Information Theory, Coding and Cryptography
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Module – 33
Space Time Codes
Lecture – 33

Hello and welcome to our next lecture on Space Time Codes.

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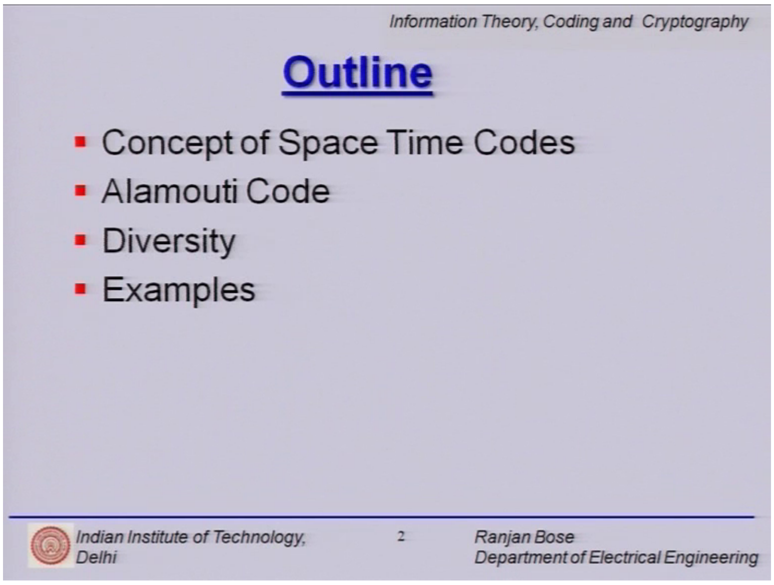

Let us start with a brief outline of today's talk; we will start with the concept of space time codes. We had originally looked at multi antenna systems and we would justify the use of that in the context of space time codes. Then we would look at the classic Alamouti code, then we will talk about diversity and diversity gains plus we will have some examples to illustrate the concepts.

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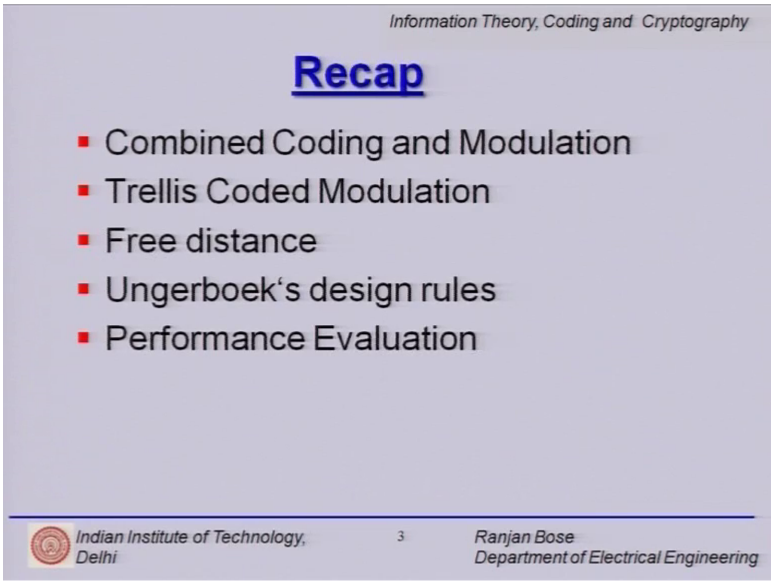

So, let us see what we have done in the previous lecture; we have looked at TCM schemes, looked at combined coding and modulation, defined free distance, design rules, performance evaluations and finally, we went over to space time trellis codes; where we started using 2 antennas. So, that was a logical extension of space trellis coded modulation going on to space time trellis code.

In today's lecture, we would look at the space time block codes, where we do not have so, to say trellis to help us encode the sequences.

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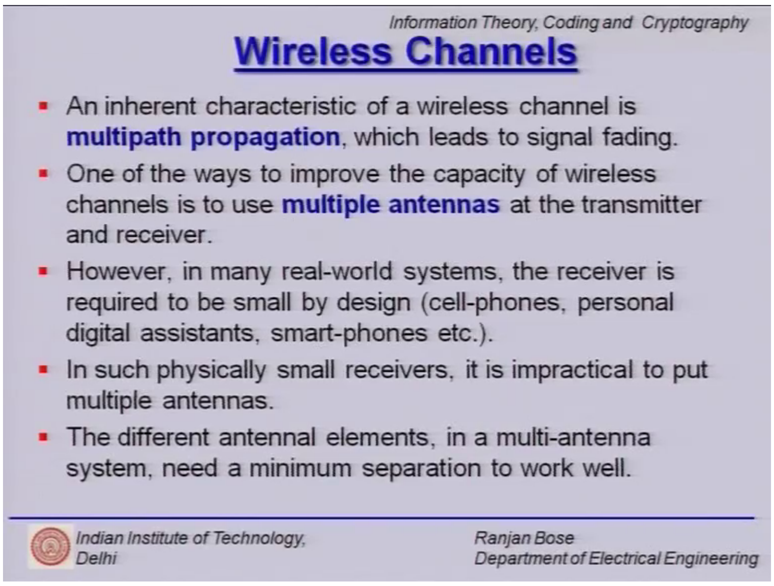

But before we do so, we start with a quick motivation; we start with wireless channels because this is where we will be using space time codes frequently. So, the inherent characteristic of any wireless channel is multipath propagation; we have seen this before and this leads to signal fading.

Now, one of the ways actually an easy way to improve the capacity of wireless channel is to use multiple antennas and the transmitter and receivers ok. So, multiple antennas can provide several things including the diversity gain that we talked about in the last class, but the problem is that the receiver could be your handset and you do not have enough space to put multiple antennas. So, it is logical to put more number of antennas at the base station.

So, what we would like to do is we would have to also have a minimum separation between the antenna elements for them to be feeding relatively independently. So, the requirement of spacing between antenna elements and the place, a space available on the receiver handset restricts putting multiple number of antenna elements at the receiver end, but from the base stations perspective this can easily be done.

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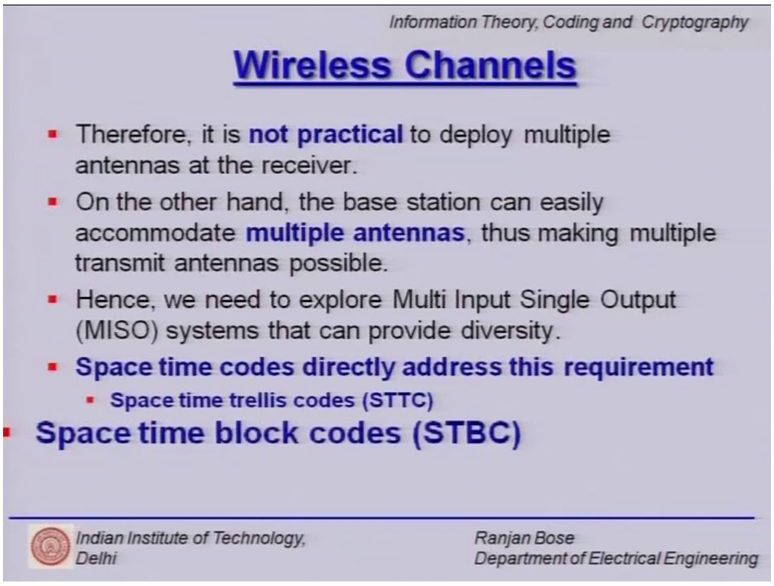

So, as we mentioned not practical to deploy multiple antennas at the receiver, but on the other hand base stations can easily accommodate multiple antennas. So, this is the basic premise with which we will start, we will have multiple antennas at the transmit site. So, we are going to look at multi input single output or MISO systems.

Now space time coding basically allows different symbols to be sent from the different antenna elements in different time slots. So, we have already looked at Space Time Trellis Codes; STTC as a natural extension of trellis coded modulation and today we would delve into space time block codes; that is the subject for today.

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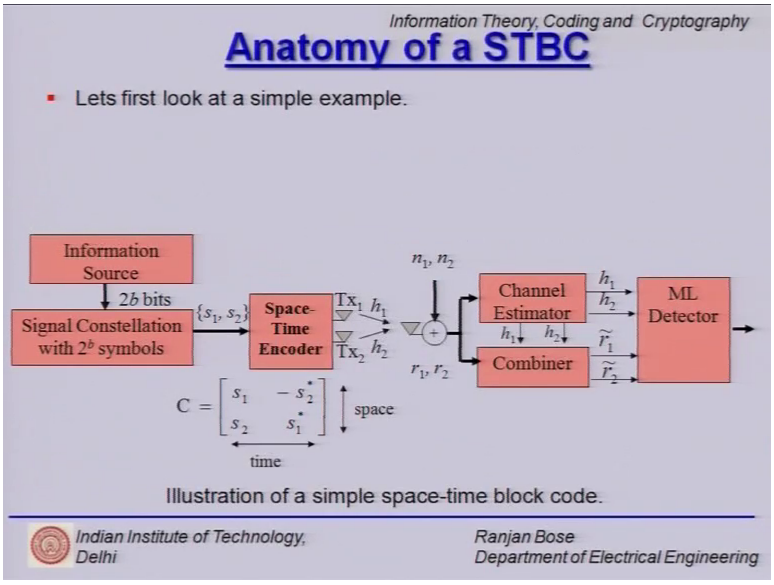

Now let us start with the anatomy of a space time block codes shortly we will discuss the details. So, we are talking about STBC; Space Time Block Codes let us start from the first block the information source. And this source is generating enough bits and what we do is we take 2 bits at a time and in general we can start with b bits at a time. So, if you club it into 2 b bits then we can look at a signal constellation with 2 b symbols.

So, let us quickly look at it here.

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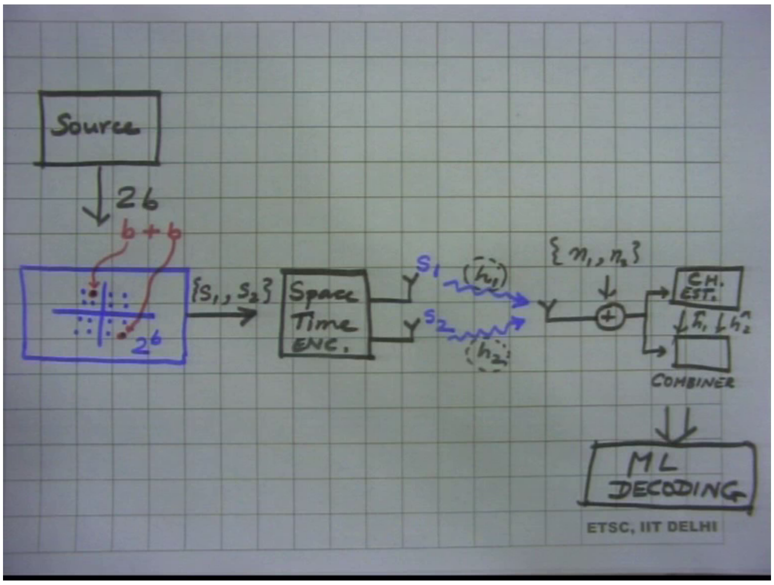

So, the source will generate 2 b which is nothing, but b and b; now what we do is we have a constellation this is predefined, this constellation has a certain number of points in the constellation diagram.

So, suppose we choose any constellation and in this there are 2 raised power b points. So, for the first b, I will go ahead and pick any one point and for the second b; I will go ahead and pick another point. Because b bits will corresponds to any one of points in the constellation diagram and once we pick up these 2 points then we say that this is your S 1 and this one is your S 2.

So, we have a pair S 1, S 2 pair this pair has to be sent to the space time encoder. So, this is a space time encoder and this guy has 2 antenna elements and what do these 2 antenna elements do? Well they have got these 2 symbols S 1 and S 2. So, they will take S 1 and S 2 and transmit it out.

But we observed that the receiver may not have the luxury of multiple antennas. So, we have only a single antenna at the receiver right and the receiver receives S 1 and S 2 simultaneously plus there will be some noise also coming. So, we will have n 1 and n 2 for the 2 signals S 1 and S 2 and this will then be fed to your channel estimator right and of course, I would also have based on the channel estimation a combiner.

So, I will get the estimates of the channels now what are these channels? Well we can represent it as h 1 and h 2 these are complex; so, these represent my channel gains alright. So, these are complex numbers and the signal S 1 gets multiplied with h 1 and gets added with noise n 1 and similarly S 2 gets multiplied by h 2 and then gets added with n 2. So, the estimates of these to h 1 hat and h 2 hat have to be given to the combiner and here after that we will take this for maximum likelihood decoding.

So, if we go and look at the slides once again we have the same concept exemplified here. You have this information source you take 2 b bits at a time, separate out these 2 bits into b bits and b bits and each one picks up a point in the constellation diagram. So, I have a pair S 1 and S 2 which is fed to the space time encoder. Now this antenna spacing is providing the space we have not talked about that time part yet.

So, we have now the time slot; so, in time slot 1, we send out S 1 and S 2. So, if you see there is a matrix right want if you want to look at this as a time axis; then this there will be symbols along this and this will be this space. So, this is corresponding to antenna element 1 and S 2 corresponds to antenna element 2.

But what we do is we then also do something more interesting in the next time slot. So, this is the axis and I already have S 2 and S 1 available with me and they are complex because they are coming from a constellation diagram. So, they have a amplitude and phase; I do a simple computation, I do minus S 2 star for example, and S 1 star. So, I take this complex conjugate multiplied with a negative and again loaded this guy on to antenna element 1 and the second person on to antenna element 2 and retransmit.

So, please note this 2 b bits resulted in the transmission of S 1 and S 2 from antenna elements 1 and 2 in the first time slot. And then now we have not taken any more bits in the second time slot what we have done is we have sent something different based on S 1 S 2; so, that is a rule. So, regardless of what S 1 and S 2 are the next 2 symbols to be transmitted simultaneously will be minus S 2 star and S 1 star.

This is just an example, this is a strategy this is my coding strategy and therefore, this can represent my space time block code. Why this will work? How good is it? Why are we doing it in this all of these questions will be answered in the subsequent slide, but the method is pretty easy; it is mechanical, but the beauty lies in the decoding part.

So, let us go on and see what really is the gain that we get from doing this kind of encoding.

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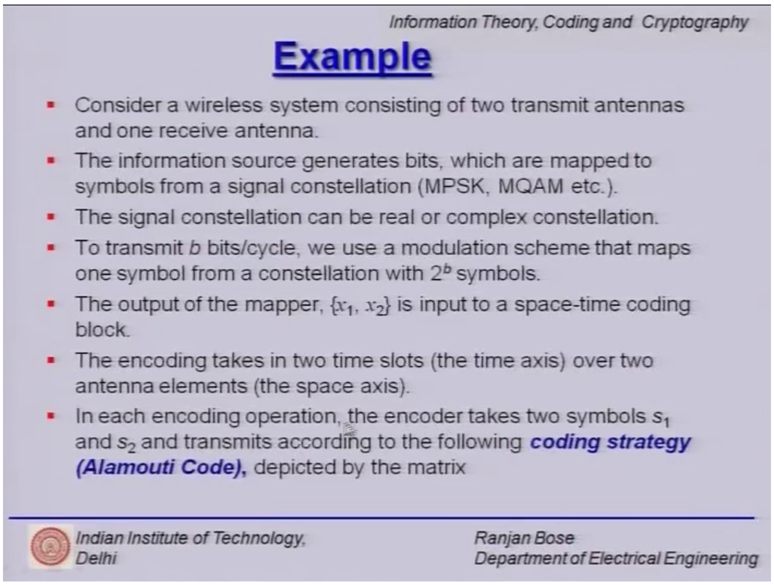

So, let us look at a simple example consider a wireless system with 2 transmit antennas and 1 receive antennas. So, these 2 transmit antennas could very well be seated on the base station and receive antenna is your handset. And we have to have some constellation diagram signal constellation available to us it could be MPSK, MQAM what have you right and this constellation could be real or complex ok. So, there is no restriction that it has to be complex.

Now, we will transmit b bits per cycle right and we will use a modulation scheme that maps one symbol from the constellation with 2 raised power b symbols and output will be x 1 x 2 and as we mentioned this encoding takes 2 time slots right.

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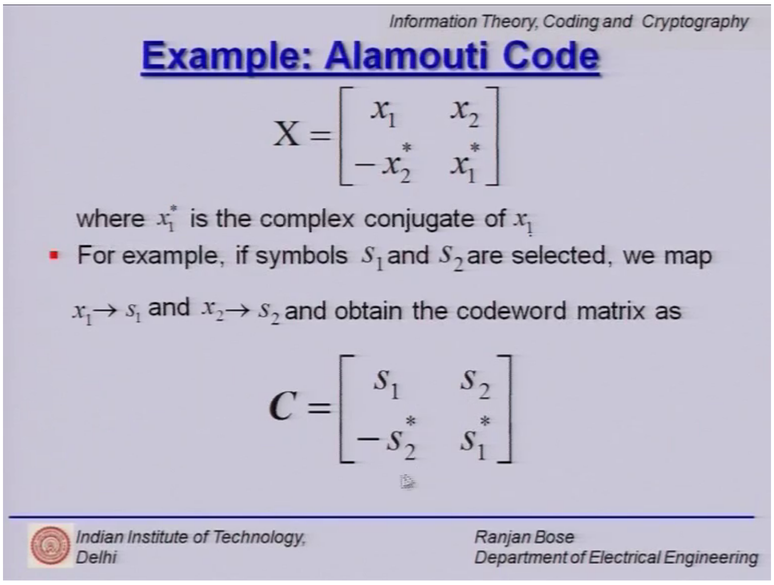

And we have this coding strategy; so, this particular way of coding has a name based on the person who first proposed it is called the Alamouti code and this is the depiction for the Alamouti code. So, if you have this as the coding strategy then the codeword matrix C as we just now observed could be written as follows S 1 S 2 minus S 2 star S 1.

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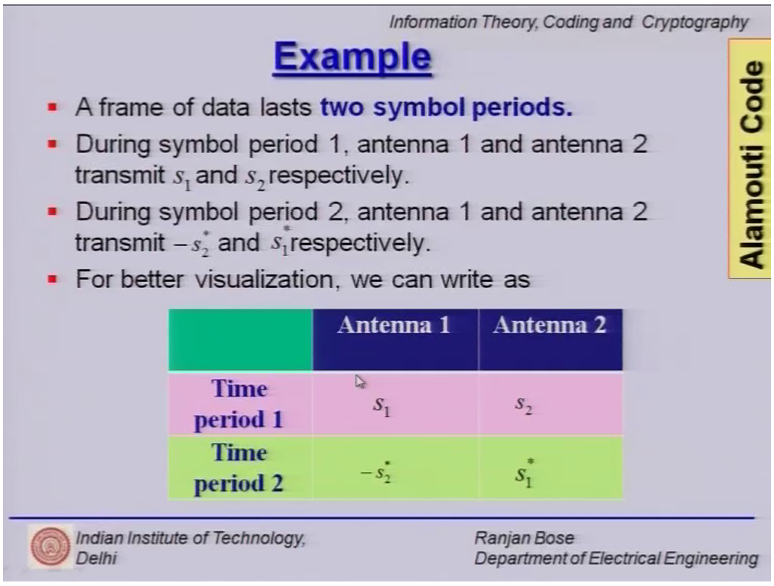

So, this method of doing space time coding can be expressed in the form of a table. Here on this axis we have the space antenna 1 and antenna 2, they are spatially apart here we have the time period 1 and time period 2. So, this is the time axis and as we mentioned in the first time slot we use antenna 1 to send out S 1 and use antenna 2 to send out S 2. And then in time period 2, we do minus S 2 star and S 1 star load them onto the antennas and transmit. So, this is in a nutshell the depiction of the Alamouti code ok; so, far it is pretty easy.

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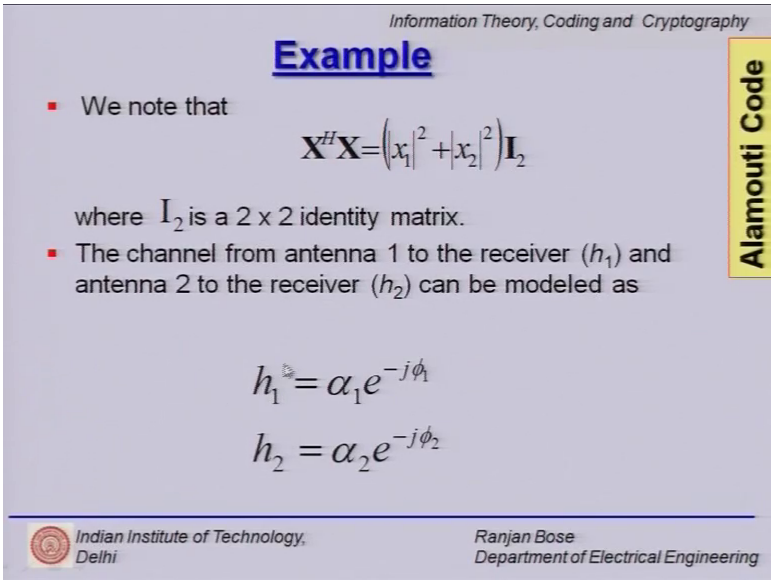

But now let us make some interesting observations. Why has it been chosen like this? What is so, great about putting a minus S 2 star and S 1 star here? So, the first observation is that X hermitian X, where X was given by this is just showing a general coding strategy. And if the exam symbols were S 1 and S 2 then coding strategy will put S 1 and S 2 in the case of x 1 and x 2. So, x represents the code and this is the code word matrix. So, X hermitian X is x 1 squared plus x 2 squared right absolute value squared into this identity matrix.

Now as we mentioned after we transmit the symbols they go through the wireless channel. So, the symbol S 1 gets a channel gain h 1 it is complex can be represented easily by alpha 1 e raised power 1 and h 2 is the channel gain encountered by symbol S 2 equal to alpha 2 e raised to power minus j phi 2.

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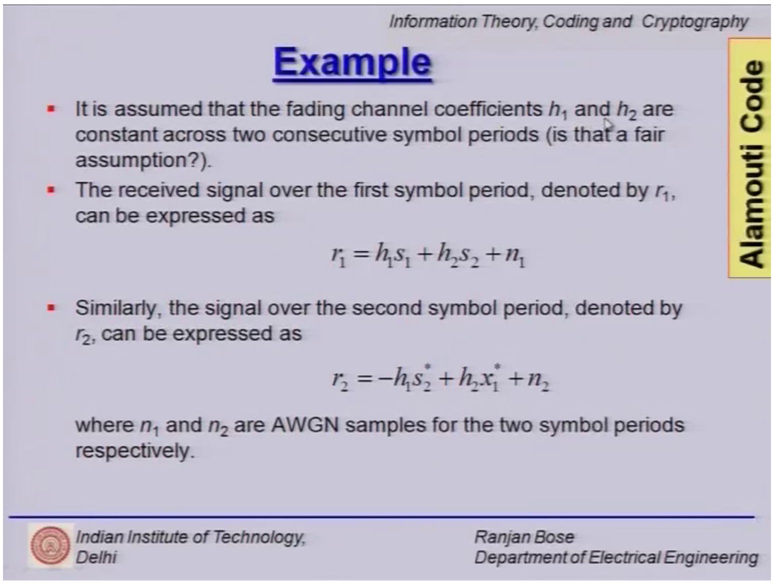

So, let us make some more assumptions it is assumed that the fading channel coefficients h 1 and h 2 are constant across 2 consecutive symbol periods; it is not a bad assumption at the data rates we have the symbol rates that we employ the channel does not change significantly between the 2 time slots. So, is that a fair assumption? Yes it is a fair assumption.

Now, the received signal over the first 2 symbol period denoted by r 1 can be represented as r 1 is h 1, s 1 plus h 2, s 2 plus n 1. Now if you look at the second symbol period right. So, this is time slot 1 because in the time slot 1 both transmit antenna are transmitting simultaneously. So, transmit antenna 1 sends S 1 transmit antenna 2 sends S 2 both get multiplied by the respective channel gains and in the first time slot I have got the noise n 1.

And similarly in the second time slot I have got noise n 2, but what has happened for the next time slot, the received signal that I get in the second time slot is I sent out from antenna 1 minus s 2 star and from antenna 2 s 1 star. So, this should be s 1 s 1 star and we have this r 1 and r 2.

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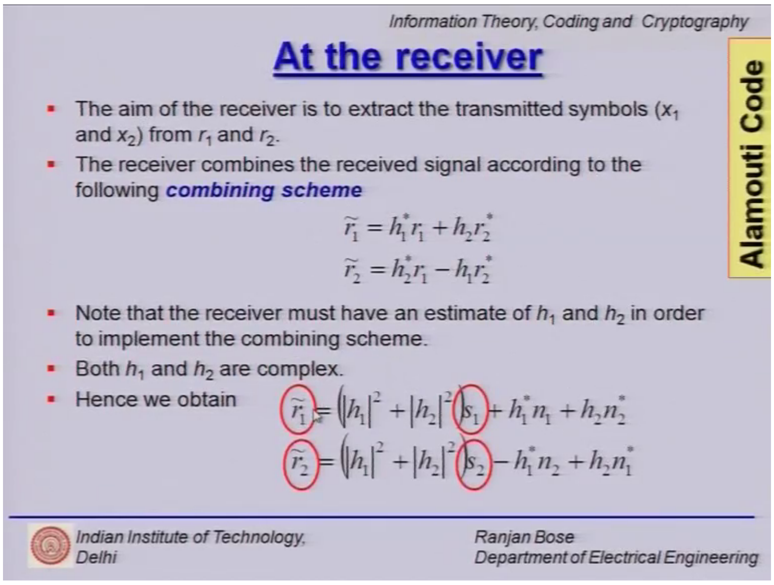

Now the aim of the receiver is to extract the transmitted symbols x 1 and x 2 from r 1 and r 2 that is the aim; so, the receiver requires some kind of a combining scheme. So, we have a strategy in place based on the channel estimates. So, we assume that we have some knowledge about the channel gains h 1 and h 2 and the combining scheme r 1 tilde is h 1 star r 1 plus h 2 r 2 star and r 2 tilde is equal to h 2 star r 1 minus h 1 r 2 star.

Now, what will all of this do? Because r 1 already we know is given by this and r 2 is given by this. So, this and complex conjugate would probably help us simplify things ok, but we make this observation that h 1 and h 2 is known. So, there is a channel estimator which tells us the channel gains and of course, h 1 and h 2 are complex. So, now, if you plug in the values of r 1 and r 2 let us see what we can get for our one tilde n r 2 tilde. So, r 1 tilde is if you do the math and it is not to complicate it you get h 1 absolute value squared plus h 2 absolute value squared into s 1 plus h 1 star n 1 plus h 2 n 2 star and similarly r 2 tilde has a expression available for it.

So, what does it bias? Well the first interesting observation is r 1 depends only on s 1. Earlier if you note everything was coupled the received signal clearly depended on s 1 and s 2; why? Both the transmitter antennas were sending together. So, it is, but obvious that you will get a jumble, you will get a mixture of s 1 and s 2 multiplied by the channel gains, but I do some smart processing at the receiver and suddenly some magic happens and this r 1 only depends on s 1.

Now we tell you the advantage of that and r 2 only depends on s 2 of course, I have the estimates of h 1 and h 2 and these are nothing, but some values multiplying s 1 and s 2, but this is the most interesting fall out of this mathematics that has come out; r 1 tilde depending only on s 1 r 2 tilde depending on only an s 2 this has big ramifications.

Because at the end my job is to guess what was transmitted s 1 and s 2 ok. So, we have to do a search because this is a maximum likelihood decoding and it is now a question of matter of reducing the search space.

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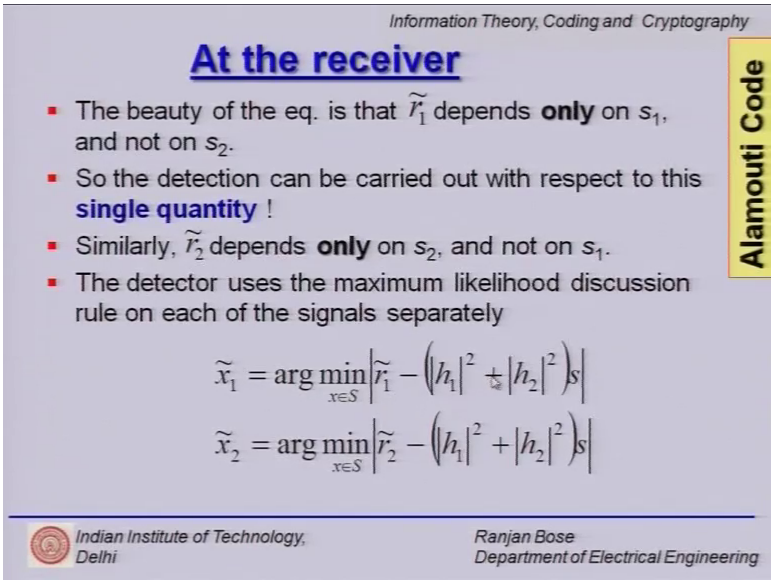

So, the beauty of the equation is that r 1 tilde depends only on s 1 and not on s 2. So, the detection can be carried out only with respect to the single quantity. So, when I going to search and find out which is the most likely symbol transmitted, I only do on the possible points in the constellation diagrams s 1. Similarly for s 2 I only do search only for s 2; if they were coupled then I have to have the number of searches s 1 and s 2 together. And that would require me to search if there are 64 points in the constellation diagram, then there the pairs number of pairs would be 64 into 64 as opposed to only 64.

So, that is the biggest advantage that we are going to get here; the detector users maximum likelihood decision and so, x 1 tilde is nothing, but this is the maximum likelihood r 1 tilde we just now minus this h 1 absolute value square plus h 2 squared s. Similarly x 2 estimate is minimum over all the possible constellations points r 2 tilde minus h 1 squared plus h to absolute value squared S ok, this is just the maximum likelihood ml decoding.

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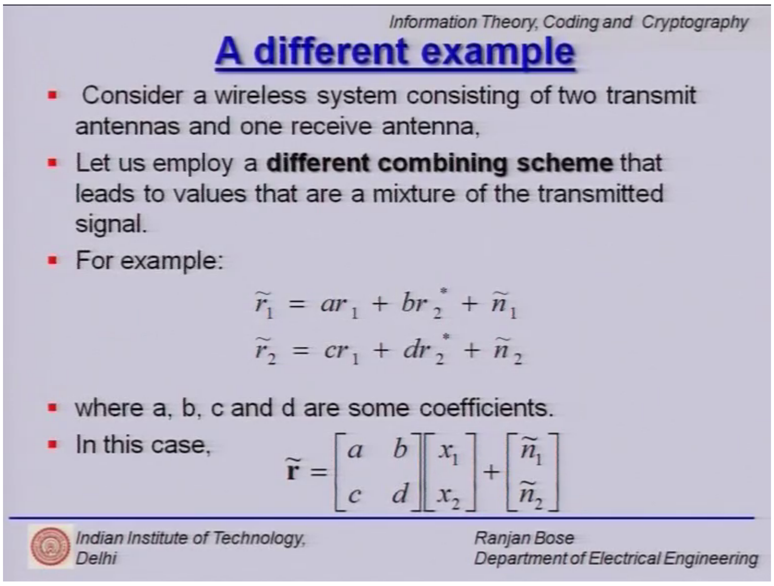

So, that is the beauty of the Alamouti scheme; we have really reduced the complexity the decoding complexity. Now will it work in general? Let us consider a slightly different example; again we have 2 transmit antennas and 1 receive antennas, nothing is different except that this time we employ a slightly different combining scheme and this will lead to a mixture of the transmitted signals.

So, for example, let r 1 tilde we sum ar 1 plus br 2 complex conjugate and r 2 tilde could be some c r 1 plus d r 2 star plus this noise part. So, if a b c and d are some coefficients then the vector r tilde can be expressed as this a b c d x 1 x 2 plus n 1 tilde n 2 tilde.

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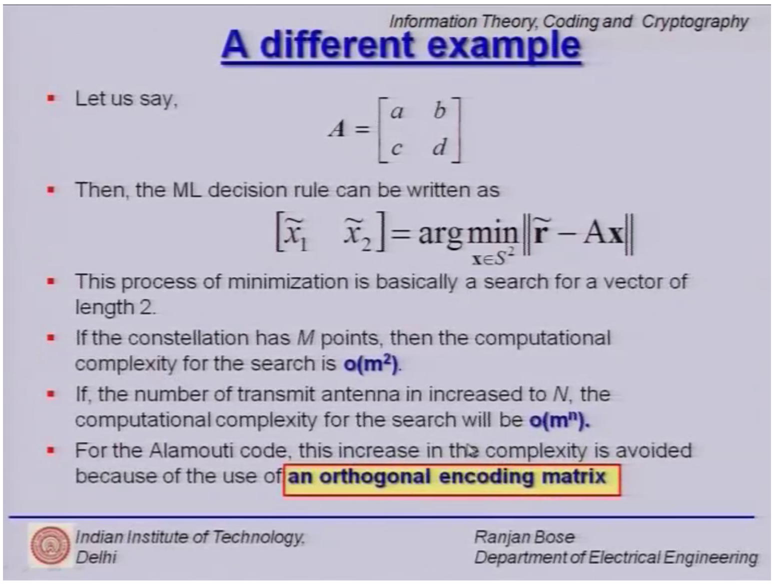

So, if a matrix can be represented as a b c d then the ml decision rule can be written x 1 tilde x 2 tilde is minimum over this vector x over S squared.

So, now this is the process of minimizing is a search for a vector of length 2 as opposed to a single symbol search. So, if the constellation has M points then the computational complexity of the search is of the order m squared. If the number of transmit antennas is increased to n the computational complexity of the search will be m raised power n. So, early we only talking about 2 therefore, I was sending symbol s 1 and s 2, if there were 3 transmit antennas then the search will be over a 3 tuple is there n transmit antennas in the search will over m raised power n.

So, this is the biggest problem the complexity will forbid the use of any general arbitrarily designed scheme. This A has to be designed very carefully, but for the Alamouti scheme that we saw this increase in complexity is avoided because of the use of an orthogonal encoding matrix. So, that was the beauty of the Alamouti code the coding matrix was orthogonal therefore, it could d couple the s 1 and s 2. So, this is the crux this is the reason why it is such an efficient decoding algorithm.

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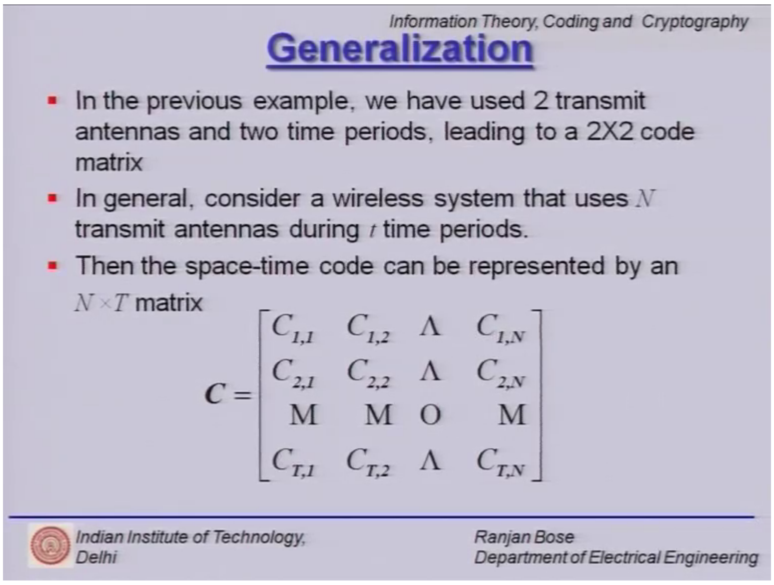

Now, in the previous example we have used 2 transmit antennas and 2 time periods leading to a 2 cross 2 code matrix right this is obvious space time. In general we can consider a wireless system that uses N transmit antennas, clearly the gain with increasing the number of transmit antennas is phenomenal. So, we have again a certain number of time periods and certain number of N transmit antennas.

Now if this and then the time slots is T, then we have an N cross T matrix which will represent the code just like when 2 transmit antennas and 2 time slots were there; so, N was 2 and times loss T were2; so, you had a 2 cross 2 matrix.

So, in general we will have this representation again this is the epsilons dot dot dot.

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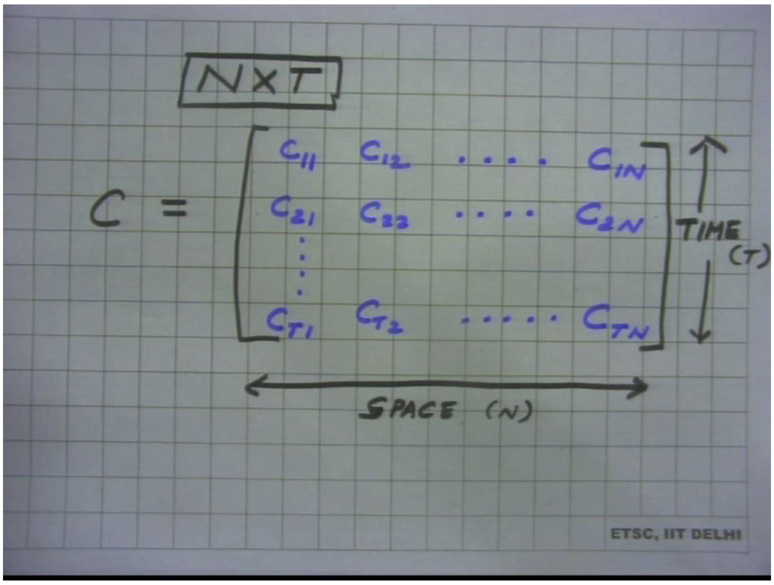

So, if we look at in general if you look at the code for this N cross T encoding matrix, then C will be written as C 11, C 12 to C 1N, then C 21, C 22, C 2 N and you can keep going and you have T time slots. So, T 1, C T 2, C T N and if you see here we have N. So, this is the space axis and this is a time axis right. So, this is space and you have N and this is T; so, this gives you the N cross T.

So, this is a general notation for a space time encoding matrix we can represent it.

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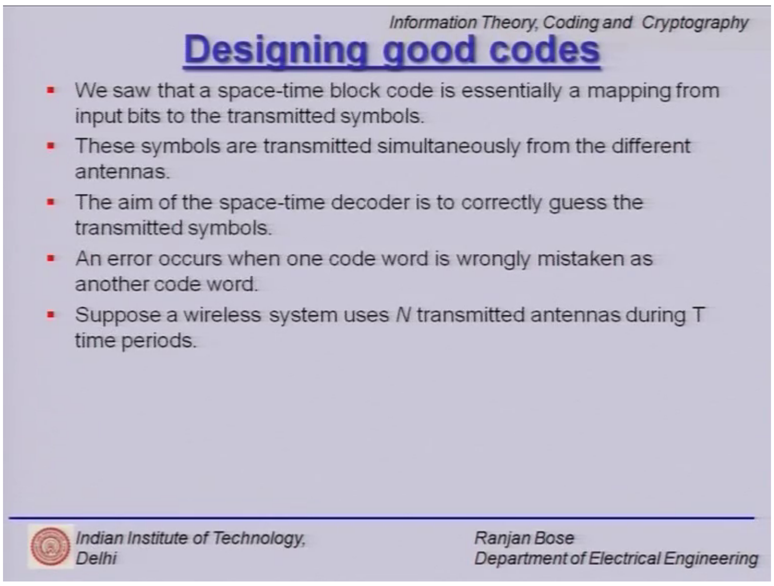

So, now the question is; what is a good space time block code? How do we design one? We need some design rules. So, we saw that a space time block code is essentially a mapping from input bits to a transmitted symbols. So, we have to play the game there, these symbols are transmitted simultaneously from the different antenna elements ok.

And therefore, they couple at the receiver; the aim of the space time decoder is to correctly guess the transmitted symbols. And error occurs when one code word is wrongly mistaken as another code word. So, suppose the wireless system use uses N transmitted antennas and T time periods, then the code word we have already looked at looked at as C 1.

Now the error is set to be the decoder is set to make an error if it decides that a different code word was indeed transmitted. So, even though C 1 was transmitted we say no it is a C 2 that is sent out. So, if you now look at; so, we already have C 1 which was sent out.

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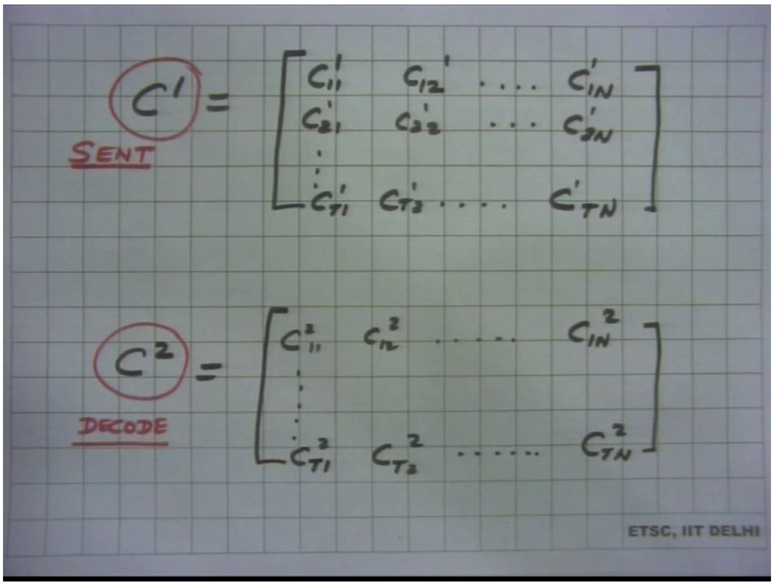

C 11 C 12 so and so forth; C 1N C 21, C 22 up to C 2 N and going on for C T 1, C T 2 up to C T N, this one was transmitted, but what we received is C 2. So, we can differentiate it as C 2 11, c and so and so forth as opposed to what was sent. So, I can put a superscript here and I can distinguish these 2.

. So, this C 1 was sent and this is what we decode and wrongly. So, there is an error in the decoding. So, let us look at the slides again.

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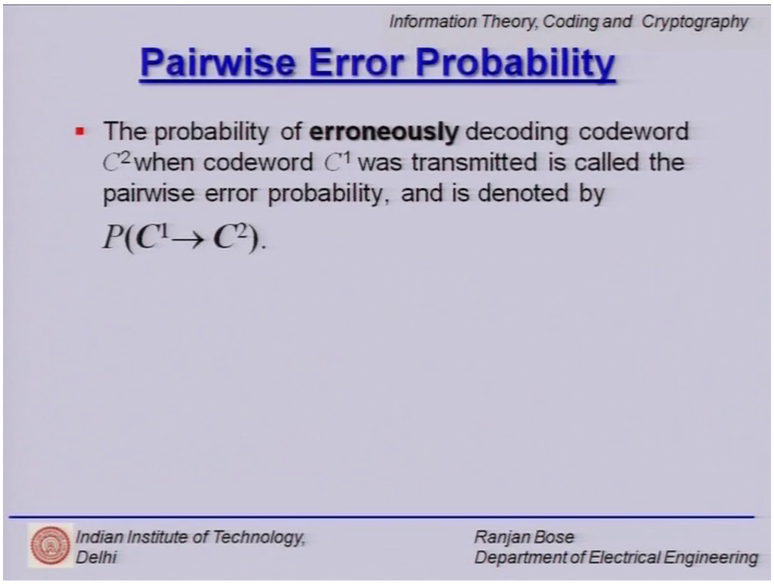

The probability of erroneously decoding code word C 2 when it is coded with C 1 was transmitted it is called the pair wise error probability. So, this is denoted by P C 1 was sent and C 2 received and this is the notation we will be using.

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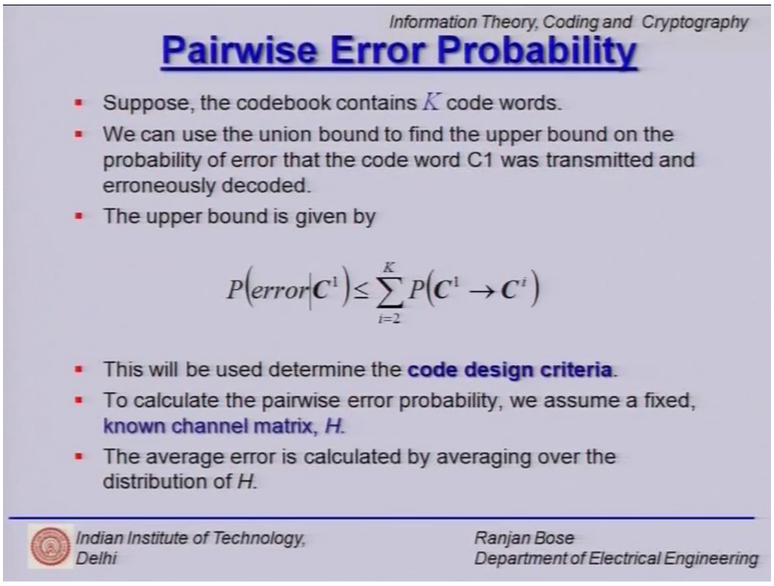

Now, suppose the codebook contains K code words. So, we can use union bind to find the upper bound on the probability of error that the code word C 1 was transmitted erroneous decoded. And the union bound is simply the probability of error given C 1 is less than all possible. So, i is equal to 2 to K because i is equal to 1 represents the correct decoding, C 1 goes to C i. So, this pair wise error probability the upper bound on that will be used for our code design criteria.

So, to calculate the pair wise error probability; we assume a fixed known channel matrix H. So, the average error is calculated by averaging over the distribution of H. So, for different H we will have a different kind of a average error calculation.

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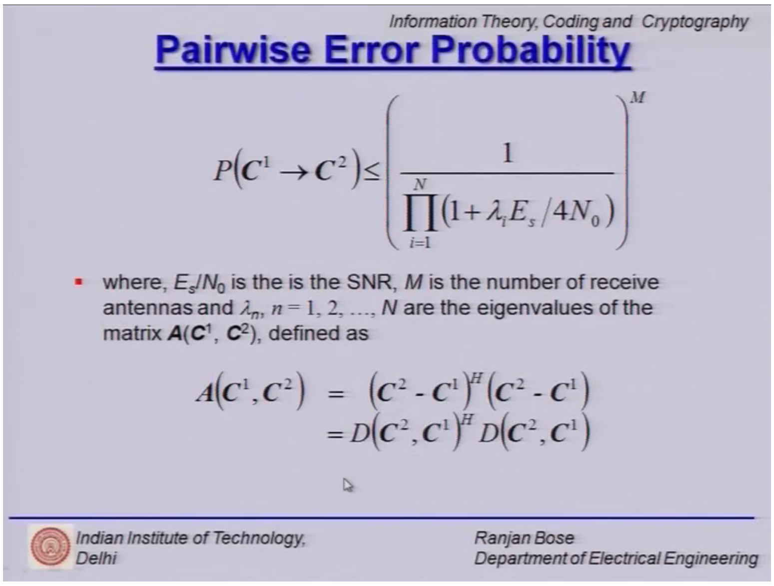

So, after some mathematics we can show that the pair wise error probability is upper bounded by this expression ok. Now here N of course, is a number of transmit antennas right and we have M as a number of receive antennas. So, we have a MIMO system in place and lambda n; n is equal to 1, 2, 3 up to N are the eigenvalues of the matrix A C 1 comma C 2. Now what is this matrix? Let us define this. So, this matrix will play a central role in our design criteria. So, A C 1 comma C 2 is defined as this C 2 this was a matrix minus C 1 this was the sent matrix hermitian into C 2 minus C 1. So, do we define it like this ok.

So, what is this C 2 minus C 1? It is like the distance, but remember what are the elements of this matrix? Well these are symbols in the constellation diagram. So, it is nothing, but a distance; so we represent it with D; so, this is the distance between C 2 and C 1; 1 2 1 hermitian and distance between C 2 and C 1; so, this is a definition of this A matrix.

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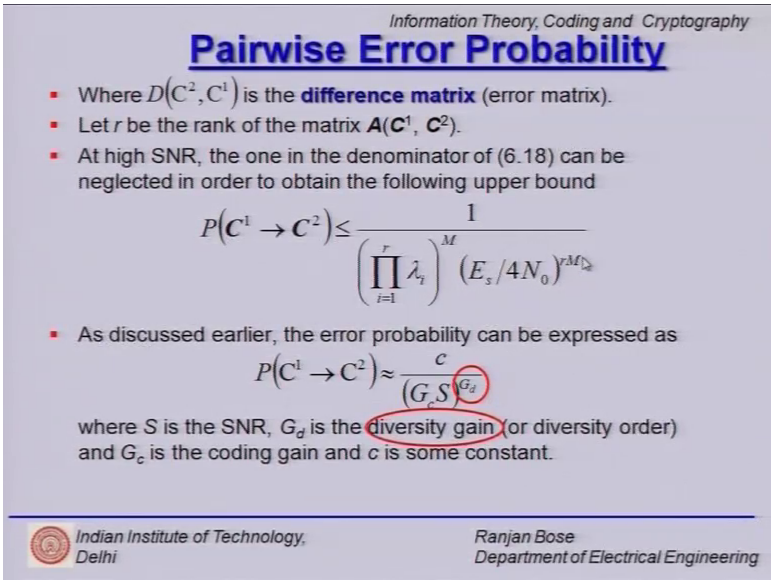

So, and this D is the difference matrix. Now, let us talk about the rank of the matrix A right; so, at high SNR what we can do is you can neglect the previous equation the denominator and so, the pair wise error probability is upper bounded by 1 over; this are the eigenvalues product M and this is the indicator for SNR; E S is energy per symbol and power noise power is denoted here and raised power r M, M is the number of receive antennas, small r is the rank rank of this matrix A.

So, if you recall we had earlier said to the pair wise error probability; if represented in general as follows some constant over G c S raised power G d. Then G d is the diversity order of the diversity gain this term diversity gain G d and G c is of course, the coding gain alright. So, if you compare these 2; we can quickly see that there is this r into M corresponding to G d right because S is the SNR here S is energy per symbol over for N naught. So, this is an indicated for the S; SNR and G d is r M.

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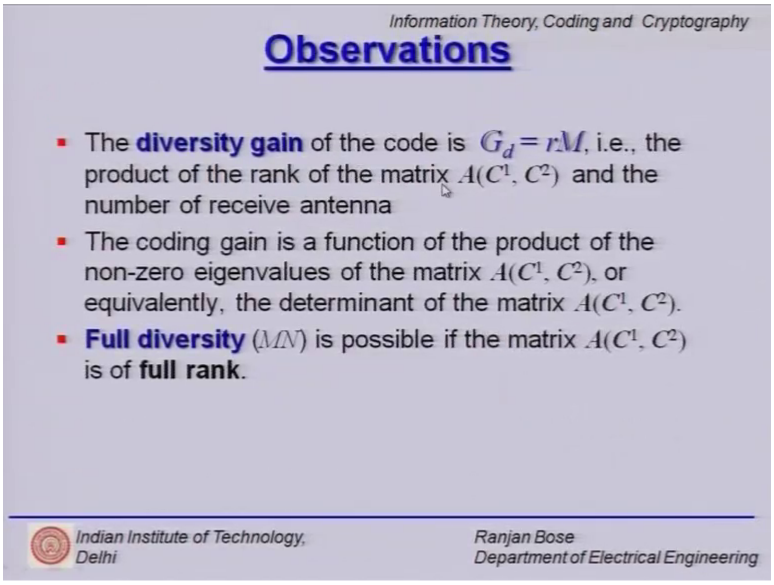

So, the diversity gain of the code G d is r M, but what is r? r is the rank of the matrix A C 1 comma C 2. So, it appears that this matrix plays a critical role and we have to design the C 1 such that this rank is maximized. What is M? M is the number of receive antennas; earlier M was 1, in that case this diversity gain would be limited to r right.

So, the coding gain is a function of the product of the nonzero eigenvalues of the matrix A, this one where is this coming from? We go back and see that this G c provides the coding gain. So, this term is essentially the coding gain and it depends on the eigenvalues of this A matrix very interesting.

So, the coding gain is a function of the product of the nonzero eigenvalues the matrix A or equivalently the determinant of the matrix A, which means the full diversity is possible and it will be M into N the matrix A is of full rank.

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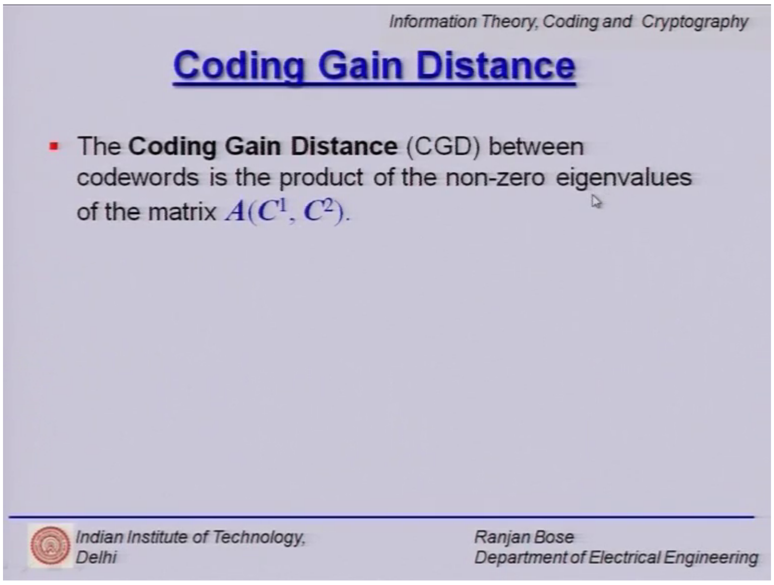

So, we have to design this essentially gives us the method to design good space time block codes. We have to design A with full rank, coding gain we have studied earlier also, but we revisit it.

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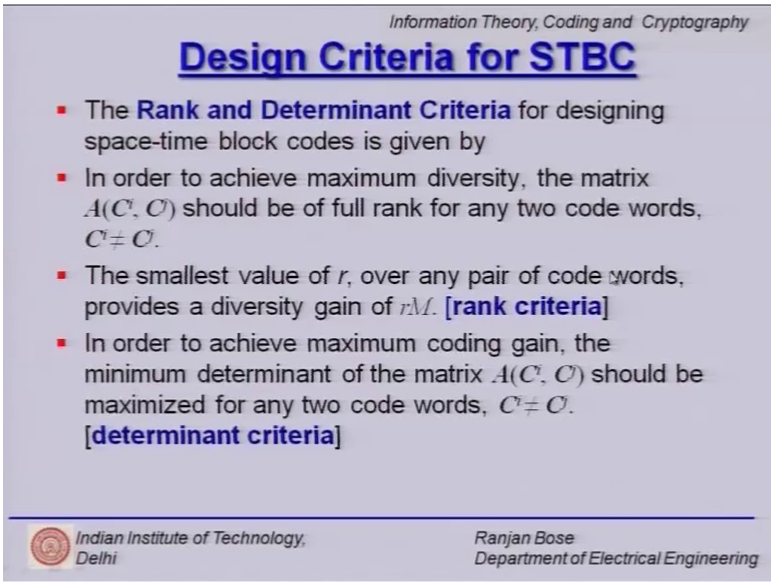

So, the coding gain distance between codeword is the product of the nonzero eigenvalues of the matrix A. So, we see from this expression that the block space time block codes can provide you definitely with the diversity gain and coding gain and depending upon how you have designed this matrix A.

So, let us now summarize these design criteria for the space time block codes. So, of course, 2 things played an important role the rank and the determinant. So, we talk about the rank and the determinant criteria for designing space time block code and what is it? In order to achieve maximum diversity and diversity is so, important in we will take up an example to show the importance.

In order to achieve maximum diversity, the matrix A should be full rank for any 2 code words C i not equal to C j alright. And the smallest value of r over any pair of code words provides a diversity gain of r times M; this is called the rank criteria M being the number of received antennas.

Now in order to maximize the coding gain ok; so, we have already talked about the diversity gain. In order to maximize the coding gain, the minimum determinant of the matrix A should be maximized for any 2 code words C i not equal to C j. So, this is talking about the determinant and hence it is called the determinant criteria. So, together they are called the rank and determinant criteria for designing good space time block codes ok.

So, we can actually do a simple computers search to look at all possible combinations and whichever gives this high rank and the maximizes the minimum determinant right we get that.

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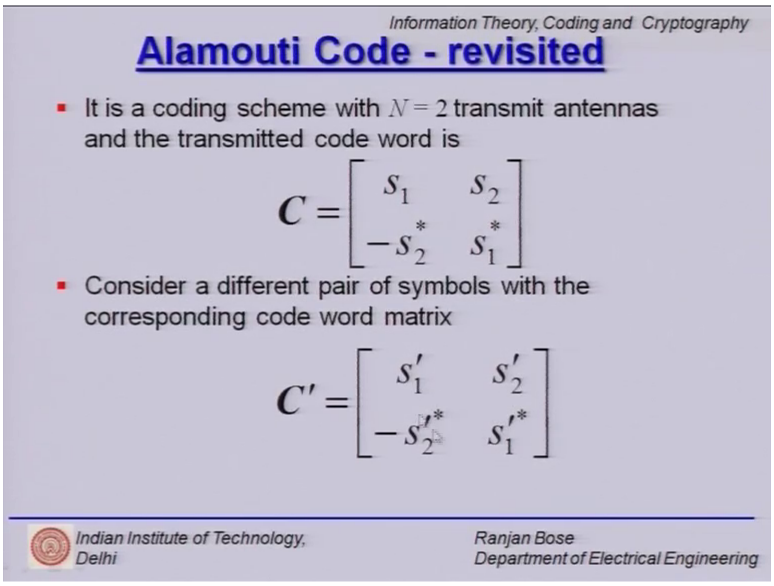

So, let us revisit Alamouti code in this slide; so, we have seen that there are 2 transmit antennas right; so, N equal to 2 and the code word we have seen earlier.

Now, let us consider a different pair of symbols with the corresponding code word matrix C prime alright.

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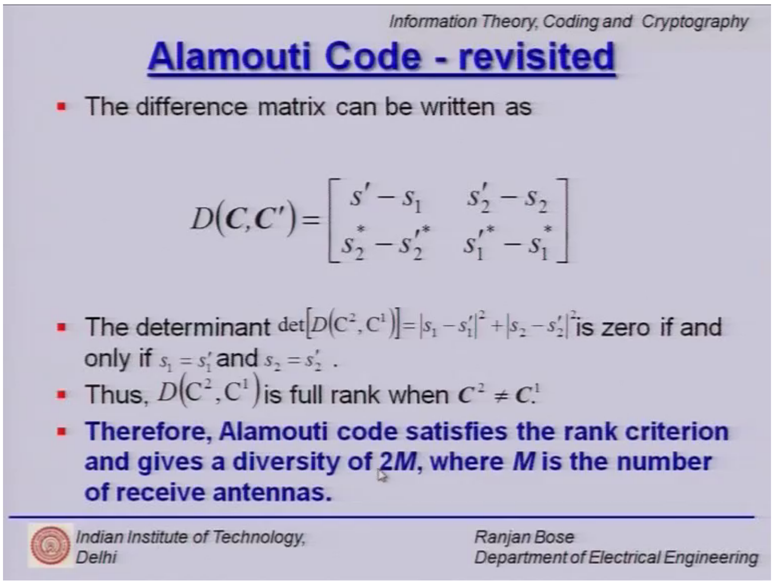

So, the difference matrix is nothing, but C 1 minus C prime. So, if you find out the determinant of this is given by this and this is 0 if and only if S 1 is equal to S 1 prime and S 2 is equal to S 2 prime in all other cases the determinant will be nonzero. Consequently this difference matrix D is full rank right, when C 2 is not equal to C 1. So, that is the beauty of the design; it is a full rank.

So, Alamouti code satisfies the rank criteria and gives a diversity of 2 M, where M is the number of receive antennas. Earlier we had talked about M equal to 1 for Alamouti code; so, only one receive antenna. So, at least it will give a diversity gain of 2, I will just now see what do we mean by a diversity gain of 2 is it good does it help.

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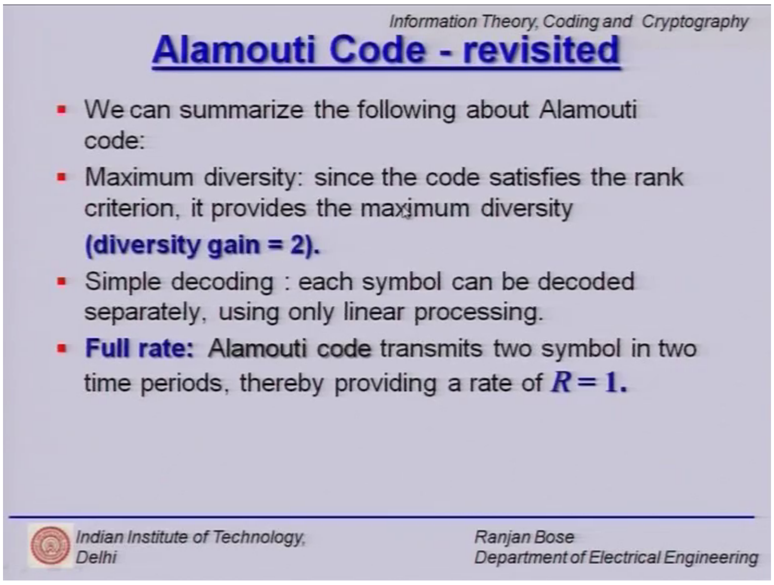

So, let us understand a little bit more about the Alamouti code; it was one of the milestones. So, maximum diversity since the code satisfies the rank criteria, it provides the maximum possible diversity of 2 when the number of receive antennas is 1. So, diversity gain is 2 it provides symbol decoding; so, single symbol decoding, it is simple each symbol can be decoded separately using a linear processing this we have established because of its orthogonal nature.

And then it is full rate that is another interesting criteria what do we mean by full rate? Well we transferred it to symbols into 2 2 time slots right; so, effectively we are sending 1 symbol per time slot; so, the rate is 1. So, we are not saying that we are slowing down the communication, we did not take more time, we did not slow down the symbol rate ok, we did not increase the decoding complexity, but at the same time we got a diversity gain of 2.

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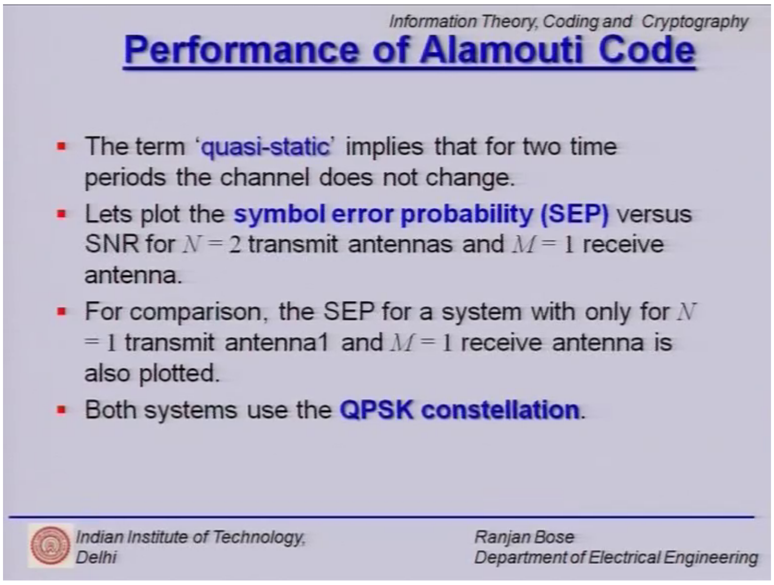

So, let us quickly look at the performance of Alamouti code. So, we have made this assumption of quasi static implying that 2 time periods are such that the channel does not change significantly; so, H value is maintained.

So, let us plot the symbol error probability versus the SNR for N is equal to 2 transmit antennas an M is equal to 1 receive antenna. So, we had already studied this scheme in detail the Alamouti scheme and let us see how the symbol error probability curve looks like, but how do we compare, did we gain anything? So, we consider the case when it was a single antenna system. So, N is equal to 1 right; so, we plot that curve also and compare how much is a Alamouti giving us let us say both of them are using this QPSK constellation.

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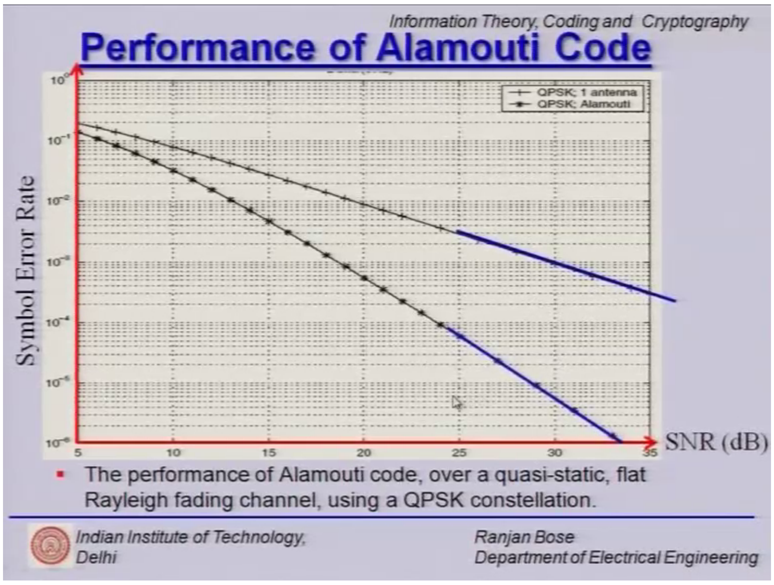

So, let us plot the axis; so, on the x axis we are going to plot SNR, on the y axis will go to plot the symbol error rate. So, if you do so if you see the stuff curve is the single antenna transmit antenna and single receive antenna case, the second curve is Alamouti scheme; both employing this QPSK constellation. And if you can see that there is a shift in the slope, the gradient of this changes indicating the diversity gain and also please note that these 2 diverge, which means as we go at a higher SNR regime the diversity gain increases.

So, it brings home the very interesting point that a diversity gain becomes more and more effective as we go to higher and higher SNRs. Therefore, if you are working with systems which inherently work in low SNRs scenarios, it is really not worth it to look at schemes that provide diversity, diversity gain is best employed when reasonably good SNR is available ok.

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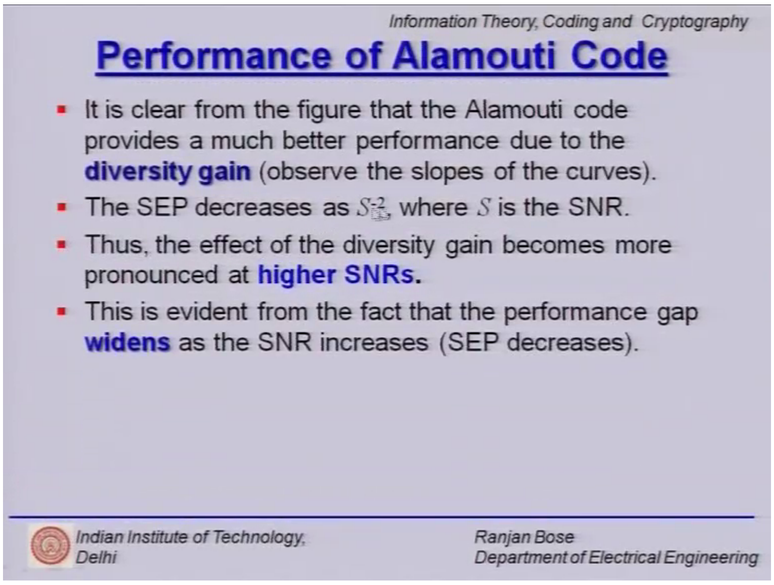

So, from the figure we observe that the symbol error probability decreases S is power minus 2 where S is the SNR which confirms that the diversity gain is of the order 2 and the effect of the diversity gives becomes more pronounced at higher SNRs and it as it is evident because the performance gap widens as SNR increases. And if you look at the slopes of the 2 curves alright, you will see that the slope is the indicator of the diversity gain that we get and typically asymptotically you can see.

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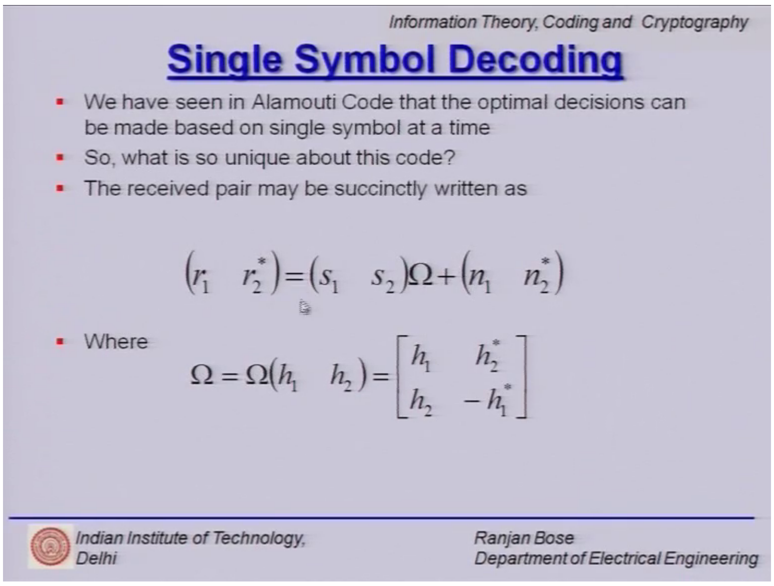

Now just a few more comments about this single symbol decoding and then we will try to explain intuitively why diversity happens, why diversity gain is happening? So, we have talked about this Alamouti code and the optimal decisions can be made based on single symbols at a time. And what we have is this r 1, r 2 star pair can be simply written as S 1, S 2 times omega; what is that? Well omega is this h matrix h 1, h 2 right n 1, n 2. So, we have just repositioned these parameters and we have rewritten the received pair as follows.

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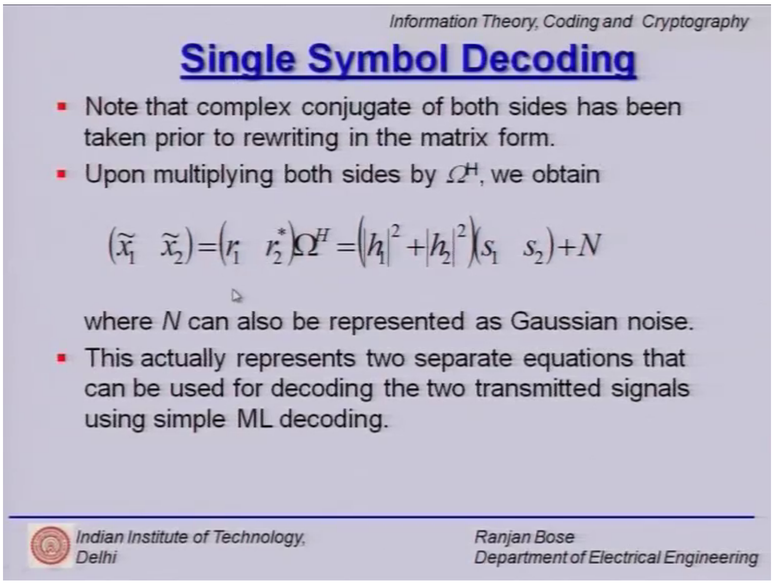

So, if you see we have this r 1, r 2 star that we had rewritten and if you multiply it with the hermitian of this omega; then you simply get h 1 absolute value squared plus h 2 absolute value squared times S 1, S 2 n. So, you can see that upon multiplying both sides with this omega h herniation; you decouple the decoding part. So, it is effectively 2 separate equations that can be solved separately ok. So, that is the reason why it is a single symbol decoding.

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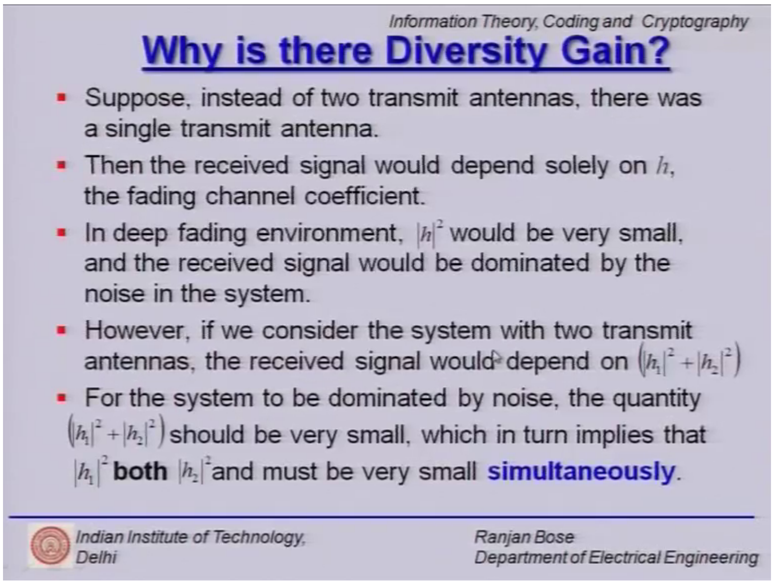

Now, let us spend just 2 more minutes on why there is diversity gain intuitively; mathematics we saw slope of the curve we saw, simulation results we saw, but intuitively why are we getting? What is the physical reason? Can we link this mathematics to any physical intuition?

So, let us suppose that instead of 2 antennas there was only 1, then the received signal would depend only on the h the fading coefficient which can vary drastically and what does h do? When the fading is bad the value of h is small. So, h squared is even small right and then the noise will dominate because the signal is just out of the picture, but if we have 2 transmit antennas the receiver signal depends as we have seen on h 1 squared plus h 2 squared we have seen this therefore, here see the received signal pair depends on h 1 squared absolute value and h 2 absolute value squared right.

Now, when will the system be dominated by noise here is the noise. The system here will be dominated by noise if both h 1 and h 2 are small; only then this signal will become insignificant with respect to noise and system becomes dominated by noise both of them not only both of them both of them must simultaneously be small. If one is small the other is large, but the signal is still there both of them must simultaneously drop down, but that is bad because h 1 is for channel 1, h 2 is for channel 2.

And somewhere we said these guys are supposed to be independent, they are independently fading. So, the probability that both h 1 and h 2; together simultaneously very small is very rare and that is where we are getting the diversity gain.

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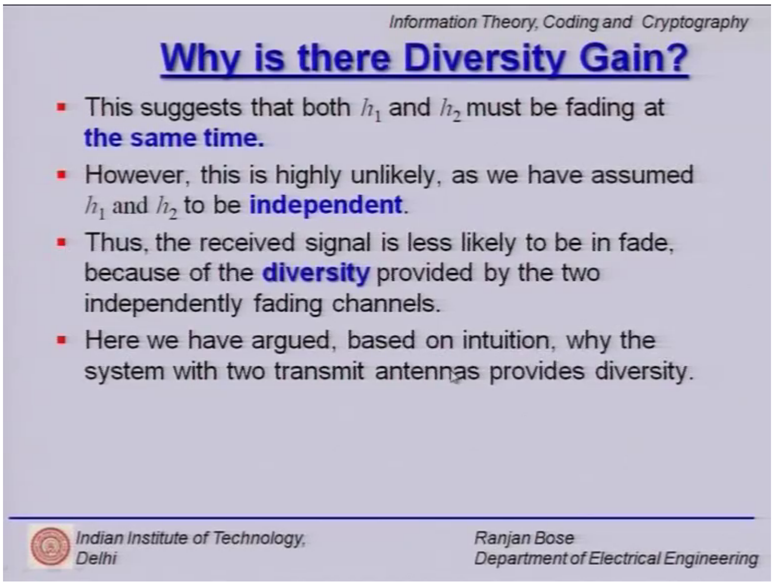

So, if noise must dominate then h 1, h 2 must be fading at the same time, but it is contrary to our assumption that h 1 and h 2 are fading independently. This intuitively the received signal is less like to likely to be in fade because of the diversity provided by the 2 independently fading channels. So, thus we have argued purely based on intuition why the system with 2 transmit antennas provides diversity ok. So, there is a very physical feel to the whole thing.

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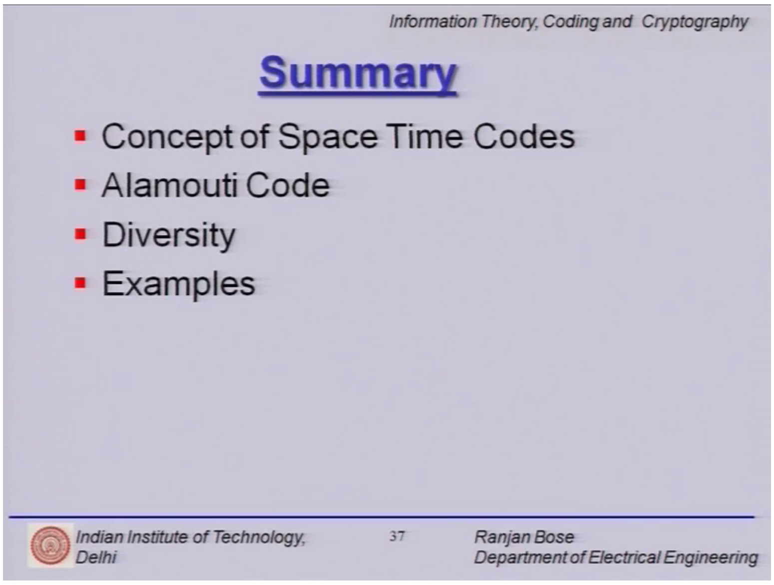

So, let us summarize what we have learned today; we have introduced this concept of space time codes and we brought it out with the help of this wonderful coding scheme called Alamouti code. We looked at why Alamouti code works and we also looked at this rank and determinant criteria, for designing good space time block codes. Then we spent some time talking about diversity and intuitively explained why diversity happens in this multi antenna systems; we also had some examples.

So, with that we come to the end of this lecture.