Nonlinear and Adaptive Control

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Lecture – 07
Robust Model Reference Adaptive Control part-1

Welcome everyone to lecture number 7 of this course on nonlinear and adaptive control. So far what we have seen is the basic framework of adaptive control. We have seen directed adaptive control, indirect adaptive control which started off with the scalar case and then we extended that to the vector case for plants which have parametric uncertainty that is where the system matrices or the control input matrix are unknown.

It is easier if the control input matrix is known as we saw in the vector case where the matrix b was required to be known for the analysis to be done in attractable way, but there are cases where you could do analysis and control design for systems of both A and B matrixes are unknown so there is left for you as an exercise. So today in this class we will focus on systems with disturbances, unmolded dynamics, unstructured, uncertainties. So their does the MRAC that we have designed so far work where we have plants with such effects.

So we will start off with slight detour so I get these questions many times. Students ask why do a nonlinear control analysis for linear system? So for in this course we have covered LTI system, MRAC basically concerns control of these linear systems, but we have used nonlinear analysis and design tools for developing the MRAC. So natural question arise is if it’s a linear system why not use linear control tools so I just want to clarify that question in this class before we move on to Robust adaptive control.

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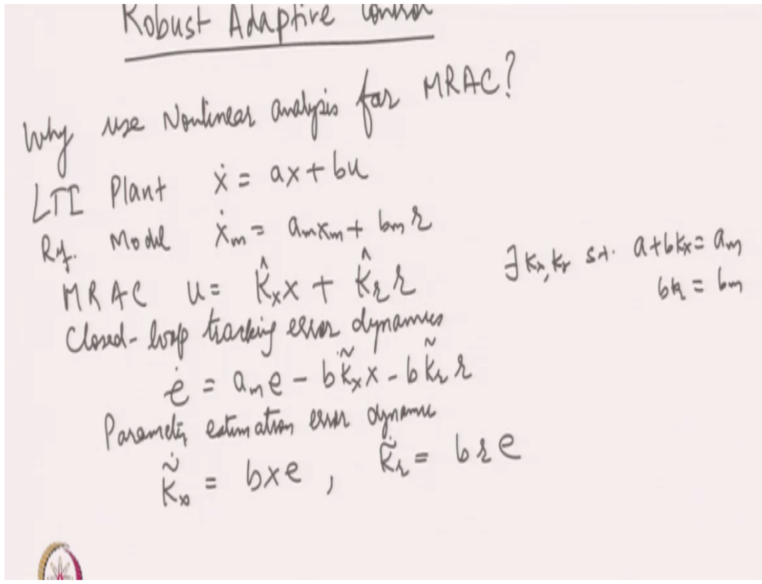

So why use nonlinear analysis for MRAC? So you have an LTI plant. I am taking scalar case X dot = ax + bu and the reference model is again LTI system xn dot is anXm + bmr. So we have done this many time in this course so this system should be familiar to you now. So now this is open loop linear system so we have not really designed the controller yet and it is an LTI system.

Now if we design our u as a modern reference adaptive controller such as what we have done this class before Kx hat * X + Kr hat * r where we have these usual matching conditions and there exists Kx and Kr such that a + bKx = am and bkr = bm. This is just for completeness, but the point that I want to make is that when you write down the closed loop error system that is when things get interesting.

So we define the error as x - xm and then we write down the expression for the aerodynamics so the closed loop in fact the tracking error should be specific here because we have the parameter estimation errors well in this case so tracking error dynamics are given by e dot = ame - bkx tilde * X - bkr tilde okay. So we can also go ahead and write down the dynamics for the parameter estimation errors.

So parameter because once we introduce these parameter estimates through our controller we get these 2 extra states Kx tilde and Kr tilde and when we consider the overall system we need to consider the parameter estimation errors also as overall system dynamics. So Kx tilde dot is given as bx series what we have designed and Kr tilde dot is given by bre. So this is you mean that b is known. So here we want to express this as state space system.

So we look at the right hand side. Suppose we know that the states are e Kx tilde and Kr tilde. Now if we can express the right hand side in the form ax + b + br or something like that then we can say it is a linear system and we could use linear design tools for stability analysis. So let us see what the right hand side here comes out to be. So what we see here is that we can replace certain terms on the right hand side.

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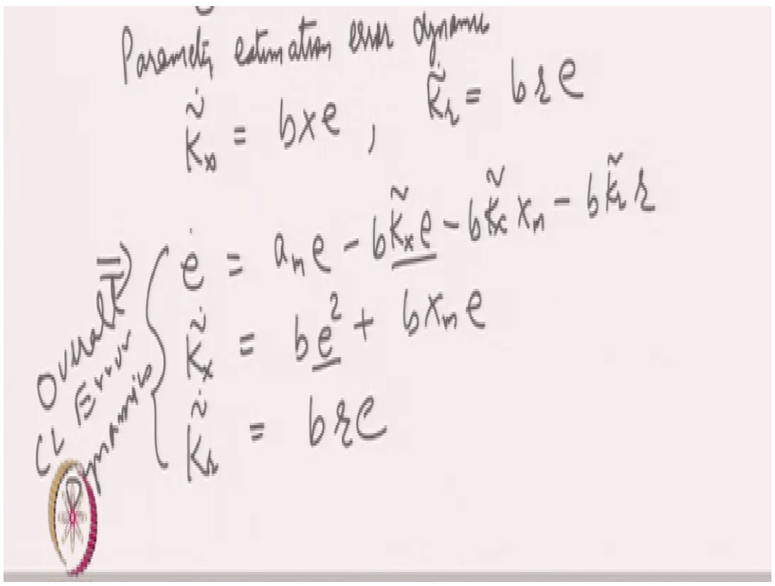

So let us rewrite the tracking error dynamics in terms of e so we replace x by e + xm so what we get is - bkx tilde e - bkx tilde Xn - b Kr tilde r and Kx tilde dot also we similarly modify so we replace x by e + xm and what we get is b square + bxne and Kr tilde dot as bre. So this is the overall closed loop error dynamics and to prove that e is bounded or e goes to 0. We need to also consider the dynamics of the parameter estimation error Kx tilde and Kr tilde and the cause now they are included in the extended state space.

We have to see whether the right hand side even did a linear system and this we have terms were like be square, Kx tilde * e, a product of states. So these terms indicate that this is no longer a linear system and we cannot go ahead with linear design tool unless you want to linearize but here we want to do the analysis in a global manner so we want to go ahead with nonlinear control tools.

So this explains why the overall state space, the extended state space indicates nonlinear system and that motivates the need for doing nonlinear design in this case. So I hope it is clear why we are using Lyapunov stability analysis instead of say looking at just the Eigen values of the a matrix for a linear system. So now we can move on to the topic of this class.

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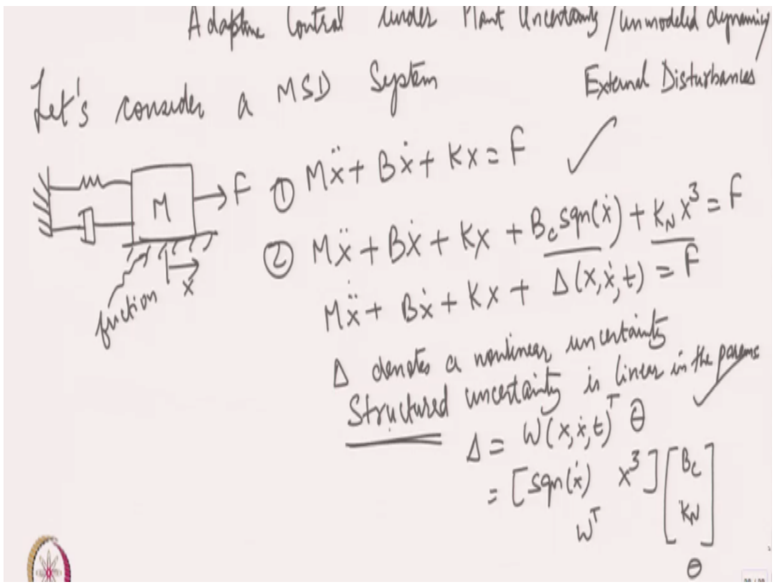

So let me write the topic. So this is adaptive control under plant uncertainty unmodeled dynamics and external disturbances. So let us consider a mass spring damper system so this is probably one of the simpler systems that you could consider, but that just helps in illustrating the point that I am trying to make here. It has got a damper, we have got a spring the mass and there is a force which is acting on this mass which is denoted by F.

So we know that this is a linear system then we could write down the equation of motion as Bx dot + Kx = F. So I ask you to do an adaptive controller for this system such that the state X which is a position of the mass tracks Xd which is a desire trajectory then you could use what we had done in the last class were we had developed an adaptive controller for a command tracking problem instead of an MRAC problem.

You could use similar method to design an adaptive controller for this command tracking problem and it would be a straight forward use of what we had done last time. So here we consider that M, D, K which are the mass, the damping constant, the spring constant are unknown constants and we can still go ahead and design the adaptive controller for this case. So this case, we can design an adaptive controller.

Now we know that real systems are not exactly linear. There is some nonlinearity or the other. So now if I modify this mass spring damper system and include some uncertain terms like say friction and say a nonlinear spring so let us see the dynamics get modified as Mx double dot + Bx dot + Kx so in addition to that we also have say some friction affecting the movement of this mass and let us see this is a Coulomb friction so we model this as Bc sine of x dot and then we have a nonlinear spring term.

So let us say that these can be this can be written as some delta. So this delta denotes a nonlinear uncertainty. Can we design an adaptive controller for this case using the methods that we have done so far? So we have to look closely here so this delta consists of the Coulomb friction term as well as this nonlinear spring term, so what we know here is that this serve in deed the coulomb friction and we have modeled it like this and the nonlinear spring as this cubic term in it.

The constants Bc and Kn are in fact unknown but the signum of x dot and x cube are things that we know. So this kind of an uncertainty falls in the quite category of a structure uncertainty. So the structure is known. The structure of the uncertainty is known. In fact, we can further qualify this by saying that this structure uncertainty is linear in the parameters. So this is linear in the parameters.

What do we mean by that is that we can separate out the unknown constants in a linear way. So we can write delta as w of x * x dot transpose theta whether this w is a regressors of known terms so here what are the known terms. The known terms here are signum(x dot) so these terms may be nonlinear and x cubes these are nonlinear terms, but we can separate out the unknown constant terms from these known terms in a linear way.

So the unknown constants come out as theta and this is W transpose. So, have we handled such kind of uncertainties before? Yes, we have in the previous class we considered a case where we had a linear system and in addition to that we had a structured uncertainty which is linear in the parameters. So such cases can also be handled using our classical adaptive control approaches. So we can say that structure uncertainty which is linear in the parameters can also be handed with our existing tools or the tools that we have done in the class so for.

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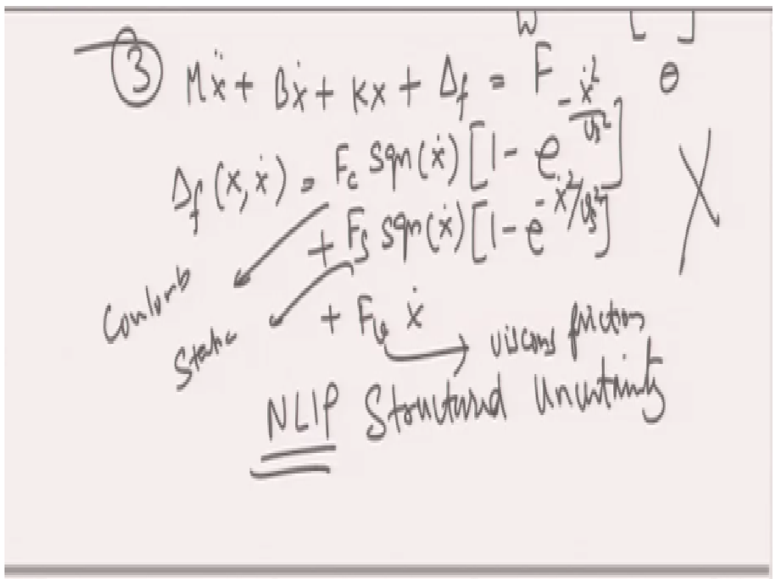

So number 3 could be we have delta which includes a slightly complicated model of friction. So let us say we have Mx double dot + Bx dot + Kx + delta f = F. So delta f refers to another model of friction which is a slightly more complicated but it is commonly used and it is given by Fe signum(x dot) [1 - exponential of - x dot square/vs] + Fs signum(x dot)[1 - e to the power - x dot square/vs square] and then we have Fv x dot.

So the first term represents the coulomb friction. So this is coulomb friction. This term represents static friction and this term represents the viscous friction. So we have 4 unknown constant here Fc, Fs, Fv and vs. vs is a stripe back parameter. So all these unknown constants are present in this model of friction but can we separate out these unknown constants from these known terms in a linear way. So the answer to that question is no.

So in this case because we have so the viscous friction can be linearly parameterized so we have Fv * x dot. So this is in the linear in the parameter form. So this can be handled with the tools that we have done so far, but what about these 2 terms so signum(x dot) signum(x dot) can be factored out but then that gets multiplied by this exponential term which has this unknown parameter vs which unfortunately cannot be factored out in a linear way.

So this kind of model of friction is called nonlinear in the parameter and in a short this is NLIP the 1 of the data above was LIP linear in the parameter. This is nonlinear in the parameter and it is not straight forward to design adaptive control loss for such cases. So we have a recent paper where we do a adaptive control design for nonlinear in the parameterized plant so if you are interested you can search on Google and you will find the paper, but the results are not very common.

There has been some research and they are not very elegant, not as elegant as say MRAC or the other rapid control methods. So this still falls so this is still NLIP structure of uncertainty so we know a structure of this uncertainty so is still called this structured uncertainty although in this case this is a NLIP. So we cannot handle this case using the adaptive controllers that we have done so far. In literature, there have been works which tackle this kind of uncertainty, but not using the tools that we have done.

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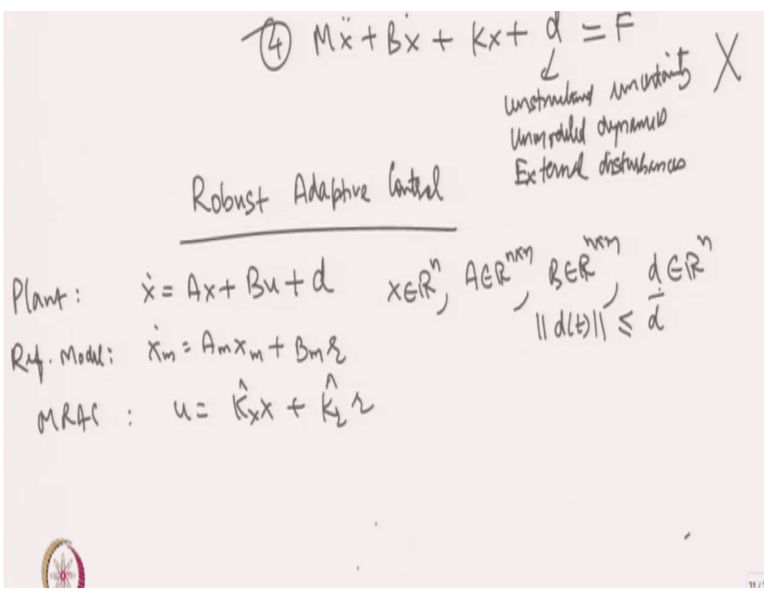

So then we come to say number 4. So number 4 could be unstructured uncertainty so where we do not exactly know the structure of an uncertainty you could also talk about unmodeled dynamics so there are systems for which we cannot model certain parts and we then call them as the unmodeled dynamics. They may be neglected under normal operating conditions, but may be under high frequency these modes also may get exited, but for normal frequency range we may neglect certain dynamics.

So those dynamics which we cannot which we not capable of modeling or which we can safely ignore for the operating conditions we call them as unmodeled dynamics and you can also these unstructured uncertainty unmodeled dynamics and external perturbations they can actually be clubbed under say d. Because this represents unstructured uncertainty unmodeled dynamics or external disturbances.

So of course this is very simplistic case where I have considered just an additive uncertainty or an additive disturbance. There may be cases where you have more complicated dynamics involved for now let us just focus on this additive disturbance. So there are various ways in which such unmolded dynamics so uncertainty with channel disturbances can be handled and the most popular robust control methods so we take the worst case scenario for such uncertainty or such dynamics and then we design controllers which account for this worst case scenario and still be still able to maintain stability.

So that is the class of robust controllers. Can we use the adaptive control techniques that we have studied so far for this scenario? So where we do not know anything we do not know the structure of this disturbance d. So in a mass spring damper system this could be like if you are saying pushing the mass you are giving some external stimulus to the mass and those forces we cannot model. We do not know what the structure of those external perturbations is so we club everything under this term d.

So can we design rapid controller for such a scenario. So using the existing techniques of course since we do not know the structure of d we cannot design so that is the focus of this lecture that we would like to design adaptive control loss which can work in scenarios where you have external disturbances and that is a fairly realistic scenario because these are terms where which in real systems you do not know these term, but may be by some experimentation we can say that we can upper bound these terms.

So if I say that the upper bound of such term is known we may be able to handle such terms using robust adaptive control methods. So let us begin talking about these robust adaptive controllers in a general setting. So again we go back to the plant model that we have x dot = Ax + Bu. So this is the plant. The reference model is given by copy paste of that we have done many times before. For this plant and for this reference model we know what is the MRAC?

We design as Kx + Kr at r. So if I ask the question that suppose this plant experiences certain channel perturbation we denote that by d. So let us just also make the small complete by mentioning the dimensions here and d is Rn. So all we know about this disturbance term d is that it is bounded. It is uniformly bounded. This d is and d bar is a positive constant okay and here Am and Bm have appropriate dimensions Am is hurwitz and r is a bounded reference signal okay.

So and we also follow the matching conditions and we write that again. Now let us see that if the same model reference adaptive control that we have designed for the disturbance free case works in the case the plant experiences the disturbance. So as a control engineer I do not say I design the controller thinking that there is no disturbance and the model of the plant is this.

X dot = Ax + Bu even though A and B may be unknown, but I do not expect this model to be something other than this, but in real life this may not be true so the controller that I design for this situation how does it fair when the plant in fact experiences external disturbances so that is the analysis that we are going to do now.

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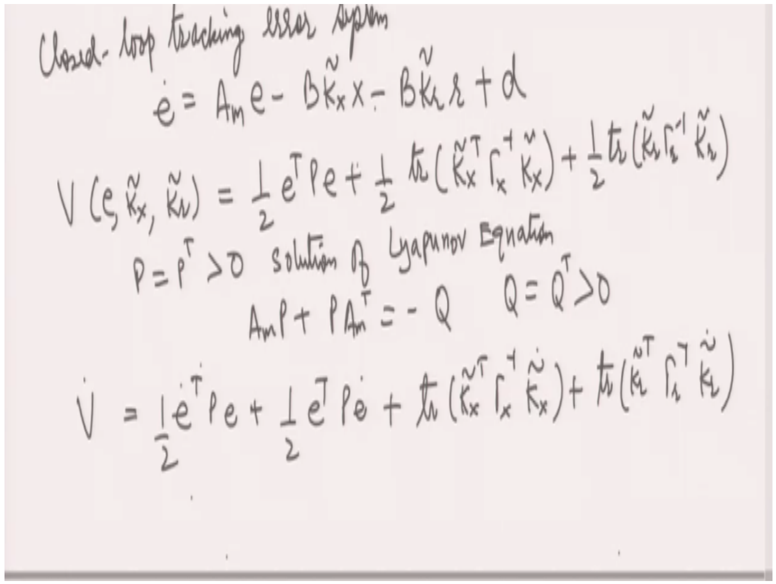

So again using the similar analysis as before we can write down the closed loop tracking error system e dot. So this was what we had gotten previously because of this disturbance term we have an extra term in our tracking error system which is d. So now we do Lyapunov analysis. So let us consider the same Lyapunov function candidate that we had considered before of course you can do a different Lyapunov function candidate.

So this Lyapunov function candidate has worked for us so far so let us try this out. This is a trace operator. We have discussed as before. So this is the Lyapunov function and here p is symmetric positive definite solution of the Lyapunov equation. AmP + pAm transpose = - Q and Q is also positive definite and symmetric and since Am is Hurwitz this will have resolution and that is given by P. So now let us check the so this function is positive definite with the unbounded decrescent we go ahead and take the time derivative.

So I will skip a few steps here Kx tilde dot, Kr tilde transpose gamma r and s and Kr tilde dot. So these gamma x and gamma r are also positive definite matrices. So now we can go ahead and substitute for the error dynamics and all these terms so we substitute for e dot in the first 2 terms and then we design Kx hat dot and Kr hat dot. So I am going to skip a few steps here and I will come to the expression for V dot.

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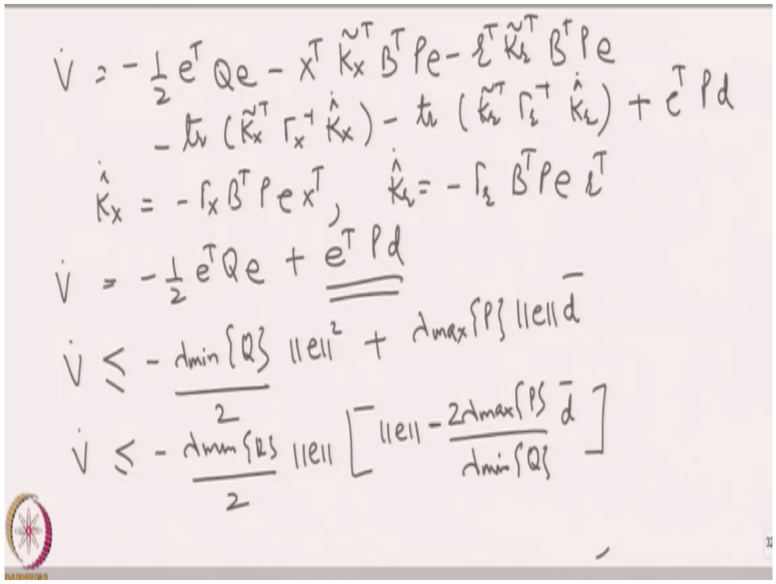

So V dot is given by - half e transpose Qe - X transpose Kx tilde transpose * B transpose Pe - r. You can go back to the notes and check if we get the same expression. - trace of Kx tilde expose * gamma x inverse, Kx hat dot - trace of Kr expose and so this is what we had obtained so last time, but there is an additional term this time which is + e transpose pd. So when you substitute for the error dynamics where we see that there is term d in the error system and that shows up here Lyapunov function derivative.

So and here of course the control designer assumed that this a disturbance free case so the control design is already done. We have already done the adaptive laws and we have given us - gamma x B transpose Pe x transpose Kr hat dot = - gamma r * B transpose P e r transpose alright okay. So once we substitute for these in the expression these expressions are including trace what we get is V dot as - half e transpose Qe.

So these update law designs will cancel the second and the third terms and what we get is simply is e transpose Pd. So suppose d was 0 then we would have gotten - 1/2 e transpose Qe and you would have concluded that this was negative. Semi definite and this would lead to Lyapunov stability. We could go ahead and prove that e goes to 0 using the Bob -lemma all signals are bounded and we are happy.

But just in the case that the plant experiences an external disturbance is the controller that we have designed robust to search external perturbations that is the question that we are trying to answer here. So if we look at this term we do not know the sign of this term, because e can have any sign it is the error, it is a state, it can have any sign positive, negative, we do not know. The p is of course a constant positive definite matrix. d is a disturbance term which is a function of time again it can have any signs.

So this term is in fact sign indeterminate. We cannot say what the sign is so the best that we can do with such terms is to upper bound it so we say that V dot is upper bounded as - 1/2 e transpose Qe + so in fact what we can do here is that also upper bound the first term so we what we get is - lambda min so the minimum Eigen value of Q. So this expression will upper bound the first term and then upper bounding the second term we have.

Because here we have a positive sign so the maximum Eigen value of the p matrix * the norm of e * norm of 10 * the norm of d which can be upper bounded as d bar. So this is what we get here. So what can we say here? The first time of course is not a problem because it is negative. The second term is a positive term. Now what we have to see here is can we dominate the second term with the first term.

If we can completely dominate then of course we can still say that V dot is <= 0 and we could still conclude Lyapunov stability. Let us see if we can do that. It does not seem very clear how we would be able to do that so V dot what we can say is that we take the lambda max. So this is what we get. So what we can see here is that if the term in the square bracket is positive then we can say that V dot <= 0 and we can conclude Lyapunov stability. So we can consider 2 cases here.

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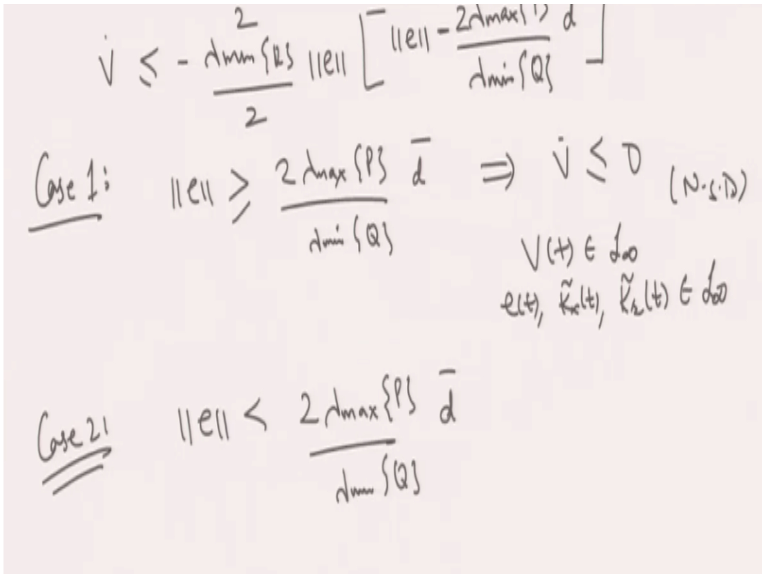

So case 1 when norm of e is > 2 lambda max/lambda min * d bar. Suppose this case this is true so what this would lead to is V dot <= 0 so which means that V dot is negative sign definite and we can conclude that V is bounded because V is positive and V dot <= 0 so V is bounded and since then V is bounded we can further say that e of t, Kx tilde of t and Kr tilde of t are also bounded.

So this is a good situation to begin, but what happens when e is <= 2 lambda max in fact I think what we can do is we can have the quality here so what if this case 2 is true that the tracking error is the norm of the tracking error is < this constant. All the terms here are constant terms. We have the maximum Eigen values p minimum Eigen value of Q and the upper bound on the disturbance.

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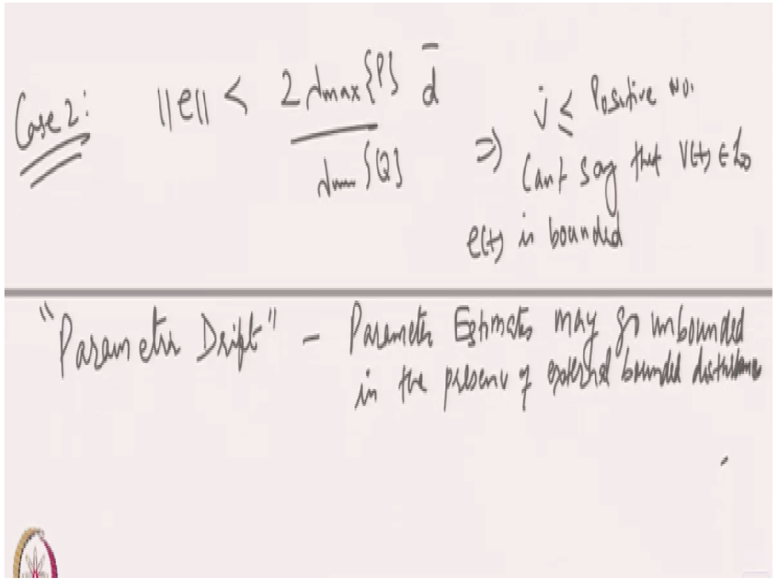

So if the tracking error goes below this value then the term in the bracket in fact becomes < 0 and then V dot becomes <= some positive number right. So what that means is that we cannot really conclude about V dot it could be a negative, it could be positive. All we know is that V dot is <= some positive number so we cannot conclude from here and we cannot say that V of t is bounded. We cannot say that.

We cannot guarantee that V is in fact bounded but what we can say from this case is that the tracking error is bounded because that is the case that we are talking about that the tracking error goes below a certain value so e of t is bounded, but what about the parameter estimation errors Kx tilde and Kr tilde in this case. So in this case we cannot really guarantee that the parameter estimation errors are bounded.

In fact, we could say that the parameter estimation errors can go and bounded in the presence of disturbances so we cannot guarantee using this Lyapunov analysis that they are in fact bounded. In case 1 we could say that both the tracking error and the parameter estimation errors are bounded, but in case 2 which can also happen we cannot say that the parameter estimation errors are bounded.

And what that means is that the controller which consists of the estimates the parameter estimates can also go unbounded which is highly undesirable scenario we of course do not found any signals in our system to be unbounded. Here in fact the control can also go unbounded so it is a very it is a scenario that should not happen and this phenomena is in fact called as parameter drift so the adaptive controller that we have designed is not robust to disturbances.

These are the parameter estimates may drift away and go unbounded in the presence of external bounded disturbances. So this phenomenon came to light many years ago when a NASA flight crashed and there were lots of investigation into the reason why that could have happened and one of the things that came out of that investigation was that if the model is not perfect.

And there are some unmodeled dynamics or some external disturbances that can lead to certain parameters to go unbounded and can lead to a catastrophe which is what happened. So this is a very important case in adaptive control we need to be able to ensure that the adaptive control are robust.

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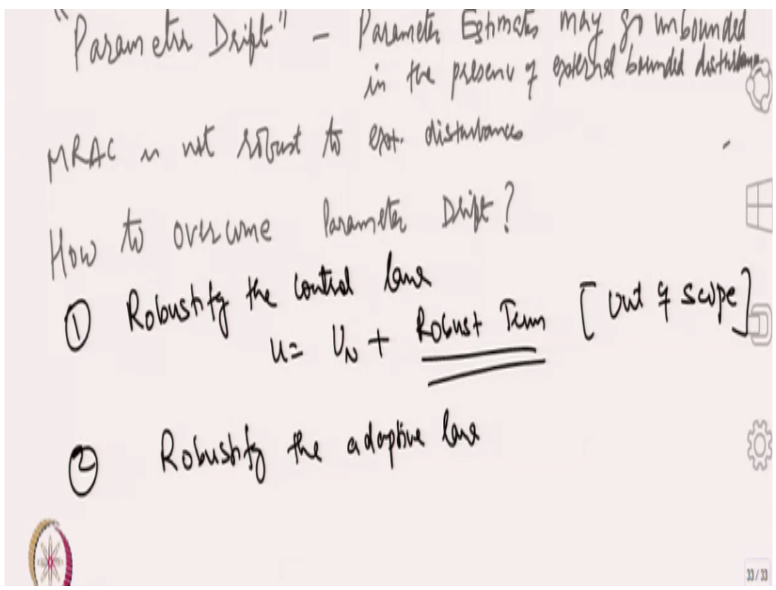

So this MRAC that we have design is of course not robust is not robust to external disturbances unfortunately. So how to overcome this parameter drift that is the next question to ask. So one way that we can overcome this parameter drift is somehow we could cancel the effect of this disturbance term from the closed loop error dynamics so if we go back to our closed loop error system so we have this term d in e dot.

So if we can say add an additional term here in the controller which we call as a robust term and that cancels the effect of the disturbance then maybe we can handle such scenarios that means we have to consider a non certainty equivalence controller and we can surely design such controllers where we have an extra robust term in addition to the nominal MRAC so such robust term for example sliding mode can cancel the effect of these disturbances you can still guarantee stability.

So that is out of scope of this class because we are only concerned with adaptive controllers here so we are not going to use a robust control law rather we are going to use a robust adaptive law. So there are 2 ways in which to handle such cases 1 of course is that we robustify the control law so we add in addition to the nominal we add some robust term and how do you design this robust term that is not the focus of this course so out of scope of this course.

So number 2 is we robustify the adaptive law. So when I say robustify I mean that we consider the worst case scenario for this disturbance and we develop a controller which is highly conservative so the performance might degrade but more important than performances is stability. So the first requirement for a control system is that it should be stable and it should be stable under uncertainty and the disturbance and then of course we look at performance.

So a robust adaptive law may not be as well performing as the laws that we have designed previously, but because of this robust element we might be able to account for the effect of the external disturbances so which is what we will do in the next lecture or 2 so the first technique that we will consider is dead zone.

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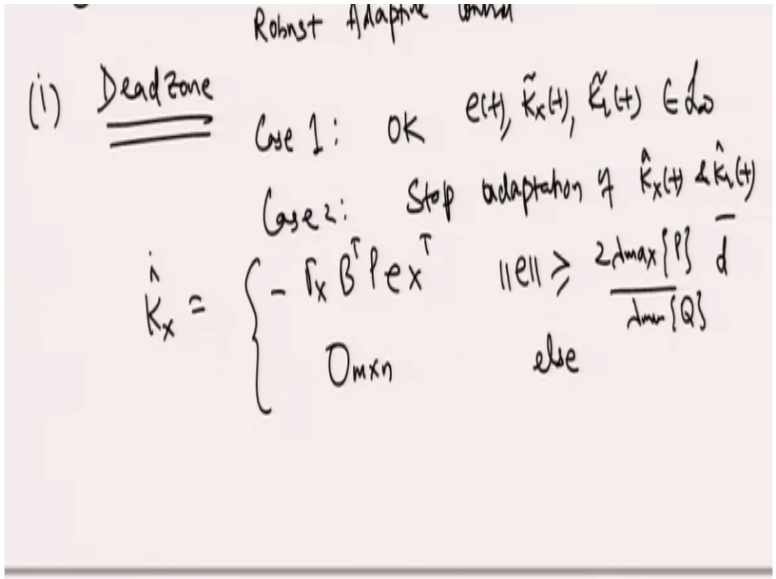

So this is we are trying to modify the adaptive laws so these are techniques which fall under the category of robust adaptive control. So that is also a very intuitive idea so if we go back to the analysis that we have done previously about how disturbance effects and MRAC system we see that case 1 is alright because here we could prove that all the errors are bounded. The problem is with case 2 where we could not prove that the parameter estimation errors are in fact bounded.

So Kx tilde and Kr tilde being bounded we do not know okay all that we could say that tracking error is bounded and because we cannot guarantee this one way in which we can make sure that in this case the estimates are bounded is by stopping the adaptation for case 2. So in case 1 we just follow the adaptive law that came out from the Lyapunov analysis and when the error goes below a certain bound then we stop the adaptation.

So the adaptive esteem the parameter esteem is they stay wherever they are and we do not go ahead with their adaptation so that will make sure that these error estimates are in fact not going to go unbounded and so we prevent the case of parameter drift by introducing a dead zone, a zone in which there is no adaptation. So let us just do a recap. So case 1 was okay because we could guarantee that the tracking error and the parameter estimation errors they are all bounded.

So this case was okay. For case 2, we stop the adaptation of Kx hat and Kr hat. So what that means is that Kx hat and Kr hat they stay constant in case 2 and which means that they will not be unbounded so what we have to do is we have to modify the update law as Kx hat dot will be now in 2 cases so we follow the same law that was used before so here we say that this is the case where the tracking error is larger than a certain thresholds and we call that as a dead zone.

We can of course take a dead zone which is larger than this. So this was we say that Kx hat dot is in fact 0 and this Kx hat is a matrix. It is an m x m matrix so we just use those dimensions here. So the tracking error is larger than the dead zone that we have used here then if we follow the adaptive law that classical adaptive law and when the tracking error falls below that value then we say that Kx hat dot = 0.

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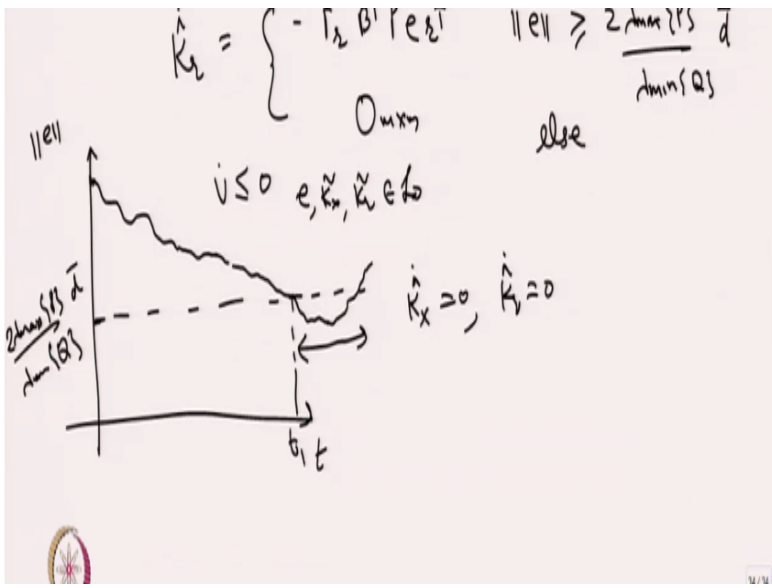

And similarly we do it for Kr hat dot so we use a similar adaptive law as we had run for Kx hat. So what happens to the adaptation in both the scenario so let us just look at a pictorial representation here so this is norm of e versus time and in the case where so let us also have the dead zone so this is 2 lambda max/lambda min of Q * d bar so this is our dead zone. So anything above this suppose the error starts above the dead zone then of course we know.

We will follow the classical adaptive laws and v dot in this case is <= 0 which means that e, Kx tilde, and Kr tilde are bounded okay. Now when the tracking error tries to go below the dead zone that is when we say that we stop the updation so in this case we say that Kx hat dot is 0 and

Kr hat dot is 0 so at this time instant say t1 whatever value that Kx hat and Kr hat had they will stay the same as long as the error stays within this dead zone.

When the error moves out of the dead zone we again start the updation of Kx hat and Kr hat. So during this time the value of Kx hat will be what it was at t1 and the value of Kr hat will be what it was at time t1 okay. Of course when the error goes outside of the dead zone we start the updation adaptation of Kx hat and Kr hat using the classical update laws, the unmodified update laws. So with this strategy what we have made sure is that for all time the tracking error and the parameter estimation errors are bounded.

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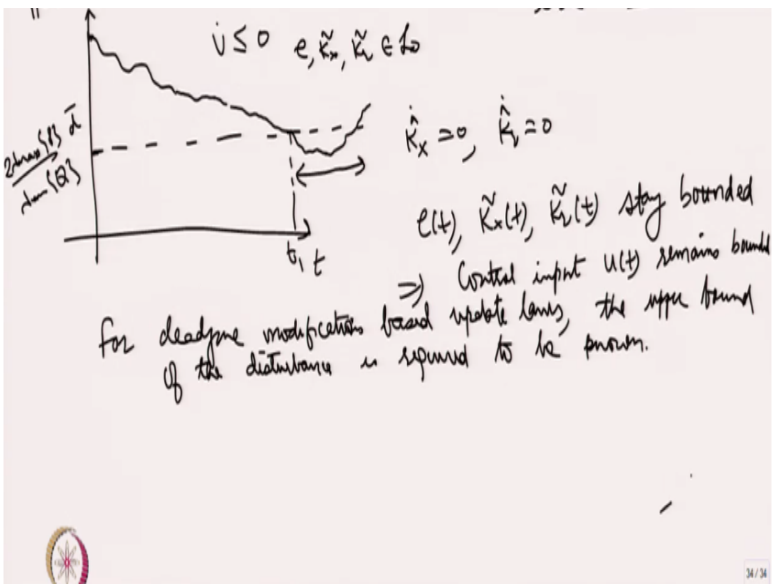

So e of t Kx tilde of t and Kr tilde of t may stay bounded so which means that the control input U(t) also remains bounded. So here interesting thing to note here is that because our case is here depend upon the evaluation of the dead zone and for that we need to know the upper bounded disturbance so we say that for this dead zone modification of the adaptive laws we need to know the upper bound on the disturbance so for dead zone modification based update laws the upper bound of the disturbance is required to be known.

So another curious aspect of this dead zone modification is that suppose we make this modification of the update law and thinking that they would be a disturbance d whose upper bound is d bar, but say in reality there is no such disturbance so the plant just follows x dot = Ax

+ bu and the control is of course hard that might be a disturbance with an upper bound d bar so the adverse effect of in the disturbance free case is that we no longer would be able to guarantee that the tracking error asymptotically goes to 0.

So this is what we had achieved using the unmodified adaptive laws done previously where we used the Barbalat's lemma to prove that e(t) goes to 0 as t goes to infinity. So here since we have modified the update laws even in the case where say the disturbance is not present in the disturbance free case we will not be able to guarantee the cause of this modification that the tracking error goes to 0 okay.

So this of course degrades the performance of the adaptive controller especially in the disturbance free case, but what it provides us its robustness to external disturbances whose upper bound we know. So this is the first of the 4 modifications that we will make to the adaptive laws to make it robust to the effects of external disturbances. So we will continue with the robust adaptive control in the next class.