Nonlinear and Adaptive Control

Dr. Shubhendu Bhasin
Department of Electrical Engineering
Indian Institute of Technology – Delhi

Lecture - 01
Introduction

Okay welcome everyone to the first class of adaptive control. So today we will talk about, firstly we talk about the different textbooks that will be useful for this course. I have already mentioned that in the course webpage, but I will just mention that again.

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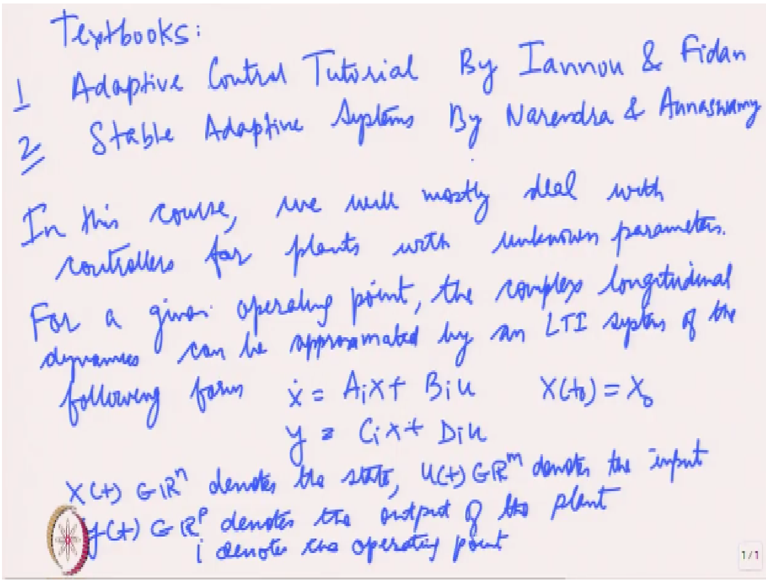

So the important textbooks to follow for this course are as follows. Adaptive Control Tutorial this is by Iannou and Fidan. Then we have Stable Adaptive Systems by Narendra and Annaswamy. So these are the 2 main text books that I will be following for this course. In addition to these there are a couple of other text books that I have mentioned on the course webpage which are by Sastry and Bodson which is again on adaptive control.

And then there are 2 texts on Nonlinear Systems and Control by Khalil and by Sastry. So in the introductory video I had mentioned that basic familiarity with nonlinear systems and Lyapunov stability methods is required for this course. So in case you do not have background in that area I encourage that you read these very standard text by Khalil and Slotine, okay.

So now we can start looking at other aspects of this course. So this is a 4-week course and we will primarily cover adaptive control for linear systems in this 10-hour course but the same methods that we develop for this course can also be utilized to study adaptive control for nonlinear systems. Okay so now the main keyword in adaptive control is adapt, and adapt literally means to change ones behaviour.

You know when you have change circumstances or new circumstances. So any controller which is able to tune itself based on the change plant dynamics can be considered to be an adaptive controller in general and in this course we will be talking about developing techniques or approaches for controlling plants where the parameters are unknown or where the parameters change in an unpredictable way, okay.

So in this course we will mostly deal with controllers for plants with unknown parameters. So when I say plants with unknown parameters I mean that plants which were the dynamics can be parameterized and in the parameters of those dynamics are not know, okay. So how do you then design a controller for such a system. So this idea of adaptive control is an old one, it originated in the 1950s when people started looking at designing autopilots for high performance aircrafts.

So these aircrafts are highly nonlinear, time varying plants where they operate in wide range of altitudes and speeds. So let us consider that for a given operating point, when I say for a given operating point I mean that for a given operating point of altitude and speed, the complex longitudinal dynamics can be approximated by an LTI system of the following form. Where the initial conditions are given by x of t0 = x0.

Here x of t denotes the state, u of t denotes the input, y of t denotes the output of the plant and the plant here is the longitudinal dynamics of an aircraft. So i here refers to, i denotes the new operating point. Example, i can be one to denote one operating point and i can vary from one to say k where the aircraft operates in k different operating points and for each operating point you have different plant dynamics.

So typically what would you do? How would you design a controller u to stabilize a plant like an aircraft? So if you were to use a fixed gain controller.

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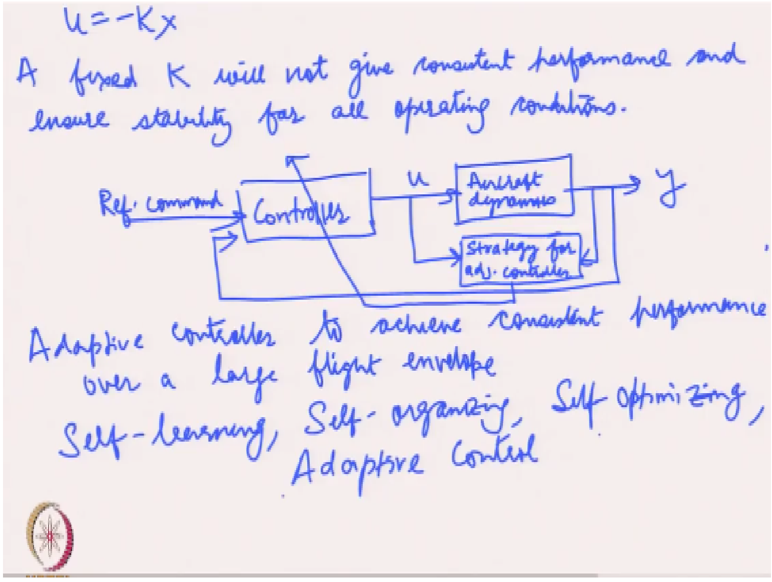

Say u = -kx and this k can actually be tuned or it can be found out by some pole placement method or by using LQR, but you would find that this k would work really well for a certain operating condition, but when the aircraft goes into a different operating condition the same k would not give you the same consistent performance and stability that it would give for which this k was designed right.

So a fixed k will not give consistent performance and ensure stability for all operating conditions. So if you were to use this kind of a controller for this plant and let us try and make a block diagram for that. So this is the aircraft dynamics. So it has an input u and an output y and then you connect a controller, this is a general fixed gain controller let us not specify what it actually is.

So the input to the controller is a reference command. So this output also is an input to the controller so where the controller block would then compare the actual output with the reference command and generate an error signal which can then be used to control the aircraft. So this is what a typical block diagram for a fixed gain controller would be.

So now as I mentioned that a fixed gain controller may not give you consistent performance and ensure stability for all kinds of operating conditions. So a better method to design controller would be to design an adaptive controller to achieve consistent performance over a large flight envelope. So the adaptive controller would be able to automatically tune the gains of the controller and give you consistent performance for a wide range of operating conditions for the aircraft.

So how it would be able to achieve that is say we have another block here, which denotes the strategy for adjusting the controller. So it probably would use an input to the plant and the output coming out from the plant and then the output from such a block would then be used to tune the gains of the controller automatically. So you see how this controller would be able to make appropriate adjustments to accommodate changes in a plant whose operating conditions are changing, right.

So that is the power that you get when you design an adaptive controller, okay. So this was way back in the 1950s and at that time there were various terms which were introduced to denote these controllers and so the people used terms like self-learning, self-organizing, self-optimizing adaptive controllers. So all these terms were used to refer to the same kind of controllers, which would automatically adjust their control parameters for different plant parameters okay.

So that was a basic motivation for adaptive control. We will consider more examples just to illustrate why these adaptive controllers are useful. So let us consider another example which I would also mention in the introductory video.

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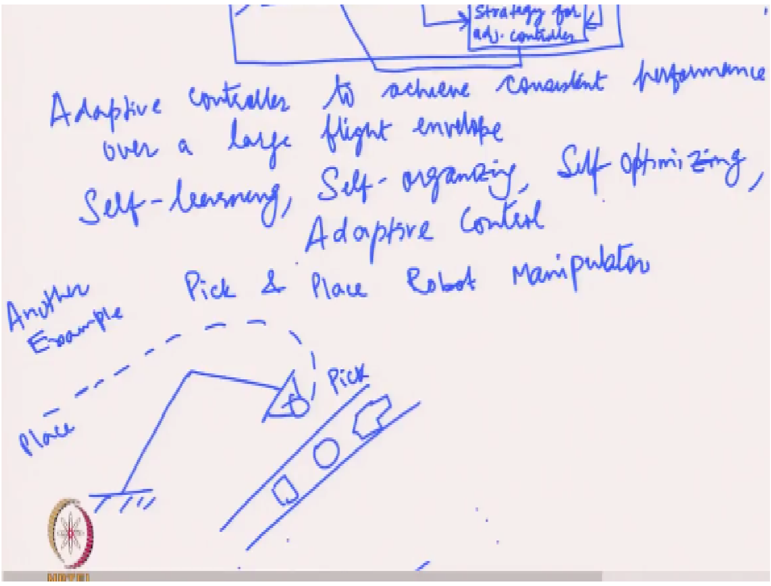

So another example of a pick and place robot manipulator. So let us consider this stick diagram of a robot with a gripper at the end and it is lifting objects on the conveyer, which come in different shapes and sizes and different mass distributions and it is lifting them up and then say let us have an object at the end of it, so the robot is lifting it and then placing it at some desired location.

So it is picking it from the conveyer and placing it here, okay. So again if you were to use classical methods to design controllers for such a task you would try and tune your control gains to get an optimum performance and ensure stability, but that would probably work in a situation where you are lifting just a certain kind of pay loads. Now in this conveyer you have various kinds of payloads.

So for each payload your robot dynamics will change for example the inertial parameter of the robot will change each time it picks a different payload and because the initial parameters will change the performance and stability property of a system will also change. So the same fixed gain controller that you designed for one kind of payload may not work as well for the other payloads.

So here that calls for designing and adaptive controller where you do not have to manually tune your gains for every kind of payload rather the controller is intelligent enough by using input, output data from the system to tune itself even if you know even when there are multiple payloads, right. So this is another example just to illustrate you know why adaptive control is useful in situations where it could be useful.

Okay, so I want to consider another example because this is the first class so I do not want to make it so involved in the beginning. So let us go slowly and introduce the topic by motivating the need for it.

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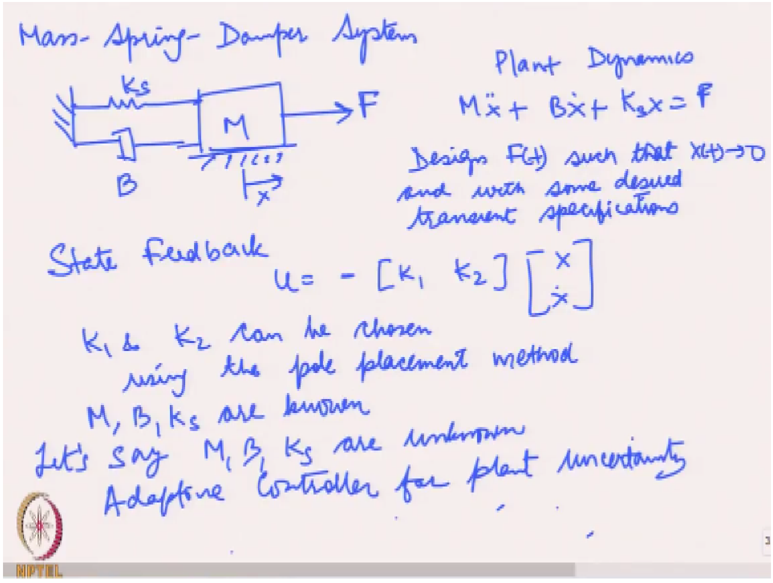

So let us consider another example of a mass spring damper system. Okay so we have a spring, a damper connected to a mass then you have a force, which is acting on the mass. So the spring constant is denoted by Ks, the damping constant is denoted by B and we can easily write down the plant dynamics as Mx double dot + Bx dot + Ksx = F. So x here is say the position of the mass from this point on.

Okay, so here the objective is to design a controller F of t such that x of t goes to 0 and with some desired transient specifications, okay. So I hope the objective here is cleared. So we would like to design a force F of t which is we control input to the system such that when this mass is perturbed from it is initial position it comes back to x = 0 with some desired transient specification for example some desired settling time with some desired over shoot.

So these are some transient specifications that we want a system to meet. So how would you go about designing such a controller so from our classical methods we know that we could design straight feedback, pole placement, controller of the form, so u is given by – K1 K2 x and x dot right. So K1 and K2 can be chosen using the pole placement method. Now an assumption that we are making here is that all the parameters of the system M, B and Ks are known.

And using those parameters we use the pole placement method to come up with K1 and K2 which can achieve this objective. Now if I tell you that, so let us say that M, B and Ks are unknown parameters. So now how would you design a controller F of t to stabilize the system. So using fixed gain controllers can you do the same task. Of course you cannot use pole placement or LQR now because all of these methods they require the system parameters to be known.

You could try using a PD controller here, but you probably not be able to get the same kind of performance. So this further illustrates the point that adaptive controllers can be used in the situations where you have systems with parameters uncertainty. So you could use adaptive controllers for plants, so here the plant is parameterized with the parameters M, B and Ks and because there is uncertainty in these parameters we would like to design adaptive controller for achieving this objective.

Okay, so hopefully this is adequate motivation to study this area and I stop at these examples and now I will take a more concrete mathematical case just to again illustrate the limitation of the fixed gain controllers or the classical controllers and motivate the need for adaptive control. Okay so let us consider a very simple scalar system.

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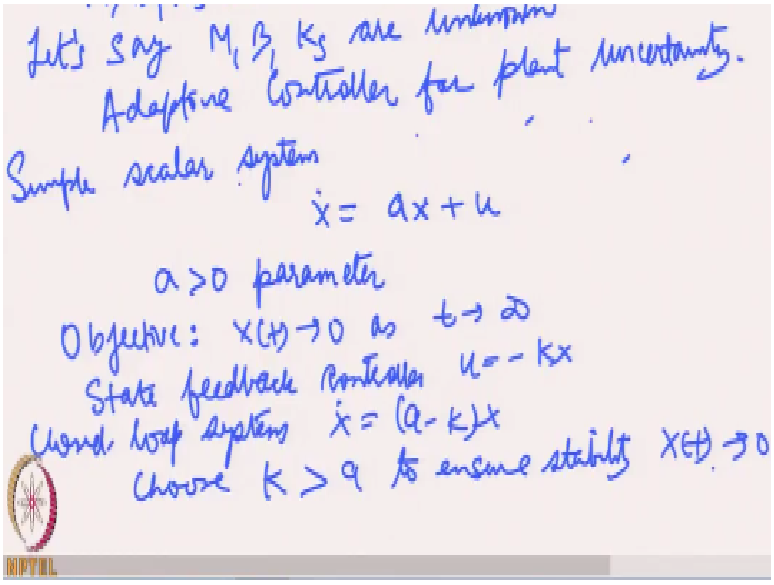

X dot = ax + u, so again here x is the state, u is the input, a is positive parameter and the task, the objective here is that x of t goes to 0 as t goes to infinity. Okay, so to meet this objective let us design a state feedback controller u = -kx okay. So the closed loop system in this case becomes x dot = a – kx okay. So we need to make sure that x tends to 0 as t tends to infinity. So how would you go about designing k to meet this objective?

So of course you would want to choose the Eigen values to be on the left hand side, so in order to do that you would have to choose k to be greater than a to ensure stability and to make sure that x of t goes to 0, x t goes to infinity right. Now this again is based on the assumption that the parameter a is known to us right.

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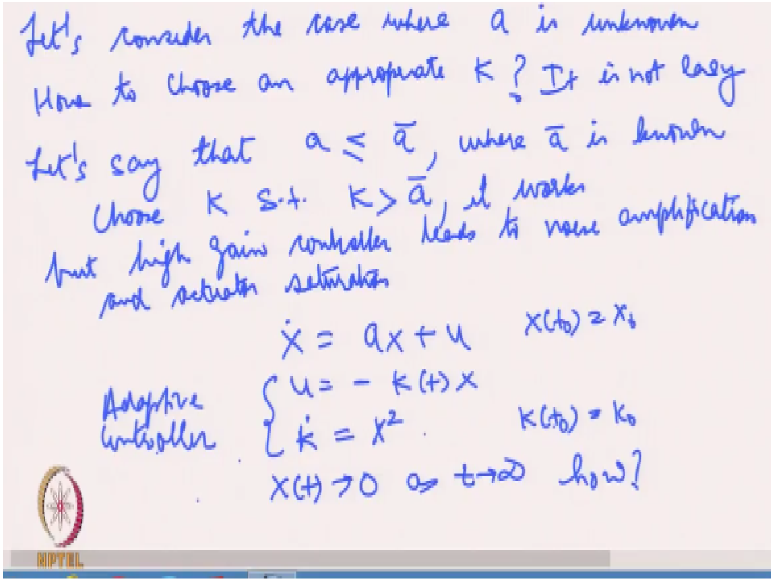

So now let us consider the case where a is unknown, then how would you design k right. It is not very straightforward to come up with the k for an unknown a right. So for this case we want to choose k right. So it is not easy right. It is not straightforward. So let us relax this little bit and say that let us say that we have an upper bound for a, where a bar is known. So although a is not known, but the upper bound of a which is a bar is known.

So we can still think about designing k in this case, so we can choose k such that k is greater than a bar right. So if you choose k to be greater than a bar we meet our objective right. So that is the case where what is the problem with this case, I mean what is one problem with this case although we can still meet our objective. So one is that here we consider the worst case scenario. We consider k to be greater than the upper bound of a.

Now a can be 2, but the upper bound can be say 100 and then we will have to choose k greater than 100 and we know what is the problem with these high gain controllers. So if the designer controller with the really high gain we know that it will amplify noise and it will also saturate our actuators. So it is not a very prudent design; however, it works in this case okay. So this works but high gain controller which are very conservative leads to noise amplification and actuator saturation, okay.

So it turns out that you could design an adaptive controller in this case and I can in fact write down the adaptive controller that we could design. So for the system x dot = ax + u we could design u to be = - k of tx. So here k is time varying not the fixed k in that we had considered earlier and the way this k of t is changing is from this differential equation which is given by, so of course we can write down the initial condition, okay.

So using this as our controller we could potentially make sure that we could prove that x of t goes to 0 as t goes to infinity. So how this is possible is something that we will consider later, but right now you can just take this on faith that an adaptive controller like this can be designed without knowing the parameter a, but at the same time it can still give you the, it can still satisfy the control objectives okay.

Alright so this was the case where this k appropriately changes with time to make sure that you have stability okay. So now the question is how to design such controllers. So as I had mentioned before in the introductory video a prerequisite to this course is background and nonlinear control or nonlinear systems or familiarity with Lyapunov stability methods. So if you do not have this background I would like that you go through the standard text.

But I would still like to go over some preliminary material just so that you know what to study in detail. I will just go over it in brief, okay.

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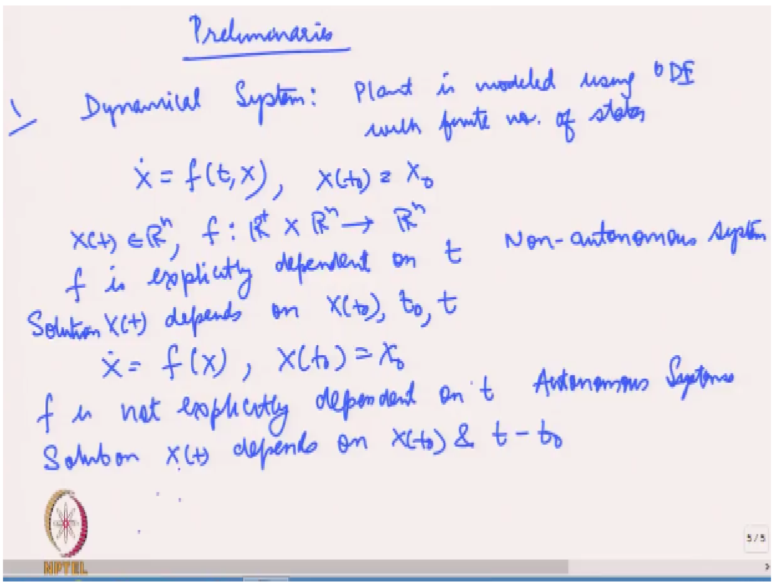

So let us look at some preliminaries to be covered by you okay. I will only discuss that in brief. So first dynamical systems, so we consider the plant to be a dynamical system, so plant is modeled using ordinary differential equations with finite number of states, okay and the way you would represent a dynamical system in general which is represented using ODE is using this equation.

X dot = f of t, x where f represents any nonlinear function. X as we all know is a state, t is the time and system starts with some initial condition at t0, which is given by x0. So x of t gross to Rn, f is the function which takes the arguments R + gross Rn to Rn okay so here you can see that the function f is explicitly dependent on t and these systems are called as non-autonomous systems.

If you solve this equation you will find that x of t, which is the solution of this differential equation will depend on the initial state x of t0, the initial time t0 and the current time t right. Now there were also systems where this function f is not explicitly a function of time t. So those systems can be represented using x dot = f of x. So here there is no explicit dependence on t. So these systems are called as, so f is not explicitly dependent on t.

The system is called as autonomous systems and the solution for these systems x of t depends on the initial state x of t0 and t – t0. So that is very important to see that the solution does not depend on t0 independently, it depends on t – t0 the difference between the current and the initial time. So this difference is important because as we will see later when we study stability of the systems we will see that their stability properties vary depending on whether you are looking at an autonomous system or a non-autonomous system.

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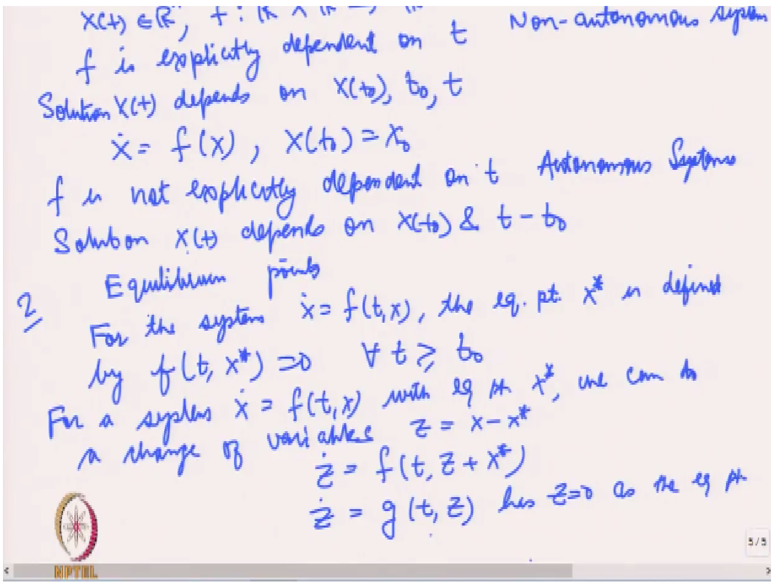

Okay, so the next topic is equilibrium points. So for the system x dot = f of t, x. the equilibrium point x star is defined by f of t, x star 0 for all t >/= t0. So x star is the point which satisfies this equation f of t, x star = 0 for all time t >/= t0. It just means that if the system solution or the system trajectory is at the equilibrium point then, it stays there for all time because at x = x star the right hand side of this differential equation is 0 which means that x dot = 0.

Which means that if you are at x = x star you would remain there for all future time, so that is the definition of an equilibrium point. Okay so now suppose for system x dot = f of t, x with equilibrium point denoted by x star we can always do a change of variables and convert the system in to a system where the equilibrium point is the origin. So we can consider z as a new variable which is noted by x – x star.

And then we can write down that dot as f of t, z + x star then we can further write this as z dot as sum function g, t, z right. So here the equilibrium point for this system has the origin z = 0 as the equilibrium point. So this is just to show that given system with a nonzero equilibrium point you could always do a change of variables and convert that into a system where the origin in equilibrium point.

This will be useful later on when we do the definitions of stability. Okay so let us consider an example of a pendulum. So we all are familiar with this example.

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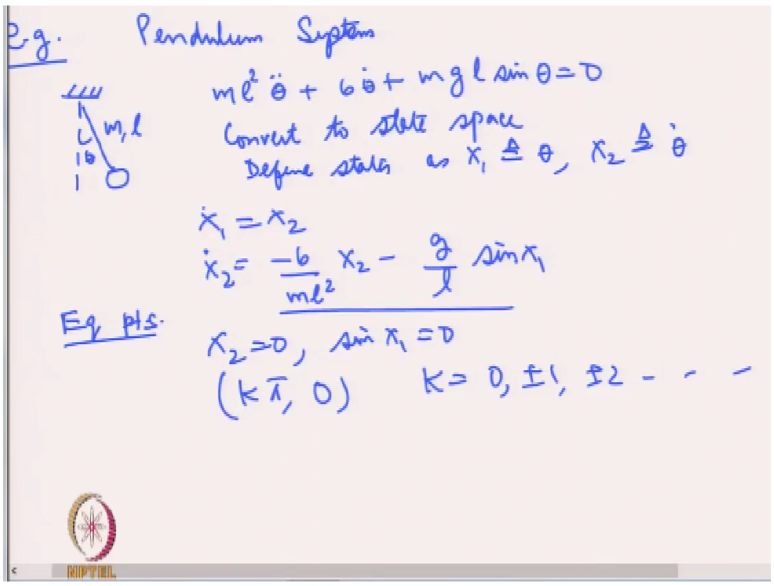

So we consider the pendulum system with the mass M and length L and we can write down the dynamics as ml square theta double dot + b theta dot + mgl sin theta = 0 okay and the question is determined the equilibrium points for the system okay. So let us follow the procedure that we laid down previously and then see if it agrees with our physical intuition that we have, okay.

So convert to state space so that would be the first step. So we can define the states as x1 is defined to be theta and x2 is defined to be theta dot so then we can write down set of first ordered differential equations as x1 dot = x2 and x2 dot = -b over ml square x2 – g over l sin x1 alright. So we have represented the dynamics in the straight space form. So now we can find out the equilibrium points by setting the right hand side of these differential equations to be equal to 0.

So which means that x2 = 0 and so if we set the right hand side of this equation to be equal to 0 what we will end up with sin of x1 to be equal to 0 because x2 is 0. So which means that the equilibrium point, there are multiple equilibrium points in this pendulum system and they are given by k pi, 0 where k pi corresponds to x1 and 0 corresponds to x2 and this k is 0 + -1 + -2 and so on.

Okay so this agree with our intuition and it does right because we know that the pendulum system has one set of equilibrium points which denote the vertically downward position and another set of equilibrium points which denote the vertically upward position right. So this is just an example to illustrate that you could, if you know the differential equation of any dynamical system you could find out the equilibrium points.

So for linear systems x dot = ax for example where a is non-singular, you would only have one equilibrium point x = 0, but for nonlinear systems you could have multiple equilibrium points. You could have systems which have no equilibrium points, so the behaviour of these nonlinear system is very different from the linear counter parts and that is why the motivation to study and analyze these nonlinear systems. Okay so now we go on to the next topic which is concern with stability of equilibrium points.

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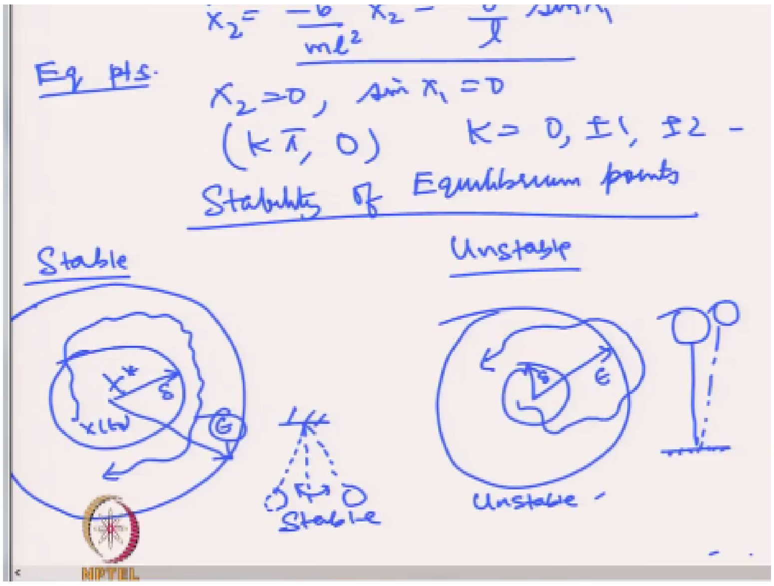

So stability, I am pretty sure you would have some concept of stability. So in your classical control classes you must have looked at bounded input, bounded output stability in the context of linear systems, but there is also a notion of stability in the sense of Lyapunov or where you are looking at the stability with respect to the equilibrium points.

So here the notion of stability is very tightly tied to the equilibrium points. In fact, we never say that a system is stable or unstable. We talk about if the equilibrium point of a system is stable or not. So if I ask you for the pendulum system can you analyse the stability of the pendulum. So of course it will not be possible to say whether the pendulum is stable or unstable, you would have to look at the equilibrium point that we are referring to.

So for the vertically downward position we know that the pendulum is stable whereas for the vertically upward position the pendulum is unstable. So how can we say that, how can we state this in a better way. So you could of course look at the mathematical definitions of stability from the book, but here I will just give you a flavor of that. So let us say that we start x star is the equilibrium point and we start in the neighbourhood of the equilibrium point so let us say we start from x, from here okay.

So it is a little bit away from the equilibrium point and as times goes on we evolve, the state of the system evolves and if it stays with an another ball say of radius epsilon then we say that the system is stable. So what we are trying to say here is that you could stay close to the origin or the equilibrium point for all time provided you start sufficiently close to it.

So this is what these 2 balls are representing here that given any epsilon that is the ball where you want your trajectories to lie, so given any epsilon if you could find a delta ball where you start from, then you could say that your system is stable okay. So this is you know how you illustrate the concept of stability, so which just mean that small pertubation from the equilibrium point result in small deviation from the equilibrium point.

Or the trajectory can stay close to the equilibrium point by starting sufficiently close to it, so if you consider the example of this pendulum, so this is the vertically downward position and we start say here and then we let this pendulum go, suppose there is no friction in the system then we would keep oscillating back and forth, but we would stay close to the equilibrium point.

And we can in fact stay arbitrary close to the equilibrium point by starting you know sufficiently close to the equilibrium point. So this is what you call as a stable equilibrium point which is the vertically downward scenario. So for unstable it is pretty clear that you could always find. If you could find an epsilon ball for which there is no delta such that if you start inside of it then you always stay within the epsilon ball.

So if you could find even a single epsilon for which this delta is not possible to be found out then you say that the system is unstable and to illustrate this we could consider the pendulum again and we could look at the vertically upward position of the pendulum, so if we start sufficiently close to it we would not be able to keep close to the equilibrium point no matter how close we start from it.

So this is an example which illustrate the unstable behaviour of the equilibrium point. Okay so there is another definition before we close there is another definition which I want to talk about which is asymptotic stability.

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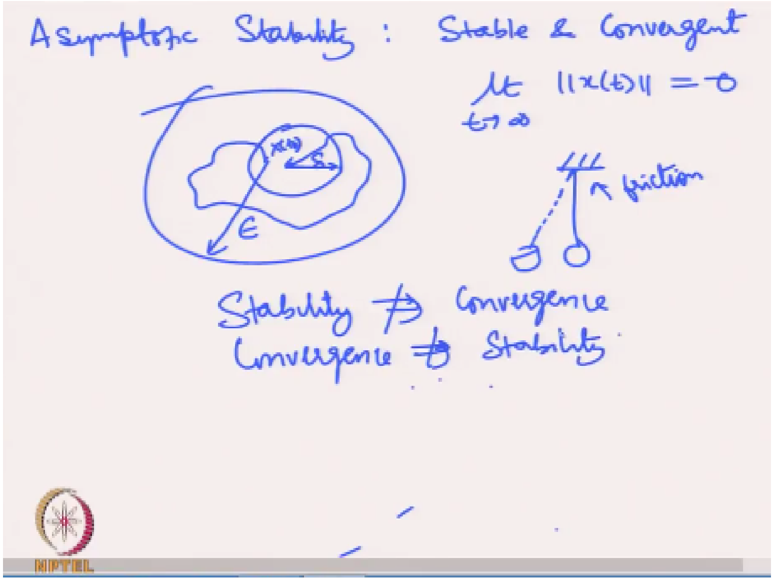

So here, so this just means that you are stable and you are convergent. Assume that the state trajectories they start somewhere inside this delta ball and for all epsilon you could find a delta such that if you start within delta then you stay within epsilon for all future time. In addition, these trajectories as t goes to infinity would converge to the equilibrium point.

So that is the additional condition that we have for asymptotic stability that as t goes to infinity the solution trajectories of the system would go to 0. Where 0 denotes the equilibrium point and without loss of generality we can consider 0 to be the equilibrium point because we could always do a change of variables. Okay so how would we consider the pendulum situation for the scenario.

So let us consider pendulum with friction and if we start and we look at the vertically downward position and we start somewhere close to the equilibrium point and because there is friction the system would continuously dissipate energy and could settle at the equilibrium point. So this is an example of equilibrium point which is not just stable that is, it is not just you know it stays close to the origin, but eventually it converges to the equilibrium point.

So this equilibrium point is in fact asymptotically stable. So just to be clear we can say that stability in general does not imply convergence and convergence also does not imply stability. Okay so we can close at this point and we will continue these definitions in the next class where we will consider some more definitions and then talk about some concepts which will be useful for us to design adaptive controllers, okay. Thank you.