

So, trying to think about this we get expected value of V 1 V 2 this is what we were deriving is nothing but it is integration g 1 t u t d t, where u t we said is nothing but it is the convolution of these two functions ok.

Now, let us invoke Parseval's theorem. So, let me remind Parseval's theorem. Parseval's theorem says that the inner product remains preserved when going from time domain to frequency domain. So, let me write that. So, if I am interested in taking the inner product of two signals in time domain. So, this bar is for conjugation. So, this is the inner product of the two signals in time domain. And this inner product will be same as this inner product, where x 1 f is the Fourier transform of x 1 t, and x 2 f is the Fourier transform of x 2 t. So, I have these two time domain signals I have obtained their frequency domain signals. So, if I take the inner product of the signal in time domain or if I take the inner product of the signals in frequency domain, this inner product remains preserved ok. So, and this is the idea behind Parseval's theorem.

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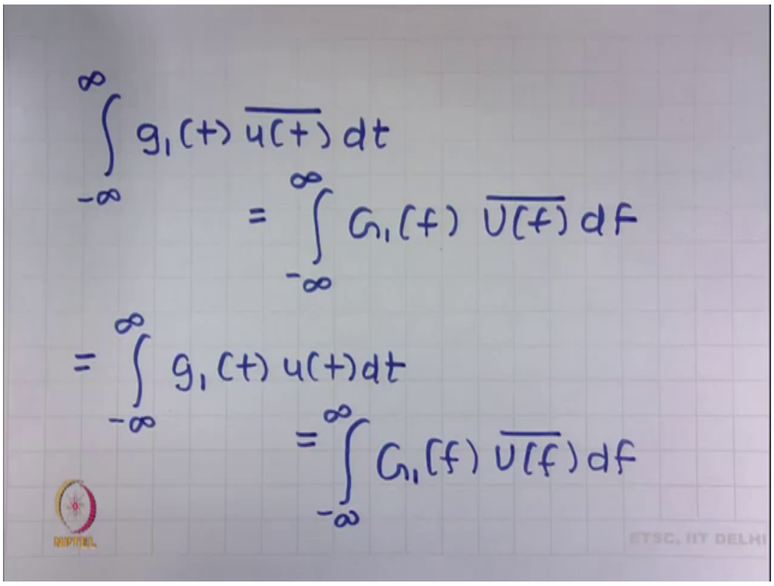

So, using Parseval's theorem, I can write g 1 t u t conjugate d t is nothing but it is G 1 f U f conjugate d f. And as expected G 1 f is the Fourier transform of g 1 t, and U f is the Fourier transform of u t. As u t is a real signal, it does not matter whether you take a conjugate or not. So, this is same as this. So, we can say in this quantity is same as this quantity ok. So, let us summarize what we have obtained so far.

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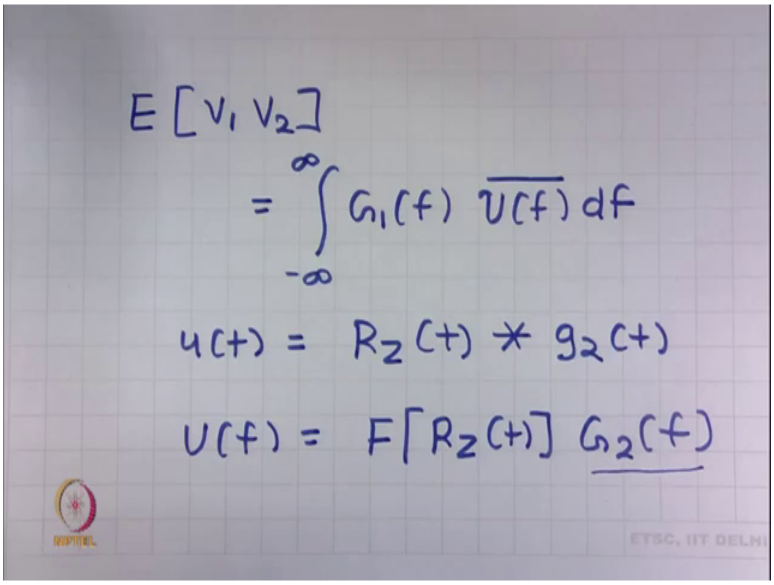

So, we have got that the expected value of two linear functionals of the random processZ of t is nothing but it is G 1 f into U f conjugate d f right. Now, let us look at what is this U f conjugate, to think about this let me remind what was u t, so u t was the convolution of autocorrelation function and g 2 t. So, the Fourier transform of u t is nothing but it is the Fourier transform of autocorrelation function multiplied by the Fourier transform of G 2 t. So, G 2 t has a Fourier transform G 2 f. And we are assuming that the Fourier transform of this autocorrelation function is this thing ok. And we are also using a property that convolution in time domain results to multiplication in frequency domain.

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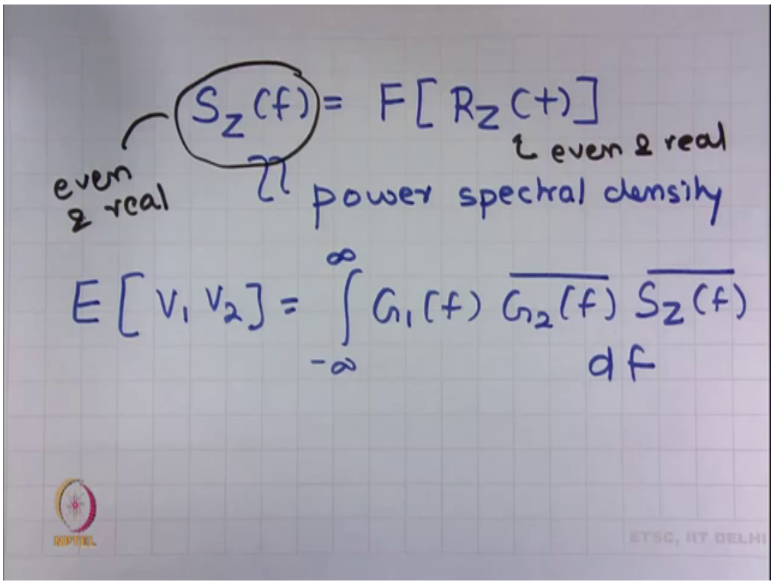

Now, let me define a very useful quantity which is S z of f which is nothing but it is the Fourier transform of autocorrelation function. Now, this S z of f is known as the power spectral density. And we will look this in more detail, but at this point it is sufficient to think it as the Fourier transform of autocorrelation function. Now, let us substitute this value of U f in this expression. So, we get expected value of V 1 V 2 is G 1 f and U f conjugate is nothing but it is G 2 f conjugate into S z f conjugate d f. So, this is the expected value of V 1 and V 2.

Now, let us also realize that because power spectral density is a Fourier transform of autocorrelation function, and because this function is even and real this we have seen in the previous lectures that autocorrelation function is even in real. Power spectral density by the properties of Fourier transform is also even and real. So, you must have seen in the course on Signals and System that the Fourier transform of real and even quantity is also real and even. And hence we can say that the power spectral density is also real an even, because autocorrelation function is real and even. So, if we realize that then whether you take a conjugate in here or you do not take conjugate does not matter, because this is a real quantity ok.

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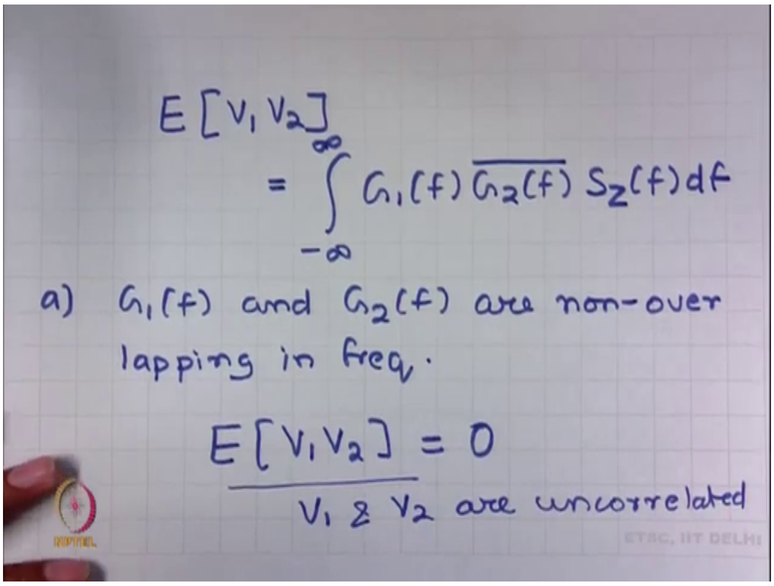

So, in that context, we can rewrite this expression as this ok. So, we have obtained the relationship that we wanted to obtain. And let us now try to think more about this expression, and let us try to reason out does it tell us something important. So, the first point that we can realize if we look at this expression is if G 1 f and G 2 f are non-overlapping in frequency that means if they are in different frequency bands, so they do not have any region of overlap, then this quantity is going to be 0 ok. Then expected value of V 1 V 2 is going to be 0 that means, these two random variables are uncorrelated right, they are 0 mean. So, we can say from here that V 1 and V 2 are uncorrelated.

Furthermore if we assume the underlying process to be a Gaussian process, and we know that if the underlying process is the Gaussian process, then V 1 and V 2 are jointly Gaussian random variables. And we have seen that if V 1 and V 2 are uncorrelated and they are jointly Gaussian, then we know that they will also be statistically independent.

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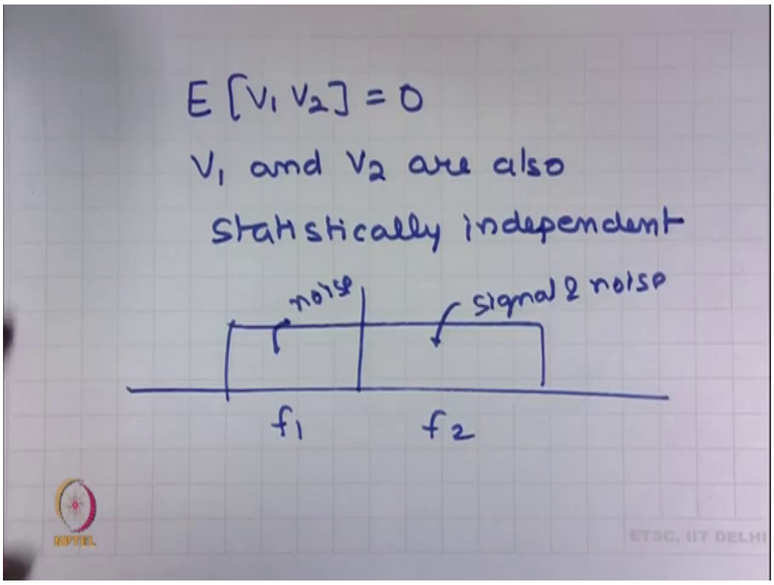

So, what we are saying is if the expected value of V 1 and V 2 is 0, and if V 1 and V 2 are the linear functionals of a Gaussian process, then V 1 and V 2 will also be statistically independent. And the most interesting process that we have already said is the Gaussian process is not it? Thus, if the noise that you are assuming is a Gaussian noise, and if you are collecting two linear functionals of the noise, and you are collecting in such a way that the deterministic function has the non-overlapping spectrum, then what you can think is that these two linear functionals will be statistically independent.

What are these implications in the context of digital communication? If implication is that suppose if you have the noise which is over two bands f 1 and f 2. And let us assume that in this band we have signal and noise. And in this band, we simply have noise. So, one strategy that you can think is can I observe this noise in this band to learn anything about this noise in this band. And the answer is no, this strategy will fall flat because noise in different frequency bands is completely independent. And thus by observing a noise in one band, we do not learn anything about the noise in another band. And thus such strategies cannot be used in the context of digital communication ok, so that is one thing.

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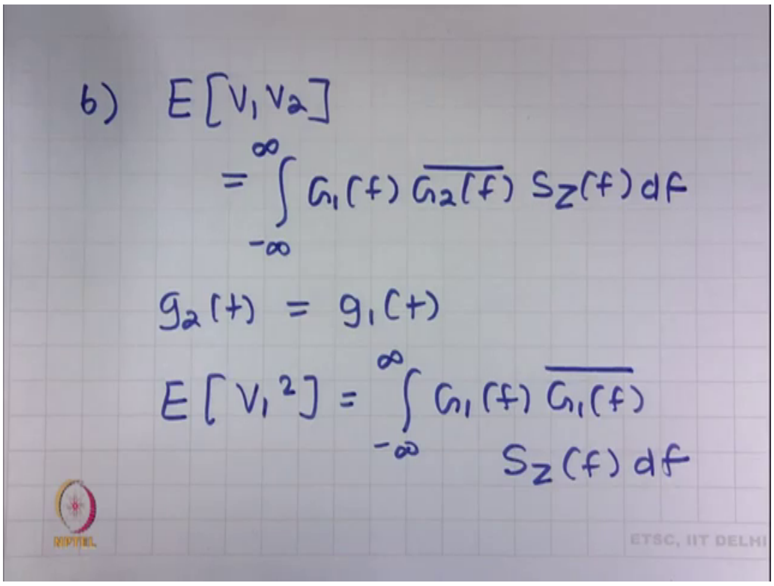

The second thing is so let me write that expression again expected value V 1 V 2, we have said is. Now, let me assume one thing that g 2 t is same as g 1 t, actually we are talking about the same linear functional. So, we get expected value of V 1 square is nothing but G 1 f G 1 f conjugate S z f d f.

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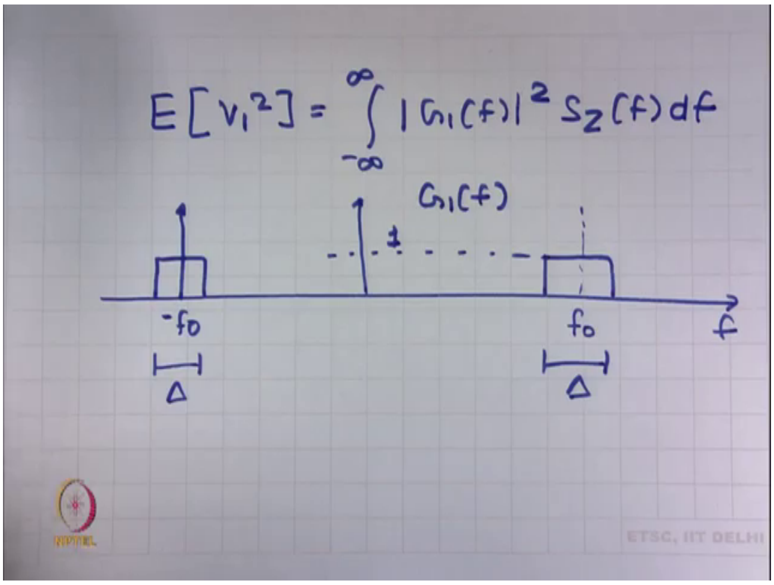

And from here what we will get is expected value of V 1 square is this thing. Now, because G 1 f is the Fourier transform of a deterministic function g 1 t, I can choose any g 1 t and thus I can choose any G 1 f. And so let me propose a G 1 f which looks like this. So, 0 everywhere except let us say at a band around f naught so, it only has nonzero value and a band around f naught, and this value for simplicity let me assume S 1. So, this is let us say is the spectrum of G 1 f and this band also let us assume is a very small band and we call the width of the band as delta. So, let us try to understand what is this quantity when you pass the random processZ t through such a filter.

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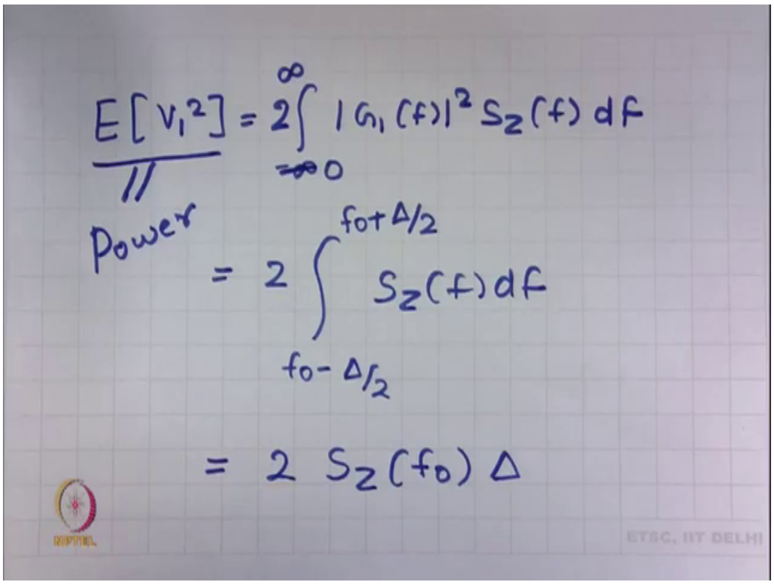

What we will get is first of all we see that this is a symmetric function or any even function. We already know that this is also an even function. So, in that case instead of integrating from minus infinity to plus infinity, we can simply integrate from 0 to infinity and multiply this thing with a factor 2, so that is first thing that we see. Second observation is this is 0 everywhere else other than frequency around f naught. So, let me just take the limit of integration between f naught minus delta by 2 to f naught plus delta by 2. And in this band G 1 f is 1. So, I end up with this.

And if I assume this delta is very small, and also assume that as that f is constant in that band because delta is very small, I can pull this out of this integration. And what I get is 2 S z f naught, because this is the value of the power spectral density at f naught ok. So, what we are assuming is that the value of the power spectral density at the frequency f naught will be S z f naught and we will have delta. And we have already seen that this quantity represents the power ok.

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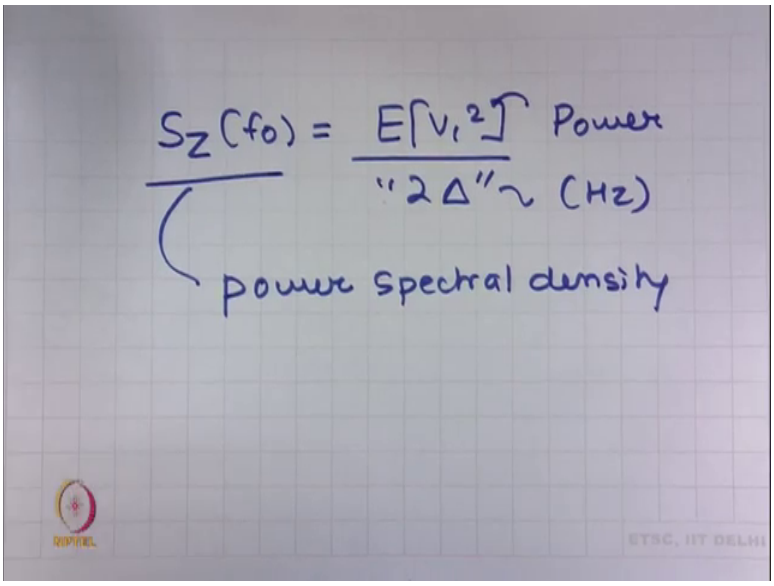

So, what we get is S z f naught is nothing but it is the expected value of V 1 square divided by 2 delta. We know that this is the power and delta has a unit of hertz. So, this quantity is nothing but it is the power spectral density. And we can also think about this as power per unit bandwidth. So, this is the physical meaning of this power spectral density.

So, we have developed in this section two important ideas. And the idea is we have started by looking into the linear functionals and then we have been investigating what is the expected value of V 1 times V 2. So, there we have seen that if the deterministic functions through which these V 1 and V 2 S are obtained are the functions whose spectrum is non-overlapping in frequency, then these random variables are uncorrelated, furthermore if the involved random process is a Gaussian process, then we also know that these random variables are statistically independent.

And the second thing that we have caught is that this quantity corresponds to the power, and the Fourier transform of autocorrelation function which we define as the power spectral density turns out to be of this form. So, next what we have to do is to look into the properties of power spectral density. So, the first thing that we have said about the power spectral density already that power spectral density is nothing but it is the Fourier transform of autocorrelation function.

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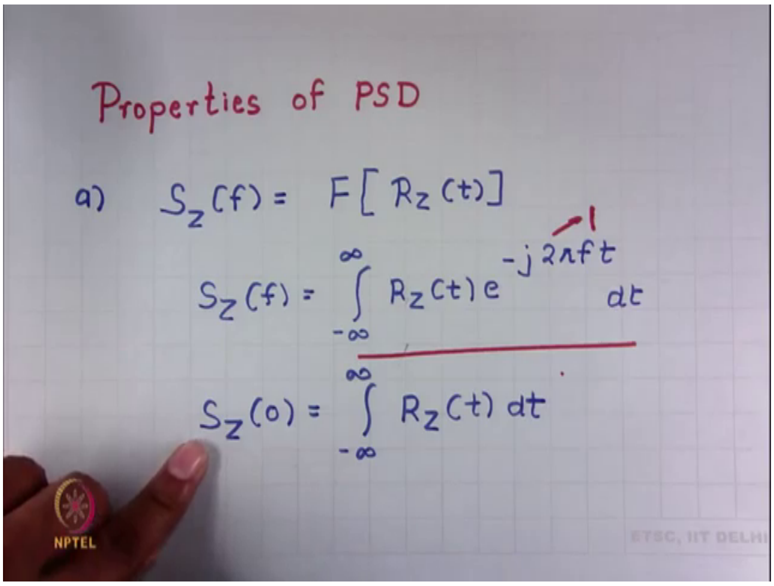

So, if we look at these expressions, the power spectral density is the Fourier transform of autocorrelation function. So, you know that this gives me the Fourier transform of a quantity. And furthermore if I put f equals to 0, and use something like moments theorem, then this goes to 1. So, we end up with this. That means, we are saying if you take the area under the autocorrelation function, you get the power spectral density at frequency 0. So, this is an important idea which you can use.

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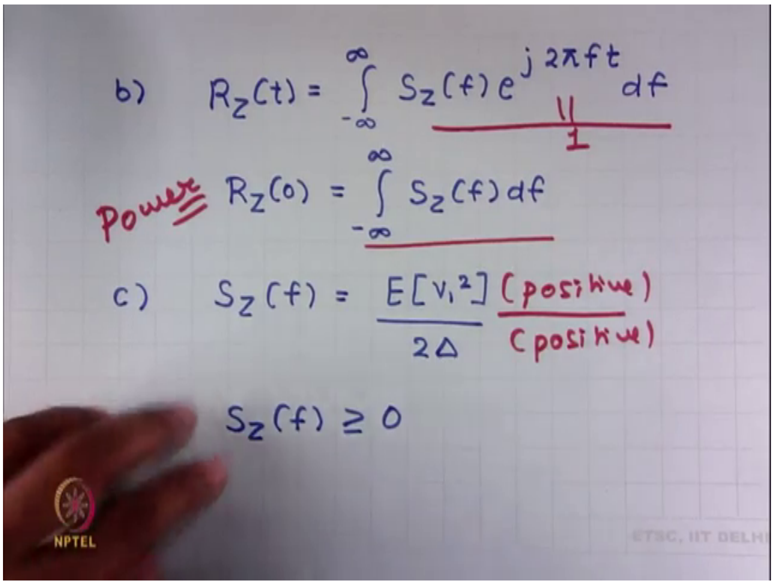

Let us look at other properties. For example we also know that if power spectral density is the Fourier transform of autocorrelation function, autocorrelation function will be the inverse Fourier transform of power spectral density ok. So, this is the relation to get the inverse Fourier transform. Similarly, I can substitute t as 0, this term goes to 1, and what I end up with is this factor that means we are saying that area under the power spectral density is nothing but R z 0 and R z 0 is nothing but is the power in the random process this we have seen before.

That means, power of a random process can be obtained by integrating the power spectral density from minus infinity to plus infinity and that is obvious. If you have a power spectral density, if you integrate from minus infinity to plus infinity, you should get the total power in the process.

So, as we have already said that this power spectral density is nothing but it is the power per unit bandwidth and because it is power per unit bandwidth it has to be strictly non-negative, because this quantity is positive this is also a positive quantity. So, what you can have is a positive quantity or at most 0, it can never be a negative quantity right. So, power spectral density is always in non-negative quantity. We have also seen that because power spectral density is a Fourier transform of autocorrelation function, and because this autocorrelation function is even and real, the power spectral density is also even and real.

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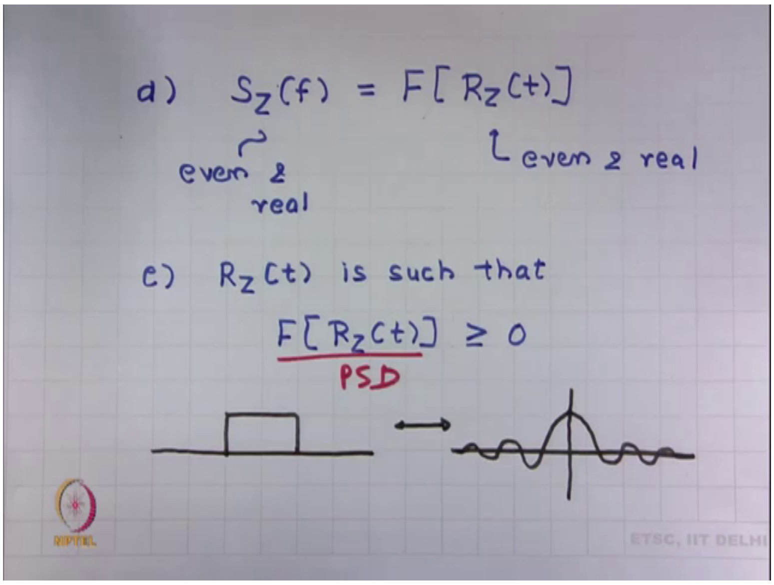

Then we can also impose one more condition that the autocorrelation function should be such that if you take its Fourier transform, this should give me power spectral density. And because power spectral density is strictly non-negative, you can have a function as an autocorrelation function only F its Fourier transform is non-negative. That means, suppose if someone says is this a valid autocorrelation function, the answer is no, because its Fourier transform gives me a sinc function and sinc also obtains negative values, and hence this is not a valid autocorrelation function.

So, the examples of valid autocorrelation functions are those functions whose Fourier transform is strictly non-negative. So, these are certain properties that are important in the context of power spectral density.

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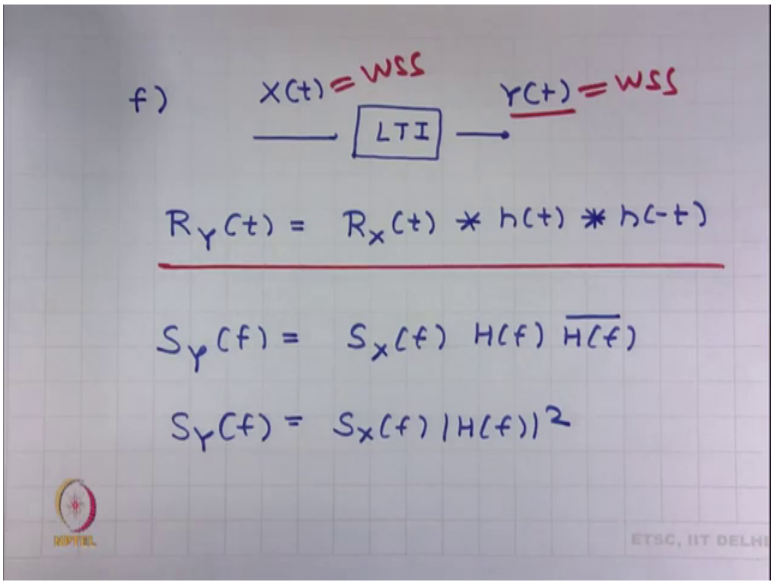

You can also look at the last property. And this follows from what we did in the last lecture. We have seen that autocorrelation function of an output process. So, we have input process which is a wide sense stationary process, output process we also assume is a wide sense stationary process. We have already derived that the autocorrelation function of the output process is nothing but it is the autocorrelation function on the input process convolved with the impulse response of an LTI system convolved with the h of minus t, where h of t is them pulse response of an LTI system. So, this relationship we derived in the last lecture.

Now, we know that you can go from autocorrelation function to power spectral density by just taking the Fourier transform. So, I take the Fourier transform of this, I get the power spectral density of the output process. So, this leads to power spectral density of the input process because we have convolution in time domain, in frequency domain, it would be substituted by simple multiplication h of t has the Fourier transform of H of f, h of minus t will have the Fourier transform H f conjugate.

And thus we have that the output power spectral density is nothing but it is input power spectral density multiplied by mod square of H f ok. So, this is how you can estimate the power spectral density of the output random process if the input power spectral density is given to you and the frequency response or the filter is given to you. This is an important idea. Remember that here we are considering impulse response to be a real function ok.

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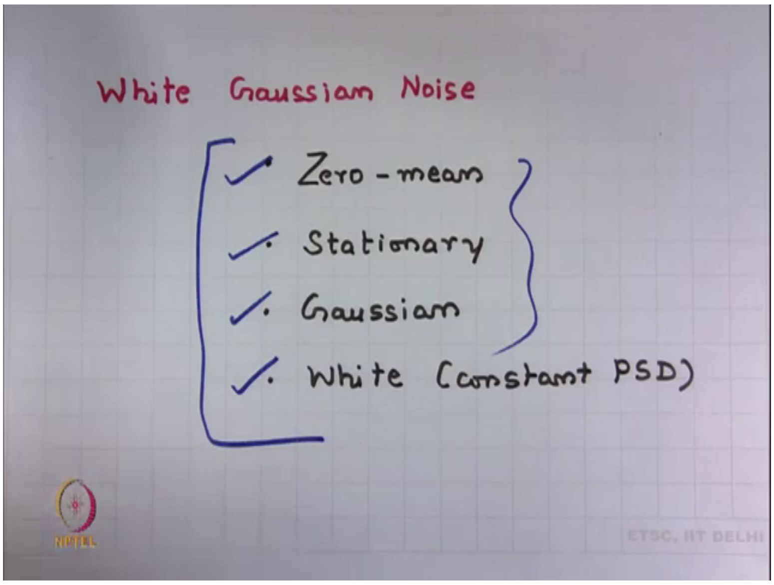

So, let us now move to the next topic and that is white Gaussian noise. So, far while discussing about the noise we have said that a practical noise process should be a zero mean noise process. We have said that the practical noise process should be stationary random process. We have seen that a practical random process should be a Gaussian random process ok. Now, what we are saying more is the practical random process must also be a white random process. With white random process I mean that it has the constant power spectral density ok.

So, if a random process has a constant power spectral density, this random process is referred to as a white random process. And what is this white Gaussian noise, so white Gaussian noise is a random process which satisfies all these four practical constraints, that means, its zero-mean, its stationary, its Gaussian, its white ok. These three properties we have already seen and this is the new addition to the properties of a practical random process.

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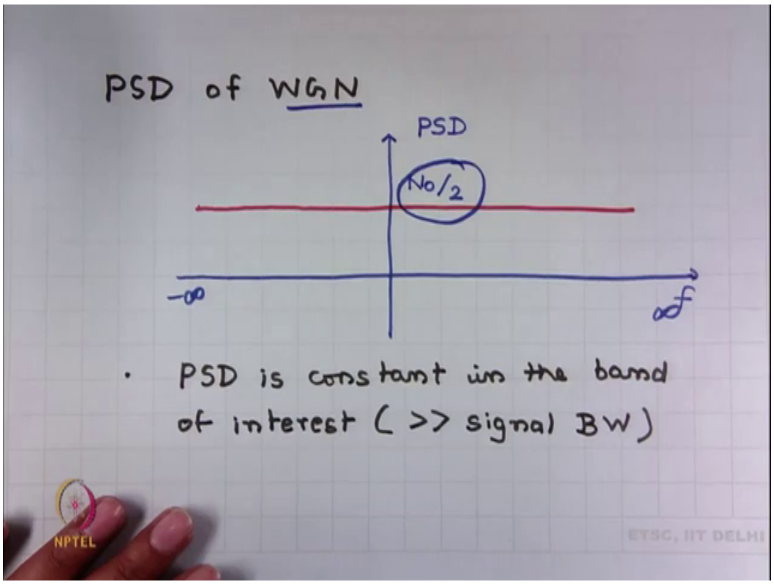

So, what we are saying is the power spectral density of a white Gaussian noise which is abbreviated as WGN. So, it is a very common abbreviation. You should get familiar with this abbreviation WGN. W refers to White; G refers to Gaussian and N refers to Noise. So, the power spectral density of a white Gaussian noise is constant ok.

So, here you can see that this is constant, and the value of this constant is N naught by 2. So, this is pi notation and we will talk why this N naught by 2 in a while, but let us not worry about this at the moment. Thus let us think that the power spectral density is some constant and this value of this constant is N naught by 2.

When we are thinking about this white Gaussian noise, when we say this power spectral density is constant, and though I have drawn as if it is constant from minus infinity to plus infinity mathematically this is so, but in practice it simply means that it is constant over the band of interest. For example, if the power spectral density of noise is constant for the range of frequencies which is much larger than the bandwidth of the signal, then we can treat that power spectral density of that process as practically constant. And this is the meaning of white. It does not mean that the noise process has the power spectral density constant from minus infinity to plus infinity. It simply means that the noise power spectral density is constant over the band of interest right. And then we do not worry about what it tests in the outer band ok. So, this is some practical constraints ok.

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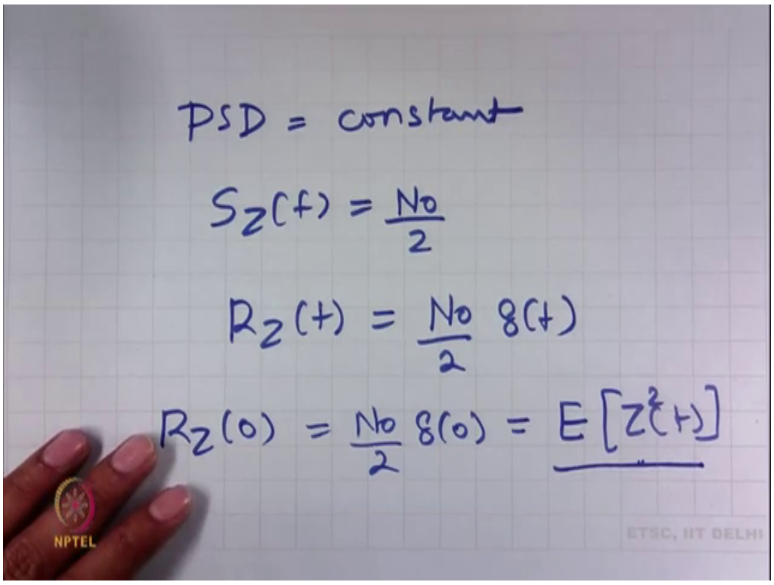

So, if the noise has a power spectral density which is constant ok and this we have said is the case in the white Gaussian noise. So, S z of f is N naught by 2, what is this autocorrelation function? So, its autocorrelation function would be an impulsive function. You must have seen in a course in Signals and System that the Fourier transform of a constant is an impulse, so Fourier transform of a power spectral density which is constant is an impulse alright.

Now, so what is this R z of 0, R z of 0 is N naught by 2 del of 0. And R z of 0 is also expected value of Z square t, where Z t is the random process. That means, we are saying that this quantity is infinite because this is infinite and this is the power of the process. So, we are saying that we have a process whose power is infinite and that is not practically true, and also we will have some mathematical difficulties in defining what this said t is because this value is infinity ok. So, we end up with some mathematical intricacies when we try to model noise as white noise, where white here means that it is white over all frequencies of interest, because we do not want to answer the band over which it is white.

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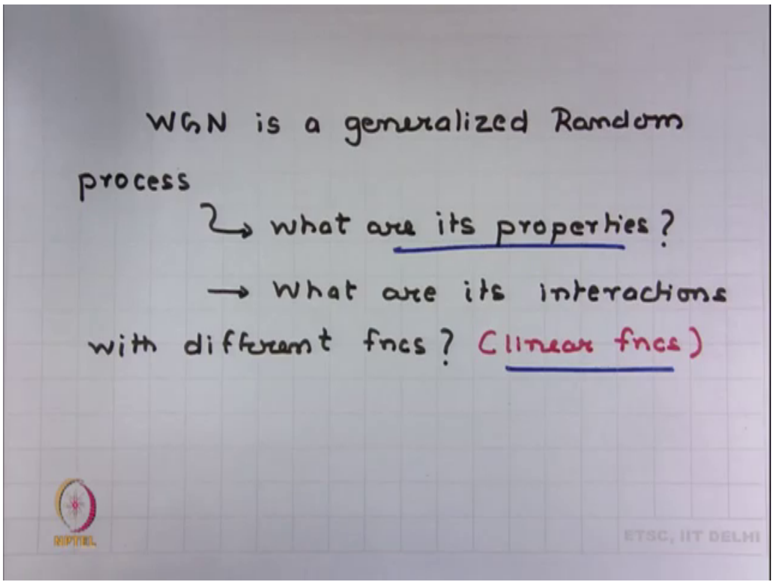

So, to solve this we say that white Gaussian noise is not a random process, but it is a generalized random process ok. What we mean by this generalized random process is simply that instead of worrying about what it is and defining it exactly we should just worry about learning its properties. So, it is defined based on the properties, and it is defined based on its interactions with deterministic functions or based on its linear functions. So, its defined based on its properties and it is defined based on its linear functionals right. It is not so important to define it as we define a random process.

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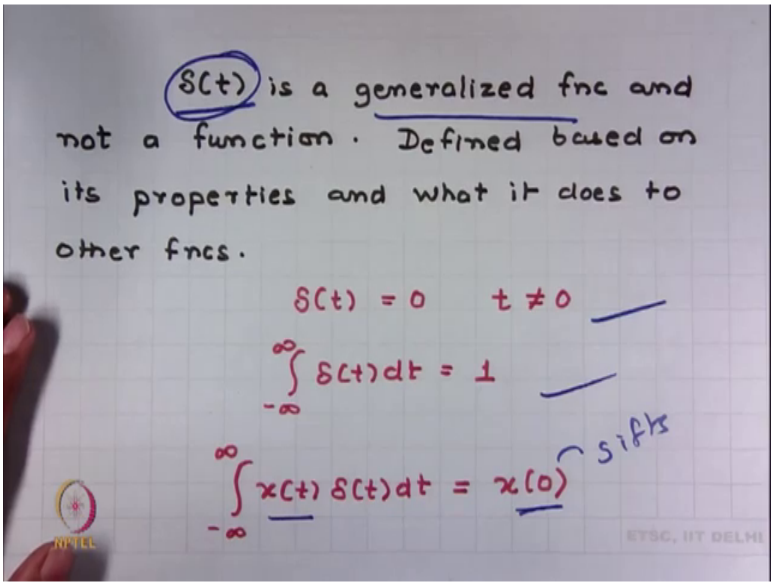

And this is the problem that you must have seen before. For example, delta t or impulse function is also not a function, but it is a generalized function. So, you cannot tell me about this delta t by specifying the values of delta t at all points in times rather if you want to talk about delta t you have to talk about its properties ok. So, for example, you can specify the properties that it is 0 for t not equals to 0, you can say that its area is 1 and so on and so forth. You can think about it how it interacts with other functions.

For example, if you multiply x t with delta t and you carry out this integration you get x of 0 right. It is sifts out the value of x t at t equals to 0. So, these are the properties based on which we understand already rather than thinking it in terms of conventional functions because it is not a function set generalized function. Similarly when we want to think about the white Gaussian noise, it is not so important to define it exactly, but you have to understand it in terms of its properties ok.

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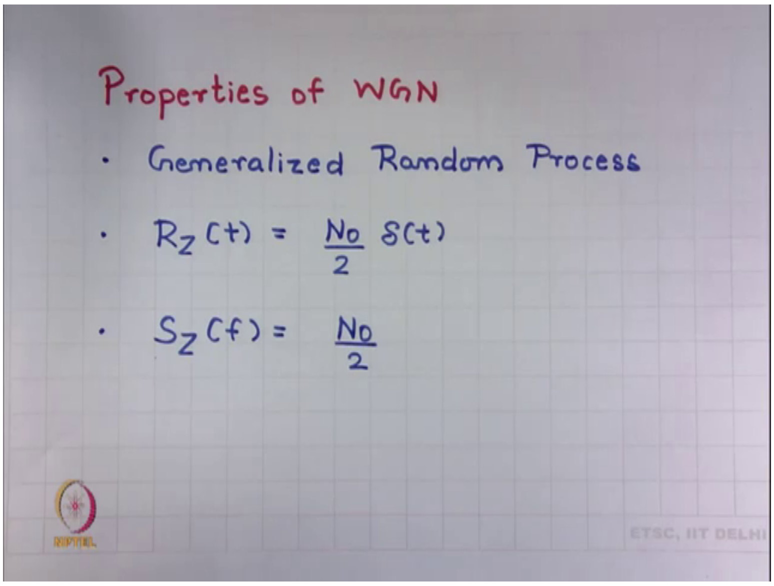

So, what are the properties of white Gaussian noise that is the next question? And the properties of the white Gaussian noise is, it is a generalized random process. It has an auto correlation function which is N naught by 2 times delta t. It has a power spectral density which is constant, and the value of this constant is N naught by 2.

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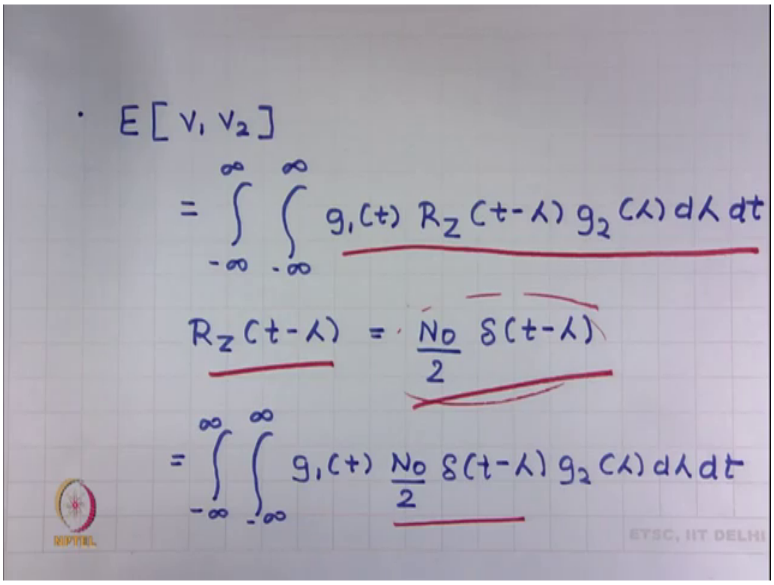

Now, let us evaluate this quantity expected value of V 1 V 2. And it is not difficult. We can follow the reasoning that we have used before to find out this quantity. From basics this is nothing but given by this expression. Remember that white Gaussian noise is also stationary noise. So, we can convert this autocorrelation function pairs in two arguments into the autocorrelation function single argument. And this in the case of white Gaussian noise is nothing but this quantity right. So, substituting this value of this function in this place, we get this ok.

Now, you see that this integration would be 0 for all lambdas which are not same as t right. And hence the contribution of this integration whenever lambda is different from t is 0. And hence we have to consider the case when lambda is same as t. And once you do that instead of having double integration, you wind up with single integration alright.

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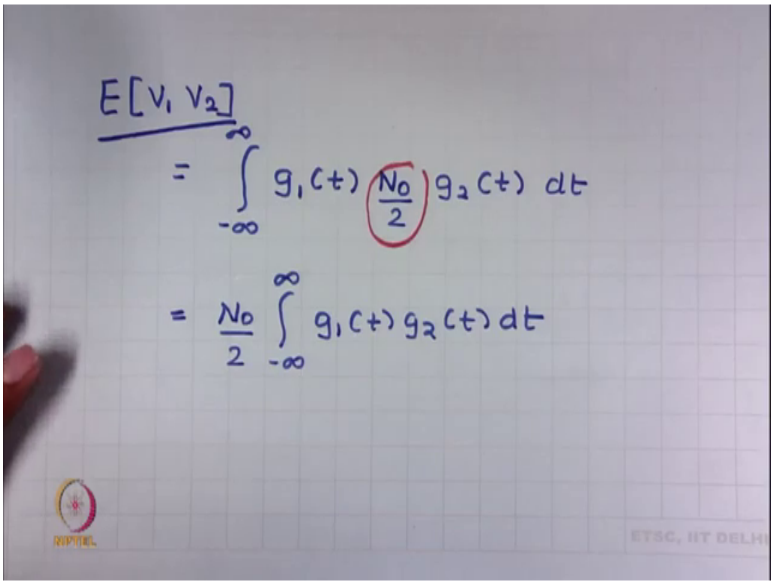

And what you get is this function. So, you have g 1 t times N naught by 2 g 2 t and d t, because this is not a function of time I can anyway pull this out, and I get this formula good. Now, so this what we have got.

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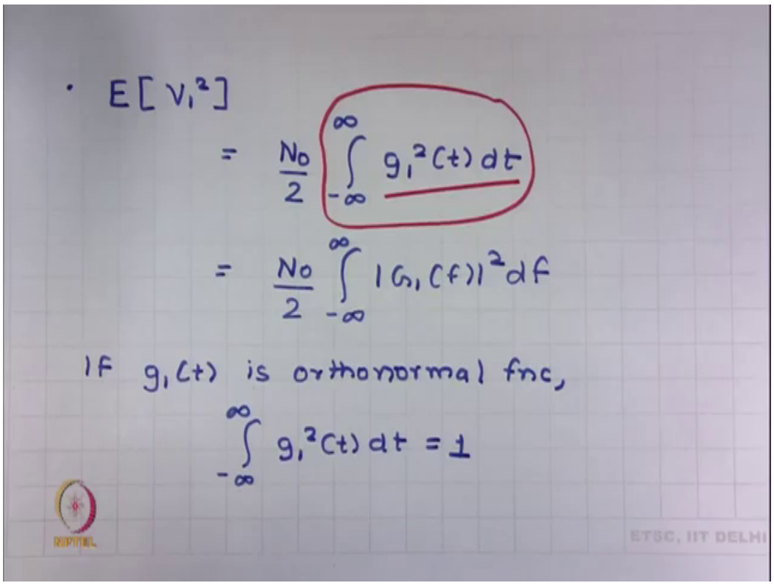

Let us first see what is the expected value of V 1 square. So, to think about this you just have to replace g 2 t with g 1 t. And in that case you get g 1 is square t d t. And then we can also use Parseval's theorem where instead of thinking about this, I can think about this integration in this way. So, this is by Parseval's theorem, we have carried out this integration several times and this is what it is.

Now, let us consider the case when this g 1 t is orthonormal function if this is an orthonormal function its energy will be 1. And this we know represents the energy of the quantity. So, if it is an orthonormal function, we can substitute this as 1 and this is an important result.

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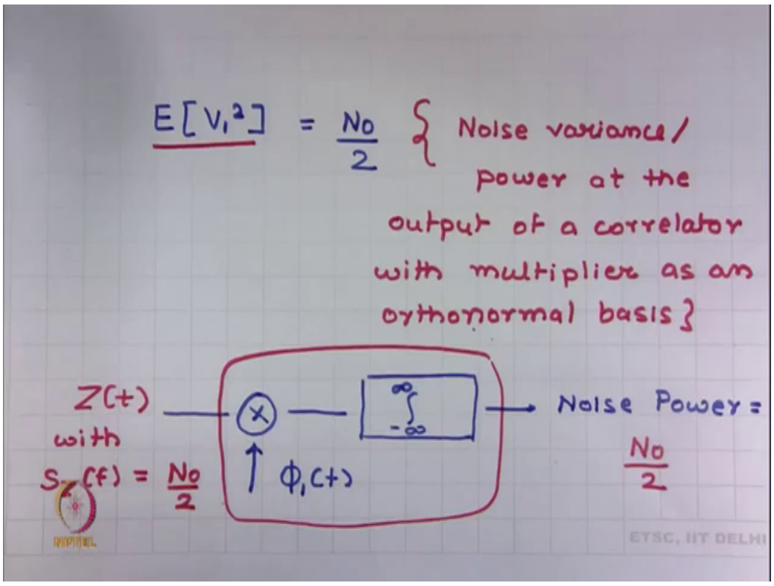

Because what we are saying is expected value of V 1 square, if I substitute this as 1 will be simply and N naught by 2 once you pass this white Gaussian noise through an orthonormal function. And this quantity we have seen before is nothing but it is the noise variance. And if it is the noise process with zero mean it is also the noise power. So, what we are saying is the noise variance or noise power available at the output of a correlator. So, we have already seen what the correlator is, correlated carries out the inner product of process with a function. And if the involved function is an orthonormal function, then the noise power or noise variance available at the output of a correlator is nothing but N naught by 2.

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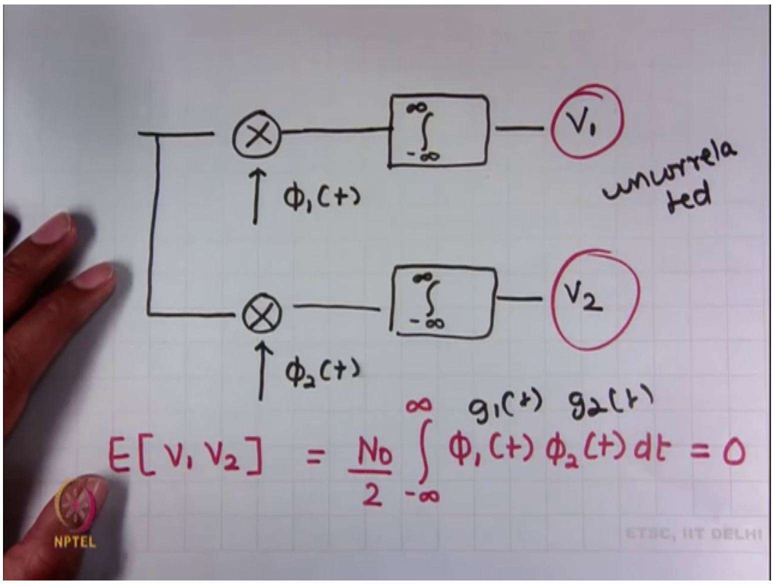

Also what we can do is if we pass this white Gaussian noise through two orthonormal functions, we get two random variables as before V 1 and V 2. And if these are orthonormal functions, so this is remember this was g 1 t and this is g 2 t. If they are orthonormal what we get is 0. So, the expected value of V 1 V 2 is 0. And thus we can say that these will be uncorrelated. And because they are the random variables obtained through Gaussian process, we also know that they will be statistically independent.

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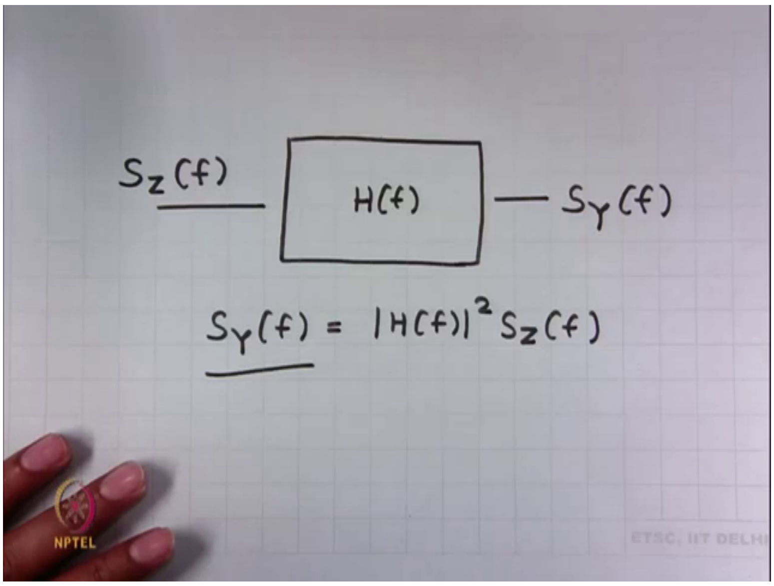

Final point, what happens if you pass this white Gaussian noise through a filter? We already have seen that the output power spectral density is nothing but the input power spectral density multiplied by mod square of the frequency response of the filter. And this property can be used to generate Gaussian random processes with different power spectral densities. You can really manipulate the power spectral density of the Gaussian random processes.

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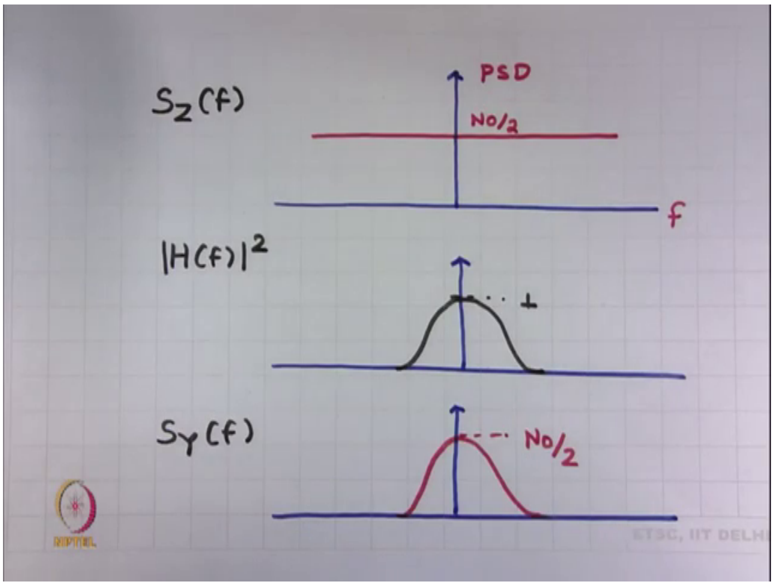

For example, if you have this as the power spectral density of the input random process, particularly in this case we are talking about white Gaussian noise. So, white Gaussian noise has this power spectral density. If it passes through a filter whose mod square of frequency response is such a response, then the output power spectral density is shaped by this function.

And because this function is a new control, you can generate the output power spectral density the way you like to generate. And this is an important idea ok. So, using white Gaussian noise and using an appropriate filter, you can obtain different Gaussian processes with different power spectral densities.

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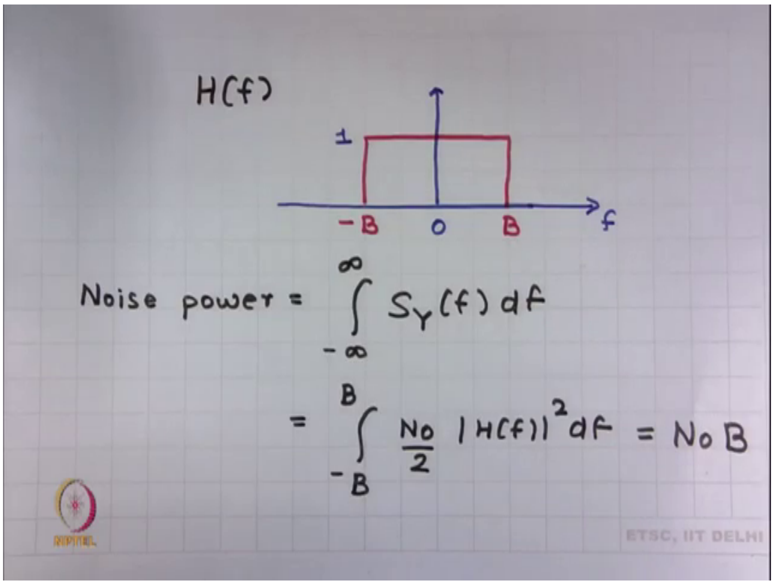

Finally, the last point where we will like to talk about this noise power and try to understand the idea of N naught by 2 is let us now pass this noise through this filter, this is a rectangular shaped filter whose magnitude is 1, and it has a bandwidth of B. So, noise power can be obtained by integrating the output power spectral densities.

So, this power spectral density as we have seen is nothing but the power spectral density of white Gaussian noise which is N naught by 2 multiplied by the mod square of the frequency response of the filter. And you have to integrate this thing from minus B to B to obtain the total noise power. And here in this range this is 1. So, what we get is N naught times B. So, the total noise power that is available at the output of a filter is N naught times B.

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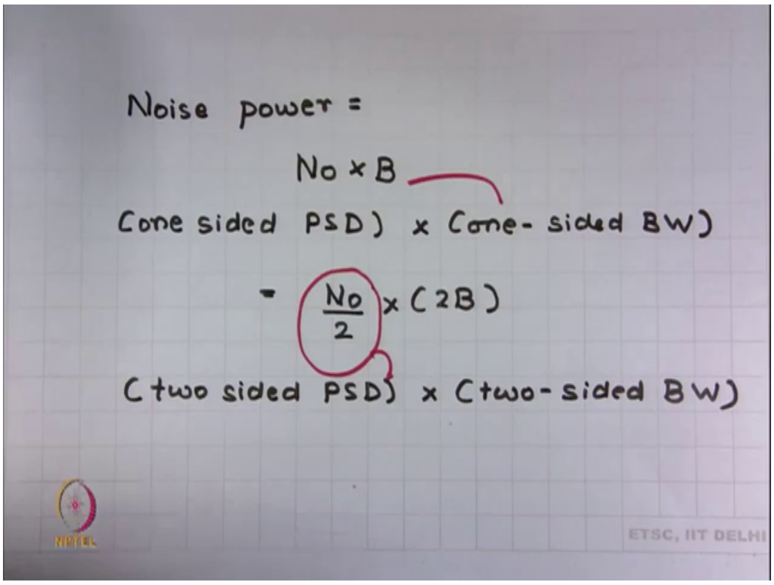

So, what we are saying is noise power available at the output of a filter is N naught times B, we have already seen that this B is refer to as one-sided bandwidth. So, while thinking about the one sided bandwidth we have only to look at the positive side of the spectrum and this n naught is one-sided power spectral density.

So, you can obtain the noise power by multiplying the one-sided power spectral density with one-sided bandwidth or you can multiply the two-sided bandwidth with two-sided power spectral density. This N naught by 2 is actually two-sided power spectral density. So, if you are using the two-sided power spectral density, you have to multiply this with the two-sided bandwidth to get the noise power.

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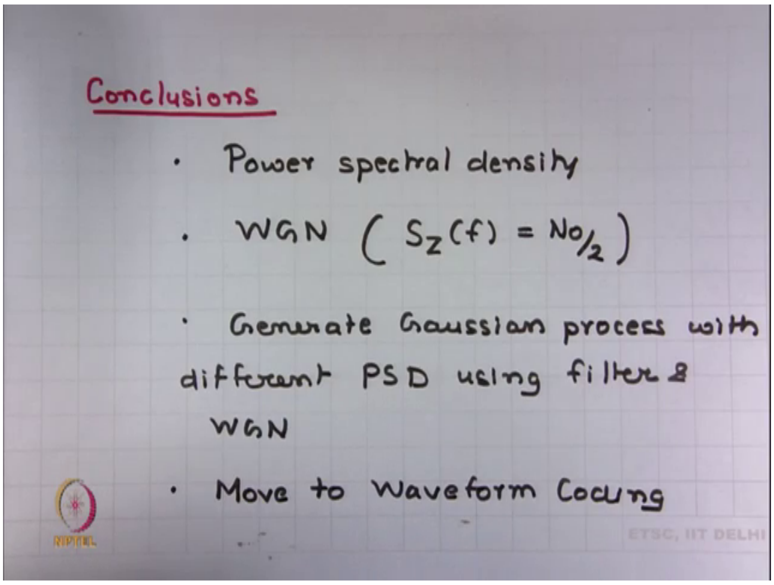

So, finally, we have come to the conclusions of today's lecture. Today we have learnt about this power spectral density. And we have defined what is this white Gaussian noise. We have also seen that you can generate a Gaussian process with different power spectral densities using filter and white Gaussian noise. From next lecture we will be moving towards the domain of waveform coding, we will see how can you convert these waveforms to binary sequence.

Thank you.