

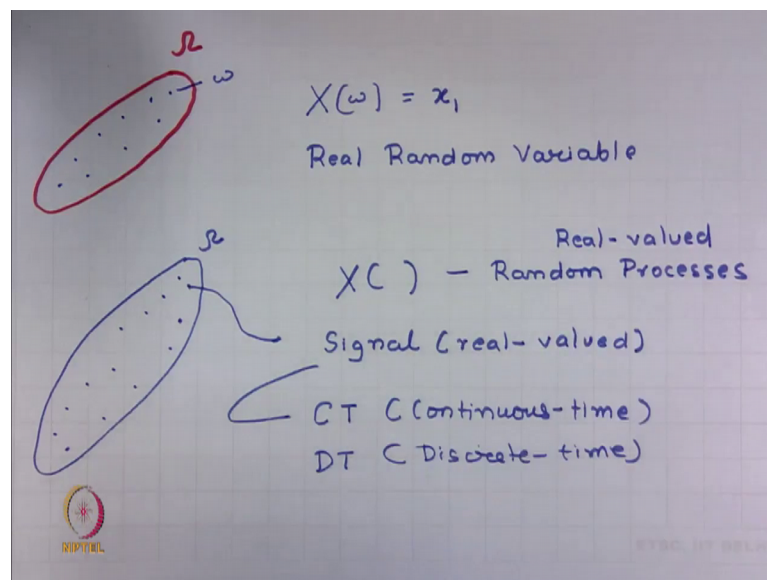
Principles of Digital Communication
Prof. Abhishek Dixit
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 11
Random Variables & Random Processes:
Introduction to Random Process

Good morning. Welcome to another lecture on Random Processes. So, we are on lecture number 5, and in this lecture we will start looking into the random processes. So, first we will define what a random processes. And then we will see some examples of random processes. So, as we have said random processes are used to model noise and information sources and they are very central to the study of digital communication.

And finally, after is taking 4 lectures on random variables we are here to discuss random processes. So, let us start, before starting discussion on random process we will revisit what are random variables.

(Refer Slide Time: 00:59)



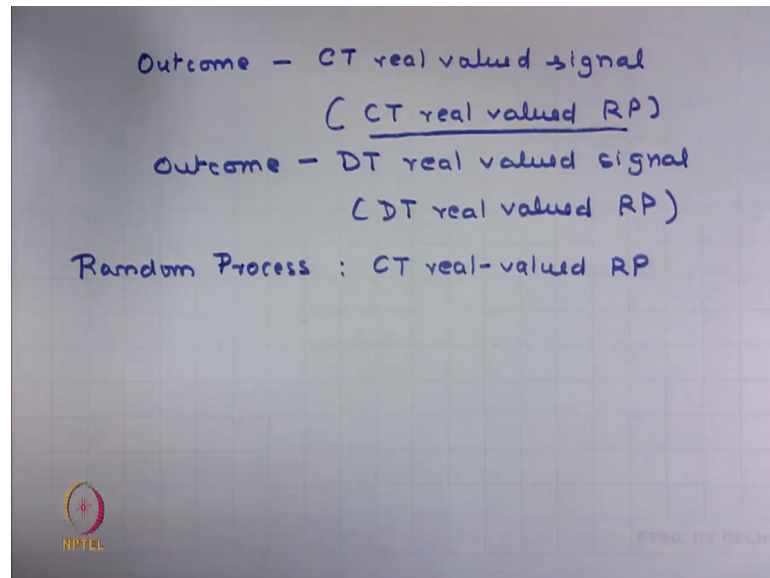
So, as I have said in the case of random variables we have a sample space which is denoted by this capital omega, and within a sample space you have various outcomes of an experiment. So, these dots represents outcome of an experiment and we use little omega to represent the outcome of an experiment. Now, in the random variable if you remember, a random variable is a function which takes in outcome of an experiment little

omega and it provides a numerical value to this outcome of an experiment. So, basically, we have set this numerical values are mostly real numbers and in this case we have a real random variable, ok. So, a real random variable or we also call this as a random variable is a function which maps the sample space to the set of real numbers.

Similarly, we can define random process almost we have the same analogy again we have the sample space, and again in the sample space you can have various outcomes of an experiment. And now again same as in this case we can have a function which takes in an outcome of an experiment, but now this function maps an outcome of an experiment to a signal and then this function is known as the random process. So, let us differentiate or letters appreciate the difference between a random variable and a random process. Random variable is a function that provides a mapping between the sample space and the set of real numbers whereas, a random process is a function which provides mapping between the sample space and set of signals. And these signals can also be real valued signals.

For most of the discussion we will assume the signals to be real valued signals and thus the process these random processes can also be called as real valued random processes, ok. We will focus our attention basically to these real valued random processes where the signals to which these outcomes of an experiment are mapped are also real valued. So, we will always assume a real valued random processes. Now, if you see that, if you revise from the basic signals and systems you can see that the signal is also of two kinds, you can have continuous time signals or you can have a discrete time signals, alright. So, if my signal is a continuous time signal then this random process is also known as CT random process.

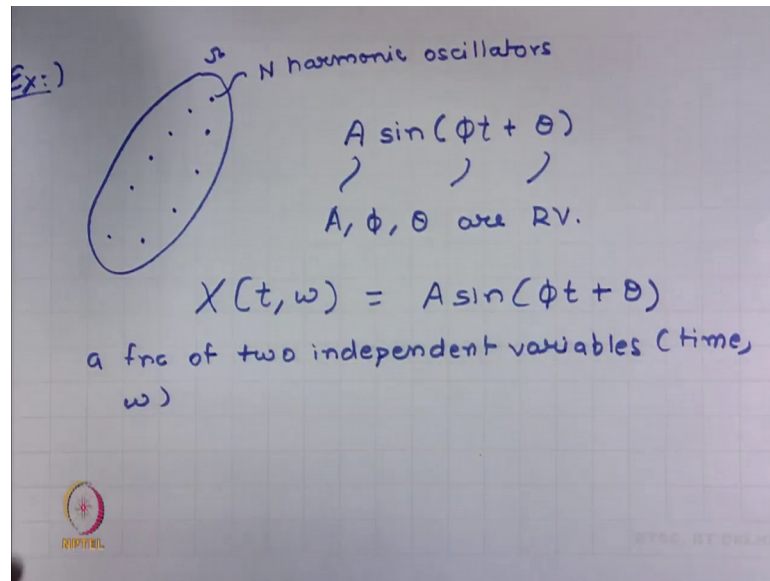
(Refer Slide Time: 04:25)



So, if outcome is mapped to a CT real valued signal, my random process is also known as CT real valued random process. And if my outcome is mapped to a DT: Discrete Time real valued signal, my process is known as DT real valued random process, ok. So, the random processes can be of several kinds it can be complex valued random process, it can be a CT random process, it can be a DT random process and so on and so forth. So, but for this course in discussion we always assume that my random process is a CT real valued random process and rather than always saying this CT real valued random process I will just say it as random process.

So, random process when I say random process in the back of your head you should think this as a CT real valued random process, ok. If there are other special instances that we will see of other kinds of random processes I will explicitly mention it right, but the random process try to think this only as a CT real valued random process, ok.

(Refer Slide Time: 06:21)



Now, there is nothing better than taking an example to understand further this random process. And for this example, let me assume that I have a sample space, and this sample space consist of N harmonic oscillators. So, I have N harmonic oscillators in my sample space. Now, a given harmonic oscillator produces a sin wave. Let us assume that a given harmonic oscillator produces a completely deterministic sinusoidal waveform, where A is an amplitude of this sin wave, ϕ is the frequency of this sin wave and θ is the phase of the sin wave.

Now, for a given harmonic oscillator these parameters are fixed that means, a given harmonic oscillator produces a specific effect sinusoidal waveform. But if you are thinking about N harmonic oscillators depending upon which harmonic oscillator you may choose the value of amplitude, frequency and phase may differ. That means, the amplitude frequency and phase depends upon which harmonic oscillator you choose but for a given harmonic oscillator these quantities are deterministic, right.

So, because these values depends upon the outcome of an experiment or it depends upon which harmonic oscillator you choose, these quantities A , ϕ and θ are random variables, ok. A , ϕ and θ are random variables because they are real numbers first of all and these values of these real numbers or the values of this constant depends upon the outcome of an experiment.

Now, if you want to think similarly what is a random process. So, I can write a random process as $A \sin(\omega t + \theta)$. Now, first thing that you should appreciate is that this random process is a function of two independent variables. So, it depends upon time and it depends upon ω , ω represents the outcome of an experiment. So, random process is a function of two independent variables, it is a function of time and it is a function of ω , right. So, what I mean with this is it is a function of time right, you see that there is a time sitting over here and it is a function of ω because the values of A , ϕ and θ depends upon the outcome of the experiment.

Remember, contrast this from the from random variable where random variable is not a function of time it is just a number right, it is not a function of time So, this is the main difference between a random process and a random variable that random process is a function of time whereas, the random variable is not a function of time, ok.

(Refer Slide Time: 09:35)

$$X(t_1, \omega) = A \sin(\phi t_1 + \theta)$$

number
(random variable)

$$t \in \mathbb{R}$$

$t_1 = \text{specific instance of } t$

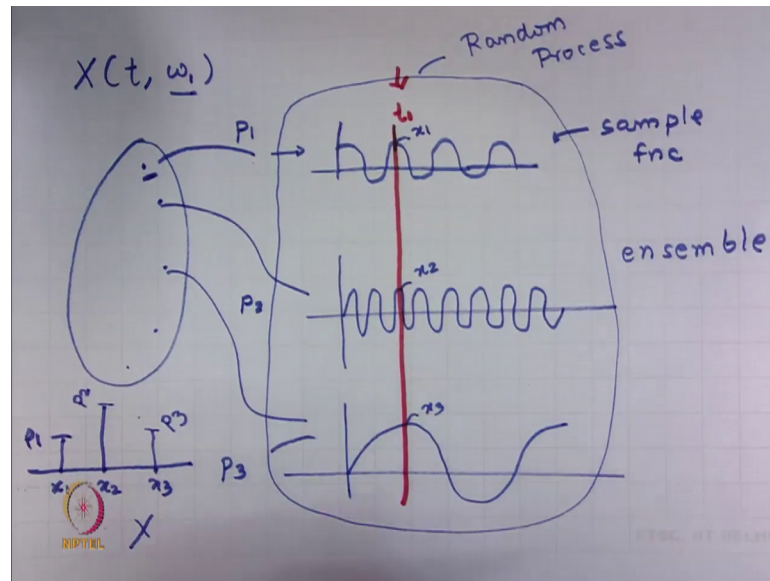
So, to understand more about this let me pose a question to you, that what do you think is this quantity X of t_1 ω . Now, before you can start thinking about what is the meaning of this quantity let me explain the meaning of the notations that I have chosen. In most of the digital communication books and digital communication is a hard subject mainly because of inconsistency in notation. So, it is very important that you clearly understand what notations are used right.

So, here in this course the notation that we use is suppose I choose t , now t is an independent variable. So, t can take any value in the set of real numbers. So, t belongs to the set of real numbers, so t is a variable. But when I write this as t_1 , so now, 1 in the subscript this means in this context or we would assume the meaning of this t_1 to be the, t_1 is a specific instance of t , right. So, t_1 is a number right it is not a variable anymore when I am writing X of t_1 omega that means, I am interested in the random process at a specific time instance. This was a specific time instance. This is not at time this is not denoting that this is a function of time but we are asking what is this quantity, what is this random process, when you are looking this random process at a specific time instances or at t_1 .

Now, to think about this what we are saying is suppose I have different sample functions as I had; so what I am saying is what happens if I evaluate or if I look down this random process at a specific time instances that is t_1 . So, let us see what happens if I put t_1 in the earlier expression. So, in the earlier expression X of t omega was $A \sin \phi t$ plus theta. Now, I am trying to think that what is this quantity at a specific time instances, that is t_1 . So, I just replace t by t_1 . Now, if you look at this quantity, this quantity is not a function of time anymore this is just in number. So, this is just a number. And this number depends upon the outcome of an experiment because this number would be decided by what values A , ϕ n theta takes and the value of A , ϕ and theta depends upon the outcome of an experiment.

So, in this context this is not a process but this is a random variable, ok. So, the difference between this quantity, this quantity was a function of time, this we say is a random process, but when you evaluate a random process at a specific time instance this does not remain a function of time but this becomes a number and this now becomes a random variable. So, what I am saying is if you look a random process at a specific time instance what you would end up with is a random variable. Now, let me ask a different question what happens if you evaluate or if you look this random process at a specific outcome?

(Refer Slide Time: 13:21)



So now, I have frozen omega, so now, I am looking this random process at a specific outcome. So, the meaning is suppose you have a sample space which was consisting of different outcomes, and here let us say that the function corresponding to this sample space. So, you have got some function corresponding to this outcome. Let me assume that a function corresponding to this is a different sinusoidal waveform. Let me assume if function corresponding to this is some other sinusoidal waveform.

So, what I am saying is if I have frozen one outcome that means, I have said that this outcome has happened what I get is just one function, right and we call this as a sample function. If you look at the random process from this direction that means, if you are looking outcome of an experiment a specific outcome of an experiment the outcome that you get is known as a sample function. Now, as I can see that a random process is built up of a family of sample functions right, there is a sample function corresponding to this outcome, there is a sample function corresponding to this outcome and so on so forth. So, you get a family of sample functions and this family of sample functions is also known as ensemble.

So, what is ensemble? Ensemble is nothing but it is a family of sample functions. Now, if I have got this ensemble which is a family of sample functions and then if I say or define a probability rule which tells me what is the probability that the sample function might occur. So, let us say this the probability is P 1, this probability P 1 tells me the ability that

this sample function occurs. And similarly, I can say that the probability that this sample function occurs let us assume that is P_2 the probability that this sample function occurs let us assume that this is P_3 .

So, what I am saying is I have now a probability rule which tells me the probability of each sample function occurring and then I have an ensemble which is a family of sample functions, and collectively combining both of these things together that means, having a probability rule and having an ensemble leads to a random process. This is another way in which you can think about a random process.

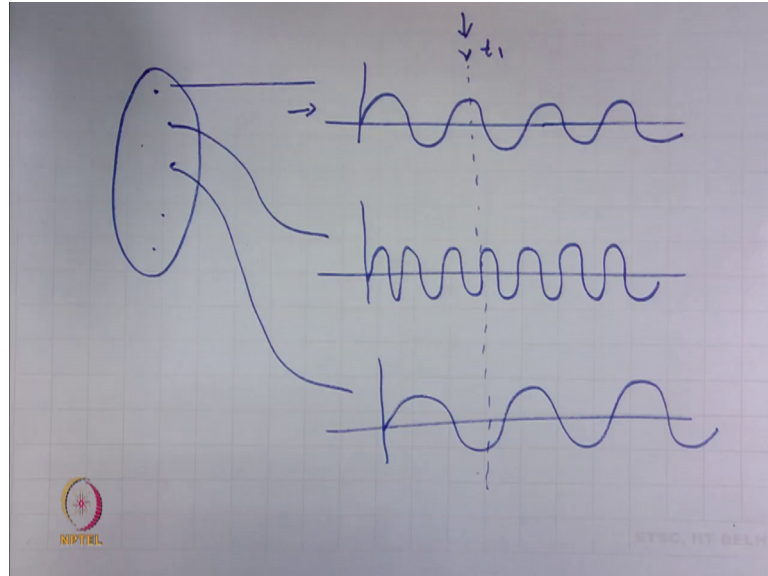
So, what I have said is random process consists of a family of sample functions. This family of sample functions is known as ensemble. And then I should also tell you what is the probability, that this sample function might occur. The probability of this sample function to occur is the same as probability of this outcome to occur. So, you have a probability for each sample function once, then it tells me everything about the random process. So, let me now do one trick here, it is not a trick we are just trying to understand this better. Let me now choose a specific time instances. Let me choose specific time instances, let me choose t_1 .

So, if I choose and I look down this ensemble at a specific time instances t_1 , what I would end up with is I would end up with some numerical value here x_1 . Let us say that numerical value is x_1 and I may get another numerical value corresponding to this sample function which is x_2 , I get another numerical value corresponding to this sample function which is x_3 . Now, I know that if I am interested in plotting the probability mass function that means, let us say that I have. Now, got 3 numerical values x_1 , x_2 and x_3 and let us say that x_1 happens with the probability P_1 , x_2 happens with the probability P_2 and x_3 happens with a probability P_3 . So, if the probability that this sample function occurs happens with the probability P_1 this is the probability is P_1 then the probability that I get a numerical value x_1 is also P_1 .

The probability that I get a numerical value x_2 is P_2 , probability that I get a numerical value x_3 is P_3 . That means, I have now created a probability mass function, and this is nothing but a random variable. So, if I investigate this thing at a particular time instances, I can collect various numerical values and to each numerical value I can assign a probability and I can construct a probability mass function. So, this is nothing but if I

investigate this random process at a specific time instances this is nothing but a random variable.

(Refer Slide Time: 19:03)

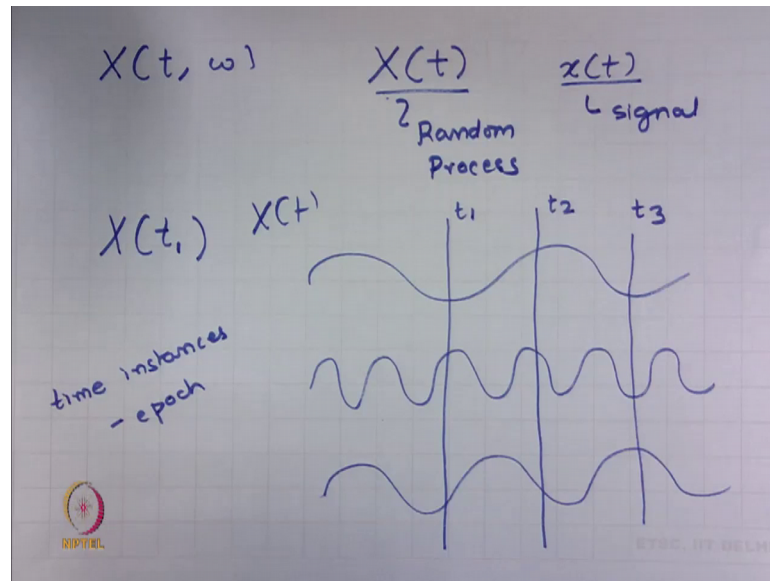


So, to conclude what we have defined is I have a sample space. The sample space may lead to different sample functions may consist of family of sample functions. Sample function is nothing but it is an outcome of a given experiment, right. So, for each outcome I have some sample function, right. So, sample function is nothing but it is the outcome of an experiment, right, it is a function corresponding to the outcome of an experiment.

If you look down to the ensemble which is a family of sample functions at a specific time instance right, then what you get is a random variable. You get several numerical values and you can obtain the probabilities with which these numerical values might occur. So, this is this will help us to construct a random variable. So, looking down the random process in this direction we get a random variable, and looking down the random processes from this direction we get sample functions, ok.

Now, let us introduce few more notations. There are several notations that run in this course. Another notation is that now we have said that a random process is a function of two independent variables, means it is a function of time and it is a function of the outcome of an experiment. But because we do not like to write this all the time I use or we can use a simpler notation like this and this notation stands for a random process.

(Refer Slide Time: 20:33)



Now, I have omitted or ignored this omega, I have not retained this omega what I just now have caught is that this random process is just a function of time. But at the back of your head you should remember random process is not only a function of time it is also the function of outcome of an experiment, right. Now, you might wonder that this is also a function of time. This is a signal, and this is a random process. How do we differentiate between the two? So, the notation that we have used for the random process is that we have used a capital letter X and for the signals we use generally a small letter x. So, this will tell me that this is a signal and this is a random process, ok.

So, let us keep our notation the state and now then what would be this X of t 1. X of t 1 would tell you that if you have a random process which is built up of different sample functions. So, this is X of t. So, we have ensemble and then you also know: what is the probability with which each sample function occurs and if you evaluate this random process at a specific time instances t 1, as I have said you get a random variable. So, X of t 1 is nothing but it is a random variable.

Now, similarly you can choose other time instances the time instances are also known as epochs, there is another word that is commonly used as epoch. So, I can choose another epoch t 2, I can choose another epoch at t 3 and so what I end up with is I get several random variables, all right.

(Refer Slide Time: 23:07)

The image shows a handwritten derivation of the joint cumulative distribution function (CDF) for three random variables. At the top, it is titled "Joint CDF". The first equation defines the joint CDF $F_{X(t_1), X(t_2), X(t_3)}(x_1, x_2, x_3)$ as the probability that $X(t_1) \leq x_1$, $X(t_2) \leq x_2$, and $X(t_3) \leq x_3$ simultaneously. The second equation generalizes this to n random variables, defining $F_{X(t_1), X(t_2), \dots, X(t_n)}(x_1, x_2, \dots, x_n)$ as the probability that $X(t_1) \leq x_1$, $X(t_2) \leq x_2$, ..., $X(t_n) \leq x_n$ simultaneously. A small logo is visible in the bottom left corner of the slide.

$$\begin{aligned} & \text{Joint CDF} \\ & F_{X(t_1), X(t_2), X(t_3)}(x_1, x_2, x_3) \\ & = P\{X(t_1) \leq x_1, X(t_2) \leq x_2, \\ & \quad X(t_3) \leq x_3\} \\ & F_{X(t_1), X(t_2), \dots, X(t_n)}(x_1, x_2, \dots, x_n) \\ & = P\{X(t_1) \leq x_1, X(t_2) \leq x_2, \dots, X(t_n) \\ & \quad \leq x_n\} \end{aligned}$$

Now, as we have already learned about the joint CDF. So, we can construct a joint cumulative distribution function also in this case. So, if I am interested let us say in this picture as I have said let us assume that I have collected 3 random variables. So, I can define the joint cumulative distribution function for these 3 random variables, this is a random variable X of t_1 .

So, what does this mean? It means, it tells me what is the probability that random variable X t_1 takes in a value less than equals to x_1 ? What is the probability that random variable X t_2 takes in a value less than equals to x_2 ? What is the probability that random variable X t_3 takes in a value less than or equals to x_3 ? And we want all these things should happen at the same time, ok. I can also put a comma here.

So, this tells me this is a joint cumulative distribution function. And how have you obtained these 3 random variables? By looking at the random process at different epochs, right. You can generalize this and you can define a joint CDF at countable epochs, which will tell me what is the probability. So, you can obtain a joint cumulative distribution function by taking countable epoch. So, here for example, let us assume that l is some positive integer. So, let l have assumed l random variables and I can construct a joint cumulative distribution function for this random process.

So, if you look at this expression, this expression might have started to give you sleepless nights. This expression looks very scary because now I have l random variables

involved, and to fully characterize a random process what we have to do is we have to write such an expression for all possible values of l and for all possible epochs t_1, t_2, \dots, t_l , and for all possible arguments does writing down such an expression to fully characterize a random processes difficult. But before thinking about how can we simplify all this let me conclude this by deriving its probability density function.

(Refer Slide Time: 26:31)

The image shows a handwritten derivation on a grid background. At the top, the joint probability density function $f_{X(t_1), X(t_2), \dots, X(t_l)}(x_1, x_2, \dots, x_l)$ is equated to the partial derivative of the joint cumulative distribution function $F_{X(t_1), X(t_2), \dots, X(t_l)}(x_1, x_2, \dots, x_l)$ with respect to each variable x_i . The derivative is written as $\frac{\partial^l F_{X(t_1), X(t_2), \dots, X(t_l)}(x_1, x_2, \dots, x_l)}{\partial x_1 \partial x_2 \dots \partial x_l}$. Below the equation, the text "first-moment / Second-moment" is written. In the bottom left corner, there is a logo for "RIIPTEL" and in the bottom right corner, "IIT DELHI" is visible.

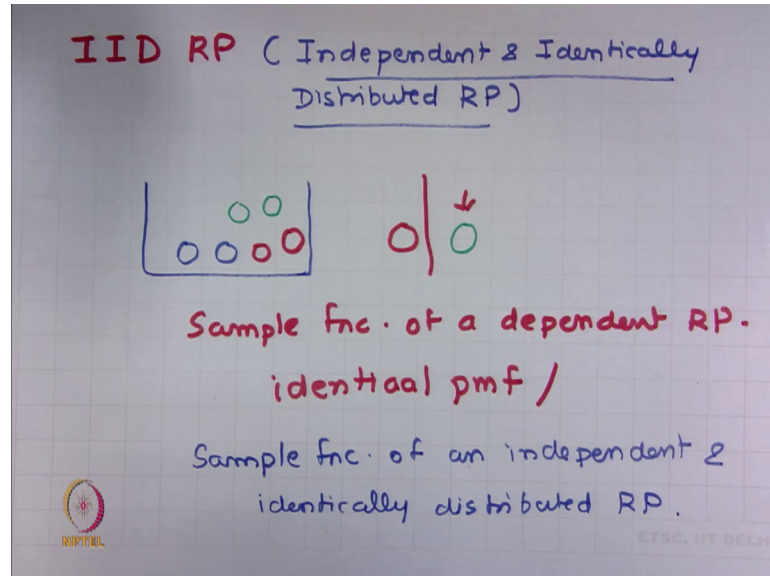
So, if you write the probability density function remember that probability density function can be obtained by partial derivative of CDF. So, this can be obtained through partial derivative of joint CDF, ok. So, we have obtained an expression for joint probability density function. Now, this expression is also as scary as the last one. This does not, is not a simplified expression or whatsoever it is still very impractical to use such an expression to characterize a random process.

The good news but is that most of the random processes that we deal in this course or are of interest to us can be completely characterized by the first moment and second moment of the random process. So, we do not have to worry about writing down the joint probability density function. The first moment and second moment would be enough most of the time to fully characterize the random process, ok.

So, now we have defined the random process and it is the time to look into some examples of random processes. And the first important example that we would like to

look into is IID random process. IID stands for Independent and Identically Distributed Random Process.

(Refer Slide Time: 28:31)



So, let us try to understand what I mean by IID random process. And to understand that let us do a small experiment. So, in this experiment let us assume that you have a bag and this bag contains several balls. These balls are of different colors. So, you have blue balls, red balls, and if we have let us say green balls.

Now, what do you do is you reach out to this bag and you randomly select a ball, ok. So, this is the experiment, it is a simple experiment, reaching out to the bag and selecting a ball. So, if you randomly select a ball let us assume that this ball that you get is a red colored ball, you took this red colored ball and you have kept in a different bag, ok. So, this is the experiment simple trivial experiment. Now, you do this experiment again you searched the bag and you again look for a ball and then you probably get a different color ball, at the second time you get it let us assume that you get a green color ball.

Now, what you can observe or you can understand appreciate whatever is that when you are doing experiment the number of different colored balls have changed. So, number of different colored balls have changed from experiment 1 to experiment 2, in experiment 2 you have a fewer red balls and in experiment 1. So, because the number have changed its probability mass function has also changed, and because the probability mass function has changed the outcome of experiment will depend upon the previous outcome. So, the

outcome in the second experiment is a function of what you obtained in the first experiment, right.

So, similarly if you can if you keep on doing this experiment the number of different colored balls will keep on changing, the probability mass function also keeps on changing and those what you would end up with is that the outcome of a given experiment begins to depend upon the previous experiment. And if you envision this process to create a sample function of random process what you would get is, you would generate a sample function of a dependent, of a dependent random process. Dependent in the sense that the outcome of a current experiment depends upon the previous outcomes because, every outcome alters the probability mass function, alright.

Now, let us assume that I do this experiment in a different way. So, I pick a ball of a of a specific color. I note down the color of that ball but after noting down the color of the ball I put that ball back in the bag, right. So, I am not taking away the ball from the bag. So, what it would do is that before every experiment the number of different colored ball remains same, hence the probability mass function is not changing with my experiment. So, every time I go and reach out to this bag, I will see identical let me write this it is important, I will see an identical pmf, right. So, every time I am doing an experiment I see an identical pmf.

So, now, the outcome of this experiment will not depend upon the previous outcome, and what this will lead to is you can use this process to generate sample function of an independent, an identically distributed. Identically because the pmf remains identical you see the same pmf every time. So, this process or this method or this experiment can generate a sample function of an independent and identically distributed random process, all right.

So, this is a trivial example but I think that you probably do often is you must have used a random number generator in the MATLAB. This random number generator always generates a number based on an identical pmf. So, the probability mass function remains unchanged right or you can start thinking about this random number generator as a generator which generates a sample function of an IID random process. And thus most of the modeling and simulation techniques use this IID random process and this is an important random process, ok.

(Refer Slide Time: 33:51)

$$f_{X(t_1), X(t_2), \dots, X(t_n)}(\alpha_1, \alpha_2, \dots, \alpha_n) \\ = \frac{\partial^n F_{X(t_1), X(t_2), \dots, X(t_n)}(\alpha_1, \alpha_2, \dots, \alpha_n)}{\partial \alpha_1 \partial \alpha_2 \dots \partial \alpha_n}$$

first-moment /
second-moment

So, let me now revise when we talked about two random variables, and if these two random variables were statistically independent what I said was that the joint probability density function of these two random variables can be written down as just a product of their marginal probability density functions. So, if x and y are statistically independent the joint pdf can be written down as a product of the marginal pdfs of these random variables. Similarly, in this case if I am thinking about an IID random process, I already have written down the joint pdf for such a random process then I can write down this joint pdf as a product of marginal pdfs of each random variable, alright. So, this will happen if my process is IID random process.

So, before looking into some examples of random processes let me answer 3 annoying questions that are sometimes posed in the context of random processes. The first question is that suppose I make him receiver and this receiver would just see a sample function right, it would just see one sample function. So, why do we worry about ensemble of sample functions? Why do we discuss about random processes? Because a mobile receiver or a digital communication receiver would, just see one sample function in its lifetime. The answer is a very simple, that when you are manufacturing a mobile phones you should manufacture them in such a way that they accommodate the multiplicity of sample functions, where they should be able to handle a multiplicity of sample function, they should not be manufactured to handle a specific sample function because you then

do not know which sample function that mobile phone or digital communication receiver would experience.

The second thing that you should notice here is the randomness lies in this specific context, lies only which outcome would turn up. So, what we say is before an experiment or a priori there is some randomness because you do not know which outcome will turn up. But a posteriori that means, after an experiment has happened because you know which outcome has occurred then the outcome of this random process is completely deterministic because as we have seen that this sinusoidal waveform is completely deterministic. There is no randomness here; so a posteriori you know that which outcome has happened. So, there is no randomness after this outcome has happened but a priori there is randomness involved because we do not know which one of these outcomes would happen.

The third point here rather is that you can envisage a random process. So, we have seen several pictures of random process. We have seen a picture where we said that a random process is made up of the sample functions, it is an ensemble and then there is some probability rule, that is the one way you can think about the random process. The second way that you can think about random process if you look at this expression carefully then you know that a random process can also be thought as an ensemble of random variables. So, you have a random variable happening at t_1 you can have a random variable happening at t_2 and t_3 and so on so forth. So, you can construct or you can think about a random process also as an ensemble of random variables, ok.

Let us now take a second example and probably the most important example in this context, second example of a random process well let me first write what I am coming to.

(Refer Slide Time: 38:19)

2) $Z(t) = \sum_k Z_k \phi_k(t)$

Annotations:
- $\phi_k(t)$ is labeled as "Orthogonal fncs"
- Z_k is labeled as "Random Variables"

Useful 2 provides a framework which allows us to model physical noise sources.

Logo: IIT DELHI

So, I have a random process Z of t which I write in terms of orthogonal functions. So, $\phi_k(t)$ are orthogonal functions and Z_k s are the coefficients of these orthogonal functions and these that Z_k s are random variables and you have to do this summation for all k . So, I can think about a random process as a sum of random variables times orthogonal functions, ok.

This is a very important concept, and this is also known as a signal space in which you are trying to think about a random process in terms of orthogonal functions. So, this idea that you can think about a random process like this is very useful, and it provides a framework which allows us to model physical noise sources. So, this is really very important idea of trying to think random processes is the sum of random variables times orthogonal functions or trying to express random process in terms of its orthogonal expansion.

(Refer Slide Time: 40:15)

$$x(t) = \sum_k a_k \phi_k(t)$$
$$P_1: z_1(t) = \sum_k z_{1k} \phi_k(t)$$
$$P_2: z_2(t) = \sum_k z_{2k} \phi_k(t)$$
$$Z(t) = \sum_{k=1}^{\infty} z'_k \phi_k(t)$$

So, let us see why this can be correct. From the basics in signals and systems we know that we can express any given signal. Of course, it has to be a finite energy signal and things like that, but let us assume that for broad class of signals we can expand a signal in terms of the coefficients and orthogonal functions. So, for a series for example, spins around this idea; so, we can take a signal, a finite energy signal, and we can express a finite energy signal as a sum of coefficients times orthogonal functions, right. This is the basic idea that you must have learned in signals and systems and we have also revised it in the unit one.

Now, taking the same idea forward, now I can take a sample function, sample function is also a function also a signal I can express a sample function in terms of the coefficients times the orthogonal functions, essentially the same idea. I can take another sample function and I would end up with different coefficients and I can express this sample function again using the same orthogonal functions.

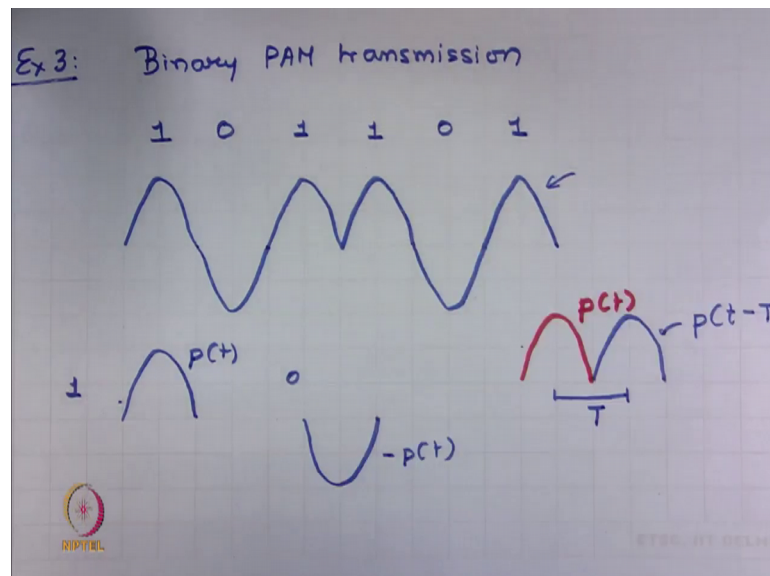
Now, how can I think about the random processes in terms of the sample functions. Now, you see that when we are talking about the random process we should also know: what is the probability that this sample function should happen, right. So, there is certain probability let us say the probability P_1 that the sample function happens, the probability P_2 there this sample function happens. So, when you are thinking about this together what would happen is this Z_{1k} or the coefficient for the sample functions $Z_1(t)$ would

happen with a certain probability, Z_2^k will happen with a certain probability. So, you can sum this entire thing you can construct a random process out of this by replacing all these coefficients and their probabilities with a random variable.

So, Z_k is a random variable. So, it tells me that that Z_1^k happens to the probability P_1 , Z_2^k happens with a probability P_2 . So, this is a random variable. So, I can express a random process as a sum of the random variables times orthogonal functions or I can do an orthogonal expansion of my random process. Now, one small point here can be that k in general varies from 1 to infinity. Now, the point is if k varies from 1 to infinity, this summation might not converge, right.

So, this is not always guaranteed that this summation converge, but for this course because we are only dealing with simple things we assume that this summation converges, ok. So, for most of the processes that we assume this summation will always converge, ok. So, we will see again and again that this model or this idea of expressing a random process in terms of orthogonal function is a very useful idea and it simplifies almost everything. So, let me just take this idea a little bit forward and use this. So, this is let us say this is an example number third for a binary PAM transmission.

(Refer Slide Time: 43:55)



So, when we are talking about a binary PAM signal, let us assume that I have some digital signal and this digital signal as you can see is composed of two bits 1 and 0. Let me assume that to transmit 1 I use a pulse shape like this and to transmit 0 I use a pulse

shape like this. So, this pulse shape is nothing but it is a negative of this pulse. For 1 again I use pulse like this. So, I can construct a binary PAM signal by thinking about that I have used two pulses. So, I have used basically a pulse like this to represent 1 and to represent 0 I have used a pulse shape like this. So, if I call this as $P(t)$ this will be minus $P(t)$, ok.

Now, one point that you can see is that if suppose let me use a different color because this is just to revise certain things that you must have studied, suppose I have a pulse $P(t)$ and I want to shift this pulse let us say I want to shift this by T units. So, I have a red pulse $P(t)$ and I have just shifted it by T units in time to produce a blue pulse which is exactly the same pulse as $P(t)$ but just shift it in time. So, this pulse can be expressed like this, ok. I can I have to shift this $P(t)$, this t should change to $t - T$ and then the pulse shifts to the right by T units.

(Refer Slide Time: 46:37)

$$b(t) = \sum_k b_k p(t - kT)$$

$$1 \times p(t) + -1 p(t - T) + 1 \times p(t - 2T)$$

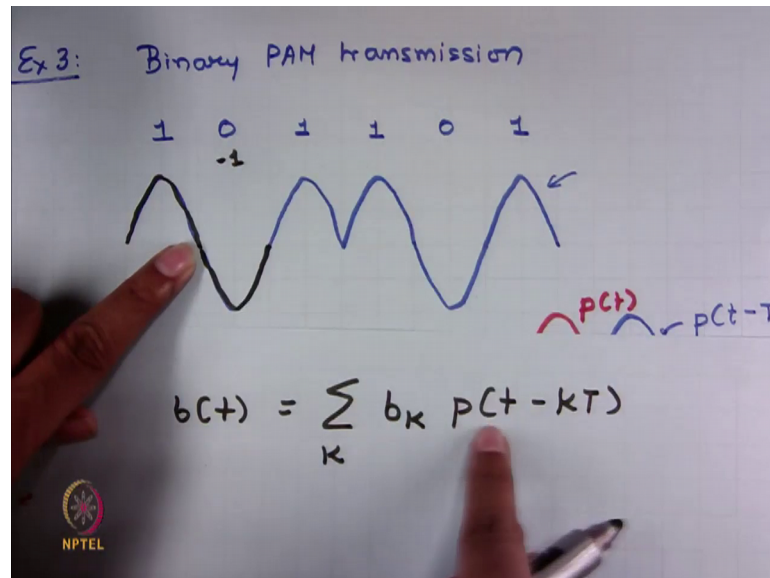
$$b_k \begin{cases} 1 \\ -1 \end{cases}$$

Bernoulli RV

$$B(t) = \sum_k B_k p(t - kT)$$

So, I can now think about this signal as; so if I am writing an expression for a binary PAM signal the binary PAM signal can be thought as if it is a sum of B_k s times $P(t - kT)$.

(Refer Slide Time: 47:01)



So, what I am saying with this is I need suppose I have a pulse then, I shift it by T units to get this pulse and I shift it by T units and I also multiply it with minus 1. So, in this case I need to multiply it with minus 1 to get an inverted pulse. Then to get this pulse my expression will be 1 times $P(t - T)$ and if I have to think about this would be nothing but minus 1 times $P(t - T)$, because I need to shift it by T units. Then I have another pulse this could be expressed as plus 1 times $P(t - 2T)$ because now this pulse is shifted by $2T$ and I can go on and on and construct a binary PAM signal by thinking about this as this.

Remember that b_k takes in two values, either takes a value 1 or it takes a value minus 1. It takes a value minus 1 whenever it is representing a 0, and it takes a value 1 whenever it is representing a value 1. So, I can write an expression for binary PAM transmission or binary PAM signal like this. Now, this is just a sample function of binary PAM system. Now, I have to if I want to convert this into a process, the idea remains the same. The b_k will become a random variable let us let us call this as capital B_k . So, you can convert a sample function to a random process simply by changing these coefficients with the random variable, ok.

(Refer Slide Time: 49:27)

The image shows handwritten mathematical derivations on a grid background. At the top, a Bernoulli random variable B_k is defined with two possible values: 1 with probability P and -1 with probability $1-P$. A note specifies $P = 1/2$. Below this, the expected value $E[B_k]$ is calculated as $1 \times P - 1 \times (1-P) = 0$. The variance $\text{Var}(B_k)$ is given as 1. The sinc function is defined as $\text{sinc}(t/T)$. Finally, the signal $B(t)$ is expressed as a sum over k of $B_k \text{sinc}(\frac{t - kT}{T})$. A small logo is visible in the bottom left corner of the slide.

$$B_k \begin{cases} 1 & P \\ -1 & 1-P \end{cases} \quad P = 1/2$$
$$E[B_k] = 1 \times P - 1 \times (1-P) = 0$$
$$\text{Var}(B_k) = 1$$
$$p(t) = \text{sinc}\left(\frac{t}{T}\right)$$
$$B(t) = \sum_k B_k \text{sinc}\left(\frac{t - kT}{T}\right)$$

Now, if you look at B_k we have seen in the past as well this B_k is a Bernoulli random variable, this B_k is a Bernoulli random variable. So, if I think about this B_k it takes in either value 1 or minus 1. Let us assume that it takes a value 1 with the probability P and does it will take a value minus 1 with a probability $1 - p$. If I find the expected value of B_k this would be 1 times P minus 1 times $1 - p$, and if you assume that 1 and 0 are equally likely that means, P is half you would get expected value of B_k as 0.

Similarly, I leave it to you to prove that variance of B_k is 1. Try to work this out. So, here we are saying that the variance of B_k is 1. So, this is another example. Yet another example as how thinking about a process in terms of orthogonal expansion helps us. So, we can also visualize and understand a binary tram PAM transmission by thinking about this in terms of orthogonal functions.

So, let us now try to make this thing little bit more specific by assuming that the $P(t)$ that I choose is nothing but the sinc function. So, I have chosen as sinc pulse and then I can express $B(t)$ as, all right. So, this is a specific case of binary PAM transmission when you have chosen the pulses which are sinc pulses.

We will continue with this in the next lecture, and we will see some other examples of random processes some more useful examples of random processes, and then we will define what is mean and covariance of a random process and we will think about a Gaussian processes. So, we have lot to cover in the next lecture. So, see you there.

Thank you.