

Information Theory, Coding and Cryptography
Dr. Ranjan Bose
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Module - 30
Lecture - 30
Trellis Coded Modulation

Hello and welcome to our lecture on Trellis Coded Modulation; let us start with the brief outline for today's talk.

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Information Theory, Coding and Cryptography

Outline

- Combined Coding and Modulation
- Trellis Coded Modulation
- Free distance
- Ungerboeck's design rules
- Examples

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We would introduce this interesting notion of combining coding and modulation; so we will look at combined coding and modulation, then we will move on to the idea of trellis coded modulation, we will then characterize the free distance and come up with interesting idea proposed by Ungerboeck in terms of his design rules; of course we look at some examples as we go along.

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So far ...

- Block Codes
- Convolutional Codes
- Turbo Codes

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So, so far what have we done? We have looked at block codes, linear block codes, we look we have looked at cyclic codes. BCH codes, then we changed gears and looked at codes with memory and we looked at convolutional codes and turbo codes and now we move on to a completely different area where we look at coding and modulation combined together.

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Trade-off: Bandwidth and Performance

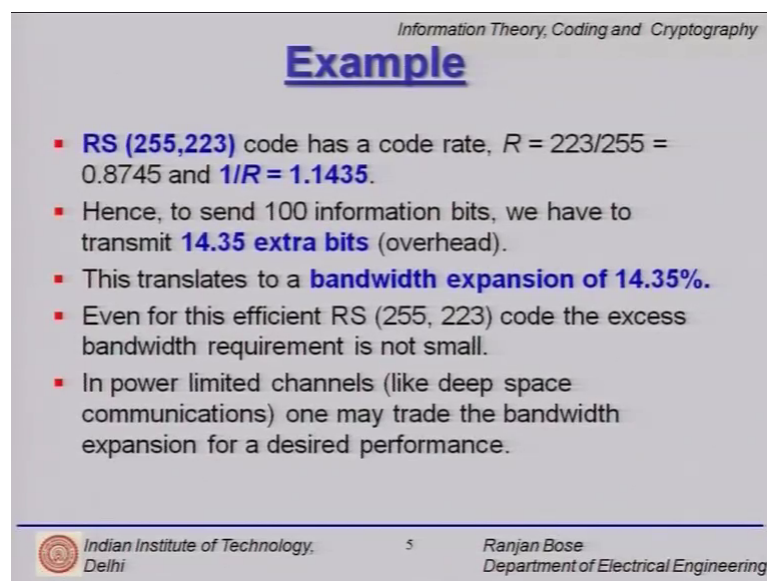
- In error control coding techniques, extra bits are added to the information bits in a **known** manner.
- However, the improvement in the bit error rate is obtained at the **expense of bandwidth**, because of these extra bits.
- This bandwidth expansion is equal to the reciprocal of the code rate.
- For example, a RS (255,223) code has a code rate, $R = 223/255 = 0.8745$ and $1/R = 1.1435$.

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But why do we do that? So first let us look at the tradeoff between bandwidth and performance; we know that in all error control coding techniques extra bits are added to the information bits in a known manner therefore, the receiver can decode it.

But this is not free additional bits come at the expense of additional bandwidth and as we know bandwidth is not cheap, the bandwidth expansion is equal to the reciprocal of the code rate for example, if you have the Reed Solomon 255 comma 223 code then there is a $1/R$ equal to 1.14 so as a 14 percent expansion.

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Example

- **RS (255,223)** code has a code rate, $R = 223/255 = 0.8745$ and $1/R = 1.1435$.
- Hence, to send 100 information bits, we have to transmit **14.35 extra bits** (overhead).
- This translates to a **bandwidth expansion of 14.35%**.
- Even for this efficient RS (255, 223) code the excess bandwidth requirement is not small.
- In power limited channels (like deep space communications) one may trade the bandwidth expansion for a desired performance.

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Now, this tantamounts to for a every 100 information bits we have about 14.35 overhead bits on an average.

So, a 14.35 percent expansion of bandwidth this is non-trivial even for an efficient code like Reed Solomon 255 comma 223. So in power limited channel one may trade the bandwidth expansion for the desired performance, but for bandwidth limited channels we may not have this luxury. So can we do better for example, telephone lines are bandwidth limited and if we want to push data over telephone lines we need to work more smartly.

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The slide is titled "Coding and Modulation" in a large, bold, blue font. Above the title, in a smaller font, is the text "Information Theory, Coding and Cryptography". Below the title, there are three bullet points, each starting with a red square. The first bullet point states that traditionally, coding and modulation are considered as two separate parts of a digital communications system. The second bullet point explains that the input message stream is first channel encoded (with extra bits added) and then converted into an analog waveform by the modulator. The third bullet point states that the objective of both the channel encoder and the modulator is to correct errors resulting from a non-ideal channel. At the bottom of the slide, there is a footer containing the IIT Delhi logo, the text "Indian Institute of Technology, Delhi", the number "6", and the name "Ranjan Bose, Department of Electrical Engineering".

Information Theory, Coding and Cryptography

Coding and Modulation

- Traditionally, coding and modulation have been considered as two **separate** parts of a digital communications system.
- The input message stream is first channel encoded (extra bits are added) and then these encoded bits are converted into an analog waveform by the modulator.
- The **objective of both the channel encoder and the modulator is to correct errors** resulting due to the non-ideal channel.

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So, let us look at coding and modulation; how can we talk about coding and modulation on the same slides. Well traditionally coding and modulations have been considered as two separate parts of a digital communication system; you first do coding and then follow it up by modulation right. First the input stream is channel coded and then we convert into an analog waveform by the modulator, but if you look at it more carefully the objective of both the channel encoder and the modulation is to correct errors and this is because of the imperfections in the channel, so both of them are doing the same job.


If you look at an error control code, we find out what is the probability of residual error; if we look at the characteristic of a modulator we always plot the BER versus the SNR curve; the standard waterfall curve, so again we are talking about bit error rate.

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Coding and Modulation

- Both these blocks (the encoder and the modulator) are **optimized independently** even though their objective is the same, that is, to correct errors introduced by the channel!
- As we have seen, a higher performance is possible by lowering the code rate at the cost of bandwidth expansion and increased decoding complexity.
- It is also possible to obtain coding gain **without** bandwidth expansion if the channel encoder is integrated with the modulator.

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So, now both these blocks are optimized independently even though it seems their objective is the same. Now a high performance is possible by lowering the code rate at the cost of the bandwidth expansion and increased decoding complexity that is what we have learned so far.


So, if you need a stronger code, you have to have a poorer code rate because that is how you will be able to recover from more number of errors. Now the question is; is it possible to obtain this coding gain without the bandwidth expansion, is there a free lunch, can we get something out of nothing; that is the question we are asking whether both coding and modulation can be looked at together to give you this gain without additional bandwidth requirement.

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Example

- Consider data transmission over a channel with a throughput of 2 bits/s/Hz.
- One possible solution is to use **uncoded QPSK**.
- Another possibility is to first use a rate $2/3$ convolutional encoder (which converts 2 uncoded bits to 3 coded bits) and then use an 8-PSK signal set which has a throughput of 3 bits/s/Hz.
- This coded 8-PSK scheme yields the same information data throughput of the **uncoded QPSK** (2 bits/s/Hz).
- Note that both the QPSK and the 8-PSK schemes require the **almost the same bandwidth**.
- But we know that the symbol error rate for the 8-PSK is worse than that of QPSK for the same energy per symbol.
- However, the $2/3$ convolutional encoder would provide some **coding gain**.

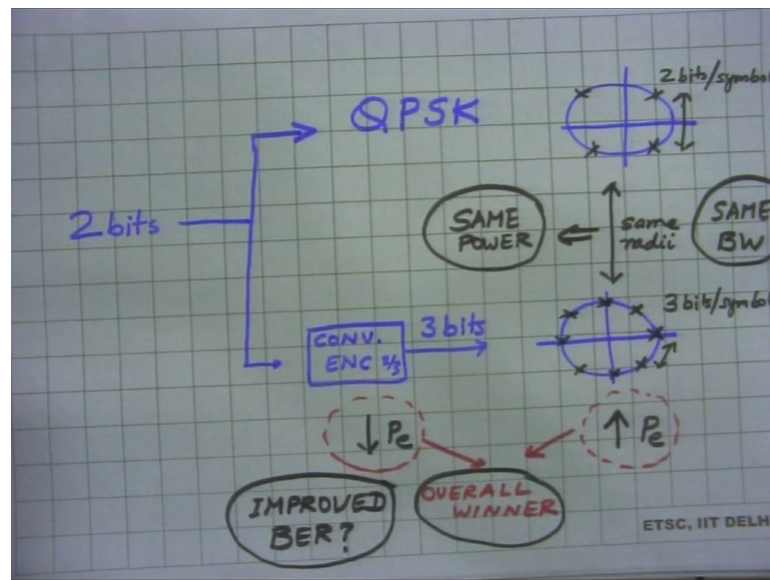
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So, let us understand this with a simple example; suppose we are looking at data transmission over a channel of throughput 2 bits per second per hertz ok. So the moment I see 2 bits per second per hertz my mind jumps to uncoded QPSK; where we have 2 bits per symbol, but we say wait a minute we can do this differently we first use a 2 by 3 rate convolution encoder and it takes the 2 bits of raw data and converts into 3 bits of coded data, and then 3 bits have to be transported and we use now an 8-PSK because I need 3 bits per symbol because I do not need additional bandwidth.

So, this coded 8-PSK schemes yield the same information data throughput as the uncoded QPSK which was sending 2 bits per second per Hertz. We assume that QPSK and 8-PSK are consuming roughly the same bandwidth it is not a bad assumption, but we know that the symbol error rate for 8-PSK is worse off than that of QPSK for the same energy per symbol, but on the other hand the 2 by 3 convolution encoder has been able to provide some coding gain.

So, if you look at it graphically, this is what we did, we had this 2 bits coming in now if you look at these 2 bits these 2 bits can be either sent using QPSK where you know there is a constellation diagram and we have these 4 possible symbols in the constellation diagram and you have got 2 bits per symbol.

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But we say no wait a minute this 2 bits can be passed through a convolution encoder which is rate 2 by 3, so 2 bits go in and 3 bits come out.

So, what do I do I have to modulate and send so I need a a higher order modulation scheme so I need probably an 8-PSK so if you look at it this is my 8-PSK this is the constellation diagram. Here I have got 3 bits per symbol please note that this radius of the circle is proportional to the energy per symbol; so we do not want additional power so the radii are the same; so the same radii implies that this circle has these symbols closer together as opposed to the distance.

So, the Euclidean distance goes down, the Euclidean distance going down may lead to an increase in probability of error, but you already have an error control code put into place whose job is to reduce the error, so this block is going to reduce the probability of error. So on one side we have a mechanism that tries to push down the error the other side may lead to increase of error.

So, if you play our cards right then we can come out an overall winner; so this trade off will decide whether we finally, end up having a better BER. Please note we have not touched power, so same power and here these two modulation schemes we have not really asked for additional bandwidth so we have same bandwidth. Now with the same power and same bandwidth do we have an improved BER that is the question; did we really gain something out of nothing.

We know that from Shannon's theorem you give me more power I will give you improved BER, you give me more bandwidth I will give you an improved BER, but I ask for no additional power, no additional bandwidth and am I going to get an improved BER; so the answer is yes and let us see how we can do it. So we go back to our slide and what we have just mentioned it could be possible that the coding gain provided by the encoder outweighs the performance loss because of the 8-PSK signal set as we are looking in our slides.

Now if the coded modulation scheme performs superior to the uncoded one at the same SNR we can claim that an improvement is achieved without sacrificing either data rate or bandwidth.

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The slide is titled "Example" and is part of a presentation on "Information Theory, Coding and Cryptography". It contains four bullet points discussing coding gain, SNR, and Trellis Coded Modulation (TCM). The slide footer includes the IIT Delhi logo, the name "Ranjan Bose", and his department "Department of Electrical Engineering".

Information Theory, Coding and Cryptography

Example

- It could be possible that the **coding gain** provided by the encoder outweighs the performance loss because of the 8-PSK signal set.
- If the coded modulation scheme performs superior to the uncoded one at the same SNR, we can claim that an improvement is achieved **without** sacrificing either the data rate or the bandwidth.
- In this example we have combined a trellis encoder with the modulator.
- Such a scheme is called a **Trellis Coded Modulation (TCM)** scheme.

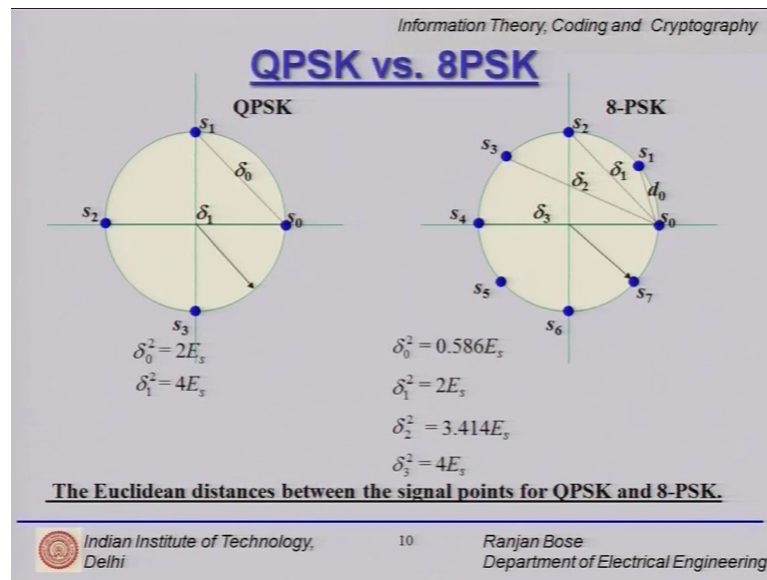
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So please note we did not sacrifice data rate in either case we got those 2 bits per second through the channel. So in this example we have combined a Trellis encoder with a modulator and hence we have looked at combined coding and modulation; this scheme is called a Trellis Coded Modulation or TCM for short.

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So, let us look at QPSK and 8-PSK a little bit more carefully; if you see that both the circles are of the same radius which means that energy per symbol is the same. Now we also know that the probability of error depends on the Euclidean distance, here the smallest Euclidean distance between s_1 and s_0 will decide what is the error rate. So I put δ_0 as the first distance if the radius of the circle is under root E_s then the distance between s_1 and s_0 δ_0 squared would be $2E_s$ and similarly the second possible distance between s_2 and s_0 will be $4E_s$.


On the other hand if you look at 8-PSK the distances are shorter, so the smallest distance possible δ_0 is $0.586E_s$, where E_s represents the energy per symbol and the radius of the circle is under root E_s . Here δ_1 squared is between s_0 and s_2 same as δ_0 squared in QPSK, but we have 2 other distances δ_2 and δ_3 listed here. So, this smaller distance leads to a higher error for 8-PSK.

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Observations

- We observe that the **expansion of the signal set** in order to provide redundancy results in the shrinking of the Euclidean distance between the signal points, if the average signal energy is to be kept constant.
- This reduction in the Euclidean distance **increases the error rate** which should be compensated with coding (increase in the Hamming distance), if the coded modulation scheme is to provide an improvement.
- We also know that the use of a **hard-decision demodulation** prior to decoding in a coded scheme causes an irreversible loss of information.
- This translates to a **loss of SNR**.

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So, we make some observations, we observe that the expansion of the signal set in order to provide redundancy results in the shrinking of the Euclidean distance; this leads to an increase in the error rate which must be compensated with the coding gain; that is the increase in the Hamming distance. We also know that the note that the use of hard decision demodulation prior to decoding in a coded scheme causes an irreversible loss of information. So, these are the observations we make and finally, this hard decision decoding leads to loss of SNR.

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Information Theory, Coding and Cryptography

Observations

- For coded modulation schemes, where the expansion of the signal set implies a power penalty, the use of **soft-decision decoding is imperative**.
- As a result, **demodulation and decoding should be combined** in a single step, and the decoder should operate on the *soft* output samples of the channel.
- For **maximum likelihood decoding using soft-decisions**, the optimal decoder chooses that code sequence which is nearest to the received sequence in terms of the Euclidean distance.
- Hence, an efficient coding scheme should be designed based on **maximizing the minimum Euclidean distance** between the coded sequences rather than the Hamming distance.

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Some more observations so for coded modulation schemes where the expansion of the signal set implies power penalty, use of soft decision decoding is imperative ok; as a result demodulation and decoding should be combined. So not only the modulation and coding is combined the demodulation and decoding itself is combined as the receiver side. For the maximum likelihood decoding using soft decision the optimal decoder chooses that sequence which is nearest to the received sequence this time in terms of the Euclidean distance and not the Hamming distance that we saw in the last time.

So, what is an efficient coding scheme? It should be based on maximizing the minimum Euclidian distance between coded sequence rather than the Hamming distance.

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Free Euclidean Distance

- **Definition** The minimum Euclidean distance between any two paths in the trellis is called the **free Euclidean distance**, d_{free} of the TCM scheme.
- Due to **linearity** of convolutional codes, the minimum free distance in terms of Hamming weight could be calculated as the minimum weight of a path that deviates from the all zero path and later merges back
- However, in order to calculate the free Euclidean distance for a TCM scheme, which is **non-linear**, all pairs of paths have to be evaluated.

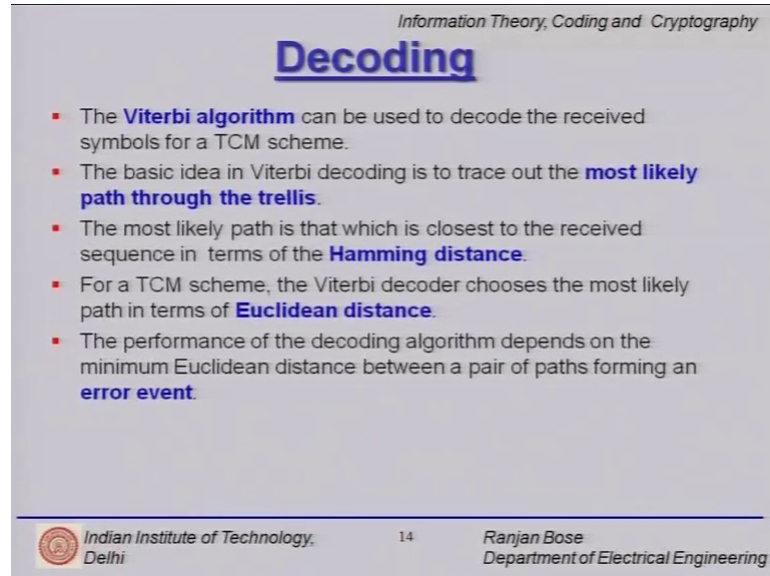
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We have looked at free distance earlier in terms of convolution codes we define the free Euclidean distance; so the minimum Euclidian distance between any 2 paths in the trellis is called the free Euclidean distance d_{free} of the TCM scheme. Please note here since there is a convolution encoder present they will be a trellis diagram, but the labels of the branches instead of being bits should be symbols because, all the bits have been mapped to symbols.

Now, you must remember in convolution codes we had the linearity constrained; so we were working with linear codes we could take the all 0 paths and find out the d_{free} with respect to the all 0 path. Here is a word of caution for TCM; TCM is non-linear therefore, in order to find the d_{free} one must look at all possible pairs within the trellis

diagram ok. We are looking at non-linear codes this mapping mapping of the bits to the symbols introduces the non-linearity.

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The slide is titled "Decoding" and is part of a presentation on "Information Theory, Coding and Cryptography". It contains a bulleted list of five points regarding the Viterbi algorithm and TCM schemes. The footer includes the Indian Institute of Technology Delhi logo, the slide number 14, and the name of the speaker, Ranjan Bose, from the Department of Electrical Engineering.

Information Theory, Coding and Cryptography

Decoding

- The **Viterbi algorithm** can be used to decode the received symbols for a TCM scheme.
- The basic idea in Viterbi decoding is to trace out the **most likely path through the trellis**.
- The most likely path is that which is closest to the received sequence in terms of the **Hamming distance**.
- For a TCM scheme, the Viterbi decoder chooses the most likely path in terms of **Euclidean distance**.
- The performance of the decoding algorithm depends on the minimum Euclidean distance between a pair of paths forming an **error event**.

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So let us look at the decoding; please note there is a viterbi algorithm associated with it simply because we have a trellis diagram which is used for encoding. So we do the decoding using the viterbi algorithm which we have studied already in the context of convolutional codes. What does the viterbi decoding do? It looks for the most likely path through the trellis, but this time instead of finding out the most likely path closest to the received sequence in terms of Hamming distance we should be using the Euclidean distance for TCM.

The performance of the decoding algorithm depends on the minimum Euclidean distance between the pairs of paths forming an error event. What is an error event? First the paths diverge and then they merge back ok.

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Information Theory, Coding and Cryptography

Example

- Consider the convolutional encoder followed by a modulation block performing natural mapping ($000 \rightarrow s_0, 001 \rightarrow s_1, \dots, 111 \rightarrow s_7$)
- The **rate** of the encoder is $2/3$.
- It takes in two bits at a time (a_1, a_2) and outputs three encoded bits (c_1, c_2, c_3).
- The three output bits are then mapped to one of the eight possible signals in the **8-PSK** signal set.

▪ The TCM scheme

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So we come back to our slide; we have done all of so let us look at this example now, let us consider a simple example where if you see a trellis encoder in the beginning it takes in 2 bits a_1 and a_2 and gives out 3 coded bits c_1, c_2, c_3 .


But the moment I get these 3 bits I use a natural mapping on 8-PSK and it gives me a symbol. So if you look at the combined coding and modulation 2 bits lead to 1 symbol; what is natural mapping? It is 000 is s_0 , 001 bit means transmit s_1 , these are the symbols 010 , transmit s_2 up to 111 means transmit s_7 symbol from the constellation diagram. The rate is clearly $2/3$ and it takes in 2 bits and converts it into 3 bits and the constellation is 8-PSK.

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Information Theory, Coding and Cryptography

Example

- This combined encoding and modulation can also be represented using a trellis with its branches labeled with the output symbol (s_j).
- This is a fully connected trellis. Each branch is labeled by a symbol from the 8-PSK constellation diagram.
- In order to represent the symbol allocation unambiguously, the assigned symbols to the branches are written at the front end of the trellis.
- The convention is as follows. Consider state 1.
- The branch from state 1 to state 1 is labeled with s_0 , branch from state 1 to state 2 is labeled with s_7 , branch from state 1 to state 3 is labeled with s_5 and branch from state 1 to state 4 is labeled with s_2 .

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So, if you see that this combined encoding and modulation can be represented using a trellis diagram, but this time we label the branches with the output symbols and this is a fully connected trellis where each branch is labeled by the symbols from 8-PSK and encoding is a simple process as we had learnt earlier; 1 comes in you take the upper branch, 0 comes in take the lower branch and you read out what is written on top of the branch.


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Information Theory, Coding and Cryptography

Example

- So, the 4-tuple $\{s_0, s_7, s_5, s_2\}$ in front of state 1 represents the branch labels emanating from state 1 in sequence.
- To encode any incoming bit stream, we follow the same procedure as for convolutional encoder.
- However, in the case of TCM, the output is a sequence of symbols rather than a sequence of bits.
- Suppose we have to encode the bit stream
101110001001

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So, if you have to look at the trellis diagram you just take in the input bit stream which tells you which path in the trellis you have to take.

Let us take a simple example of a streak sequence of bits to be encoded using the trellis coded modulation scheme, say the bit stream is 1 0 1 1 1 0 0 0 1 0 0 1 this has to be encoded using the trellis coded modulation scheme.

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Corresponding Trellis

State 0: s_0, s_7, s_5, s_2

State 1: s_6, s_2, s_0, s_7

State 2: s_3, s_4, s_6, s_1

State 3: s_6, s_1, s_3, s_4

8-PSK

- The path in the trellis corresponding to the input sequence 10 11 10 00 ...
- As in the case of convolutional encoder, in TCM too, every encoded sequence corresponds to a unique path in the trellis.
- The objective of the decoder is to recover this path from the trellis diagram.

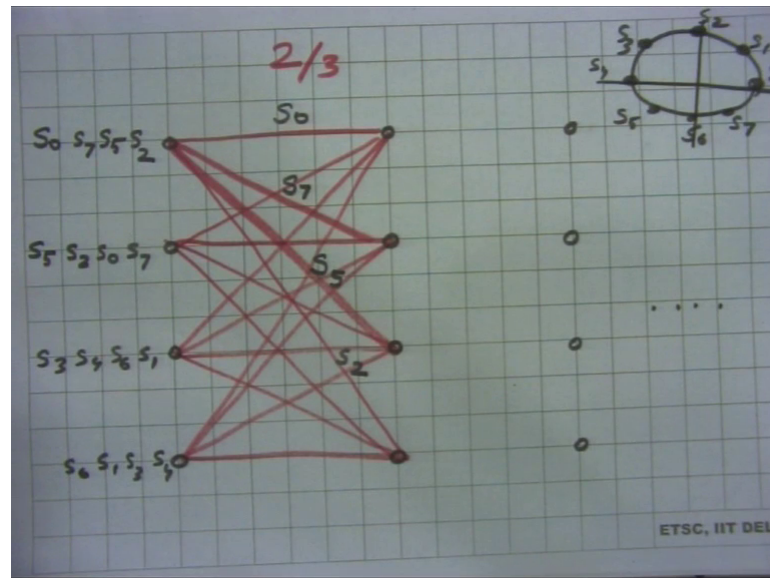
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So, here is the corresponding trellis for the TCM schemes; now please note that instead of labeling each branch and it becomes a little confusing we write the symbols here so s_0 corresponds to the first branch label, s_7 corresponds to the second branch, s_5 third and s_2 fourth. So, let us look at it graphically here so we draw this trellis diagram.

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There are 4 states in the trellis, because if you see you have 2 memory units leading to 4 states and it is a fully connected trellis which means that each node is connected to every other node and so and so forth. Now for any bit that comes in and this is a rate 2 by 3; so 2 bits come out so they have possible 4 possible transitions 0 0 comes in take this branch, 0 1 take this branch, 1 0 take this branch, 1 1 take the last branch.

What happens once you take the branch you get an output bit, but you encoded using and then you map it back using the 8-PSK natural mapping. So the mapping strategy that we have given is s 0 for 1 what is this s 0 well if you look at your 8-PSK these are your symbols; so this is your s 0 this is your s 1 so this s 0 is this s 0, but suppose you go for this transition then you label it with s 7. This is a design that we have come up with I could have labeled it something else, but we are not yet looking at the design criteria and then the third branch is s 5, the fourth branch is s 2.

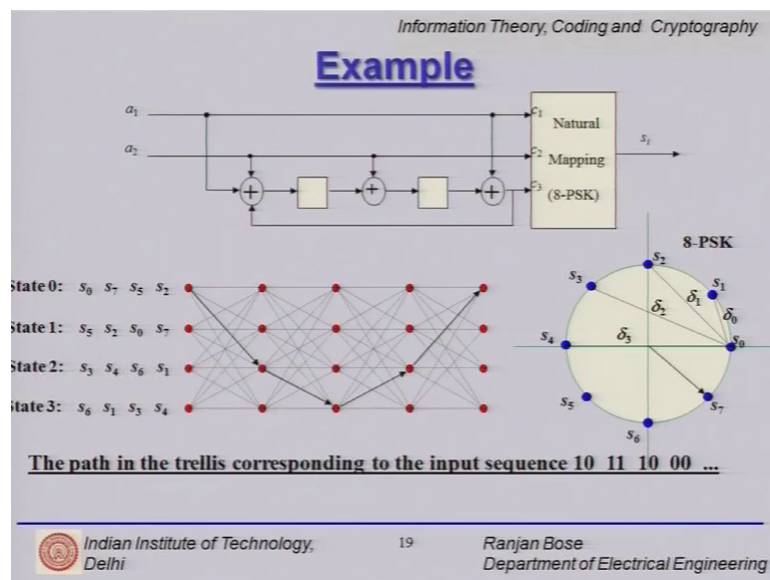
If I start labeling all of them it gets a little congested so we can say this is s 0 s 7 s 5 and s 2 so this is the convention. Similarly for the next 1 I can write s 5 s 2 s 0 s 7 of course, it will be easy to start connecting them and labeling and so and so forth. And similarly the third one and the fourth one and this repeats, so we just fix it and write out the trellis s 4 s 6 s 1 similarly s 6 s 1 s 3 and s 4 right.

So, we can always write a trellis diagram in a compact form; so let us look at the slides again and if you see this fully connected trellis then we must now find out a way to

encode the path in the trellis. So it says the bits input bit stream is 1 0, 1 1, 1 0 and 0 0 in the slide. So what we do is look at the first 2 bits since encoder and decoder are friends we know that 2 bits are encoded at one time and the encoder then takes 1 0 means takes the third path then 1 1 take this path and then so this basically gives which is the direction to take which sequence of bits will define that a certain path in the trellis must be traversed.

And what we do is simply read out what is written off on top of each of the branches. The objective of the decoder again is to recover this particular path from the trellis diagram.

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So, here is in a nutshell all of it if you see this is the hardware followed by the mapping, so I can describe my TCM scheme as a diagram with a circuit implementation and a mapping or equivalently I can define my trellis using the states and a trellis diagram they are the same you give me 1 and I will give you the other.


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Path in the Trellis

- We first group the input sequence in pairs because the input is two bits at a time.
- The grouped input sequence is
 $10\ 11\ 10\ 00\ \dots$
- The TCM encoder output can be obtained simply by following the path in the trellis as dictated by the input sequence.
- The first input pair is 10.
- Starting from the first node in state 0, we traverse the third branch emanating from this node as dictated by the input 01.
- This takes us to state 2.

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So, if you look at the path in the trellis this slide tells me how to go from the input sequence to the path on the trellis and the encoder takes 2 bits at a time and follows the consequent paths in the trellis.


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Path in the Trellis

- The symbol output for this branch is s_5 .
- From state 2 we move along the fourth branch as determined by the next input pair 11.
- The symbol output for this branch is s_1 .
- In this manner, the output symbols corresponding to the given input sequence is
 $s_5, s_1, s_3, s_3 \dots$

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And reads out the corresponding symbol written on top of that branch so this slide basically tells you that first branch has s_5 written on it so read out s_5 second branch has written s_1 on it so read out s_1 s_3 and s_3 and so on and so forth.

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Finding the free distance

- The free Euclidean distance, d_{free} of the TCM scheme can be found by inspecting all possible pairs of paths in the trellis.
- The two paths that are separated by the minimum squared Euclidean distance (which yields the d_{free}) are shown with bold lines in the trellis diagram given in Fig. 7.4.

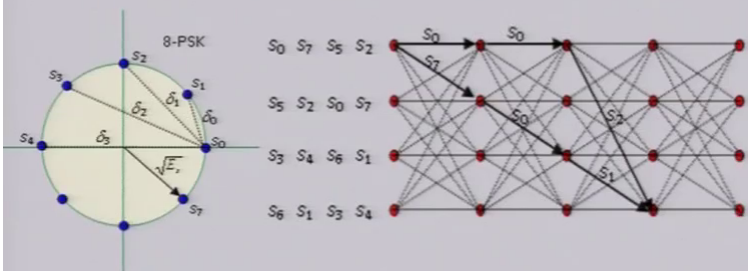
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So, now we go to the job of finding the free distance; we know that free distance is the single most important parameter which describes how good this trellis code is. So the free Euclidean distance of the TCM scheme can be found by inspecting all possible pairs of path in the trellis not just from the all 0 path this is because we are dealing with a non-linear code.


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Finding the free distance



The two paths in the trellis that have the free Euclidean distance, d_{free}^2

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So let us look at this diagram and this is the same trellis coded modulation scheme described earlier here after a lot of inspection we have been able to identify 1 pair of path which leads to the minimum distance called the free distance.

So we have $s_0 s_0 s_2$ and $s_7 s_0 s_1$ that they have been shown using bold lines and clearly the distance is not coming from the all 0 path. In fact, the all 0 path does not lead to the calculation of d_{free} here at all.

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Information Theory, Coding and Cryptography

Finding the free distance

$$d_{free}^2 = d_E^2(s_0, s_7) + d_E^2(s_0, s_0) + d_E^2(s_2, s_1)$$

$$= \delta_0^2 + 0 + \delta_0^2 = 2 \delta_0^2 = 1.172 E_s$$

- It can be seen that in this case, the error event that results in d_{free} does not involve the all zero sequence.
- As mentioned before, in order to find the d_{free} , we must evaluate **all possible pairs of paths in the trellis**.
- Its **not sufficient** just to evaluate the paths diverging from and later merging back into the all zero path because of the non-linear nature of TCM.

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So, if you add those free distances, so it is Euclidean distance so if you see first we will take the distance between s_0 and s_7 plus between s_0 and s_0 plus distance between s_2 and s_1 , but what distances are we adding? We are adding the Euclidean distances and therefore, we take the squared Euclidean distance because we can add the squared Euclidean distance overall.


So, we take the distance between s_0 and s_7 square Euclidean distance, square Euclidean distance between s_0 and s_0 and squared Euclidean distance between s_2 and s_1 . If you add them up you get answer equal to $1.172 E_s$; what is E_s ? Well the radius of the circle is under root E of s so it can be seen that in this case the error event that results in the d_{free} does not involve the all 0 path in the trellis and we looked at all possible pairs in the trellis to find out d_{free} not just the all 0 path.

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Information Theory, Coding and Cryptography

Coding Gain

- The difference between the values of the SNR for the coded and uncoded schemes required to achieve the same error probability is defined as the **coding gain**, g .
$$g = SNR|_{\text{uncoded}} - SNR|_{\text{coded}}$$
- At high SNR, the coding gain can be expressed as
$$g_{\infty} = g|_{SNR \rightarrow \infty} = 10 \log \frac{(d_{\text{free}}^2 / E_s)_{\text{coded}}}{(d_{\text{free}}^2 / E_s)_{\text{uncoded}}}$$
- where g_{∞} represents the **asymptotic coding gain** and E_s is the average signal energy.
For uncoded schemes, d_{free} is simply the minimum Euclidean distance between the signal points.

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Now, we define something called as coding gain the difference between the values of the SNR for the coded and the uncoded schemes required to achieve the same error probability is called the Coding Gain. What is this gain? This g is SNR uncoded minus SNR coded we had seen this definition earlier also. So at high SNR this asymptotic coding gain g_{∞} where SNR tends to infinity is defined as $10 \log$ to the base 10 d_{free}^2 over E_s of coded scheme versus d_{free}^2 over E_s of uncoded scheme this is called the Asymptotic Coding Gain.

So, for uncoded scheme d_{free} simply the minimum Euclidian distance between the signal points, so uncoded is in the denominator.

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Information Theory, Coding and Cryptography

Example

- Consider the TCM scheme discussed earlier.
- If we were to send uncoded bits, we would employ QPSK.
- The d_{free} for the uncoded scheme (QPSK) is $2E_s$.
- We have $d_{free} = 1.172E_s$ for our TCM scheme.
- The asymptotic coding gain is then given by

$$g_{\infty} = 10 \log \frac{1.172}{2} = -2.3 \text{ dB.}$$

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So, let us look at the TCM scheme we considered earlier, we had calculated d_{free} as $1.172 E_s$ for our scheme, but for QPSK the distance between any 2 signal points is $2 E_s$; so g_{∞} is $10 \log 1.17$ divided by 2 equal to minus 2.3 dB; which means that we have actually lost out the gain provided by the encoder function encoder did not more than enough compensate the loss because of movement to 8-PSK. So, we did not kind of win the game we can do better.

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Information Theory, Coding and Cryptography

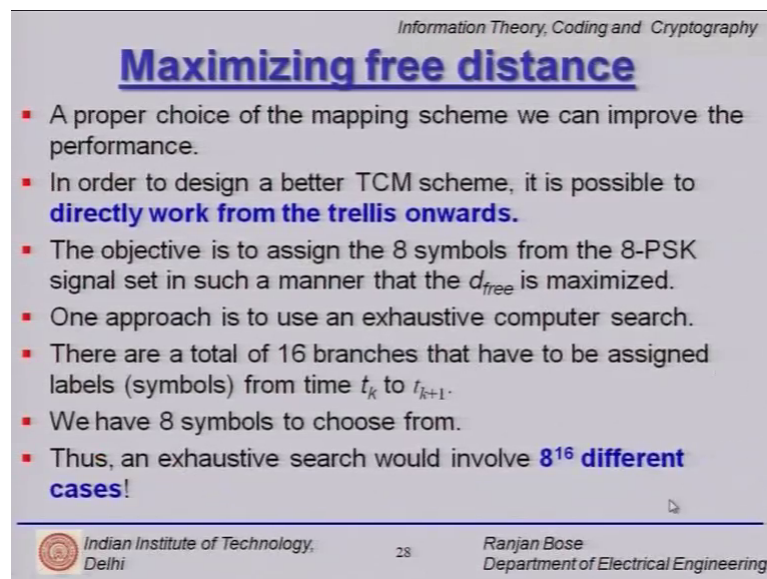
Example

- This implies that the performance of our TCM scheme is actually *worse* than the uncoded scheme.
- A quick look at the convolutional encoder used in this example suggests that it has good properties in terms of Hamming distance.
- In fact, it can be verified that this convolutional encoder is optimal in the sense of maximizing the free Hamming distance.
- **However, the encoder fails to perform well for the case of TCM.**
- This illustrates the point that TCM schemes must be designed to **maximize the Euclidean distance** rather than the Hamming distance.

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And that is what we have to look at so our performance is actually worse than the uncoded scheme, so it is a 2 edged sword and it tells us that we have not we have used the good properties of the encoder in terms of Hamming distance, but not the Euclidean distance. So the aim of the design of good TCM scheme is to maximize the Euclidean distance; so we did not do well in terms of TCM; so we must now look at maximizing the Euclidean distance.

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Information Theory, Coding and Cryptography

Maximizing free distance

- A proper choice of the mapping scheme we can improve the performance.
- In order to design a better TCM scheme, it is possible to **directly work from the trellis onwards.**
- The objective is to assign the 8 symbols from the 8-PSK signal set in such a manner that the d_{free} is maximized.
- One approach is to use an exhaustive computer search.
- There are a total of 16 branches that have to be assigned labels (symbols) from time t_k to t_{k+1} .
- We have 8 symbols to choose from.
- Thus, an exhaustive search would involve **8^{16} different cases!**

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So how do we do that; well a proper choice of mapping scheme can improve the performance and it is better to design the TCM scheme working from the trellis itself because finally, it is a d_{free} that we have to maximize and d_{free} is calculated using the trellis so why not work with that trellis itself. So the objective is to assign 8 symbols of the 8-PSK signal set in such a manner that d_{free} is maximized ok. Now in the last example there were 16 branches because there were 4 states 4 branches were coming out from each.

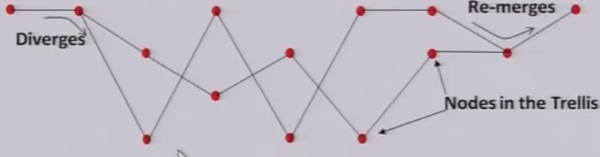
So, there were 16 branches we have to choose 8 symbols; so how many ways we can do it for an exhaustive search well 8^{16} different cases that is just too large even a supercomputer, network supercomputer, distributed computing cannot solve that so we need better heuristic methods to solve it. A computer search will not work why are we making this point; for convolution encoders efficient computer searches had helped solve the problem, but not for this case.

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Information Theory, Coding and Cryptography

Maximizing free distance

- Another approach is to assign the symbols to the branches in the trellis in a heuristic manner so as to increase the d_{free} .
- We know that an **error event** consists of a path diverging in one state and then merging back after one or more transitions
- The Euclidean distance associated with such an error event can be expressed as
$$d_{total}^2 = d_E^2 \text{ (diverging pair of paths)} + \dots + d_E^2 \text{ (re-merging pair of paths)}.$$



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So, let us look at what is an error event will error event we have seen when 2 paths diverge they keep away from each other and then merge back in the trellis after sometimes. This red dots are the nodes in the trellis, so we have just shown 2 paths which diverge and merge path this is constitutes an error event and the one with the minimum distance between these 2 paths distance in terms of Euclidean distance is comes out as the d_{free} . So at least what we can do we can maximize the two branches where it diverges and merges back and rest all will add to it, but we do not have much control how it jumps up and down in the trellis.

So, we will choose the Euclidean distance associated with at least the diverging branches and the merge in back branches to give us something and we also know that there will not be too many hop in 2 3 hops it combines back.


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Information Theory, Coding and Cryptography

Maximizing free distance

- Thus, in order to design a TCM scheme with a large d_{free} , we can at least ensure that the $d_{\bar{E}}$ (diverging pair of paths) and the $d_{\bar{E}}$ (re-merging pair of paths) are as large as possible.
- In TCM schemes, a redundant 2^{m+1} ary signal set is often used to transmit m bits in each signaling interval.
- The m input bits are first encoded by a $m/(m+1)$ convolutional encoder.
- The resulting $m+1$ output bits are then mapped to the signal points of the 2^{m+1} -ary signal set.
- The mapping is done in such a manner so as to maximize the minimum Euclidean distance between the different paths in the trellis.
- This is done using a rule called mapping by **Set Partitioning**.

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So, it will not be a bad choice so we look at the diverging and re-emerging path and we have a redundancy of 2^{m+1} m -ary signal set is used to transmit m bits; why because we had this 8-PSK and then we went to from QPSK we went to 8-PSK.


So, we have m over $m+1$ convolutional encoder in the last case we had 2 by 3 so m was 2 so m shows that uncoded and $m+1$ is the coded so this is a typical case you can have a 4 by 5, 7 by 8 convolution encoder. So the resulting $m+1$ output bits are then mapped so we will use this concept to come up with some design methodology in terms of increasing the Euclidean distance, but the method to do so is called set partitioning and let us discuss how it is done.

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Information Theory, Coding and Cryptography

Set Partitioning

- The mapping by set partitioning is based on successive partitioning of the expanded 2^{m+1} ary signal set into subsets with **increasing minimum Euclidean distances**.
- Each time we partition the set, we reduce the number of the signal points in the subset, but increase the **minimum** distance between the signal points in the subset.

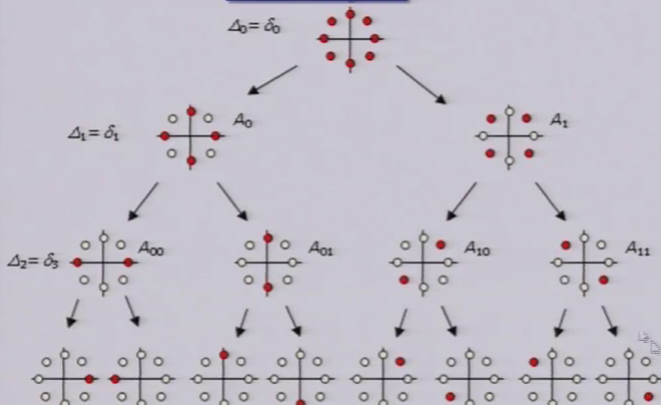
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So, the aim of set partitioning is to progressively increase the Euclidean distance and then use that partitioned set to map onto the trellis; each time we partition the set we reduce the number of signals points, but increase the distance between the signal points in the subset.


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Example



▪ Set partitioning of the 8-PSK signal set.

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Let us look at an example let us start with 8-PSK on the top you can see 8-PSK they are all placed on a circle. So we now partition this is a set of 8 signal points which partition

it into 2 4 and 4, but see the distance has increased so the minimum distance has gone up, but why should we stop here we can partition it further.


Again there is more than 1 way to do the partitioning, but we partition it in such a way that the minimum distance in this partition set between 2 constellation points have gone up so here there were 8 points now we have 4 points and 4 points highlighted by red and then 2 points and 2 points 2 points and 2 points and then of course, you have single point in the in the sets. So we have done the set partitioning of 8-PSK signal set it is pretty mechanical you can do it for any large constellation diagram.

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Information Theory, Coding and Cryptography

Example

- Consider the set partitioning of 8-PSK.
- Before partitioning, the minimum Euclidean distance of the signal set is $D_0 = d_0$.
- In the first step, the 8 points in the constellation diagram are subdivided into two subsets, A_0 and A_1 , each containing 4 signal points
- As a result of this first step, the minimum Euclidean distance of each of the subsets is now $D_1 = d_1$, which is larger than the minimum Euclidean distance of the original 8-PSK.
- We continue this procedure and subdivide the sets A_0 and A_1 into two subsets each, $A_0 \rightarrow \{A_{00}, A_{01}\}$ and $A_1 \rightarrow \{A_{10}, A_{11}\}$.
- As a result of this second step, the minimum Euclidean distance of each of the subsets is now $D_2 = d_2$.
- Further subdivision results in one signal point per subset.

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So, now the aim is how to use this partitioned set and map it to the trellis that we have here; so what we do is we basically come up with a rule as to how to apply these points from the different levels of constellation diagram because level decides what is the minimum distance in terms of the diverging and merging paths.

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Information Theory, Coding and Cryptography

Example

- Consider the expanded 2^{m+1} -ary signal set used for TCM.
- In general it is not necessary to continue the process of set partitioning until the last stage.
- The partitioning can be stopped as soon as the minimum distance of a subset is larger than the desired minimum Euclidean distance of the TCM scheme to be designed.

Suppose the desired Euclidean distance is obtained just after the $\tilde{m}+1^{\text{th}}$ set partitioning step ($\tilde{m} \leq m$).

It can be seen that after $\tilde{m}+1$ steps we have $2^{\tilde{m}+1}$ subsets and each subset contains $2^{m-\tilde{m}}$ signal points.

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So, again consider the expanded 2^{m+1} -ary signal set used for TCM we do not necessarily have to continue the process of partitioning till the last stage.


Suppose the desired Euclidean distance to be obtained just after the $\tilde{m}+1^{\text{th}}$ set partitioning step is $\tilde{m} \leq m$; then it can be seen that $\tilde{m}+1$ steps we have got $2^{\tilde{m}+1}$ subsets each subset containing $2^{m-\tilde{m}}$ signal points. So as you go down the number of signal points reduces and you have to only go down to those many points for which you have the branches to assign to so you do not have to go right into the bottom of the parameter.

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Information Theory, Coding and Cryptography

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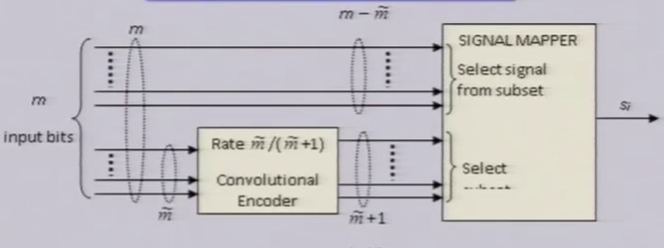
- A general structure of a TCM encoder is given in Fig. 7.7.
- It consists of m input bits of which the \tilde{m} bits are fed into a rate $\tilde{m}/(\tilde{m}+1)$ convolutional encoder while the remaining $m - \tilde{m}$ bits are left uncoded.
- The $\tilde{m} + 1$ output bits of the encoder along with the $m - \tilde{m}$ uncoded bits are then input to the signal mapper.
- The signal mapper uses the $\tilde{m} + 1$ bits from the convolutional encoder to select one of the possible $2^{\tilde{m}+1}$ subsets.
- The remaining $m - \tilde{m}$ uncoded bits are used to select one of the $2^{m-\tilde{m}}$ signals from this subset.
- Thus, the input to the TCM encoder are m bits and the output is a signal point chosen from the original constellation.

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Information Theory, Coding and Cryptography

General Structure



- Let us denote the minimum Euclidean distance between parallel transitions by $\Delta_{\tilde{m}-1}$ and the minimum Euclidean distance between non-parallel paths of the trellis by $d_{free}(\tilde{m})$.
- The free Euclidean distance of the TCM encoder shown can then be written as

$$d_{free} = \min[\Delta_{\tilde{m}-1}, d_{free}(\tilde{m})].$$

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So, now let us look at a simple example of a general structure, suppose you have got m bits so first you have a convolutional encoder here it is systematic so you have $m - \tilde{m}$ bits going directly to the signal mapping and then $\tilde{m} / (\tilde{m} + 1)$ convolutional encoder giving you the coded bit streams. The first $m - \tilde{m}$ bits are used to select the signal from the subset and these are used to select the subset so you have the subset and within the subsets you have signals.

So, this is a strategy how to choose the subset and then a particular signal from the subset and this will give you a minimum d free in terms of the deltas.

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Information Theory, Coding and Cryptography

General Structure

- For the TCM encoder we observe that $m - \tilde{m}$ uncoded bits have no effect on the state of the convolutional encoder because the input to the convolutional encoder is not being altered.
- Thus, we can change the first $m - \tilde{m}$ bits of the total m input bits *without* changing the encoder state.
- This implies that $2^{m-\tilde{m}}$ **parallel transitions** exist between states.
- These parallel transitions are associated with the signals of the subsets in the lowest layer of the set partitioning tree.
- For the case of $m = \tilde{m}$, the states are joined by single transitions.

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So, this is the general structure that we have just described how you can use a TCM encoder.

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Information Theory, Coding and Cryptography

Ungerboeck's Scheme

- Consider the TCM scheme proposed by Ungerboeck.
- It is designed to maximize the free Euclidean distance between coded sequences.
- It consists of a rate 2/3 convolutional encoder coupled with an 8-PSK signal set mapping.
- The encoder is given below

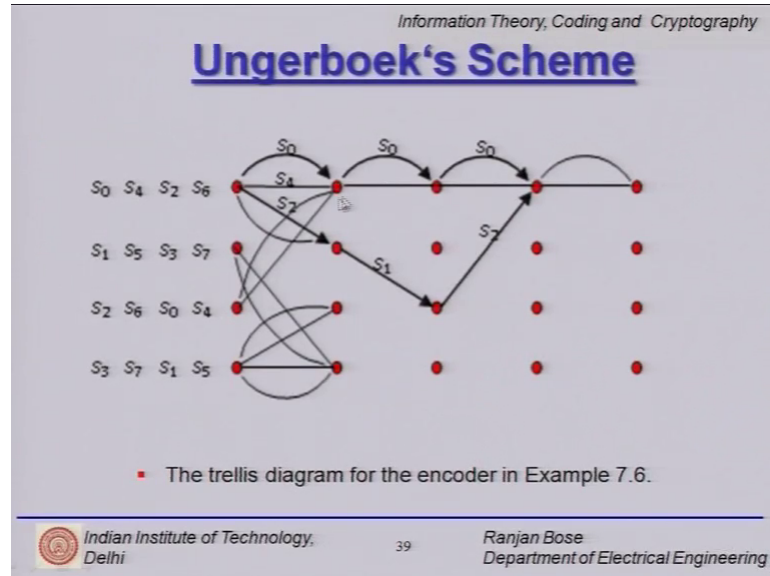
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Now, we go on to Ungerboeck scheme which tells us how to assign the symbols on to the trellis having done the set partitioning so the aim is to maximize the free Euclidean distance between coded sequences so just consider an example of a rate 2 by 3

convolutional encoder very simple 2 by 3 encoder natural mapping we have shown below.

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And for that we are going to do an Ungerboeck scheme and here you can see that there are parallel paths in this case because you know if this input a 1 does not change then the state transition does not change and for a 2 between 1 and 0 there are 2 possible outputs so there will be 2 paths from any 1 state to the next state, but the question is how to assign the symbols from the 8-PSK.

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Information Theory, Coding and Cryptography

Ungerboeck's Scheme

- For this encoder $m = 2$ and $\tilde{m} = 1$, which implies that there are $2^{m-\tilde{m}} = 2^1 = 2$ parallel transitions between each state.
- The minimum squared Euclidean distance between parallel transitions is

$$\Delta_{m+1}^2 = \Delta_2^2 = \delta_3^2 = 4E_s.$$
- The minimum squared Euclidean distance between non-parallel paths in the trellis, $d_{free}^2(\tilde{m})$, is given by the error event shown in the Trellis by bold lines.
- From the figure we have

$$\begin{aligned} d_{free}^2(\tilde{m}) &= d_E^2(s_0, s_2) + d_E^2(s_0, s_1) + d_E^2 \\ &= \delta_1^2 + \delta_0^2 + \delta_1^2 = 4.586 E_s. \end{aligned}$$

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So, here we have used the Ungerboeck scheme which we will discuss how to assign the different symbols from the;

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Ungerboeck's Scheme

- The error events associated with the parallel paths have the minimum squared Euclidean distance among all the possible error events.
- Therefore, the minimum squared Euclidean distance for the TCM scheme is

$$d_{free}^2 = \min[d_{m+1}^2, d_{free}^2(\tilde{m})] = 4E_s.$$

- The **asymptotic coding gain** of this scheme is

$$g_{\infty} = 10 \log \frac{4}{2} = 3 \text{ dB.}$$

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So once you do a mapping as described earlier then we can calculate directly the d_{free} as the minimum of d_{m+1}^2 and $d_{free}^2(\tilde{m})$ which is $E_s/4$ comes from the maximally separated points on the symbol set and this tells you that the d_{free} is larger than the uncoded one giving us an asymptotic gain of 3 dB. So we have been able to recover and we have been able to give you a 3 dB gain using a TCM scheme this called the Ungerboeck TCM scheme.

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Information Theory, Coding and Cryptography

Ungerboeck's Scheme

- This shows that the TCM scheme proposed by Ungerboeck shows an improvement of 3 dB over the uncoded QPSK.
- This example illustrates the point that the combined coded modulation scheme can compensate for the loss from the expansion of the signal set by the coding gain achieved by the convolutional encoder.
- Further, for the non-parallel paths
$$d_{total}^2 = d_E^2 \text{ (diverging pair of paths)} + \dots + d_E^2 \text{ (re-merging pair of paths)}$$
$$= \delta_1^2 + \dots + \delta_1^2 = (\delta_1^2 + \delta_1^2) + \dots = \delta_3^2 + \dots = 4E_s + \dots$$
- However, the minimum squared Euclidean distance for the parallel transition is $\delta_3^2 = 4E_s$.
- **Hence, the minimum squared Euclidean distance of this TCM scheme is determined by the parallel transitions.**


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Information Theory, Coding and Cryptography

Ungerboeck's Design Rules

- In 1982 Ungerboeck proposed a set of design rules for maximizing the free Euclidean distance for TCM schemes.
- These design rules are based on heuristics.
- **Rule 1:** Parallel transitions, if present, must be associated with the signals of the subsets in the lowest layer of the set partitioning tree.
These signals have the minimum Euclidean distance
- **Rule 2:** The transitions originating from or merging into one state must be associated with signals of the first step of set partitioning.
- The Euclidean distance between these signals is at least
- **Rule 3:** All signals are used with equal frequency in the trellis diagram.

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So, let us look at the design rules first and then revisit the example; so these were proposed in 1982 by Ungerboeck these are heuristic design rules; what are the rules? Well they tell us how to assign symbols from the set partitioning to the trellis. So if a trellis has parallel transitions right then they must be associated with the signals of the subset in the lowest layer of the set partitioning tree that is they are maximally apart they have the maximum minimum Euclidean distance.


Rule 2 is that the transition originating from or merging into 1 state must be associated with signals from the first step of the set partitioning we already have done the set partitioning and then rule number 3 is all signals are used with equal frequency in the trellis diagram so these are heuristic rules which will ensure a good design of your trellis.

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Information Theory, Coding and Cryptography

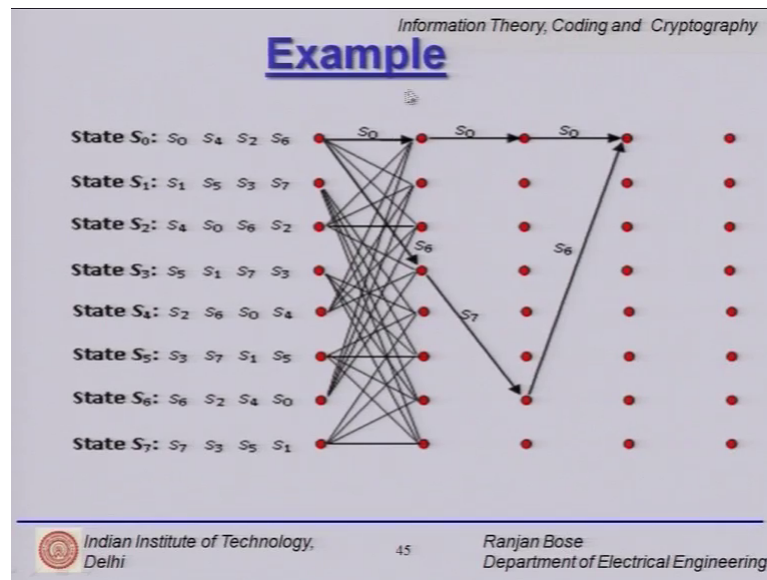
Example

- We next wish to improve upon the TCM scheme proposed by Ungerboeck
- We observed that the parallel transitions limit the d_{free}^2
- Therefore, we must come up with a trellis that has no parallel transitions.
- The absence of parallel paths would imply that the d_{free}^2 is not limited to δ_3^2 - the maximum possible separation between two signal points in the 8-PSK constellation diagram.
- Consider a trellis that has 8 states.

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So, we want to improve the trellis scheme we just proposed and we want to improve the performance of that scheme using these design rules. So, let us look at a trellis with 8 states, let us look at an example where we will execute all the points proposed by Ungerboeck.

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So, this is a trellis with 8 states and as you can see that there are 4 branches emanating from every node ok. So that 2 bits coming in for this convolutional encoder and there are clearly s_0 to s_7 so there are 8 points in the constellation diagram so 3 bits are being mapped to the symbols so it is a rate 2 by 3 encoded that has resulted in this trellis.


Since there are 3 memory elements therefore, there are 8 states in the trellis so the number of input bits is pretty much independent of the states in the trellis which depends on the memory unit and the number of output bits basically decide that the constellation size that we will use; so this is the trellis that we have at disposal.

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Information Theory, Coding and Cryptography

Example

- Since there are no parallel transitions here, we start directly with Ungerboeck's second rule.
- We must assign the transitions originating from or merging into one state with signals from the first step of set partitioning.
- We will refer to the set partitioning diagram for 8-PSK.
- The first step of set partitioning yields two subsets, A_0 and A_1 , each consisting of four signal points.
- We first focus on the diverging paths.
- Consider the topmost node (state S_0).
- We assign to these four diverging paths the signals s_0, s_4, s_2 and s_6 .
- Note that they all belong to the subset A_0 .
- For the next node (state S_1), we assign the signals s_1, s_5, s_3 and s_7 belonging to the subset A_1 .
- For the next node (state S_2), we assign the signals s_4, s_0, s_6 and s_2 belonging to the subset A_0 .

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So, there are no parallel transitions we start directly with the Ungerboeck second rule of assigning; so in this trellis we have assigned s_0, s_4, s_2 and s_6 from for these 2 for these 4 branches which are diverging out.

So, this is from the lower most but 1 level of the set partitioning that we have done already. Similarly we have taken the next one s_1, s_5, s_3 and s_7 these are the other 4 maximally separated points in the trellis and so and so forth and we have make sure that these are assigned with equal frequencies. Same care has been taken for the diverging and the merging back paths; so if you look at the points which are merging back they have been also assigned with that same care.

So, in this slide we are looking at the diverging path and this slide describes why s_0, s_4, s_2 and s_6 symbol have been assigned to that they belong to the subset A_0 and for the next node we have done s_1, s_5, s_3 and s_7 as we just not discussed ok.


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Information Theory, Coding and Cryptography

Example

- The order has been shuffled to ensure that at the re-merging end we still have with signals from the first step of set partitioning.
- If we observe the four paths that merge into the node of state S_0 , their branches are labeled s_0, s_4, s_2 and s_6 , which belong to A_0 .
- This clever assignment has ensured that the transitions originating from or merging into one state are labeled with signals from the first step of set partitioning, thus satisfying rule 2.
- It can be verified that all the signals have been used with equal frequency.
- We did not have to make any special effort to ensure that.

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So we have done all of this and the order has been shuffled to ensure that the re the at the reemerging and we still have the signals with the first step of the set partitioning that is the care we have taken to do so and we have also ensured that the all the symbols have been used with equal frequency.

So, we have been able to use Ungerboeck design rule to assign symbols to this trellis diagram with 8 states.

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Information Theory, Coding and Cryptography

Example

- The error even corresponding to the squared free Euclidean distance is shown in the trellis diagram with bold lines.
- The squared free Euclidean distance of this TCM scheme is


$$d_{free}^2 = d_E^2(s_0, s_6) + d_E^2(s_0, s_7) + d_E^2(s_0, s_6)$$
$$= \delta_1^2 + \delta_0^2 + \delta_1^2 = 4.586 E_c$$

- In comparison to uncoded QPSK, this translates to an asymptotic coding gain of

$$g_\infty = 10 \log \frac{4.586}{2} = 3.60 \text{ dB}$$

- Thus, at the cost of **added encoding and decoding complexity**, we have achieved a 0.6 dB gain over the TCM scheme discussed earlier

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Lets quickly calculate the d_{free} , so d_{free} we had already highlighted the 2 paths which result in d_{free} and the d_{free} calculation gives the g_{∞} 2.3 point 6 db therefore, this has a even better point 6 db gain over the TCM scheme discussed earlier by Ungerboeck.

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Information Theory, Coding and Cryptography

Summary

- Combined Coding and Modulation
- Trellis Coded Modulation
- Free distance
- Ungerboeck's design rules
- Examples

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Department of Electrical Engineering

So, let us now kind of conclude this lecture what we have looked at is the notion of combined coding and modulation then we introduced the idea about trellis coded modulation and how we can get improved performance without additional power or bandwidth then we talked about the free distance and the Ungerboeck's designed rules we also followed it up with some examples with that we come to the end of this lecture.