Information Theory, Coding and Cryptography
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Module – 34
Space Time Codes
Lecture – 34

Hello and welcome, to our next lecture on Space Time Codes.

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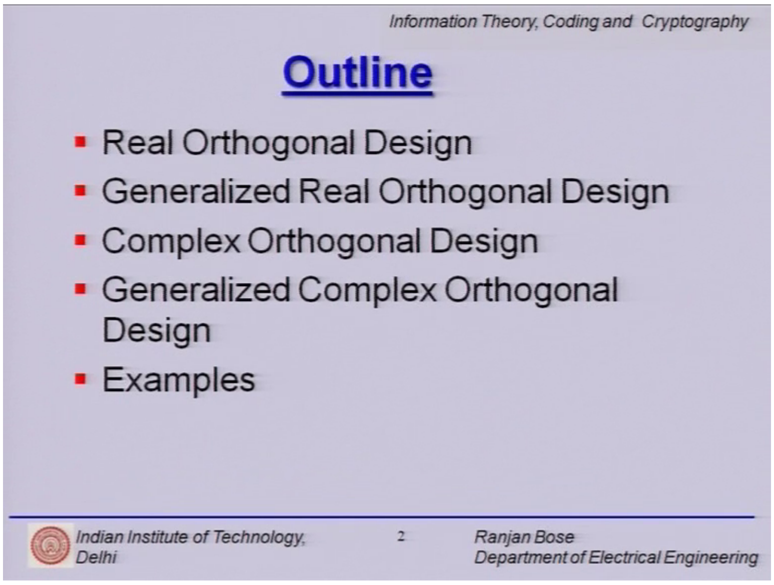

Let us start with a brief outline of today's lecture. What we will study today are real orthogonal designed for space time block codes, then we will look at the generalization of real orthogonal designs, then we will move on to the domain of complex orthogonal design and then we would like to generalize that concept. Finally, we will look at some examples.

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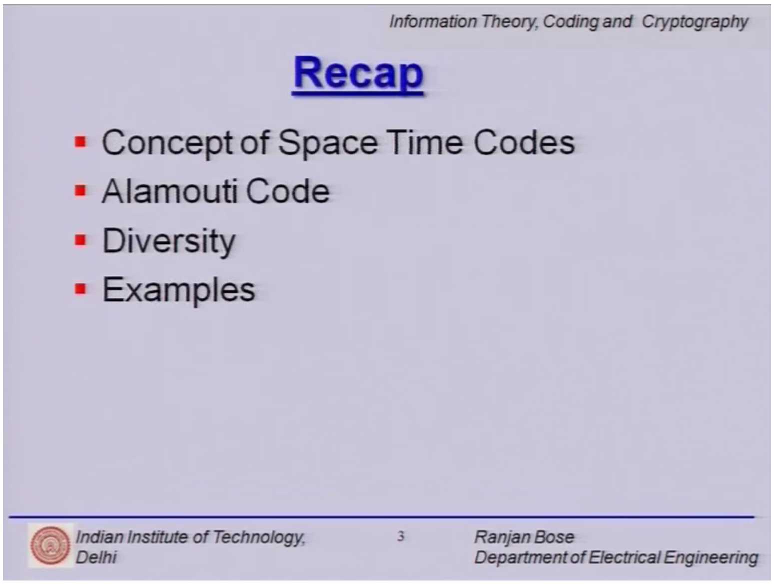

Let us do a quick recap as to what we have done so far. We have already introduced the concept of space time codes and how it gives diversity gain. We specifically looked at Alamouti code as a very interesting example of space time block codes. We dealt into diversity gain and coding gain and we looked at some examples along the way.

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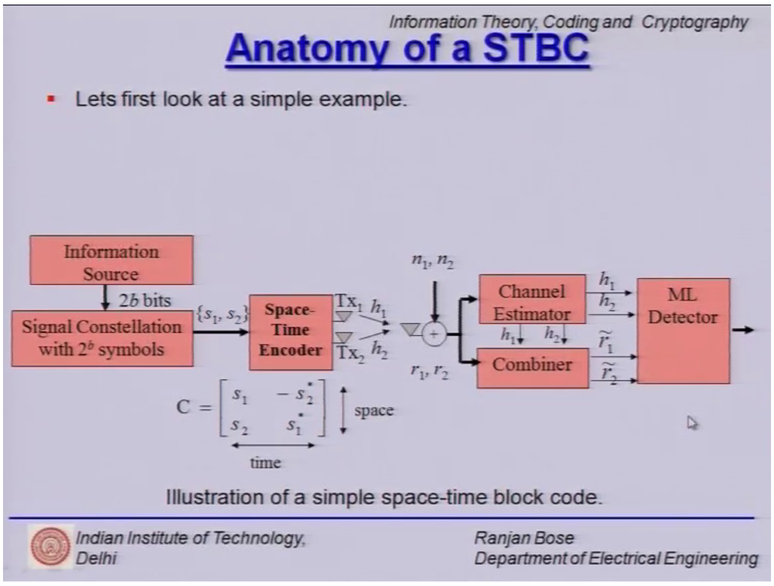

Let us revisit the anatomy of a space time block code STBC for short and what we realized was that an information source pumping in bits of information what we do is we take 2b bits at a time and of this first b bit is used to select one signal from the signal constellation and then the next b bits are used to pick up another signal s 2 from the signal constellation. So, I have s 1 corresponding to the first b bits and s 2 corresponding to the next b bits, thus we have a tuple s 1, s 2 which is coming out from this signal constellation block and this is fed to the space time encoder.

In this example we have two transmit antennas therefore, we picked up 2 symbols s 1 and s 2. What they do is load symbol s 1 onto the element 1 and load s 2 onto it antenna element 2. So, Tx 1 sends out s 1 Tx 2 sends out x 2. There is a only one receiver because we have argued that in most cases it is difficult to put multiple antennas on the receiver, but you can have multiple antennas. In this example we are only looking at one single receiver antenna element. So, the s 1 goes through a channel gain of h 1 whereas, s 2 undergoes a channel gain of h 2 and we receive it and of course, we have additive white Gaussian noise added up n 1.

But, this we do in the first time slot as a name suggests it is a space time block code. So, the space element is coming from the antenna and the time is the different time slots that we are going to send out. So, we sent out s 1 and s 2 from the two antenna elements in the first time slot. Now, for those same 2b bits we have picked up s 1 and s 2, but in the second time slot what we do is we send out minus s 2 star and s 1 star. s 1 and s 2 belong to a complex constellation. So, we have the complex conjugates here.

So, what we do is we send out from antenna element 1 minus s 2 star and s 1 star this is just an example I can have other ways of doing it and then we send out. These two go through the channel gains and we get noise n 2 added and so, we have two received signals in time slot 1 and time slot 2 of course, we need the estimates of the channels h 1 and h 2 and then we have a combining strategy which goes through the ML decoder.

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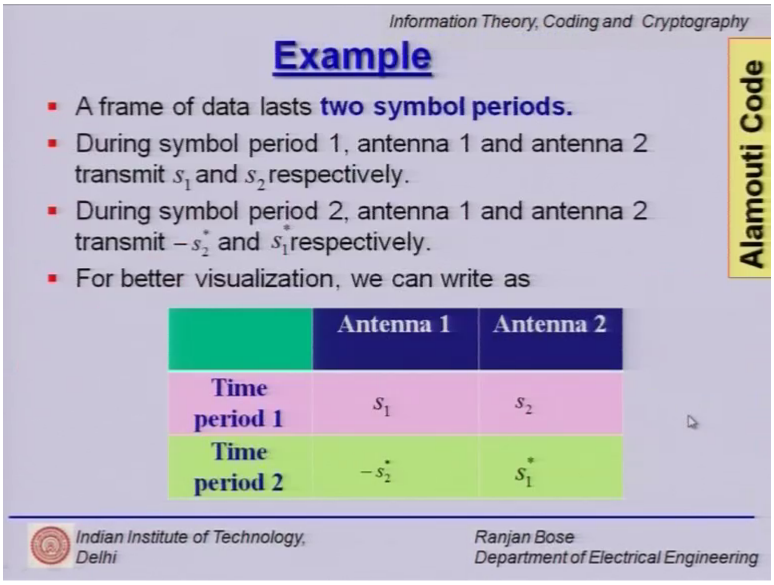

So, in this example we have talked about the space antenna 1 and antenna 2 and time; time period 1 and time period 2, so, we have this matrix of symbols that we want to transmit and this is kind of the code the space time block code that is being used.

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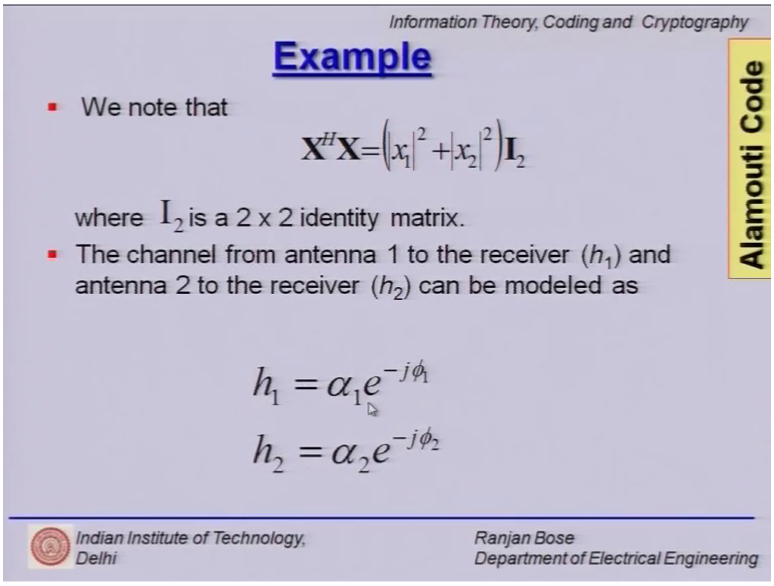

Now, we make some observations because if you represent x with those matrix then X Hermitian X happens to be x 1 absolute value squared plus x 2 absolute value squared I 2, where I 2 is the identity matrix.

Now, the channel gains h 1 and h 2 are complex and they can be modeled as alpha 1 e raise power j phi 1 and alpha 2 e raise power minus j phi 2.

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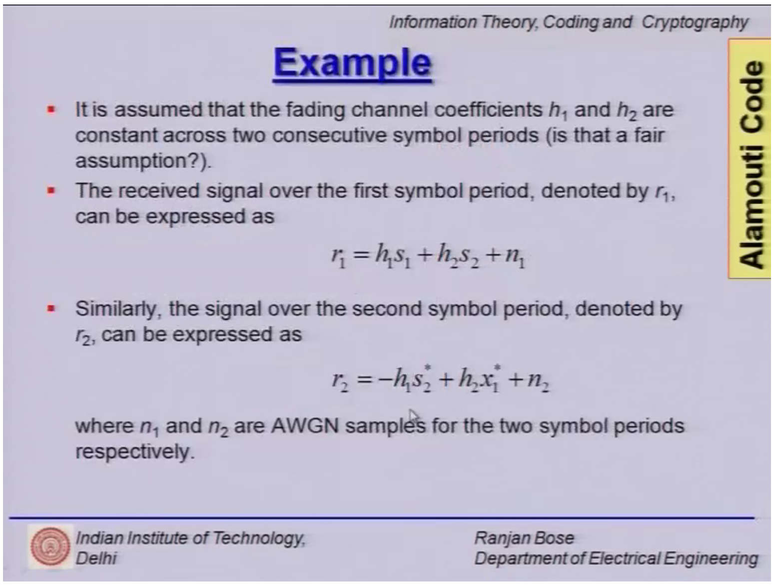

And, we can multiply it out to get the first received signal in time slot 1 as a r 1, this is simply s 1 with channel gain h 1 plus s 2 with channel gain h 2 plus n 1. Similarly, in the second time slot we have minus h 1 s 2 star plus h 2 x 1 star plus n 2 here it should be s 1. So, we have this received signal r 2 in the second time slot. Now, the observation right here is that r 1 depends both on s 1 and s 2 and r 2 also depends on s 2 and s 1, simultaneously.

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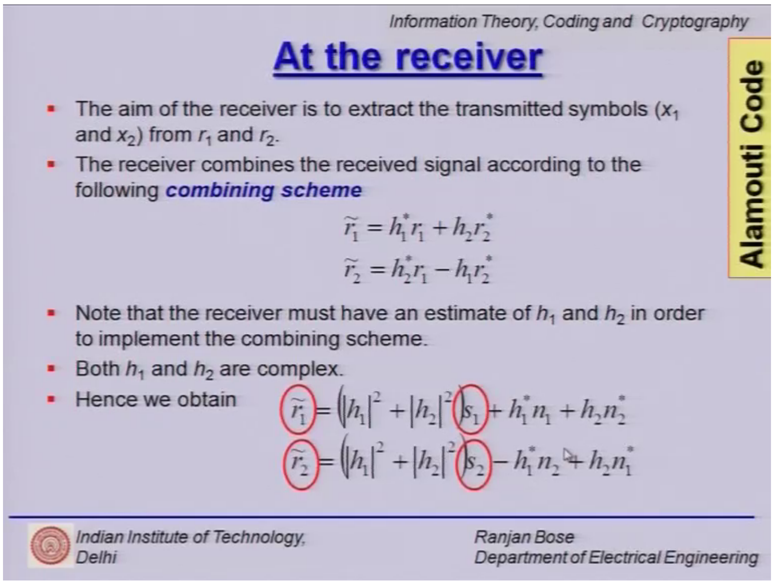

But we have this interesting combining scheme wherein we have r 1 tilde equal to h 1 complex value, complex conjugate r 1 plus h 2 into r 2 star, which means that if we have the knowledge about the channel gains h 1 and h 2 which are complex we can use this combining scheme to get r 1 tilde and similarly, r 2 tilde.

But, if you workout this then r 1 tilde comes out to be this expression and r 2 tilde correspondingly gets this one. The interesting observation is r 1 tilde only depends on s 1. So, suddenly with this combining scheme we have been able to decouple the decoding. So, r 1 is only dependent on s 1 and r 2 is only dependent on s 2. So, we can use the maximum likelihood decoding strategy and recover using the single symbol decoding.

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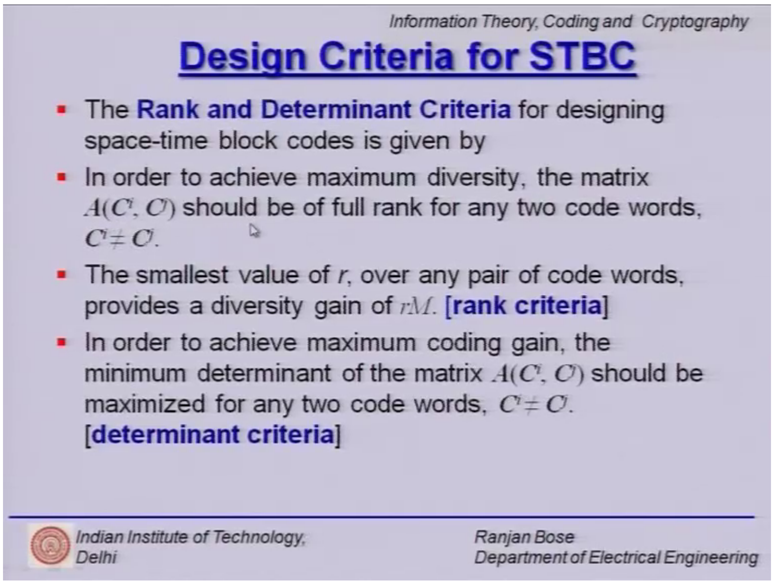

In the last class, we therefore, came up with this rank and determinant criteria for designing space time block codes and what we came up with is the rank and the determinant criteria. So, we defined this matrix A and we said that it should be full rank for any two code words C i not equal to C j and the smallest value of the r rank over any pair of code words provides a diversity gain of r into M, where M is the number of received antennas.

This is the rank criteria and the determinant criteria says that in order to achieve maximum coding gain, right. So, we have already talked about the diversity gain, but in order to achieve the maximum coding gain the minimum determinant of the matrix A should be maximized for any two code words which are not equal.

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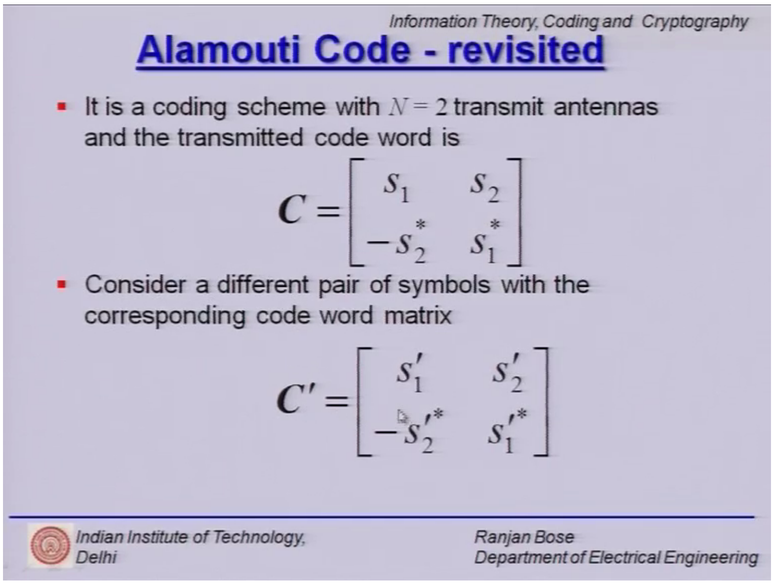

So, we quickly revisit the Alamouti scheme with N is equal to two transmit antennas and this is the code further Alamouti scheme and if we have another pair of symbols and this matrix is denoted by C prime as follows.

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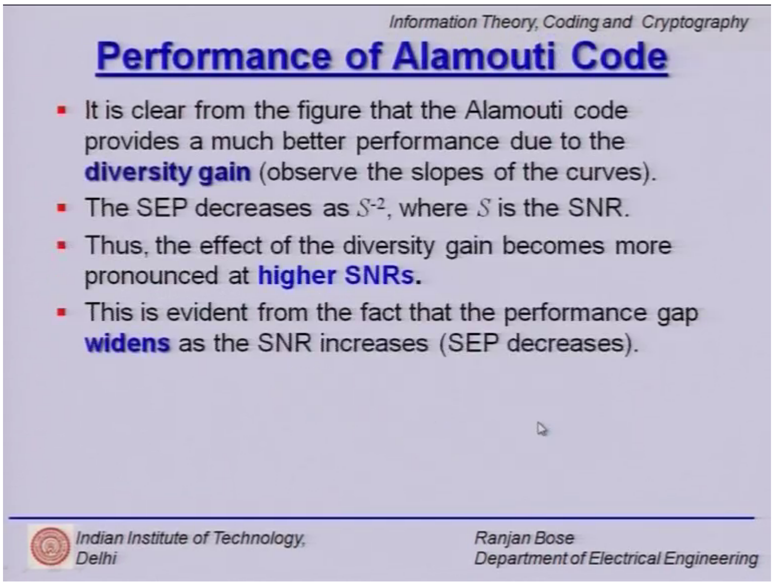

Then, we can go and find out how the diversity gain is obtained from the Alamouti code. So, diversity gain is obtained simply by looking at the slopes of the BER curves. So, on the x axis we can have the SNR, y axis we have the symbol error rate, if you will and then if you look at with and without Alamouti code you see a distinct change in the slope which indicates the a diversity gain.

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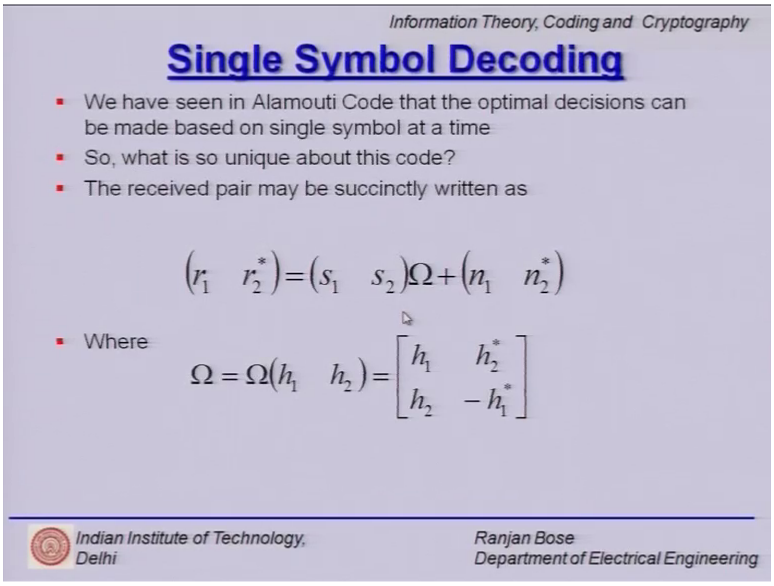

Now, a few words about single symbol decoding; We have realized the uniqueness of this Alamouti code because this pair r 1, r 2 star is dependent only on s 1 and s 2 independently.

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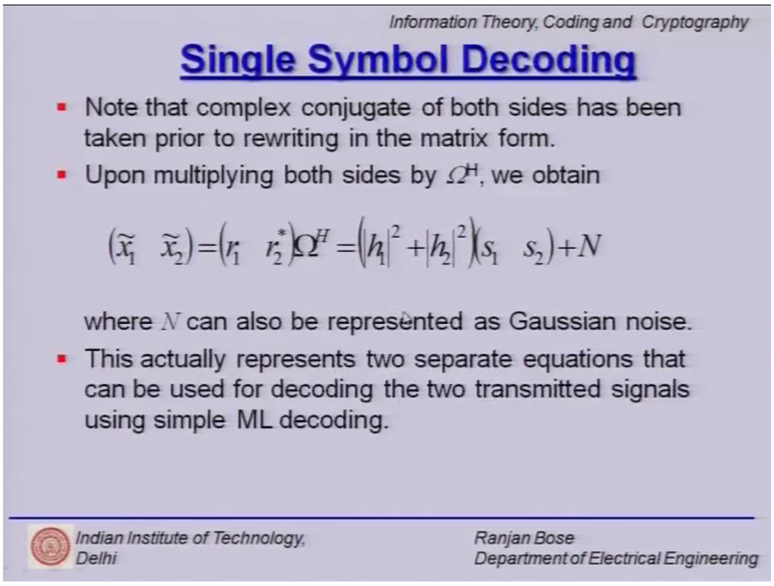

So, you can have this estimates of x 1 and x 2 as simply as h 1 absolute value squared plus h 2 absolute value squared s 1, s 2. So, basically what it means is that your x 1 tilde depends only on s 1 and x 2 tilde depends only on s 2, ok. So, this is the single symbol decoding which reduces drastically the complexity and the decoder.

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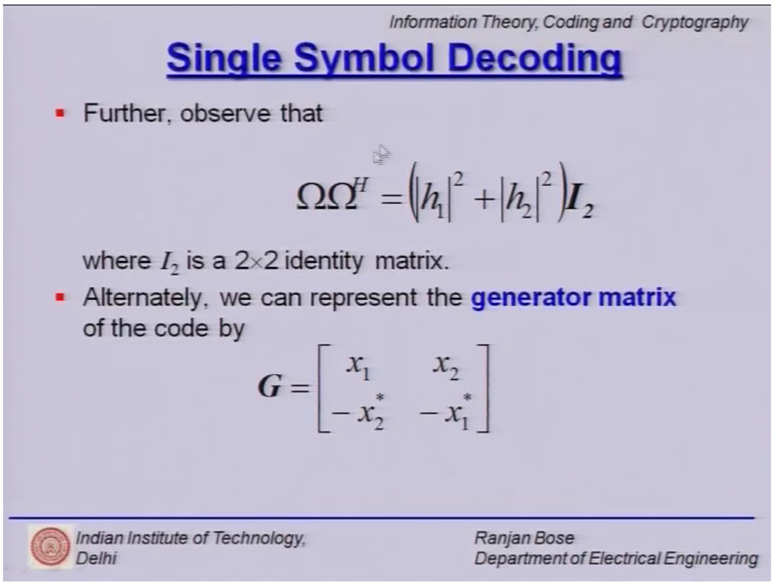

So, you observe that this sigma sigma Hermitian is nothing, but h 1 squared plus h 2 squared I 2 and the generator matrix for the code is simply given by G. So, throughout this lecture this is how we will denote the generator matrix of the code for different cases. This is a 2 cross 2 complex design, but we will look at real designs and complex designs in our subsequent slides.

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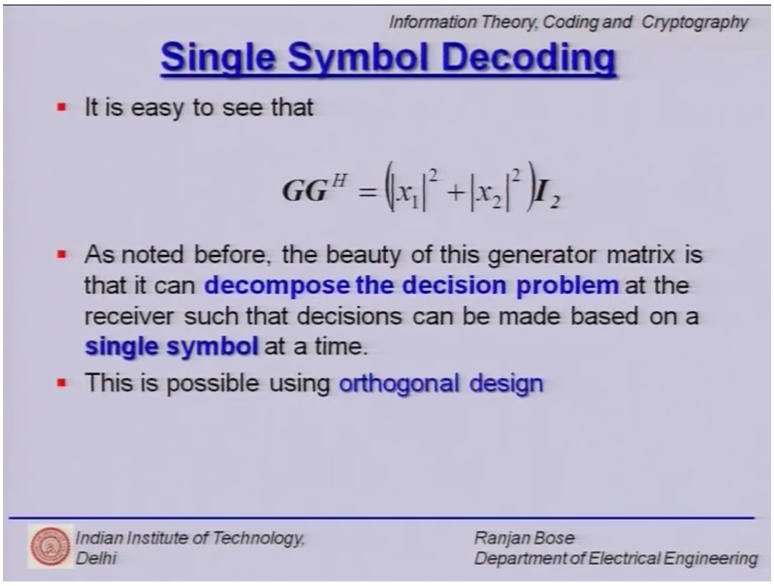

So, what is interesting about this generator matrix is if you take GG Hermitian you come up with a scaled identity matrix and this interesting fact allows us to decouple the decoding problem and we do symbol by symbol decoding we have one single symbol at a time which is required to take the decision. And the reason is because we have an orthogonal design. So, GG Hermitian is simply this identity matrix multiplied by this term.

So, the key part is this orthogonal design and now, let us focus on what good orthogonal designs are available, how to go about doing it, do orthogonal designs of all size exist or not let us look at these questions.

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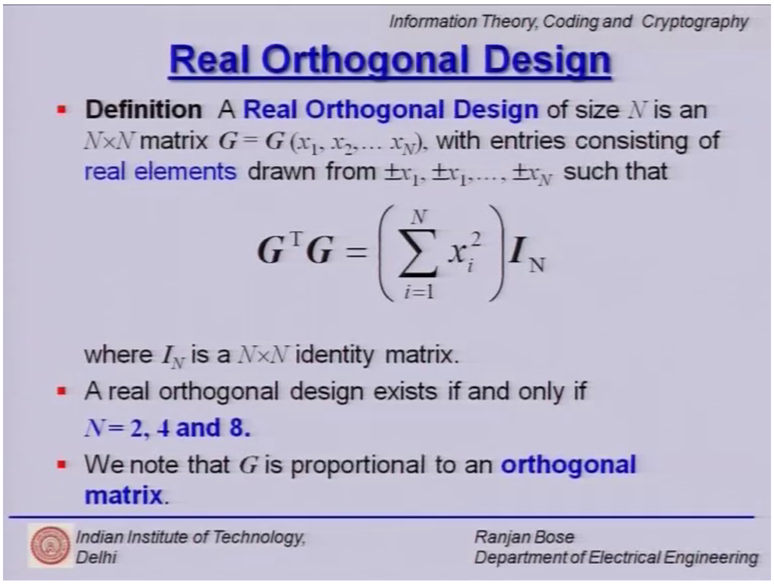

So, let us now start on real orthogonal design. So, let us start with the definition a real orthogonal design of size N is an N cross N matrix G. So, the moment we are trying to define a space time block code which is a real orthogonal we would simply represent it with a matrix G which is the generator matrix with entries consisting of only real elements drawn from plus minus x 1, x x 2 so and so forth, till x N such that G transpose G is summation I is equal to 1 through N x i squared times this identity matrix of size N cross N.

So, what is very interesting is it can be shown that only when N is 2, 4 or 8 do you have a really orthogonal design that this is possible otherwise you simply cannot have this condition being satisfied. We also note that G is proportional to an orthogonal matrix, ok. So, I can have a proportionality a constant also ahead in front of you.

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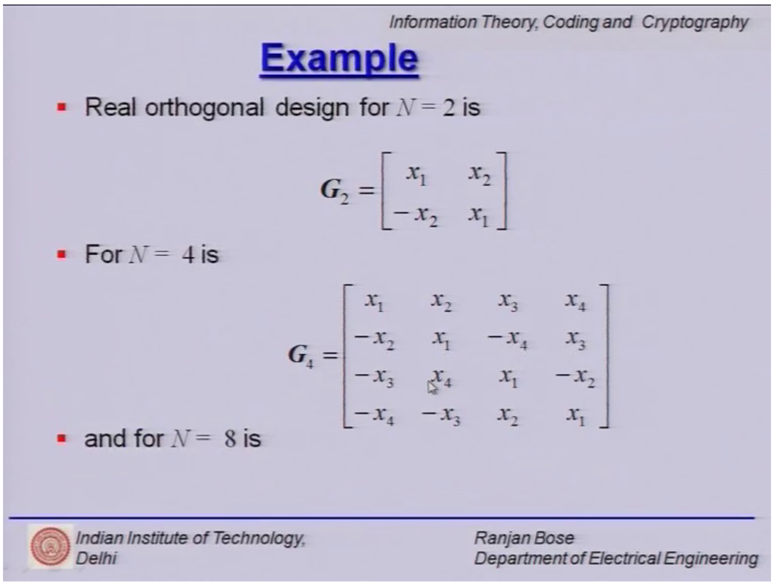

Let us look at these possibilities for N is equal to 2, 4 and 8. So, if you have generator matrix G 2 as follows then you can satisfy yourself by a taking G 2 transpose into G 2 and you will get that there will be terms only along the diagonal and there will be 0's of diagonal.

Similarly, this is the design for N is equal to 4. So, please note the convention is the same this axis is a space axis, this axis is the time axis. So, if I were to implement a real life system with G 4 I will divide my implementation into four time slots. In the first time slot I will send out x 1 through antenna element 1, x 2 through antenna element 2, x 3 from antenna element 3 and x 4 from antenna element 4 and then wait for the next time slot wherein I will send out minus x 2 x 1 minus x 4 and x 3 and so and so forth for the four time slot.

So, in four time slot I have been able to send out 4 symbols x 1 x 2 x 3 and x 4. So, therefore, the rate is also 1, I have not compromised on the rate, right, but what is interesting is if you take G 4 transpose into G 4 you will again get the values along the diagonal and there will be 0 terms of diagonal. So, again this is an orthogonal design which means you can happily go ahead and do single symbol decoding. So, the decoding complexity is again quite low.

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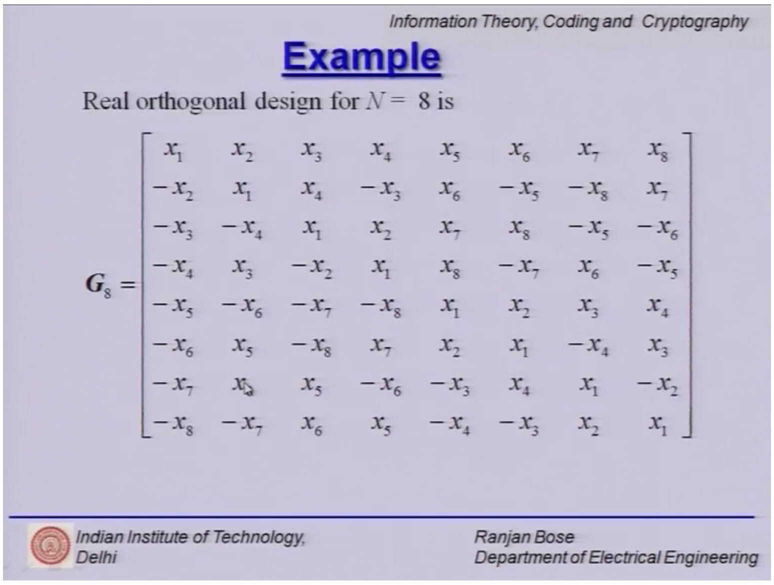

If you look at for N is equal to 8 again this is a real design and it is a real orthogonal design you can verify G 8 transpose into G 8 will again give you a only non-zero elements along the diagonal. And this is again the simple strategy I have got 8 antenna elements right and here are 8 time slots and this precisely tells you what to send at what time slot on what antenna, ok. Again, single symbol decoding is possible. Unfortunately, beyond eight we have no designs possible, it is possible to prove it.

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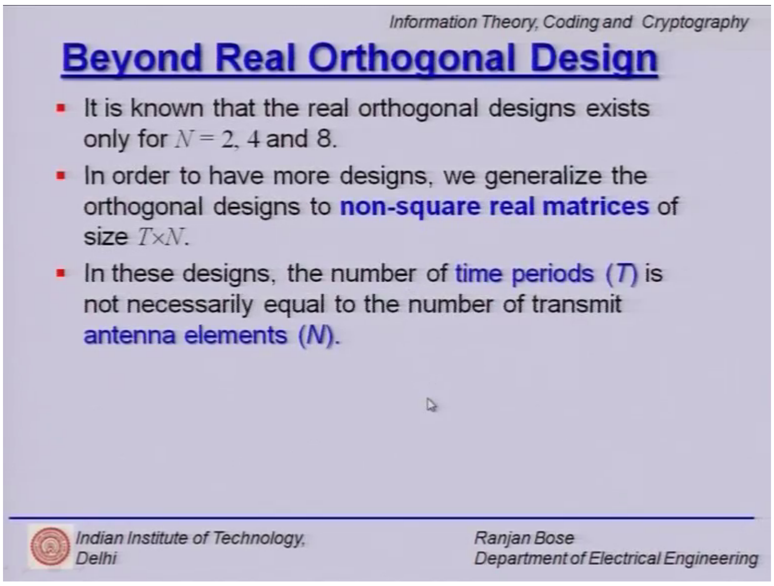

So, it is known that real orthogonal designs exists only for N N is equal to 2, 4 and 8. So, how do we go beyond that? We have to generalize the orthogonal designs to non square real matrices of size T cross N earlier it was N cross N, but suddenly I want to have T which means I want to increase the number of time slots. So, it might take a hit on the rate. So, the number of time periods is denoted by T and the number of antenna elements is N and T is not necessarily equal to N.

So, clearly for the N is equal to 2, 4 and 8 cases, the generator matrix was a square matrix, N was equal to T, but since we have no more such matrices for larger values of N we now resort to T cross N. So, these are non square matrices.

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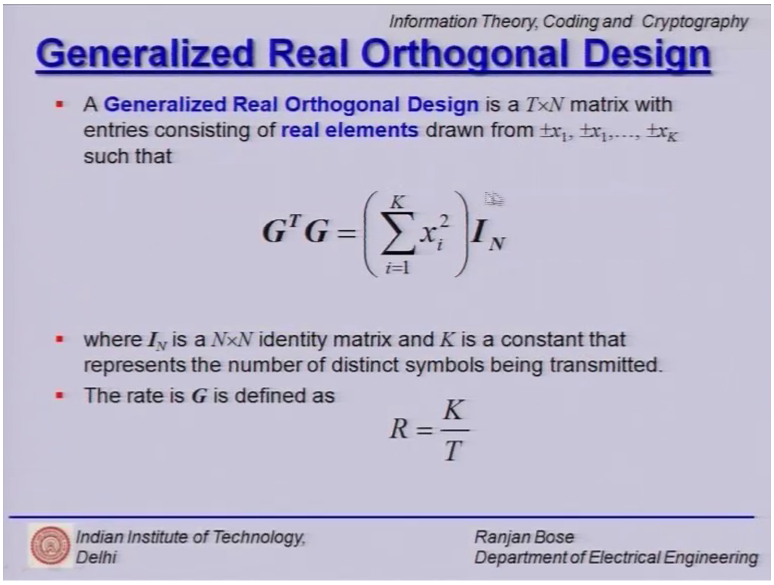

So, what is the definition? A generalized real orthogonal design is a T cross N matrix. So, if you have to design a generalized real orthogonal design we have to just come up with a matrix which is T cross N, but all the elements of this matrix all the entries are real drawn from plus minus x 1 plus minus x 2 up to plus minus x K such that G transpose G is again equal to I is equal to 1 through K x i squared I N, where IN is a N cross N identity matrix, ok.

So, again only the diagonal terms exist rest are nonzero, rest all off diagonal terms are 0 only the diagonal terms are nonzero. But, clearly, now the rate has to be defined and rate is K over T, right. So, T is of course, larger than K and therefore, it is possible that the rate will be less than 1.

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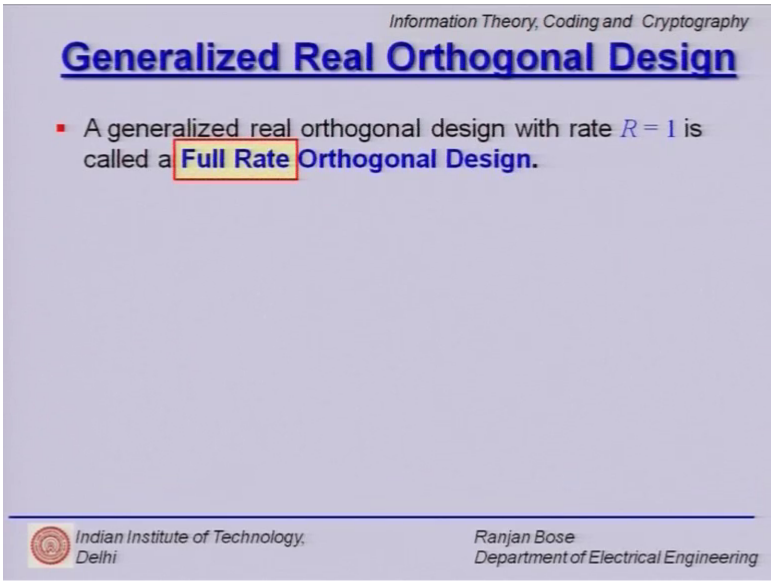

But of course, in real life we would like the rate to be as close to 1, if not 1. So, a generalized real orthogonal design with rate R is equal to 1 it is called a full rate orthogonal design, ok; so, if we are still in the domain of generalized real orthogonal design, but we are not talking about full rate design.

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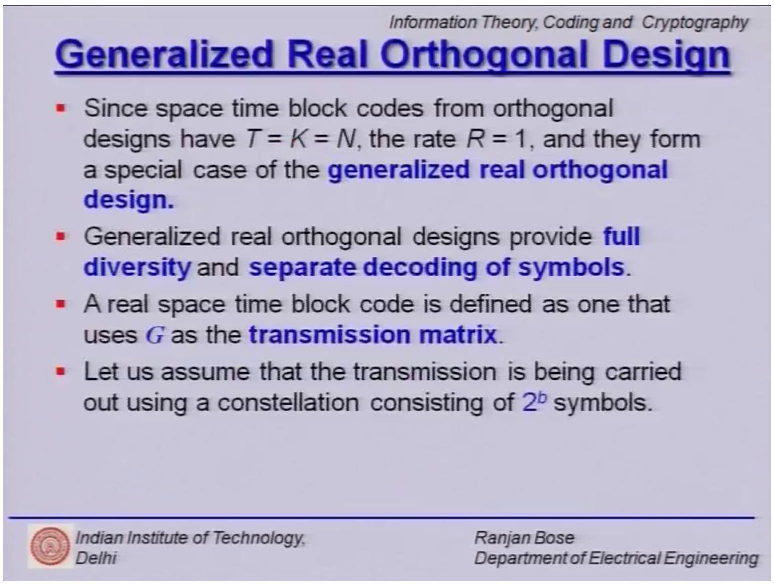

Since the space time block codes from orthogonal designs have to have T is equal to K is equal to N and the rate R is equal to one and they form a special case of the generalized real orthogonal design. So, the already the case of N is equal to 2, 4 and 8 the orthogonal designs there were a special case of the generalized real orthogonal designs with rate R is equal to 1. Generalized real orthogonal designs provide full diversity and separate decoding of symbols. So, the diversity is there as discussed earlier and you have single symbol decoding. So, separate decoding of symbols exists. So, receiver complexity, receiver time is both reduced.

A real space time block code is defined as one of as one that uses G as a transmission matrix. So, we have already looked at the properties of G. G transpose into G should be a matrix with only the diagonal elements as nonzero. So, let us assume that the transmission is been carried out using a constellation consisting of 2 raise power b symbols we have discussed this before.

So, this generalized real orthogonal design we will now make it GROD standing for Generalized Real Orthogonal Design. So, let us now talk about the steps for generalized real orthogonal design.

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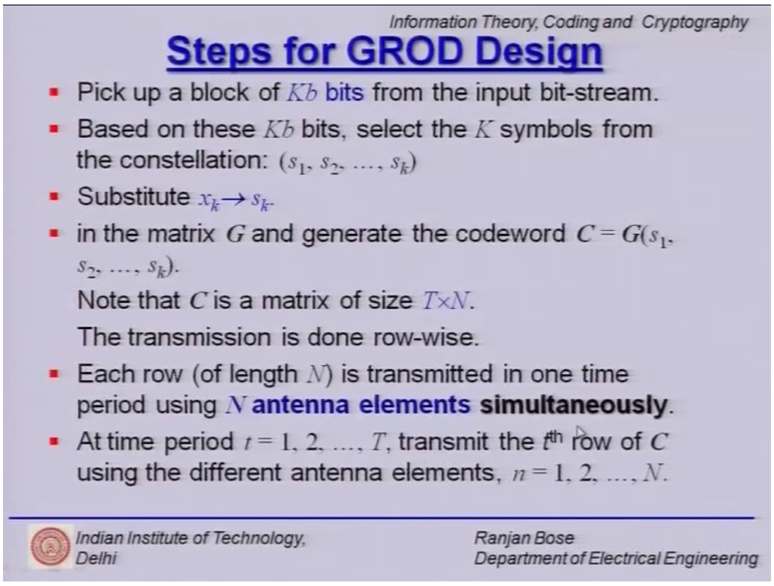

So, pick up a block of Kb bits coming from the input stream. So, we have no worries getting enough bits to process today we are generating humongous amounts of bits, but what we do is we take up a block of K into b bits. Remember, the constellation has signals coming from 2 raise power b points in the constellation.

So, based on these Kb bits select K symbols from the constellation; so, the first b bits are used to pick up first symbol s 1, the second b bits are used to pick up s 2 and the last b bits are used to pick up s k. So, we have now s 1, s 2 up to s k symbols as coming mapped out of this K b bits. So, from my generator for the space time block code I substitute x k to s k.

So, now we have the code word coming as the generator matrix consisting of s 1, s 2, up to s k. Clearly, this generator matrix is of the size T cross N. So, the transmission is done row-wise, which means in the first time slot send out row 1, in the second time slot send out row 2 because the rows represent the time axis the columns represent the antenna element the space axis. So, each row obviously, of length N is transmitted at one time period using the N antenna elements simultaneously.

At the time period t is equal to 1, 2, 3, up to T transmit the T-th row of the C; the code word C, using the different antenna elements N is equal to 1, 2, up to N. So, this much is pretty clear.

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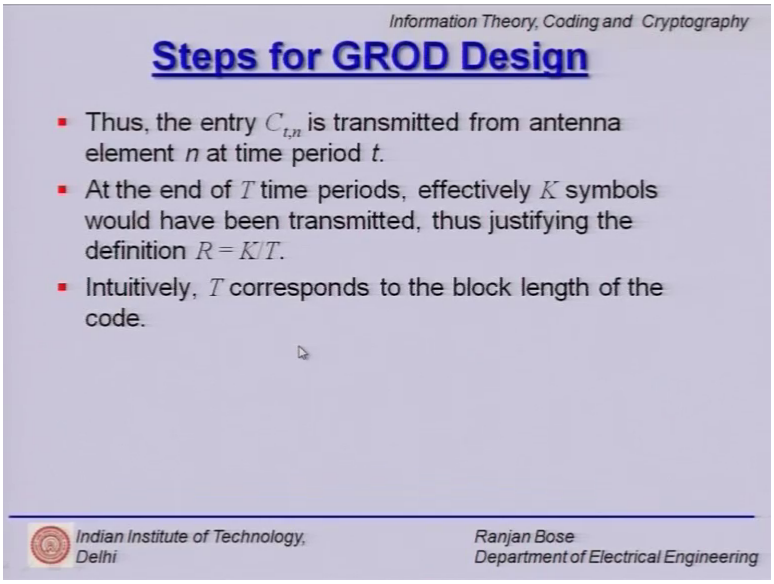

So, thus the entry C t comma n of the code is transmitted from the antenna element n at the time period t, that is the notation. At the end of T time periods effectively K symbols would have been transmitted, thus justifying the rate R is equal to K by T.

So, what does this mean? Intuitively, T corresponds to the block length of the code.

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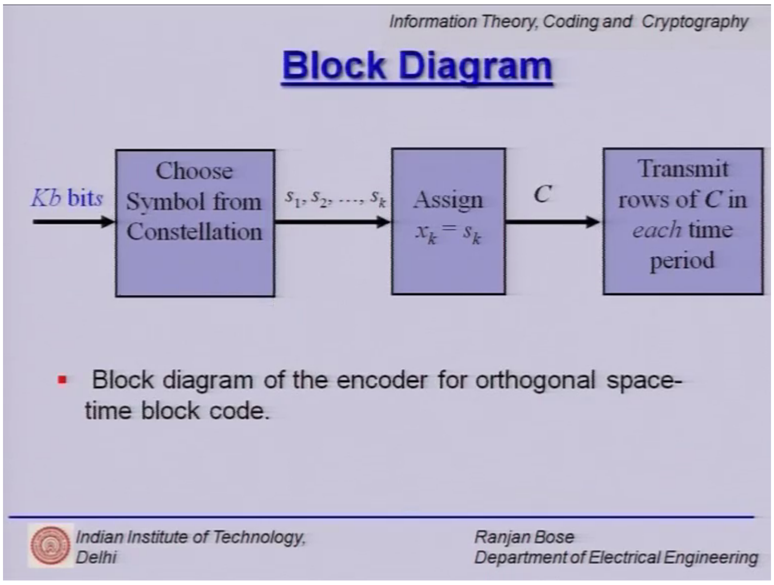

Let us show it using a block diagram for orthogonal space time block codes. What we have done is taken K bit bits coming at a time divided this Kb bits into b bits then b bits then b bits and so and so forth for each of the b bits I pick up one symbol from the constellation diagram.

So, s 1 corresponds to the first b bits s 2 to the second and so and so forth and then I assign x k equal to s k gives my generator matrix to generate the code C. Now, this is T cross N matrix and then I transmit the rows of C in each time period.

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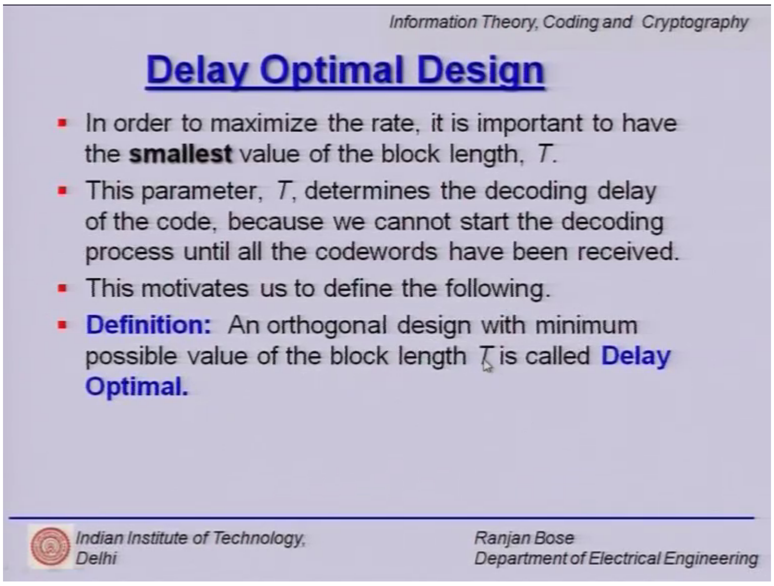

Now, let us talk about delay optimal design. Clearly as we increase the size of T, I will use more and more time slots if I use more and more time slots then my delay would increase at the receiver side to get the decoded output. So, let us talk about this delay optimal where we minimize this T. So, in order to maximize the rate it is important to have the smallest value of the block length T, this is obvious.

So, the parameter T determines the decoding delay of the code, because we cannot really start decoding until all the code words have been received, ok. So, we have to wait till the last transmission and after T time slots and we would like to reduce this to the extent possible.

So, what do we do? Let us define the delay optimal design. So, an orthogonal design with minimum possible value of the block length T is called delay optimal.

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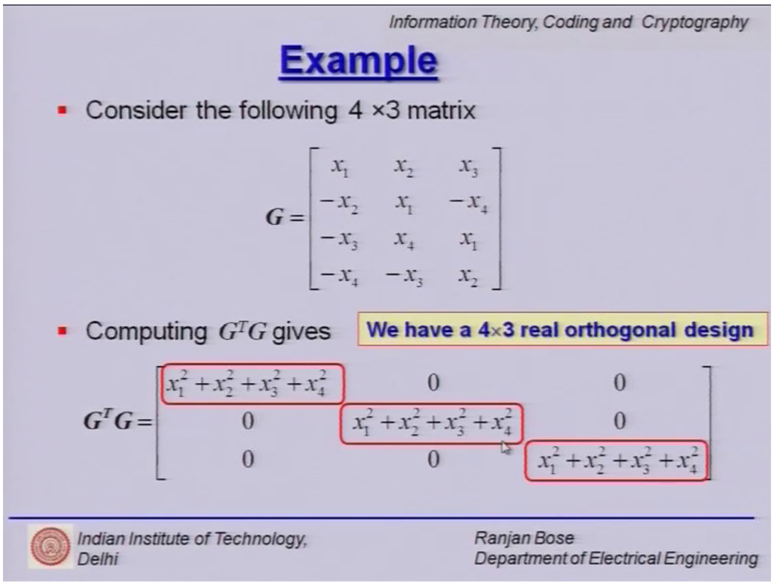

It is better to look at an example to illustrate the point. So, let us consider the following 4 cross 3 matrix. So, N is equal to 3, T is equal to 4, leading it to a T cross N matrix. Remember, this is the space axis. So, we have 3 antenna elements and this is the time axis that I will be using four time slots. So, what this generator matrix tells us is in the first time slot send x 1 from antenna 1, x 2 from antenna 2, x 3 from antenna 3, then wait for the next time slot and transmit minus x 2, x 1 minus x 4 and so and so forth in the 4 time slots.

But, we make a very interesting observation. We just do not have three symbols to transmit we have x 1 up to x 4. So, in 4 time slots we have been judiciously distributed the symbols such that x 1, x 2, x 3 and x 4 all have been placed, such that G transpose G again adheres to the definition of the orthogonal design. So, let us compute G transpose G and if you do so, for this you will be surprised to find the answer as follows and the observation is that the diagonal elements are non-zero and rest all are 0.

So, we have really in front of us a 4 cross 3 real orthogonal design, it is a generalized real orthogonal design because it is not a square matrix, right.

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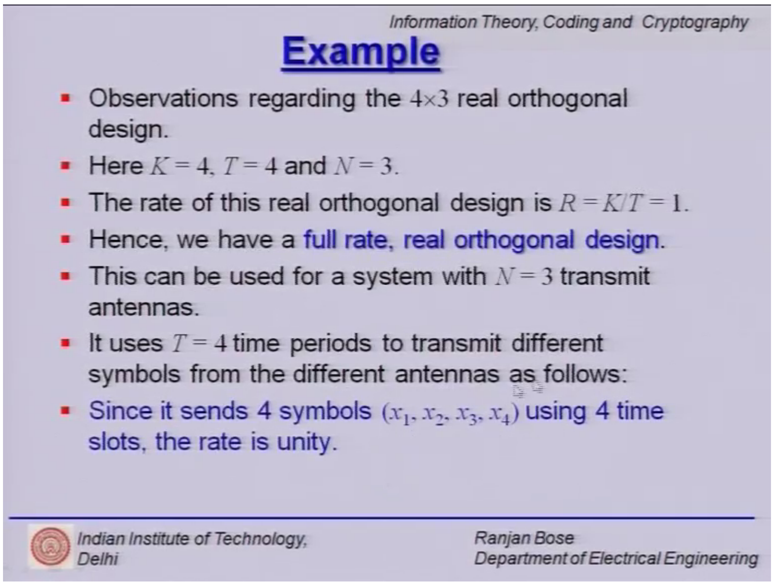

So, what are the observations regarding this design? T is equal to 4, N is equal to 3, as we observed K is equal to 4, but the rate is 1, ok. We did not compromise on the rate. So, it is a full rate generalized real orthogonal design. So, we have only used 3 transmit antennas because N is equal to 3, but we have judiciously used the 3 antenna elements in different time slots to send out the message which are single symbol decodable.

Of course, it uses 4 time periods to do. So, the delay increase a little bit, right. Please remember, we never had a 3 cross 3 solution for real orthogonal designs, we had 2 cross 2, 4 cross 4 and 8 cross 8, but when we went to the generalized domain we have a 4 cross 3 solution and that is the interesting observation. So, it sends out 4 symbols using 4 time slots and the rate is unity.

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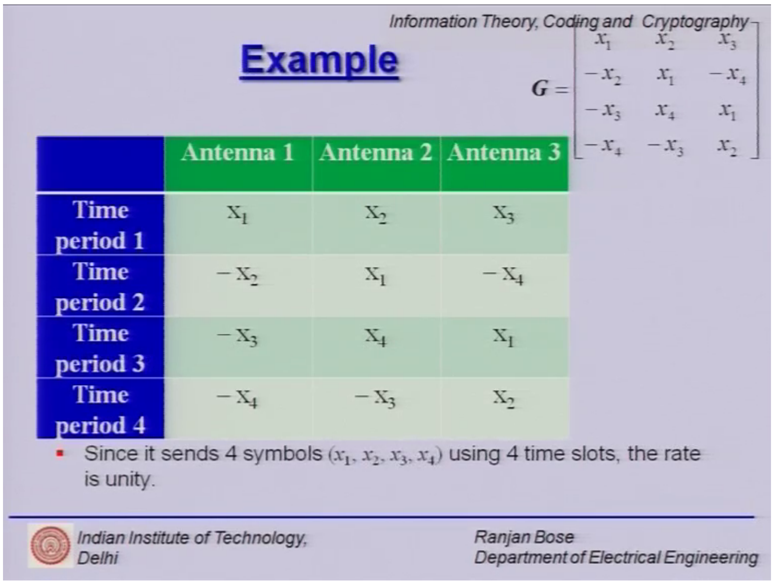

So, let us start back and analyze it a little bit further. So, you have this three antenna elements sending out this first row in time slot 1, second row in time slot 2, third row in time slot 3 and fourth row in time slot 4 and if you want to depict it more clearly you have this 3 antenna elements and four time slots and you exactly know which one you are sending in which time slots, ok. So, this is actually the recipe for sending out the code based on this design.

So, the important observation is that it is full rate we have not compromised on the rate.

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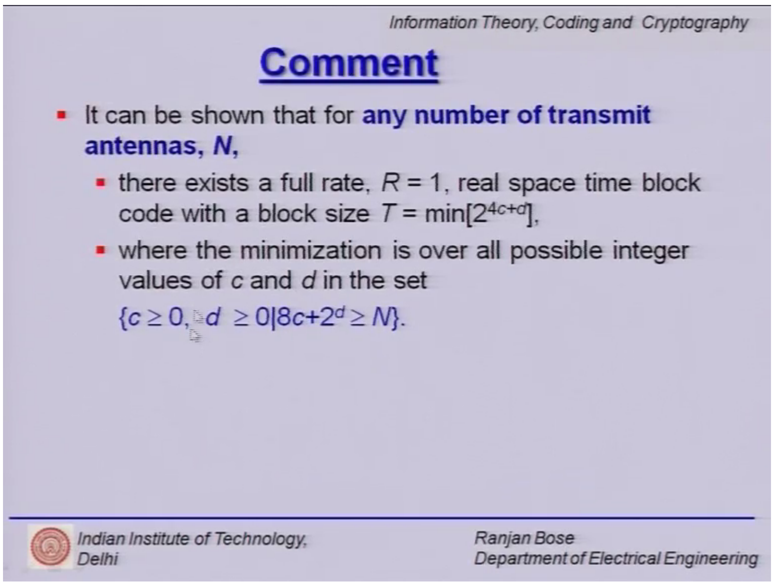

So, it can be shown that for any number of transmit antennas N, there exists a full rate real space time block code with block size T given by a minimum of 2 raise power 4 c plus d. So, this rate full rate can be guaranteed provided you have a constraint on this block size T. The minimization is all possible integer value values of c and d.

So, here this is the block size T and if you can have c and d as integers then you can probably come up with this design and we look at an example shortly. This c and d have some constraint c should be greater than or equal to 0 d should be greater than or equal to 0 and 8, c plus 2 raise power d should be greater than or equal to N. So, if you can find c and d which can satisfy these conditions then T can be found out by this minimum and then you can find out an example.

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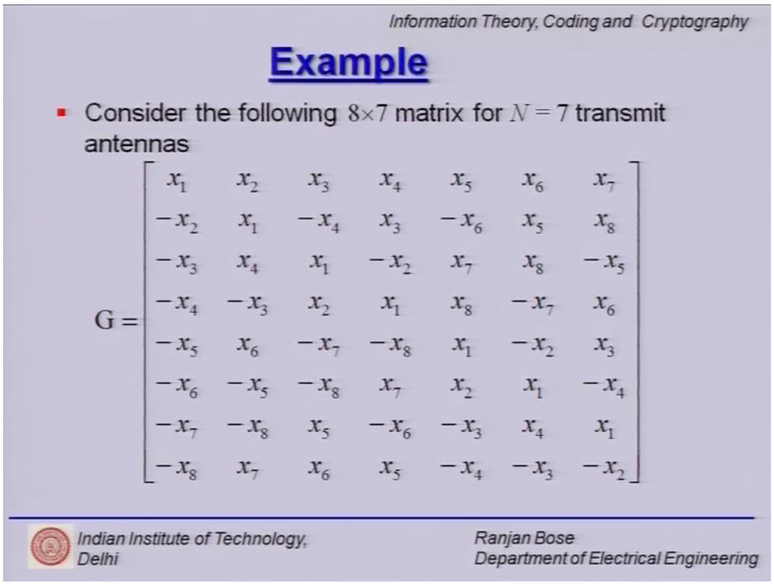

So, let us look at the following example we have a 8 cross 7 matrix for 7 transmit antennas. So, just by looking at this matrix I know that there are 7 columns so, 7 transmit antennas will be there. There are 8 rows so, I will be using 8 time slots. Obviously, in the first time slot I will be sending out this first row, second row in the second time slot and the eighth row in the eighth time slot.

Now, before we proceed any further we can quickly take G transpose into G and we will see whether only the diagonal elements are non-zero rest are all zeros which will give that it is a orthogonal design.

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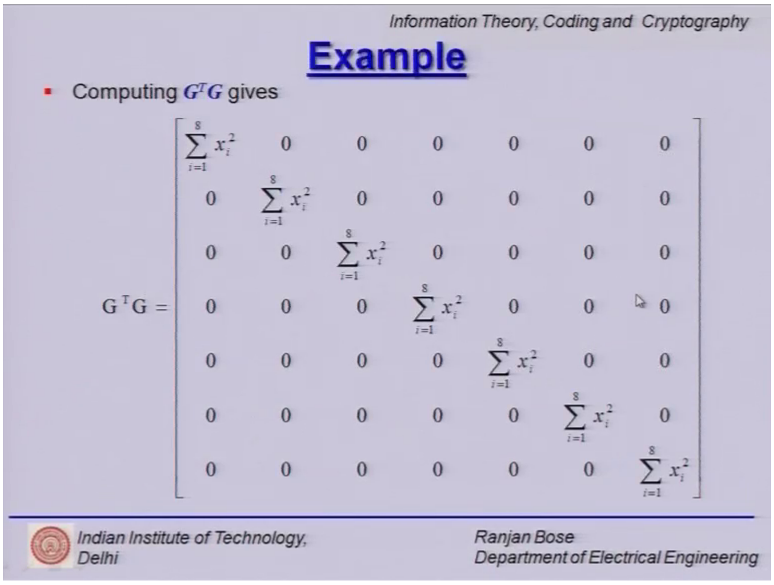

So, we do that computation G transpose G and one can easily verify that you get this expression. So, this will guarantee single symbol decoding.

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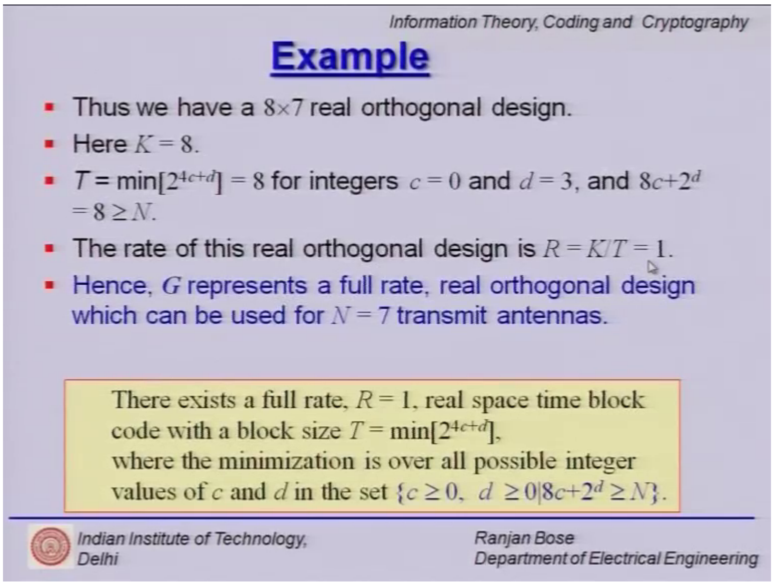

But, we also see that 8 cross 7 is a real orthogonal design with K is equal to 8 and you can do T as minimum of 2 raise power 4 c plus d equal to 8, because the integers which we found was c is equal to 0 and d is equal to 3. So, we have satisfied those conditions and we were able to get this 8 plus 7 matrix. So, not only it tells you that this T cross N matrix exists, but you already have an example of what that matrix should look like.

So, this is the condition we satisfied ourselves with that there exists a full rate R is equal to 1 real space time block code with block size given by this where the minimization is over all possible interval integer values of c and d. In this example you have R is equal to 1, because you had if you see eight time slots and you have x 1 up to x 8 if you carefully observe I am pushing through along x 1, x 2, x 3 up to x 8 in a distributed manner. So, in eight time slots effectively I have sent out eight symbols. So, my rate is indeed 1. So, G represents a full rate real orthogonal design which can be used with 7 transmit antennas.

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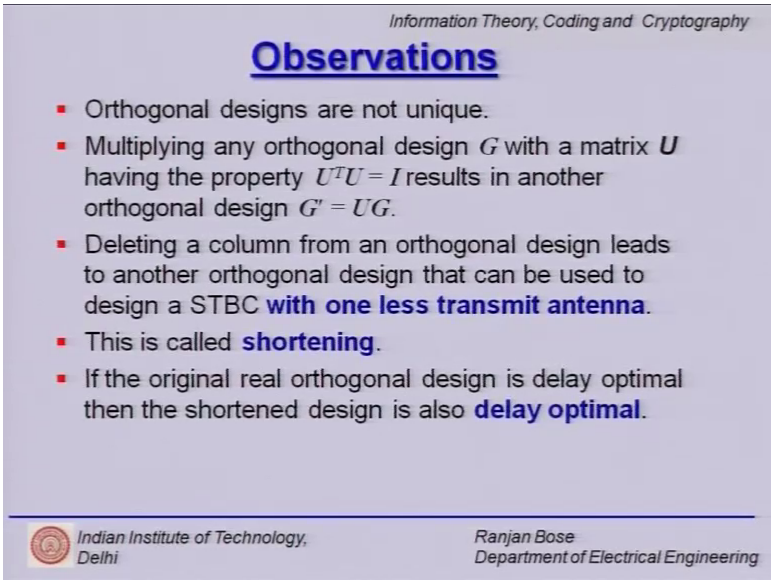

So, some observations orthogonal designs are not unique, number – 2 multiplying any orthogonal design G with a with a matrix U, right having the property U transpose U T U equal to 1. So, results in another orthogonal design ok. So, deleting a column from an orthogonal design leads to another orthogonal design that can be used to design a space time block code with a one less antenna.

So, these are practical issues that we can look at, if we have a one design we can go to another design by deleting a row or a column depending upon how you look at it. So, deleting a column basically reduces one antenna element and you still have a orthogonal design left. This process is called shortening if the original real orthogonal design is a delay optimal then the shortened design is also delay optimal.

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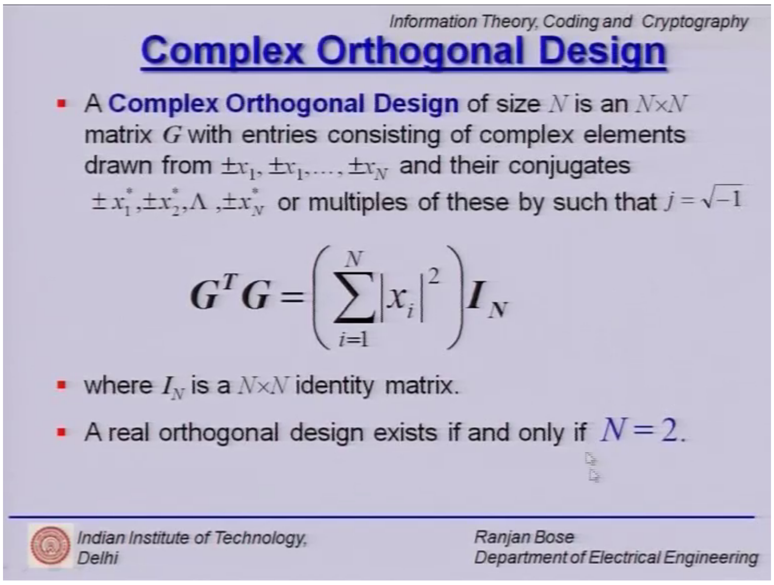

Now, we changed gears slightly and we say look lot of our signal constellations are complex. So, are there complex orthogonal designs? Of course, we had looked at Alamouti to begin with so, we know the answer is, yes. So, let us define a complex orthogonal design. A complex orthogonal design of size N is an N cross N matrix G with entries consisting of complex elements drawn from plus minus x 1, x 2 up to x N and their complex conjugates x 1 star, x 2 star up to x N star, right or multiples of these by j is equal to under root of minus 1 such that G transpose G, right should be this.

So, we actually should write G Hermitian G should be written in this form, where I N is the N cross N identity matrix and what is interesting is a real orthogonal design exists if and only if N is equal to 2. It can be shown that and this great design is that Alamouti code we have looked at so.

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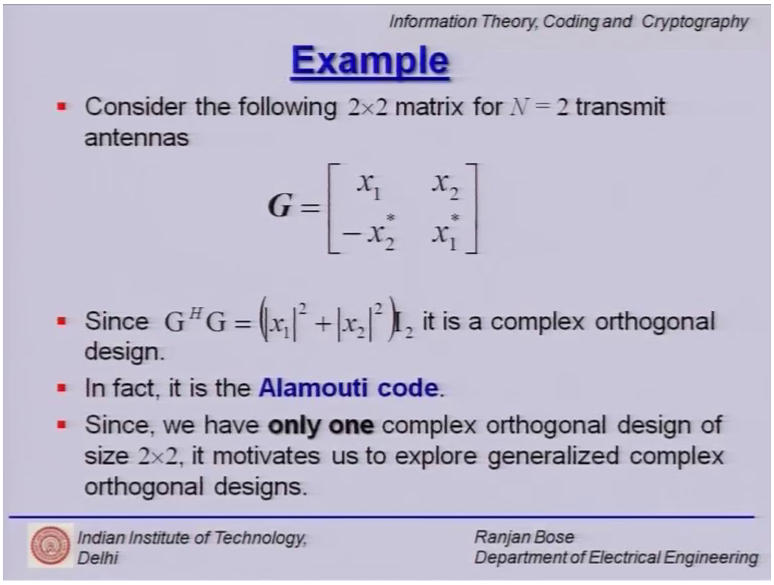

This is an example of a 2 cross 2 matrix which we have already seen as being an orthogonal design it is the Alamouti code, but this is the only one complex orthogonal design which exists size 2 cross 2.

So, it is obvious that we must graduate to the generalized complex orthogonal design.

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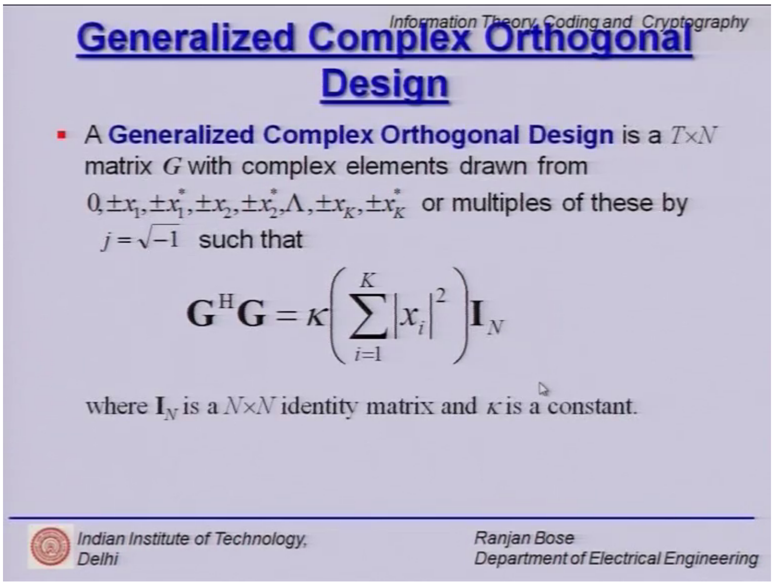

So, we define it quickly. Generalized complex orthogonal design similar to the generalized version of the real orthogonal design is a T cross N matrix G with complex elements drawn from 0, x 1, x 2, x 3 and so and so forth up to x K such that G Hermitian

G is some kappa times summation of x i absolute value squared I N, where I N is an N cross N identity matrix and kappa is a constant.

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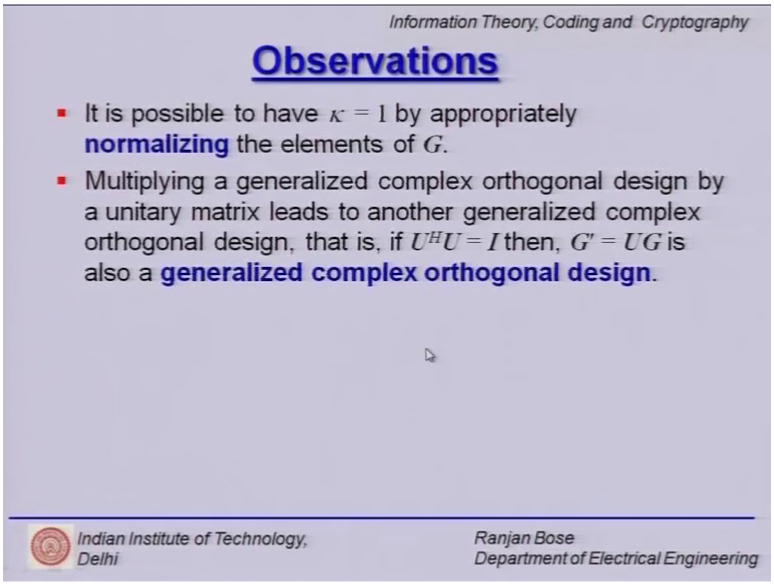

So, it is possible to have kappa is equal to 1 by appropriately normalizing the elements of G is still remains an orthogonal matrix and we can multiply as before with the unitary matrix then G prime equal to U times G is also generalized complex orthogonal design. So, multiplication by unitary matrix as in the earlier case does not change it is still remains a generalized complex orthogonal design.

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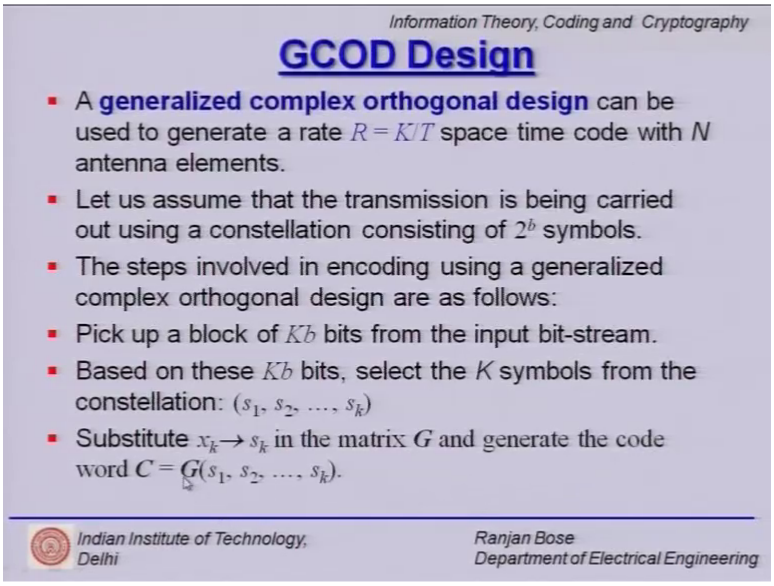

So, let us use the acronym generalized complex orthogonal design GCOD. So, a GCOD design can be used to generate a rate R is equal to K by T space time code with N antenna elements. Again, we do a similar thing we assume that we are using a constellation consisting of 2 raise power b symbols and we look at the following steps as before take a big block of Kb bits divided into b bits then again block of b bits and so and so forth gives each block to pick up the symbols s 1, s 2 up to s k we have done this before and then substitute x k for s k in the matrix G to generate the code C.

So, this G matrix always have the elements x 1, x 2, up to x k and once you substitute the G generator matrix generates a codeword, the codeword matrix becomes the C, ok.

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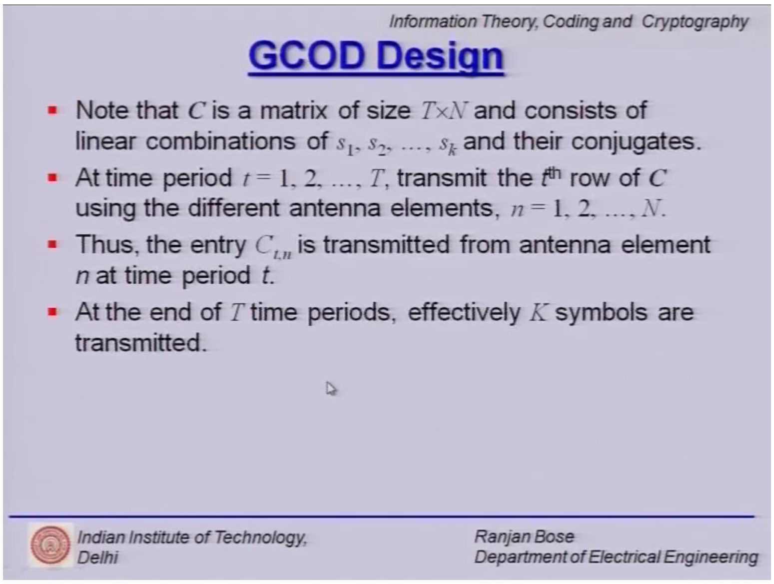

So, this C matrix is of the size T cross N and it has a linear combination of s 1, s 2, up to s k and their conjugates because it is a complex design. So, at time period t is equal to 1 through T you transmit the t-th row as before and then, clearly the C t comma n is transmitted from antenna element n at time period t as before it is pretty much the same and we have transmitted effectively K symbols at the end of T time period thereby justifying that rate.

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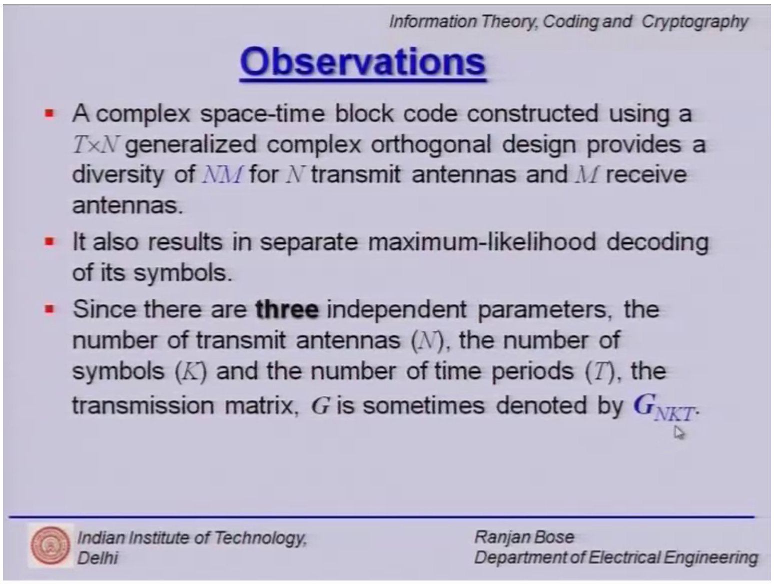

So, some simple observations a complex space time block code constructed using a T cross N generalized complex orthogonal design provides a diversity of N cross M. So, N is the number of transmit antennas M is the number of receive antennas. So, this is the diversity that is provided and it also results in a separate maximum likelihood decoding symbol by symbol decoding is possible.

But, please note there are three independent parameters, the number of transmit antennas N, the number of symbols K and the number of time periods T and so, the transmission matrix generator matrix G is sometimes denoted by G subscript NKT.

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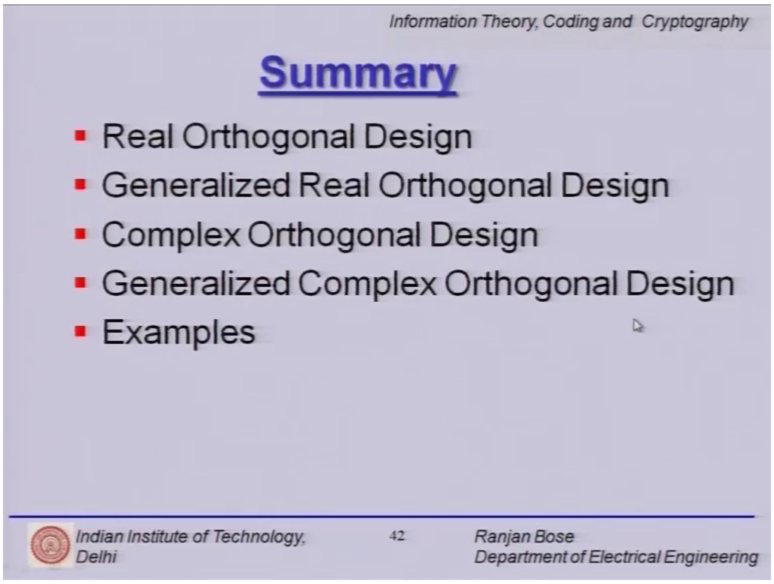

So, let us now try to conclude what we have studied in today's lecture. We started out with the notion of real orthogonal designs which provide single symbol decoding and full diversity and we realize that only N is equal to 2, 4 and 8 is possible for real orthogonal design that motivated us to look at generalized real orthogonal designs, where we are working with a matrices which are not square and so, you can have more number of time slots.

But, still it is possible to have rate R is equal to 1 and with then we defined the notion of delay optimal then we moved on to complex orthogonal design, where the elements in the generator matrix could be complex and we found out that only 2 cross 2 the Alamouti code exists as the only example of complex orthogonal design. Therefore, we moved over to the generalized complex orthogonal design. For both real orthogonal design and complex orthogonal design the generalized versions we defined the steps as to how to go about designing and sending it.

Finally, we looked at the examples for ROD, GROD, COD and GCOD. So, with that we come to the end of this lecture.